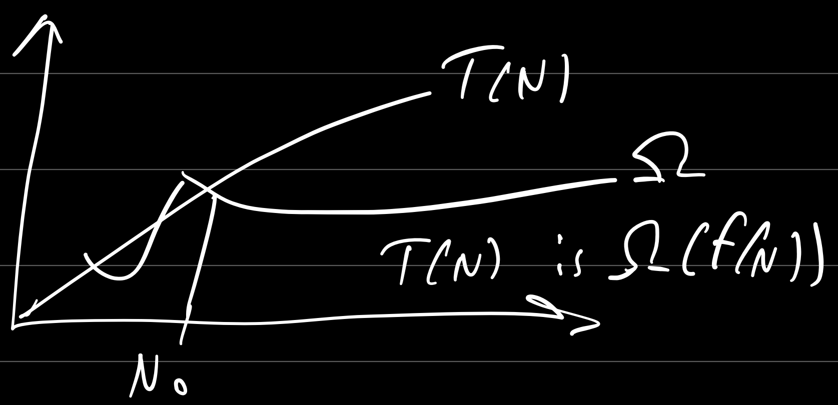
Big Omega:

$T(N)$ is $\Omega(f(N))$ if and only if there exists positive constants N_0, B, such that:

$T(N) >= B \cdot f(N)$ for any $N \geq N_0$



Proof of Ω:

Ex #1: (Plus)

$$T(N) = 3N^2 + 2N + 1 = \Omega(N^2)$$

$$\geq B \cdot N^2$$

$\therefore 3N^2 + 2N + 1 \geq 3 \cdot N^2$ when $N_0 = 0$

$\therefore T(N) = \Omega(N^2)$ with $B = 3$, $N_0 = 0$

Ex #2: (Sub)

$$T(N) = N^2 - N = \Omega(N^2)$$

$$N^2 - N = B \cdot N^2$$

When $N \geq 1$, $N^2 \geq N$

$\therefore N^2 - N \geq N^2 - N^2 = 0$

$\therefore N^2 - N \geq N^2 - \frac{1}{2}N^2 = \frac{1}{2}N^2$, $B = 1/2$

$\therefore N \geq 2$, when $N \geq 2$ we have

$\frac{1}{2}N^2 \geq N$

$\therefore N^2 - N \geq N^2 - \frac{1}{2}N^2 = \frac{1}{2}N^2$

$\therefore T(N) = N^2 - N = \Omega(N^2)$, $B = 1/2$, $N_0 = 4$

Ex #3: $T(N) = N\lg(N) + N - 100\lg(N) = \Omega(N \cdot \lg(N))$

$$N\lg(N) + N - 100\lg(N) \geq N\lg(N) - 100\lg(N)$$

Find a number to make this true

$\therefore \frac{1}{19} N\lg(N) \geq 100\lg(N)$

$N \geq 1900$

Prove: When $N \geq 1900$,

$\frac{1}{19} \cdot N \cdot \lg(N) \geq 100 \cdot \lg(N)$

$$\therefore N \cdot \lg(N) + N - 100\lg(N) \geq N\lg(N) - 100\lg(N)$$

$$\geq N\lg(N) - \frac{1}{19}N\lg(N)$$

$$= \frac{18}{19}N\lg(N)$$

$\therefore T(N) = N\lg(N) + N - 100\lg(N) = \Omega(\lg(N))$

$\therefore T(N) = N\lg(N) + N - 100\lg(N) = \Omega(N\lg(N))$

with $B = \frac{18}{19}, N_0 = 1900$

Prove 2:

$$T(N) = N \cdot \lg(N) + N - 100\lg(N) = \Omega(N\lg(N))$$

$$\therefore 2N\cancel{\lg(N)} \geq 100\cancel{\lg(N)}$$

$$N \geq 50$$

When $N \geq 50$,

$$2N\lg(N) \geq 100\lg(N)$$

$$\therefore N\lg(N) + N - 100\lg(N) \geq N\lg(N) - 2N\lg(N)$$

$$= -N\lg(N),\ B = -1$$

Since B must be greater than 0, $B > 0$, $N = 2$ does not work.

Proof 3: $N = 1/2$,

$$\frac{1}{2}N\lg(N) \geq 100\lg(N)$$

$$N \geq 200$$

$$\therefore N\lg(N) + N - 100\lg(N) \geq N\lg(N) - 1/2\, N\lg(N)$$

$$= 1/2\lg(N)$$

$\therefore B = 1/2,\ N_0 \geq 200$

Ex #4: $T(N) = 2N^2 - N\lg(N) = \Omega(N^2)$

$\therefore N^2 \geq N\lg(N)$

$\therefore 2N^2 - N\lg(N) \geq 2N^2 - N^2 = N^2$

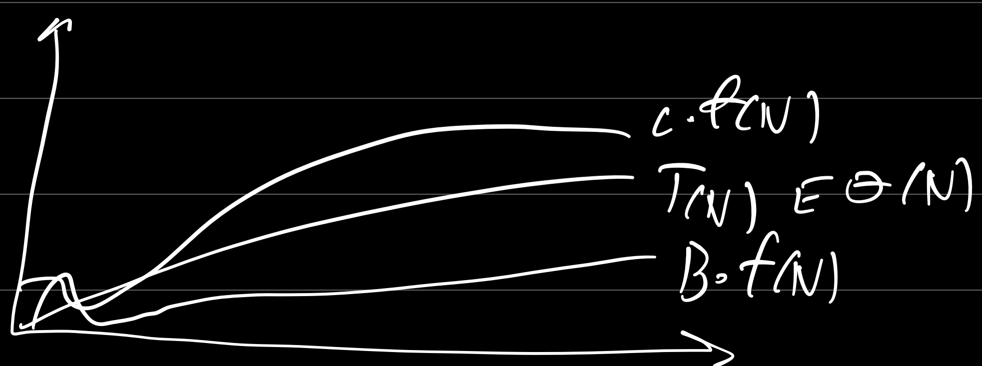
$\therefore T(N) = \Omega(N^2)$ with $B = 1$, $N_0 = 1$

Big Theta Notation:

$T(N)$ is $\Theta(f(N)) \Leftrightarrow T(N)$ is $O(f(N))$ ...

$\wedge\ T(N)$ is $\Omega(f(N))$

$\rightarrow \Omega(N) \leq T(N) \leq O(N)$



$\therefore T(N) = 3N^2 + 2N + 1 \rightarrow \Theta(N^2)$

Step 1: $T(N) \rightarrow 6N^2 \rightarrow O(N^2)$

Step 2: Ω

$\therefore 3N^2 + 2N + 1 \geq 3N^2 \rightarrow \Omega(N^2)$

Step 3: $T(N) = \Theta(N^2)$

Proof 1: $T(N) = x^2 + x$, $O(x^2)$? ✓

$O(x^3)$? ✓

$\Theta(x^2)$? ✓

$\Theta(x^3)$? ✗

$\lg(N)$

N

$N\lg(N)$

N^2

N^3

2^N

3^N

$N!$

N^N

Time Increases!

Ex: $2N^2 + 8N\lg(N) + N^7 + 4 \cdot 3^N + N = \Theta(3^N)$

Big-Theta is most important!