Probabilistic Spiking Neuron Model

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Model Description

The dynamic variable for this model of spiking neuron is the vector $\vec{\theta}$. Let us consider a system of n neurons connected via excitatory and inhibitory connections.

Let $\vec{\theta}_t$ be the n-dimensional vector of firing probabilities of each of the n neurons at the time t. By the definition of probability, each element of this vector is bounded by 0 and 1. In absence of any current input to the neurons, the probability of firing would go down as the activity decays to equilibrium. Let λ be the rate for the exponential decay.

Thus in absence of current input (external and synaptic), $\vec{\theta}$ will follow the dynamical equation:

$$\vec{\theta}_{t+1} = \lambda\vec{\theta}_t$$

But at each time step = t, we perform a sampling event to evaluate the firing of the neurons. Let $\langle\vec{\theta}\rangle_t$ be the result of the sampling event. This means that $[\langle\vec{\theta}\rangle_t]_i$ is the result of a binary coin toss with $P(1) = [\vec{\theta}_t]_i$ and $P(0) = 1 - [\vec{\theta}_t]_i$, where $i = 1, 2, 3...n$.

But whenever the neuron fires, the probability of firing again is set to zero. Thus the final dynamical equation without current input becomes

$$\vec{\theta}_{t+1} = \lambda(1 - \langle\vec{\theta}\rangle_t)\vec{\theta}_t$$

We initialize the values of $\vec{\theta}_t$ with values from a uniform random distribution and then follow the system by the rules. This will give a system of non interacting neurons that have a dynamic probability of firing.

Now we model the interactions and the current response. Let us define excitatory and inhibitory connectivity matrices $[\mathbf{E}]_{\mathbf{n\times n}}$ and $[\mathbf{I}]_{\mathbf{n\times n}}$ with the horizontal axis representing the pre-synaptic neuron and the vertical axis the post-synaptic neuron. We also define a function $I_{ext}(t) : \mathbb{R} \rightarrow \mathbb{R}^n$ that gives us the external current input to each neuron at time t. We also define two parameters e and i that describe the excitation and inhibition coupling coefficient respectively. Finally, we define a response function based on Mirollo Strogatz Model, $U : \mathbb{R} \rightarrow \mathbb{R}$ which has the following properties:

$$U'(x) > 0 \ and \ U''(x) < 0, \forall x \in [0, 1]$$

$$U(0) = 0 \ and \ U(1) = 1$$

We will use this function to update the probability based on current input. Say, the current input is $\vec{\epsilon}$ where each element $\epsilon_i \in [-1, 1]$ or any other bound that depends on the range of the response function over which the response is variable. We define the new probability using the function $H : \mathbb{R}^n \rightarrow \mathbb{R}^n$ where

$$[H(\vec{x}, \vec{\epsilon})]_i = [U^{-1}(U(x_i) + \epsilon_i)]_i \ \forall i \in \{1, 2, 3...n\}$$

One such function that satisfies these properties is:

$$U_b(x) = \frac{1}{b} ln(1 + (e^b - 1)x)$$

This function is numerically and analytically useful because the function $H_b(\vec{x}, \vec{\epsilon})$ for $U_b(x)$ becomes an affline linear map

$$H_b(\vec{x}, \vec{\epsilon}) = e^{b\epsilon} x + \frac{e^{b\epsilon} - 1}{e^b - 1}$$

Now, given a synaptic response function $F : \mathbb{R} \rightarrow [0, 1]$, we define the synaptic current I^{syn} at time t+1 as

$$I_{syn}(t + 1) = e.F(\mathbf{E} \times \langle\vec{\theta}\rangle_t) - i.F(\mathbf{I} \times \langle\vec{\theta}\rangle_t)$$

For simplicity, we can use the linear response function

$$F(x) = x/n$$

Now we define the dynamical equation with current input as

$$\vec{\theta}_{t+1} = H_b(\ \lambda(1 - \langle\vec{\theta}\rangle_t)\vec{\theta}_t \ , \ I_{ext}(t + 1) + I_{syn}(t + 1) \ )$$

This model can now be simulated.