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Differential Equation Types: Linear, Homogeneous, Autonomous

A) $\frac{d^3y}{dt^3} + y\frac{dy}{dt} + t^2y = \cos(t)$

Non-linear because of $y \cdot \frac{dy}{dt}$ term

Non-homogeneous because of $\cos(t)$

B) $\frac{dy}{dt} - \frac{1}{t^2+1} \cdot y = \sin(t)$, Linear, Non-homogeneous due to $\sin(t)$, Non-autonomous since includes $t$ terms

F) $ty^5 + t^2y^4 + t^3y^3 + t^4y'' + ty' + t^6y = 0$

Linear, Homogenous

C) $\frac{d^2u}{dx^2} = \frac{u+x^2}{x^2+4}$, $\frac{d^2u}{dx^2} - \frac{1}{x^2+4} \cdot u = \frac{x^2}{x^2+4}$

Linear, Non-homogenous due to $\frac{x^2}{x^2+4}$ term

Autonomous: When the $t$ variable only appears as a derived variable.

G) $yy' = -t$, Non-linear

D) $\frac{d^2u}{dx^2} + xe^u = e^x$, Non-linear due to $e^u$, Non-homogenous due to $e^x$

2A) $x' = tx - y^2 + \sin(2t)$

$y' = t^2 + x + y - xy$

Non-linear due to $-xy$ and $-y^2$

3B) $\frac{dy}{dt} = \cos(3t)$, $y = \int \cos(3t)\,dt$

$y(t) = \frac{\sin(3t)}{3} + C$

3D) $y' - \frac{1}{t}y = 0$, $y(t) = Ct$, IVP, $y(0) = 5$

Not solvable, no value satisfies $5(0) = y$

5A) $(t+1)y' + 3y = 0$, $y' + \frac{3}{(t+1)}y = 0$

$y' + a(t)y = 0$, $a(t) = \frac{3}{t+1}$, $A(t) = 3\ln(t+1)$

$y(t) = C \cdot e^{-A(t)} = C \cdot e^{-3\ln(t+1)}$

5ii B) $t\ln(t)y' + y = 2\ln(t)$, Non-homogeneous

$y' + \frac{1}{t\ln(t)}y = \frac{2}{t}$, $a(t) = \frac{1}{t\ln(t)}$, $f(t) = \frac{2}{t}$

$A(t) = \int \frac{1}{t\ln(t)}\,dt$, $u = \ln(t)$, $du = \frac{1}{t}dt$, $\int \frac{1}{u}\,du$

$= \ln|u| = \ln|\ln(t)|$,

$$e^{A(t)}\left(y' + a(t)y\right) = e^{A(t)} f(t)$$

$$\frac{d}{dt}\left(e^{A(t)} y\right) = e^{A(t)} f(t)$$

$$\boxed{y = e^{-A(t)} \int e^{A(t)} f(t)\,dt}$$

$$y = \frac{1}{\ln(t)} \cdot \int \ln(t) \cdot \frac{2}{t}\,dt, \quad \text{u-sub}$$

$$u = \ln(t), \; du = \frac{1}{t}, \; \hookrightarrow = \frac{1}{\ln(t)} \int 2u\,du$$

$$= \frac{1}{\ln(t)}\left(\ln(t)^2 + C\right) = \ln(t) + \frac{C}{\ln(t)}$$