JORDI GALÍ

Monetary Policy, Inflation, and the Business Cycle

An Introduction to the New Keynesian Framework and Its Applications

SECOND EDITION

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PREFACE

THE PRESENT monograph draws together some of the lecture notes for graduate-level courses on monetary economics that I have taught over the past fifteen years at different institutions, including Universitat Pompeu Fabra, the Barcelona Graduate School of Economics, the Massachusetts Institute of Technology (MIT), and the Swiss Beginner's Doctoral Program at Gerzensee. The book's main objective is to give the reader an introduction to the New Keynesian framework and some of its applications. That framework has emerged as the workhorse for the analysis of monetary policy and its implications for inflation, economic fluctuations, and welfare. It constitutes the backbone of the new generation of medium-scale models used at the Federal Reserve Board, the European Central Bank (ECB), and many other central banks. It has also provided the theoretical underpinnings to the inflation stability-oriented strategies adopted by the majority of central banks in the industrialized world.

A defining feature of the present book is the use of a single reference model throughout the chapters. That benchmark framework, which I refer to as the "basic New Keynesian model," is developed in chapter 3. It features monopolistic competition and staggered price setting in goods markets, coexisting with perfectly competitive labor markets. The "classical model" introduced in chapter 2, characterized by perfect competition in goods markets and flexible prices, can be viewed as a limiting case of the benchmark model, when both the degree of price stickiness and firms' market power vanish. The discussion of the empirical shortcomings of the classical monetary model provides the motivation for the development of the New Keynesian model, as discussed in the introductory chapter.

The implications for monetary policy of the basic New Keynesian model, including the desirability of inflation targeting, are analyzed in chapter 4. Each of the subsequent chapters then builds on the basic model, and analyzes an extension of that model along some specific dimension. Once the reader has grasped the contents of chapters 1 to 4, each of the subsequent chapters can be read independently, and in any order. Thus, chapter 5 introduces a policy tradeoff in the form of an exogenous cost-push shock, which serves as the basis for a discussion of the differences between the optimal policy with and without commitment. Chapters 6 and 7 extend the assumption of nominal rigidities to the labor market and examine the policy implications of the coexistence of sticky wages and sticky prices. Chapter 8 develops a small open economy version of the basic New Keynesian model, introducing explicitly in the analysis a number of variables inherent to open economies, including trade flows, nominal and real exchange rates, and the terms of trade. In addition to some concluding comments, chapter 9 provides a brief description of several extensions of the basic model not covered in the present monograph, and a list of key references for each one.

Chapters 2 through 8 contain a final section with a brief summary and discussion of the related literature, including references to some of the key papers. Thus, references in the main text are kept to a minimum. The reader will also find at the end of most chapters a list of exercises related to the material covered therein.

The present second edition incorporates some new material to the contents of the book, in addition to providing an improved exposition of the old material and correcting some of the errors (none fatal, as far as I know) that passed it into the first edition and that have been uncovered by myself or others. The new material includes the analysis of the optimal policy under both discretion and commitment in the presence of a zero lower bound on the nominal interest rate, a constraint that was altogether ignored in the first edition, and whose relevance has come to the forefront of the policy debate (as well as the academic literature) in light of the economic and financial crisis of recent years and the associated responses of central banks. Moreover, a new chapter has been added (chapter 7), which introduces unemployment as an additional variable in the model, allowing for an analysis of its role in the design of monetary policy, following an approach that I originally proposed in my Zeuthen lectures. Throughout the book, consideration is made of exogenous shifts in the discount factor as a source of fluctuations, in addition to the technology and monetary policy shocks already allowed for in the first edition. Variations in the discount factor lead to changes in aggregate demand, without having any direct effect on labor supply and can thus be considered as a good stand-in for more general aggregate demand shocks.

The level of the book should make it suitable for use as a reference in a graduate course on monetary theory, possibly supplemented with readings covering some of the recent extensions not treated here. Chapters 1 through 5 could also prove useful as the basis for the "monetary block" of a first-year graduate macro sequence or even an advanced undergraduate course on monetary theory. Chapters 3 through 5 can also be used as the basis for a short course that serves as an introduction to the New Keynesian framework.

Much of the material contained in the present book overlaps with that found in two other (excellent) books on monetary theory published in recent years: Carl Walsh's *Monetary Theory and Policy* (MIT Press, 3rd ed.) and Michael Woodford's *Interest and Prices* (Princeton University Press). The focus of the present book on the New Keynesian model, with the use of a single underlying framework throughout the chapters, represents the main difference from Walsh's, with the latter providing in many respects a more comprehensive, textbook-like, coverage of the field of monetary theory, with a variety of models being used. On the other hand, the main difference from Woodford's comprehensive treatise lies in the more compact presentation of the basic New Keynesian model and its implications for monetary policy found here, which may facilitate its use as a textbook in an introductory graduate course. In addition, the present book contains an analysis of unemployment and of open economy extensions of the basic New Keynesian model, which are topics not covered in Woodford's book.

Many people have contributed to this book in important ways. First and foremost, I am in special debt with Rich Clarida, Mark Gertler, and Tommaso Monacelli, with whom I coauthored the original articles underlying much of the material found in this book, in particular that in chapters 5 and 8. I am also especially thankful to Olivier Blanchard who, as a teacher and thesis advisor at MIT, made me discover the fascination of modern macroeconomics. Working with him as a coauthor in recent years has helped me sharpen my understanding of many of the issues dealt with here. My interest in monetary theory was triggered by a course taught by Mike Woodford at MIT in the fall of 1988. His work in monetary economics (and in anything else) has always been a source of inspiration.

Many other colleagues have helped me improve the present monograph either with specific comments on earlier versions of the chapters or through discussions over the years on some of the topics covered here. A nonexhaustive list includes Kosuke Aoki, Larry Christiano, José de Gregorio, Andy Levin, David López-Salido, Albert Marcet, Dirk Niepelt, Louis Phaneuf, Stephanie Schmitt-Grohé, Lars Svensson, Lutz Weinke, and Iván Werning. I am also grateful to several anonymous reviewers for useful comments.

The two editions of the book have benefited from the help of excellent research assistants. Davide Debortoli, now already a well-established macroeconomist, was the main RA behind the first edition and came to my rescue again for the second edition when I ran into some technical hurdles. Mehregan Ameri, Lien Laureys, Cristina Manea, and Alain Schlaepfer have provided invaluable help with the second edition. Many others uncovered algebra mistakes or offered suggestions on different

chapters, including Suman Basu, Sevinc Cucurova, Jose Dorich, Andrew Li, Lorenzo Magnolfi, Elmar Mertens, and Juan Carlos Odar. Needless to say, I am solely responsible for any remaining errors.

I should also like to thank Princeton University Press and, in particular, Richard Baggaley, Sarah Caro, and Hannah Paul for their continuous support on this project.

Much of the research underlying this book has received the financial support of several sponsoring institutions, which I would like to acknowledge for their generosity. They include the European Research Council, the National Science Foundation, the Spanish Ministry of Science and Technology, the Fundación Ramón Areces, the Generalitat de Catalunya, the Banque de France, and the Barcelona GSE.

Monetary Policy, Inflation, and the Business Cycle

Chapter 1

INTRODUCTION

OVER THE PAST TWO DECADES, monetary economics has been among the most fruitful research areas within macroeconomics. The efforts of many researchers to understand the relationship among monetary policy, inflation, and the business cycle have led to the development of a framework—the so called New Keynesian model—that is widely used for monetary policy analysis. The present monograph offers an overview of that framework and a discussion of its policy implications.

The need for a framework that can help us understand the links between monetary policy and the aggregate performance of an economy seems self-evident. On the one hand, and in their condition as consumers, workers, or investors, citizens of modern societies have good reason to care about developments in inflation, employment, and other economy-wide variables, for those developments affect to an important degree their opportunities to maintain or improve their standard of living. On the other hand, monetary policy has an important role in shaping those macroeconomic developments, both at the national and supranational levels. Changes in interest rates have an influence on the valuation of financial assets and their expected returns, as well as on the consumption and investment decisions of households and firms. Those decisions can in turn have consequences for GDP growth, employment, and inflation. It is thus not surprising that the interest rate decisions made by the Fed, the ECB, or other prominent central banks around the world are given so much attention, not only by market analysts and the financial press, but also by the general public. It would thus seem important to understand how those interest rate decisions end up affecting the various measures of an economy's performance, both nominal and real. A key goal of monetary theory is to provide us with an account of the mechanisms through which those effects arise, that is, the transmission mechanism of monetary policy.

Central banks do not change interest rates in an arbitrary or whimsical manner. Their decisions are meant to be purposeful, that is, they seek to attain certain objectives, while taking as given the constraints posed by the workings of a market economy, in which the vast majority of economic decisions are made in a decentralized manner by a large number of individuals and firms. Understanding what should be the objectives of monetary policy and how the latter should be conducted in order to attain those objectives constitutes another important aim of modern monetary theory, in its normative dimension.

The following chapters present a framework that helps us understand both the transmission mechanism of monetary policy and the elements that come into play in the design of rules or guidelines for the conduct of monetary policy. The framework presented is, admittedly, highly stylized and should be viewed more as a pedagogical tool than a quantitative model that can be readily taken to the data. Nevertheless, and despite its simplicity, it contains the key elements (though not all the bells and whistles) found in the models being developed and used at central banks and other policy institutions.[1]

The monetary framework that constitutes the focus of the present monograph has a core structure that corresponds to a Real Business Cycle (RBC) model, on which a number of "Keynesian features" are superimposed.[2] Each of those two influences is briefly described next, in order to provide some historical background to the framework developed in subsequent chapters.

1.1 Background: Real Business Cycle Theory and Classical Monetary Models

During the years following the seminal papers of Kydland and Prescott (1982) and Prescott (1986), Real Business Cycle (RBC) theory provided the main reference framework for the analysis of economic fluctuations, and became to a large extent the core of macroeconomics. The impact of the RBC revolution had both a methodological and a conceptual dimension.

From a *methodological* point of view, RBC theory established firmly the use of dynamic stochastic general equilibrium (DSGE) models as a central tool for macroeconomic analysis. Behavioral equations describing aggregate variables were thus replaced by first order conditions of intertemporal problems facing consumers and firms. Ad hoc assumptions on the formation of expectations gave way to rational expectations. In addition, RBC economists stressed the importance of the quantitative

[1] See, e.g., Bayoumi (2004), Christoffel, Coenen, and Warne (2008), Erceg, Guerrieri, and Gust (2006), Edge, Kiley, and Laforte (2007), Adolfson et al. (2007) for a description of some of the DSGE models used at the International Monetary Fund, the European Central Bank, and the Federal Reserve Board.

[2] That confluence of elements led some authors to label the new paradigm as the "new neoclassical synthesis." See Goodfriend and King (1997).

aspects of modeling, as reflected in the central role given to the calibration, simulation, and evaluation of their models.

The most striking dimension of the RBC revolution was, however, conceptual. It rested on three basic claims:

- *The efficiency of business cycles*. Thus, the bulk of economic fluctuations observed in industrialized countries could be interpreted as an equilibrium outcome resulting from the economy's response to exogenous variations in real forces (most importantly, technology), in an environment characterized by perfect competition and frictionless markets. According to that view, cyclical fluctuations did not necessarily signal an inefficient allocation of resources (in fact, the fluctuations generated by the standard RBC model were fully optimal). That view had an important corollary: stabilization policies may not be necessary or desirable, and they could even be counterproductive. This was in contrast with the conventional interpretation, tracing back to Keynes (1936), of recessions as periods with an inefficiently low utilization of resources, which could be brought to an end by means of economic policies aimed at expanding aggregate demand.
- *The importance of technology shocks as a source of economic fluctuations*. That claim derived from the ability of the basic RBC model to generate "realistic" fluctuations in output and other macroeconomic variables, even when variations in total factor productivity—calibrated to match the properties of the Solow residual—are assumed to be the only exogenous driving force. Such an interpretation of economic fluctuations was in stark contrast with the traditional view of technological change as a source of long-term growth, unrelated to business cycles.
- *The limited role of monetary factors*. Most importantly, given the subject of the present monograph, RBC theory sought to explain economic fluctuations with *no reference to monetary factors*, even abstracting from the existence of a monetary sector.

Its strong influence among academic researchers notwithstanding, the RBC approach had a very limited impact (if any) on central banks and other policy institutions. The latter continued to rely on large-scale macroeconometric models despite the challenges to their usefulness for policy evaluation (Lucas (1976)) or the largely arbitrary identifying restrictions underlying the estimates of those models (Sims (1980)).

The attempts by Cooley and Hansen (1989) and others to introduce a monetary sector in an otherwise conventional RBC model, while sticking to the assumptions of perfect competition and fully flexible prices and wages, were not perceived as yielding a framework that was relevant for policy analysis. As discussed in chapter 2, the resulting framework, which we refer to as the *classical* monetary model, generally predicts neutrality (or near neutrality) of monetary policy with respect to real variables. That finding is at odds with the widely held belief (certainly among central bankers) in the power of that policy to influence output and employment developments, at least in the short run. That belief is underpinned by a large body of empirical work, tracing back to the narrative evidence of Friedman and Schwartz (1963), up to the more recent work using time series techniques, as described in Christiano, Eichenbaum, and Evans (1999).[3]

In addition to the empirical challenges mentioned above, the normative implications of classical monetary models have also led many economists to call into question their relevance as a framework for policy evaluation. Thus, those models generally yield as a normative implication the optimality of the Friedman rule—a policy that requires that central banks keep the short-term nominal rate constant at a zero level—even though that policy seems to bear no connection whatsoever with the monetary policies pursued (and viewed as desirable) by the vast majority of central banks. Instead, the latter are characterized by (often large) adjustments of interest rates in response to deviations of inflation and indicators of economic activity from their target levels.[4]

The conflict between theoretical predictions and evidence, and between normative implications and policy practice, can be viewed as a symptom that some elements that are important in actual economies may be missing in classical monetary models. As discussed below, those shortcomings are the main motivation behind the introduction of some Keynesian assumptions, while maintaining the RBC apparatus as an underlying structure.

[3] An additional challenge to RBC models has been posed by the recent empirical evidence on the effects of technology shocks. Some of that evidence suggests that technology shocks generate a negative short-run comovement between output and labor input measures, thus rejecting a prediction of the RBC model that is key to its ability to generate fluctuations that resemble actual business cycles (see, e.g., Galí (1999) and Basu, Fernald, and Kimball (2006)). Other evidence suggests that the contribution of technology shocks to the business cycle has been quantitatively small (see, e.g., Christiano, Eichenbaum, and Vigfusson (2003)), though investment-specific technology shocks may have played a more important role (Fisher (2006)). See Galí and Rabanal (2004) for a survey of the empirical evidence on the effects of technology shocks.

[4] In the wake of the recent economic and financial crisis and the subsequent slow recovery, many central banks, including the Federal Reserve and the ECB, have brought down their policy rates to zero or near-zero levels. Few (if any) would interpret that policy as the result of a deliberate attempt to implement the Friedman rule. Rather, it should be viewed as an illustration of the zero lower bound on nominal interest rate becoming binding, in the face of central banks' attempt to provide further stimulus to the economy.

1.2 The New Keynesian Model: Main Elements and Features

Despite their different policy implications, there are important similarities between the RBC model and the New Keynesian monetary framework. The latter, whether in the simple versions presented below or in its more complex extensions, has at its core some version of the RBC model. This is reflected in the assumption of (i) an infinitely lived representative household, who seeks to maximize the utility from consumption and leisure, subject to an intertemporal budget constraint, and (ii) a large number of firms with access to an identical technology, subject to exogenous random shifts. Though endogenous capital accumulation, a key element of RBC theory, is absent in the basic versions of the New Keynesian model, it is easy to incorporate and is a common feature of medium-scale versions.[5] Also, as in RBC theory, an equilibrium takes the form of a stochastic process for all the economy's endogenous variables, consistent with optimal intertemporal decisions by households and firms, given their objectives and constraints, and with the clearing of all markets.

The New Keynesian modeling approach, however, combines the DSGE structure characteristic of RBC models with assumptions that depart from those found in classical monetary models. Here is a list of some of the key elements and properties of the resulting models:[6]

- *Monopolistic competition*. Prices and/or wages are set by private economic agents in order to maximize their objectives, as opposed to being determined by an anonymous Walrasian auctioneer seeking to clear all markets.
- *Nominal rigidities*. Firms are subject to some constraints on the frequency with which they can adjust the prices of the goods they sell. Alternatively, they may face some costs of adjusting those prices. The same kind of friction applies to workers—or the unions that represent them—in the presence of sticky wages.
- *Short-run non-neutrality of monetary policy*. As a consequence of the presence of nominal rigidities, changes in short-term nominal interest rates (whether chosen directly by the central bank or induced by changes in the money supply) are not matched by one-for-one changes in expected inflation, thus leading to variations in real interest rates. The latter bring about changes in consumption and investment and, as a result, on output and employment, since

[5] See, e.g., Smets and Wouters (2003, 2007).

[6] See Galí and Gertler (2007) for an extended introduction to the New Keynesian model and a discussion of its main features.

firms find it optimal to adjust the quantity of goods supplied to the new level of demand. The same holds true for workers in the presence of sticky wages. In the long run, however, all prices and wages adjust, and the economy reverts back to its natural equilibrium, that is, the equilibrium that would prevail in the absence of nominal rigidities.

It is important to note that the three ingredients above were already central to the New Keynesian literature that emerged in the late 1970s and 1980s, and which developed in parallel with RBC theory. The models used in that literature, however, were often static or used reduced form equilibrium conditions that were not derived from explicit dynamic optimization problems facing firms and households. The emphasis of much of that work was instead on providing microfoundations, based on the presence of small menu costs, for the stickiness of prices and the resulting monetary non-neutralities.[7] Other papers emphasized the persistent effects of monetary policy on output, and the role that staggered contracts played in generating that persistence.[8] The novelty of the new generation of monetary models has been to embed those features in a fully specified DSGE framework, thus adopting the formal modeling approach that has been the hallmark of RBC theory.

Not surprisingly, important differences with respect to RBC models emerge in the new framework. First, the economy's response to shocks is generally inefficient. Second, the non-neutrality of monetary policy resulting from the presence of nominal rigidities makes room for welfare-enhancing interventions by the monetary authority, in order to minimize the existing distortions. Furthermore, those models are arguably suited for the analysis and comparison of alternative monetary regimes without being subject to the Lucas critique.[9]

1.2.1 Evidence of Nominal Rigidities and Monetary Policy Non-Neutrality

The presence of nominal rigidities and the implied real effects of monetary policy are two distinctive ingredients of New Keynesian models. It would be hard to justify the introductions of those features

[7] See, e.g., Akerlof and Yellen (1985), Mankiw (1985), Blanchard and Kiyotaki (1987), and Ball and Romer (1990).

[8] See, e.g., Fischer (1977) and Taylor (1980).

[9] This will be the case at least to the extent that the economy is sufficiently stable so that the log-linearized equilibrium conditions remain a good approximation, and that some of the parameters that are taken as "structural" (including the degree of nominal rigidities) can be viewed as approximately constant.

in the absence of evidence in support of their relevance. Next we briefly describe some of that evidence and provide the reader with some relevant references.

1.2.1.1 EVIDENCE OF NOMINAL RIGIDITIES

Most attempts to uncover evidence on the existence and importance of price rigidities have generally relied on the analysis of micro data, that is, data on the prices of individual goods and services.[10] In an early survey of that research, Taylor (1999) concludes that there is ample evidence of price rigidities, with the average frequency of price adjustment being about one year. In addition, he points to the very limited evidence of synchronization of price adjustments, thus providing some justification for the assumption of staggered price setting commonly found in the New Keynesian model. The study of Bils and Klenow (2004), based on the analysis of the average frequencies of price changes for 350 product categories underlying the U.S. CPI, called into question that conventional wisdom by uncovering a median duration of prices between 4 and 6 months. Nevertheless, more recent evidence by Nakamura and Steinsson (2008), using data on the individual prices underlying the U.S. CPI and excluding price changes associated with sales, has led to a reconsideration of the Bils-Klenow evidence, with an upward adjustment of the estimated median duration to a range between 8 and 11 months. Evidence for the euro area, discussed in Dhyne et al. (2006), points to a similar distribution of price durations to that uncovered by Nakamura and Steinsson for the United States.[11] It is worth mentioning that, in addition to evidence of substantial price rigidities, most studies find a large amount of heterogeneity in price durations across sectors/types of goods, with services being associated with the largest degree of price rigidities, and unprocessed food and energy with the lowest.

The literature also contains several studies using micro data that provide analogous evidence of nominal rigidities for wages. Taylor (1999) contains an early survey of that literature and suggests an estimate of the average frequency of wage changes of about one year, the same as for prices. A significant branch of the literature on wage rigidities has focused on the possible existence of asymmetries that make wage cuts very rare or unlikely. Bewley's (1999) detailed study of firms' wage

[10] See, e.g., Cecchetti (1986) and Kashyap (1995) for early papers examining the patterns of prices of individual goods.

[11] In addition to studies based on the analysis of micro data, some researchers have conducted surveys of firms' pricing policies. See, e.g., Blinder et al. (1998) for the United States and Fabiani et al. (2005) for several countries in the euro area. The conclusions from the survey-based evidence tend to confirm the evidence of substantial price rigidities coming out of the micro-data analysis.

policies based on interviews with managers finds ample evidence of downward nominal wage rigidities. The multicountry study of Dickens et al. (2007) uncovers evidence of significant downward nominal and real wage rigidities in most of the countries in their sample. More recently, Barattieri et al. (2014) and Druant et al. (2012) use large survey-based data sets for the United States and euro area countries, respectively, and confirm the low frequency of wage adjustments, pointing to wage spells that on average last one year or longer.

1.2.1.2 EVIDENCE OF MONETARY POLICY NON-NEUTRALITIES

Monetary non-neutralities are, at least in theory, a natural consequence of the presence of nominal rigidities. As will be shown in chapter 3, if prices don't adjust in proportion to changes in the money supply (thus causing real balances to vary), or if expected inflation does not move one-for-one with the nominal interest rate when the latter varies (thus leading to a change in the real interest rate), the central bank will generally be able to alter the level aggregate demand and, as a result, the equilibrium levels of output and employment. Is the evidence consistent with that prediction of models with nominal rigidities? And if so, are the effects of monetary policy interventions sufficiently important quantitatively to be relevant?

Unfortunately, identifying the effects of changes in monetary policy is not an easy task. The reason for this is well understood: an important part of the movements in whatever variable we take as the instrument of monetary policy (e.g., the short-term nominal rate) are likely to be endogenous, that is, the result of a deliberate response of the monetary authority to developments in the economy. Thus, simple correlations of interest rates (or the money supply) on output or other real variables cannot be used as evidence of non-neutralities, for the direction of causality may go, fully or in part, from movements in the real variable (resulting from nonmonetary forces) to the monetary variable. Over the years, a large literature has developed seeking to answer such questions while avoiding the pitfalls of a simple analysis of comovements. The main challenge facing that literature lies in identifying changes in policy that could be interpreted as exogenous, that is, not the result of the central bank's response to movements in other variables. While alternative approaches have been pursued in order to meet that challenge, much of the recent literature has relied on time series econometrics techniques and, in particular, on structural (or identified) vector autoregressions.

The evidence displayed in figure 1.1, taken from Christiano, Eichenbaum, and Evans (1999), is representative of the findings in much of the recent literature seeking to estimate the effects of exogenous monetary

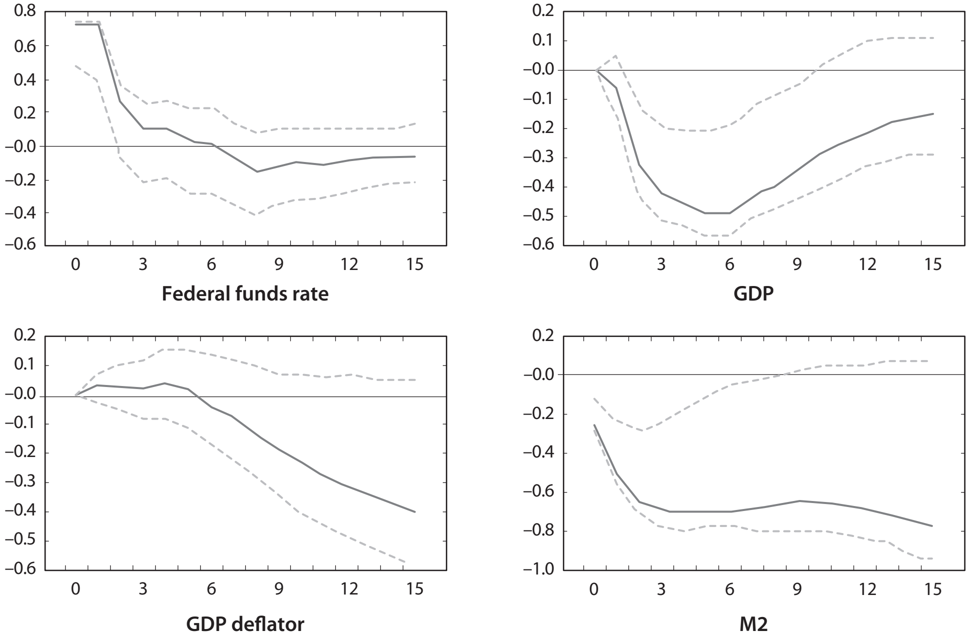

Figure 1.1. Estimated Dynamic Response to a Monetary Policy Shock
Source: Christiano, Eichenbaum, and Evans (1999).

policy shocks.[12] In the empirical model underlying figure 1.1, monetary policy shocks are identified as the residual from an estimated policy rule followed by the Federal Reserve. That policy rule determines the level of the federal funds rate (taken to be the instrument of monetary policy), as a linear function of its own lagged values, current and lagged values of GDP, the GDP deflator, and an index of commodity prices, as well as the lagged values of some monetary aggregates. Under the assumption that neither GDP nor the two price indexes can respond contemporaneously to a monetary policy shock, the coefficients of the previous policy rule can be estimated consistently with OLS, and the fitted residual can be taken as an estimate of the exogenous monetary policy shock. The response over time of any variable of interest to that shock is then given by the estimated coefficients of a regression of the

[12] Other references include Sims (1992), Galí (1992), Bernanke and Mihov (1998), Uhlig (1995), and Romer and Romer (2004), all of them using postwar U.S. data. Peersman and Smets (2003) provide similar evidence for the euro area. An alternative approach to identification, based on a narrative analysis of contractionary policy episodes, can be found in Romer and Romer (1989).

current value of that variable on the current and lagged values of the fitted residual from the first stage regression.[13]

Figure 1.1 shows the dynamic responses of the federal funds rate, (log) GDP, (log) GDP deflator, and the (log) money supply (measured by M2), to an exogenous tightening of monetary policy. The solid line represents the estimated response, with the dashed lines capturing the corresponding 95 percent confidence interval. The scale on the horizontal axis measures the number of quarters after the initial shock. Note that the path of the funds rate itself, depicted in the top-left graph, shows an initial increase of about 75 basis points, followed by a gradual return to its original level. In response to that tightening of policy, GDP declines with a characteristic hump-shaped pattern. It reaches a trough after 5 quarters at a level about 50 basis points below its original level, and then it slowly reverts back to its original level. That estimated response of GDP can be viewed as evidence of sizable persistent real effects of monetary policy shocks. On the other hand, the (log) GDP deflator displays a flat response for over a year, after which it declines. That estimated sluggish response of prices to the policy tightening is generally interpreted as evidence of substantial price rigidities.[14] Finally, note that (log) M2 displays a persistent decline in the face of the rise in the federal funds rate, suggesting that the Fed needs to reduce the amount of money in circulation in order to bring about the increase in the nominal rate. The observed negative comovement between money supply and nominal interest rates is known as the "liquidity effect." As discussed in chapter 2, it appears at odds with the predictions of a classical monetary model.

Having discussed the empirical evidence in support of the key assumptions underlying the New Keynesian framework, we end this introductory chapter with a brief description of the organization of the remaining chapters.

1.3 Organization of the Book

The book is organized in nine chapters, including this introduction. Chapters 2 through 8 develop a unified framework, with new elements

[13] In practice, the estimation of the impulse responses is carried out in a single step, using a linear transformation of an estimated vector autoregressive model that satisfies some identifying assumptions (typically in the form of predeterminedness and exclusion restrictions).

[14] Also, note that expected inflation hardly changes for several quarters and then declines. Combined with the path of the nominal rate, this implies a large and persistent increase in the real rate in response to the tightening of monetary policy, which provides another manifestation of the non-neutrality of monetary policy.

being incorporated in each chapter. Throughout the book references in the main text are kept to a minimum. The reader will find instead a section at the end of each chapter with notes on the literature, including references to some of the key papers underlying the results presented in the chapter or containing extensions not covered therein. In addition, each chapter contains a list of suggested exercises related to the material covered in the chapter. Next a brief description follows of the book's organization and the content of the different chapters.

Chapter 2 starts by introducing the assumptions on preferences and technology that are maintained, with few variations, throughout the book. The economy's equilibrium is then analyzed under the assumptions of *perfect competition* in all markets, and fully *flexible prices and wages*. Those assumptions define what is labeled the *classical monetary economy*, whose baseline specification is characterized by neutrality of monetary policy and efficiency of the equilibrium allocation, with monetary policy's influence on equilibrium outcomes restricted to the determination of nominal variables.

In the baseline model used in the first part of chapter 2, as in the rest of the book, "money" is just the unit of account, that is, the unit in terms of which prices of goods, labor services, and financial assets are quoted. Its role as a store of value (and hence as an asset in agents' portfolios) or as a medium of exchange is ignored. As a result, one generally does not need to specify a money demand function, unless monetary policy itself is specified in terms of a monetary aggregate, in which case a simple log-linear money demand schedule is postulated in an ad hoc way. In the second part of chapter 2, however, an explicit motive to hold money is introduced by assuming that real balances are an additional argument in households' utility function, and its implications are examined under the alternative assumptions of separability and non-separability of real balances. In the latter case, in particular, the result of monetary policy neutrality is shown to break down, even in the absence of nominal rigidities. The resulting non-neutralities, however, are shown to be quantitatively small and empirically little relevant.

Chapter 3 introduces the basic New Keynesian model, by adding product differentiation, monopolistic competition, and staggered price setting to the framework developed in chapter 2. Labor markets are still assumed to be perfectly competitive and wages fully flexible. The solution to the optimal price setting problem of a firm in that environment and the resulting inflation dynamics are derived. The log-linearization of the optimality conditions of households and firms, combined with some market clearing conditions, leads to the canonical representation of the model's equilibrium, which includes the New Keynesian Phillips curve, a dynamic IS equation, and a description of monetary policy. Two variables play a central role in the equilibrium dynamics: the output gap and the natural rate of interest. The output gap is defined as the log deviation between output and the natural level of output, where the latter corresponds to the equilibrium level of output in the absence of nominal rigidities. Similarly, the natural rate of interest refers to the equilibrium, real interest rate in the absence of nominal rigidities. The presence of sticky prices is shown to make monetary policy non-neutral. This is illustrated by analyzing the economy's response to three types of shocks: an exogenous monetary policy shock, a shock to households' discount rate, and a technology shock.

Chapter 4 discusses the role of monetary policy in the basic New Keynesian model from a normative perspective. In particular, it shows that, under some assumptions, it is optimal to pursue a policy that fully stabilizes the price level ("strict inflation targeting") and discuss alternative ways in which that policy can be implemented (optimal interest rate rules). The likely practical difficulties in the implementation of the optimal policy motivate the introduction and analysis of simple monetary policy rules, that is, rules that can be implemented with little or no knowledge of the economy's structure and/or realization of shocks. A welfare-based loss function that can be used for the evaluation and comparison of those rules is then derived and applied to two simple rules: a Taylor rule and a constant money growth rule.

A common criticism of the analysis of optimal monetary policy contained in chapter 4 is the absence of a tradeoff between inflation stabilization and output gap stabilization, a property that has come to be known as "the divine coincidence." In chapter 5 that criticism is addressed by showing how a meaningful policy tradeoff emerges in the presence of variations in the gap between the natural and efficient levels of output. In that context, and following the analysis in Clarida, Galí, and Gertler (1999), the optimal monetary policy is derived under two alternative assumptions, discretion and commitment, emphasizing the key role played by the forward-looking nature of inflation as a source of the gains from commitment. The final section in the chapter discusses the challenges and tradeoffs generated by the presence of a zero lower bound on the nominal interest rate, which may become binding in the face of an adverse demand shock. The relevance of the zero lower bound has become evident in the wake of the recent economic and financial crisis, when central banks' efforts to stimulate the economy through reductions in the short-term policy rate *eventually* hit that constraint, forcing them to rely on a variety of unconventional measures.

Chapter 6 extends the basic New Keynesian framework by introducing imperfect competition and staggered nominal wage setting in labor markets, in coexistence with staggered price setting, following the work of Erceg, Henderson, and Levin (2000). The presence of sticky nominal wages, and the consequent variations in wage markups, render a policy aimed at fully stabilizing price inflation suboptimal. The reason is that fluctuations in wage inflation, in addition to variations in price inflation and the output gap, generate a resource misallocation and a consequent welfare loss. Thus, the optimal policy is one that seeks to strike the right balance between stabilization of those three variables. For a broad range of parameters, however, the optimal policy can be approximated well by a policy that stabilizes a weighted average of price and wage inflation, where the proper weights are function of the relative stickiness of prices and wages.

Chapter 7 reformulates the standard New Keynesian model with staggered price and wage setting of the previous chapter in a way that allows for the explicit introduction of unemployment in the model, defined as the gap between participation and employment. The role of unemployment in the design of monetary policy is discussed. It is shown that a simple interest rate rule that responds to inflation and the unemployment rate can approximate surprisingly well the optimal monetary policy.

In chapter 8 a small open economy version of the basic New Keynesian model is developed. The analysis of the resulting model yields several results. First, it is shown that the equilibrium conditions have a canonical representation analogous to that of the closed economy, including a New Keynesian Phillips curve, a dynamic IS equation, and an interest rate rule. In general, though, both the natural level of output and the natural real rate are a function of foreign, as well as domestic, shocks. Second, and under certain assumptions, the optimal policy consists in fully stabilizing domestic inflation, while accommodating the changes in the exchange rate (and, as a result, in CPI inflation) necessary to bring about desirable changes in the relative price of domestic goods. Thus, in general, policies that seek to stabilize the nominal exchange rate, including the limiting case of an exchange rate peg, are shown to be suboptimal.

Chapter 9 concludes by reviewing some of the general lessons that can be drawn from the previous chapters. In doing so two key insights associated with the New Keynesian framework are emphasized, namely, the key role of expectations in shaping the effects of monetary policy, and the importance of the natural levels of output and the interest rate for the design of monetary policy. The chapter ends by discussing some of the limitations of the New Keynesian framework as developed here, together with a description of extensions not covered in the present book that aim at overcoming some of those limitations.

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Chapter 2

A CLASSICAL MONETARY MODEL

This chapter presents a simple model of a classical monetary economy, featuring perfect competition and fully flexible prices in all markets. As stressed below, many of the model's predictions are at odds with the evidence reviewed in chapter 1. That notwithstanding, the analysis of the classical economy provides a benchmark that will be useful in subsequent chapters when some of its strong assumptions are relaxed. It also allows for the introduction of some notation, as well as assumptions on preferences and technology that are used throughout the book.

Following much of the recent literature, the baseline classical model developed here attaches a very limited role to money. Thus, in the first four sections of this chapter, the only explicit role played by money is to serve as a unit of account. In that case, and as shown below, whenever monetary policy is specified in terms of an interest rate rule, no reference whatsoever needs to be made to the quantity of money in circulation in order to determine the economy's equilibrium. When the specification of monetary policy involves the money supply, a "conventional" money demand equation is postulated in order to close the model, without taking a stand on its microfoundations. In section 2.5, an explicit role for money is introduced, beyond that of serving as a unit of account. In particular, a version of the classical monetary model is analyzed in which real balances are assumed to generate utility to households, and the implications for monetary policy of alternative assumptions on the properties of that utility function are explored.

Independently of how money is introduced, the proposed framework assumes a representative household solving a dynamic optimization problem. That problem and the associated optimality conditions are described in section 2.1. Section 2.2 introduces the representative firm's technology and determines its optimal behavior under the assumption of price and wage taking. Section 2.3 characterizes the equilibrium, and shows how real variables are uniquely determined, independently of monetary policy. Section 2.4 discusses the determination of the price level and other nominal variables under alternative monetary policy rules. Finally, section 2.5 analyzes a version of the model with money in the utility function, and discusses the extent to which the conclusions drawn from the earlier analysis need to be modified under that assumption.

2.1 Households

The economy is assumed to be inhabited by a large number of identical households. The representative household seeks to maximize the objective function:

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t; Z_t) \tag{1}$$

where C_t is the quantity consumed of the single good available in the economy, N_t denotes hours of work or employment, and Z_t is an exogenous preference shifter.[1] Parameter $\beta \in (0, 1)$ is the discount factor. $E_t\{\cdot\}$ is the expectational operator, conditional on information at time t. The period utility function, $U(C_t, N_t; Z_t)$, is assumed to be continuous and twice differentiable. Letting $U_t \equiv U(C_t, N_t; Z_t)$, it is assumed that $U_{c,t} \equiv \frac{\partial U_t}{\partial C_t} > 0$, $U_{cc,t} \equiv \frac{\partial^2 U_t}{\partial C_t^2} \leq 0$, $U_{n,t} \equiv \frac{\partial U_t}{\partial N_t} \leq 0$, and $U_{nn,t} \equiv \frac{\partial^2 U_t}{\partial N_t^2} \leq 0$. In words, the marginal utility of consumption $U_{c,t}$ is assumed to be positive and nonincreasing, while the marginal disutility of labor, $-U_{n,t}$, is positive and nondecreasing. In addition, it is assumed that $U_{cz,t} \equiv \frac{\partial^2 U_t}{\partial C_t \partial Z_t} > 0$, that is, an increase in Z_t raises the marginal utility of consumption.

Maximization of (1) is subject to a sequence of flow budget constraints given by

$$P_t C_t + Q_t B_t \leq B_{t-1} + W_t N_t + D_t \tag{2}$$

for $t = 0, 1, 2, \ldots$ P_t is the price of the consumption good, W_t denotes the nominal wage (per hour or per worker, depending on the interpretation of N_t), and B_t represents the quantity of one-period nominally riskless discount bonds purchased in period t, and maturing in period $t+1$. Each bond pays one unit of money at maturity, and its price is Q_t. In order to keep the notation simple, the set of tradable assets is restricted to that one-period nominally riskless debt. Given the assumption of a representative household, this has no consequences for equilibrium outcomes, since no assets would be traded in equilibrium anyway. D_t represents dividends, accruing to households in their condition of firms' owners. When solving the problem above, the household is assumed to

[1] Note that N_t can be interpreted as the number of household members who are employed, assuming a "large" household and ignoring integer constraints. In much of what follows I use the term "employment" interchangeably as "number of individuals employed" or "number of work hours employed."

take as given the price of goods and bonds, as well as the wage and dividends.

In addition to (2), it is assumed that the household is subject to a solvency constraint that prevents it from engaging in Ponzi-type schemes. The following constraint is assumed

$$\lim_{T\to\infty} E_t \left\{ \Lambda_{t,T} \frac{B_T}{P_T} \right\} \geq 0 \tag{3}$$

for all t, where $\Lambda_{t,T} \equiv \beta^{T-t} U_{c,T}/U_{c,t}$ is the stochastic discount factor.[2]

The optimality conditions implied by the maximization of (1) subject to (2) are given by

$$-\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t} \tag{4}$$

$$Q_t = \beta E_t \left\{ \frac{U_{c,t+1}}{U_{c,t}} \frac{P_t}{P_{t+1}} \right\} \tag{5}$$

as well as the transversality condition

$$\lim_{T\to\infty} E_t \left\{ \Lambda_{t,T} \frac{B_T}{P_T} \right\} = 0 \tag{6}$$

for $t = 0, 1, 2, \ldots$.

The previous optimality conditions can be derived using a simple variational argument. Consider first the impact on utility of a small departure, in period t, from the household's optimal plan. That departure consists of an increase in consumption dC_t and an increase in employment dN_t, while keeping the remaining variables unchanged (including consumption and employment in other periods). If the household is following an optimal plan to begin with, it should not be possible to raise utility by departing from that plan, without violating the budget constraint. Formally, it must be the case that

$$U_{c,t} dC_t + U_{n,t} dN_t = 0$$

[2] See Woodford (2003; chap. 2) for a derivation of (3) under the assumption of complete markets, as the constraint that guarantees that debt can be repaid at any point in time by selling claims to future income.

for any pair (dC_t, dN_t) satisfying

$$P_t dC_t = W_t dN_t$$

for otherwise it would be possible to raise utility by increasing (or decreasing) consumption and employment, thus contradicting the assumption that the household is on an optimal plan. Note that the optimality condition (4) is obtained by combining both equations.

Similarly, consider the impact on expected utility as of time t of a reallocation of consumption between periods t and $t+1$, while keeping consumption in any other period and employment in all periods unchanged. If the household is optimizing it must be the case that

$$U_{c,t} dC_t + \beta E_t\{U_{c,t+1} dC_{t+1}\} = 0$$

for any pair (dC_t, dC_{t+1}) satisfying

$$P_{t+1} dC_{t+1} = -\frac{P_t}{Q_t} dC_t$$

where the latter equation determines the increase in consumption expenditures in period $t+1$ made possible by the additional savings $-P_t dC_t$ allocated into one-period bonds. Combining the two previous equations yields the intertemporal optimality condition (5).

Finally, transversality condition (6) is also a requirement of an optimal plan: It can be shown that if (3) were to hold with strict inequality, it would be feasible for the household to increase its current consumption by a discrete amount without the need to reduce it in the future, which would be inconsistent with optimality, as long as utility is strictly increasing in consumption (as assumed here).[3]

In much of what follows, it is assumed that the period utility takes the form

$$U(C_t, N_t; Z_t) = \begin{cases} \left(\frac{C_t^{1-\sigma}-1}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi}\right) Z_t & \text{for } \sigma \neq 1 \\ \left(\log C_t - \frac{N_t^{1+\varphi}}{1+\varphi}\right) Z_t & \text{for } \sigma = 1 \end{cases}$$

where $\sigma \geq 0$ and $\varphi \geq 0$ determine, respectively, the curvature of the utility of consumption and the disutility of labor. The analysis is

[3] The assumptions of a representative household and a zero net supply of one-period debt, guarantee that (6) is always satisfied in equilibrium, since $B_t = 0$ for all t. That observation allows us to ignore that condition in much of what follows.

considerably simplified by two properties of the above utility function: (i) separability, that is, $U_{cn,t} = 0$, and (ii) the implied constancy of the elasticities for the marginal utility of consumption and for the marginal disutility of labor, which leads to simple log-linearized approximations to the equilibrium conditions, as shown below. Furthermore, the way the preference shifter Z_t enters the utility function implies that it may be interpreted as a shock to the effective discount factor (which becomes $Z_t \beta^t$, for $t = 0, 1, 2, \ldots$), whose effect will be restricted to intertemporal choices (through its influence on $U_{c,t+k}/U_{c,t}$, the intertemporal marginal rate of substitution), but with no impact on intratemporal ones (since it does not affect $U_{n,t}/U_{c,t}$, the intratemporal marginal rate of substitution).

In addition, it is assumed that $z_t \equiv \log Z_t$ follows an exogenous $AR(1)$ process

$$z_t = \rho_z z_{t-1} + \varepsilon_t^z$$

where $\rho_z \in [0, 1]$.

The consumer's optimality conditions (4) and (5) thus become

$$\frac{W_t}{P_t} = C_t^\sigma N_t^\varphi \tag{7}$$

$$Q_t = \beta E_t \left\{ \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \left(\frac{Z_{t+1}}{Z_t} \right) \left(\frac{P_t}{P_{t+1}} \right) \right\} \tag{8}$$

Note, for future reference, that equation (7) can be rewritten in log-linear form as

$$w_t - p_t = \sigma c_t + \varphi n_t \tag{9}$$

where lowercase letters denote the (natural) logs of the corresponding variable (e.g., $w_t \equiv \log W_t$). The previous condition can be interpreted as a competitive labor supply schedule, determining the quantity of labor supplied as a function of the real wage, given the marginal utility of consumption (which is a function of consumption only, under the above assumption on utility). Parameter φ is the inverse (Frisch) labor supply elasticity. Note that the position of the labor supply schedule is invariant to shocks to the preference shifter z_t.

As shown in appendix 2.1, a log-linear approximation of (8) around a steady state with constant rates of inflation and consumption growth yields

$$c_t = E_t\{c_{t+1}\} - \frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} - \rho) + \frac{1}{\sigma}(1 - \rho_z) z_t \tag{10}$$

where $i_t \equiv -\log Q_t$, $\rho \equiv -\log \beta$ and $\pi_{t+1} \equiv p_{t+1} - p_t$ is the rate of inflation between t and $t+1$ (having defined $p_t \equiv \log P_t$). Notice that i_t corresponds to the log of the gross bond yield; henceforth, i_t is referred to as the *nominal interest rate*.[4] Analogously, ρ can be interpreted as the household's discount rate.[5]

While the above framework does not explicitly introduce money as an asset that households may hold (nor provides a motive for holding it), in some cases it will be convenient to postulate a demand for real balances with a log-linear form given by (up to an additive constant)

$$m_t - p_t = c_t - \eta i_t \tag{11}$$

where $\eta \geq 0$ denotes the interest semielasticity of money demand. A money demand equation similar to (11) can be derived under a variety of assumptions. For instance, in section 2.5 it is derived as an optimality condition for the household when real money balances yield utility.

2.2 Firms

A large number of identical firms are assumed to operate in the economy, producing a homogeneous consumption good. The representative firm's technology is described by the production function

$$Y_t = A_t N_t^{1-\alpha} \tag{12}$$

where A_t represents the level of technology, and $a_t \equiv \log A_t$ evolves over time according to an exogenous $AR(1)$ process

$$a_t = \rho_a a_{t-1} + \varepsilon_t^a$$

where $\rho_a \in [0, 1]$.

Each period the firm maximizes profits

$$P_t Y_t - W_t N_t \tag{13}$$

subject to (12), taking the price and wage as given.

[4] The gross yield $\mathcal{I}_t$ on the one-period bond is given by $\mathcal{I}_t \equiv 1/Q_t$. The nominal interest rate is defined here as $i_t \equiv \log \mathcal{I}_t$. Thus, i_t is a monotonic functions of the *net* yield $\mathcal{I}_t - 1$, and approximately equal to the latter for values close to zero.

[5] Note that $\rho \equiv -\log \beta \simeq \beta^{-1} - 1$, which is the usual definition of the discount rate.

Maximization of (13) subject to (12) yields the optimality condition

$$\frac{W_t}{P_t} = (1-\alpha) A_t N_t^{-\alpha} \tag{14}$$

that is, the firm hires labor up to the point where its marginal product equals the real wage. Equivalently, the marginal cost $\frac{W_t}{(1-\alpha) A_t N_t^{-\alpha}}$ must be equated to the price P_t.

In log-linear terms,

$$w_t - p_t = a_t - \alpha n_t + \log(1-\alpha) \tag{15}$$

which can be interpreted as a labor demand schedule, mapping the real wage into the quantity of labor demanded, given the level of technology.

2.3 Equilibrium

The baseline model abstracts from sources of goods demand other than consumption (like investment, government purchases, or net exports). Accordingly, the goods market clearing condition is given by

$$y_t = c_t \tag{16}$$

that is, all output must be consumed.

By combining the optimality conditions of households and firms with (16) and the log-linear aggregate production relationship

$$y_t = a_t + (1-\alpha) n_t \tag{17}$$

one can determine the equilibrium levels of employment and output:

$$n_t = \psi_{na} a_t + \psi_n \tag{18}$$

$$y_t = \psi_{ya} a_t + \psi_y \tag{19}$$

where $\psi_{na} \equiv \frac{1-\sigma}{\sigma(1-\alpha)+\varphi+\alpha}$, $\psi_n \equiv \frac{\log(1-\alpha)}{\sigma(1-\alpha)+\varphi+\alpha}$, $\psi_{ya} \equiv \frac{1+\varphi}{\sigma(1-\alpha)+\varphi+\alpha}$, and $\psi_y \equiv (1-\alpha)\psi_n$.

Furthermore, given the equilibrium process for output, (10) can be used to determine the implied real interest rate, $r_t \equiv i_t - E_t\{\pi_{t+1}\}$, as

$$\begin{aligned} r_t &= \rho + (1-\rho_z) z_t + \sigma E_t\{\Delta y_{t+1}\} \\ &= \rho + (1-\rho_z) z_t - \sigma(1-\rho_a)\psi_{ya} a_t \end{aligned} \tag{20}$$

Finally, the equilibrium real wage, $\omega_t \equiv w_t - p_t$, is given by

$$\begin{aligned}\omega_t &= a_t - \alpha n_t + \log(1-\alpha) \\ &= \psi_{\omega a} a_t + \psi_\omega \end{aligned} \tag{21}$$

where $\psi_{\omega a} \equiv \frac{\sigma+\varphi}{\sigma(1-\alpha)+\varphi+\alpha}$ and $\psi_\omega \equiv \frac{(\sigma(1-\alpha)+\varphi)\log(1-\alpha)}{\sigma(1-\alpha)+\varphi+\alpha}$.

Notice that the equilibrium levels of employment, output, and the real interest rate are determined *independently of monetary policy*. In other words, monetary policy is *neutral* with respect to those real variables. In the simple model, output, employment, and the real wage fluctuate in response to variations in technology, though they are invariant to preference shocks. In particular, output always rises in the face of a productivity increase, with the size of the increase being given by $\psi_{ya} > 0$. The same is true for the real wage. On the other hand, the sign of the employment is ambiguous, depending on whether σ (which measures the strength of the income effect on labor supply) is larger or smaller than one. When $\sigma < 1$, the substitution effect on labor supply resulting from a higher real wage dominates the negative effect caused by a smaller marginal utility of consumption, leading to an increase in employment. The converse is true whenever $\sigma > 1$. When the utility of consumption is logarithmic ($\sigma = 1$), employment remains unchanged in the face of technology variations, for substitution and income effects exactly cancel one another. Finally, and under the assumptions on the processes followed by a_t and z_t, the real interest rate goes down in response to a positive technology shock, but rises in the wake of an increase in z_t.

What about nominal variables, like inflation or the nominal interest rate? Not surprisingly, and in contrast with real variables, their equilibrium values cannot be determined independently of monetary policy. To illustrate how nominal variables are influenced by the way monetary policy is conducted, their equilibrium behavior under alternative monetary policy rules will be considered next.

2.4 Monetary Policy and Price Level Determination

Let us start by examining the implications of some *interest rate rules*. Rules that involve monetary aggregates will be introduced later. Throughout use is made of the Fisherian equation

$$i_t = E_t\{\pi_{t+1}\} + r_t \tag{22}$$

which implies that the nominal rate adjusts one-for-one with expected inflation, given a real interest rate that is determined exclusively by real factors, as in (20).

Equation (20) implies that in the steady state without growth $r = \rho$, that is, the real interest rate corresponds to the household's discount rate. Thus it follows from (22) that in the perfect foresight steady state the nominal rate and inflation will be related by

$$i = \rho + \pi$$

In what follows the analysis is restricted to nonexplosive equilibrium paths for inflation and the nominal interest rate, that is, equilibrium paths that remain within a bounded neighborhood of the steady state, for sufficiently small fluctuations in the exogenous driving forces.

2.4.1 An Exogenous Path for the Nominal Interest Rate

Let us first consider the case of a monetary policy that implies an *exogenous* path for the nominal interest rate. For concreteness, let us assume the rule

$$i_t = i + v_t$$

where $\{v_t\}$ is assumed to follow an exogenous $AR(1)$ process

$$v_t = \rho_v v_{t-1} + \varepsilon_t^v \tag{23}$$

Henceforth, the stochastic component $\{v_t\}$ in the policy rule under consideration is referred to as a monetary policy shock. It should be interpreted as a random, transitory deviation from the "usual" conduct of monetary policy as anticipated by the public, due to a change in the policymaker's preferences, a response to an unusual unanticipated event, or, simply, an error in the implementation of monetary policy.[6] Even if one views those shocks as empirically unimportant, the analysis of their effects helps uncover how monetary policy interventions are transmitted to the economy under different sets of assumptions about the structure of the latter.

A particular case of this rule corresponds to a constant interest rate, that is, $i_t = i$ for all t. Note that $\pi = i - \rho$ is the steady state inflation (or the implicit long-run inflation target) associated with the rule above.

[6] Note that in the particular case of the rule (23) considered here, the "usual" policy consists in keeping the interest rate constant at level i.

Using (22), write

$$\begin{aligned} E_t\{\pi_{t+1}\} &= i_t - r_t \\ &= \pi + v_t - \widehat{r}_t \end{aligned}$$

where $\widehat{r}_t \equiv r_t - \rho$ is determined independently of the monetary policy rule, as shown above.

Note that expected inflation is pinned down uniquely by the previous equation, since it can be written as a function of exogenous variables, using (20). But actual inflation is not. Because there is no other condition that can be used to determine inflation, it follows that any inflation path that satisfies

$$\pi_t = \pi + v_{t-1} - \widehat{r}_{t-1} + \xi_t$$

is consistent with equilibrium, where ξ_t is a shock, possibly unrelated to economic fundamentals, satisfying $E_{t-1}\{\xi_t\} = 0$ for all t. Equivalently, the price level is given by

$$p_t = \pi + p_{t-1} + v_{t-1} - \widehat{r}_{t-1} + \xi_t$$

Shocks such as ξ_t are often referred to in the literature as *sunspot shocks*. An equilibrium in which such nonfundamental factors may cause fluctuations in one or more variables is referred to as an *indeterminate equilibrium*. The example above shows how an exogenous nominal interest rate leads to *price level indeterminacy*.

Notice that the equilibrium nominal wage is equal, in logs, to the real wage, which is determined by (21), plus the price level, which is indeterminate. Thus, the nominal wage inherits the indeterminacy of the price level.

2.4.2 A Simple Interest Rate Rule

Suppose that the central bank adjusts the nominal interest rate in response to deviations of inflation from a target π, according to the interest rate rule

$$i_t = \rho + \pi + \phi_\pi(\pi_t - \pi) + v_t$$

where $\phi_\pi \geq 0$ is a coefficient determining the strength of the endogenous response of monetary policy and $\{v_t\}$ is an exogenous monetary policy shifter, as defined above.

Combining the previous rule with the Fisherian equation (22) yields

$$\phi_\pi \widehat{\pi}_t = E_t\{\widehat{\pi}_{t+1}\} + \widehat{r}_t - v_t \tag{24}$$

where $\widehat{\pi}_t \equiv \pi_t - \pi$. A distinction is made between two cases, depending on whether the inflation coefficient in the above rule, ϕ_π, is larger or smaller than one.

If $\phi_\pi > 1$, the previous difference equation has only one nonexplosive solution. That solution can be obtained by solving (24) forward, which yields

$$\widehat{\pi}_t = \sum_{k=0}^{\infty} \phi_\pi^{-(k+1)} E_t\{\widehat{r}_{t+k} - v_{t+k}\} \tag{25}$$

The previous equation fully determines inflation (and, hence, the price level) as a function of the path of the real interest rate, which in turn is a function of exogenous real forces, as shown in (20). In particular, under the assumed driving processes for technology and preference parameters, inflation can be written as

$$\pi_t = \pi - \frac{\sigma(1-\rho_a)\psi_{ya}}{\phi_\pi - \rho_a}\, a_t + \frac{1-\rho_z}{\phi_\pi - \rho_z}\, z_t - \frac{1}{\phi_\pi - \rho_v}\, v_t$$

Note that a central bank following a rule of the form considered here can influence the degree of inflation volatility through its choice of ϕ_π. The larger that coefficient is the smaller will be the impact of real shocks on inflation. Monetary policy shocks, v_t, are seen to generate "unnecessary" fluctuations in inflation.

Given equilibrium path for inflation, the price and nominal wage levels are uniquely determined by the identities $p_t = p_{t-1} + \pi_t$ and $w_t = \omega_t + p_t$, with the real wage ω_t given by (21).

Note that an exogenous tightening of monetary policy (i.e., a positive realization of v_t) reduces inflation and, if persistent (i.e., $\rho_v > 0$), the nominal interest rate as well, since $\frac{\partial i_t}{\partial \varepsilon_t^v} = -\frac{\rho_v}{\phi_\pi - \rho_v}$. Assuming a money demand function of the form (11) the (contemporaneous) response of the money supply to the same shock is given by $\frac{\partial m_t}{\partial \varepsilon_t^v} = \frac{\eta\rho_v - 1}{\phi_\pi - \rho_v}$, which can be positive or negative, depending on the values of η and ρ_v. Thus, under the assumed interest rate rule, a liquidity effect will be present in response to a monetary policy shock if and only if $\eta\rho_v > 1$, for in that case the nominal rate will decline and the money supply will increase in response to the shock. Yet, and in contrast with the evidence shown in chapter 1, the previous responses will be accompanied by a decrease in prices (given $\frac{\partial \pi_t}{\partial \varepsilon_t^v} = -\frac{1}{\phi_\pi - \rho_v}$), and no effect on output or any other real variable.

On the other hand, if $\phi_\pi < 1$, the forward solution (25) does not converge. Instead, the stationary solution to (24) takes the form

$$\pi_t = (1 - \phi_\pi)\pi + \phi_\pi \pi_{t-1} - \widehat{r}_{t-1} + v_{t-1} + \xi_t \tag{26}$$

where $\{\xi_t\}$ is, again, an arbitrary sequence of shocks, related or unrelated to fundamentals, satisfying $E_{t-1}\{\xi_t\} = 0$ for all t.

Accordingly, any process $\{\pi_t\}$ satisfying (26) is consistent with equilibrium, while remaining in a neighborhood of the steady state (for sufficiently small shocks). So, as in the case of an exogenous nominal rate, the price level (and, hence, inflation) are not determined uniquely when the interest rate rule implies a weak response of the nominal rate to deviations of inflation from target.[7]

More specifically, the condition for a determinate price level, $\phi_\pi > 1$, requires that the central bank adjusts nominal interest rates more than one-for-one in response to any change in inflation, a property known as the *Taylor principle*. The previous result can be viewed as a particular instance of the need to satisfy the Taylor principle in order for an interest rate rule to bring about a determinate equilibrium.

2.4.3 An Exogenous Path for the Money Supply

The analysis above assumed that monetary policy can be described by an interest rate rule. In that case, money supply does not play an independent role in determining the equilibrium. Instead, it just adjusts endogenously, in order to match money demand, given the path of output, the price level, and the interest rate, as implied by an equation of the form

$$m_t = p_t + y_t - \eta i_t$$

which corresponds to (11), after imposing the goods market clearing condition.

Suppose instead that the central bank sets an exogenous path for the money supply $\{m_t\}$. Using (11) (with $c_t = y_t$) to eliminate the nominal

[7] With no loss of generality one can write

$$\xi_t = \tau_a \varepsilon_t^a + \tau_z \varepsilon_t^z + \tau_v \varepsilon_t^v + \xi_t^*$$

where τ_a, τ_z, and τ_v can take any value (i.e., are indeterminate) and ξ_t^* is a "pure sunspot," i.e., a martingale-difference process orthogonal to all fundamental shocks. Thus, the response of inflation (and, as result, of any nominal variable) to any shock is indeterminate.

interest rate in (22), the following difference equation for the price level can be derived

$$p_t = \left(\frac{\eta}{1+\eta}\right) E_t\{p_{t+1}\} + \left(\frac{1}{1+\eta}\right) m_t + u_t \tag{27}$$

where $u_t \equiv (1+\eta)^{-1}(\eta r_t - y_t)$ evolves independently of $\{m_t\}$, as shown in (19) and (20).

Assuming $\eta > 0$ and solving forward obtains

$$p_t = \frac{1}{1+\eta} \sum_{k=0}^{\infty} \left(\frac{\eta}{1+\eta}\right)^k E_t\,\{m_{t+k}\} + \overline{u}_t \tag{28}$$

where $\overline{u}_t \equiv \sum_{k=0}^{\infty} \left(\frac{\eta}{1+\eta}\right)^k E_t\,\{u_{t+k}\}$ is, again, independent of monetary policy.

Thus, when the monetary policy rule takes the form of an exogenous path for the money supply, the equilibrium price level is always determined uniquely, as shown in (28).

It is often convenient to have an expression for the equilibrium price level in terms of the expected future growth rate of money:[8]

$$p_t = m_t + \sum_{k=1}^{\infty} \left(\frac{\eta}{1+\eta}\right)^k E_t\,\{\Delta m_{t+k}\} + \overline{u}_t \tag{29}$$

Given (29), as determined above, money demand equation (11) can be used to solve for the nominal interest rate

$$\begin{aligned} i_t &= \frac{1}{\eta}\,(y_t - (m_t - p_t)) \\ &= \frac{1}{\eta} \sum_{k=1}^{\infty} \left(\frac{\eta}{1+\eta}\right)^k E_t\,\{\Delta m_{t+k}\} + \underline{u}_t \end{aligned}$$

where $\underline{u}_t \equiv \eta^{-1}(\overline{u}_t + y_t)$ is independent of monetary policy.

[8] To derive (29), rewrite (27) as a difference equation in terms of $\widetilde{p}_t \equiv p_t - m_t$, i.e.,

$$\widetilde{p}_t = \left(\frac{\eta}{1+\eta}\right) E_t\{\widetilde{p}_{t+1}\} + \left(\frac{\eta}{1+\eta}\right) E_t\{\Delta m_{t+1}\} + u_t$$

which can be solved forward to yield (29).

For concreteness, consider the case in which money growth follows an $AR(1)$ process

$$\Delta m_t = \rho_m \Delta m_{t-1} + \varepsilon_t^m$$

For simplicity, assume the absence of real shocks, thus implying a constant output and a constant real rate. Without loss of generality, set $r_t = y_t = 0$ for all t. Then, it follows from (29) that

$$p_t = m_t + \frac{\eta \rho_m}{1 + \eta(1 - \rho_m)} \Delta m_t$$

Hence, in response to an exogenous monetary policy shock, and as long as $\rho_m > 0$ (the empirically relevant case, given the observed positive autocorrelation of money growth), the price level should respond more than one-for-one with the increase in the money supply, a prediction that contrasts starkly with the sluggish response of the price level observed in empirical estimates of the effects of monetary policy shocks as discussed in chapter 1.

The nominal interest rate is in turn given by

$$i_t = \frac{\rho_m}{1 + \eta(1 - \rho_m)} \Delta m_t$$

that is, in response to an expansion of the money supply, and as long as $\rho_m > 0$, the nominal interest rate is predicted to go up. In other words, the model implies the absence of a liquidity effect, in contrast with the evidence discussed in chapter 1.

2.4.4 Optimal Monetary Policy

The analysis of the baseline classical economy above has shown that while real variables are independent of monetary policy, the latter can have important implications for the behavior of nominal variables and, in particular, of prices. Yet, and given that households' utility is a function of consumption and employment only—two real variables that are invariant to the way monetary policy is conducted—it follows that no policy rule is better than any other. Thus, in the classical model above, a policy that generates large fluctuations in inflation and other nominal variables (perhaps as a consequence of following a policy rule that does not guarantee a unique equilibrium for those variables) is no less desirable from a welfare perspective than one that succeeds in stabilizing prices in the face of the same shocks.

The previous result, which is clearly extreme and empirically unappealing, can be overcome once versions of the classical monetary model are considered in which a motive to keep part of households' wealth in the form of monetary assets is introduced explicitly. Section 2.5 discusses one such model in which real balances are assumed to yield utility.

The overall assessment of the classical monetary model as a framework to understand the observed behavior of nominal and real variables and their connection to monetary policy cannot be a favorable one. The model cannot explain the observed real effects of monetary policy on real variables. Its predictions regarding the response of the price level, the nominal rate, and the money supply to exogenous monetary policy shocks are also in conflict with the empirical evidence. Those empirical failures are the main motivation behind the introduction of nominal frictions in the model, a task that will be undertaken in chapter 3.

2.5 Money in the Utility Function

In the model developed in the previous sections, and in much of the recent monetary literature, the only role played by money is to serve as a numéraire, that is, a unit of account in which prices, wages, and securities payoffs are stated. Economies with that characteristic are often referred to as *cashless economies*. Above, a simple log-linear money demand function was occasionally postulated, but it was done in an ad hoc manner and without an explicit justification for why agents would want to hold an asset that is dominated in return by bonds while having identical risk properties. Even though in the analysis of subsequent chapters the assumption of a cashless economy is maintained, it is useful to understand how the basic framework can incorporate a role for money other than that of a unit of account and, in particular, how it can generate a demand for money. The discussion in this section focuses on models that achieve the previous objective by assuming that real money holdings are an argument of the utility function.[9] The reader can find references to alternative "microfoundations" in the section on the literature at the end of the chapter.

The assumption that real money holdings (or *real balances*), M_t/P_t, enter the utility function is a convenient way of formalizing the notion that, in contrast with other assets, money provides a "transactions service" that households value. The service that any nominal amount of money can yield is likely to depend on its purchasing power, thus explaining why it is the *real* value of monetary holdings that enters the utility function.

[9] Readers not interested in this extension may skip this section and proceed to section 2.6 without any loss of continuity.

The introduction of money in the utility function requires modifying the representative household's problem in two ways. First, preferences are now given by

$$E_0 \sum_{t=0}^{\infty} \beta^t U\left(C_t, \frac{M_t}{P_t}, N_t; Z_t\right) \tag{30}$$

where M_t denotes holdings of money in period t. It is assumed that period utility is increasing and concave in real balances M_t/P_t. Second, the flow budget constraint incorporates monetary holdings explicitly, taking the form

$$P_t C_t + Q_t B_t + M_t \leq B_{t-1} + M_{t-1} + W_t N_t + D_t \tag{31}$$

Letting $\mathcal{A}_t \equiv B_t + M_t$ denote total financial wealth at the beginning of period t (i.e., after bonds mature, but before consumption and portfolio decisions are made), the previous flow budget constraint can be rewritten as

$$P_t C_t + Q_t \mathcal{A}_t + (1 - Q_t) M_t \leq \mathcal{A}_{t-1} + W_t N_t + D_t \tag{32}$$

with the solvency constraint now taking the form

$$\lim_{T \to \infty} E_t \left\{ \Lambda_{t,T} \frac{\mathcal{A}_T}{P_T} \right\} \geq 0$$

for all t.

The previous representation of the budget constraint can be thought of as equivalent to that of an economy in which all financial assets (represented by $\mathcal{A}_t$) yield a gross nominal return Q_t^{-1} $(= \exp\{i_t\})$, and where agents can "purchase" the utility-yielding services of money balances at a unit price $1 - Q_t = 1 - \exp\{-i_t\} \simeq i_t$. Thus, the implicit price for the services of money roughly corresponds to the nominal interest rate, that is, the opportunity cost of holding financial wealth in terms of monetary assets, instead of interest-bearing bonds.

Consider next the household's problem, which consists of maximizing (30) subject to (32). Two of the implied optimality conditions are the same as those obtained for the cashless model, namely, (4) and (5), with the marginal utility terms being now defined over (and evaluated at) $(C_t, \frac{M_t}{P_t}, N_t; Z_t)$. In addition to (4) and (5), there is an additional optimality condition given by

$$\begin{aligned} \frac{U_{m,t}}{U_{c,t}} &= 1 - Q_t \\ &= 1 - \exp\{-i_t\} \end{aligned} \tag{33}$$

where $U_{m,t} \equiv \frac{\partial U_t}{\partial (M_t/P_t)} > 0$.

Again, in order to derive that optimality condition a simple variational argument can be used. Suppose that a household following an optimal plan considers a deviation from the latter which consists in adjusting consumption and money holdings in period t by amounts dC_t and dM_t, respectively, while keeping all other variables unchanged at their optimal values. Optimality of the initial plan requires that utility cannot rise as a result of the deviation, which in turn requires

$$U_{c,t}dC_t + U_{m,t}\frac{1}{P_t}dM_t = 0$$

for any pair (dC_t, dM_t) satisfying

$$P_t dC_t + (1 - Q_t)dM_t = 0$$

which guarantees that the budget constraint is met without the need to adjust any other variables. Combining the previous two equations and using the definition of the nominal rate $i_t \equiv -\log Q_t$ yields the optimality condition (33).

In order to be able to make any statements about the consequences of having money in the utility function, more specific assumptions are needed about the way money balances interact with other variables in yielding utility. In particular, whether the utility function is separable or not in real balances determines the extent to which the neutrality properties derived above for the cashless economy carry over to the economy with money in the utility function. That point is illustrated by considering, in turn, two example economies with separable and nonseparable utility.

2.5.1 An Example with Separable Utility

Specifically, the household's utility function is assumed to have the functional form

$$U\left(C_t, \frac{M_t}{P_t}, N_t; Z_t\right) = \left(\frac{C_t^{1-\sigma} - 1}{1-\sigma} + \frac{(M_t/P_t)^{1-\nu} - 1}{1-\nu} - \frac{N_t^{1+\varphi}}{1+\varphi}\right) Z_t$$

Note that, given the assumed separability, neither $U_{c,t}$ nor $U_{n,t}$ depend on the level of real balances. As a result, (7) and (8), as well as their log-linear counterparts, (9) and (10), continue to hold unchanged. It follows that the equilibrium values for output, employment, the real rate, and the real wage can be determined by following the same steps as in the previous section, without any reference to monetary policy.

The introduction of money in the utility function allows a money demand equation to be derived from the household's optimal behavior. Using the above specification of utility, the optimality condition (33) can be rewritten as

$$\frac{M_t}{P_t} = C_t^{\sigma/\nu} \left(1 - \exp\{-i_t\}\right)^{-1/\nu} \tag{34}$$

which can be naturally interpreted as a demand for real balances. The latter is increasing in consumption and inversely related to the nominal interest rate, as in conventional specifications.

Using the first-order Taylor expansion $\log(1 - \exp\{-i_t\}) \simeq const. + \frac{1}{\exp\{i\}-1} i_t$, (34) can be rewritten in approximate log-linear form (up to an uninteresting constant) as

$$m_t - p_t = \frac{\sigma}{\nu} c_t - \eta i_t \tag{35}$$

where $\eta \equiv \frac{1}{\nu(\exp\{i\}-1)} \simeq \frac{1}{\nu i}$ is the implied interest semielasticity of money demand.

The particular case of $\nu = \sigma$ is an appealing one in the present context, because it implies a unit elasticity with respect to consumption. Under that assumption, a conventional log-linear demand for real balances is obtained as

$$\begin{aligned} m_t - p_t &= c_t - \eta i_t \\ &= y_t - \eta i_t \end{aligned} \tag{36}$$

where the second equality holds under the assumption (made here) that all output is consumed. The previous specification is often maintained in subsequent chapters, without the need to invoke its source explicitly.

As in the analysis of the cashless economy, the usefulness of (35), or (36), is confined to the determination of the equilibrium values for inflation and other nominal variables whenever the description of monetary policy involves the quantity of money in circulation. Otherwise, the only use of the money demand equation is to determine the quantity of money that the central bank will need to supply in order to support, in equilibrium, the nominal interest rate implied by the policy rule.

2.5.2 An Example with Nonseparable Utility

Let us consider an economy with period utility given by

$$U(X_t, N_t) = \frac{X_t^{1-\sigma} - 1}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi}$$

where X_t is a composite index of consumption and real balances defined by

$$X_t \equiv \left[(1-\vartheta)C_t^{1-\nu} + \vartheta\left(\frac{M_t}{P_t}\right)^{1-\nu}\right]^{\frac{1}{1-\nu}} \qquad for\ \nu \neq 1$$

$$\equiv C_t^{1-\vartheta}\left(\frac{M_t}{P_t}\right)^{\vartheta} \qquad for\ \nu = 1$$

with ν representing the (inverse) elasticity of substitution between consumption and real balances, and ϑ the relative weight of real balances in utility. Note that, in order to ease the algebra, preference shocks are ignored for the remainder of this section.

Notice that the marginal utilities of consumption and real balances are now given, respectively, by

$$U_{c,t} = (1-\vartheta)X_t^{\nu-\sigma}C_t^{-\nu}$$

$$U_{m,t} = \vartheta X_t^{\nu-\sigma}\left(\frac{M_t}{P_t}\right)^{-\nu}$$

whereas the marginal (dis)utility of labor is, as before, given by $U_{n,t} = -N_t^{\varphi}$. The optimality conditions of the household's problem, (4), (5), and (33), can now be written as

$$\frac{W_t}{P_t} = N_t^{\varphi}X_t^{\sigma-\nu}C_t^{\nu}(1-\vartheta)^{-1} \tag{37}$$

$$Q_t = \beta E_t\left\{\left(\frac{C_{t+1}}{C_t}\right)^{-\nu}\left(\frac{X_{t+1}}{X_t}\right)^{\nu-\sigma}\left(\frac{P_t}{P_{t+1}}\right)\right\} \tag{38}$$

$$\frac{M_t}{P_t} = C_t\,(1-\exp\{-i_t\})^{-\frac{1}{\nu}}\left(\frac{\vartheta}{1-\vartheta}\right)^{\frac{1}{\nu}} \tag{39}$$

Notice that in the particular case in which the intertemporal and intratemporal elasticities of substitution coincide (i.e., $\nu = \sigma$), optimality conditions (37) and (38) match exactly those obtained in the case of separable utility, and thus lead to the same equilibrium implications derived for that case and discussed in section 2.5.1.

In the general case, however, both the labor supply equation (37) and the Euler equation (38) depend on the level of real balances through variable X_t. The level of real balances depends, in turn, on the nominal interest rate, as implied by (39).[10] Those features imply that monetary policy is no longer neutral in the case of nonseparable utility considered here. In particular, to the extent that different monetary policy rules have different implications for the path of the nominal rate (as will generally be the case), they will also have different effects on real balances and—through the latter's influence on the marginal utility of consumption—on the position of the labor supply schedule and, hence, on employment and output. This mechanism is analyzed formally below.

Notice that the implied money demand equation (39) can be rewritten in log-linear form (and up to an additive constant) as in (36) above

$$m_t - p_t = c_t - \eta i_t \tag{40}$$

where, as above, $\eta \equiv \frac{1}{\nu(\exp\{i\}-1)}$. Thus, the implied interest semielasticity of money demand η is now proportional to the elasticity of substitution between real balances and consumption ν^{-1}.

On the other hand, (37) can be written in log-linear form as (up to an additive constant):

$$w_t - p_t = \sigma c_t + \varphi n_t + (\nu - \sigma)(c_t - x_t)$$

For the remainder of this section the analysis is restricted to a neighborhood of the zero inflation steady state.[11] Log-linearizing the definition of X_t around that steady state and combining the resulting expression with (39), yields

$$\begin{aligned} w_t - p_t &= \sigma c_t + \varphi n_t + \chi(\nu - \sigma)\,(c_t - (m_t - p_t)) \\ &= \sigma c_t + \varphi n_t + \eta\chi(\nu - \sigma) i_t \end{aligned}$$

where $\chi \equiv \frac{\vartheta^{\frac{1}{\nu}}(1-\beta)^{1-\frac{1}{\nu}}}{(1-\vartheta)^{\frac{1}{\nu}}+\vartheta^{\frac{1}{\nu}}(1-\beta)^{1-\frac{1}{\nu}}} \in [0, 1)$, and where the second equality makes use of (40).

For future reference, it is convenient to rewrite the previous optimality conditions in terms of the steady state ratio $k_m \equiv \frac{M/P}{C}$, that is, the inverse consumption velocity. Using the money demand equation, evaluated at the zero inflation steady state, we have $k_m = \left(\frac{\vartheta}{(1-\beta)(1-\vartheta)}\right)^{\frac{1}{\nu}}$. Noting that

[10] Note that the present utility specification guarantees a unit-elastic money demand with respect to consumption, independently of the values of ν and σ.

[11] This is largely for notational simplicity. It is straightforward to extend the analysis to a neighborhood of a nonzero inflation steady state.

$\chi = \frac{k_m(1-\beta)}{1+k_m(1-\beta)}$, and using the definition of η evaluated at the zero inflation steady state, the optimality condition above can be rewritten as

$$w_t - p_t = \sigma c_t + \varphi n_t + \varpi i_t \tag{41}$$

where $\varpi \equiv \frac{k_m\beta(1-\frac{\sigma}{\nu})}{1+k_m(1-\beta)}$. Thus, the sign of the effect of the nominal interest rate on labor supply is determined by the sign of $\nu - \sigma$. When $\nu > \sigma$ (implying $\varpi > 0$), the reduction in real balances induced by an increase in the nominal rate brings down the marginal utility of consumption (for any given c_t), lowering the quantity of labor supplied at any given real wage. The opposite effect is obtained when $\nu < \sigma$. Note, however, that $\nu \simeq \frac{1}{i\eta}$ is likely to be larger than σ for any plausible values of i, η, and σ. Thus, the case of $U_{cm} > 0$ (and, hence, $\varpi > 0$) appears as the most plausible one, conditional on the specification of preferences analyzed here.

In order to reflect the changes implied by nonseparable utility, the economy's log-linearized equilibrium conditions need to be modified. Thus, combining (41) with the labor demand schedule (15) yields the labor market clearing condition

$$\sigma c_t + \varphi n_t + \varpi i_t = a_t - \alpha n_t + \log(1-\alpha) \tag{42}$$

which can be rewritten, using the goods market clearing condition (16), the log-linear production relationship (17), and up to an additive constant, as:

$$y_t = \psi_{ya} a_t + \psi_{yi} i_t \tag{43}$$

where $\psi_{yi} \equiv -\frac{\varpi(1-\alpha)}{\sigma(1-\alpha)+\varphi+\alpha}$ and, as before, $\psi_{ya} \equiv \frac{1+\varphi}{\sigma(1-\alpha)+\varphi+\alpha}$.

Condition (43) points to a key implication of the nonseparability property (i.e., $\varpi \neq 0$): Equilibrium output is no longer invariant to monetary policy, at least to the extent that the latter implies variations in the nominal interest rate. In other words, monetary policy is not neutral. As a result, equilibrium condition (43) does not suffice to determine the equilibrium level of output, in contrast to the economy with separable utility analyzed above. Note that in the empirically plausible case of $\varpi > 0$, $\psi_{yi} < 0$ implying that output is inversely related to the nominal interest rate.

In order to assess the extent to which monetary policy may affect the level of output it is useful to get a sense of the magnitude of the interest rate multiplier ψ_{yi}. With that purpose, and in a way consistent with the baseline calibration that will be introduced in chapter 3, the settings

$\sigma = 1$, $\varphi = 5$, and $\alpha = 1/4$ are chosen and taken to be "empirically plausible." Using the definition of ϖ, and the fact that $\nu = \frac{1}{\eta i}$ is "large" for any reasonable values of η, it follows that $\psi_{yi} \simeq -\frac{k_m}{8}$, and so the size of the inverse velocity k_m is a key determinant of the quantitative importance of monetary non-neutralities in the model. The magnitude of k_m, however, depends crucially on the definition of money one adopts. Thus, and focusing on postwar U.S. data, $k_m \simeq 0.3$ if the monetary base is taken to be the relevant measure of money.[12] In that case $\psi_{yi} \simeq -0.04$, which implies a very small multiplier: A monetary policy intervention that raised the nominal rate by one percentage point (expressed at an annual rate) would generate a decrease in quarterly output of about 0.01 percent. Using $M2$ instead as the definition of money, $k_m \simeq 3$, and hence $\psi_{yi} \simeq -0.4$. Thus, an analogous monetary policy intervention would lower output by 0.1 percent. The latter value, while still very small, appears to be more in line with the estimated output effects of monetary policy shock found in the literature, at least in terms of order of magnitude. Yet, even in the latter case, there are other aspects of the monetary transmission mechanism associated with the model above that are clearly at odds with the evidence. The analysis below makes this point by analyzing the joint response of output, inflation, and the interest rate to a monetary policy shock.

Combining (40) and (43) (after setting $a_t = 0$ for all t, given the focus on exogenous shifts to monetary policy), one obtains

$$y_t = \Theta(m_t - p_t) \tag{44}$$

where $\Theta \equiv \frac{\varpi(1-\alpha)}{\eta[\sigma(1-\alpha)+\varphi+\alpha]+\varpi(1-\alpha)} \in [0, 1)$ under the empirically plausible assumption that $\varpi \geq 0$ (which is maintained for the remainder of the section). Also,

$$i_t = -(1/\eta)(1 - \Theta)(m_t - p_t) \tag{45}$$

Thus, in order for an exogenous tightening of monetary policy to raise the nominal rate, while lowering output and the money supply (as suggested by the evidence in chapter 1), it must also be associated with a decline in real balances, that is, the price level should *not* decline more than proportionally to the money supply. But the latter requirement is inconsistent with the predictions of the model, as shown next, at least to the extent that the policy tightening is associated with a persistent decline in money supply growth, as found in the data.

[12] This is the approach followed in Woodford (2003, chap. 2).

First, note that the corresponding log-linear approximation to (38) is given by

$$
\begin{aligned}
c_t &= E_t\{c_{t+1}\} - \frac{1}{\sigma}\,(i_t - E_t\{\pi_{t+1}\} - (\nu-\sigma)E_t\{(c_{t+1}-x_{t+1}) - (c_t - x_t)\} - \rho) \\
&= E_t\{c_{t+1}\} - \frac{1}{\sigma}\,(i_t - E_t\{\pi_{t+1}\} - \chi(\nu-\sigma)E_t\{\Delta c_{t+1} - \Delta(m_{t+1} - p_{t+1})\} - \rho) \\
&= E_t\{c_{t+1}\} - \frac{1}{\sigma}\,(i_t - E_t\{\pi_{t+1}\} - \varpi\, E_t\{\Delta i_{t+1}\} - \rho)
\end{aligned} \tag{46}
$$

where, again, the last equality makes use of (40). Thus, when $\nu > \sigma$ (and, hence, $\varpi > 0$) the anticipation of a nominal rate increase (and, hence, of a decline in real balances) lowers the expected one period ahead level of the marginal utility of consumption (for any expected c_{t+1}), which induces an increase in current consumption (in order to smooth marginal utility over time). Imposing the goods market clearing condition $y_t = c_t$ on (46), yields an equation relating output, to the nominal interest rate and expected inflation:

$$
y_t = E_t\{y_{t+1}\} - \frac{1}{\sigma}\,(i_t - E_t\{\pi_{t+1}\} - \varpi\, E_t\{\Delta i_{t+1}\} - \rho) \tag{47}
$$

Combining the previous equation with (44) and (45) one can obtain (after some algebraic manipulation) an expression relating the price level to current and expected money supply which generalizes (29) to the case of nonseparable utility:

$$
p_t = m_t + \frac{\eta}{\eta + \varpi\Lambda} \sum_{k=1}^{\infty} \left(\frac{\eta + \varpi\Lambda}{1 - \Theta + \eta + \varpi\Lambda} \right)^{k} E_t\{\Delta m_{t+k}\} \tag{48}
$$

where $\Lambda \equiv \frac{\eta(\alpha+\varphi)}{\eta[\sigma(1-\alpha)+\varphi+\alpha]+\varpi(1-\alpha)} \in [0,1)$. Thus, in order for the price level to decline less than the money supply in response to a tightening of monetary policy involving a higher nominal rate and lower output, the initial decline in the money supply would have to be followed by persistently higher than average money growth. The latter implication is at odds with the empirical evidence, as shown in chapter 1. Note that the above implication is independent of the details of monetary policy implementation (i.e., of the specific monetary policy rule).

The model also predicts that the real wage should move countercyclically in response to an exogenous shift in monetary policy, to the extent that the latter has an effect on output (i.e., if $\varpi \neq 0$). The reason is that the resulting change in employment is a consequence of a shift in the labor supply schedule, moving along a stable labor demand. That prediction is also in conflict with the empirical evidence in Christiano, Eichenbaum, and Evans (2005), which points to a mildly procyclical real wage in response to an exogenous monetary policy shock.

The analysis above has revealed important weaknesses regarding the empirical plausibility of the classical model's monetary transmission mechanism. A key concern has to do with the large and rapid price adjustment that follows a monetary shock, with the price level changing in the short run more than proportionally to the change in the money supply. The previous assessment motivates the introduction of nominal frictions in the models analyzed from the next chapter onward.

2.5.3 Optimal Monetary Policy in a Classical Economy with Money in the Utility Function

This section derives the form of the optimal monetary policy in the presence of money in the utility function. The problem of a hypothetical social planner seeking to maximize the utility of the representative household is presented and solved.

Note that, under the assumptions made above, the economy has no aggregate intertemporal links: Even though each individual household can reallocate its own consumption over time through financial markets, there are no mechanisms that make this possible for the economy as a whole. Thus, a benevolent social planner solves a sequence of static problems of the form

$$\max U\left(C_t, \frac{M_t}{P_t}, N_t; Z_t\right)$$

subject to the aggregate resource constraint

$$C_t = A_t N_t^{1-\alpha}$$

The optimality conditions for that problem are given by

$$-\frac{U_{n,t}}{U_{c,t}} = (1-\alpha) A_t N_t^{-\alpha} \tag{49}$$

$$U_{m,t} = 0 \tag{50}$$

Condition (49) requires that the marginal rate of substitution between employment and consumption be equated to the marginal product of labor. Condition (50) equates the marginal utility of real balances to the "social" marginal cost of producing real balances, which is implicitly assumed to be zero in the model.

Under what conditions does the equilibrium of the decentralized economy satisfy efficiency conditions (49) and (50)? First note that condition (49) is implied by the combined effect of profit maximization by firms (see equation (14), which equates the real wage to the marginal product of labor) and the optimal labor supply choice by the household (see equation (4), which equates the real wage to the marginal rate of substitution between hours of work and consumption). Hence, (49) will be satisfied independently of monetary policy. On the other hand, and as shown in equation (33) above, the household's optimal choice of money balances requires

$$\frac{U_{m,t}}{U_{c,t}} = 1 - \exp\{-i_t\}$$

Accordingly, efficiency condition (50) will be satisfied if and only if, $i_t = 0$ for all t. This is the policy prescription known as the *Friedman rule*. The rationale for that policy is quite intuitive: While the social cost of producing real balances is zero, the private (opportunity) cost is given by the nominal interest rate. As a result, only when the nominal interest rate is zero are the private and social costs of holding money equated. Note that such a policy implies an average (steady state) rate of inflation

$$\pi = -\rho < 0$$

that is, prices will decline on average at the rate of time preference. In other words, under the Friedman rule the economy experiences (moderate) steady state deflation, on average.

Implementation of the Friedman rule requires some discussion. First, as was the case in the cashless model, a policy rule of the form $i_t = 0$ for all t leaves the price level indeterminate in the classical model with money in the utility function. To see this note that, under that rule, (43) and (47) imply that any inflation path that satisfies

$$\pi_{t+1} = -\rho - \sigma \psi_{ya} E_t\{\Delta a_{t+1}\} + \xi_{t+1}$$

for an arbitrary martingale-difference sequence $\{\xi_t\}$ is an equilibrium path.[13]

[13] That finding can be seen as a particular case of a more general result: Under a simple interest rate rule of the form $i_t = \rho + \pi + \phi_\pi(\pi_t - \pi)$, with $\phi_\pi \geq 0$, the condition for a locally unique equilibrium is given by $\phi_\pi > 1$ (see exercise 6 below). Note that the rule analyzed in the text corresponds to the particular case $\pi = -\rho$ and $\phi_\pi = 0$.

Even though that indeterminacy should not have any welfare consequences (because consumption, employment, and real balances are pinned down uniquely as a function of technology and the nominal interest rate), a central bank could avoid *nominal* indeterminacy by following a rule of the form

$$i_t = \phi(r_{t-1} + \pi_t)$$

for any $\phi > 1$. Combined with (22), that rule implies the difference equation

$$E_t\{i_{t+1}\} = \phi i_t$$

whose only stationary solution is $i_t = 0$ for all t. Under that rule, equilibrium inflation is fully predictable and given by

$$\pi_t = -r_{t-1}$$

More generally, any rule that makes the central bank adjust its policy settings (e.g., the money supply) to guarantee that current inflation moves one-for-one, but inversely, with the lagged real interest rate, will imply a zero nominal interest rate and, thus, an optimal level of real balances.

2.6 Notes on the Literature

The modeling approach favored in much of the recent monetary literature, and the one adopted in this book (with the exception of section 2.5 of this chapter), does not incorporate money explicitly in the analysis. Under that approach the main role played by money is that of a unit of account. Such model economies can be viewed as a limiting case (the so-called *cashless limit*) of an economy in which money is valued and held by households, but in which the transaction services provided by real balances are arbitrarily small relative to the size of the economy. Woodford (2003) provides a detailed discussion and a forceful defense of that approach.

Most models in the literature that introduce monetary assets explicitly rely on one of two alternative formalisms in order to generate a demand for an asset that—as is the case with money—is dominated in its rate of return by alternative assets that have identical risk characteristics: they assume that real balances either (i) generate (indirect) utility to households or (ii) provide some "transaction services" which render them necessary—or, at least, useful—when purchasing goods.[14] The first approach can often be viewed as a reduced form of the second (exercises 7 and 9 below illustrate this point).

The first of those approaches—money in the utility function—traces back to Sidrauski (1967), who introduced that assumption in an otherwise standard neoclassical growth model (with inelastic labor supply). Woodford (2003) offers a detailed analysis of the implications of alternative assumptions on the specification of utility and, in particular, of the likely degree of monetary non-neutralities arising from the nonseparability of real balances. Walsh (2010, chap. 2) develops a model with money in the utility function, and analyzes the equilibrium properties of a calibrated version of that model. Both analyses conclude, in a way consistent with the discussion above, that even under a utility that is nonseparable in real balances, the real effects of monetary policy are quantitatively very small for plausible calibrations of the models.

A common approach to the modeling of a transaction's motive for holding money builds on the assumption, originally due to Clower (1967), that cash must be held in advance in order to purchase certain goods. Early examples of classical monetary models in which a demand for money is generated by postulating a cash-in-advance constraint can be found in the work of Lucas (1982) and Svensson (1985). Cooley and Hansen (1989) analyze an otherwise standard RBC model augmented with a cash-in-advance constraint for consumption goods, showing that monetary policy is nearly neutral for plausible calibrations of that model. Walsh (2010, chap. 3) provides a detailed description of classical monetary models with cash-in-advance constraints and their implications for the role of monetary policy. Exercises 8 and 9 below ask the reader to analyze economies with cash-in-advance constraints.

An alternative way of modeling the transaction services of money consists in assuming that the time required to purchase goods (which detracts from leisure and/or work time) is inversely related to the level of real balances. See Walsh (2010, chap. 3) for a detailed analysis of a "shopping time" model.

The practice, followed in this monograph, of appending a money demand equation to a set of equilibrium conditions that have been derived in the context of cashless economy is often found in the literature. King and Watson (1995) constitutes an early example of that practice.

[14] A third approach, seldom found in the recent literature, emphasizes the function of money as a store of value, i.e., as a way of transferring resources intertemporally. In the absence of other services or dividends associated with money, the latter can be viewed as a pure bubble under this approach. See, e.g., Samuelson (1958).

The analysis of the form of the optimal monetary policy in a classical economy goes back to Friedman (1969), where a case is made for a policy that keeps the nominal interest rate constant at a zero level. More recent treatments of the conditions under which that rule is optimal include Woodford (1990) and Correia and Teles (1999).

In the present chapter, as well as in the rest of the book, the analysis is restricted to *local* equilibrium dynamics, that is, to equilibrium paths that remain within a neighborhood of the steady state. Accordingly, the notion of uniqueness or indeterminacy used throughout the book must be understood as referring to those bounded equilibria only. A discussion of the conditions under which a classical monetary economy may display *global* multiplicity of equilibria, coexisting with a *locally* unique equilibrium, can be found in Benhabib, Schmitt-Grohé, and Uribe (2001) and Woodford (2003, chap. 2), among others. The previous authors also discuss alternative policies that guarantee that locally unique equilibria around the desired steady state are also globally unique.

Finally, the reader can find two useful discussions of the notion of monetary neutrality and its evolution in macroeconomic thinking in Patinkin (1987) and Lucas (1996).

Appendix

2.1 Log-Linear Approximation of the Euler Equation

The consumer's Euler equation can be rewritten as

$$1 = E_t\{\exp(i_t - \sigma \Delta c_{t+1} - \pi_{t+1} - \rho + \Delta x_{t+1})\} \tag{51}$$

In a perfect foresight steady state with constant inflation π and constant growth γ

$$i = \rho + \pi + \sigma\gamma$$

A first-order Taylor expansion of $\exp(i_t - \sigma \Delta c_{t+1} - \pi_{t+1} - \rho + \Delta x_{t+1})$ around that steady state yields

$$\begin{aligned}
&\exp(i_t - \sigma \Delta c_{t+1} - \pi_{t+1} - \rho + \Delta x_{t+1}) \\
&\quad \simeq 1 + (i_t - i) - \sigma(\Delta c_{t+1} - \gamma) - (\pi_{t+1} - \pi) + \Delta x_{t+1} \\
&\quad = 1 + i_t - \sigma \Delta c_{t+1} - \pi_{t+1} - \rho + \Delta x_{t+1}
\end{aligned}$$

which can be used in (51) to obtain, after some rearrangement of terms, the log-linearized Euler equation

$$c_t = E_t\{c_{t+1}\} - \frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} - \rho - (1 - \rho_x)x_t)$$

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Exercises

2.1. **Optimality conditions under nonseparable leisure**
Derive the log-linearized optimality conditions of the household problem under the following specification of the period utility function:

$$U(C_t, N_t) = \frac{[C_t(1-N_t)^{\nu}]^{1-\sigma} - 1}{1-\sigma}$$

2.2. **Alternative interest rules for the classical economy**
Consider the classical economy described in the text, with equilibrium conditions

$$y_t = E_t\{y_{t+1}\} - \frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} - \rho)$$

and

$$\begin{aligned} r_t &\equiv i_t - E_t\{\pi_{t+1}\} \\ &= \rho + \sigma E_t\{\Delta y_{t+1}\} \end{aligned}$$

where y_t and, hence, r_t, are determined independently of monetary policy. Analyze the implications of the following alternative monetary policy rules. When relevant, assume that the money demand function takes the form

$$m_t - p_t = y_t - \eta i_t + \varepsilon_t^m$$

where ε_t^m is a stochastic money demand disturbance.

a. *Rule #1: Strict inflation targeting.*

   i. Derive an interest rate rule that guarantees full stabilization of inflation, i.e., $\pi_t = \pi^*$ for all t where π^* is an inflation target.
   ii. Determine the behavior of money growth implied by that rule.
   iii. Explain why a policy characterized by a constant rate of money growth $\Delta m_t = \pi^*$ will generally not succeed in stabilizing inflation in that economy.

b. *Rule #2: Price level targeting*

   i. Consider the interest rate rule

$$i_t = \rho + \pi^* + \phi_p(p_t - p_t^*)$$

where $\phi_p > 0$ and $p_t^* = p_0^* + \pi^* t$ is a time-varying target for the (log) price level consistent with an average inflation π^*. Determine the equilibrium behavior of the price level under this rule. (Hint: you may find it useful to introduce a new variable $\widehat{p}_t \equiv p_t - p^*$, the deviation of the price level from target, to ease some of the algebraic manipulations.)

ii. Consider instead the money supply rule

$$\Delta m_t = \pi^*$$

Determine the equilibrium behavior of the price level under this rule, assuming that $m_0 = p_0^*$.

iii. Show that the money supply rule considered in (ii) can be combined with the money market clearing condition and rewritten as a price level targeting rule of the form

$$i_t = \psi(p_t - p_t^*) + u_t$$

for some constant ψ and stochastic process u_t to be determined.

iv. Suppose that the central bank wants to minimize the volatility of the price level. Discuss the advantages and disadvantages of the interest rate rule in (i) versus the money supply rule in (ii) in light of your findings above.

2.3. **Equilibrium indeterminacy and interest rate rules** (based on Woodford (2003))
Consider a classical economy with an exogenous real interest rate process $\{r_t\}$. Discuss the conditions under which the equilibrium will be (locally) unique and solve for the equilibrium level of inflation when the central bank follows the rules:

a. *Rule #1: Partial adjustment*

$$i_t = \phi_i i_{t-1} + (1 - \phi_i) i_t^*$$

where $\phi_i \in [0, 1]$ and i_t^* is a reference interest rate given by

$$i_t^* = \rho + \phi_\pi \pi_t$$

b. *Rule #2: Moving average inflation targeting*

$$i_t = \rho + \phi_\pi \overline{\pi}_t$$

where

$$\overline{\pi}_t = (1-\delta)\sum_{k=0}^{\infty} \delta^k \pi_{t-k}$$

c. Show the equivalence between the two rules

2.4. **On the welfare costs of inflation** (based on Lucas (2000))
Consider an endowment economy where the representative consumer's period utility is given by

$$U\left(C_t, \frac{M_t}{P_t}\right) = \frac{\left[C_t \psi\left(\frac{M_t/P_t}{C_t}\right)\right]^{1-\sigma} - 1}{1-\sigma}$$

where $\psi(z) \equiv z/(\chi + z)$. The household's budget constraint is given by

$$P_t C_t + Q_t B_t + M_t \leq B_{t-1} + M_{t-1} + P_t Y_t$$

where $\{Y_t\}$ is an exogenous stochastic endowment, which is nonstorable.

a. Derive the household's optimality conditions, including the demand for real balances.
b. Show that the Friedman rule is the welfare-maximizing monetary policy
c. Focusing on a steady state with $Y = 1$, define the welfare cost of inflation as the permanent percent increase in consumption that would make the household indifferent between a steady state inflation rate π and zero inflation.
d. Determine, up to a second order approximation, the welfare losses resulting from fluctuations in the nominal interest rate, relative to a constant interest rate policy.

2.5. **Optimal monetary policy in a classical economy with an exact equilibrium representation**
Consider a version of the classical economy with money in the utility function, where the representative consumer maximizes $E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, \frac{M_t}{P_t}, N_t)$ subject to the sequence of budget constraints

$$P_t C_t + M_t + Q_t B_t \leq M_{t-1} + B_{t-1} + W_t N_t + D_t$$

Assume a period utility given by

$$U\left(C_t, \frac{M_t}{P_t}, N_t\right) = \log C_t + \log \frac{M_t}{P_t} - \frac{N_t^{1+\varphi}}{1+\varphi} \tag{52}$$

Suppose there is a representative perfectly competitive firm, producing the single consumption good. The firm has access to the linear production function $Y_t(i) = A_t N_t(i)$, where productivity evolves according to

$$\frac{A_t}{A_{t-1}} = (1+\gamma_a)\exp\{\varepsilon_t^a\}$$

with $\{\varepsilon_t^a\}$ an i.i.d. random process, normally distributed, with mean 0 and variance σ_a^2.

The money supply varies exogenously according to the process

$$\frac{M_t}{M_{t-1}} = (1+\gamma_m)\exp\{\varepsilon_t^m\} \tag{53}$$

where $\{\varepsilon_t^m\}$ is an i.i.d., normally distributed process with mean 0 and variance σ_m^2. Assume that $\{\varepsilon_t^m\}$ evolves exogenously, outside the control of the monetary authority (e.g., could reflect shocks in the monetary multiplier that prevent the monetary authority from fully controlling the money supply). Finally, assume that all output is consumed, so that in equilibrium $Y_t = C_t$ for all t.

a. Derive the optimality conditions for the problem of households and firms.
b. Determine the equilibrium levels of aggregate employment, output, and inflation (Hint: guess that the equilibrium involves a constant velocity $\frac{P_t Y_t}{M_t} = V$ for all t).
c. Discuss how utility depends on the two parameters describing monetary policy, γ_m and σ_m^2 (recall that the nominal interest rate is constrained to be non-negative, i.e., $Q_t \leq 1$ for all t). Show that the optimal policy must satisfy the Friedman rule ($i_t = 0$ for all t).

2.6. **Equilibrium uniqueness in the classical model with money in the utility function**

Consider the classical model equilibrium, as described by conditions (43), (47), and the simple interest rate rule

$$i_t = \rho + \phi_\pi \pi_t$$

Show that the equilibrium is locally unique if and only if $\phi_\pi > 1$.

2.7. **A shopping time model**

Assume that the transactions technology is such that consuming C_t requires a quantity of shopping time $N_t^s = S(C_t, \frac{M_t}{P_t})$, given real balances $\frac{M_t}{P_t}$, where $S_c > 0$ and $S_m \leq 0$. The period utility is given by $V(C_t, L_t)$ where $L_t = 1 - N_t - N_t^s$ denotes leisure time and N_t time at work.

a. Derive the implied utility function $U(C_t, \frac{M_t}{P_t}, N_t)$, i.e., in terms of consumption, hours, and real balances and discuss its properties (sign of first and second derivatives) as a function of those of V and S
b. Using the finding in part a), derive the optimality conditions of the household's problem, under the sequence of budget constraints (31), and show that under the optimal plan $\frac{W_t}{P_t} s_m \simeq i_t$. Interpret the latter condition.
c. Derive the money demand function under the assumption that $V(C_t, L_t) = \log C_t + \chi \log L_t$ and $S\left(C_t, \frac{M_t}{P_t}\right) = Av_t + B/v_t - 2\sqrt{AB}$ where $v_t \equiv P_t C_t / M_t$ (based on Schmitt-Grohé and Uribe (2011))

2.8. **A model with a cash-in-advance constraint**

Consider a monetary economy where households' maximize $\sum_{t=0}^{\infty} \beta^t (\log C_t - \frac{1}{1+\varphi} N_t^{1+\varphi})$ subject to the sequence of budget constraints

$$P_t C_t + Q_t B_t + M_t \leq B_{t-1} + M_{t-1} + W_t N_t + T_t$$

and the cash-in-advance (CIA) constraint $P_t C_t \leq M_{t-1} + T_t$, where T_t is a lump-sum monetary transfer to households. Output is produced by a representative perfectly competitive firm with technology $Y_t = N_t$. The money supply grows at a constant growth rate γ_m. There is no uncertainty.

a. Derive the household's optimality conditions.
b. Determine how money growth affects equilibrium output.
c. What is the optimal rate of inflation?

2.9. **A model with cash and credit goods**

Assume that the utility of the representative household is given by

$$V(C_{1t}, C_{2t}, N_t) \tag{54}$$

where C_{1t} denotes consumption of a "cash good" (i.e., a good that requires cash in order to be purchased), C_{2t} is consumption of a "credit good" (which does not require cash), and N_t is labor supply. For simplicity, assume that the price of the two goods is identical and equal to P_t (this will be the case if the production function of the representative firm is given by $Y_{1t} + Y_{2t} = N_t$ and there is perfect competition). Purchases of cash goods have to be settled in cash, whereas credit goods can be financed by issuing one-period riskless nominal bonds.

The budget constraint is given by

$$P_t \,(C_{1t} + C_{2t}) + Q_t \, B_t + M_t = B_{t-1} + M_{t-1} + W_t N_t + T_t$$

Finally, the cash-in-advance (CIA) constraint is given by

$$P_t \, C_{1t} \leq M_{t-1} + T_t$$

where, in equilibrium, $T_t = \Delta M_t$, i.e., transfers to households correspond to money transfers made by the central bank, which consumers take as given. For simplicity, assume no uncertainty.

a. Derive the first-order conditions associated with the household's problem
b. Note that whenever the CIA constraint is binding reduced form period utility can be defined as

$$U\left(C_t, \frac{M_t}{P_t}, N_t\right) \equiv V\left(\frac{M_t}{P_t}, C_t - \frac{M_t}{P_t}, N_t\right)$$

where $C_t = C_{1t} + C_{2t}$. Show that $U_m \geq 0$, given the optimality conditions derived in (a).

Chapter 3

THE BASIC NEW KEYNESIAN MODEL

The present chapter describes the key elements of the baseline model that will be used as a reference framework in the remainder of the book. The model involves two main departures from the assumptions of the classical monetary economy developed in chapter 2. First, imperfect competition in the goods market is introduced by assuming that each firm produces a differentiated good for which it sets the price (instead of taking the price as given), given a demand constraint. Second, a form of price stickiness is introduced by assuming that only a fraction of firms can reset their prices in any given period. In particular, and following much of the literature, a model of staggered price setting due to Calvo (1983) and characterized by random price durations is adopted.[1] The resulting framework is referred to as the *basic New Keynesian model*. As discussed in chapter 1, that model has become in recent years the workhorse for the analysis of monetary policy, fluctuations, and welfare.

The introduction of differentiated goods requires that the household problem be modified slightly relative to the one considered in the previous chapter. That modification is discussed before turning to the firms' optimal price-setting problem and the implied inflation dynamics.

3.1 Households

Once again, assume a representative infinitely lived household seeking to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t; Z_t)$$

where C_t is now a consumption index given by

$$C_t \equiv \left(\int_0^1 C_t(i)^{1-\frac{1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}}$$

[1] The resulting inflation dynamics can also be derived under the assumption of quadratic costs of price adjustment. See, e.g., Rotemberg (1982).

with $C_t(i)$ denoting the quantity of good i consumed by the household in period t. A continuum of goods represented by the interval [0, 1] is assumed. The period budget constraint now takes the form

$$\int_0^1 P_t(i)C_t(i)di + Q_t B_t \leq B_{t-1} + W_t N_t + D_t$$

for $t = 0, 1, 2...$, where $P_t(i)$ is the price of good i, and where the remaining variables are defined as in chapter 2: N_t denotes employment or hours worked, W_t is the nominal wage, B_t represents purchases of one-period discount bonds (at a price Q_t), and D_t denotes dividends from the ownership of firms. The above sequence of period budget constraints is supplemented with a solvency constraint of the form $\lim_{T\to\infty} E_t \left\{\Lambda_{t,T} \frac{B_T}{P_T}\right\} \geq 0$ for all t, where $\Lambda_{t,T} \equiv \beta^{T-t} U_{c,T}/U_{c,t}$. Note that monetary holdings are not modeled explicitly, so one can think of the present framework as the cashless limit of an economy with money in the utility function, with the latter being additively separable.[2]

In addition to the consumption/savings and labor supply decisions already analyzed in chapter 2, the household now must decide how to allocate its consumption expenditures among the different goods. Optimal behavior requires that the consumption index C_t be maximized for any given level of expenditures $\int_0^1 P_t(i)C_t(i)di$. As shown in appendix 3.1, the solution to that problem yields the set of demand equations

$$C_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} C_t \tag{1}$$

for all $i \in [0, 1]$, where $P_t \equiv \left(\int_0^1 P_t(i)^{1-\epsilon} di\right)^{\frac{1}{1-\epsilon}}$ is an aggregate price index. Furthermore, and conditional on such optimal behavior

$$\int_0^1 P_t(i)C_t(i)di = P_t C_t$$

i.e., total consumption expenditures can be written as the product of the price index times the quantity index. Plugging the previous expression into the budget constraint yields

$$P_t C_t + Q_t B_t \leq B_{t-1} + W_t N_t + D_t$$

[2] See section 5 of chapter 2 for a discussion.

which is formally identical to the constraint facing households in the single good economy analyzed in chapter 2. Hence, the optimal consumption/savings and labor supply decisions are identical to the ones derived therein, and described by the conditions

$$-\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t}$$

$$Q_t = \beta E_t \left\{ \frac{U_{c,t+1}}{U_{c,t}} \frac{P_t}{P_{t+1}} \right\}$$

As in chapter 2, it is assumed that the household's period utility is given by

$$U(C_t, N_t; Z_t) = \begin{cases} \left(\frac{C_t^{1-\sigma}-1}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right) Z_t & \text{for } \sigma \neq 1 \\ \left(\log C_t - \frac{N_t^{1+\varphi}}{1+\varphi} \right) Z_t & \text{for } \sigma = 1 \end{cases}$$

where $z_t \equiv \log Z_t$ follows an exogenous $AR(1)$ process

$$z_t = \rho_z z_{t-1} + \varepsilon_t^z$$

with $\rho_z \in [0, 1)$.

As shown in chapter 2, the resulting log-linear versions of the above optimality conditions take the form

$$w_t - p_t = \sigma c_t + \varphi n_t \tag{2}$$

$$c_t = E_t\{c_{t+1}\} - \frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} - \rho) + \frac{1}{\sigma}(1 - \rho_z) z_t \tag{3}$$

where $i_t \equiv -\log Q_t$ is the short-term nominal interest rate and $\rho \equiv -\log \beta$ is the discount rate, and where lowercase letters are used to denote the logs of the original variables. As before, the previous conditions are supplemented, when necessary, with an ad hoc log-linear money demand equation of the form

$$m_t - p_t = y_t - \eta i_t \tag{4}$$

where m_t denotes (log) nominal money holdings.

3.2 FIRMS

Assume a continuum of firms indexed by $i \in [0, 1]$. Each firm produces a differentiated good, but they all use an identical technology, represented by the production function

$$Y_t(i) = A_t N_t(i)^{1-\alpha} \tag{5}$$

where A_t represents the level of technology, assumed to be common to all firms and to evolve exogenously over time according to the process

$$a_t = \rho_a a_{t-1} + \varepsilon_t^a \tag{6}$$

with $\rho_a \in [0, 1]$.

All firms face an identical isoelastic demand schedule given by (1), and take the aggregate price level P_t and aggregate consumption index C_t as given.

Following the formalism proposed in Calvo (1983), each firm may reset its price only with probability $1-\theta$ in any given period, independent of the time elapsed since it last adjusted its price. Thus, in each period a measure $1-\theta$ of producers reset their prices, while a fraction θ keep their prices unchanged. As a result, the average duration of a price is given by $\frac{1}{1-\theta}$. In this context, θ becomes a natural index of price stickiness.

3.2.1 Aggregate Price Dynamics

As shown in appendix 3.2, the above environment implies that the aggregate price dynamics are described by the equation

$$\Pi_t^{1-\epsilon} = \theta + (1-\theta)\left(\frac{P_t^*}{P_{t-1}}\right)^{1-\epsilon} \tag{7}$$

where $\Pi_t \equiv \frac{P_t}{P_{t-1}}$ is the gross rate of inflation between $t-1$ and t, and P_t^* is the price set in period t by firms reoptimizing their price in that period. Notice that, as shown below, all firms resetting their price in any given period will choose the same price because they face an identical problem. It follows from (7) that in a steady state with zero inflation ($\Pi = 1$), $P_t^* = P_{t-1} = P_t$ for all t. Furthermore, a log-linear approximation to the aggregate price index around that steady state yields

$$\pi_t = (1-\theta)(p_t^* - p_{t-1}) \tag{8}$$

or, equivalently, after rearranging terms:

$$p_t = \theta p_{t-1} + (1-\theta) p_t^*$$

i.e., the current price level is a weighted average of last period's price level and the newly set price, all in logs, with the weights given by the fraction of firms that do not and do adjust prices, respectively.

The previous equations make clear that, in the present setup, variations in the price level result from the fact that firms reoptimizing in any given period choose a price that differs from the economy's average price in the previous period. Hence, and in order to understand the evolution of inflation over time, one needs to analyze the factors underlying firms' price setting decisions, a question which is discussed next.

3.2.2 Optimal Price Setting

A firm reoptimizing in period t will choose the price P_t^* that maximizes the current market value of the profits generated while that price remains effective. As shown in appendix 3.3, this corresponds to solving the following problem:

$$\max_{P_t^*} \sum_{k=0}^{\infty} \theta^k E_t \left\{ \Lambda_{t,t+k} (1/P_{t+k}) \left(P_t^* Y_{t+k|t} - \mathcal{C}_{t+k}(Y_{t+k|t}) \right) \right\}$$

subject to the sequence of demand constraints

$$Y_{t+k|t} = \left(\frac{P_t^*}{P_{t+k}} \right)^{-\epsilon} C_{t+k} \tag{9}$$

for $k = 0, 1, 2, \ldots$ where $\Lambda_{t,t+k} \equiv \beta^k U_{c,t+k}/U_{c,t}$ is the stochastic discount factor, $\mathcal{C}_t(\cdot)$ is the (nominal) cost function, and $Y_{t+k|t}$ denotes output in period $t+k$ for a firm that last reset its price in period t. Note that it is implicitly assumed that the firm always meets the demand for its good at the current price. That assumption, which is maintained throughout the analysis below, requires, in turn, that the average price markup is sufficiently large and/or that the shifts in demand resulting from a variety of shocks are not too large.

The optimality condition associated with the problem above takes the form

$$\sum_{k=0}^{\infty} \theta^k E_t \left\{ \Lambda_{t,t+k} Y_{t+k|t} (1/P_{t+k}) \left(P_t^* - \mathcal{M} \Psi_{t+k|t} \right) \right\} = 0 \tag{10}$$

where $\Psi_{t+k|t} \equiv \mathcal{C}'_{t+k}(Y_{t+k|t})$ denotes the (nominal) marginal cost in period $t+k$ for a firm which last reset its price in period t and $\mathcal{M} \equiv \frac{\epsilon}{\epsilon-1}$.

Note that in the limiting case of no price rigidities ($\theta = 0$), the previous condition collapses to the familiar optimal price setting condition for a monopolistic competitor under flexible prices

$$P_t^* = \mathcal{M}\Psi_{t|t}$$

which allows us to interpret $\mathcal{M}$ as the optimal markup in the absence of constraints on the frequency of price adjustment. Henceforth, $\mathcal{M}$ is often referred to as the “desired,” “natural,” or “frictionless” markup. That markup is constant given the assumption of a time-invariant elasticity of substitution ϵ.

Next, the optimal price setting condition (10) is log-linearized around the perfect foresight zero inflation steady state. Note that the latter is characterized by $\Lambda_{t,t+k} = \beta^k$ and $P_t^*/P_{t-k} = P_t/P_{t-k} = 1$. Thus, all firms produce the same quantity of output, and face the same marginal cost, that is, $Y_{t+k|t} = Y$, and $\Psi_{t+k|t} = \Psi_{t+k} = \Psi_t$ for all t and $k = 0, 1, 2, 3, \ldots$ Thus, (10) implies that $P_t = \mathcal{M}\Psi_t$ for all t in that steady state, i.e., actual markups, common to all firms, coincide with the frictionless markup.

A first-order Taylor expansion of (10) around the zero inflation steady state yields, after some manipulation,

$$p_t^* = \mu + (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\{\psi_{t+k|t}\} \tag{11}$$

where $\psi_{t+k|t} \equiv \log \Psi_{t+k|t}$ is the (log) marginal cost and $\mu \equiv \log \mathcal{M}$ is the log of the desired gross markup (which, for $\mathcal{M}$ close to one, is approximately equal to the net markup $\mathcal{M}-1$). Hence, and to the extent that prices are sticky ($\theta > 0$), firms set prices in a forward-looking way. In particular, they choose a price that corresponds to their desired markup over a weighted average of their current and expected (nominal) marginal costs, with the weights being proportional to the probability of the price remaining effective at each horizon, θ^k, times the cumulative discount factor, β^k.

3.3 Equilibrium

Market clearing in the goods market requires that the quantity produced of each good matches the quantity demanded. In the stylized model analyzed here consumption is the only source of demand for goods.

Thus, in equilibrium,

$$Y_t(i) = C_t(i)$$

for all $i \in [0, 1]$ and all t.

Letting aggregate output be defined as $Y_t \equiv \left(\int_0^1 Y_t(i)^{1-\frac{1}{\epsilon}} di\right)^{\frac{\epsilon}{\epsilon-1}}$ it follows that

$$Y_t = C_t \tag{12}$$

must hold for all t.

Aggregate consumption C_t (which corresponds to aggregate demand in the present model) is in turn determined by the consumer's Euler equation (3). Combining the latter with (12) yields an expression for current (log) output as a function of the latter's expected one-period-ahead value, the real interest rate, and the demand shifter:

$$y_t = E_t\{y_{t+1}\} - \frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} - \rho) + \frac{1}{\sigma}(1 - \rho_z)z_t \tag{13}$$

Note that (13) can be solved forward to yield

$$y_t = \frac{1}{\sigma}z_t - \frac{1}{\sigma}\sum_{k=0}^{\infty} E_t\{i_{t+k} - E_t\{\pi_{t+1+k}\} - \rho\} + \lim_{T\to\infty} E_t\{y_{t+T}\}$$

Thus, an exogenous shock will impact output only to the extent that it meets one or more of the following conditions: (i) it shifts the preference parameter z_t, (ii) it has a permanent effect on the level of output, or (iii) it leads to a deviation of the real interest rate from the discount rate, current or anticipated.

Aggregate employment is given by the sum of employment across firms:

$$N_t = \int_0^1 N_t(i)di$$

Using (5)

$$N_t = \int_0^1 \left(\frac{Y_t(i)}{A_t} \right)^{\frac{1}{1-\alpha}} di$$
$$= \left(\frac{Y_t}{A_t} \right)^{\frac{1}{1-\alpha}} \int_0^1 \left(\frac{P_t(i)}{P_t} \right)^{-\frac{\epsilon}{1-\alpha}} di$$

where the second equality follows from (1) and the goods market clearing condition. Taking logs,

$$(1-\alpha)n_t = y_t - a_t + d_t$$

where $d_t \equiv (1-\alpha)\log \int_0^1 (P_t(i)/P_t)^{-\frac{\epsilon}{1-\alpha}}\, di$ is a measure of price (and, hence, output) dispersion across firms. In appendix 3.4 it is shown that, in a neighborhood of the zero inflation steady state, d_t is equal to zero up to a first-order approximation. Hence, one can write the following approximate relation

$$n_t = \frac{1}{1-\alpha}(y_t - a_t) \tag{14}$$

which can be thought of as determining aggregate employment, given aggregate output and technology.

Note that the (log) marginal cost for an individual firm that last set its price in period t is given by:[3]

$$\psi_{t+k|t} = w_{t+k} - mpn_{t+k|t}$$
$$= w_{t+k} - (a_{t+k} - \alpha n_{t+k|t} + \log(1-\alpha))$$

where $mpn_{t+k|t}$ and $n_{t+k|t}$ respectively denote the (log) marginal product of labor and (log) employment in period $t+k$ for a firm that last reset its price in period t.

[3] Marginal cost equals the labor needed to produce an additional unit of output, $1/MPN_{t+k|t}$, times the cost of each unit of that labor, i.e., the wage W_{t+k}. Formally, $\Psi_{t+k|t} = W_{t+k}/MPN_{t+k|t}$.

Letting $\psi_t \equiv \int_0^1 \psi_t(i)di$ represent the (log) average marginal cost, it follows that

$$\psi_t = (1-\theta)\sum_{k=0}^{\infty}\theta^k \psi_{t|t-k}$$
$$= w_t - (a_t - \alpha n_t + \log(1-\alpha))$$

where the second equality makes use of the relation $n_t = \int_0^1 n_t(i)di$, which holds up to a first order approximation about a symmetric equilibrium.

Thus the following relation holds between firm-specific and economy-wide marginal costs:

$$\begin{aligned}\psi_{t+k|t} &= \psi_{t+k} + \alpha(n_{t+k|t} - n_{t+k}) \\ &= \psi_{t+k} + \frac{\alpha}{1-\alpha}(y_{t+k|t} - y_{t+k}) \\ &= \psi_{t+k} - \frac{\alpha\epsilon}{1-\alpha}(p_t^* - p_{t+k}) \qquad (15)\end{aligned}$$

where the second equality follows from (14) and the third equality results from combining demand schedule (9) and goods market clearing condition (12) (after taking logs). Notice that under the assumption of constant returns to scale ($\alpha = 0$), marginal cost is independent of the level of production and, hence, it is common across firms, that is, $\psi_{t+k|t} = \psi_{t+k}$.

Substituting (15) into (11) and rearranging terms yields

$$p_t^* = (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\left\{p_{t+k} - \Theta\widehat{\mu}_{t+k}\right\}$$

where $\widehat{\mu}_t \equiv \mu_t - \mu$ is the deviation between the average and desired markups, with $\mu_t \equiv p_t - \psi_t$ and $\Theta \equiv \frac{1-\alpha}{1-\alpha+\alpha\epsilon} \in (0, 1]$.

The above expression for p_t^* can be rewritten as a recursive equation:

$$p_t^* = \beta\theta E_t\{p_{t+1}^*\} + (1-\beta\theta)(p_t - \Theta\widehat{\mu}_t) \qquad (16)$$

Finally, combining (8) and (16) yields the inflation equation

$$\pi_t = \beta E_t\{\pi_{t+1}\} - \lambda\widehat{\mu}_t \qquad (17)$$

where

$$\lambda \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}\,\Theta$$

is strictly decreasing in the index of price stickiness θ, in the measure of decreasing returns α, and in the demand elasticity ϵ.[4] Note that $\Theta = 1$ in the particular case of constant returns to labor ($\alpha = 0$).

Solving (17) forward, inflation is expressed as the discounted sum of current and expected future deviations of average markups from their desired levels:

$$\pi_t = -\lambda \sum_{k=0}^{\infty} \beta^k E_t\{\widehat{\mu}_{t+k}\}$$

Thus, inflation will be positive when firms expect average markups to be below their desired level μ, for in that case firms that have the opportunity to reset prices will, on average, choose a price above the economy's average price level in order to realign their markup closer to its desired level.

It is worth emphasizing here that the mechanism underlying fluctuations in the aggregate price level and inflation laid out above has little in common with the one at work in the classical monetary economy. Thus, in the present model, inflation results from the aggregate consequences of purposeful price-setting decisions by firms, which adjust their prices in light of current and anticipated cost conditions. By contrast, in the classical monetary economy analyzed in chapter 2, inflation is a consequence of the changes in the aggregate price level that, given the monetary policy rule in place, are consistent with an equilibrium allocation that is independent of the evolution of nominal variables, with no account given of the mechanism (other than an "invisible hand") that will bring about those price level changes.

Next, a relation is derived between the economy's average price markup and aggregate output. Notice that independently of the nature

[4] An alternative representation of the same equation in terms of the real marginal cost is often found in the literature (see, e.g., Galí and Gertler (1999)). Defining the (log) average real marginal cost as $mc_t = \psi_t - p_t$ and noting that $mc_t = -\mu_t$ we can write (17) as:

$$\pi_t = \beta E_t\{\pi_{t+1}\} + \lambda \widehat{mc}_t$$

of price setting, the average price markup can be expressed as

$$\begin{aligned}\mu_t &= p_t - \psi_t \\ &= -(w_t - p_t) + (a_t - \alpha n_t + \log(1-\alpha)) \\ &= -(\sigma y_t + \varphi n_t) + (a_t - \alpha n_t + \log(1-\alpha)) \\ &= -\left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right) y_t + \left(\frac{1+\varphi}{1-\alpha}\right) a_t + \log(1-\alpha) \qquad (18)\end{aligned}$$

where derivation of the second and third equalities make use of the household's optimality condition (2) and the relation between aggregate employment and output (14).

As discussed above, under *flexible prices* ($\theta = 0$) the average markup is constant and equal to the desired one, μ. Defining the *natural level of output*, denoted by y_t^n, as the equilibrium level of output under flexible prices, and evaluating (18) at the flexible price equilibrium yields:

$$\mu = -\left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right) y_t^n + \left(\frac{1+\varphi}{1-\alpha}\right) a_t + \log(1-\alpha) \qquad (19)$$

thus implying

$$y_t^n = \psi_{ya} a_t + \psi_y \qquad (20)$$

where $\psi_y \equiv -\frac{(1-\alpha)(\mu-\log(1-\alpha))}{\sigma(1-\alpha)+\varphi+\alpha} > 0$ and $\psi_{ya} \equiv \frac{1+\varphi}{\sigma(1-\alpha)+\varphi+\alpha}$. Notice that when $\mu = 0$ (perfect competition) the natural level of output corresponds to the equilibrium level of output in the baseline classical monetary economy analyzed in chapter 2. The presence of market power by firms has the effect of lowering the average level of output, without affecting its response to changes in technology. Note also that the natural level of output is independent of monetary policy, and invariant to preference shocks $\{z_t\}$, two properties that also characterized the equilibrium of the baseline classical monetary model of chapter 2.

Subtracting (19) from (18) yields

$$\widehat{\mu}_t = -\left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)(y_t - y_t^n) \qquad (21)$$

that is, the markup gap is proportional to the log deviation of output from its flexible price counterpart. Following convention, the latter deviation is referred to as the *output gap*, and denoted by $\widetilde{y}_t \equiv y_t - y_t^n$.

By combining (21) with (17) one can obtain an equation relating inflation to its one period ahead forecast and the output gap

$$\pi_t = \beta E_t\{\pi_{t+1}\} + \kappa \widetilde{y}_t \tag{22}$$

where $\kappa \equiv \lambda\left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)$. Equation (22) is often referred to as the *New Keynesian Phillips curve* (or NKPC, for short), and constitutes one of the key building blocks of the basic New Keynesian model.

The second key equation describing the equilibrium of the New Keynesian model can be obtained by rewriting (13) in terms of the output gap as

$$\widetilde{y}_t = -\frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} - r_t^n) + E_t\{\widetilde{y}_{t+1}\} \tag{23}$$

where r_t^n is the *natural rate of interest*, that is, the real interest rate that would prevail in the flexible price equilibrium. Under the assumed processes for $\{z_t\}$ and $\{a_t\}$ it is given by

$$r_t^n = \rho - \sigma(1-\rho_a)\psi_{ya} a_t + (1-\rho_z) z_t \tag{24}$$

Henceforth (23) is referred to as the *dynamic IS equation* (or DIS, for short). Under the assumption that the effects of nominal rigidities vanish asymptotically, $\lim_{T\longrightarrow\infty} E_t\{\widetilde{y}_{t+T}\} = 0$. In that case one can solve equation (23) forward to yield the expression

$$\widetilde{y}_t = -\frac{1}{\sigma}\sum_{k=0}^{\infty} E_t\{r_{t+k} - r_{t+k}^n\} \tag{25}$$

where $r_t \equiv i_t - E_t\{\pi_{t+1}\}$ is the expected real return on a one period bond (i.e., the real interest rate). The previous expression emphasizes the fact that the output gap is inversely related to the sum of current and anticipated deviations between the real interest rate and its natural counterpart.

Equations (22) and (23), together with an equilibrium process for the natural rate r_t^n (given by (24) under the assumptions made above), constitute the *nonpolicy* block of the basic New Keynesian model. That block has a simple structure: The NKPC determines inflation given a path for the output gap, whereas the DIS equation determines the output gap given a path for the (exogenous) natural rate and the actual real rate, with the latter being a function of the nominal rate and (expected) inflation.

In order to close the model, those two equations must be supplemented with one or more equations determining how the nominal interest rate i_t evolves over time, that is, with a description of how monetary policy is conducted. Thus, and in contrast with the baseline classical model analyzed in chapter 2, the presence of sticky prices implies that the equilibrium path of real variables *cannot* be determined independently of monetary policy. In other words, monetary policy is non-neutral.

In order to illustrate the workings of the basic model just developed, two alternative specifications of monetary policy are considered and their equilibrium implications are analyzed.

3.4 Equilibrium Dynamics under Alternative Monetary Policy Rules

3.4.1 *Equilibrium under a Simple Interest Rate Rule*

The equilibrium is first analyzed under a simple interest rate rule of the form

$$i_t = \rho + \phi_\pi \pi_t + \phi_y \widehat{y}_t + v_t \tag{26}$$

where $\widehat{y}_t \equiv y_t - y$ denotes the deviation of output from its steady state value, and v_t is an exogenous monetary policy shock that evolves according to the $AR(1)$ process:

$$v_t = \rho_v v_{t-1} + \varepsilon_t^v$$

where $\rho_v \in [0, 1)$. A positive (negative) realization of ε_t^v should be interpreted as a contractionary (expansionary) monetary policy shock, leading to a rise (decline) in the nominal interest rate, *given* inflation and output.

Coefficients ϕ_π and ϕ_y are chosen by the monetary authority, and assumed to be non-negative. Note that the choice of the intercept ρ makes the rule consistent with a zero inflation steady state. The specification of interest rate rule (26) is broadly consistent with the rule proposed by Taylor (1993, 1999) as a good description of Fed monetary policy under the tenure of Chairman Greenspan, and widely known as the "Taylor rule."[5]

[5] Taylor's original rule takes the form:

$$i_t^F = 4 + 1.5(\overline{\pi}_t - 2) + 0.5(y_t - y_t^*)$$

where i_t^F is the (annualized) federal funds rate, $\overline{\pi}_t$ is year-on-year inflation, and y_t^* is trend (log) GDP, fitted by means of an estimated linear function of time. Note that the Taylor

Note that (26) can be rewritten in terms of the output gap as follows:

$$i_t = \rho + \phi_\pi \pi_t + \phi_y \widetilde{y}_t + \phi_y \widehat{y}_t^n + v_t$$

where $\widehat{y}_t^n \equiv y_t^n - y$.

After combining (22), (23), and (26), the equilibrium conditions can be represented by means of the following system of difference equations,

$$\begin{bmatrix} \widetilde{y}_t \\ \pi_t \end{bmatrix} = \mathbf{A}_T \begin{bmatrix} E_t\{\widetilde{y}_{t+1}\} \\ E_t\{\pi_{t+1}\} \end{bmatrix} + \mathbf{B}_T u_t \tag{27}$$

where

$$\begin{aligned} u_t &\equiv \widehat{r}_t^n - \phi_y \widehat{y}_t^n - v_t \\ &= -\psi_{ya}(\phi_y + \sigma(1-\rho_a))a_t + (1-\rho_z)z_t - v_t \end{aligned}$$

and

$$\mathbf{A}_T \equiv \Omega \begin{bmatrix} \sigma & 1 - \beta\phi_\pi \\ \sigma\kappa & \kappa + \beta(\sigma + \phi_y) \end{bmatrix} \quad ; \quad \mathbf{B}_T \equiv \Omega \begin{bmatrix} 1 \\ \kappa \end{bmatrix}$$

with $\Omega \equiv \frac{1}{\sigma + \phi_y + \kappa\phi_\pi}$.

Given that both the output gap and inflation are non-predetermined variables, the solution to (27) is locally unique, if and only if, $\mathbf{A}_T$ has both eigenvalues within the unit circle.[6] Under the assumption of non-negative coefficients (ϕ_π, ϕ_y) it can be shown that a necessary and sufficient condition for uniqueness is given by [7]

$$\kappa(\phi_\pi - 1) + (1-\beta)\phi_y > 0 \tag{28}$$

which is assumed to hold, unless stated otherwise. An economic interpretation to the previous condition will be offered in chapter 4.

The method of undetermined coefficients can be used in order to solve for the economy's response to different exogenous shocks, under the assumption of a unique equilibrium. Assume that u_t follows an $AR(1)$

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rule above implicitly assumed an 2 percent annual inflation target and a 2 percent steady state real interest rate.

[6] See, e.g., Blanchard and Kahn (1980).

[7] See Bullard and Mitra (2002) for a proof.

process, with autoregressive coefficient $\rho_u \in [0,1)$ (this assumption is relaxed below, when the different shocks are analyzed separately). The stationary solution to (27) is conjectured to be of the form $\widetilde{y}_t = \psi_y u_t$ and $\pi_t = \psi_\pi u_t$. Imposing the conjectured relations into (27) allows one to solve for the undetermined coefficients ψ_y and ψ_π:

$$\psi_y = (1-\beta\rho_u)\Lambda_u \tag{29}$$

$$\psi_\pi = \kappa\Lambda_u \tag{30}$$

where $\Lambda_u \equiv \frac{1}{(1-\beta\rho_u)[\sigma(1-\rho_u)+\phi_y]+\kappa(\phi_\pi-\rho_u)} > 0$.

The following sections make use of the previous result to analyze the economy's equilibrium response to different types of exogenous shocks, when the central bank follows the interest rate rule (26).

3.4.1.1 THE EFFECTS OF A MONETARY POLICY SHOCK

The focus is here on the economy's response to a shift in v_t, with $a_t = z_t = 0$ for all t. Note first that neither the natural rate of interest, r_t^n, nor the natural level of output, y_t^n, depends on the monetary policy shock, v_t. Using (29) and (30) and the fact that $\frac{\partial u_t}{\partial v_t} = -1$, it follows that

$$y_t = \widetilde{y}_t = -(1-\beta\rho_v)\Lambda_v v_t$$

and

$$\pi_t = -\kappa\Lambda_v v_t$$

where Λ_v takes the same form as Λ_u above, with ρ_u replaced by ρ_v (a similar rule applies in the definition of Λ_a below). Thus, both output and inflation decline in response to an exogenous increase in the interest rate. The same is true for employment, given (14), and the real wage, given (2).

One can use (23) to obtain an expression for the real interest rate as a function of the monetary policy shock

$$\widehat{r}_t = \sigma(1-\rho_v)(1-\beta\rho_v)\Lambda_v v_t$$

which is thus shown to increase unambiguously in response to an exogenous increase in the nominal rate.

The response of the nominal interest rate combines both the direct effect of v_t and the variation induced by lower output and inflation. It is given by

$$\widehat{i}_t = \widehat{r}_t + E_t\{\pi_{t+1}\} = [\sigma(1-\rho_v)(1-\beta\rho_v) - \rho_v\kappa]\Lambda_v v_t$$

Note that if the persistence of the monetary policy shock ρ_v is sufficiently high, the nominal rate will decline in response to a rise in v_t. In that case the downward adjustment in the nominal rate induced by the decline in inflation and output more than offsets the direct effect of a higher v_t. But even in that case, the increase in v_t has a contractionary effect on output, because the latter is inversely related to the *real* rate, which goes up unambiguously, as seen above.

Finally, one can use (4) to determine the change in the money supply required to bring about the desired change in the interest rate:

$$\begin{aligned} m_t &= p_t + y_t - \eta i_t \\ &= p_{t-1} - \Lambda_v[(1-\beta\rho_v)(1+\eta\sigma(1-\rho_v)) + (1-\eta\rho_v)\kappa]v_t \end{aligned}$$

The sign of the change in the money supply that is needed to support the exogenous policy intervention is, in principle, ambiguous. Note, however, that $di_t/dv_t > 0$ is a *sufficient* condition for a contraction in the money supply in response to an exogenous monetary policy tightening. It is also a sufficient condition for the presence of a liquidity effect (i.e., a negative short-run comovement of the nominal rate and the money supply in response to an exogenous monetary policy shock).

The previous analysis can be used to quantify the effects of a monetary policy shock, given numerical values for the model's parameters. A baseline calibration of the model is presented next.

All quantitative results throughout the book are based on calibrations that take each period in the model to correspond to a quarter. In the baseline calibration of the model's preference parameters it is assumed $\beta = 0.99$, which implies a steady state real (annualized) return on financial assets of about 4 percent. It is also assumed $\sigma = 1$ (log utility) and $\varphi = 5$ (which implies a Frisch elasticity of labor supply of 0.2), $\alpha = 1/4$, and $\epsilon = 9$ (implying $\mathcal{M} = 1.125$, i.e., a steady state markup of a 12.5 percent). These are values broadly similar to those found in the business cycle literature. The interest semielasticity of money demand, η, is set to equal 4.[8] In addition it is assumed $\theta = 3/4$, which implies

[8] The calibration of η is based on the estimates of an OLS regression of (log) M2 inverse velocity on the 3 month Treasury Bill rate (quarterly rate, per unit), using quarterly data over the period 1960:1–1988:1. The focus is on that period because it is characterized by a highly stable relationship between velocity and the nominal rate, as implied by the model.

an average price duration of four quarters, a value consistent with much of the empirical evidence.[9] As to the interest rate rule coefficients, it is assumed $\phi_\pi = 1.5$ and $\phi_y = 0.5/4$, in a way consistent with Taylor's original rule.[10] Finally, $\rho_v = 0.5$, a setting associated with a moderately persistent shock.

Figure 3.1 illustrates the dynamic effects of a contractionary monetary policy shock on a number of macro variables. The shock takes the form of an increase of 25 basis points in ε_t^v, which, in the absence of a further change induced by the response of inflation or the output gap, would imply an increase of 100 basis points in the annualized nominal rate, on impact. The responses of inflation and the two interest rates shown in figure 3.1 (and in all subsequent figures) are expressed in annual terms, that is, they are obtained by multiplying by 4 the responses of π_t, i_t, and r_t implied by the quarterly model.

In a way consistent with the analytical results above it is seen that the policy shock generates a decrease in inflation, output (whose response corresponds to that of the output gap, because the natural level of output is not affected by the monetary policy shock), employment, and the real wage. Note that under the baseline calibration the nominal rate goes up, though by less than its exogenous component—as a result of the downward adjustment induced by the decline in inflation and output. Note also that the increase in the real rate is larger than that of the nominal rate as a result of the decrease in expected inflation. That persistent increase in the real rate is the factor behind the decline of consumption and output, as implied by (25), and given the constancy of the natural rate.

In order to bring about the observed rise in the nominal interest rate, the central bank must engineer a large short-run reduction in the money supply. The calibrated model thus displays a liquidity effect. Note also that the negative response of the price level to the tightening of monetary policy builds up gradually, and is much more muted in the short run than that of the money supply, which instead overshoots its new permanent plateau. In the long run, however, the price level and the money experience a permanent decline of identical size, since real balances revert back to their initial level.

Overall, the dynamic responses to a monetary policy shock shown in figure 3.1 are similar, at least in a qualitative sense, to those estimated

[9] See, in particular, the estimates in Galí, Gertler, and López-Salido (2001) and Sbordone (2002), based on aggregate data and the discussion of the micro evidence in chapter 1.

[10] See, e.g., Taylor (1999). Note that empirical interest rate rules are generally estimated using inflation and interest rate data expressed in annual rates. Conversion to quarterly rates requires that the output gap coefficient be divided by 4.

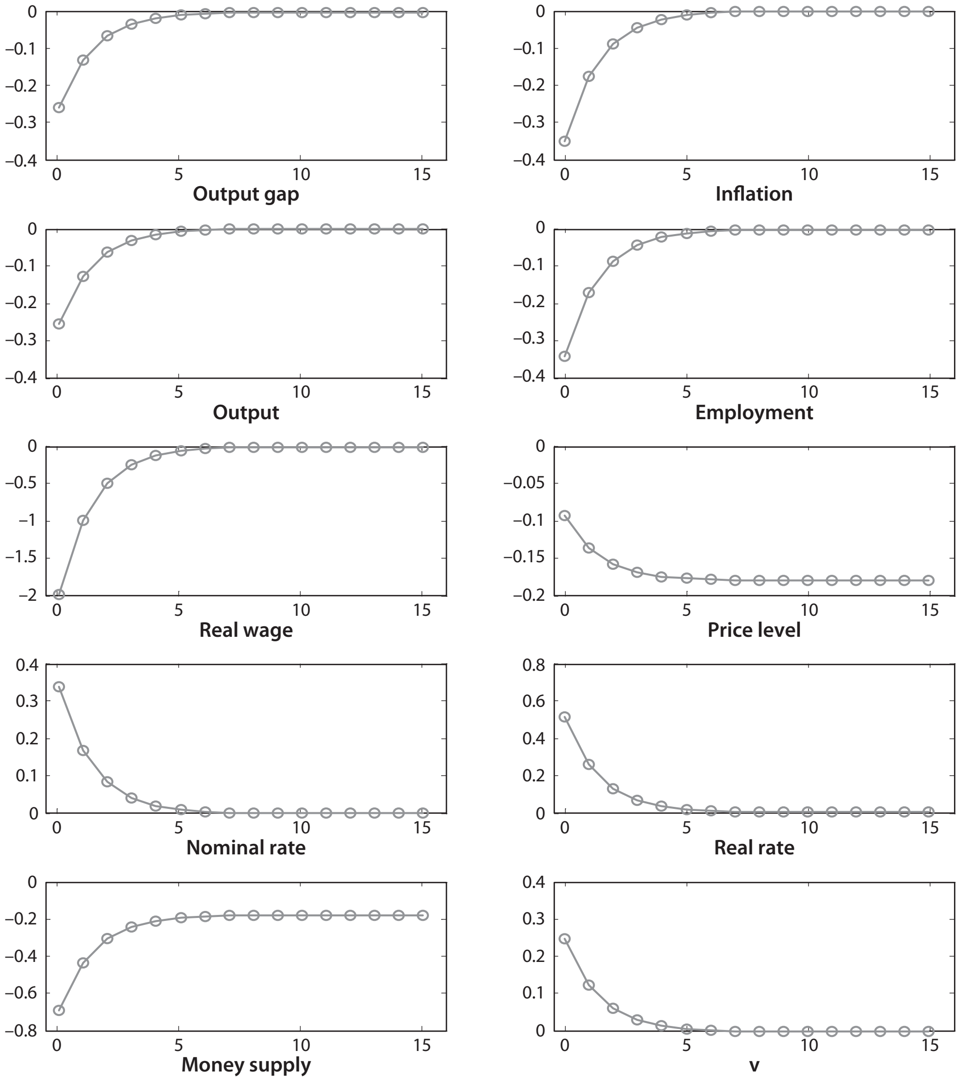

Figure 3.1. Dynamic Responses to a Monetary Policy Shock: Interest Rate Rule.

using structural vector autoregressive (VAR) methods, as described in chapter 1. Nevertheless, and as emphasized in Christiano, Eichenbaum, and Evans (2005), among others, matching some of the quantitative features of the empirical impulse responses requires that the basic New Keynesian model be enriched in a variety of dimensions.

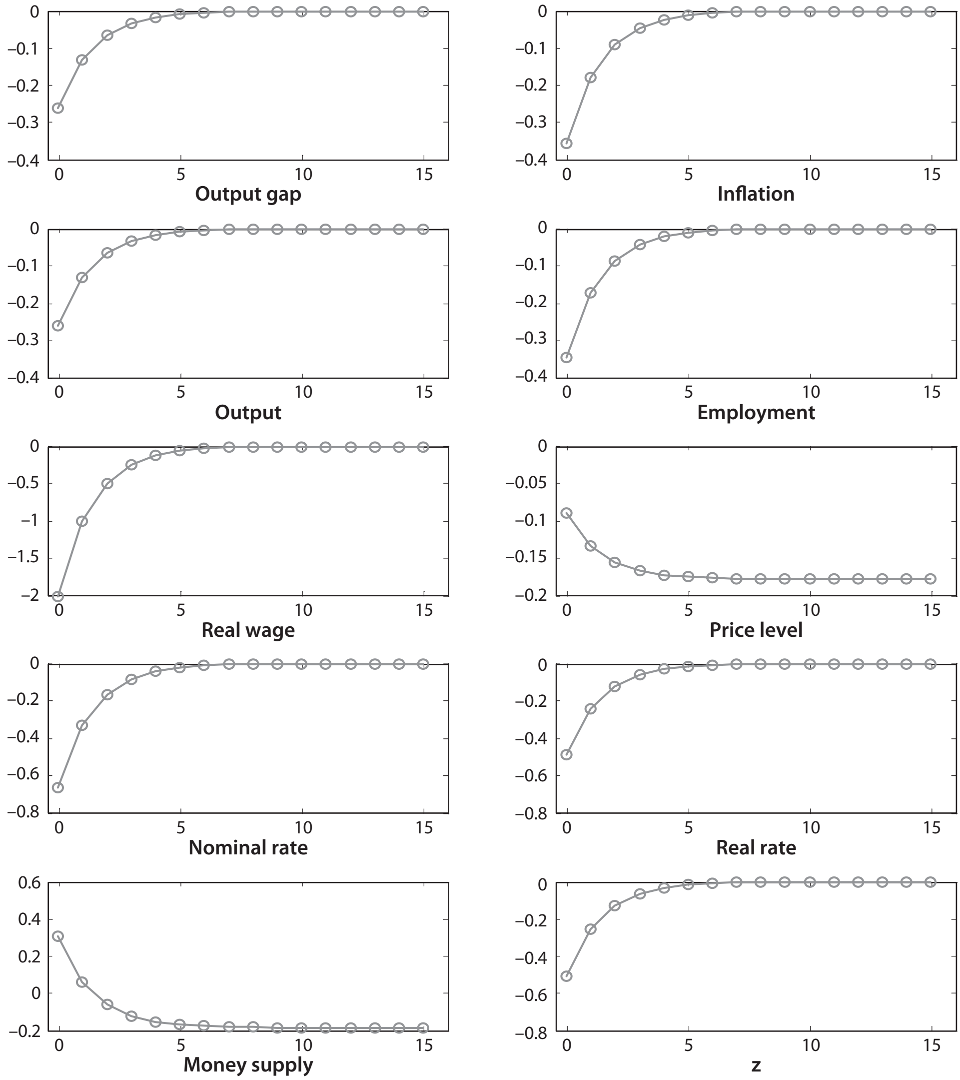

Figure 3.2. Dynamic Responses to a Discount Rate Shock: Interest Rate Rule.

3.4.1.2 THE EFFECTS OF A DISCOUNT RATE SHOCK

Figure 3.2 displays the dynamic responses of different macro variables to a discount factor shock, in the form of a decrease in z_t. Like in the case of the monetary policy shock, the corresponding autoregressive coefficient is set to $\rho_z = 0.5$. The size of the initial shock ε_t^z is normalized to -0.5

percentage points, so that $\frac{\partial r_t^n}{\partial \varepsilon_t^z} = (1-\rho_z)\varepsilon_t^z = -0.25$, implying an increase on impact of one percentage point in the (annualized) natural rate of interest. A decline in z_t should be interpreted as a reduction in the weight that households give to current utility relative to future utility. That shift in preferences induces a decline in consumption and hence in aggregate demand.

As shown in figure 3.2, the decline in z_t leads to a contraction in output, employment, inflation, and the real wage. In fact, and given the normalization of the size of the shock, the response of those variables is identical to that describing the effects of a monetary policy tightening, as shown in figure 3.1. Formally, the reason for this is that $(1-\rho_z)z_t$ and v_t enter symmetrically (though with opposite sign) in the system (27) describing the equilibrium dynamics for inflation and the output gap under the interest rate rule considered here. Given that z_t doesn't affect the natural level of output either, the effects on output, employment, and the real wage are also identical.

Discount rate shocks and monetary policy shocks differ in two respects, however: (i) shifts in the discount factor have an effect on the natural rate of interest r_t^n, whereas monetary policy shocks don't (see (24)), and (ii) monetary policy shocks lead to changes in the nominal interest rate, for any given levels of inflation and output, whereas discount factor shocks don't (see (26)). As a result, the effects of the two shocks on the nominal and real interest rates are very different, as a comparison of figures 3.1 and 3.2 makes clear. In particular, the nominal rate falls in response to a negative discount factor shock, due to the decline in inflation and output. The decline in the nominal rate drags the real rate down, given the sluggish change in expected inflation. The decline in the real interest rate, however, is not sufficient to prevent the overall contraction in economic activity. Note also that the different response of the nominal rate is associated with a very different pattern in the response of the money supply, which now rises in the short run (due to the dominant effect of a lower nominal rate), before it declines and settles at a permanently lower level (the same as the price level, which in turn corresponds to that implied by the monetary policy shock).

3.4.1.3 THE EFFECTS OF A TECHNOLOGY SHOCK

Next, the effects of exogenous changes in a_t are considered. Given (27) and the analysis of its solution above, it follows that

$$\widetilde{y}_t = -\psi_{ya}(\phi_y + \sigma(1-\rho_a))(1-\beta\rho_a)\Lambda_a a_t$$

and

$$\pi_t = -\psi_{ya}(\phi_y + \sigma(1-\rho_a))\kappa\Lambda_a a_t$$

where Λ_a is equal to Λ_u above, with ρ_a replacing ρ_u. Thus, a positive technology shock has an unambiguous negative effect on the output gap and inflation.

Output is now given by

$$\begin{aligned} y_t &= \widetilde{y}_t + y_t^n \\ &= \psi_{ya}\kappa(\phi_\pi - \rho_a)\Lambda_a a_t \end{aligned}$$

It is easy to check that $\kappa(\phi_\pi - \rho_a)\Lambda_a \in (0,1)$. Thus, a positive technology shock always increases output, but by less than its natural counterpart, thus generating a negative output gap, as seen above. The response of employment is given by

$$\begin{aligned} n_t &= \left(\frac{1}{1-\alpha}\right)(y_t - a_t) \\ &= \left[\frac{(1-\sigma)\kappa(\phi_\pi - \rho_a)}{\sigma(1-\alpha)+\varphi+\alpha} - (\phi_y + \sigma(1-\rho_a))(1-\beta\rho_a)\right]\Lambda_a a_t \end{aligned}$$

Hence, the sign of the response of employment to a positive technology shock is in general ambiguous, depending on the configuration of parameter values, including the interest rate rule coefficients. Note, however, that $\sigma \geq 1$ is a sufficient condition for a technological improvement to cause a decline in employment. The same is true if ϕ_y is sufficiently large, that is, if the central bank puts enough weight on output stabilization, even in the face of fluctuations in natural output resulting from changes in technology. Such a response of employment is consistent with much of the recent empirical evidence on the effects of technology shocks.[11]

Figure 3.3 shows the dynamic response of several macro variables to a one percent increase in technology, under the baseline calibration described above, and the assumption of $\rho_a = 0.9$. In a way consistent with the analysis above, the output gap and inflation are shown to decline persistently in response to the positive technology shocks, even though output itself expands.[12] Under the baseline calibration employment

[11] See, e.g., Galí (1999) and Basu, Fernald, and Kimball (2004), among others. Galí and Rabanal (2004) provide a survey of that empirical evidence.

[12] Under the assumed calibration $\varphi_{ya} = 1$, implying that the response of the natural level of output corresponds to that of technology, a_t. Note that the response of output falls short of the technology response.

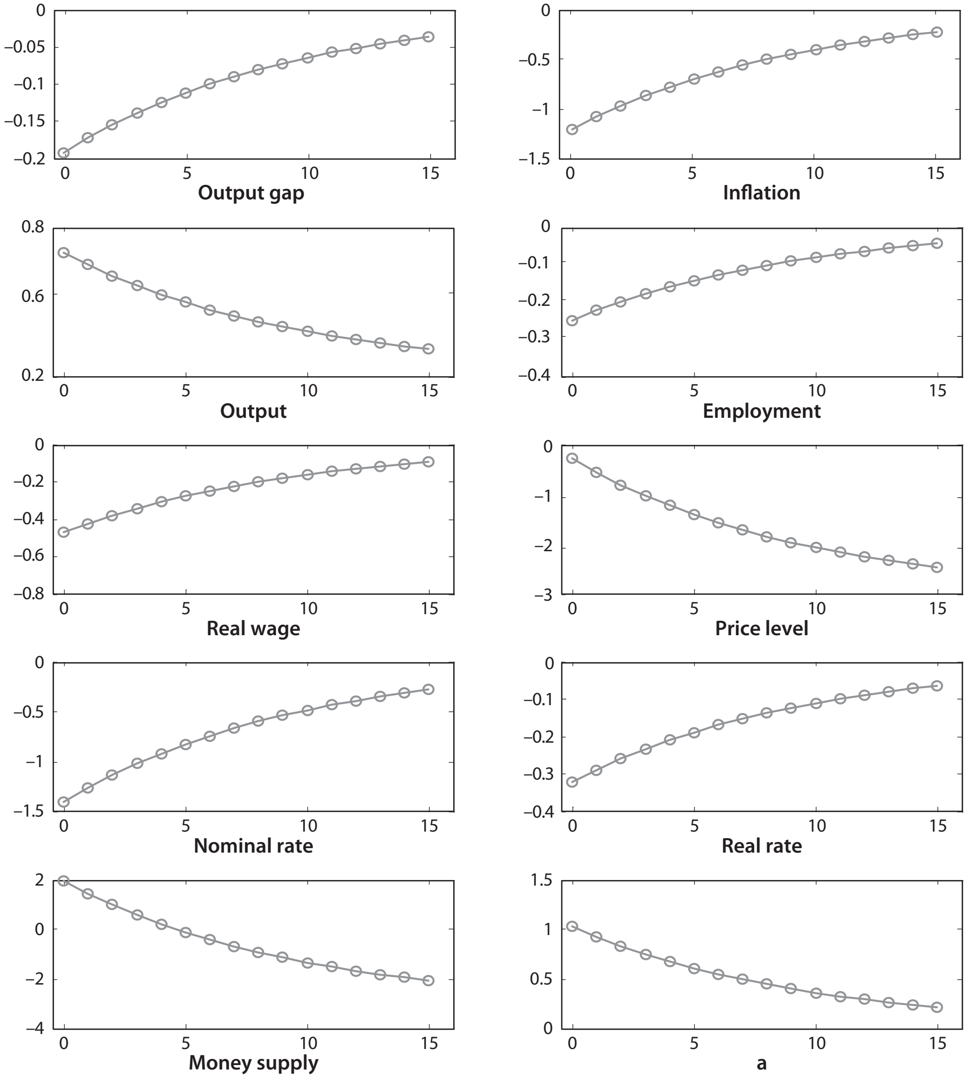

Figure 3.3. Dynamic Responses to a Technology Shock: Interest Rate Rule.

experiences a large and persistent decline. Thus, technology shocks generate a negative comovement between output and employment, which contrasts with the large and positive unconditional correlation between the cyclical components of output and employment (or work hours) observed in the data, but is consistent with the negative estimates of that correlation *conditional* on technology shocks.[13]

The nominal interest rate goes down substantially, due to the dominant influence of lower inflation in shaping the central bank's response. As a result, the real rate also declines, but given the response of the output gap (and (25)) we can infer that such decline is not sufficient to match that of its natural rate counterpart. Finally, note that the money supply increases in the short run in order to support the lower nominal rate and to accommodate the increase in money demand resulting from higher output. Over time, however, those two requirements vanish, while the permanently lower price level requires a permanent reduction in the quantity of money in circulation, as shown in figure 3.3.

3.4.2 Equilibrium under an Exogenous Money Supply

Next the equilibrium dynamics of the basic New Keynesian model are analyzed under the assumption of an exogenous path for the growth rate of the money supply, Δm_t. As discussed in chapter 2, the fact that the policy rule is specified in terms of the money supply requires that the money demand equation (4) is included in the set of conditions needed to solve for the model's equilibrium (i.e., beyond its usefulness to back out the quantity of money that supports a given interest rate policy, as in the previous subsection).

As a preliminary step, it is useful to rewrite money demand condition in terms of the output gap

$$l_t = \widetilde{y}_t - \eta i_t + y_t^n \tag{31}$$

where $l_t \equiv m_t - p_t$ denotes real balances. Substituting the latter equation into (23) to eliminate the interest rate yields

$$(1 + \sigma\eta)\,\widetilde{y}_t = \sigma\eta E_t\{\widetilde{y}_{t+1}\} + \widehat{l}_t + \eta E_t\{\pi_{t+1}\} + \eta\widehat{r}_t^n - \widehat{y}_t^n \tag{32}$$

where, as usual, a "^" symbol on top of a variable refers to deviations of the latter from steady state.

Note also that real balances are related to inflation and money growth through the identity

$$\widehat{l}_{t-1} = \widehat{l}_t + \pi_t - \Delta m_t \tag{33}$$

[13] See Galí (1999).

Hence, the equilibrium dynamics for real balances, output gap, and inflation are described by equations (32) and (33) together with the NKPC equation (22). They can be summarized compactly by the system

$$\mathbf{A}_{\mathbf{M},0}\begin{bmatrix}\widetilde{y}_t\\ \pi_t\\ \widehat{l}_{t-1}\end{bmatrix}=\mathbf{A}_{\mathbf{M},1}\begin{bmatrix}E_t\{\widetilde{y}_{t+1}\}\\ E_t\{\pi_{t+1}\}\\ \widehat{l}_t\end{bmatrix}+\mathbf{B}_{\mathbf{M}}\begin{bmatrix}\widehat{r}^n_t\\ \widehat{y}^n_t\\ \Delta m_t\end{bmatrix} \tag{34}$$

where

$$\mathbf{A}_{\mathbf{M},0}\equiv\begin{bmatrix}1+\sigma\eta & 0 & 0\\ -\kappa & 1 & 0\\ 0 & -1 & 1\end{bmatrix};\quad \mathbf{A}_{\mathbf{M},1}\equiv\begin{bmatrix}\sigma\eta & \eta & 1\\ 0 & \beta & 0\\ 0 & 0 & 1\end{bmatrix};\quad \mathbf{B}_{\mathbf{M}}\equiv\begin{bmatrix}\eta & -1 & 0\\ 0 & 0 & 0\\ 0 & 0 & -1\end{bmatrix}$$

and with $\widehat{r}^n_t$ and $\widehat{y}^n_t$ defined as in (24) and (20) as a function of the exogenous technology and discount factor shocks.

The system above has one predetermined variable (l_{t-1}) and two nonpredetermined variables ($\widetilde{y}_t$ and π_t). Accordingly, a stationary solution will exist and be unique, if and only if, $\mathbf{A}_{\mathbf{M}} \equiv \mathbf{A}_{\mathbf{M},0}^{-1}\mathbf{A}_{\mathbf{M},1}$ has two eigenvalues inside and one outside (or on) the unit circle. The latter condition can be shown to be always satisfied so, in contrast with the interest rate rule discussed above, the equilibrium is always determined under an exogenous path for the money supply.[14]

Next the equilibrium responses of the economy to different exogenous shocks are examined.

3.4.2.1 THE EFFECTS OF A MONETARY POLICY SHOCK

In order to illustrate how the economy responds to an exogenous shock to the money supply, assume that its growth rate Δm_t follows the AR(1) process

$$\Delta m_t = \rho_m \Delta m_{t-1} + \varepsilon^m_t \tag{35}$$

where $\rho_m \in [0, 1)$ and $\{\varepsilon^m_t\}$ is white noise.

The economy's response to a monetary policy shock can be obtained by determining the stationary solution to the dynamical system consisting of (34) and (35) tracing the effects of a shock to ε^m_t (while setting $\widehat{r}^n_t = \widehat{y}^n_t = 0$, for all t).[15] In doing so, it is assumed that $\rho_m = 0.5$,

[14] That result is based on numerical analysis of the eigenvalues for a broad range of calibrations of the model's parameter values.

[15] See, e.g., Blanchard and Kahn (1980) for a description of a solution method.

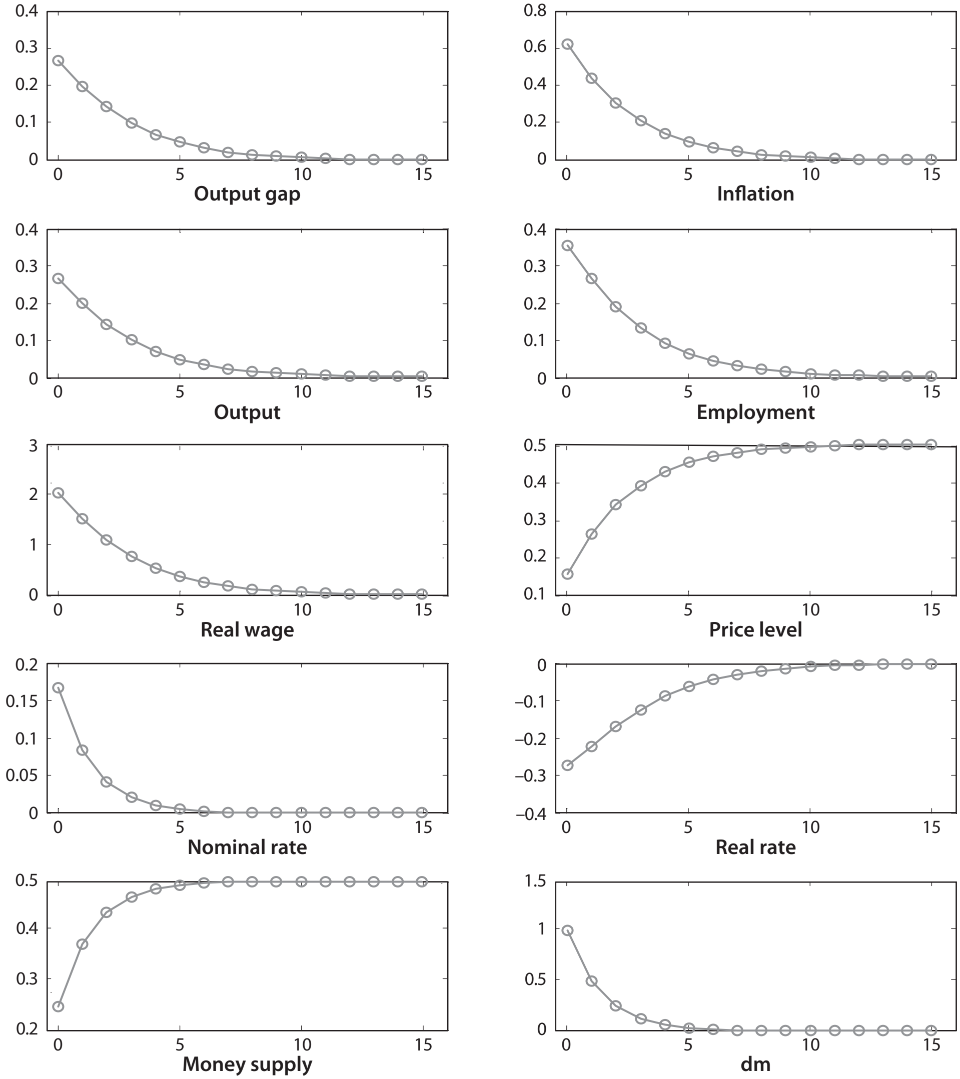

Figure 3.4. Dynamic Responses to a Monetary Policy Shock: Money Supply Rule.

a value roughly consistent with the first-order autocorrelation of money growth in postwar U.S. data.

Figure 3.4 displays the dynamic responses of several variables of interest to an expansionary monetary policy shock that takes the form of a positive realization of ε_t^m of size 0.25. That impulse corresponds to a one percent increase, on impact, in the annualized rate of money growth. The sluggish adjustment of prices implies that real balances rise in response to the increase in the money supply. As a result, clearing of the money market requires either a rise in output and/or a decline in the nominal rate. Under the calibration considered here, output (and the output gap, since natural output is unaffected) increases by about a third of a percentage point on impact, after which it slowly reverts back to its initial level. The nominal rate, however, shows a slight increase. Hence, and in contrast with the case of the interest rate rule considered above, a liquidity effect does not emerge here. Note, however, that the rise in the nominal rate does not prevent the real rate from declining persistently (due to higher expected inflation), leading in turn to an expansion in aggregate demand and output, as implied by (25), and, as a result, a persistent rise in inflation, which follows from (22).

It is worth noting that the absence of a liquidity effect is not a necessary feature of the exogenous money supply regime considered here, but instead a consequence of the calibration used. To see this, note that one can combine equations (4) and (23) to obtain the difference equation

$$i_t = \frac{\eta}{1+\eta} E_t\{i_{t+1}\} + \frac{\rho_m}{1+\eta} \Delta m_t + \frac{\sigma - 1}{1+\eta} E_t\{\Delta y_{t+1}\}$$

whose forward solution yields

$$i_t = \frac{\rho_m}{1+\eta(1-\rho_m)} \Delta m_t + \frac{\sigma-1}{1+\eta} \sum_{k=0}^{\infty} \left(\frac{\eta}{1+\eta} \right)^k E_t\{\Delta y_{t+1+k}\}$$

Note that when $\sigma = 1$, as in the assumed baseline calibration, the nominal rate always comoves positively with money growth. Nevertheless, and given that quite generally the summation term on the right-hand side will be negative (since for most calibrations output tends to adjust monotonically to its original level after the initial increase), a liquidity effect emerges given a value for σ sufficiently above one combined with a sufficiently low value for ρ_m.[16]

3.4.2.2 THE EFFECTS OF A DISCOUNT RATE SHOCK

Figure 3.5 displays the effects of a contractionary discount rate shock, identical to the one analyzed above, but now under the assumption of an exogenous money growth rule, implying the absence of a response in the money supply to the shock, that is, $\Delta m_t = 0$ for all t.

[16] See Galí (2003) for a detailed analysis.

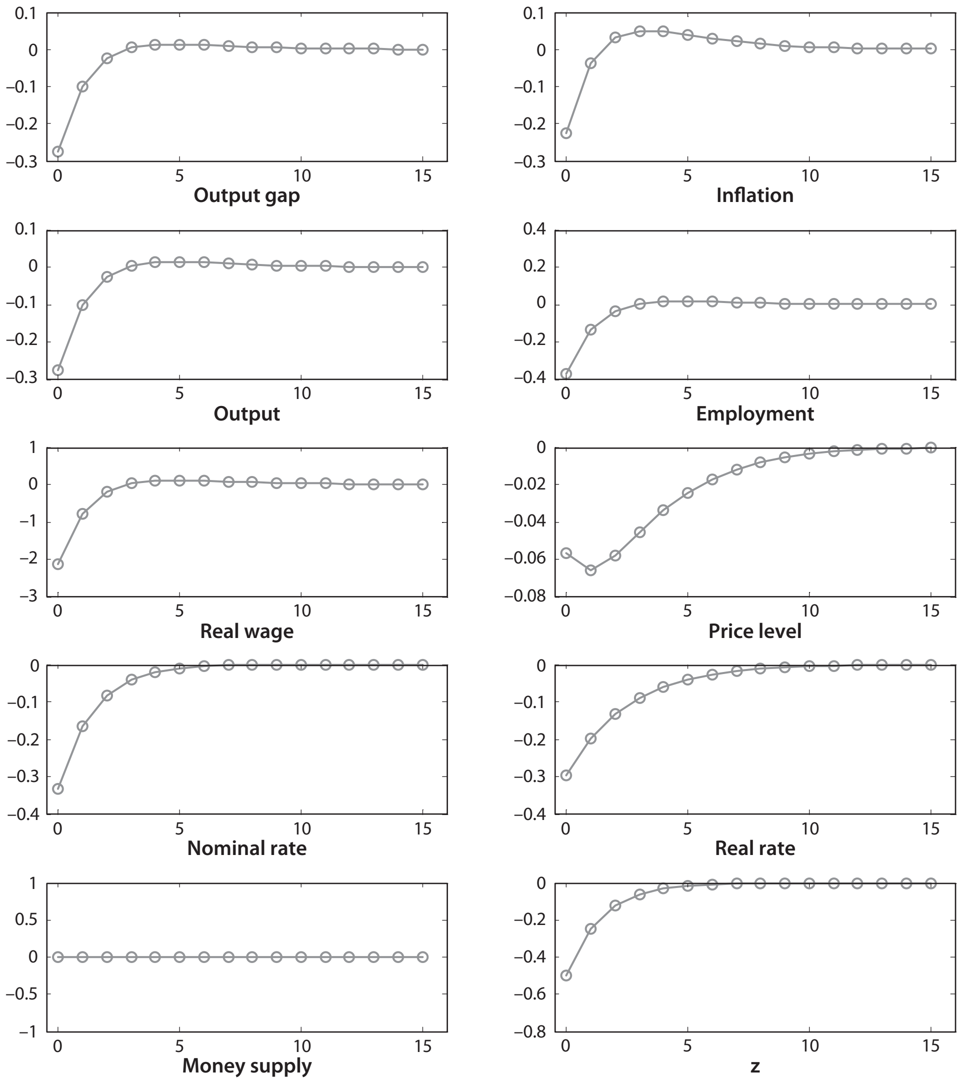

Figure 3.5. Dynamic Responses to a Discount Rate Shock: Money Supply Rule

Note that the dynamic effects on output and employment are, at first sight, qualitatively similar to those derived under an interest rate rule. But this is not the case, strictly speaking. The reason is simple: under monetary targeting the price level returns to its original value, which requires that inflation eventually turns positive after its initial decline, as shown in figure 3.5. It follows that output (as well as employment and the real wage, as a result) must also overshoot its initial level, which in turn requires that the real rate eventually stays below its natural counterpart (none of these effects can be easily seen in the Figure, however).

Intuitively, the reason for the different effects of the discount rate shock under the monetary targeting rule is that the latter is effectively equivalent to an interest rate rule that targets the price level (as opposed to inflation). To see this, note that we can rewrite (4) (ignoring constants) as

$$\begin{aligned} i_t &= \left(\frac{1}{\eta}\right)(p_t - m_{t-1}) + \left(\frac{1}{\eta}\right)\widehat{y}_t + v_t \\ &= \left(\frac{1}{\eta}\right)(p_{t-1} - m_{t-1}) + \left(\frac{1}{\eta}\right)\pi_t + \left(\frac{1}{\eta}\right)\widehat{y}_t + v_t \end{aligned}$$

where $v_t \equiv -(1/\eta)\Delta m_t$. Note that the previous rule is of the same form as (26), with the additional term involving $(p_{t-1} - m_{t-1})$ appended. Thus, and in the absence of a change in the money supply, the interest rate will adjust endogenously to (lagged) deviations between the price level from the implicit price level target which is given by m_{t-1}. That systematic response guarantees the stationarity of the price level in response to any shock (other than a shock to the growth rate of money itself).

3.4.2.3 THE EFFECTS OF A TECHNOLOGY SHOCK

Finally, the effects of a technology shock under an exogenous money supply are analyzed next. Once again, assume the technology parameter a_t follows the stationary process given by (6). In a way consistent with the assumption of exogenous money, it is assumed that $\Delta m_t = 0$ for all t for the purpose of the present exercise.

Figure 3.6 displays the dynamic responses to a one percent increase in the technology. A comparison with the responses shown in figure 3.2 (corresponding to the analogous exercise under an interest rate rule) reveals some similarities: In both cases the output gap (and, hence, inflation) displays a negative response to the technology improvement, as a result of output failing to increase as much as its natural level. Note, however, that in the case of exogenous money the gap between output and its natural level is much larger, which also explains the larger decline in employment. This is due to the upward response of the real rate implied by the unchanged money supply, which contrasts with its decline (in response to the negative response of inflation and the output gap) under the interest rate rule. Because the natural real rate also declines in response to the positive technology shock (in order to support the transitory increase in output and consumption), the response of interest rates generated under the exogenous money regime becomes highly contractionary, relative to the equilibrium under flexible prices. As in the case of a discount rate shock analyzed above (but in contrast with the response to a technology shock under the simple interest rate rule) the price level is stationary and returns gradually to its original level after its initial decline, which requires that inflation eventually turn positive.

The previous simulations have served several goals. First, they have helped illustrate the workings of the New Keynesian model, that is, how the model can be used to answer some specific questions about the behavior of the economy under different assumptions regarding the monetary policy rule in place. Second, under a plausible calibration, it was seen how the simulated responses to some shocks (monetary policy shocks, in particular) display notable similarities (at least qualitative) with the empirical evidence on their effects. Third, the previous analysis has made clear that monetary policy in the New Keynesian model can have large and persistent effects on both real and nominal variables. The latter feature leads one to raise a natural question, which is the focus of the next chapter: How should monetary policy be conducted?

3.5 Notes on the Literature

Early examples of microfounded monetary models with monopolistic competition and sticky prices can be found in Akerlof and Yellen (1985), Mankiw (1985), Blanchard and Kiyotaki (1987), and Ball and Romer (1990).

An early version and analysis of the baseline New Keynesian model can be found in Yun (1996), which used a discrete-time version of the staggered price-setting model originally developed in Calvo (1983). King and Wolman (1996) provide a detailed analysis of the steady state and dynamic properties of that model. King and Watson (1996) compare its predictions regarding the cyclical properties of money, interest rates, and prices with those of flexible price models. Woodford (1996) incorporates a fiscal sector in the model and analyzes its properties under a non-Ricardian fiscal policy regime.

An inflation equation identical to the New Keynesian Phillips curve can be derived under the assumption of quadratic costs of price adjustment, as shown in Rotemberg (1982). Hairault and Portier (1993) developed and analyzed an early version of a monetary model with quadratic costs of price adjustment and compared its second-moment predictions with those of the French and U.S. economies.

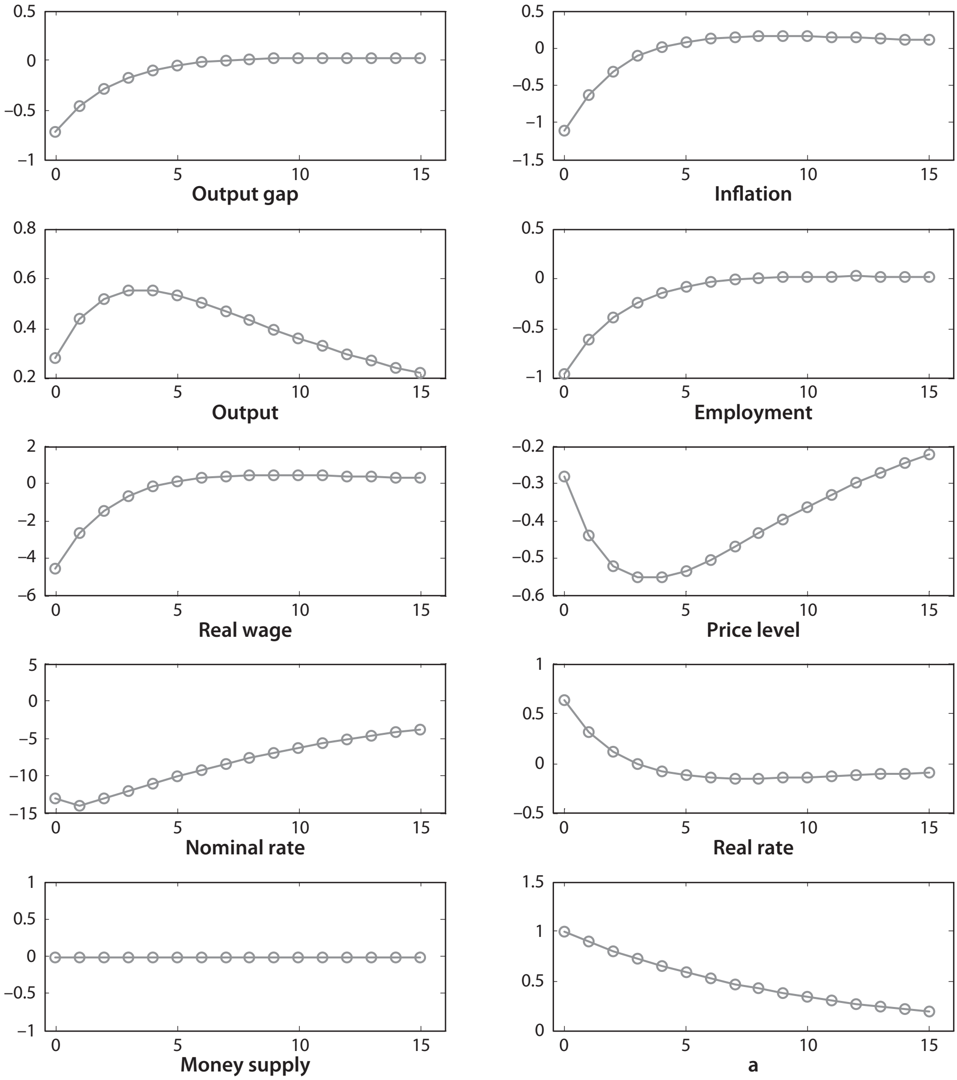

Figure 3.6. Dynamic Responses to a Technology Shock: Money Supply Rule.

The New Keynesian Phillips curve derived in the main text is based on a log-linearization of the optimal price setting condition and the law of motion for the aggregate price level around the zero inflation steady state. Ascari (2004), Ascari and Ropele (2009), and Cogley and Sbordone (2008) analyze the consequences for inflation dynamics of a steady state with nonzero inflation.

Two main alternatives to the Calvo random price duration model can be found in the literature. The first one is given by staggered price-setting models with a deterministic price duration, originally proposed by Taylor (1980) in the context of a non-microfounded model. A microfounded version of the Taylor model can be found in Chari, Kehoe, and McGrattan (2000), who analyzed the output effects of exogenous monetary policy shocks. An alternative price-setting structure is given by state dependent models in which the timing of price adjustments is influenced by the state of the economy. A quantitative analysis of a state dependent pricing model can be found in Dotsey, King, and Wolman (1999) and, more recently, in Golosov and Lucas (2007), Gertler and Leahy (2008), and Midrigan (2011). Klenow and Kryvtsov (2008) and Nakamura and Steinsson (2008) examine the ability of state dependent models to account for the price patterns found in the micro evidence.

The empirical performance of the New Keynesian Phillips curve has always been controversial. An early critical assessment can be found in Fuhrer and Moore (1995). Mankiw and Reis (2002) give a quantitative review of the perceived shortcomings of the NKPC and propose an alternative price-setting structure based on the assumption of sticky information. Galí and Gertler (1999), Sbordone (2002), and Galí, Gertler, and López-Salido (2001) provide favorable evidence of the empirical fit of the equation relating inflation to marginal costs, and discuss the difficulties in estimating or testing the NKPC given the unobservability of the output gap. Mavroeidis, Plagborg-Moller, and Stock (2013) overview the empirical literature on the NKPC, focusing on the problem of weak identification and its consequences.

Taylor (1993, 1999) put forward the notion that a simple interest rate rule that responds systematically to inflation and detrended output approximates fairly well, when its coefficients are chosen appropriately, the path of the federal funds rate in the Greenspan years. Empirical evidence on the stability of the Taylor rule over time and across different countries can be found in Clarida, Galí, and Gertler (1998, 2000) and Orphanides (2003), among others.

Rotemberg and Woodford (1999) and Christiano, Eichenbaum, and Evans (1999, 2005) provide empirical evidence on the effects of monetary policy shocks, and discuss a number of modifications of the baseline New Keynesian model aimed at improving the model's ability to match the estimated impulse responses.

Evidence on the effects of technology shocks and their implications for the relevance of alternative models can be found in Galí (1999) and Basu, Fernald, and Kimball (2004), among others. Recent evidence as well as alternative interpretations are surveyed in Galí and Rabanal (2004).

Appendix

3.1 Optimal Allocation of Consumption Expenditures

The problem of maximization of C_t for any *given* expenditure level

$$X_t \equiv \int_0^1 P_t(i) C_t(i) di$$

can be formalized by means of the Lagrangian

$$\mathcal{L} = \left(\int_0^1 C_t(i)^{1-\frac{1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}} - \lambda \left(\int_0^1 P_t(i) C_t(i) \, di - X_t \right)$$

The implied first-order conditions are

$$C_t(i)^{-\frac{1}{\epsilon}} C_t^{\frac{1}{\epsilon}} = \lambda P_t(i)$$

for all $i \in [0, 1]$. Thus, for any two goods (i, j),

$$C_t(i) = C_t(j) \left(\frac{P_t(i)}{P_t(j)} \right)^{-\epsilon}$$

which can be substituted into the expression for consumption expenditures to yield

$$C_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon} \frac{X_t}{P_t}$$

for all $i \in [0, 1]$. The latter condition can then be substituted into the definition of C_t to obtain

$$\int_0^1 P_t(i) C_t(i) di = P_t C_t$$

Combining the two previous equations yields the demand schedule

$$C_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon} C_t$$

3.2 Aggregate Price Level Dynamics

Let $S(t) \subset [0, 1]$ represent the set of firms not reoptimizing their posted price in period t. Using the definition of the aggregate price level and the fact that all firms resetting prices will choose an identical price P_t^*,

$$P_t = \left[\int_{S(t)} P_{t-1}(i)^{1-\epsilon} di + (1-\theta)(P_t^*)^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}}$$
$$= \left[\theta (P_{t-1})^{1-\epsilon} + (1-\theta)(P_t^*)^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}}$$

where the second equality follows from the fact that the distribution of prices among firms not adjusting in period t corresponds to the distribution of effective prices in period $t-1$, though with total mass reduced to θ.

Dividing both sides by P_{t-1},

$$\Pi_t^{1-\epsilon} = \theta + (1-\theta) \left(\frac{P_t^*}{P_{t-1}} \right)^{1-\epsilon} \tag{36}$$

3.3 Firm's Objective Function

The value of a firm in period t, expressed in terms of current consumption is given by

$$V_t(i) = \sum_{k=0}^{\infty} E_t\{\Lambda_{t,t+k}(D_{t+k}(i)/P_{t+k})\}$$

where $D_t(i) \equiv P_t(i) Y_t(i) - \mathcal{C}_t(Y_t(i))$.

Note that for a firm resetting its price in period t,

$$E_t\{\Lambda_{t,t+k}(D_{t+k}(i)/P_{t+k})\} = \theta^k E_t\{\Lambda_{t,t+k}(D_{t+k|t}/P_{t+k})\}$$
$$+ (1-\theta) \sum_{h=1}^{k} \theta^{k-h} E_t\{\Lambda_{t,t+k}(D_{t+k|t+h}/P_{t+k})\} \tag{37}$$

where $D_{t+k|t} \equiv P_t^* Y_{t+k|t} - \mathcal{C}_{t+k}(Y_{t+k|t})$ denotes period $t+k$ dividends conditional on the price having been last reset in period t. Note that the second term on the right-hand side of (37) is independent of P_t^* since it involves states of nature for which the price has been reset at least once after period t.

Thus, the value of a firm resetting its price in period t is given by

$$V_{t|t} = \sum_{k=0}^{\infty} \theta^k E_t\{\Lambda_{t,t+k}(D_{t+k|t}/P_{t+k})\} + \Upsilon_t$$

where Υ_t is a term independent of P_t^* and hence can be ignored when choosing the latter, as reflected in the objective function in the main text.

3.4 Price Dispersion

Let $\widehat{p}_t(i) \equiv p_t(i) - p_t$. Notice that,

$$\begin{aligned}\left(\frac{P_t(i)}{P_t}\right)^{1-\epsilon} &= \exp\left[(1-\epsilon)\widehat{p}_t(i)\right] \\ &= 1 + (1-\epsilon)\widehat{p}_t(i) + \frac{(1-\epsilon)^2}{2}\widehat{p}_t(i)^2\end{aligned}$$

Note that from the definition of P_t, $1 = \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{1-\epsilon} di$. A second-order approximation to this expression thus implies

$$E_i\{\widehat{p}_t(i)\} = \frac{\epsilon - 1}{2} E_i\{\widehat{p}_t(i)^2\}$$

In addition, a second-order approximation to $\left(\frac{P_t(i)}{P_t}\right)^{-\frac{\epsilon}{1-\alpha}}$ yields

$$\left(\frac{P_t(i)}{P_t}\right)^{-\frac{\epsilon}{1-\alpha}} = 1 - \frac{\epsilon}{1-\alpha}\,\widehat{p}_t(i) + \frac{1}{2}\left(\frac{\epsilon}{1-\alpha}\right)^2 \widehat{p}_t(i)^2$$

Combining the two previous results, it follows that

$$\begin{aligned}\int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\frac{\epsilon}{1-\alpha}} di &= 1 + \frac{1}{2}\left(\frac{\epsilon}{1-\alpha}\right)\frac{1}{\Theta}\, E_i\{\widehat{p}_t(i)^2\} \\ &= 1 + \frac{1}{2}\left(\frac{\epsilon}{1-\alpha}\right)\frac{1}{\Theta}\, var_i\{p_t(i)\}\end{aligned}$$

where $\Theta \equiv \frac{1-\alpha}{1-\alpha+\alpha\epsilon}$, and where the last equality follows from the observation that, up to second order,

$$\int_0^1 (p_t(i) - p_t)^2 \, di \simeq \int_0^1 (p_t(i) - E_i\{p_t(i)\})^2 \, di$$
$$\equiv var_i\{p_t(i)\}$$

Finally, using the definition of d_t and up to a second-order approximation,

$$d_t \equiv (1-\alpha) \log \int_0^1 \left(\frac{P_t(i)}{P_t} \right)^{-\frac{\epsilon}{1-\alpha}} di \simeq \frac{\epsilon}{2\Theta} var_i\{p_t(i)\}$$

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Exercises

3.1. **Interpreting discrete-time records of data on price adjustment frequency**

Suppose firms operate in continuous time, with the *pdf* for the duration of the price of an individual good being $f(t) = \phi \exp(-\phi t)$, where $t \in \mathbb{R}^+$ is expressed in month units.

a. Show that the implied instantaneous probability of a price change is constant over time and given by ϕ.

b. What is the *mean* duration of a price? What is the *median* duration? What is the relationship between the two?
c. Suppose that the prices of individual goods are recorded once a month (say, on the first day, for simplicity). Let λ_t denote the fraction of items in a given goods category whose price in month t is different from that recorded in month $t-1$ (Note: of course, the price may have changed more than once since the previous record.) How would you go about estimating parameter ϕ?
d. Given information on monthly frequencies of price adjustment, how would you go about calibrating parameter θ in a quarterly Calvo model?

3.2. **Introducing government purchases in the basic New Keynesian model**
Assume that the government purchases quantity $G_t(i)$ of good i, for all $i \in [0,1]$. Let $G_t \equiv \left(\int_0^1 G_t(i)^{1-\frac{1}{\epsilon}}\, di\right)^{\frac{\epsilon}{\epsilon-1}}$ denote an index of public consumption, which the government seeks to maximize for any level of expenditures $\int_0^1 P_t(i)G_t(i)di$. Assume government expenditures are financed by means of lump-sum taxes.

a. Derive an expression for total demand facing firm i.
b. Derive a log-linear aggregate goods market clearing condition that is valid around a steady state with a constant public consumption share $S_G \equiv \frac{G}{Y}$.
c. Derive the corresponding expression for average real marginal cost as a function of aggregate output, government purchases, and technology and provide some intuition for the effect of government purchases.
d. How is the equilibrium relationship linking interest rates to current and expected output affected by the presence of government purchases?

3.3. **Government purchases and sticky prices**
Consider a model economy with the following equilibrium conditions. The household's log-linearized Euler equation takes the form

$$c_t = -\frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} - \rho) + E_t\{c_{t+1}\}$$

where c_t is consumption, i_t is the nominal rate, and $\pi_{t+1} \equiv p_{t+1} - p_t$ is the rate of inflation between t and $t+1$ (Note: as in the text, lowercase letters denote the logs of the original variable.) The household's log-linearized labor supply is given by

$$w_t - p_t = \sigma c_t + \varphi n_t$$

where w_t denotes the nominal wage, p_t is the price level, and n_t is employment.

Firms' technology is given by

$$y_t = n_t$$

The time between price adjustments follows a geometric distribution, which gives rise to an inflation equation

$$\pi_t = \beta E_t\{\pi_{t+1}\} + \kappa \widetilde{y}_t$$

where $\widetilde{y}_t \equiv y_t - y_t^n$ is the output gap (with y_t^n representing the natural level of output). Assume that in the absence of constraints on price adjustment firms would set a price equal to a constant markup over marginal cost given by μ (in logs).

Suppose that the government purchases a fraction τ_t of the output of each good, with τ_t varying exogenously. Government purchases are financed through lump-sum taxes.

a. Derive a log-linear version of the goods market clearing condition of the form $y_t = c_t + g_t$, where $g_t \equiv -\log(1 - \tau_t)$.
b. Determine the behavior of the natural level of output y_t^n as a function of g_t and discuss the mechanism through which a fiscal expansion leads to an increase in output when prices are flexible.
c. Assume that $\{g_t\}$ follows a simple AR(1) process with autoregressive coefficient $\rho_g \in [0, 1)$. Derive the dynamic IS equation

$$\widetilde{y}_t = E_t\{\widetilde{y}_{t+1}\} - \frac{1}{\sigma}\,(i_t - E_t\{\pi_{t+1}\} - r_t^n)$$

together with an expression for the natural rate r_t^n as a function of g_t.

d. Determine the economy's response to an exogenous increase in g_t when the central bank follows an interest rate rule given by

$$i_t = \rho + \phi_\pi \pi_t$$

3.4. **Indexation and the New Keynesian Phillips curve**
Consider the Calvo model of staggered price setting with the following modification: In the periods between price reoptimizations firms mechanically adjust their prices according to some indexation rule. Formally, a firm that reoptimizes its price in period t (an event that occurs with probability $1-\theta$) sets a price P_t^* in that period. In subsequent periods (i.e., until it reoptimizes prices again), its price is adjusted according to the following indexation rule:

$$P_{t+k|t} = P_{t+k-1|t}(\Pi_{t+k-1})^{\omega}\Pi^{1-\omega}$$

for $k = 1, 2, 3, \ldots$ and

$$P_{t,t} = P_t^*$$

where $P_{t+k|t}$ denotes the price effective in period $t+k$ for a firm that last reoptimized its price in period t, $\Pi_t \equiv \frac{P_t}{P_{t-1}}$ is the aggregate gross inflation rate, Π the latter's steady state value, and $\omega \in [0, 1]$ is an exogenous parameter that measures the degree of indexation to lagged inflation.

Suppose that all firms have access to the same constant returns to scale technology and face a demand schedule with a constant price elasticity ϵ.

The objective function for a firm reoptimizing its price in period t (i.e., choosing P_t^*) is given by

$$\max_{P_t^*} \sum_{k=0}^{\infty} \theta^k E_t \left\{ \Lambda_{t,t+k}(1/P_{t+k}) \left(P_{t+k|t} Y_{t+k|t} - \mathcal{C}_{t+k}(Y_{t+k|t}) \right) \right\}$$

subject to a sequence of demand constraints (9), and the indexation rule described above. All variables are defined as in the main text.

a. Using the definition of the price level index $P_t \equiv \left(\int_0^1 P_t(i)^{1-\epsilon} di \right)^{\frac{1}{1-\epsilon}}$, derive a log-linear expression for the evolution of inflation π_t as a function of the newly set price p_t^*.

b. Derive the first-order condition for the firm's problem that determines the optimal price P_t^*.
c. Log-linearize the first-order condition around the corresponding steady state and derive an expression for p_t^* (i.e., the approximate log-linear price-setting rule).
d. Combine the results of (a) and (c) to derive an inflation equation of the form

$$\widehat{\pi}_t = \gamma_b \widehat{\pi}_{t-1} + \gamma_f E_t\{\widehat{\pi}_{t+1}\} - \lambda \widehat{\mu}_t$$

where $\widehat{\pi}_t \equiv \pi_t - \pi$.

3.5. **Optimal price setting and equilibrium dynamics in the Taylor model**
Assume a continuum of firms indexed by $i \in [0, 1]$. Each firm produces a differentiated good, with a technology

$$Y_t(i) = A_t N_t(i)$$

where A_t represents the level of technology, and $a_t \equiv \log A_t$ evolves exogenously according to some stationary stochastic process.

During each period a fraction $\frac{1}{N}$ of firms reset their prices, which then remain effective for N periods. Hence, a firm i setting a new price P_t^* in period t will seek to maximize

$$\max_{P_t^*} \sum_{k=0}^{N-1} E_t \left\{ \Lambda_{t,t+k}(1/P_{t+k}) \left(P_t^* Y_{t+k|t} - \mathcal{C}_{t+k}(Y_{t+k|t}) \right) \right\}$$

subject to

$$Y_{t+k|t} = (P_t^*/P_{t+k})^{-\epsilon} C_{t+k}$$

where $\Lambda_{t,t+k} \equiv \beta^k \left(\frac{C_{t+k}}{C_t} \right)^{-\sigma}$

a. Show that P_t^* must satisfy the first-order condition

$$\sum_{k=0}^{N-1} E_t \left\{ \Lambda_{t,t+k} Y_{t+k|t} (P_t^* - \mathcal{M} \Psi_{t+k} \right\} = 0$$

where $\Psi_t \equiv \mathcal{C}_t'(\cdot)$ is the nominal marginal cost and $\mathcal{M} \equiv \frac{\epsilon}{\epsilon - 1}$.

b. Derive the following log-linearized optimal price-setting rule (around a zero inflation steady state)

$$p_t^* = \mu + \sum_{k=0}^{N-1} \omega_k E_t \{\psi_{t+k}\}$$

where $\omega_k \equiv \frac{\beta^k(1-\beta)}{1-\beta^N}$, $\psi_t \equiv \log \Psi_t$ and $\mu \equiv \log \mathcal{M}$. Show that in the limiting case of $\beta = 1$ (no discounting) the above equation can be rewritten as

$$p_t^* = \mu + \frac{1}{N} \sum_{k=0}^{N-1} E_t \{\psi_{t+k}\}$$

How does the previous price-setting rule differ from the one generated by the Calvo model?

c. Recalling the expression for the aggregate price index $P_t \equiv \left(\int_0^1 P_t(i)^{1-\epsilon}\, di\right)^{\frac{1}{1-\epsilon}}$, show that around a zero inflation steady state the (log) price level will satisfy

$$p_t = \left(\frac{1}{N}\right) \sum_{k=0}^{N-1} p_{t-k}^*$$

d. Consider the particular case of $N = 2$ and $\beta = 1$ and assume that the consumer's (log) marginal rate of substitution between labor and consumption is given by $\sigma c_t + \varphi n_t$. Assume also that all output is consumed. Show that:

$$p_t^* = \frac{1}{2} p_{t-1}^* + \frac{1}{2} E_t\{p_{t+1}^*\} + \delta(\widetilde{y}_t + E_t\{\widetilde{y}_{t+1}\})$$

where $\delta \equiv \sigma + \varphi$.

e. Assume that money demand takes the simple form $m_t - p_t = y_t$ and that both m_t and a_t follow (independent) random walks, with innovations ε_t^m and ε_t^a, respectively. Derive a closed-form expression for the output gap, employment, and the price level as a function of the exogenous shocks.

f. Discuss the influence of δ on the persistence of the effects of a monetary shock, and provide some intuition for that result.

3.6. **The Mankiw-Reis model: Inflation dynamics under predetermined prices**
Suppose that each period a fraction of firms $1-\theta$ gets to choose *a path of future prices* for their respective goods (a "price plan"), while the remaining fraction θ keeps their current price plans. Let $\{P_{t,t+k}\}_{k=0}^{\infty}$ denote the price plan chosen by firms that get to revise that plan in period t. Firm's technology is given by $Y_t(i) = \sqrt{A_t} N_t(i)$. Consumer's period utility is assumed to take the form $U(C_t, N_t) = C_t - \frac{N_t^2}{2}$, where $C_t \equiv \left(\int_0^1 C_t(i)^{1-\frac{1}{\epsilon}} di\right)^{\frac{\epsilon}{\epsilon-1}}$. The demand for real balances is assumed to be given by $\frac{M_t}{P_t} = C_t$. All output is consumed.

a. Let $P_t \equiv \left(\int_0^1 P_t(i)^{1-\varepsilon} di\right)^{\frac{1}{1-\varepsilon}}$ denote the aggregate price index. Show that, up to a first-order approximation,

$$p_t = (1-\theta) \sum_{j=0}^{\infty} \theta^j \, p_{t-j,t} \tag{38}$$

b. A firm i, revising its price plan in period t will seek to maximize

$$\sum_{k=0}^{\infty} \theta^k E_t \left\{ \Lambda_{t,t+k} (1/P_{t+k}) Y_{t+k}(i) \left(P_{t,t+k} - \frac{W_{t+k}}{\sqrt{A_{t+k}}} \right) \right\}$$

Derive the first-order condition associated with that problem, and show that it implies the following approximate log-linear rule for the price plan

$$p_{t,t+k} = \mu + E_t\{\psi_{t+k}\} \tag{39}$$

for $k = 0, 1, 2, \ldots$ and where $\psi_t = w_t - \frac{1}{2} a_t$ is the nominal marginal cost.

c. Use the optimality conditions for the consumer's problem, and the labor market clearing condition to show that (i) the *natural* level of output satisfies $y_t^n = -\mu + a_t$, and (ii) the (log) real marginal cost (in deviation from its perfect foresight steady state value) equals the output gap, i.e.,

$$\widehat{mc}_t = \widetilde{y}_t$$

for all t, where $\widetilde{y}_t \equiv y_t - y_t^n$.

d. Using (38) and (39), show how one can derive the following equation for inflation

$$\pi_t = \frac{1-\theta}{\theta}\widetilde{y}_t + \frac{1-\theta}{\theta}\sum_{j=1}^{\infty}\theta^j E_{t-j}\{\Delta\widetilde{y}_t + \pi_t\} \tag{40}$$

e. Suppose that the money supply follows a random walk process $m_t = m_{t-1} + u_t$, where $m_t \equiv \log M_t$ and $\{u_t\}$ is white noise. Determine the dynamic response of output, employment, and inflation to a money supply shock. Compare the implied response to the one obtained under the standard New Keynesian Phillips curve, where $\pi_t = \beta E_t\{\pi_{t+1}\} + \kappa\widetilde{y}_t$ (Hint: use the fact that in equilibrium $y_t = m_t - p_t$, substitute for $\widetilde{y}_t$ in (40) in order to obtain a difference equation for the (log) price level.)

f. Suppose that technology is described by the random walk process $a_t = a_{t-1} + \varepsilon_t$, where $a_t \equiv \log A_t$, and $\{\varepsilon_t\}$ is white noise. Determine the dynamic response of output, the output gap, employment, and inflation to a technology shock. Compare the implied response to one obtained under the standard New Keynesian Phillips curve, where $\pi_t = \beta\ E_t\{\pi_{t+1}\} + \kappa\ \widetilde{y}_t$ (Hint: same as (e) above).

3.7. **Technology shocks and the Taylor rule**

Consider an economy with Calvo-type staggered price setting, where a continuum of monopolistically competitive firms have access to a technology $Y_t = A_t N_t$, where Y_t is output, N_t denotes hours of work, and A_t is an exogenous technology parameter. The representative consumer has a period utility $U(C_t, N_t) = \log C_t - \frac{N_t^{1+\varphi}}{1+\varphi}$, where C_t is a CES function of the quantities consumed of the different types of goods. The technology parameter $a_t \equiv \log A_t$ is assumed to follow a random walk process, i.e. $a_t = a_{t-1} + \varepsilon_t$, where $\{\varepsilon_t\}$ is white noise. Firms' desired markups are constant. All output is consumed. The labor market is perfectly competitive.

The implied equilibrium conditions take the form

$$\widetilde{y}_t = E_t\{\widetilde{y}_{t+1}\} - (i_t - E_t\{\pi_{t+1}\} - \rho) \tag{41}$$

$$\pi_t = \beta E_t\{\pi_{t+1}\} + \kappa\widetilde{y}_t \tag{42}$$

where $\widetilde{y}_t \equiv y_t - y_t^n$ is the output gap, y_t is (log) output, y_t^n is the (log) natural level of output, $\pi_t \equiv p_t - p_{t-1}$ is the rate of inflation, and i_t is the short-term nominal rate.

a. Determine the natural level of output y_t^n as a function of the technology parameter.
b. Suppose that the monetary authority adopts a simple interest rate rule of the form

$$i_t = \rho + \phi_\pi \pi_t \tag{43}$$

where $\phi_\pi > 1$. Determine the equilibrium path of inflation, the output gap, and output.
c. How would your answer to (b) change if the technology process was instead given by $a_t = \rho_a a_{t-1} + \varepsilon_t$ with $\rho_a \in (0, 1)$? Derive your result analytically and explain it intuitively.

3.8. **A New Keynesian economy with linear utility**
Consider an economy with Calvo-type staggered price setting, where a continuum of monopolistically competitive firms have access to a technology $Y_t = A_t N_t$, where Y_t is output, N_t denotes employment, and A_t is an exogenous technology parameter. The representative consumer maximizes $E_0 \left\{\sum_{t=0}^{\infty} \beta^t U(C_t, N_t)\right\}$ where the period utility is given by $U(C_t, N_t) = C_t - \frac{N_t^{1+\varphi}}{1+\varphi}$, where C_t is a CES function of the quantities consumed of the different types of goods. Firms' desired markup is constant. All output is consumed. The labor market is perfectly competitive.

a. Determine the (log) natural level of employment $\left(n_t^n\right)$ and output $\left(y_t^n\right)$ as a function of technology.
b. Explain, using a labor market diagram, why natural employment *always* rises in response to a positive technology shock under the assumptions above.
c. Derive the consumer's Euler equation and show that it implies a constant real interest rate. Give some intuition for that result.
d. Write down a *simple* interest rate rule (with finite coefficients and a function of observables only) which will replicate the natural (i.e., flexible price) allocation. Explain why (short proof).

3.9. **A NKPC with backward-looking firms** (based on Galí and Gertler (1999)).
Consider the following modification to the basic New Keynesian model developed in the text: a fraction $\omega \in [0, 1]$ of firms set

prices according to the following *backward-looking* rule:

$$p_t^b = p_{t-1}^* + \pi_{t-1}$$

where

$$p_t^* = \omega p_t^b + (1-\omega) p_t^f$$

is the average newly set (log) price in period t, with p_t^f denoting the (log) price set by forward-looking firms. The latter is given as in the standard model by:

$$p_t^f = \mu + (1-\beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k E_t\{\psi_{t+k}\}$$

where, for simplicity, constant returns have been assumed ($\alpha = 0$).

Both backward- and forward-looking firms can reset their prices with probability $1-\theta$ each period. Thus, the average (log) price evolves according to:

$$p_t = \theta p_{t-1} + (1-\theta) p_t^*$$

Assume, as in the standard model, that the following relationship between the average markup and the output gap holds:

$$\widehat{\mu}_t = -(\sigma + \varphi)\widetilde{y}_t$$

a. Show that the New Keynesian Phillips curve will now take the form

$$\pi_t = \gamma_b \pi_{t-1} + \gamma_f E_t\{\pi_{t+1}\} + \kappa \widetilde{y}_t$$

and determine the values of coefficients γ_b, γ_f, and κ as a function of the model's parameters.

b. Discuss the implications for the costs of disinflation, compared to the standard New Keynesian Phillips curve.

Chapter 4

MONETARY POLICY DESIGN IN THE BASIC NEW KEYNESIAN MODEL

This chapter addresses the question of how monetary policy should be conducted, using as a reference framework the basic New Keynesian model developed in chapter 3. To start, that model's efficient allocation is characterized and shown to correspond to the equilibrium allocation of the decentralized economy under monopolistic competition and flexible prices once an appropriately chosen subsidy is in place. As it will be demonstrated, when prices are sticky, that allocation can be attained by means of a policy that fully stabilizes the price level.

The objectives of the optimal monetary policy are first determined, and then the issues pertaining to its implementation are addressed. Examples of interest rules that implement the optimal policy are provided. But an argument is given that none of those rules seems a likely candidate to guide monetary policy in practice, for they all require that the central bank responds contemporaneously to changes in a variable—the natural rate of interest—that is not observable in actual economies. That observation motivates the introduction of rules that a central bank could arguably follow in practice (labeled as *simple rules*), and the development of a welfare-based criterion to evaluate the relative desirability of those rules. An illustration of that approach to policy evaluation is provided by analyzing the properties of two examples of such simple rules: a Taylor rule and a constant money growth rule.

4.1 The Efficient Allocation

The efficient allocation associated with the model economy described in chapter 3 can be determined by solving the problem facing a benevolent social planner seeking to maximize the representative household's welfare, given technology and preferences. Thus, for each period the optimal allocation must maximize the household's utility

$$U\,(C_t, N_t; Z_t)$$

where, as in the previous chapter, $C_t \equiv \left(\int_0^1 C_t(i)^{1-\frac{1}{\epsilon}} di\right)^{\frac{\epsilon}{\epsilon-1}}$ is a consumption index, N_t denotes employment (or work hours), and Z_t is an exogenous preference shifter. That maximization is subject to the resource constraints

$$C_t(i) = A_t N_t(i)^{1-\alpha}$$

for all $i \in [0, 1]$ and

$$N_t = \int_0^1 N_t(i) di$$

The associated optimality conditions are

$$C_t(i) = C_t, \text{ all } i \in [0, 1] \tag{1}$$

$$N_t(i) = N_t, \text{ all } i \in [0, 1] \tag{2}$$

$$-\frac{U_{n,t}}{U_{c,t}} = MPN_t \tag{3}$$

where $MPN_t \equiv (1 - \alpha) A_t N_t^{-\alpha}$ denotes the economy's average marginal product of labor (which in the case of the symmetric allocation considered above also happens to coincide with the marginal product for each individual firm).

Thus, it is optimal to produce and consume the same quantity of all goods and to allocate the same amount of labor to all firms. That result is a consequence of all goods entering the utility function symmetrically, combined with concavity of utility and production functions, identical for all goods. Once that symmetric allocation is imposed, the remaining condition defining the efficient allocation, equation (3), equates the marginal rate of substitution between consumption and employment to the corresponding marginal rate of transformation (which in turn equals the marginal product of labor). Note also that the latter condition coincides with the one determining the equilibrium allocation of the classical monetary model (with perfect competition and fully flexible prices) analyzed in chapter 2.

Next, the factors that make the equilibrium allocation in the baseline model suboptimal are discussed.

4.2 Sources of Suboptimality in the Basic New Keynesian Model

The basic New Keynesian model developed in chapter 3 is characterized by two distortions, whose implications are worth considering separately. The first distortion is the presence of market power in goods markets, exercised by monopolistically competitive firms. That distortion is unrelated to the presence of sticky prices, that is, it is operative even under the assumption of flexible prices. The second distortion results from the assumption of infrequent price adjustment by firms. Next, both types of distortions and their implications for the efficiency of equilibrium allocations are discussed.

4.2.1 Distortions Unrelated to Sticky Prices: Monopolistic Competition

The fact that each firm perceives the demand for its differentiated product to be imperfectly elastic endows it with some market power and leads to pricing-above-marginal cost policies. To isolate the role of monopolistic competition as a source of an inefficient allocation, let us suppose for the time being that prices are fully flexible, that is, each firm can adjust freely the price of its good each period. In that case, the profit maximizing price is identical across firms. In particular, under an isoelastic demand function (with price elasticity ϵ), the optimal price-setting rule is given by

$$P_t = \mathcal{M} \frac{W_t}{MPN_t}$$

where $\mathcal{M} \equiv \frac{\epsilon}{\epsilon - 1} > 1$ is the (gross) optimal markup chosen by firms and $\frac{W_t}{MPN_t}$ is the marginal cost. Accordingly,

$$-\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t} = \frac{MPN_t}{\mathcal{M}} < MPN_t$$

where the first equality corresponds to the household's intratemporal optimality condition. Hence, it is seen that the presence of a nontrivial price markup implies that condition (3) characterizing the efficient allocation is violated. Because, in equilibrium, the marginal rate of substitution $-U_{n,t}/U_{c,t}$ and the marginal product of labor are, respectively, increasing and decreasing (or nonincreasing) in N_t, the presence of a markup distortion leads to an *inefficiently low* level of employment and output.

The above inefficiency, resulting from the presence of market power, can be eliminated through an employment subsidy, financed by means of lump-sum taxes. Let τ denote the rate at which the cost of employment is subsidized. Then, under flexible prices, $P_t = \mathcal{M}\frac{(1-\tau)W_t}{MPN_t}$. Accordingly,

$$-\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t} = \frac{MPN_t}{\mathcal{M}(1-\tau)}$$

Hence, the optimal allocation can be attained if $\mathcal{M}(1-\tau) = 1$ or, equivalently, by setting $\tau = \frac{1}{\epsilon}$. In much of the analysis below it is assumed that such an optimal subsidy is in place. By construction, the equilibrium under flexible prices is efficient in that case.

4.2.2 Distortions Related to Sticky Prices

The assumed constraints on the frequency of price adjustment constitute a source of inefficiency on two different grounds. First, the fact that firms do not adjust their prices continuously implies that the economy's average markup will vary over time in response to shocks, and will generally differ from the constant frictionless markup $\mathcal{M}$. Formally, letting $\mathcal{M}_t$ denote the economy's average markup, and allowing for an optimal employment subsidy as introduced above,

$$\mathcal{M}_t = \frac{P_t}{(1-\tau)(W_t/MPN_t)} = \frac{P_t\mathcal{M}}{W_t/MPN_t}$$

where the second equality follows from the assumption that the subsidy in place exactly offsets the monopolistic competition distortion, which allows the isolation of the role of sticky prices. In that case,

$$-\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t} = MPN_t\,\frac{\mathcal{M}}{\mathcal{M}_t}$$

which violates efficiency condition (3) to the extent that $\mathcal{M}_t \neq \mathcal{M}$, as will generally be the case in the presence of constraints on price adjustment. The efficiency of the equilibrium allocation can be restored only if the economy's average markup is stabilized at its frictionless level.

In addition to the above distortion, which generates an inefficient level of aggregate employment and output, the presence of *staggered* price setting is a source of a second type of inefficiency. The latter has to do with the fact that the relative prices of different goods will vary in a way unwarranted by changes in preferences or technologies, as a result of the lack of synchronization in price adjustments. Thus, generally

$P_t(i) \neq P_t(j)$ for any pair of goods (i, j) whose prices do not happen to have been adjusted in the same period. Such *relative price distortions* will lead, in turn, to different quantities of the different goods being produced and consumed, that is, $C_t(i) \neq C_t(j)$, and, as a result, different quantities of labor employed by different firms, that is, $N_t(i) \neq N_t(j)$ for some (i, j). That outcome violates efficiency conditions (1) and (2). Attaining the efficient allocation requires that the quantities produced and consumed of all goods are equalized, which in turn requires that their prices and marginal costs are equalized.

Next, the policy that attains those objectives is characterized.

4.3 Optimal Monetary Policy in the Basic New Keynesian Model: The Case of an Efficient Natural Allocation

In the present section the optimal monetary policy in the basic New Keynesian model is derived for the case in which the equilibrium allocation under flexible prices—henceforth referred to as the *natural* allocation—is efficient (e.g., due to the presence of an optimal subsidy). The analysis of the optimal policy in the case of an inefficient natural allocation is dealt with in chapter 5.

An optimal subsidy is assumed that exactly offsets the market power distortion. As discussed above, this is sufficient to render the natural allocation of the basic New Keynesian model efficient. In addition, and in order to keep the analysis simple, the analysis is restricted to the case where there are *no inherited relative price distortions*, that is, it is assumed that $P_{-1}(i) = P_{-1}$ for all $i \in [0, 1]$.[1] Under the previous assumptions, the efficient allocation can be attained by a policy that stabilizes firms' marginal cost at a level consistent with their desired markup $\mathcal{M}$, *at unchanged prices*. If that policy is expected to remain in place indefinitely, no firm has ever an incentive to adjust its price, because it is currently charging its optimal markup and expects to keep doing so in the future. As a result, $P_t^* = P_{t-1}$ and, hence, $P_t = P_{t-1}$ for $t = 0, 1, 2, \ldots$ In other words, the aggregate price level is fully stabilized and no relative price distortions emerge. In addition, $\mathcal{M}_t = \mathcal{M}$ for all t, and output and employment match their counterparts in the flexible price equilibrium allocation (which, in turn, corresponds to the efficient allocation, given the assumed subsidy).

[1] The case of a nondegenerate initial distribution of prices is analyzed in Yun (2005). In that case, the optimal monetary policy converges to the one described here after a transition period.

Using the notation introduced in chapter 3, the optimal policy requires that

$$y_t = y_t^n$$

or, equivalently,

$$\widetilde{y}_t = 0 \tag{4}$$

for all t, that is, the output gap should be closed at all times. In that case, the New Keynesian Phillips curve implies

$$\pi_t = 0 \tag{5}$$

for all t, that is, inflation is kept constant at a zero rate (or, equivalently, the aggregate price level is fully stabilized).

The dynamic IS equation then implies

$$i_t = r_t^n$$

for all t, that is, the equilibrium nominal interest rate (which equals the real rate, given zero inflation) must be equal to the natural rate of interest.

Two features of the optimal policy are worth emphasizing. First, stabilizing output is not desirable in and of itself. Instead, output should vary one-for-one with the natural level of output, that is, $y_t = y_t^n$ for all t. There is no reason, in principle, why the natural level of output should be constant or take the form of a smooth trend, because all kinds of real shocks are a potential source of variation in its level.[2] In that context, policies that stress output stability (possibly around a smooth trend) may generate potentially large deviations of output from its natural level and, thus, be suboptimal. This point is illustrated below, in the context of the analysis of a simple policy rule.

Second, price stability emerges as a feature of the optimal policy even though, a priori, the policymaker does *not* attach any weight to such an objective. But under the assumptions made, price stability implies an efficient level of output, and vice versa. The previous finding, often referred to as the *divine coincidence* in the literature, implies that a central bank doesn't need to know or worry about what the efficient level of output is at each point in time, for the latter can be attained automatically, as a byproduct of a successful price stabilization policy.[3]

[2] This is arguably one of the main lessons from the Real Business Cycle literature.
[3] See Blanchard and Galí (2007).

The intuition behind the desirability of zero inflation in the case of an efficient natural allocation can be conveyed as follows: if price stability is attained, then it must be the case that no firm is adjusting its price even when having the option of doing so, from which it follows that the constraints on price setting are not binding and, hence, that the equilibrium allocation corresponds to that of an economy with flexible prices (which is, under the assumptions made here, efficient).

4.3.1 *Implementation: Optimal Interest Rate Rules*

Next, some candidate rules for implementing the optimal policy are considered. All of them are consistent with the desired equilibrium outcome, given by (4) and (5). Some, however, are *also* consistent with other suboptimal outcomes. In order to analyze its equilibrium implications, each candidate rule is embedded in the two equations describing the nonpolicy block of the basic New Keynesian model introduced in chapter 3. Those two key equations are shown here again for convenience:

$$\widetilde{y}_t = E_t\{\widetilde{y}_{t+1}\} - \frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} - r_t^n) \tag{6}$$

$$\pi_t = \beta E_t\{\pi_{t+1}\} + \kappa \widetilde{y}_t \tag{7}$$

where $r_t^n = \rho - \sigma(1-\rho_a)\psi_{ya} a_t + (1-\rho_z) z_t$ is the natural rate of interest.

4.3.1.1 AN EXOGENOUS INTEREST RATE RULE

Consider the candidate interest rate rule

$$i_t = r_t^n \tag{8}$$

for all t. This is a rule that instructs the central bank to adjust the nominal rate one-for-one with variations in the natural rate, and only in response to variations in the latter. Such a rule would seem a natural candidate to implement the optimal policy since (8) was shown earlier to hold in an equilibrium that attains the optimal allocation.

After substituting (8) into (6) and rearranging terms, the equilibrium conditions under rule (8) can represented by means of the system

$$\begin{bmatrix} \widetilde{y}_t \\ \pi_t \end{bmatrix} = \mathbf{A_O} \begin{bmatrix} E_t\{\widetilde{y}_{t+1}\} \\ E_t\{\pi_{t+1}\} \end{bmatrix} \tag{9}$$

where

$$\mathbf{A_O} \equiv \begin{bmatrix} 1 & \frac{1}{\sigma} \\ \kappa & \beta + \frac{\kappa}{\sigma} \end{bmatrix} \tag{10}$$

Note that $\widetilde{y}_t = \pi_t = 0$ for all t –the outcome associated with the efficient allocation—is *a* solution to (9). That solution, however, is *not* unique: It can be shown that one of the two (real) eigenvalues of $\mathbf{A_O}$ always lies in the interval (0, 1), while the second is strictly greater than unity. Given that both $\widetilde{y}_t$ and π_t are non-predetermined, the existence of an eigenvalue outside the unit circle implies the existence of a (local) multiplicity of equilibria in addition to $\widetilde{y}_t = \pi_t = 0$ for all t.[4] In that case nothing guarantees that the latter outcome will be precisely the one that will emerge as an equilibrium. That shortcoming leads to the consideration of rules alternative to (8).

4.3.1.2 AN INTEREST RATE RULE WITH AN ENDOGENOUS COMPONENT

Let us consider next the following interest rate rule

$$i_t = r_t^n + \phi_\pi \pi_t + \phi_y \widetilde{y}_t \tag{11}$$

where ϕ_π and ϕ_y are non-negative coefficients determined by the central bank, and that describe the strength of the interest rate response to deviations of inflation and the output gap from their zero target levels.

Substituting the nominal rate out of (6) using the assumed interest rate rule, and rearranging terms, the equilibrium dynamics can be represented by the system of difference equations

$$\begin{bmatrix} \widetilde{y}_t \\ \pi_t \end{bmatrix} = \mathbf{A_T} \begin{bmatrix} E_t\{\widetilde{y}_{t+1}\} \\ E_t\{\pi_{t+1}\} \end{bmatrix} \tag{12}$$

where

$$\mathbf{A_T} \equiv \Omega \begin{bmatrix} \sigma & 1 - \beta\phi_\pi \\ \sigma\kappa & \kappa + \beta(\sigma + \phi_y) \end{bmatrix} \tag{13}$$

and $\Omega \equiv \frac{1}{\sigma + \phi_y + \kappa\phi_\pi}$.

[4] See, e.g., Blanchard and Kahn (1980).

Once again, the desired outcome ($\widetilde{y}_t = \pi_t = 0$ for all t) is always *a* solution to the dynamical system (12) and, hence, *an* equilibrium of the economy under rule (11). Yet, in order for that outcome to be the only (stationary) equilibrium, that is, the only equilibrium that remains within a small neighborhood of the steady state, both eigenvalues of matrix $\mathbf{A_T}$ should lie within the unit circle. The size of those eigenvalues now depends on the policy coefficients (ϕ_π, ϕ_y), in addition to the nonpolicy parameters. Under the assumption of non-negative values for (ϕ_π, ϕ_y), a necessary and sufficient condition for $\mathbf{A_T}$ to have two eigenvalues within the unit circle and, hence, for the equilibrium to be unique, is given by[5]

$$\kappa(\phi_\pi - 1) + (1 - \beta)\phi_y > 0 \tag{14}$$

Thus, and in order to guarantee the successful implementation of the optimal policy, the monetary authority should respond to deviations of inflation and the output gap from their target levels by adjusting the nominal rate with "sufficient strength." Figure 4.1 illustrates graphically the regions of parameter space for (ϕ_π, ϕ_y) associated with determinate and indeterminate equilibria, as implied by condition (14).

Interestingly, and somewhat paradoxically, if condition (14) is satisfied, both the output gap and inflation will be zero and, hence, $i_t = r_t^n$ for all t will hold ex-post. But in contrast with the case considered above (in which the equilibrium outcome $i_t = r_t^n$ was also taken to be the policy rule), the presence of a "threat" of a strong response to an eventual deviation of the output gap and inflation from target suffices to rule out any such deviation in equilibrium.

Condition (14) has a simple interpretation: it implies that, in the absence of permanent change in the natural rate, and in response to a permanent increase in inflation, the central bank will eventually increase more than one-for-one the nominal interest rate. Formally,

$$\begin{aligned} \lim_{k\to\infty} \frac{di_{t+k}}{d\pi_t} &= \phi_\pi + \phi_y \lim_{k\to\infty} \frac{d\widetilde{y}_{t+k}}{d\pi_t} \\ &= \phi_\pi + \frac{\phi_y(1-\beta)}{\kappa} \end{aligned} \tag{15}$$

where the change in inflation $d\pi_t$ is taken to be permanent and where the second equality makes use of the long-term relationship between inflation and the output gap implied by (7). Note that condition (14)

[5] See Bullard and Mitra (2002) for a proof.

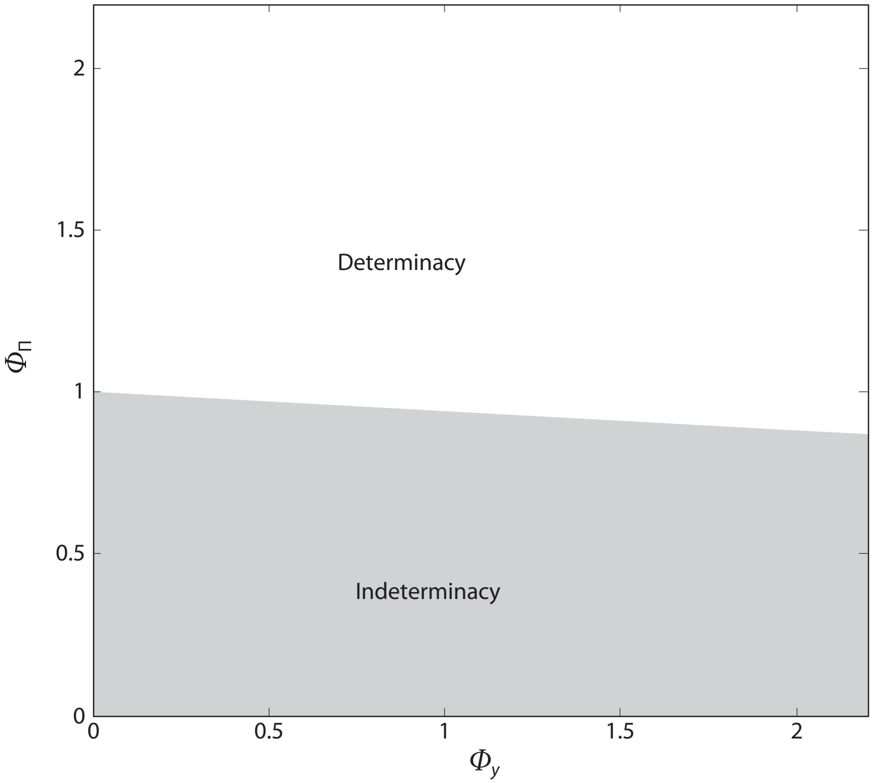

Figure 4.1. Determinacy and Indeterminacy Regions: Standard Taylor Rule.

is equivalent to the right-hand term in (15) being greater than one. Thus, the equilibrium will be unique under interest rate rule (11) whenever ϕ_π and ϕ_y are sufficiently large to guarantee that the real rate eventually rises in the face of a permanent increase in inflation. The previous property is often referred to as the *Taylor principle* and, to the extent that it prevents the emergence of multiple equilibria, it is naturally viewed as a desirable feature of any interest rate rule.[6]

4.3.1.3 A FORWARD-LOOKING INTEREST RATE RULE

In order to illustrate the existence of a multiplicity of policy rules capable of implementing the optimal policy, let us consider the following

[6] See Woodford (2001) for a discussion.

forward-looking rule

$$i_t = r_t^n + \phi_\pi E_t\{\pi_{t+1}\} + \phi_y E_t\{\widetilde{y}_{t+1}\} \tag{16}$$

which has the monetary authority adjust the nominal rate in response to variations in *expected* inflation and the *expected* output gap, as opposed to their current values, as assumed in (11).

Under (16) the implied dynamics are described by the system

$$\begin{bmatrix} \widetilde{y}_t \\ \pi_t \end{bmatrix} = \mathbf{A_F} \begin{bmatrix} E_t\{\widetilde{y}_{t+1}\} \\ E_t\{\pi_{t+1}\} \end{bmatrix}$$

where

$$\mathbf{A_F} \equiv \begin{bmatrix} 1 - \frac{\phi_y}{\sigma} & \frac{1-\phi_\pi}{\sigma} \\ \kappa(1 - \frac{\phi_y}{\sigma}) & \beta - \frac{\kappa\phi_\pi}{\sigma} \end{bmatrix}$$

In this case, the conditions for a (locally) unique equilibrium (i.e., for both eigenvalues of $\mathbf{A_F}$ lying within the unit circle) are twofold and given by[7]

$$\kappa(\phi_\pi - 1) + (1 - \beta)\phi_y > 0 \tag{17}$$

$$\kappa(\phi_\pi - 1) + (1 + \beta)\phi_y < 2\sigma(1 + \beta) \tag{18}$$

Note that (17) corresponds to the condition for determinacy in the case of the "contemporaneous" rule considered above, and has an identical interpretation in terms of the Taylor principle. But (local) determinacy of equilibrium under the present forward-looking rule also requires that the central bank does not reacts "too strongly" to deviations of inflation and/or the output gap from target, as made clear by condition (18).

Figure 4.2 represents the determinacy/indeterminacy regions in (ϕ_π, ϕ_y) space, under the baseline calibration for the remaining parameters. The figure suggests that the kind of "overreaction" that would be conducive to indeterminacy would require rather extreme values of the inflation and/or output gap coefficients, well above those characterizing empirical interest rate rules.

[7] Bullard and Mitra (2002) lists a third condition, given by the inequality $\phi_y < \sigma(1 + \beta^{-1})$, as necessary for uniqueness. But it can be easily checked that the latter condition is implied by the two conditions (17) and (18). I thank Davide Debortoli for this observation.

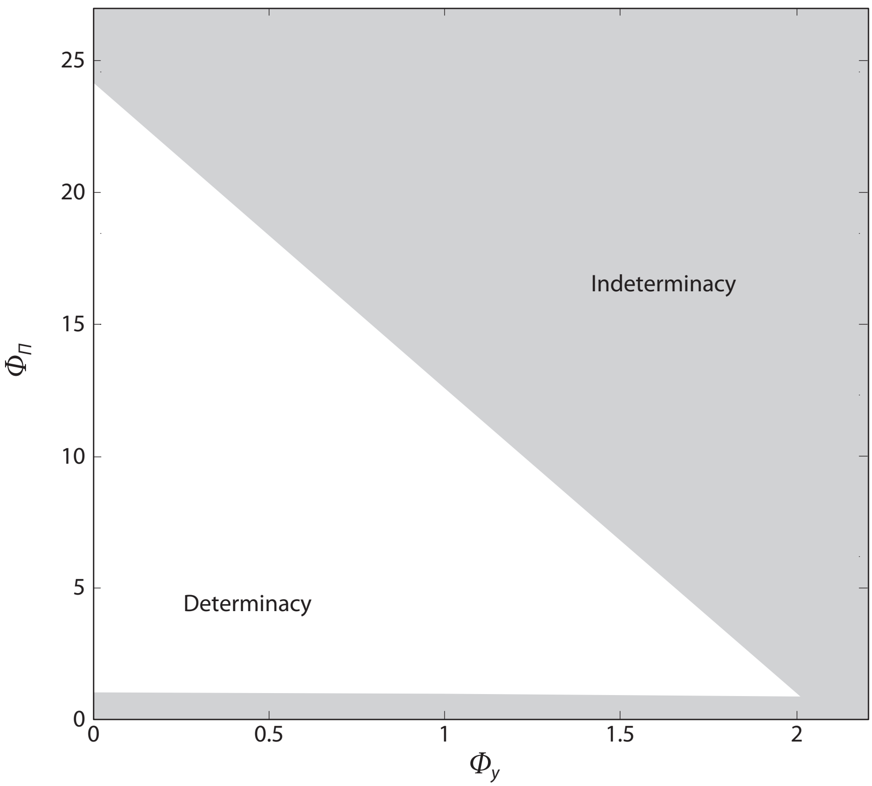

Figure 4.2. Determinacy and Indeterminacy Regions: Forward Looking Taylor Rule.

4.3.2 Shortcomings of Optimal Policy Rules

The previous subsection has provided two examples of interest rate rules that implement the optimal policy, thus guaranteeing that the efficient allocation is attained as the unique equilibrium outcome. While such optimal interest rate rules appear to take a relatively simple form, there exists an important reason why they are unlikely to provide useful practical guidance for the conduct of monetary policy. The reason is that they both require that the policy rate is adjusted one-for-one with the natural rate of interest, thus implicitly assuming observability of the latter variable. That assumption is plainly unrealistic because determination of the natural rate and its movements requires an exact knowledge of (i) the economy's "true model," (ii) the values taken by all its parameters, and (iii) the realized value (observed in real time) of all the shocks that affect the natural rate.

Note that a similar requirement would have to be met if, as implied by (11) and (16), the central bank should also adjust the nominal rate in response to deviations of output from the natural level of output, because the latter is also unobservable. That requirement, however, is not nearly as binding as the unobservability of the natural rate of interest, for nothing prevents the central bank from adopting a version of the optimal rule that does not require a systematic response to changes in the output gap. Formally, ϕ_y in (11) or (16) could be set to zero, with uniqueness of equilibrium being still guaranteed by the choice of an inflation coefficient greater than unity in the case of a rule responding to contemporaneous inflation (and no greater than $1+2\sigma(1+\beta)/\kappa$ in the case of the forward-looking rule).

The practical shortcomings of optimal interest rate rules discussed above have led many authors to propose a variety of "simple rules" —understood as rules that a central bank could arguably adopt in practice—and to analyze their properties.[8] In that context, an interest rate rule is generally considered "simple" if it makes the policy instrument a function of observable variables only, and does not require any precise knowledge of the exact model or the values taken by its parameters. The desirability of any given simple rule is thus given to a large extent by its robustness, that is, its ability to yield a good performance across different models and parameter configurations.

In the following section, two such simple rules are analyzed —a simple Taylor-type rule and a constant money growth rule—and their performance is assessed in the context of our baseline New Keynesian model.

4.4 Two Simple Monetary Policy Rules

This section provides an illustration of how the basic New Keynesian model developed in chapter 3 can be used to assess the performance of two policy rules. A formal evaluation of the performance of a simple rule (relative, say, to the optimal rule or to an alternative simple rule) requires the use of some quantitative criterion. Following the seminal work of Rotemberg and Woodford (1999), much of the literature has adopted a welfare-based criterion, relying on a second-order approximation to the utility losses experienced by the representative consumer as a consequence of deviations from the efficient allocation. As shown in appendix 4.1, under the assumptions made in this chapter (which guarantee the

[8] The volume edited by John Taylor (1999b) contains several important contributions in that regard.

optimality of the flexible price equilibrium), that approximation yields the following *welfare loss function*

$$\mathbb{W} = \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left[\left(\sigma + \frac{\varphi + \alpha}{1 - \alpha} \right) \widetilde{y}_t^2 + \frac{\epsilon}{\lambda} \pi_t^2 \right]$$

where losses are expressed in terms of the equivalent permanent consumption decline, measured as a fraction of steady state consumption.

The average welfare loss per period is thus given by a linear combination of the variances of the output gap and inflation

$$\mathbb{L} = \frac{1}{2} \left[\left(\sigma + \frac{\varphi + \alpha}{1 - \alpha} \right) \; var(\widetilde{y}_t) + \frac{\epsilon}{\lambda} \; var(\pi_t) \right]$$

Note that the relative weight of output gap fluctuations in the loss function is increasing in σ, φ, and α. The reason is that larger values of those "curvature" parameters amplify the effect of any given deviation of output from its natural (efficient) level on the size of the gap between the marginal rate of substitution and the marginal product of labor, which is a measure of the economy's aggregate inefficiency. On the other hand, the weight of inflation fluctuations is increasing in the elasticity of substitution among goods ϵ, for the latter amplifies the dispersion in the quantities of goods consumed of different varieties caused by any given price dispersion. The inflation weight is also increasing in the degree of price stickiness θ (which is inversely related to λ), since a greater stickiness amplifies the degree of price dispersion associated with any given deviation from zero inflation.

Given a policy rule and a calibration of the model's parameters, one can determine the implied variance of inflation and the output gap and the corresponding welfare losses associated with that rule (relative to the optimal allocation). That procedure is illustrated next through the analysis of two simple rules.

4.4.1 A Taylor Rule

Let us first consider the following interest rate rule

$$i_t = \rho + \phi_\pi \pi_t + \phi_y \widehat{y}_t \tag{19}$$

where $\widehat{y}_t \equiv \log(Y_t/Y)$ denotes the log deviation of output from its steady state and where $\phi_\pi > 0$ and $\phi_y > 0$ are assumed to satisfy determinacy condition (14). Again, the choice of intercept $\rho \equiv -\log \beta$ is consistent with a zero inflation steady state. A numerical rule of the same form as (19) with $\phi_\pi = 1.5$ and $\phi_y = 0.125$ was proposed by Taylor (1993) as an approximation to the behavior of the Fed in the early years of Alan Greenspan's tenure.[9] This explains why rule (19) is often referred to as a "Taylor rule."

Note that (19) can be rewritten in terms of the output gap as

$$i_t = \rho + \phi_\pi \pi_t + \phi_y \widetilde{y}_t + v_t \tag{20}$$

where $v_t \equiv \phi_y \widehat{y}_t^n$. The resulting equilibrium dynamics are thus identical to those of the interest rate rule analyzed in chapter 3, with v_t now interpreted as a term proportional to the deviations of natural output from steady state, instead of an exogenous monetary policy shock. Note that the variance of "pseudo-shock" v_t is no longer exogenous, but increasing in ϕ_y, the coefficient determining the response of the monetary authority to fluctuations in output. Formally, the equilibrium dynamics are described by the system

$$\begin{bmatrix} \widetilde{y}_t \\ \pi_t \end{bmatrix} = \mathbf{A}_\mathrm{T} \begin{bmatrix} E_t\{\widetilde{y}_{t+1}\} \\ E_t\{\pi_{t+1}\} \end{bmatrix} + \mathbf{B}_\mathrm{T}(\widehat{r}_t^n - v_t)$$

where, as in chapter 3, $\mathbf{A}_\mathrm{T}$ and $\mathbf{B}_\mathrm{T}$ are defined by

$$\mathbf{A}_T \equiv \Omega \begin{bmatrix} \sigma & 1 - \beta\phi_\pi \\ \sigma\kappa & \kappa + \beta(\sigma + \phi_y) \end{bmatrix}; \qquad \mathbf{B}_T \equiv \Omega \begin{bmatrix} 1 \\ \kappa \end{bmatrix}$$

with $\Omega \equiv \frac{1}{\sigma + \phi_y + \kappa\phi_\pi}$.

Using the expressions for $\widehat{y}_t^n$ and $\widehat{r}_t^n$ one can write

$$\begin{aligned} \widehat{r}_t^n - v_t &= -\sigma\psi_{ya}(1 - \rho_a)a_t - \phi_y\psi_{ya}a_t + (1 - \rho_z)z_t \\ &= -\psi_{ya}(\sigma(1 - \rho_a) + \phi_y)a_t + (1 - \rho_z)z_t \end{aligned}$$

[9] The original Taylor rule was given by

$$i_t = 4 + 1.5(\pi_t - 2) + 0.5\widehat{y}_t$$

with both the nominal rate and inflation expressed in annualized terms and $\widehat{y}_t$ representing the log deviation of output from a deterministic trend. Note that the rule implies an inflation target of 2 percent and a steady state real interest rate of 2 percent (both annualized). The $\phi_y = 0.125$ value used in the text results from rewriting the original Taylor rule in terms of quarterly interest and inflation rates.

TABLE 4.1
Evaluation of Simple Rules: Taylor Rule

	Technology				*Demand*			
ϕ_π	1.5	1.5	5	1.5	1.5	1.5	5	1.5
ϕ_y	0.125	0	0	1	0.125	0	0	1
$\sigma(y)$	1.85	2.07	2.25	1.06	0.59	0.68	0.28	0.31
$\sigma(\widetilde{y})$	0.44	0.21	0.03	1.23	0.59	0.68	0.28	0.31
$\sigma(\pi)$	0.69	0.34	0.05	1.94	0.20	0.23	0.09	0.10
$\mathbb{L}$	1.02	0.25	0.006	7.98	0.10	0.13	0.02	0.02

where, as in chapter 3, $\psi_{ya} \equiv \frac{1+\varphi}{\sigma+\varphi+\alpha(1-\sigma)} > 0$. Note that under the present rule, and in contrast with the rule analyzed in chapter 3 that had the output gap $\widetilde{y}_t$ as an argument, changes in coefficient ϕ_y have an effect on the variance of the driving force term $\widehat{r}_t^n - v_t$.

Table 4.1 reports the standard deviation of output, the output gap and inflation for different configurations of coefficients ϕ_π and ϕ_y in rule (19). The analysis is conducted conditional on technology and demand/preference shocks separately. The standard deviation of the innovations in both the technology and preference processes is set to one percent. For each shock, the first column reports results based on the original calibration proposed by Taylor (1993). The second and third columns are based on a rules involving no response to output fluctuations, with a very aggressive anti-inflation stance in the case of the third rule ($\phi_\pi = 5$). Finally, the fourth rule assumes a strong output-stabilization motive ($\phi_y = 1$). The remaining parameters are calibrated at their baseline values, introduced in chapter 3.

For each version of the Taylor rule, table 4.1 shows the implied standard deviations of output, the output gap, and (quarterly) inflation, all expressed in percentage terms, as well as the welfare losses resulting from the deviations from the efficient allocation, expressed as a percentage of steady state consumption. Several results stand out. First, when technology shocks are the source of fluctuations, a tradeoff emerges between stabilization of output on the one hand, and stabilization of inflation and the output gap on the other: increasing the value of coefficient ϕ_y leads to a reduction in the volatility of output, but to higher volatility in the output gap and inflation and, hence, larger welfare losses. Those losses increase substantially when the output coefficient ϕ_y is set to unity, relative to Taylor's original calibration. Second, the smallest welfare losses are attained when the monetary authority responds to changes in inflation only. Furthermore, those losses (as well as the underlying fluctuations in the output gap and inflation) become smaller as the strength of that response increases.

When the analysis is conditioned on demand shocks being the source of fluctuations, the previous tradeoff vanishes: the natural level of output remains unchanged, implying that output stabilization is equivalent to output gap stabilization. Thus, increases in either ϕ_π or ϕ_y appear to be effective at stabilizing the welfare-relevant variables and reducing welfare losses.

Taking all the findings above into account, it can be concluded that in the context of the basic New Keynesian model considered here, a simple Taylor-type rule that responds aggressively to movements in inflation can approximate arbitrarily well the optimal policy.[10]

4.4.2 A Constant Money Growth Rule

Next, a simple rule consisting of a constant growth rate for the money supply is considered, a rule generally associated with Friedman (1960). Without loss of generality, a zero money supply growth rate is assumed, which is consistent with zero inflation in the steady state (given the absence of secular growth). Formally,

$$\Delta m_t = 0$$

for all t.

Once again, the assumption of a monetary rule requires that equilibrium conditions (6) and (7) be supplemented with a money demand equation. The latter is taken to be of the form

$$l_t = y_t - \eta i_t - \zeta_t$$

where $l_t \equiv m_t - p_t$ denotes (log) real balances and ζ_t is an exogenous money demand shock following the process

$$\Delta\zeta_t = \rho_\zeta \Delta\zeta_{t-1} + \varepsilon_t^\zeta$$

where $\rho_\zeta \in [0, 1)$.

It is convenient to rewrite the money demand equation in terms of deviations from steady state as

$$\widehat{l}_t = \widetilde{y}_t + \widehat{y}_t^n - \eta \widehat{i}_t - \zeta_t$$

[10] It is easy to check that the variances of the output gap and inflation both converge to zero as $\phi_\pi \to +\infty$.

Letting $l_t^+ \equiv l_t + \zeta_t$ denote (log) real balances adjusted by the exogenous component of money demand,

$$\widehat{i}_t = \frac{1}{\eta}(\widetilde{y}_t + \widehat{y}_t^n - \widehat{l}_t^+)$$

In addition, using the definition of l_t^+ together with the assumed rule $\Delta m_t = 0$, and imposing clearing of the money market:

$$\widehat{l}_{t-1}^+ = \widehat{l}_t^+ + \pi_t - \Delta\zeta_t$$

Combining the previous two equations with (6) and (7) to substitute out the nominal rate, the equilibrium dynamics under a constant money growth rule can be summarized by the system

$$\mathbf{A_{M,0}} \begin{bmatrix} \widetilde{y}_t \\ \pi_t \\ \widehat{l}_{t-1}^+ \end{bmatrix} = \mathbf{A_{M,1}} \begin{bmatrix} E_t\{\widetilde{y}_{t+1}\} \\ E_t\{\pi_{t+1}\} \\ \widehat{l}_t^+ \end{bmatrix} + \mathbf{B_M} \begin{bmatrix} \widehat{r}_t^n \\ \widehat{y}_t^n \\ \Delta\zeta_t \end{bmatrix}$$

where $\mathbf{A_{M,0}}$, $\mathbf{A_{M,1}}$, and $\mathbf{B_M}$ are defined as in chapter 3. Thus, under the monetary targeting rule considered here, money demand shocks will generally lead to fluctuations in the output gap and inflation, in a way formally identical to exogenous money supply shocks in the analysis of chapter 3. Note that this is in contrast with a Taylor rule or any other interest rate rule, for in that case the money supply is endogenous and gets adjusted one-for-one with any changes in money demand, insulating the interest rate and leaving the remaining variables unaffected.

Table 4.2 (see end of Appendix 4.1) reports the standard deviation of output, the output gap and inflation, as well as the implied welfare losses, under a constant money growth rule. Statistics are reported conditional on each of the three shocks considered here. The parameters of the process for the money demand shock are calibrated on the basis of an $AR(1)$ model estimated using the residual of an empirical money demand function for U.S. M2.[11] This yields $\rho_\zeta = 0.2$ and $\sigma_\zeta = 0.01$. The calibration of the process for the technology and preference shifters is the same as above.

[11] Specifically, a time series for the money demand shifter is

$$\zeta_t = y_t + p_t - m_t - 3.77 i_t$$

where 3.77 is the estimated interest rate semielasticity using OLS over the period 1960:1–1988:4, which is characterized by a relatively stable money demand.

Note that the constant money growth rule performs reasonably well relative to the basic Taylor rule, conditional on technology and demand shocks. In particular the implied welfare losses are below those generated by the original Taylor rule. Yet, it performs worse than the version of the Taylor rule that responds aggressively to inflation but not at all to output. In other words, it is not difficult to come up with an interest rate rule that improves on the performance of a constant money growth rule.

On the other hand the fluctuations and welfare losses associated with money demand disturbances, nonexistent in the case of an interest rate rule, are substantial when a constant money growth is adopted. Thus, the degree of stability of money demand is a key element in determining the desirability of a rule that focuses on the control of a monetary aggregate.

4.5 Notes on the Literature

An early detailed discussion of the case for price stability in the basic New Keynesian model can be found in Goodfriend and King (1997). Svensson (1997) contains an analysis of the desirability of inflation targeting strategies, using a not-fully-microfounded model.

When deriving the optimal policy no inherited dispersion of prices across firms was assumed. A rigorous analysis of the optimal monetary policy in the case of an initial nondegenerate price distribution can be found in Yun (2005).

Taylor (1993) introduced the simple formula commonly known as the *Taylor rule*, as one that provided a good approximation to Fed policy in the early Greenspan years. Judd and Rudebusch (1998) and Clarida, Galí, and Gertler (2000) estimate alternative versions of the Taylor rule, and examine its (in)stability over the postwar period. Taylor (1999a) uses the rule calibrated for the Greenspan years as a benchmark for the evaluation of monetary policy during other episodes over the postwar period. Orphanides (2003) argues that the bulk of the deviations from the baseline Taylor rule observed in the pre-Volcker era may have been the result of large biases in real-time measures of the output gap. The contributions in Koenig, Leeson and Kahn (2012) provide some historical perspective on the influence of Taylor-type rules in both research and policy.

Key contributions to the literature on the properties of alternative simple rules can be found in the papers contained in the volume edited by Taylor (1999b). In particular, the paper by Rotemberg and Woodford (1999) derives a second-order approximation to the utility of the representative consumer. Chapter 6 in Woodford (2003) provides a detailed discussion of welfare-based evaluation of policy rules.

Appendix

4.1 A Second-Order Approximation to a Household's Welfare: The Case of an Undistorted Steady State

This appendix derives a second-order approximation to the utility of the representative consumer when the economy remains in a neighborhood of an efficient steady state, in a way consistent with the assumptions made in this chapter. The generalization to the case of a distorted steady state is left for chapter 5.

A second-order approximation of utility is derived around a given steady state allocation. Frequent use is made of the following second-order approximation of relative deviations in terms of log deviations

$$\frac{X_t - X}{X} \simeq \widehat{x}_t + \frac{1}{2}\widehat{x}_t^2$$

where $\widehat{x}_t \equiv x_t - x \equiv \log(X_t/X)$ is the log deviation from steady state for a generic variable X_t. All along it is assumed that utility is separable in consumption and hours (i.e., $U_{cn} = 0$). In order to lighten the notation, define $U_t \equiv U(C_t, N_t; Z_t)$, $U_t^n \equiv U(C_t^n, N_t^n; Z_t)$, and $U \equiv U(C, N; Z)$. As in chapter 3, utility is assumed to take the form $U(C_t, N_t; Z_t) = \overline{U}(C_t, N_t)Z_t$.

The second-order Taylor expansion of U_t around a steady state (C, N) yields

$$\begin{aligned} U_t - U \simeq\ & U_c C\left(\frac{C_t - C}{C}\right) + U_n N\left(\frac{N_t - N}{N}\right) + \frac{1}{2}U_{cc}C^2\left(\frac{C_t - C}{C}\right)^2 \\ & + \frac{1}{2}U_{nn}N^2\left(\frac{N_t - N}{N}\right)^2 + U_c C\left(\frac{C_t - C}{C}\right)\left(\frac{Z_t - Z}{Z}\right) \\ & + U_n N\left(\frac{N_t - N}{N}\right)\left(\frac{Z_t - Z}{Z}\right) + t.i.p. \end{aligned}$$

where $t.i.p.$ stands for *terms independent of policy*.

In terms of log deviations, and ignoring terms independent of policy.

$$\begin{aligned} U_t - U \simeq\ & U_c C\left(\widehat{y}_t(1 + z_t) + \frac{1-\sigma}{2}\,\widehat{y}_t^2\right) \\ & + U_n N\left(\widehat{n}_t(1 + z_t) + \frac{1+\varphi}{2}\,\widehat{n}_t^2\right) + t.i.p. \end{aligned}$$

where $\sigma \equiv -\frac{U_{cc}}{U_c}C$ and $\varphi \equiv \frac{U_{nn}}{U_n}N$, and where use of the market clearing condition $\widehat{c}_t = \widehat{y}_t$ has been made.

The next step consists in rewriting $\widehat{n}_t$ in terms of output. Using the fact that $N_t = \left(\frac{Y_t}{A_t}\right)^{\frac{1}{1-\alpha}} \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\frac{\epsilon}{1-\alpha}} di$,

$$(1-\alpha)\widehat{n}_t = \widehat{y}_t - a_t + d_t$$

where $d_t \equiv (1-\alpha)\log \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\frac{\epsilon}{1-\alpha}} di$. The following lemma shows that d_t is proportional to the cross-sectional variance of relative prices.

***Lemma 1*:** in a neighborhood of a symmetric steady state, and up to a second-order approximation, $d_t = \frac{\epsilon}{2\Theta}\ var_i\{p_t(i)\}$, where $\Theta \equiv \frac{1-\alpha}{1-\alpha+\alpha\epsilon}$.

Proof: see the appendix in chapter 3.4

Now, the period t utility can be rewritten (ignoring terms of third or higher order) as

$$\begin{aligned} U_t - U = U_c C &\left(\widehat{y}_t(1+z_t) + \frac{1-\sigma}{2}\,\widehat{y}_t^2\right) \\ &+\frac{U_n N}{1-\alpha}\left(\widehat{y}_t(1+z_t) + \frac{\epsilon}{2\Theta}\ var_i\{p_t(i)\} + \frac{1+\varphi}{2(1-\alpha)}(\widehat{y}_t - a_t)^2\right) \\ &+t.i.p. \end{aligned}$$

Efficiency of the steady state implies $-\frac{U_n}{U_c} = MPN$. Thus, and using the fact that $MPN = (1-\alpha)(Y/N)$ and $Y = C$,

$$\begin{aligned} \frac{U_t - U}{U_c C} &\simeq -\frac{1}{2}\left[\frac{\epsilon}{\Theta}\ var_i\{p_t(i)\} - (1-\sigma)\widehat{y}_t^2 + \left(\frac{1+\varphi}{1-\alpha}\right)(\widehat{y}_t - a_t)^2\right] + t.i.p. \\ &= -\frac{1}{2}\left[\frac{\epsilon}{\Theta}\ var_i\{p_t(i)\} + \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)\widehat{y}_t^2 - 2\left(\frac{1+\varphi}{1-\alpha}\right)\widehat{y}_t a_t\right] + t.i.p. \\ &= -\frac{1}{2}\left[\frac{\epsilon}{\Theta}\ var_i\{p_t(i)\} + \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)(\widehat{y}_t^2 - 2\widehat{y}_t\widehat{y}_t^n)\right] + t.i.p. \\ &= -\frac{1}{2}\left[\frac{\epsilon}{\Theta}\ var_i\{p_t(i)\} + \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)\widetilde{y}_t^2\right] + t.i.p. \end{aligned}$$

where $\widehat{y}_t^n \equiv y_t^n - y^n$, and where the fact was used that $\widehat{y}_t^n = \frac{1+\varphi}{\sigma(1-\alpha)+\varphi+\alpha}\, a_t$ and $\widehat{y}_t - \widehat{y}_t^n = \widetilde{y}_t$.

Accordingly, a second-order approximation to the consumer's welfare losses can be written and expressed as a fraction of steady state consumption (and up to additive terms independent of policy) as

$$\mathbb{W} = -E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{U_t - U}{U_c C} \right)$$

$$= \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{\epsilon}{\Theta}\, var_i\{p_t(i)\} + \left(\sigma + \frac{\varphi + \alpha}{1 - \alpha} \right) \widetilde{y}_t^2 \right)$$

The final step consists in rewriting the terms involving the price dispersion variable as a function of inflation. In order to do so, make use of the following lemma

Lemma 2: $\sum_{t=0}^{\infty} \beta^t\, var_i\{p_t(i)\} = \frac{\theta}{(1-\beta\theta)(1-\theta)} \sum_{t=0}^{\infty} \beta^t \pi_t^2$

Proof: Woodford (2003, chap. 6)

Using the fact that $\lambda \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}\, \Theta$, the previous lemma can be combined with the expression above to obtain

$$\mathbb{W} = \frac{1}{2}\, E_0 \sum_{t=0}^{\infty} \beta^t \left[\left(\frac{\epsilon}{\lambda} \right) \pi_t^2 + \left(\sigma + \frac{\varphi + \alpha}{1 - \alpha} \right) \widetilde{y}_t^2 \right]$$

TABLE 4.2
Evaluation of Simple Rules: Constant Money Growth

	Technology	*Demand*	*Money Demand*
$\sigma(y)$	1.72	0.59	1.07
$\sigma(\widetilde{y})$	0.92	0.59	1.07
$\sigma(\pi)$	0.35	0.12	0.55
$\mathbb{L}$	0.29	0.04	0.69

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Exercises

4.1. Some indeterminacy proofs

a. Prove that matrix $\mathbf{A}_0$ in (10) has two real positive eigenvalues, but only one of them is greater than unity.
b. Prove that matrix $\mathbf{A}_T$ in (13), with $\phi_\pi \geq 0$ and $\phi_y \geq 0$, has its two eigenvalues outside the unit circle if and only condition (14) is satisfied.

4.2. Inflation targeting with noisy data

Consider a model economy whose output gap and inflation dynamics are described by the system

$$\pi_t = \beta E_t\{\pi_{t+1}\} + \kappa \widetilde{y}_t$$
$$\widetilde{y}_t = -\frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} - r_t^n) + E_t\{\widetilde{y}_{t+1}\}$$

where all variables are defined as in the text. The natural rate r_t^n is assumed to follow the exogenous process

$$r_t^n - \rho = \rho_r(r_{t-1}^n - \rho) + \varepsilon_t$$

where $\{\varepsilon_t\}$ is a white-noise process and $\rho_r \in [0, 1)$.

Suppose that inflation is measured with some *i.i.d.* error ξ_t, i.e., $\pi_t^o = \pi_t + \xi_t$ where π_t^o denotes measured inflation. Assume that the central bank follows the rule

$$i_t = \rho + \phi_\pi \pi_t^o$$

a. Solve for the equilibrium processes for inflation and the output gap under the previous rule (Hint: you may want to start analyzing the simple case of $\rho_r = 0$).
b. Describe the behavior of inflation, the output gap, and the nominal rate when ϕ_π approaches infinity.
c. Determine the size of the inflation coefficient ϕ_π that minimizes the variance of actual inflation.

4.3. Monetary policy and the effects of technology shocks

Consider a New Keynesian economy with equilibrium conditions

$$y_t = E_t\{y_{t+1}\} - \frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} - \rho)$$
$$\pi_t = \beta E_t\{\pi_{t+1}\} + \kappa(y_t - y_t^n)$$

where all variables are defined as in the text.

Monetary policy is described by a simple rule of the form

$$i_t = \rho + \phi_\pi \pi_t$$

where $\phi_\pi > 1$. Labor productivity is given by

$$y_t - n_t = a_t$$

where a_t is an exogenous technology parameter that evolves according to

$$a_t = \rho_a a_{t-1} + \varepsilon_t$$

where $\rho_a \in [0, 1)$ and $\{\varepsilon_t\}$ is an i.i.d. process.
The natural level of output is assumed to be proportional to technology

$$y_t^n = \psi_y a_t$$

where $\psi_y > 1$.

a. Determine the equilibrium response of output, employment, and inflation to a technology shock. (Hint: Guess that each endogenous variable will be proportional to the contemporaneous value of technology.)
b. Describe how those responses depend on the value of ϕ_π and κ. Provide some intuition. What happens when $\phi_\pi \longrightarrow \infty$? What happens as the degree of price rigidities changes?
c. Analyze the joint response of employment and output to a technology shock and discuss briefly the implications for assessment of the role of technology as a source of business cycles.

4.4. **Interest rate versus money supply rules**
Consider an economy described by the equilibrium conditions

$$\widetilde{y}_t = E_t\{\widetilde{y}_{t+1}\} - \frac{1}{\sigma}\,(i_t - E_t\{\pi_{t+1}\} - r_t^n)$$
$$\pi_t = \beta E_t\{\pi_{t+1}\} + \kappa\,\widetilde{y}_t$$

$$m_t - p_t = y_t - \eta i_t$$

where all variables are defined as in the text. Both y_t^n and r_t^n evolve exogenously, independent of monetary policy.
The central bank seeks to minimize a loss function of the form

$$\vartheta\; var(\widetilde{y}_t) + var(\pi_t)$$

a. Show how the optimal policy could be implemented by means of an interest rate rule.
b. Show that a rule requiring a constant money supply will generally be suboptimal. Explain. (Hint: derive the path of money under the optimal policy.)

c. Derive a money supply rule that would implement the optimal policy.

4.5. **Optimal monetary policy with price setting in advance**
Consider an economy where the representative consumer maximizes

$$E_0 \sum_{t=0}^{\infty} \beta^t U\left(C_t, \frac{M_t}{P_t}, N_t\right)$$

subject to a sequence of dynamic budget constraints

$$P_t C_t + M_t + Q_t B_t \leq M_{t-1} + B_{t-1} + W_t N_t + D_t$$

and where all variables are defined as in the text.
Assume that period utility is given by

$$U(C_t, \frac{M_t}{P_t}, N_t) = \log C_t + \log \frac{M_t}{P_t} - \frac{N_t^{1+\varphi}}{1+\varphi} \tag{21}$$

Firms are monopolistically competitive, each producing a differentiated good whose demand is given by $Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} Y_t$. Each firm has access to the linear production function

$$Y_t(i) = A_t N_t(i) \tag{22}$$

where productivity evolves according to

$$\frac{A_t}{A_{t-1}} = (1+\gamma_a) \exp\{\varepsilon_t^a\}$$

with $\{\varepsilon_t^a\}$ being an i.i.d. normally distributed process with mean zero and variance σ_a^2.
The money supply varies exogenously according to the process

$$\frac{M_t}{M_{t-1}} = (1+\gamma_m) \exp\{\varepsilon_t^m\} \tag{23}$$

where $\{\varepsilon_t^m\}$ is an i.i.d. normally distributed process with mean zero and variance σ_m^2.

Finally, assume that all output is consumed, so that in equilibrium $Y_t = C_t$ for all t.

a. Derive the optimality conditions for the problem facing the representative consumer.
b. Assume that firms are monopolistically competitive, each producing a differentiated good. Each period, after observing the shocks, firms set the price of their good in order to maximize current profit

$$Y_t(i)\left(P_t(i) - \frac{W_t}{A_t}\right)$$

subject to the demand schedule above. Derive the optimality condition associated with the firm's problem.
c. Solve (exactly) for the equilibrium levels of aggregate employment, output, and inflation
d. Discuss how utility depends on the two parameters describing monetary policy, γ_m and σ_m^2. Show that the optimal policy must satisfy the Friedman rule and discuss alternative ways of supporting that rule in equilibrium.
e. Next, assume that for each period firms have to set the price in advance, i.e., before the realization of the shocks. In that case they will choose a price in order to maximize the discounted profit

$$E_{t-1}\left\{\Lambda_{t-1,t}\frac{Y_t(i)}{P_{t+1}}\left(P_t(i) - \frac{W_t}{A_t}\right)\right\}$$

subject to the demand schedule $Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} Y_t$, where $\Lambda_{t-1,t} \equiv \beta\frac{C_{t-1}}{C_t}$ is the relevant stochastic discount factor. Derive the first-order condition of the firm's problem and solve (exactly) for the equilibrium levels of employment, output and real balances.
f. Evaluate expected utility at the equilibrium values of output, real balances and employment.
g. Consider the class of money supply rules of the form (23) with $\varepsilon_t^m = \phi_a \varepsilon_t^a + \phi_v \nu_t$, where $\{\nu_t\}$ is a normally distributed i.i.d. process with zero mean and unit variance, and independent of $\{\varepsilon_t^a\}$ at all leads and lags. Notice that within that family of rules, monetary policy is fully described by three parameters: γ_m, ϕ_ε, and ϕ_v. Determine the values of those parameters

that maximize expected utility, subject to the constraint of a non-negative nominal interest rate. Show that the resulting equilibrium under the optimal policy replicates the flexible price equilibrium analyzed above.

4.6. **A price level based interest rate rule**
Consider an economy described by the equilibrium conditions

$$\widetilde{y}_t = E_t\{\widetilde{y}_{t+1}\} - \frac{1}{\sigma}\,(i_t - E_t\{\pi_{t+1}\} - r_t^n)$$

$$\pi_t = \beta E_t\{\pi_{t+1}\} + \kappa\,\widetilde{y}_t$$

Show that the interest rate rule

$$i_t = r_t^n + \phi_p \widehat{p}_t$$

where $\widehat{p}_t \equiv p_t - p^*$, where p^* is a price level target, generates a unique stationary equilibrium, if and only if, $\phi_p > 0$.

Chapter 5

MONETARY POLICY TRADEOFFS: DISCRETION VERSUS COMMITMENT

5.1 Monetary Policy Tradeoffs: Discretion versus Commitment

In chapter 4 the optimal monetary policy problem was analyzed in the context of a baseline model in which the presence of staggered price setting was the only relevant distortion that the central bank had to confront. It was shown that a policy that seeks to replicate the flexible price equilibrium allocation is both feasible and optimal in that context. That policy requires that the central bank responds to shocks so that the price level is fully stabilized. The rationale for such a policy is easy to summarize: With zero inflation output equals its natural level, which in turn, under the assumptions made in chapter 4, is also the efficient level. In such an environment the central bank does not face a meaningful policy tradeoff and "strict inflation targeting" emerges as the optimal policy.

The analysis of such an environment and its implications for the design of monetary policy is useful from a pedagogical point of view, but is not realistic. The reason is that, in practice, central banks view themselves as facing significant tradeoffs, at least in the short run. As a result, even central banks that call themselves "inflation targeters" do not claim to be seeking to stabilize inflation completely month by month or quarter by quarter, regardless of the consequences that this would entail for the evolution of real variables like output and employment. Instead, the presence of short-run tradeoffs have led inflation targeting central banks to pursue a policy that allows for a partial accommodation of inflationary pressures in the short run. This is in order to avoid an excessive instability of output and employment, while remaining committed to a medium-term inflation target. A policy of that kind is often referred to in the literature as *flexible inflation targeting*.[1]

[1] The term *flexible inflation targeting* was coined by Lars Svensson, to refer to the monetary policies that result from the minimization of a central bank loss function that attaches a nonzero penalty to output gap fluctuations, in the presence of a tradeoff between stabilization of inflation and stabilization of the output gap.

In this chapter, the optimal design of monetary policy is analyzed in environments in which the central bank faces a nontrivial tradeoff. Three such environments are considered. Section 5.1 studies the optimal monetary policy problem when the gap between the natural and the efficient levels of output varies over time, but both variables coincide in the steady state, that is, the steady state is efficient. Section 5.2 analyzes an analogous problem for an economy for which the steady state itself is inefficient, possibly due to firms' market power. Finally, section 5.3 studies the challenges facing an optimizing monetary authority resulting from the presence of a zero lower bound on the nominal interest rate. For each of the three cases the optimal monetary policy problem is solved under two alternative assumptions regarding the central bank's ability to commit to a given plan. Thus, under the optimal *discretionary* policy each period the central bank makes whatever decision is optimal at that time, without feeling bound by earlier promises. By contrast, the optimal policy with commitment assumes that the central bank can commit to a state-contingent policy plan, which generally involves actions that are suboptimal ex-post, but whose anticipation leads to an improvement of the policy tradeoffs (and higher welfare).

5.2 Monetary Policy Tradeoffs under an Efficient Steady State

When nominal rigidities coexist with *real* imperfections, the flexible price equilibrium allocation is generally inefficient. In that case, it is no longer optimal for the central bank to seek to replicate that allocation. On the other hand, any deviation of economic activity from its natural (i.e., flexible price) level generates variations in inflation, with consequent relative price distortions.

A special case of interest arises when the possible inefficiencies associated with the flexible price equilibrium do not affect the steady state, which remains efficient. This section analyzes the optimal monetary policy problem under that assumption. In contrast with the analysis in chapter 4, however, here *short-run* deviations are allowed for between the natural level of output, y_t^n, and its efficient counterpart, denoted by y_t^e. More precisely, the gap between the two is assumed to follow a stationary process with a zero mean. Implicitly, the presence is assumed of some real imperfections that generate a time-varying gap between output and its efficient counterpart even in the absence of price rigidities.

In that case, and as shown in appendix 5.1, the welfare losses experienced by the representative household are, up to a second-order approximation, proportional to

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\pi_t^2 + \vartheta x_t^2\right) \tag{1}$$

where $x_t \equiv y_t - y_t^e$ can be interpreted as the *welfare-relevant output gap*, with y_t^e denoting the (log) *efficient* level of output. As before, $\pi_t \equiv p_t - p_{t-1}$ denotes the rate of inflation between periods $t-1$ and t. Coefficient ϑ represents the weight of output gap fluctuations (relative to inflation) in the loss function, and is given by $\vartheta = \kappa/\epsilon$, where κ is the coefficient on the output gap in the New Keynesian Phillips curve (as derived in chapter 3) and ϵ is the elasticity of substitution between goods. More generally, and stepping beyond the welfare-theoretic justification for (1), one can interpret ϑ as the weight attached by the central bank to deviations of output from its efficient level (relative to price stability) in its own loss function, which does not necessarily have to coincide with the household's.

A structural equation relating inflation and the welfare-relevant output gap can be derived by using the identity $\widetilde{y}_t \equiv (y_t - y_t^e) + (y_t^e - y_t^n)$ to substitute for the output gap $\widetilde{y}_t$ in the NKPC relationship derived in chapter 3. This yields the following structural equation for inflation

$$\pi_t = \beta E_t\{\pi_{t+1}\} + \kappa x_t + u_t \tag{2}$$

where $u_t \equiv \kappa(y_t^e - y_t^n)$.

Hence, the central bank will seek to minimize (1) subject to the sequence of constraints given by (2). Two features of that problem are worth stressing. First, note that, under the previous assumptions, the disturbance u_t is exogenous with respect to monetary policy, because the latter can influence neither the natural nor the efficient level of output. As a result, the central bank will always take the current and anticipated values of u_t as given when solving its policy problem.

Second, and most important, time variations in the gap between the efficient and natural levels of output—reflected in fluctuations in u_t—generate a tradeoff for the monetary authority, because they make it impossible to attain simultaneously zero inflation and an efficient level of activity. This is a key difference from the model analyzed in chapter 4, where $y_t^n = y_t^e$ for all t, thus implying $u_t = 0$ for all t. In appendix 5.2 several potential sources of variation in the gap between the efficient and natural levels of output are discussed, including exogenous changes in desired markups, as well as fluctuations in labor income taxes. Nevertheless, at least for the purposes of the analysis in this chapter, knowledge of the specific source of that gap is not important.

Following much of the literature, the disturbance u_t in (2) is referred to as a *cost-push shock*. Also, and for the remainder of this chapter, assume that u_t follows the exogenous AR(1) process

$$u_t = \rho_u u_{t-1} + \varepsilon_t^u \tag{3}$$

where $\rho_u \in [0, 1)$, and $\{\varepsilon_t^u\}$ is a white-noise process with constant variance σ_u^2.

The forward-looking nature of constraint (2) in the policy problem requires the specification of the extent to which the central bank can credibly commit in advance to future policy actions. As will be clear below, the reason is that by committing to some future policies the central bank is able to influence expectations in a way that improves its short-run tradeoffs. The next two subsections characterize the optimal monetary policy under two alternative (and extreme) assumptions regarding the central bank's ability to commit to future policies.

5.2.1 Optimal Discretionary Policy

Start by considering a case in which the central bank cannot commit itself to any future action. Under that assumption the above problem becomes one of sequential optimization, that is, each period the central bank makes whatever decision is optimal at that time, without feeling bound by any earlier promises. That case is often referred to in the literature as *optimal policy under discretion*.

More specifically, each period the monetary authority is assumed to choose output and inflation in order to minimize the period losses

$$\pi_t^2 + \vartheta x_t^2$$

subject to the constraint

$$\pi_t = \kappa x_t + \upsilon_t$$

where the term $\upsilon_t \equiv \beta E_t\{\pi_{t+1}\} + u_t$ is taken as given by the monetary authority, because u_t is exogenous and $E_t\{\pi_{t+1}\}$ is a function of expectations about *future* output gaps (as well as future u_t's) which, by assumption, cannot be currently influenced by the policymaker.[2]

[2] To be precise, the term $E_t\{\pi_{t+1}\}$ can be treated as given by the central bank because there are no endogenous state variables (e.g., past inflation) affecting current inflation. Otherwise the central bank would have to take into account the influence that its current actions, through their impact on those state variables, would have on future inflation.

The optimality condition for the problem above is given by

$$x_t = -\frac{\kappa}{\vartheta}\,\pi_t \tag{4}$$

for $t = 0, 1, 2, \ldots$ The previous condition has a simple interpretation: In the face of inflationary pressures resulting from a cost-push shock the central bank must respond by driving output below its efficient level, thus creating a negative output gap, with the objective of dampening the rise in inflation. The central bank carries out such a "leaning against the wind" policy up to the point where condition (4) is satisfied. Thus, one can view that condition as a relation between target variables that the discretionary central bank will seek to maintain at all times, and it is in that sense that it may be labeled a "targeting rule."[3]

Using (4) to substitute for x_t in (2), yields the following difference equation for inflation

$$\pi_t = \frac{\vartheta\beta}{\vartheta + \kappa^2}\,E_t\{\pi_{t+1}\} + \frac{\vartheta}{\vartheta + \kappa^2}\,u_t$$

Iterating the previous equation forward, an expression is obtained for equilibrium inflation under the optimal discretionary policy

$$\pi_t = \frac{\vartheta}{\kappa^2 + \vartheta(1 - \beta\rho_u)}\,u_t \tag{5}$$

Combining (4) and (5) obtains an analogous expression for the output gap:

$$x_t = -\frac{\kappa}{\kappa^2 + \vartheta(1 - \beta\rho_u)}\,u_t \tag{6}$$

Thus, under the optimal discretionary policy, the central bank lets the output gap and inflation fluctuate in proportion to the current value of the cost-push shock. This is illustrated graphically by the circled lines in figures 5.1 and 5.2, which represent the responses under the optimal discretionary policy of the (welfare-relevant) output gap, inflation, and the price level to a one percent increase in u_t. In figure 5.1, the cost-push shock is assumed to be purely transitory ($\rho_u = 0$), whereas in figure 5.2 it

[3] See, e.g., Svensson (1999) and Svensson and Woodford (2005) for a discussion of "targeting" vs. "instrument" rules as alternative approaches to implementation of inflation targeting policies.

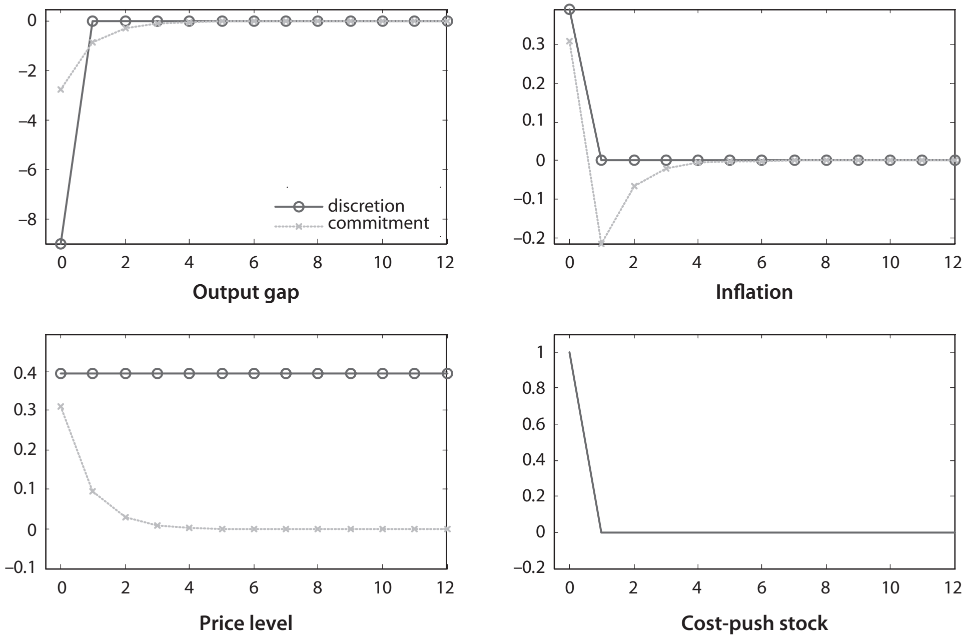

Figure 5.1. Discretion vs. Commitment: Responses to a Transitory Cost-Push Shock.

is assumed to have a positive autocorrelation ($\rho_u = 0.8$). The remaining parameters are set at the values assumed in the baseline calibration of chapter 3.

The path of the cost-push shock u_t, after a one percent increase, is displayed in the bottom-right plot of figures 5.1 and 5.2. In both cases the central bank finds it optimal to accommodate partly the inflationary pressures resulting from the cost-push shock, and thus let inflation rise. Note, however, that the increase in inflation is smaller than the increase that would be obtained if the output gap remained unchanged. In the latter case it is easy to check that inflation would be given by

$$\pi_t = \frac{1}{1 - \beta\rho_u} u_t$$

thus implying a larger response of inflation (in absolute value) at all horizons in response to the cost-push shock. Instead, under the optimal discretionary policy, the impact on inflation is dampened by the negative response of the output gap, also displayed in figures 5.1 and 5.2. Finally, it is seen that the implied response of inflation leads naturally to a

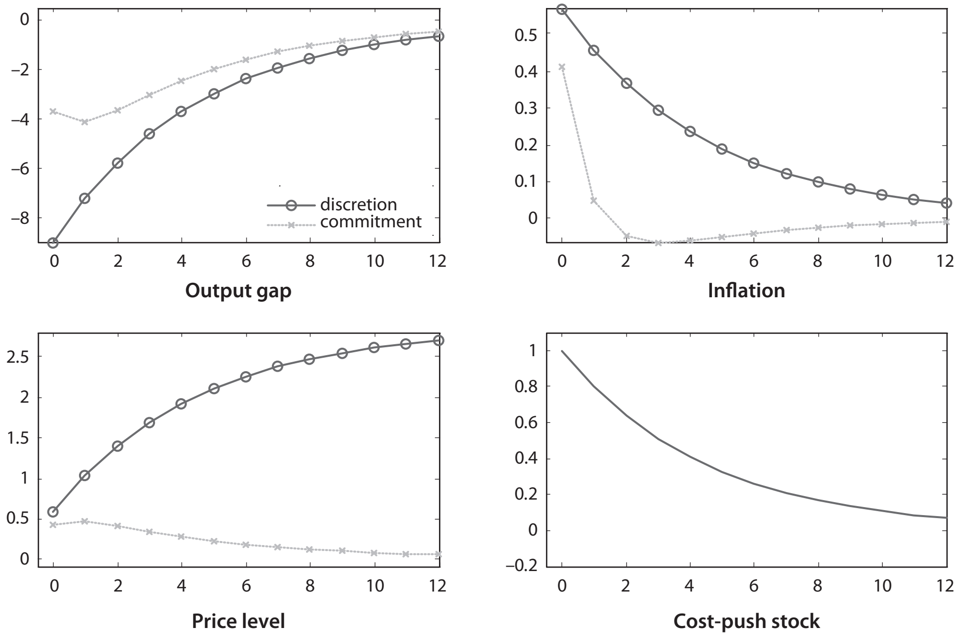

Figure 5.2. Discretion vs. Commitment: Responses to a Persistent Cost-Push Shock.

permanent change in the price level, whose size is increasing in the persistence of the shock.

The analysis above implicitly assumes that the monetary authority can choose its desired level of inflation and the output gap at each point in time. Of course, in practice, a central bank cannot directly set either variable. One possible approach to implementing that policy is to adopt an interest rate rule that guarantees that the desired outcome is attained. Before deriving the form that such a rule may take it is convenient to determine the *equilibrium* interest rate under the optimal discretionary policy as a function of the exogenous driving forces. In order to do so, note first that the consumer's intertemporal optimality condition can now be written (after imposing goods market clearing and some elementary algebraic manipulation) as:

$$x_t = E_t\{x_{t+1}\} - \frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} - r_t^e) \tag{7}$$

where the natural interest rate $r_t^e \equiv \rho + \sigma E_t\{\Delta y_{t+1}^e\} + (1 - \rho_z)z_t = \rho - \sigma(1 - \rho_a)\psi_{ya} a_t + (1 - \rho_z)z_t$ is the real interest rate consistent with the efficient level of output.

Thus, combining (7) and (5) with (6) yields:

$$i_t = r_t^e + \Psi_i u_t \tag{8}$$

where $\Psi_i \equiv \frac{\vartheta\rho_u + \sigma\kappa(1-\rho_u)}{\kappa^2 + \vartheta(1-\beta\rho_u)} > 0$.

Applying the arguments of chapter 3, it is easy to see that (8) cannot be viewed as a desirable interest rate rule, for it does not guarantee a unique equilibrium and, hence, the attainment of the desired outcome. In particular, if "rule" (8) is used to eliminate the nominal rate in (7), the resulting equilibrium dynamics are represented by the system

$$\begin{bmatrix} x_t \\ \pi_t \end{bmatrix} = \mathbf{A}_\mathbf{O} \begin{bmatrix} E_t\{x_{t+1}\} \\ E_t\{\pi_{t+1}\} \end{bmatrix} + \mathbf{B}_\mathbf{O} u_t \tag{9}$$

where

$$\mathbf{A}_\mathbf{O} \equiv \begin{bmatrix} 1 & \frac{1}{\sigma} \\ \kappa & \beta + \frac{\kappa}{\sigma} \end{bmatrix} \quad ; \quad \mathbf{B}_\mathbf{O} \equiv \begin{bmatrix} -\frac{\Psi_i}{\sigma} \\ 1 - \frac{\kappa\Psi_i}{\sigma} \end{bmatrix}$$

As argued in chapter 4, matrix $\mathbf{A}_\mathbf{O}$ has always one eigenvalue outside the unit circle, thus implying that (9) has a multiplicity of solutions, only one of which corresponds to the desired outcome given by (5) and (6).

In the context of the present model, one can always derive a rule that guarantees equilibrium uniqueness (independently of parameter values), by appending to the expression for the equilibrium nominal rate under the optimal discretionary policy (given by (8)), a term proportional to the deviation between inflation and the equilibrium value of the latter under that policy, with a coefficient of proportionality greater than one (in order to satisfy the Taylor principle). Formally,

$$\begin{aligned} i_t &= r_t^e + \Psi_i u_t + \phi_\pi \left(\pi_t - \frac{\vartheta}{\kappa^2 + \vartheta(1-\beta\rho_u)} u_t \right) \qquad (10) \\ &= r_t^e + \Theta_i u_t + \phi_\pi \pi_t \end{aligned}$$

where $\Theta_i \equiv \frac{\sigma\kappa(1-\rho_u) - \vartheta(\phi_\pi - \rho_u)}{\kappa^2 + \vartheta(1-\beta\rho_u)}$ and for an arbitrary inflation coefficient satisfying $\phi_\pi > 1$.

In practice, an interest rate rule like (10) is not easy to implement, for the reasons spelled out in chapter 4: It requires knowledge of the model's parameters, and real-time observation of variations in the cost-push shock and the efficient interest rate. Those difficulties have led some authors to emphasize "targeting rules" like (4) as practical guides for monetary policy, as opposed to "instrument rules" like (10). Under a targeting rule, the central bank would adjust its instrument until a certain optimal relation between target variables is satisfied. In the aforementioned example, however, following such a targeting rule requires that the efficient level of output y_t^e be observed in real time in order to determine the output gap, thus limiting its practical appeal.

5.2.2 Optimal Policy under Commitment

After having analyzed the optimal policy under discretion, next is the case of a central bank that is assumed to be able to commit, with full credibility, to a *policy plan*. In the context of the model, such a plan consists of a specification of the desired levels of inflation and output at all possible dates and states of nature, current and future. More specifically, the monetary authority is assumed to choose a state-contingent sequence $\{x_t, \pi_t\}_{t=0}^{\infty}$ that minimizes

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\pi_t^2 + \vartheta x_t^2\right)$$

subject to the sequence of constraints

$$\pi_t = \beta E_t\{\pi_{t+1}\} + \kappa x_t + u_t$$

and where $\{u_t\}$ follows the exogenous process (3).

In order to solve the previous problem it is useful to write down the associated Lagrangian, which is given by

$$\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{2}\left(\pi_t^2 + \vartheta x_t^2\right) + \xi_t(\pi_t - \kappa x_t - \beta \pi_{t+1})\right] + t.i.p.$$

where $\{\xi_t\}_{t=0}^{\infty}$ is a sequence of Lagrange multipliers, and where the law of iterated expectations has been used to eliminate the conditional expectation that appeared in each constraint.

Differentiating the Lagrangian with respect to y_t and π_t yields the optimality conditions

$$\vartheta x_t - \kappa \xi_t = 0$$

$$\pi_t + \xi_t - \xi_{t-1} = 0$$

that must hold for $t = 0, 1, 2, \ldots$ and where $\xi_{-1} = 0$, because the inflation equation corresponding to period -1 is not an effective constraint for the central bank choosing its optimal plan in period 0.

Combining the two optimality conditions to eliminate the Lagrange multipliers yields

$$x_0 = -\frac{\kappa}{\vartheta}\,\pi_0 \tag{11}$$

and

$$\Delta x_t = -\frac{\kappa}{\vartheta}\,\pi_t \tag{12}$$

for $t = 1, 2, 3, \ldots$

Note that (11) and (12) can be jointly represented by the single equation in "levels"

$$x_t = -\frac{\kappa}{\vartheta}\,\widehat{p}_t \tag{13}$$

for $t = 0, 1, 2, \ldots$ where $\widehat{p}_t \equiv p_t - p_{-1}$ is the (log) deviation between the price level and an "implicit target" given by the price level prevailing one period before the central bank chooses its optimal plan. Thus, (13) can be viewed as a "targeting rule" that the central bank must follow period by period in order to implement the optimal policy under commitment.

It is worth pointing out the difference between (13) and the corresponding targeting rule for the discretionary case given by (4). The optimal discretionary policy requires that the central bank keeps output below (above) its efficient level as long as inflation is positive (negative). By way of contrast, under the optimal policy with commitment the central bank sets the sign and size of the output gap in proportion to the deviations of the price *level* from its implicit target. As is discussed next, this has important consequences for the economy's equilibrium response to a cost-push shock.

By combining optimality condition (13) with (2), after rewriting the latter in terms of the price level, the stochastic difference equation satisfied by $\widehat{p}_t$ under the optimal policy is derived

$$\widehat{p}_t = \gamma\,\widehat{p}_{t-1} + \gamma\beta E_t\{\widehat{p}_{t+1}\} + \gamma u_t \tag{14}$$

for $t = 0, 1, 2, \ldots$ where $\gamma \equiv \frac{\vartheta}{\vartheta(1+\beta)+\kappa^2}$.

The stationary solution to the previous difference equation is given by

$$\widehat{p}_t = \delta \widehat{p}_{t-1} + \frac{\delta}{1 - \delta\beta\rho_u} u_t \tag{15}$$

for $t = 0, 1, 2, \ldots$ where $\delta \equiv \frac{1-\sqrt{1-4\beta\gamma^2}}{2\gamma\beta} \in (0, 1)$.[4] Then (13) is used to derive the equilibrium process for the output gap

$$x_t = \delta x_{t-1} - \frac{\kappa\delta}{\vartheta(1 - \delta\beta\rho_u)} u_t \tag{16}$$

for $t = 1, 2, 3, \ldots$, with the response at the time of the shock ($t = 0$) being given by

$$x_0 = -\frac{\kappa\delta}{\vartheta(1 - \delta\beta\rho_u)} u_0$$

The lines with crosses in figure 5.1 show the equilibrium responses of the output gap, inflation, and the price level to a one percent transitory cost-push shock. Analogous responses for the case of a persistence cost-push shock are displayed in figure 5.2 . In both cases those responses are shown side by side with the responses implied by the optimal discretionary policy (represented by the circled lines described earlier), thus facilitating comparison of the two regimes' outcomes.

A look at the case of a transitory cost-push shock illustrates the difference most clearly. In the case of discretionary policy, both the output gap and inflation return to their zero initial value once the shock has vanished (i.e., one period after the shock). By contrast, and as implied by (15) and (16), under the optimal policy with commitment the deviations in the output gap and inflation from target persist well beyond the life of the shock, that is, those variables display endogenous or intrinsic persistence. Given that a zero inflation, zero output gap outcome is feasible once the shock has vanished, why does the central bank find it optimal to maintain a persistently negative output gap and inflation? The reason is simple: By committing to such a response, the central bank manages to improve the output gap/inflation tradeoff in

[4] To derive the stationary solution, first guess that it takes the form

$$\widehat{p}_t = \delta \widehat{p}_t + b u_t$$

for some pair (δ, b), with $|\delta| < 1$. The previous guess can be verified and coefficients (δ, b) determined by plugging the above equation into (14) and using the method of undetermined coefficients.

the period when the shock occurs. In the case illustrated in figure 5.1 it lowers the initial impact of the cost-push shock on inflation (relative to the discretionary case), while incurring smaller output gap losses in the same period. This is possible because of the forward-looking nature of inflation, which can be highlighted by iterating (2) forward to yield

$$\pi_t = \kappa x_t + \kappa \sum_{k=1}^{\infty} \beta^k E_t\{x_{t+k}\} + \frac{1}{1-\beta\rho_u}\, u_t$$

Hence, it is seen that the central bank can offset the inflationary impact of a cost-push shock by lowering the current output gap x_t, but also by committing to lower future output gaps (or, equivalently, future reductions in the price level). If credible, such “promises” will bring about a downward adjustment in the sequence of expectations $E_t\{x_{t+k}\}$ for $k = 1, 2, 3, \ldots$ As a result, and in response to a positive realization of the cost-push shock u_t, the central bank may achieve any given level of current inflation π_t with a smaller decline in the current output gap x_t. That is the sense in which the output gap/inflation tradeoff is improved by the possibility of commitment. Given the convexity of the loss function in inflation and output gap deviations, the dampening of those deviations in the period of the shock brings about an improvement in overall welfare relative to the case of discretion, because the implied benefits are not offset by the (relatively small) losses generated by the deviations in subsequent periods (and which are absent in the discretionary case).

Figure 5.2 displays analogous impulse responses under the assumption that $\rho_u = 0.8$, that is, allowing for a persistent cost-push shock. Note that in this case the economy reverts back to the initial position only asymptotically, even under the optimal discretionary policy (because the inflationary pressures generated by the shock remain effective at all horizons, albeit with a declining influence). Yet, some of the key qualitative features emphasized above are still present: In particular, the optimal policy with commitment manages once again to attain both lower inflation and a smaller output gap (in absolute value) at the time of the shock, relative to the optimal discretionary policy. Note also that under the optimal policy with commitment the price level reverts back to its original level, albeit at a slower rate than in the case of a transitory shock. As a result inflation displays some positive short-run autocorrelation, illustrating the fact that the strong negative short-run autocorrelation observed in the case of a purely transitory shock is not a necessary implication of the policy with commitment.

In all cases, a feature of the economy’s response under discretionary policy is the attempt to stabilize the output gap in the medium term more than the optimal policy under commitment calls for, without internalizing the benefits in terms of short-term stability that result from allowing larger deviations of the output gap at future horizons. This characteristic, which is most clearly illustrated by the example of a purely transitory cost-push shock represented in figure 5.1, is often referred to as the *stabilization bias* associated with the discretionary policy.[5]

As in the case of discretion, one might be interested in deriving an interest rate rule that would bring about the paths of output gap and inflation implied by the optimal policy under commitment. Next, such a rule is derived for the special case of serially uncorrelated cost push shocks ($\rho_u = 0$). In that case, combining (7), (15), and (16) yields the process describing the equilibrium nominal rate under the optimal policy with commitment

$$\begin{aligned} i_t &= r_t^e - (1-\delta)\left(1 - \frac{\sigma\kappa}{\vartheta}\right)\widehat{p}_t \\ &= r_t^e - (1-\delta)\left(1 - \frac{\sigma\kappa}{\vartheta}\right)\sum_{k=0}^{t}\delta^{k+1}u_{t-k} \end{aligned}$$

Thus, one possible rule that would bring about the desired allocation as the unique equilibrium is given by

$$i_t = r_t^e - \left(\phi_p + (1-\delta)\left(1 - \frac{\sigma\kappa}{\alpha_x}\right)\right)\sum_{k=0}^{t}\delta^{k+1}u_{t-k} + \phi_p\widehat{p}_t$$

for any $\phi_p > 0$. Note that under the previous formulation the central bank stands ready to respond to any deviation of the price level from the path prescribed by (15), though this will not be necessary in equilibrium.[6]

5.3 The Monetary Policy Problem: The Case of a Distorted Steady State

Next, consider the case in which the presence of uncorrected real distortions generates a permanent gap between the natural and the

[5] That stabilization bias must be distinguished from the inflation bias that arises when the zero inflation steady state is associated with an inefficiently low level of activity. The stabilization bias is obtained independent of the degree of inefficiency of the steady state, as discussed below.

[6] An interest rate rule that displays a positive response to the price level can be shown to generate a unique equilibrium in the basic New Keynesian model. See exercise 4.5 in chapter 4.

efficient levels of output, which is reflected in an inefficient steady state. The size of the steady state distortion is measured by a parameter Φ representing the wedge between the marginal product of labor and the marginal rate of substitution between consumption and hours, both evaluated at the steady state. Formally, Φ is defined by

$$-\frac{U_n}{U_c} = (1 - \Phi) MPN$$

Below, it is assumed $\Phi > 0$, which implies that the steady state levels of output and employment are below their respective efficient levels. The presence of firms' market power in the goods market as assumed in the basic model of chapter 3 constitutes an example of the kind of distortion that, if uncorrected through an appropriate subsidy, would generate an inefficiently low level of activity. In that case, and as implied by the analysis of chapter 4, $\Phi \equiv 1 - \frac{1}{\mathcal{M}} > 0$, where $\mathcal{M}$ is the steady state gross markup.

Under the assumption of a "small" steady state distortion (i.e., when Φ has the same order of magnitude as fluctuations in the output gap or inflation), and as shown in appendix 5.1, the component of the welfare losses experienced by the representative household that can be affected by policy is approximately proportional, in a neighborhood of the zero inflation steady state, to the expression

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{2} \left(\pi_t^2 + \vartheta \widehat{x}_t^2 \right) - \Lambda \widehat{x}_t \right] \tag{17}$$

where $\Lambda \equiv \Phi\lambda/\epsilon > 0$ and $\widehat{x}_t \equiv x_t - x$, with $x \equiv y^n - y^e$ measuring the size of the welfare-relevant output gap in the zero inflation steady state. Note that the linear term in $\widehat{x}_t$ captures the fact that any marginal increase in the output gap relative to its steady state value has a positive first-order effect on welfare (thus decreasing welfare losses), because output is assumed to be below its efficient level at that steady state, that is, $x < 0$.

Similarly, the inflation equation can be written in terms of as

$$\pi_t = \beta E_t\{\pi_{t+1}\} + \kappa \widehat{x}_t + u_t \tag{18}$$

where now $u_t \equiv \kappa(\widehat{y}_t^e - \widehat{y}_t^n)$. Thus, the monetary authority will seek to minimize (17) subject to the sequence of constraints given by (18) for $t = 0, 1, 2, \ldots$

Note that under the assumption of the "small" steady state distortion made above, the linear term $\Lambda \widehat{y}_t$ is already of second order, thus giving the central bank's problem the convenient linear-quadratic format.[7]

As in the previous section, the solution to the central bank's problem under discretion is first analyzed, before turning to the optimal policy with commitment.

5.3.1 Optimal Discretionary Policy

In the absence of a commitment technology, the monetary authority chooses $(\widehat{x}_t, \pi_t)$ in order to minimize the period losses

$$\frac{1}{2}(\pi_t^2 + \vartheta \widehat{x}_t^2) - \Lambda \widehat{x}_t$$

subject to the constraint

$$\pi_t = \kappa \widehat{x}_t + \upsilon_t$$

where, once again, $\upsilon_t \equiv \beta E_t\{\pi_{t+1}\} + u_t$ is taken as given by the policy-maker.

The associated optimality condition is

$$\widehat{x}_t = \frac{\Lambda}{\vartheta} - \frac{\kappa}{\vartheta}\,\pi_t \tag{19}$$

Note that (19) implies, for any given level of inflation, a more expansionary policy than that given in the absence of a steady state distortion. This is a consequence of the desire by the central bank to partly correct for the inefficiently low average level of activity.

Plugging (19) into (18) and solving the resulting difference equation yields the following expression for equilibrium inflation

$$\pi_t = \frac{\Lambda\kappa}{\kappa^2 + \vartheta(1-\beta)} + \frac{\vartheta}{\kappa^2 + \vartheta(1-\beta\rho_u)}\,u_t \tag{20}$$

[7] In the presence of a large distortion, the linear term in (17) would require the use of a second-order approximation to the equilibrium condition connecting output and inflation.

Combining (19) and (20) yields the corresponding expression for the equilibrium output gap

$$\widehat{x}_t = \frac{\Lambda(1-\beta)}{\kappa^2 + \vartheta(1-\beta)} - \frac{\kappa}{\kappa^2 + \vartheta(1-\beta\rho_u)}\, u_t$$

Thus, it is seen that the presence of a distorted steady state does not affect the response of the output gap and inflation to cost-push shocks under the optimal discretionary policy. It has, however, an effect on the average levels of inflation and the output gap around which the economy fluctuates. In particular, when the natural level of output and employment are inefficiently low ($\Lambda > 0$), the optimal discretionary policy leads to positive average inflation as a consequence of the central bank's incentive to push output above its natural steady state level. Thus, in steady state:

$$\pi = \frac{\Lambda\kappa}{\kappa^2 + \vartheta(1-\beta)}$$

and

$$\widehat{x} = \frac{\Lambda(1-\beta)}{\kappa^2 + \vartheta(1-\beta)}$$

The incentive to push output above its natural steady state increases with the degree of inefficiency of the natural steady state, which explains the fact that the average inflation is increasing in Φ (and hence in Λ), giving rise to the *classical inflation bias* phenomenon.

5.3.2 Optimal Policy under Commitment

As in the case of an efficient steady state, the optimal policy under commitment is solved by setting up the Lagrangean corresponding to the central bank's problem, which in this case is given by

$$\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{2}\left(\pi_t^2 + \vartheta \widehat{x}_t^2\right) - \Lambda \widehat{x}_t + \xi_t(\pi_t - \kappa \widehat{x}_t - \beta \pi_{t+1}) \right] + t.i.p.$$

where $\{\xi_t\}$ are the Lagrange multipliers associated with the sequence of constraints (18), for $t = 0, 1, 2, \ldots$

The corresponding optimality conditions are given by

$$\vartheta \widehat{x}_t - \kappa \xi_t - \Lambda = 0$$

$$\pi_t + \xi_t - \xi_{t-1} = 0$$

which must hold for $t = 0, 1, 2, \ldots$ and where $\xi_{-1} = 0$. The previous conditions can be combined into a single condition, after eliminating the Lagrange multiplier:

$$\vartheta \widehat{x}_t = -\kappa \widehat{p}_t + \Lambda \tag{21}$$

for $t = 0, 1, 2, \ldots$ where, as above, $\widehat{p}_t \equiv p_t - p_{-1}$. Combining the previous condition with (18) yields the following difference equation for the (log) price level

$$\widehat{p}_t = \gamma \widehat{p}_{t-1} + \gamma \beta E_t\{\widehat{p}_{t+1}\} + \frac{\gamma \kappa \Lambda}{\vartheta} + \gamma u_t \tag{22}$$

where and $\gamma \equiv \frac{\vartheta}{\vartheta(1+\beta)+\kappa^2} \in (0, 1)$.

The stationary solution to the previous difference equation describes the evolution of the equilibrium price level under the optimal policy with commitment. It takes the form

$$\widehat{p}_t = \delta \widehat{p}_{t-1} + \frac{\delta}{1 - \delta\beta\rho_u} u_t + \frac{\delta}{1 - \delta\beta} \left(\frac{\kappa \Lambda}{\vartheta} \right) \tag{23}$$

for $t = 0, 1, 2, \ldots$ where $\delta \equiv \frac{1-\sqrt{1-4\beta\gamma^2}}{2\gamma\beta} \in (0, 1)$.[8] Solving the previous equation backward yields:

$$\widehat{p}_t = \left(\frac{1 - \delta^{t+1}}{1 - \delta} \right) \left(\frac{\delta}{1 - \delta\beta} \right) \left(\frac{\kappa \Lambda}{\vartheta} \right) + \frac{\delta}{1 - \delta\beta\rho_u} \sum_{k=0}^{t} \delta^k u_{t-k} \tag{24}$$

[8] To derive the stationary solution the method of undetermined coefficients can again be used, based on the following guess as to the form of the solution:

$$\widehat{p}_t = \delta \widehat{p}_{t-1} + b u_t + c$$

for some values of (δ, b, c), with $|\delta| < 1$.

Combining (21) and (23), the corresponding path for output can be derived (after some algebra):

$$\widehat{x}_t = \delta \widehat{x}_{t-1} - \frac{\kappa \delta}{\vartheta(1 - \delta\beta\rho_u)} u_t \tag{25}$$

for $t = 1, 2, 3, \ldots$ with the response at $t = 0$ being given by

$$\widehat{x}_0 = -\frac{\kappa \delta}{\vartheta(1 - \delta\beta\rho_u)} u_0 + \frac{\Lambda(1 - \delta)}{\vartheta} \tag{26}$$

Equivalently, solving (25) backward and combining it with (26):

$$\widehat{x}_t = \frac{\Lambda(1 - \delta)\delta^k}{\vartheta} - \frac{\kappa \delta}{\vartheta(1 - \delta\beta\rho_u)} \sum_{k=0}^{t} \delta^k u_{t-k} \tag{27}$$

A glance at (24) and (27) reveals the main implication of the presence of a distorted steady state on the path of the price level and output under the optimal policy with commitment: it leads to some persistent, deterministic transitional dynamics. Thus, in the absence of cost-push shocks, the monetary authority chooses an output gap and inflation persistently above their steady state levels, and converging to the latter only asymptotically. In the presence of shocks, that deterministic component is added to the stochastic one (involving the responses to shocks), which is otherwise identical to the case of an efficient steady state. That modification thus generates a persistent inflationary bias, resulting from the benefits from a higher output, even at the cost of higher inflation. In the long run, however, the marginal benefits of increasing output (Λ) equal the costs of (cumulative) inflation ($\lim_{T\to\infty} \kappa \widehat{p}_T$), so it is optimal to keep output at its natural level ($\lim_{T\to\infty} x_T = x$).

The deterministic component of inflation and output resulting from the optimal policy under commitment also differs in an important way from its discretionary policy counterpart. As shown above, in the case of discretion that component takes the form of a constant positive mean, resulting from the period-by-period incentive to close the gap between output and its efficient level, which results in a permanent inflation bias (and a permanent increase of output above its natural level). In the case of commitment, however, it is seen that the price level converges asymptotically to a constant, given by $\lim_{T\to\infty} p_T = p_{-1} + \frac{\delta}{(1-\delta\beta)(1-\delta)} \left(\frac{\kappa\Lambda}{\vartheta}\right)$. Hence, under the optimal plan the economy eventually converges to an equilibrium characterized by zero average inflation and a zero average output gap (relative to natural output). In that sense it is (asymptotically) observationally equivalent to the outcome of an economy with an efficient steady state. The desirability of zero long-run inflation can be explained by the benefits arising from its anticipation by the public, which leads to an improvement in the short-run tradeoff facing the central bank, allowing it to raise output above its natural level (with the consequent welfare improvement) with more subdued effects on inflation (since the public anticipates a gradual return of output to its natural level). Thus, the central bank's ability to commit avoids (at least asymptotically) the inflation bias that characterizes the outcome of the discretionary policy. On the other hand, and as was the case under the discretionary policy, the response to a cost-push shock under the optimal policy with commitment is not affected by the presence of a distorted steady state. Hence, the impulse responses displayed in figures 5.1 and 5.2 illustrating the economy's response to a cost-push shock under discretion and under commitment remain valid in the present context, independently of the degree of steady state inefficiency.

5.4 Optimal Monetary Policy under a Zero Lower Bound on the Nominal Interest Rate

In market economies, the existence of monetary assets (like currency) generate no nominal payoffs, but otherwise have risk properties identical to short-term nominal debt but, imposes a floor on the nominal return i_t that such debt yields. That floor is commonly known as the zero lower bound (ZLB, for short) on the nominal rate. It can be represented by a simple inequality constraint:

$$i_t \geq 0 \tag{28}$$

for all t. The present section analyzes some of the consequences of the ZLB for the design of monetary policy. It is not meant to provide a comprehensive treatment of this (nontrivial) issue, but instead to illustrate by means of an example some of the complications that its presence may raise for the design and implementation of monetary policy.

In order to focus on the difficulties created by the existence of the zero lower bound itself, the following analysis abstracts from the presence of cost-push shocks or an inefficient steady state. In other words, the assumption that $y_t^n = y_t^e$ for all t, is maintained for the remainder of this chapter. Thus the nonpolicy block of the model can be represented by the two equations:

$$\pi_t = \beta E_t\{\pi_{t+1}\} + \kappa x_t \tag{29}$$

$$x_t = E_t\{x_{t+1}\} - \frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} - r_t^n) \tag{30}$$

where the natural rate r_t^n follows an exogenous deterministic path. More specifically, it is assumed that the natural rate remains constant at its steady state level $\rho > 0$ up to (and including) period -1. In period 0 it unexpectedly drops to a negative value $r_t^n = -\epsilon < 0$ and remains at that level up to period t_Z. From period $t_Z + 1$ onward it takes again its steady state value ρ. Once the unexpected change in period 0 occurs, the subsequent path of the natural rate is assumed to be known with certainty by all agents. The question posed here is the following: How should the central bank conduct its monetary policy in response to the natural rate shock, given the non-negativity constraint on the nominal interest rate?[9]

It is important to note first that the ZLB constraint restricts the set of feasible equilibrium paths. In particular, that constraint prevents, while binding, the attainment of the optimal allocation, characterized by zero inflation and a zero output gap at all times. This is the case, even though inflation equation (29) is in principle consistent with such an outcome. The reason is that, as discussed in chapter 4, supporting the efficient outcome as an equilibrium requires that $i_t = r_t^n$ for all t, which violates (28) whenever $r_t^n < 0$, as in the example considered here. The optimal policy will thus necessarily involve a second best outcome.

As earlier in the chapter, the cases of optimal policy under discretion and under commitment are analyzed separately. In both cases it is assumed that up to period 0, when the unexpected shift in the natural rate occurs, the economy's equilibrium involved $\pi_t = x_t = 0$ and $i_t = \rho$, for all $t < 0$. Given the absence of cost-push shocks, such an outcome is consistent with the optimal policy under both discretion and commitment, as shown earlier in this chapter.

[9] The example considered here is related (but different in details) to the one analyzed in Jung, Teranishi, and Watanabe (2005) and Eggertsson and Woodford (2003). See the section at the end of the chapter for further references to the ZLB literature.

5.4.1 Optimal Discretionary Policy in the Presence of a ZLB Constraint

Starting from $t = 0$, each period the central bank minimizes the loss function

$$\pi_t^2 + \vartheta x_t^2$$

subject to the constraints

$$\pi_t = \kappa x_t + \upsilon_{0,t} \tag{31}$$

$$x_t \leq \upsilon_{1,t} \tag{32}$$

for $t = 0, 1, 2, \ldots$ where $\upsilon_{0,t} \equiv \beta \pi_{t+1}$ and $\upsilon_{1,t} \equiv x_{t+1} + (1/\sigma)(\pi_{t+1} + r_t^n)$ are taken as given by the central bank, and where (32) combines (28) and (30) in a single constraint.

The Lagrangean for the above problem takes the form:

$$\mathcal{L} = \frac{1}{2}\left(\pi_t^2 + \vartheta x_t^2\right) + \xi_{1,t}(\pi_t - \kappa x_t - \upsilon_{0,t}) + \xi_{2,t}(x_t - \upsilon_{1,t})$$

with the corresponding optimality conditions given by:

$$\pi_t + \xi_{1,t} = 0$$

$$\vartheta x_t - \kappa \xi_{1,t} + \xi_{2,t} = 0$$

and the slackness conditions:

$$\xi_{2,t} \geq 0 \ ; \ i_t \geq 0 \ ; \ \xi_{2,t} i_t = 0$$

combined with equilibrium conditions (29) and (30), all for $t = 0, 1, 2, \ldots$ Note that we can combine the first two optimality conditions to write:

$$\vartheta x_t = -\kappa \pi_t - \xi_{2,t} \tag{33}$$

The solution to the previous problem is easy to characterize. From $t_Z + 1$ onward,

$$i_t = \rho > 0$$

implying $x_t = -(\kappa/\vartheta)\pi_t$, which combined with (29) implies:

$$x_t = \pi_t = 0$$

which attains the first best, once the ZLB constraint is no longer binding.[10]

On the other hand, for $t = 0, 1, 2, \ldots\ t_Z$, the ZLB constraint is binding, implying

$$i_t = 0\ ;\ \xi_{2,t} > 0$$

The equilibrium path for inflation and the output gap over that period can be determined recursively backward using the system of difference equations (describing the equilibrium under $i_t = 0$)

$$\begin{bmatrix} x_t \\ \pi_t \end{bmatrix} = \mathbf{A} \begin{bmatrix} x_{t+1} \\ \pi_{t+1} \end{bmatrix} - \mathbf{B}\epsilon \tag{34}$$

with terminal conditions $x_{t_Z+1} = \pi_{t_Z+1} = 0$ and where

$$\mathbf{A} \equiv \begin{bmatrix} 1 & \frac{1}{\sigma} \\ \kappa & \beta + \frac{\kappa}{\sigma} \end{bmatrix}; \qquad \mathbf{B} \equiv \begin{bmatrix} \frac{1}{\sigma} \\ \frac{\kappa}{\sigma} \end{bmatrix}$$

It can also be checked that along that equilibrium path, $x_t < 0$ and $\pi_t < 0$ for $t = 0, 1, 2, \ldots, t_z$ thus guaranteeing (given (33)) that $\xi_{2,t} > 0$, consistent with the assumption of a binding ZLB during this period.

The lines with circles in figure 5.3 represent the paths of the output gap, inflation, the price level, and the nominal interest rate implied by the optimal discretionary policy, together with the exogenous path for the natural rate of interest. It is assumed that the unexpected drop in the natural rate, from 1 percent to −1 percent (4 to −4 percent in annualized rates) lasts 6 quarters (i.e., from $t = 0$ to $t_z = 5$). The remaining parameters are set at their baseline values. Note that both and the output gap and inflation experience a large decline on impact and remain below

[10] As discussed above, in order to guarantee that the desired outcome is not only consistent with equilibrium but also the only possible equilibrium outcome, the central bank could adopt an interest rate rule of the form

$$i_t = \rho + \phi_\pi \pi_t$$

with $\phi_\pi > 1$.

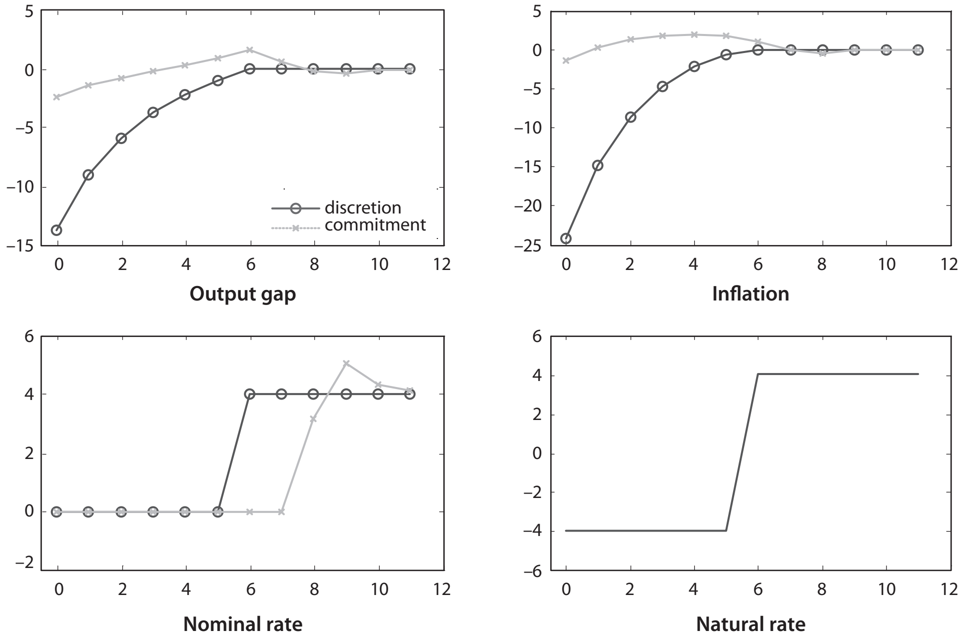

Figure 5.3. Discretion vs. Commitment in the Presence of a ZLB.

their optimal values until the negative shock vanishes. The presence of the zero lower bound is the ultimate source of the welfare losses resulting from the adverse demand shock. Those losses cannot be fully avoided, but are considerably reduced when the central bank can commit credibly to a future policy plan, as shown next.

5.4.2 *Optimal Policy under Commitment in the Presence of a ZLB Constraint*

In period $t = 0$, once the fall in the natural rate has been materialized, the monetary authority chooses the path of for the output gap and inflation $\{x_t, \pi_t\}_{t=0}^{\infty}$ that minimizes

$$\sum_{t=0}^{\infty} \beta^t \left(\pi_t^2 + \vartheta x_t^2\right)$$

subject to the sequence of constraints

$$\pi_t = \beta\pi_{t+1} + \kappa x_t$$

$$x_t \leq x_{t+1} + \frac{1}{\sigma}(\pi_{t+1} + r_t^n)$$

where $r_t^n = -\epsilon$ for $t = 0, 1, 2, \ldots t_z$ and $r_t^n = \rho$ for $t = t_z + 1, t_z + 2, \ldots$

The associated Lagrangian is now given by

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{2}\left(\pi_t^2 + \vartheta x_t^2\right) + \xi_{1,t}(\pi_t - \kappa x_t - \beta\pi_{t+1}) \right.$$
$$\left. + \xi_{2,t}(x_t - x_{t+1} - \frac{1}{\sigma}(\pi_{t+1} + r_t^n))\right]$$

with associated first order conditions

$$\pi_t + \xi_{1,t} - \xi_{1,t-1} - \frac{1}{\beta\sigma}\xi_{2,t-1} = 0 \tag{35}$$

$$\vartheta x_t - \kappa\xi_{1,t} + \xi_{2,t} - \frac{1}{\beta}\xi_{2,t-1} = 0 \tag{36}$$

and slackness conditions

$$\xi_{2,t} \geq 0 \; ; \; i_t \geq 0 \; ; \; \xi_{2,t} i_t = 0$$

as well as initial conditions $\xi_{1,-1} = \xi_{2,-1} = 0$.

The solution is conjectured (and subsequently verified) to be of the following form. From period 0 to $t_C \geq t_Z$ the nominal rate remains at zero. It becomes positive in period $t_C + 1$ and remains positive from then onward.

The equilibrium dynamics for $t = t_C + 2, t_C + 3, \ldots$ are described by the difference equations

$$\pi_t + \xi_{1,t} - \xi_{1,t-1} = 0 \tag{37}$$

$$\vartheta x_t - \kappa\xi_{1,t} = 0 \tag{38}$$

$$\pi_t = \beta\pi_{t+1} + \kappa x_t \tag{39}$$

together with an initial condition for ξ_{1,t_C+1} (to be determined below).

Note that (37) and (38) can be combined into the single condition

$$x_t = -\frac{\kappa}{\vartheta}(p_t - p^*) \tag{40}$$

for $t = t_C + 2, t_C + 3, \ldots$ where $p^* \equiv p_{t_C+1} + \xi_{1,t_C+1}$. Combining the previous equation with (39) yields a second-order difference equation

$$\widehat{p}_t = \gamma \widehat{p}_{t-1} + \gamma\beta \widehat{p}_{t+1}$$

where $\widehat{p}_t \equiv p_t - p^*$ and $\gamma \equiv \frac{\vartheta}{\vartheta(1+\beta)+\kappa^2}$. The unique stationary solution to the previous difference equation is

$$\widehat{p}_t = \delta \widehat{p}_{t-1} \tag{41}$$

for $t = t_C + 2, t_C + 3, \ldots$, with initial condition $\widehat{p}_{t_C+1} = -\xi_{1,t_C+1} < 0$, and where $\delta \equiv \frac{1-\sqrt{1-4\beta\gamma^2}}{2\gamma\beta} \in (0, 1)$. By combining (40) and (41) the path of the output gap for $t = t_C + 2, t_C + 3, \ldots$ can then be determined. Note (41) implies

$$\widehat{p}_{t_c+2+k} = -\delta^{k+1}\xi_{1,t_C+1}$$

and

$$x_{t_c+2+k} = \frac{\kappa\delta^{k+1}}{\vartheta}\xi_{1,t_C+1} > 0 \tag{42}$$

for $k = 0, 1, 2, \ldots$ as well as

$$\pi_{t_c+2+k} = (1-\delta)\delta^k\xi_{1,t_C+1} > 0 \tag{43}$$

Thus, under the optimal policy with commitment inflation and the output gap converge to zero asymptotically.

Consider next the equilibrium conditions in period $t_c + 1$, the first period in which the ZLB is not binding. They are given by:

$$\pi_{t_c+1} + \xi_{1,t_c+1} - \xi_{1,t_c} - \frac{1}{\beta\sigma}\xi_{2,t_c} = 0 \tag{44}$$

$$\vartheta x_{t_c+1} - \kappa\xi_{1,t_c+1} - \frac{1}{\beta}\xi_{2,t_c} = 0 \tag{45}$$

$$\pi_{t_c+1} = \beta(1-\delta)\xi_{1,t_C+1} + \kappa x_{t_c+1} \tag{46}$$

Using (46) to substitute out ξ_{1,t_C+1} from (44) and (45) yields the linear relation:

$$\begin{bmatrix} x_{t_c+1} \\ \pi_{t_c+1} \end{bmatrix} = \mathbf{M} \begin{bmatrix} \xi_{1,t_c} \\ \xi_{2,t_c} \end{bmatrix} \tag{47}$$

where

$$\mathbf{M} \equiv \begin{bmatrix} -\kappa & 1+\beta(1-\delta) \\ \beta(1-\delta)+\dfrac{\kappa^2}{\vartheta} & -\dfrac{\kappa}{\vartheta} \end{bmatrix}^{-1} \begin{bmatrix} \beta(1-\delta) & \dfrac{1-\delta}{\sigma} \\ 0 & \dfrac{1-\delta}{\vartheta} \end{bmatrix}$$

Finally, consider the equilibrium trajectory between periods 0 and t_C. During this phase the ZLB is binding, with $i_t = 0$, and the equilibrium trajectory is given by

$$\begin{bmatrix} x_t \\ \pi_t \end{bmatrix} = \mathbf{A} \begin{bmatrix} x_{t+1} \\ \pi_{t+1} \end{bmatrix} - \mathbf{B}\epsilon \tag{48}$$

for $t = 0, 1, \ldots, t_z$, and

$$\begin{bmatrix} x_t \\ \pi_t \end{bmatrix} = \mathbf{A} \begin{bmatrix} x_{t+1} \\ \pi_{t+1} \end{bmatrix} + \mathbf{B}\rho \tag{49}$$

for $t = t_{z+1}, \ldots, t_c$, where $\mathbf{A}$ is defined as above. In addition, (35) and (36) describe the evolution over time of the Lagrange multipliers, which can be written in compact form as:

$$\begin{bmatrix} \xi_{1,t} \\ \xi_{2,t} \end{bmatrix} = \mathbf{H} \begin{bmatrix} \xi_{1,t-1} \\ \xi_{2,t-1} \end{bmatrix} - \mathbf{J} \begin{bmatrix} x_t \\ \pi_t \end{bmatrix} \tag{50}$$

with initial conditions $\xi_{1,-1} = \xi_{2,-1} = 0$, where

$$\mathbf{H} \equiv \begin{bmatrix} 1 & \dfrac{1}{\beta\sigma} \\ \kappa & \dfrac{1}{\beta}\left(1+\dfrac{\kappa}{\sigma}\right) \end{bmatrix}; \qquad \mathbf{J} \equiv \begin{bmatrix} 0 & 1 \\ \vartheta & \kappa \end{bmatrix}$$

Given an initial guess for t_c, the equilibrium path under the optimal policy with commitment can be determined as follows. Equations (47), (48), (49) and (50) make up a system of $4(t_c+2)$ equations with an equal number of unknowns, namely, $(x_t, \pi_t, \xi_{1,t}, \xi_{2,t})$ for $t = 0, 1, \ldots, t_c + 1$. The value for ξ_{1,t_c+1} associated with that solution can then be combined with (42) and (43) in order to determine (x_t, π_t) for $t = t_c + 2, \ldots$

Given the path of inflation and the output gap is determined, one can solve for the interest rate implied by the dynamic IS equation:

$$i_t = r_t^n + \pi_{t+1} + \sigma(x_{t+1} - x_t)$$

and check that indeed $i_t = 0$ for $t = 0, 1, \ldots, t_c$ and $i_t > 0$ for $t = t_c + 1, t_c + 2, \ldots$ If those conditions are not verified, the procedure is repeated for a different value for t_c.

The lines with crosses in figure 5.3 display the equilibrium paths for the output gap, inflation, and the nominal rate under the optimal policy with commitment, when the economy experiences the same adverse demand shock analyzed for the discretionary case. Note that the deviation of the output gap and inflation from their optimal (zero) values are now much smaller, and so are the resulting welfare losses. What makes this possible is the central bank's (credible) promise to adopt a looser monetary policy than would be warranted by an assessment of contemporaneous conditions only once the adverse demand shock is gone. This is reflected in nominal interest rate, which remains at zero for two additional periods once the natural rate is back at its normal level, and below the natural rate for a third period. The anticipation of such a policy reduces the initial impact of the excessively tight policy implied by the binding ZLB, leading to much smaller deviations of the output gap and inflation from target between $t = 0$ and $t = t_z$, which more than offset from a welfare point of view the subsequent deviations.

The previous analysis can be viewed as providing the theoretical underpinning to the so-called "forward guidance" strategy adopted by the Fed and the ECB (among other central banks) during the aftermath of the economic and financial crisis of 2008–2009 when policy rates were at zero (or nearly zero) levels, and whereby those central banks promised to keep policy rates low for a long period, even beyond the time when inflation and output would start recovering.

5.5 Notes on the Literature

This chapter follows closely Clarida, Galí, and Gertler (1999), where the optimal monetary policy in the context of the basic New Keynesian model augmented with an ad hoc cost-push shocks is analyzed, and where the outcomes under discretion and commitment are compared. That paper also contains a discussion of the classical inflation bias, whose ultimate source is modeled as a positive target for the output gap in the policymaker's loss function. The original treatment of the inflation bias and the gains from commitment, in the context of a new classical model with a Lucas supply curve, can be found in Kydland and Prescott (1980) and Barro and Gordon (1983).

Woodford (2003b) discusses a source of monetary policy tradeoffs different from cost-push shocks: that created by the presence of transaction frictions that lead to an indirect utility function in which real balances are one of the arguments, as in the model at the end of chapter 2. In that context, and in addition to variations in inflation and the output gap, variations in the nominal rate (which acts as a tax on money holdings) are a source of welfare losses. As a result, a policy that fully stabilizes the output gap and inflation by making the interest rate move one-for-one with the natural rate, while feasible, is no longer optimal because it implies excessive interest rate volatility. The optimal policy, as shown by Woodford, smooths the fluctuations in the nominal rate, at the cost of some variations in inflation and output gap.

The approximation to welfare losses focuses on the case of "small" steady state distortions. The analysis of optimal policy in the presence of "large" steady state distortions lies beyond the scope of this book. The main difficulty in that case arises from the presence of a linear term in the second-order approximation to the welfare loss function. In that context, the use of a log-linear (i.e., first-order) approximation to the equilibrium conditions to describe the evolution of endogenous variables leads to second-order terms potentially relevant to welfare being ignored (e.g., the losses associated with the steady state effects of different degrees of volatility).

Several approaches to overcoming that problem are found in the literature. A first approach consists of solving for the evolution of the endogenous variables using a second-order (or higher) approximation to the equilibrium conditions under a given policy rule, and evaluating the latter using the original second-order approximation to the welfare losses. An application of that approach to the monetary policy problem can be found in Schmitt-Grohé and Uribe (2004), among others.

The second approach, due to Benigno and Woodford (2005), makes use of a second-order approximation to the structural equations of the model in order to replace the linear terms appearing in the welfare loss function and rewriting those losses as a function of quadratic terms only. The resulting quadratic loss function can then be minimized subject to the constraints provided by log-linearized equilibrium conditions. That approach allows one to preserve the convenient structure and properties of linear-quadratic problems, including the linearity of their implied optimal policy rules.

A third approach, illustrated in Khan, King, and Wolman (2003), requires that the optimal policy be determined in a first stage using the exact structural equations and utility function, and log-linearizing the resulting equilibrium conditions (embedding the optimal policy) in order to characterize the optimal responses to shocks.

An analysis of the optimal design of monetary policy in the presence of a ZLB on the nominal rate closely related to the exercise above can be found in Jung, Teranishi, and Watanabe (2005) and Eggertsson and Woodford (2003). Both papers study the case of a fully unanticipated, once-and-for-all adverse shock to the natural rate, which pushes the optimizing central bank against the ZLB. Adam and Billi (2006, 2007) and Nakov (2008) study the implications of the ZLB for the optimal monetary policy, when that constraint is occasionally (but recurrently) binding. Nakata and Schmidt (2014) explore the advantages of delegating monetary policy to a "conservative" central banker (i.e., one that puts more weight than society on inflation stabilization) in the presence of an occasionally binding ZLB.

Benhabib, Schmitt-Grohe, and Uribe (2001a, 2001b) show that the central bank follows a Taylor rule, the presence of the ZLB on the nominal rate implies the existence of two steady states, as well as global multiplicity of equilibria. One of the steady states can be thought of as a "liquidity trap," and is characterized by a binding ZLB and inefficiently low output and inflation. See also Armenter (2014) for an analysis of the multiplicity implied by the ZLB outside the steady state.

Appendix

5.1 A Second-Order Approximation to Welfare Losses: The Case of a Small Steady State Distortion

As shown in the appendix to chapter 4, a second-order Taylor expansion to period t utility around the zero inflation steady state, combined with a goods market clearing condition, yields

$$\begin{aligned} U_t - U = {} & U_c C \left(\widehat{y}_t (1 + z_t) + \frac{1-\sigma}{2} \widehat{y}_t^2 \right) \\ & + \frac{U_n N}{1-\alpha} \left(\widehat{y}_t (1 + z_t) + \frac{\epsilon}{2\Theta} \, var_i \{ p_t(i) \} + \frac{1+\varphi}{2(1-\alpha)} (\widehat{y}_t - a_t)^2 \right) \\ & + t.i.p. \end{aligned}$$

where $t.i.p.$ stands for *terms independent of policy*.

Let Φ denote the size of the steady state distortion, implicitly defined by $-\frac{U_n}{U_c} = MPN(1-\Phi)$. Using the fact that $MPN = (1-\alpha)(Y/N)$,

$$\frac{U_t - U}{U_c C} = \widehat{y}_t(1+z_t) + \frac{1-\sigma}{2}\,\widehat{y}_t^2$$

$$-(1-\Phi)\left(\widehat{y}_t(1+z_t) + \frac{\epsilon}{2\Theta}\,var_i\{p_t(i)\} + \frac{1+\varphi}{2(1-\alpha)}(\widehat{y}_t - a_t)^2\right)$$

$$+t.i.p.$$

Under the "small distortion" assumption (so that the product of Φ with a second-order term can be taken as negligible),

$$\frac{U_t - U}{U_c C} = \Phi\widehat{y}_t - \frac{1}{2}\left[\frac{\epsilon}{\Theta}var_i\{p_t(i)\} - (1-\sigma)\widehat{y}_t^2 + \frac{1+\varphi}{1-\alpha}(\widehat{y}_t - a_t)^2\right] + t.i.p.$$

$$= \Phi\widehat{y}_t - \frac{1}{2}\left[\frac{\epsilon}{\Theta}var_i\{p_t(i)\} + \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)\widehat{y}_t^2 - 2\left(\frac{1+\varphi}{1-\alpha}\right)\widehat{y}_t a_t\right]$$

$$+t.i.p.$$

$$= \Phi\widehat{y}_t - \frac{1}{2}\left[\frac{\epsilon}{\Theta}var_i\{p_t(i)\} + \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)(\widehat{y}_t^2 - 2\widehat{y}_t\widehat{y}_t^e)\right] + t.i.p.$$

$$= \Phi\widehat{y}_t - \frac{1}{2}\left[\frac{\epsilon}{\Theta}var_i\{p_t(i)\} + \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)(\widehat{y}_t - \widehat{y}_t^e)^2\right] + t.i.p.$$

where $\widehat{y}_t^e \equiv y_t^e - y^e$, using the fact that $\widehat{y}_t^e = \frac{1+\varphi}{\sigma(1-\alpha)+\varphi+\alpha}\,a_t$.

Accordingly, a second-order approximation can be written to the consumer's welfare losses (up to additive terms independent of policy), and expressed as a fraction of steady state consumption as

$$\mathbb{W} = -E_0\sum_{t=0}^{\infty}\beta^t\left(\frac{U_t - U}{U_c C}\right)$$

$$= -E_0\sum_{t=0}^{\infty}\beta^t\left[\Phi\widehat{y}_t - \frac{1}{2}\left(\frac{\epsilon}{\Theta}var_i\{p_t(i)\} + \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)(\widehat{y}_t - \widehat{y}_t^e)^2\right)\right]$$

Using lemma 2 in appendix 4.1 in chapter 4, the welfare losses can finally be written as

$$\mathbb{W} = -E_0 \sum_{t=0}^{\infty} \beta^t \left[\Phi \widehat{y}_t - \frac{1}{2} \left(\frac{\epsilon}{\lambda} \pi_t^2 + \left(\sigma + \frac{\varphi + \alpha}{1 - \alpha} \right) (\widehat{y}_t - \widehat{y}_t^e)^2 \right) \right] + t.i.p.$$

$$= -E_0 \sum_{t=0}^{\infty} \beta^t \left[\Phi \widehat{x}_t - \frac{1}{2} \left(\frac{\epsilon}{\lambda} \pi_t^2 + \left(\sigma + \frac{\varphi + \alpha}{1 - \alpha} \right) \widehat{x}_t^2 \right) \right] + t.i.p.$$

where $\widehat{x}_t \equiv \widehat{y}_t - \widehat{y}_t^e$

Note that in the particular case of an efficient zero inflation steady state, $\Phi = 0$ and $\widehat{x}_t \equiv \widehat{y}_t - \widehat{y}_t^e = y_t - y_t^e \equiv x_t$. Moreover, if in addition $y_t^n = y_t^e$ for all t (as assumed in chapter 4), then $\widehat{x}_t \equiv \widehat{y}_t - \widehat{y}_t^e = \widetilde{y}_t$.

5.2 Sources of Cost-Push Shocks

This appendix describes two possible sources of cost-push shocks: exogenous variations in desired price markups and exogenous variations in wage markups.

Exogenous variations in desired price markups. Assume that the elasticity of substitution among goods varies over time according to some stationary stochastic process $\{\epsilon_t\}$. Let the associated desired markup be given by $\mu_t^n \equiv \log \frac{\epsilon_t}{\epsilon_t - 1}$. Assuming for simplicity constant returns to labor ($\alpha = 0$), the log-linearized price-setting rule is then given by

$$p_t^* = (1 - \beta\theta) \sum_{k=1}^{\infty} (\beta\theta)^k E_t\{\mu_{t+k}^n + \psi_{t+k}\}$$

$$= (1 - \beta\theta) \sum_{k=1}^{\infty} (\beta\theta)^k E_t\{\mu_{t+k}^n - (p_{t+k} - \psi_{t+k}) + p_{t+k}\}$$

$$= (1 - \beta\theta) \sum_{k=1}^{\infty} (\beta\theta)^k E_t\{p_{t+k} - (\mu_{t+k} - \mu_{t+k}^n)\}$$

The resulting inflation equation then becomes

$$\begin{aligned}\pi_t &= \beta E_t\{\pi_{t+1}\} - \lambda(\mu_t - \mu_t^n) \\ &= \beta E_t\{\pi_{t+1}\} - \lambda(\mu_t - \mu) + \lambda\widehat{\mu}_t^n \\ &= \beta E_t\{\pi_{t+1}\} + \kappa(y_t - y_t^e) + u_t\end{aligned}$$

where $\widehat{\mu}_t^n \equiv \mu_t^n - \mu$, $u_t \equiv \lambda\widehat{\mu}_t^n$, and where it has been assumed the existence of a subsidy that corrects the steady state markup μ, thus implying $\mu_t - \mu \equiv -(\sigma+\varphi)(y_t - y_t^e)$.

Exogenous variations in wage markups. Again, $\alpha = 0$ is assumed for simplicity. As in chapter 3, the inflation equation is given by

$$\pi_t = \beta E_t\{\pi_{t+1}\} - \lambda(\mu_t - \mu)$$

though now

$$\begin{aligned}\mu_t &= p_t - (w_t - a_t) \\ &= -\mu_{w,t} - mrs_t + a_t \\ \mu_t &= \mu_{w,t} - (\sigma+\varphi)y_t + (1+\varphi)a_t\end{aligned}$$

where $\mu_{w,t} \equiv (w_t - p_t) - mrs_t$ represents a time-varying, exogenous wage markup. Let there be a constant subsidy that corrects the distortions associated with the steady state price and wage markups, μ and μ_w. Then, it follows that

$$\mu = \mu_w - (\sigma+\varphi)y_t^e + (1+\varphi)a_t$$

Combining the two previous equations:

$$\mu_t - \mu_t = -(\sigma+\varphi)(y_t - y_t^e) + (\mu_{w,t} - \mu_w)$$

which can be plugged into the inflation equation to yield

$$\pi_t = \beta E_t\{\pi_{t+1}\} + \kappa(y_t - y_t^e) + u_t$$

where $u_t \equiv -\lambda(\mu_{w,t} - \mu_w)$.

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EXERCISES

5.1. **An optimized Taylor rule**

Consider an economy with Calvo-type staggered price setting whose equilibrium dynamics are described by the system

$$x_t = E_t\{x_{t+1}\} - \frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} - \rho) + z_t$$

$$\pi_t = \beta E_t\{\pi_{t+1}\} + \kappa x_t + u_t$$

where $\{z_t\}$ and $\{u_t\}$ are i.i.d., mutually uncorrelated, demand and supply disturbances with variances given by σ_z^2 and σ_u^2 respectively.

Assume that the monetary authority adopts a simple Taylor rule of the form

$$i_t = \rho + \phi_\pi \pi_t$$

a. Solve for the equilibrium processes for output and inflation, as a function of the exogenous supply and demand shocks.
b. Determine the value of the inflation coefficient ϕ_π that minimizes the central bank's loss function

$$\vartheta \; var(x_t) + var(\pi_t)$$

c. Discuss and provide intuition for the dependence of the optimal inflation coefficient on the weight ϑ and the variance ratio $var(z)/var(u)$. What assumptions on parameter values would warrant an aggressive response to inflation implemented through a large ϕ_π? Explain.

5.2. **Optimal Markovian policy**
Consider an economy where inflation is described by the New Keynesian Phillips curve

$$\pi_t = \beta E_t\{\pi_{t+1}\} + \kappa x_t + u_t$$

where $\{u_t\}$ is an exogenous cost-push shock following a stationary AR(1) process

$$u_t = \rho_u u_{t-1} + \varepsilon_t^u$$

In period 0, the central bank chooses once and for all its policy among the class of Markovian policies of the form $x_t = \psi_x u_t$ and $\pi_t = \psi_\pi u_t$ for all t, in order to minimize the loss function

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\pi_t^2 + \vartheta x_t^2\right)$$

subject to the sequence of constraints describing the evolution of inflation.

a. Determine the optimal values of ψ_x and ψ_π.
b. Compare the resulting optimal policy to the optimal discretionary policy analyzed in this chapter. Which one is more desirable from a welfare point of view? Explain
c. Compare the resulting optimal policy to the optimal policy under commitment analyzed in this chapter. Which one is more desirable from a welfare point of view? Explain.

5.3. **Optimal monetary policy in the presence of transaction frictions** *(based on Woodford (2003b))*
As shown in Woodford (2003b), in the presence of real balances as a source of indirect utility in an otherwise standard New Keynesian model, a second-order approximation to the representative household's welfare is proportional to

$$-\frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left(\pi_t^2 + \vartheta \widetilde{y}_t^2 + \alpha_i i_t^2\right)$$

Consider the problem of choosing the state-contingent policy $\{\widetilde{y}_t, \pi_t\}_{t=0}^{\infty}$ that maximizes welfare subject to the sequence of constraints

$$\pi_t = \beta E_t\{\pi_{t+1}\} + \kappa \widetilde{y}_t$$

$$\widetilde{y}_t = -\frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} - r_t^n) + E_t\{\widetilde{y}_{t+1}\}$$

for $t = 0, 1, 2, \ldots$ where the natural rate r_t^n is assumed to follow an exogenous process.

a. Determine the optimality conditions for the problem described above.
b. Show that the implied optimal policy can be implemented by means of an interest rate rule of the form

$$i_t = (1 + \frac{\kappa}{\sigma\beta})i_{t-1} + \frac{1}{\beta}\Delta i_{t-1} + \frac{\kappa}{\alpha_i \sigma}\ \pi_t + \frac{\vartheta}{\alpha_i \sigma}\ \Delta\widetilde{y}_t$$

that is independent of r_t^n and its properties.

5.4. **Inflation persistence and monetary policy** *(based on Steinsson (2003))*
As shown in Steinsson (2003), in the presence of partial price indexation by firms the second-order approximation to the household's welfare losses takes the form

$$\frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t [\vartheta x_t^2 + (\pi_t - \gamma \pi_{t-1})^2]$$

where γ denotes the degree of price indexation to past inflation. The equation describing the evolution of inflation is now given by

$$\pi_t - \gamma \pi_{t-1} = \kappa x_t + \beta E_t\{(\pi_{t+1} - \gamma \pi_t)\} + u_t$$

where u_t represents an exogenous i.i.d. cost-push shock.

a. Determine the optimal policy under discretion, i.e., under the assumption that the monetary authority seeks to minimize each period the short-term losses $\vartheta x_t^2 + (\pi_t - \gamma \pi_{t-1})^2$.
b. Determine the optimal policy under commitment.
c. Discuss how the degree of indexation γ affects the optimal responses to a transitory cost-push shock under the previous two scenarios.

5.5. **Monetary policy, optimal steady state inflation, and the zero lower bound**

Consider a New Keynesian model with equilibrium conditions given by

$$x_t = E_t\{x_{t+1}\} - \frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} - \rho) + z_t$$

and

$$\pi_t - \pi = \beta E_t\{(\pi_{t+1} - \pi)\} + \kappa x_t + u_t$$

where x_t is the (welfare-relevant) output gap, π_t denotes inflation, i_t is the nominal rate, and π is steady state inflation. The disturbances z_t and u_t represent demand and cost-push shocks, and are assumed to follow independent and serially uncorrelated normal distributions with zero mean and variances σ_z^2 and σ_u^2 respectively.

Assume that the loss function for the monetary authority is given by

$$\Theta\pi + E_0 \sum_{t=0}^{\infty} \beta^t \left[\vartheta x_t^2 + (\pi_t - \pi)^2\right]$$

where the first term is assumed to capture the costs of steady state inflation.

a. Derive the optimal policy under discretion (i.e., the time-consistent policy resulting from period-by-period maximization), including the choice of steady state inflation π subject to the constraint that the interest rate hits the zero-bound constraint with only a 5 percent probability.
b. Derive an interest rate rule that would implement the optimal allocation derived in (a) as the unique equilibrium.

Chapter 6

A MODEL WITH STICKY WAGES AND PRICES

Throughout the previous chapters the labor market has been modeled as a perfectly competitive market, in which households and firms take the wage as given. This chapter departs from that assumption by introducing some imperfections in the labor market and analyzing their consequences for monetary policy. In particular, it is assumed that households/workers have some monopoly power, which allows them to set the wage for the differentiated labor services they supply. Furthermore, as was done with the price-setting firms in chapter 3, the assumption here is that workers face Calvo-type constraints on the frequency with which they can adjust nominal wages.

A key result emerges from the analysis of the model with sticky wages and prices: fully stabilizing price inflation is no longer optimal. Instead, the central bank should be concerned about both price and wage stability, because fluctuations in both price and wage inflation, as well as in the output gap, are a source of inefficiencies in the allocation of resources that result in welfare losses for households. Accordingly, the optimal policy seeks to strike a balance between three different objectives, with the relative weights attached to them being a function of the underlying parameter values.

The chapter is organized as follows. Section 6.1 describes a benchmark model in which both sticky wages and sticky prices coexist. Section 6.2 derives the model's log-linearized equilibrium conditions. Section 6.3 discusses the welfare-based criterion and analyzes the limiting cases of full price flexibility and full wage flexibility. Section 6.4 derives and characterizes the optimal monetary policy, while section 6.5 studies the performance of alternative simple rules and their merits as an approximation to the optimal policy. Section 6.6 concludes with some bibliographical notes.

6.1 A Model with Staggered Wage and Price Setting

This section lays out a model of an economy in which nominal wages, as well as prices, are sticky. Following Erceg, Henderson, and Levin (2000), wage stickiness is introduced in a way analogous to price stickiness, as modeled in chapter 3. In particular, a continuum of differentiated labor services is assumed, all of which are used by each firm. Each period only the workers specialized in a (randomly drawn) subset of labor types can adjust their posted nominal wage. As a result, the aggregate nominal wage responds sluggishly to shocks, generating inefficient variations in the average wage markup. In addition, wage inflation, combined with the staggering of wage adjustments, brings about relative wage distortions and an inefficient allocation of labor in a way symmetric to the relative price distortions generated by price inflation in the presence of staggered price setting.

Next, the problem facing firms and households in this environment is described.

6.1.1 Firms

As in chapter 3, a continuum of firms is assumed, indexed by $i \in [0, 1]$. Each firm produces a differentiated good with a technology represented by the production function

$$Y_t(i) = A_t N_t(i)^{1-\alpha} \tag{1}$$

where $Y_t(i)$ denotes the output of good i, A_t is an exogenous technology parameter common to all firms, and $N_t(i)$ is an index of labor input used by firm i and defined by

$$N_t(i) \equiv \left(\int_0^1 N_t(i,j)^{1-\frac{1}{\epsilon_w}} \, dj \right)^{\frac{\epsilon_w}{\epsilon_w - 1}} \tag{2}$$

where $N_t(i, j)$ denotes the quantity of type-j labor employed by firm i in period t. Note that parameter ϵ_w represents the elasticity of substitution among labor varieties. Note also the assumption of a continuum of labor types, indexed by $j \in [0, 1]$.

Let $W_t(j)$ denote the nominal wage for type-j labor prevailing in period t, for all $j \in [0, 1]$. As discussed below, nominal wages are set by workers of each type (or a union representing them) and taken as given by firms. Given the wages effective at any point in time for the different types of labor services, cost minimization yields a corresponding set of demand schedules for each firm i and labor type j, given the firm's total employment $N_t(i)$

$$N_t(i, j) = \left(\frac{W_t(j)}{W_t} \right)^{-\epsilon_w} N_t(i) \tag{3}$$

for all i, $j \in [0, 1]$, where

$$W_t \equiv \left(\int_0^1 W_t(j)^{1-\epsilon_w} dj \right)^{\frac{1}{1-\epsilon_w}} \tag{4}$$

is an aggregate wage index. Substituting (3) into the definition of $N_t(i)$, one can obtain the convenient aggregation result

$$\int_0^1 W_t(j) N_t(i,j) dj = W_t N_t(i)$$

that is, the wage bill of any given firm can be expressed as the product of the wage index W_t and that firm's employment index $N_t(i)$.

Hence, and conditional on an optimal allocation of the wage bill among the different types of labor implied by (3), a firm adjusting its price in period t will solve the following problem, which is identical to the one analyzed in chapter 3, namely:

$$\max_{P_t^*} \sum_{k=0}^{\infty} \theta_p^k E_t \left\{ \Lambda_{t,t+k} (1/P_{t+k}) \left(P_t^* Y_{t+k|t} - \mathcal{C}_{t+k}(Y_{t+k|t}) \right) \right\}$$

subject to the sequence of demand constraints

$$Y_{t+k|t} = \left(\frac{P_t^*}{P_{t+k}} \right)^{-\epsilon_p} C_{t+k} \tag{5}$$

for $k = 0, 1, 2, \ldots$ where $\Lambda_{t,t+k} \equiv \beta^k U_{c,t+k}/U_{c,t}$ is the stochastic discount factor, $\mathcal{C}_t(\cdot)$ is the (nominal) cost function, and $Y_{t+k|t}$ denotes output in period $t + k$ for a firm that last reset its price in period t. Note that a subscript/superscript p is added to price-related variables, to distinguish them from their wage-related counterparts.

As shown in chapter 3, the aggregation of the resulting price-setting rules yields, to a first-order approximation and in a neighborhood of the zero inflation steady state, the following equation for price inflation $\pi_t^p \equiv p_t - p_{t-1}$,

$$\pi_t^p = \beta E_t\{\pi_{t+1}^p\} - \lambda_p \widehat{\mu}_t^p \tag{6}$$

where, as in chapter 3, $\widehat{\mu}_t^p \equiv \mu_t^p - \mu^p$ is the deviations of the average (log) price markup from its flexible price counterpart, and

$\lambda_p \equiv \frac{(1-\theta_p)(1-\beta\theta_p)}{\theta_p}\frac{1-\alpha}{1-\alpha+\alpha\epsilon_p}$. Hence, and as discussed in chapter 3, the presence (or anticipation) of average price markups below their desired levels (i.e., a negative $\widehat{\mu}_t^p$) leads firms that currently adjust prices to raise the latter, thus generating positive inflation.

6.1.2 Households

The economy is populated by a large number of identical households. Each household is made up of a continuum of members, each specialized in a different labor service, and indexed by $j \in [0, 1]$. Income is pooled within each household, which thus acts as risk sharing mechanism. A typical household seeks to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, \{\mathcal{N}_t(j)\}; Z_t)$$

subject to a sequence of period budget constraints

$$\int_0^1 P_t(i)C_t(i)di + Q_t B_t \leq B_{t-1} + \int_0^1 W_t(j)\mathcal{N}_t(j)di + D_t$$

for $t = 0, 1, 2 \ldots$, where $\mathcal{N}_t(j)$ is employment (or work hours) of type-j labor (with $W_t(j)$ the corresponding wage), $C_t \equiv \left(\int_0^1 C_t(i)^{1-\frac{1}{\epsilon_p}} di\right)^{\frac{\epsilon_p}{\epsilon_p-1}}$ is the consumption index, and the remaining variables are defined as in chapter 3. In contrast with the model in that chapter, however, here each household takes as given labor income $\int_0^1 W_t(j)\mathcal{N}_t(j)dj$, since, individually, it has no influence on wages (set by unions) or employment (determined by firms). Thus, the only decisions made by households involve the optimal allocation of consumption expenditures among different good varieties and the optimal intertemporal allocation of consumption. As in chapter 3, those decisions give rise, respectively, to the optimality conditions

$$C_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon_p} C_t \tag{7}$$

for all $i \in [0, 1]$, and

$$Q_t = \beta E_t \left\{ \frac{U_{c,t+1}}{U_{c,t}} \frac{P_t}{P_{t+1}} \right\} \tag{8}$$

The period utility is assumed to be given by:

$$U(C_t, \{\mathcal{N}_t(j)\}; Z_t) = \begin{cases} \left(\dfrac{C_t^{1-\sigma} - 1}{1-\sigma} - \displaystyle\int_0^1 \dfrac{\mathcal{N}_t(j)^{1+\varphi}}{1+\varphi} dj \right) Z_t & \text{for } \sigma \neq 1 \\ \left(\log C_t - \int_0^1 \dfrac{N_t(j)^{1+\varphi}}{1+\varphi} dj \right) Z_t & \text{for } \sigma = 1 \end{cases}$$

where $z_t \equiv \log Z_t$ follows an exogenous $AR(1)$ process

$$z_t = \rho_z z_{t-1} + \varepsilon_t^z$$

Hence the intertemporal optimality condition can be rewritten as:

$$Q_t = \beta E_t \left\{ \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \left(\frac{Z_{t+1}}{Z_t} \right) \left(\frac{P_t}{P_{t+1}} \right) \right\}$$

which can be represented in log-linearized form as:

$$c_t = E_t\{c_{t+1}\} - \frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}^p\} - \rho) + \frac{1}{\sigma}(1 - \rho_z) z_t \tag{9}$$

using the notation introduced in chapters 2 and 3.

Next the problem of wage setting is described.

6.1.2.1 WAGE SETTING

Let us first consider how workers specialized in a given type of labor (or the union representing them) set their wage. Wage rigidities are introduced in a way analogous to the goods market. Specifically, workers specialized in any given labor type can reset their nominal wage only with probability $1 - \theta_w$ each period, independently of the time elapsed since they last adjusted their wage. Equivalently, each period the nominal wage for workers of any given type remains unchanged with probability θ_w.

Consider thus a union resetting its members' wage in period t, and let W_t^* denote the newly set wage. The union chooses W_t^* in a way consistent with utility maximization of its members' households, taking as given the decisions of other unions as well as all the path for aggregate consumption and prices. Specifically, the union will seek to maximize

$$E_t \sum_{k=0}^{\infty} (\beta\theta_w)^k \left(C_{t+k}^{-\sigma} \frac{W_t^*}{P_{t+k}} N_{t+k|t} - \frac{N_{t+k|t}^{1+\varphi}}{1+\varphi} \right) Z_{t+k} \tag{10}$$

subject to the sequence of labor demand schedules

$$N_{t+k|t} = \left(\frac{W_t^*}{W_{t+k}}\right)^{-\epsilon_w} \left(\int_0^1 N_t(i)di\right)$$

for $k = 0, 1, 2, 3, \ldots$ where $N_{t+k|t}$ denotes the level of employment in period $t+k$ among workers that last reset their wage in period t.

The first-order condition associated with the problem above is given by

$$\sum_{k=0}^{\infty}(\beta\theta_w)^k E_t \left\{ N_{t+k|t} Z_{t+k} \left(C_{t+k}^{-\sigma} \frac{W_t^*}{P_{t+k}} + \mathcal{M}_w N_{t+k|t}^{\varphi} \right) \right\} = 0$$

where $\mathcal{M}_w \equiv \frac{\epsilon_w}{\epsilon_w - 1}$.

Letting $MRS_{t+k|t} \equiv C_{t+k}^{\sigma} N_{t+k|t}^{\varphi}$ denote the marginal rate of substitution between household consumption and employment in period $t+k$ relevant to the workers resetting their wage in period t, the optimality condition above can be rewritten as

$$\sum_{k=0}^{\infty}(\beta\theta_w)^k E_t \left\{ N_{t+k|t} Z_{t+k} C_{t+k}^{-\sigma} \left(\frac{W_t^*}{P_{t+k}} - \mathcal{M}_w MRS_{t+k|t} \right) \right\} = 0 \qquad (11)$$

Note that in the limiting case of full wage flexibility ($\theta_w = 0$),

$$\frac{W_t^*}{P_t} = \frac{W_t}{P_t} = \mathcal{M}_w MRS_{t|t}$$

for all t. Thus, $\mathcal{M}_w$ is the wedge between the real wage and the marginal rate of substitution that prevails in the absence of wage rigidities, that is, the *desired* or frictionless gross wage markup.

Note also that in a perfect foresight zero inflation steady state

$$\frac{W^*}{P} = \frac{W}{P} = \mathcal{M}_w MRS$$

Log-linearizing (11) around that steady state yields, after some algebraic manipulation, the following approximate wage setting rule

$$w_t^* = \mu^w + (1 - \beta\theta_w) \sum_{k=0}^{\infty}(\beta\theta_w)^k E_t \left\{ mrs_{t+k|t} + p_{t+k} \right\} \qquad (12)$$

where $\mu^w \equiv \log \mathcal{M}_w$ and $mrs_{t+k|t} \equiv \sigma c_{t+k} + \varphi n_{t+k|t}$.

The intuition behind wage setting rule (12) is straightforward. First, w_t^* is increasing in expected future prices, because households care about the purchasing power of their nominal wage. Second, w_t^* is increasing in the expected average marginal disutility of labor (in terms of goods) over the life of the wage, because households want to adjust their expected average real wage accordingly.

Letting $mrs_{t+k} \equiv \sigma c_{t+k} + \varphi n_{t+k}$ define the economy's *average* marginal rate of substitution, where $n_{t+k} \equiv \log \int_0^1 \int_0^1 N_t(i,j) d_j d_i$ denotes (log) aggegrate employment. Then, and up to a first order approximation:

$$\begin{aligned} mrs_{t+k|t} &= mrs_{t+k} + \varphi(n_{t+k|t} - n_{t+k}) \\ &= mrs_{t+k} - \epsilon_w \varphi(w_t^* - w_{t+k}) \end{aligned}$$

Hence, (12) can be written as

$$\begin{aligned} w_t^* &= \frac{1-\beta\theta_w}{1+\epsilon_w\varphi} \sum_{k=0}^{\infty} (\beta\theta_w)^k E_t\{\mu_w + mrs_{t+k} + \epsilon_w \varphi w_{t+k} + p_{t+k}\} \\ &= \frac{1-\beta\theta_w}{1+\epsilon_w\varphi} \sum_{k=0}^{\infty} (\beta\theta_w)^k \; E_t\left\{(1+\epsilon_w\varphi) w_{t+k} - \widehat{\mu}_{t+k}^w\right\} \\ &= \beta\theta_w E_t\{w_{t+1}^*\} + (1-\beta\theta_w)\left(w_t - (1+\epsilon_w\varphi)^{-1} \widehat{\mu}_t^w\right) \end{aligned} \tag{13}$$

where $\widehat{\mu}_t^w \equiv \mu_t^w - \mu^w$ denotes the deviations of the economy's (log) average wage markup $\mu_t^w \equiv (w_t - p_t) - mrs_t$ from its steady state level μ^w.

6.1.2.2 WAGE INFLATION DYNAMICS

Given the assumed wage setting structure, the evolution of the aggregate wage index (4) is given by

$$W_t = \left(\theta_w W_{t-1}^{1-\epsilon_w} + (1-\theta_w)(W_t^*)^{1-\epsilon_w}\right)^{\frac{1}{1-\epsilon_w}}$$

The previous equation can be log-linearized around the zero (wage) inflation steady state to yield

$$w_t = \theta_w w_{t-1} + (1-\theta_w) w_t^* \tag{14}$$

Combining (13) and (14), and letting $\pi_t^w = w_t - w_{t-1}$ denote wage inflation yields, after some manipulation, the baseline wage inflation equation

$$\pi_t^w = \beta E_t\{\pi_{t+1}^w\} - \lambda_w \widehat{\mu}_t^w \tag{15}$$

where $\lambda_w \equiv \frac{(1-\theta_w)(1-\beta\theta_w)}{\theta_w(1+\epsilon_w\varphi)}$. Note that this wage inflation equation has a form analogous to (6), the equation describing the dynamics of price inflation. The intuition behind it is identical: When the average wage in the economy is below the level consistent with maintaining (on average) the desired markup, workers readjusting their nominal wage will tend to increase the latter, thus generating positive wage inflation.

In this model wage inflation equation (15) replaces condition $w_t - p_t = mrs_t$, one of the optimality conditions associated with the household's problem used extensively in previous chapters. The imperfect adjustment of nominal wages implies that, in response to shocks, the average real wage will not move one-for-one with the average marginal rate of substitution, leading to variations in the average wage markup and in wage inflation, as implied by (15).

6.2 Equilibrium

As in the basic New Keynesian model of chapter 3, goods market clearing requires $Y_t(i) = C_t(i)$ for all t and $i \in [0, 1]$. This in turn implies $Y_t = C_t$ for all t, where $Y_t \equiv \left(\int_0^1 Y_t(i)^{1-\frac{1}{\epsilon_p}} di\right)^{\frac{\epsilon_p}{\epsilon_p - 1}}$ is an index of aggregate output. In logs,

$$y_t = c_t$$

for all t. Aggregate employment is given by

$$N_t \equiv \int_0^1 \int_0^1 N_t(i, j) dj di$$

$$= \int_0^1 N_t(i) \int_0^1 \frac{N_t(i, j)}{N_t(i)} dj di$$

$$= \Delta_{w,t} \int_0^1 N_t(i) di$$

$$= \Delta_{w,t} \left(\frac{Y_t}{A_t} \right)^{\frac{1}{1-\alpha}} \int_0^1 \left(\frac{Y_t(i)}{Y_t} \right)^{\frac{1}{1-\alpha}} di$$

$$= \Delta_{w,t} \Delta_{p,t} \left(\frac{Y_t}{A_t} \right)^{\frac{1}{1-\alpha}}$$

where $\Delta_{w,t} \equiv \int_0^1 \left(\frac{W_t(j)}{W_t} \right)^{-\epsilon_w} dj$ and $\Delta_{p,t} \equiv \int_0^1 \left(\frac{P_t(i)}{P_t} \right)^{\frac{-\epsilon_p}{1-\alpha}} di$. Thus, the following relation between (log) aggregate output and (log) aggregate employment holds

$$(1-\alpha) n_t = y_t - a_t + d_{w,t} + d_{p,t}$$

where $d_{w,t} \equiv (1-\alpha) \log \int_0^1 \left(\frac{W_t(j)}{W_t} \right)^{-\epsilon_w} dj$ and $d_{p,t} \equiv (1-\alpha) \log \times \int_0^1 \left(\frac{P_t(i)}{P_t} \right)^{\frac{-\epsilon_p}{1-\alpha}} di$. As shown in appendix 4.1 of chapter 4, $d_{p,t} \simeq \frac{\epsilon_p}{2\Theta}\ var_i\{p_t(i)\}$. Using an analogous derivation, one can show $d_{w,t} \simeq \frac{(1-\alpha)\epsilon_w}{2}\ var_j\{w_t(j)\}$. Hence, and up to a first order approximation, one can write:

$$(1-\alpha) n_t = y_t - a_t$$

which can be thought of as a labor demand equation, determining employment as a function of output, given technology.

Next a version of the equations for price and wage inflation in terms of the output gap $\widetilde{y}_t \equiv y_t - y_t^n$ is derived. Importantly, the concept of natural output y_t^n used in this chapter is to be understood as referring to the equilibrium level of output when *both* prices and wages are fully flexible. A new variable, the *real wage gap,* is introduced and denoted by $\widetilde{\omega}_t$ and formally defined as

$$\widetilde{\omega}_t \equiv \omega_t - \omega_t^n$$

where $\omega_t \equiv w_t - p_t$ denotes the real wage, and where ω_t^n is the *natural real wage*, that is, the real wage that would prevail in the absence of nominal rigidities, and which is given by

$$\begin{aligned} \omega_t^n &= \log(1-\alpha) + (a_t - \alpha n_t^n) - \mu^p \\ &= \log(1-\alpha) + \psi_{wa} a_t - \mu^p \end{aligned} \tag{16}$$

where $\psi_{wa} \equiv \frac{1-\alpha\psi_{ya}}{1-\alpha} > 0$ and $\psi_{ya} \equiv \frac{1+\varphi}{\sigma(1-\alpha)+\varphi+\alpha}$ (as derived in chapter 3).

First, the average price markup is related to the output and real wage gaps. Using the fact that $\mu_t^p = mpn_t - \omega_t$,

$$\begin{aligned}
\widehat{\mu}_t^p &= (mpn_t - \omega_t) - \mu^p \\
&= (\widetilde{y}_t - \widetilde{n}_t) - \widetilde{\omega}_t \\
&= -\frac{\alpha}{1-\alpha}\widetilde{y}_t - \widetilde{\omega}_t
\end{aligned} \tag{17}$$

Hence, combining (6) and (17) yields the following equation for price inflation as a function of the output and real wage gaps

$$\pi_t^p = \beta E_t\{\pi_{t+1}^p\} + \varkappa_p \widetilde{y}_t + \lambda_p \widetilde{\omega}_t \tag{18}$$

where $\varkappa_p \equiv \frac{\alpha\lambda_p}{1-\alpha}$.

Similarly,

$$\begin{aligned}
\widehat{\mu}_t^w &= \omega_t - mrs_t - \mu^w \\
&= \widetilde{\omega}_t - (\sigma\widetilde{y}_t + \varphi\widetilde{n}_t) \\
&= \widetilde{\omega}_t - \left(\sigma + \frac{\varphi}{1-\alpha}\right)\widetilde{y}_t
\end{aligned} \tag{19}$$

Combining (15) and (19) yields an analogous version of the wage inflation equation in terms of the output and real wage gaps

$$\pi_t^w = \beta E_t\{\pi_{t+1}^w\} + \varkappa_w \widetilde{y}_t - \lambda_w \widetilde{\omega}_t \tag{20}$$

where $\varkappa_w \equiv \lambda_w \left(\sigma + \frac{\varphi}{1-\alpha}\right)$.

In addition, there is an identity relating the changes in the wage gap to price inflation, wage inflation, and the natural wage

$$\widetilde{\omega}_t \equiv \widetilde{\omega}_{t-1} + \pi_t^w - \pi_t^p - \Delta\omega_t^n \tag{21}$$

In order to complete the nonpolicy block of the model, equilibrium conditions (18), (20), and (21) must be supplemented with a dynamic IS equation familiar from earlier chapters, and which can be derived by combining the goods market clearing condition $y_t = c_t$ with the Euler equation (9). The resulting expression is rewritten in terms of the output gap as

$$\widetilde{y}_t = -\frac{1}{\sigma}(i_t - E_t\{\pi^p_{t+1}\} - r^n_t) + E_t\{\widetilde{y}_{t+1}\} \tag{22}$$

where the natural interest rate $r^n_t \equiv \rho - \sigma(1-\rho_a)\psi_{ya}a_t + (1-\rho_z)z_t$, derived as in chapter 3, should now be understood as the prevailing real rate in an equilibrium with flexible wages *and* flexible prices.

Finally, and in order to close the model, how the interest rate is determined must be specified. This is done by postulating an interest rate rule of the form

$$i_t = \rho + \phi_p \pi^p_t + \phi_w \pi^w_t + \phi_y \widehat{y}_t + v_t \tag{23}$$

where v_t is an exogenous component normalized to have zero mean.

Plugging (23) into (22) to eliminate the interest rate, the resulting equation and the remaining conditions (18), (20), and (21), the equilibrium dynamics can be represented by means of a system of the form

$$\mathbf{A}^w_0 \mathbf{x}_t = \mathbf{A}^w_1 E_t\{\mathbf{x}_{t+1}\} + \mathbf{B}^w_0 \mathbf{u}_t \tag{24}$$

where $\mathbf{x}_t \equiv [\widetilde{y}_t, \pi^p_t, \pi^w_t, \widetilde{\omega}_{t-1}]'$, $\mathbf{u}_t \equiv [\widehat{r}^n_t - v_t - \phi_y \widehat{y}^n_t, \Delta\omega^n_t]'$,

$$\mathbf{A}^w_0 \equiv \begin{bmatrix} \sigma + \phi_y & \phi_p & \phi_w & 0 \\ -\varkappa_p & 1 & 0 & 0 \\ -\varkappa_w & 0 & 1 & 0 \\ 0 & -1 & 1 & 1 \end{bmatrix}$$

$$\mathbf{A}^w_1 \equiv \begin{bmatrix} \sigma & 1 & 0 & 0 \\ 0 & \beta & 0 & \lambda_p \\ 0 & 0 & \beta & -\lambda_w \\ 0 & 0 & 0 & 1 \end{bmatrix}; \quad \mathbf{B}^w_0 \equiv \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}$$

Note that $\{\mathbf{u}_t\}$ can in turn be expressed as a linear transformation of the exogenous technology, preference, and monetary policy shifters $\{a_t, z_t, v_t\}$.

An important property of (24) is worth emphasizing at this point: In general, the system does *not* have a solution satisfying $\widetilde{y}_t = \pi^p_t = \pi^w_t = 0$ for all t. This is the case even under the assumption that the intercept of the interest rate rule adjusts one-for-one to variations in the natural rate of interest ($v_t = \widehat{r}_t^n$, for all t) combined with $\phi_y = 0$, in contrast with the basic model analyzed in chapters 3 and 4. An implication of that result is that the allocation associated with the equilibrium with flexible prices and wages cannot be attained in the presence of nominal rigidities in both goods and labor markets. The intuition for the previous result rests on the idea that in order for the constraints on price and wage setting not to be binding (and hence, not to distort the equilibrium allocation) all firms and workers should view their current prices and wages as the desired ones. This makes any adjustment unnecessary and leads to constant aggregate price and wage levels, that is, zero inflation in both markets. Note, however, that such an outcome implies a constant real wage, which will generally be inconsistent with the flexible price/flexible wage allocation. Only when the natural wage is constant (so that $\Delta\omega_t^n = 0$ for all t) and $\phi_y = 0$, and as long as the central bank adjusts the nominal rate one-for-one with changes in the natural rate (i.e., $v_t = \widehat{r}_t^n$ for all t) the outcome $\widetilde{y}_t = \pi_t^p = \pi_t^w = 0$ for all t is a solution to (24) and, hence, is consistent with equilibrium. Note, however, that the natural wage will not be constant as long as technology shocks are present (as (16) makes clear).

A second question of interest relates to the conditions that the rule (23) must satisfy to guarantee a unique stationary equilibrium or, equivalently, a unique stationary solution to the system of difference equations (24). Given that vector $\mathbf{x}_t$ contains three non-predetermined variables and one predetermined variable, (local) uniqueness requires that three eigenvalues of $\mathbf{A}^w \equiv (\mathbf{A}_0^w)^{-1}\mathbf{A}_1^w$ lie inside, and one outside, the unit circle.[1] Under the assumption of non-negative values for coefficients (ϕ_p, ϕ_w, ϕ_y), the condition for uniqueness is given by:[2]

$$\phi_p + \phi_w + \phi_y \left(\frac{1-\beta}{\sigma + \frac{\alpha+\varphi}{1-\alpha}} \right) \left(\frac{1}{\lambda_p} + \frac{1}{\lambda_w} \right) > 1$$

Note that in the special case of $\phi_y = 0$ the previous condition simplifies to

$$\phi_p + \phi_w > 1$$

or, what is equivalent, the central bank must adjust the nominal rate more than one-for-one in response to variations in any arbitrary weighted

[1] See Blanchard and Kahn (1980).
[2] See Flaschel, Franke and Proaño (2008) and Blasselle and Poissonier (2013) for a proof.

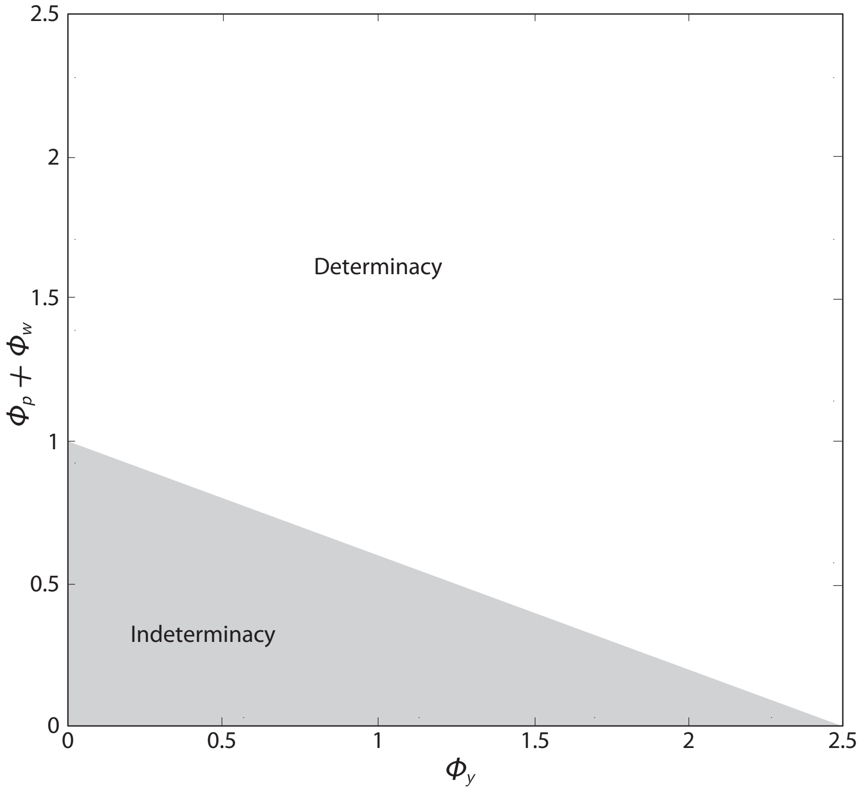

Figure 6.1. Determinacy and Indeterminacy Regions.

average of price and wage inflation. The previous condition can be viewed as extending the Taylor principle requirement discussed in earlier chapters to the case where the central bank is allowed to respond to wage inflation in addition to price inflation. As ϕ_y increases, the lower bound on $\phi_p + \phi_w$ consistent with a unique equilibrium declines, as illustrated in figure 6.1, using the baseline calibration introduced below.

6.2.1 Dynamic Responses to a Monetary Policy Shock

Not surprisingly, the presence of staggered wage setting influences the economy's equilibrium response to different shocks. Figure 6.2 illustrates this point by displaying the responses of the output gap, price inflation, wage inflation, and real wages to a monetary policy shock. Both the policy intervention (a persistent increase in the interest rate rule

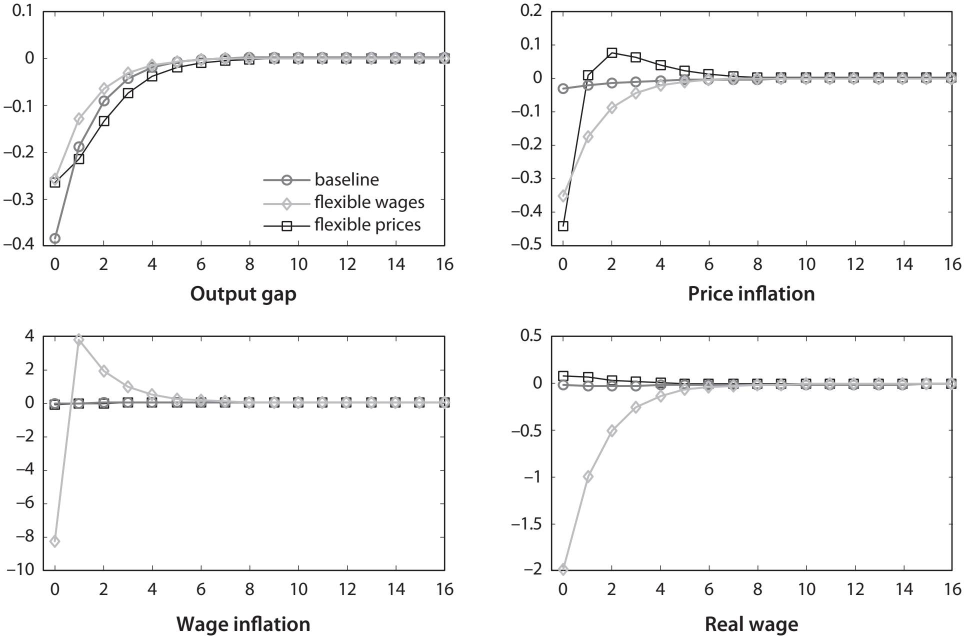

Figure 6.2. Dynamic Responses to a Monetary Policy Shock.

shifter v_t) and the model's calibration are identical to the analogous experiment described in chapter 3. In particular, a simple policy rule of the form (23) is assumed with $\phi_p = 1.5$ and $\phi_y = 0.125$. The only difference here is to allow for sticky wages, introduced as described above. In order to disentangle the role played by each type of rigidity, results are shown for three alternative calibrations of θ_p and θ_w. The first calibration corresponds to an economy in which price and wage rigidities coexist. As in our baseline model of chapter 3, it is assumed that $\theta_p = 3/4$. In addition, $\theta_w = 3/4$ is set, which implies an average duration of wage spells of four quarters. The latter assumption seems to accord with the empirical evidence.[3] The second calibration assumes sticky prices and flexible wages ($\theta_p = 3/4$, $\theta_w = 0$) and, hence, corresponds to the basic model introduced in chapter 3. Finally, the third calibration corresponds to an economy with flexible prices and sticky wages ($\theta_p = 0$, $\theta_w = 3/4$). The policy intervention consists of an increase of 0.25 percentage points in the exogenous component of the interest rate rule. That change would lead, in the absence of an endogenous component in the interest rate

[3] See, e.g., Barattieri, Basu, and Gottschalk (2014).

rule, to an impact increase of one percentage point in the (annualized) nominal interest rate. In addition, we set $\epsilon_w = 4.5$, a value that is justified in the next chapter as being consistent with an average unemployment rate of 5 percent. As in the analogous experiment of chapter 3, assume an autoregressive coefficient $\rho_v = 0.5$ in the AR(1) process followed by the interest rate shifter. The remaining parameters are set at their baselines values, as introduced in chapter 3.

In order to interpret the results shown in figure 6.2, it is useful to take the responses under sticky prices and flexible wages—already discussed in chapter 3 and represented here by the lines with diamonds—as a benchmark. The presence of both sticky wages and prices (responses shown by the lines with circles) generates, not surprisingly, a more muted response of wage inflation. The latter partly explains the sluggish response of the real wage, which in turn reduces the impact of the decline in activity on price markups and, hence, the limited size of the inflation response. As a result, there is only a moderate endogenous response of the monetary authority to the lower inflation, thus implying persistently higher interest rates, which in turn account for the larger decline in output. By contrast, in the flexible wage economy the decline in activity leads to an (implausibly) large and persistent reduction in the real wage, which amplifies the size of the price inflation drop and the endogenous reaction of the monetary authority, leading to a more muted response of output.

Consider next the consequences of assuming the coexistence of sticky wages and flexible prices (impulse responses represented by the lines with squares). Again, the presence of sticky wages dampens the response of wage inflation to the contractionary monetary policy shock. But now, and given the absence of constraints on price adjustment, price inflation falls considerably in response to the decline in activity and the ensuing lower marginal costs. The large decline in prices in turn leads to a rise in the average real wage that, in turn, dampens (and eventually overturns) the effects of the activity decline on price inflation.

Neither the large negative response of wage inflation and the real wage in the sticky price/flexible wage model, nor the large initial decline in price inflation in the sticky wage/flexible price model appears to be consistent with existing estimates of the dynamic effects of exogenous monetary policy shocks. The latter estimates are instead more in line with the predictions of the model with both sticky prices and wages.[4]

[4] See, e.g., Christiano, Eichenbaum, and Evans (2005).

6.3 Monetary Policy Design with Sticky Wages and Prices

This section explores some of the normative implications of the coexistence of sticky prices and sticky wages, as modeled in the framework above, for the conduct of monetary policy. In so doing, and in order to keep the analysis as simple as possible, necessary assumptions are made to guarantee that the *natural* allocation, that is, the equilibrium allocation in the absence of nominal rigidities, is also the efficient allocation. Given the absence of mechanisms (e.g., capital accumulation) for the economy as a whole to transfer resources across periods, the efficient allocation corresponds to the solution of a sequence of static social planner problems of the form

$$\max U(C_t, \{\mathcal{N}_t(j)\}; Z_t)$$

where $C_t \equiv \left(\int_0^1 C_t(i)^{1-\frac{1}{\epsilon_p}} di\right)^{\frac{\epsilon_p}{\epsilon_p - 1}}$, subject to (1) and (2), as well as the usual market clearing conditions. The optimality conditions for that problem are given by

$$C_t(i) = C_t, \ \text{all } i \in [0, 1] \tag{25}$$

$$N_t(i, j) = \mathcal{N}_t(j) = N_t(i) = N_t, \ \text{all } i, j \in [0, 1] \tag{26}$$

$$-\frac{U_{n,t}}{U_{c,t}} = MPN_t \tag{27}$$

where $MPN_t \equiv (1-\alpha) A_t N_t^{-\alpha}$. Note that demand shocks $\{Z_t\}$ do not affect the efficient allocation.

In the decentralized economy, if *all* firms and households reoptimize their prices each period they will all choose the same prices and wages and, hence, (25) and (26) will be satisfied. On the other hand, optimal price and wage setting in the absence of nominal rigidities implies

$$\frac{W_t}{P_t} = -\frac{U_{n,t}}{U_{c,t}} \mathcal{M}_w$$

and

$$P_t = \mathcal{M}_p \frac{(1-\tau) W_t}{MPN_t}$$

where τ is an employment subsidy, funded through lump sum taxes. Note that by setting $\tau = 1 - \frac{1}{\mathcal{M}_p \mathcal{M}_w}$, condition (27) is also satisfied, thus guaranteeing the efficiency of the flexible price/flexible wage equilibrium allocation. The latter property is assumed to hold for the remainder of this chapter.

Appendix 6.1 derives a second-order approximation to households' discounted utility in the economy with sticky wages and prices, resulting from fluctuations around a steady state with zero wage and price inflation. When the latter is efficient, as is the case under the optimal subsidy derived above, welfare losses, expressed as a fraction of steady state consumption, are given by

$$\mathbb{W} = \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left[\left(\sigma + \frac{\varphi + \alpha}{1 - \alpha} \right) \widetilde{y}_t^2 + \frac{\epsilon_p}{\lambda_p} (\pi_t^p)^2 + \frac{\epsilon_w (1 - \alpha)}{\lambda_w} (\pi_t^w)^2 \right] + t.i.p. \tag{28}$$

where $t.i.p.$ collects various terms that are independent of policy. Thus, by ignoring the latter terms, the average period welfare loss can be written as a linear combination of the variances of the output gap, price inflation, and wage inflation given by

$$\mathbb{L} = \frac{1}{2} \left[\left(\sigma + \frac{\varphi + \alpha}{1 - \alpha} \right) var(\widetilde{y}_t) + \frac{\epsilon_p}{\lambda_p} \, var(\pi_t^p) + \frac{\epsilon_w (1 - \alpha)}{\lambda_w} \, var(\pi_t^w) \right] \tag{29}$$

Note that the relative weight of each of the variances is a function of the underlying parameter values. The weights associated with output gap and price inflation fluctuations are identical to those derived and discussed in chapter 4 for a version of the model economy with sticky prices and flexible wages. The presence of sticky wages implies an additional source of welfare losses, associated with wage inflation fluctuations. The contribution of wage inflation volatility to the welfare losses is increasing in (i) the elasticity of substitution among labor types (ϵ_w), (ii) the elasticity of output with respect to labor input $1-\alpha$, and (iii) the degree of wage stickiness θ_w (which is inversely related to λ_w). Note that (i) and (ii) amplify the negative effect on aggregate productivity of any given dispersion of wages across labor types, while (iii) raises the degree of wage dispersion resulting from any given rate of wage inflation different from zero.

In general, and as argued above, the lower bound of zero welfare losses that characterizes an allocation where $\widetilde{y}_t = \pi_t^p = \pi_t^w = 0$ for all

t is not attainable. The optimal policy will thus have to strike a balance in stabilizing the three abovementioned variables.

In the limiting case of flexible wages, $\lambda_w \longrightarrow +\infty$, and the term in the loss function associated with wage inflation volatility vanishes (i.e., wage inflation is no longer costly). Note that in that case the wage markup is constant and hence,

$$\begin{aligned}\widetilde{\omega}_t &= \sigma \widetilde{c}_t + \varphi \widetilde{n}_t \\ &= \left(\sigma + \frac{\varphi}{1-\alpha}\right) \widetilde{y}_t\end{aligned}$$

which, substituted into (18), yields a New Keynesian Phillips curve identical to that derived in chapter 3, namely,

$$\pi_t^p = \beta E_t\{\pi_{t+1}^p\} + \kappa_p \widetilde{y}_t$$

where $\kappa_p \equiv \lambda_p \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)$. Accordingly, and as shown in chapter 3, there is no longer a tradeoff between stabilization of price inflation and stabilization of the output gap with the optimal policy requiring that $\pi_t^p = \widetilde{y}_t = 0$ for all t.

Similarly, in the limiting case of flexible prices (but sticky wages), $\lambda_p \longrightarrow +\infty$ so that only the terms associated with fluctuations in the output gap and wage inflation remain a source of welfare losses. In that case, and using the fact that price markups will be constant,

$$\begin{aligned}\widetilde{\omega}_t &= \widetilde{y}_t - \widetilde{n}_t \\ &= -\frac{\alpha}{1-\alpha}\, \widetilde{y}_t\end{aligned}$$

which, substituted into (20), yields the wage-inflation equation

$$\pi_t^w = \beta E_t\{\pi_{t+1}^w\} + \kappa_w \widetilde{y}_t$$

where $\kappa_w \equiv \lambda_w \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)$. In that case, the optimal policy will attain the zero lower bound for the welfare losses by fully stabilizing the output gap and wage inflation, i.e., $\pi_t^w = \widetilde{y}_t = 0$ for all t.

Thus, with the exception of the limiting case of full wage flexibility, a policy that seeks to stabilize price inflation completely (i.e., a strict price inflation targeting policy) will be suboptimal. The same is true for a strict wage inflation targeting policy, with the exception of an economy with fully flexible prices.

6.4 Optimal Monetary Policy

Next, the optimal monetary policy is characterized in the economy in which both prices and wages are sticky. For concreteness, the analysis is restricted to the case of full commitment. The central bank seeks to minimize (28) subject to (18), (20), and (21) for $t = 0, 1, 2, \ldots$ Let $\{\zeta_{1,t}\}$, $\{\zeta_{2,t}\}$, and $\{\zeta_{3,t}\}$ denote the sequence Lagrange multipliers associated with the previous constraints, respectively. The optimality conditions for the optimal policy problem are thus given by

$$\left(\sigma + \frac{\varphi + \alpha}{1 - \alpha}\right) \widetilde{y}_t + \varkappa_p \zeta_{1,t} + \varkappa_w \zeta_{2,t} = 0 \tag{30}$$

$$\frac{\epsilon_p}{\lambda_p} \pi_t^p - \Delta\zeta_{1,t} + \zeta_{3,t} = 0 \tag{31}$$

$$\frac{\epsilon_w (1 - \alpha)}{\lambda_w} \pi_t^w - \Delta\zeta_{2,t} - \zeta_{3,t} = 0 \tag{32}$$

$$\lambda_p \zeta_{1,t} - \lambda_w \zeta_{2,t} + \zeta_{3,t} - \beta E_t\{\zeta_{3,t+1}\} = 0 \tag{33}$$

for $t = 0, 1, 2, \ldots$ which, together with the constraints (18), (20), and (21) given $\zeta_{1,-1} = \zeta_{2,-1} = 0$ and an initial condition for $\widetilde{\omega}_{-1}$, characterize the solution to the optimal policy problem. The equilibrium under the optimal policy is represented in a compact way as the stationary solution to the dynamical system

$$\mathbf{A}_0^* \mathbf{x}_t = \mathbf{A}_1^* E_t\{\mathbf{x}_{t+1}\} + \mathbf{B}_0^* \Delta a_t$$

where $\mathbf{x}_t \equiv [\widetilde{y}_t,\ \pi_t^p,\ \pi_t^w, \widetilde{\omega}_{t-1},\ \zeta_{1,t-1},\ \zeta_{2,t-1},\ \zeta_{3,t}]'$ and where $\mathbf{A}_0^*$, $\mathbf{A}_1^*$, and $\mathbf{B}_0^*$ are defined in appendix 6.2.

6.4.1 *The Case of Demand Shocks*

The optimal policy when demand shocks are the only source of fluctuations is considered first. Note that in that case the equilibrium corresponds to the stationary solution to the system

$$\mathbf{A}_0^* \mathbf{x}_t = \mathbf{A}_1^* E_t\{\mathbf{x}_{t+1}\}$$

Thus, starting from a steady state (i.e., under the assumption that $\widetilde{\omega}_{-1} = 0$), the optimal policy requires full stabilization of the output and wage gaps, price inflation, and wage inflation at a zero level, that is, $\mathbf{x}_t = 0$ for all t.[5] Since neither the natural level of output nor the natural

[5] In the case $\widetilde{\omega}_{-1} \neq 0$ the optimal policy implies some transitional dynamics (needed to bring back the real wage to its steady state level), with $\mathbf{x}_t = 0$ holding only asymptotically.

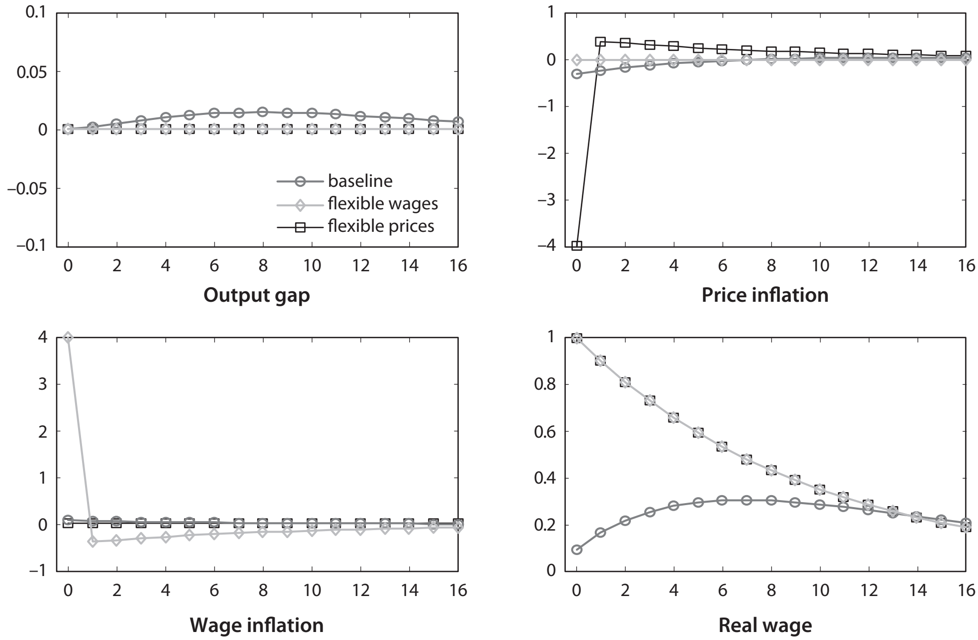

Figure 6.3. Dynamic Responses to a Technology Shock under the Optimal Monetary Policy.

wage is affected by demand shocks, that outcome is associated with a constant real wage and output.

In order to implement the optimal policy, the central bank can follow a rule that adjusts the nominal rate one-for-one to changes in the natural rate (which is affected by the demand shocks). Thus, the rule

$$i_t = r_t^n + \phi_p \pi_t^p$$

where $r_t^n = \rho + (1 - \rho_z) z_t$, and $\phi_p > 1$ would implement the optimal policy under demand shocks, with the second term on the right-hand side included in order to guarantee a unique equilibrium. The optimal policy's outcome would correspond to the efficient allocation.

6.4.2 The Case of Technology Shocks

Figure 6.3 displays the responses of the output gap, price and wage inflation, and the real wage to a positive technology shock under the optimal policy, for the three parameter calibrations considered earlier. Note that, as shown in chapter 4, with sticky prices but flexible wages (lines with diamonds) the optimal policy implies full stabilization of the price level and thus no effect on price inflation. Because that policy replicates the flexible price/flexible wage equilibrium allocation, the responses of both output and the real wage correspond to their natural counterparts, with the necessary adjustment of the real wage attained through large and persistent wage inflation that, given the assumed flexibility of wages, causes no distortions.

When prices are flexible but wages are sticky (represented by the lines with squares), and in a way consistent with the discussion in section 6.3, the natural allocation can also be attained, though now it requires full stabilization of nominal wages and, hence, zero wage inflation. The latter requirement in turn implies that the adjustment in the real wage be achieved through negative price inflation that, given the assumption of flexible prices, is no longer costly in terms of welfare.

When both prices and wages are sticky the natural allocation can no longer be attained. In that case, the optimal policy strikes a balance between attaining the output and real wage adjustments warranted by the rise in productivity and, on the other hand, keeping wage and price inflation close to zero to avoid the distortions associated with nominal instability. As a result, and in response to a positive technology shock, the real wage rises but not as much as the natural wage (note that the latter coincides with the response under the two previous calibrations). Given the convexity of welfare losses in price and wage inflation, it is optimal to raise the real wage smoothly, through a mix of negative price inflation and positive wage inflation. Note finally that the output gap deviates from zero in response to the shock, though such deviations are quantitatively very small.

Next, a particular configuration of parameter values is examined for which the optimal policy takes a simple form that can be characterized analytically.

6.4.3 A Special Case with an Analytical Solution

Consider the special case with $\varkappa_p = \varkappa_w \equiv \varkappa$ and $\epsilon_p = \epsilon_w(1-\alpha) \equiv \epsilon$. In that case optimality conditions (30), (31), and (32) in the monetary policy problem can be rewritten as:[6]

$$\pi_t = -\frac{1}{\epsilon}\,\Delta \widetilde{y}_t \tag{34}$$

[6] The derivation of that result makes use of the fact, obtained by combining the definitions of $\varkappa_w$ and $\varkappa_p$, that $\varkappa = \frac{\lambda_p \lambda_w}{\lambda_p + \lambda_w}\left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)$.

for $t = 1, 2, 3, \ldots$ as well as

$$\pi_0 = -\frac{1}{\epsilon}\, \widetilde{y}_0 \tag{35}$$

where

$$\pi_t \equiv \frac{\lambda_w}{\lambda_p + \lambda_w}\, \pi_t^p + \frac{\lambda_p}{\lambda_p + \lambda_w}\, \pi_t^w$$

is a weighted average of price and wage inflation, henceforth referred to as *composite* inflation, in which the relative weight associated with wage (price) inflation is increasing in the degree of wage (price) rigidities.

Note also that, independent of parameter values, one can always combine the wage and price inflation equations (18) and (20) to obtain the following version of the New Keynesian Phillips curve in terms of composite inflation

$$\pi_t = \beta E_t\{\pi_{t+1}\} + \varkappa \widetilde{y}_t \tag{36}$$

Thus, (36) implies that there is no tradeoff between stabilization of the output gap and stabilization of the particular composite measure of inflation introduced above. Thus it is possible to fully stabilize composite inflation and the output gap at zero, that is,

$$\pi_t = 0$$

and

$$\widetilde{y}_t = 0$$

for $t = 0, 1, 2, \ldots$ But under the special case considered here, such a policy it is not only feasible but also optimal, since it satisfies the optimality conditions (34) and (35).

Thus, in the particular case considered here, the optimal policy takes a simple form: The central bank should focus uniquely on targeting and fully stabilizing a weighted average of price and wage inflation, with the weights determined by underlying parameters. In particular, the relative weight of price (wage) inflation is increasing in the degree of price (wage) stickiness.

A nice feature of the optimal policy in the particular case analyzed above is that its implementation by the monetary authority does not rely on the output gap being observable: It suffices that the monetary authority keeps track of the composite inflation measure, and responds (aggressively) to any deviation from zero of that measure. This is thus an instance in which the divine coincidence result introduced in chapter 4 obtains.

Of course, and as seen above for the general case, the optimal policy does not have such a simple characterization, requiring instead that the central bank follows a much more complicated target rule satisfying (18), (20), (30), (31), (32), and (33) simultaneously. In that context, it is of interest to know to what extent different simple monetary policy rules may be able to approximate the optimal policy, an issue that is attended to in the following section.

6.5 Evaluation of Simple Rules under Sticky Wages and Prices

This section considers a number of simple monetary policy rules and provides a quantitative evaluation of their impact on welfare. That evaluation is based on the average period losses implied by each simple rule, given by (29).

Six different simple rules are analyzed. The first rule, which is referred to as *strict price inflation targeting*, requires that price inflation be zero at all times ($\pi_t^p = 0$ for all t). Also assumed is an analogous rule for wage inflation, that is, a *strict wage inflation targeting* rule ($\pi_t^w = 0$ for all t). The third rule stabilizes the weighted average of price and wage inflation introduced in the previous section. That rule is referred to as a *strict composite inflation targeting* rule. As shown above, that rule is optimal only under a very specific configuration of the model's parameters (which is not the case for the baseline calibration). But even when those conditions are not satisfied, that rule has a special interest because, as implied by (36), it is equivalent to a rule that fully stabilizes the output gap.

The remaining three rules considered take the form of a simple interest rate rule

$$i_t = 0.01 + 1.5\pi_t$$

where π_t refers, respectively, to price inflation, wage inflation, or composite inflation. These rules are referred to as *flexible (price, wage, or composite) inflation targeting* rules.

The top panel of table 6.1 reports the main findings of that exercise when variations in the technology shifter a_t are the only source of

TABLE 6.1
Evaluation of Simple Rules

	Optimal	*Strict Targeting*			*Flexible Targeting*		
		Price	*Wage*	*Composite*	*Price*	*Wage*	*Composite*
Technology shocks							
$\sigma(\pi^p)$	0.11	0	0.13	0.12	0.29	0.24	0.24
$\sigma(\pi^w)$	0.03	0.26	0	0.02	0.23	0.16	0.16
$\sigma(\widetilde{y})$	0.04	3.38	0.20	0	0.84	1.18	1.11
$\mathbb{L}$	0.0330	0.78	0.039	0.0337	0.47	0.305	0.307
Demand shocks							
$\sigma(\pi^p)$	0	0	0	0	0.02	0.04	0.03
$\sigma(\pi^w)$	0	0	0	0	0.05	0.06	0.06
$\sigma(\widetilde{y})$	0	0	0	0	1.08	1.05	1.06
$\mathbb{L}$	0	0	0	0	0.061	0.067	0.066

fluctuations. That variable is assumed to follow an AR(1) process with an autoregressive coefficient $\rho_a = 0.9$ and a standard deviation of 0.01 for its innovation. The remaining parameters are set at their baseline values. For each simple rule, the implied standard deviation of price inflation, wage inflation, and the output gap is reported, as well as the corresponding average period welfare loss. In addition to the simple rules, the table also reports the corresponding statistics for the optimal policy, which provides a useful benchmark.

A number of results are worth highlighting. The optimal policy implies near-constancy of the output gap, and a standard deviation of wage inflation that is about a third that of price inflation. The implied welfare losses (relative to the unattainable first-best allocation) are very small, about 1/30 of a percent of steady state consumption. Among the simple rules, the one that fully stabilizes composite inflation does, for practical purposes, as well as the optimal policy, generating a very similar pattern of volatilities of the three welfare-relevant variables. Given that, under the baseline calibration, wage inflation has a weight of 0.85 in composite inflation, it is perhaps not surprising that strict wage inflation targeting ranks second among the simple rules considered, with implied losses only slightly above those of the optimal policy. Interestingly, under this baseline calibration, strict price inflation targeting rules are the worst, largely due to the large fluctuations in wage inflation and the output gap that result from following those rules. Among the flexible targeting rules, the one that responds to price inflation does worse, though it improves on its strict targeting counterpart. Both wage and composite inflation targeting, in their flexible form, perform worse than their strict targeting counterparts.

The bottom panel of table 6.1 reports the corresponding results conditional on demand shocks. Specifically, z_t is assumed to follow an $AR(1)$ process with autoregressive coefficient 0.5 and a standard deviation of 0.01 for its innovation. As discussed above, the optimal policy is consistent with the efficient allocation in this case, implying no welfare losses. That policy can be implemented by fully stabilizing any of the three inflation measures considered, implying in turn the full stabilization of the output gap and hence a form of divine coincidence. The flexible targeting policies, on the other hand, fall short of attaining the efficient allocation, generating a non-negligible volatility in the output gap.

Overall, the message conveyed by the exercise of this section can be viewed as twofold. First, in the presence of sticky wages (coexisting with sticky prices), policies that focus exclusively on stabilizing price inflation are clearly suboptimal, especially in the presence of technology shocks. Second, when technology shocks are present and in the absence of further imperfections, a policy that responds aggressively to an appropriate weighted average of price and wage inflation emerges as a most desirable one. Of course, choosing the appropriate weights remains a challenge. A policy that gives a slightly dominant weight to wage inflation in the definition of that composite performs as well as the optimal policy for all practical purposes. Interestingly, that conclusion appears at odds with the practice of most central banks, which seem to attach little weight to wage inflation as a target variable, with the interest in that variable often limited to its ability to influence (and thus help predict) current and future price inflation developments.

6.6 Notes on the Literature

Early examples of nonoptimizing rational expectations models with nominal wage rigidities can be found in the work of Fischer (1977) and Taylor (1980). Cooley and Cho (1995) and Bénassy (1995) were among the first papers that embedded the assumption of sticky nominal wages in a dynamic stochastic general equilibrium model, and examined its implications for the properties of a number of variables in the presence of both real and monetary shocks.

Erceg, Henderson, and Levin (2000) developed the New Keynesian model with both staggered price and staggered wage contracts à la Calvo that has become the framework of reference in the literature, and on which much of this chapter builds. The focus of their paper was, like this chapter, on the derivation of the implications for monetary policy. A similar focus, including a discussion of the special case in which targeting a weighted average of wage and price inflation is optimal, can be found in Woodford (2003, chap. 6) and Giannoni and Woodford (2003). Other work has focused instead on the impact of staggered wage setting on the persistence of the effects of monetary policy shocks. See, for example, Huang and Liu (2002) and, especially, Woodford (2003, chap. 3) for a detailed discussion of the role of wage stickiness in that regard.

Staggered wage setting is also a common feature of medium-scale models like those of Kim (2000), Smets and Wouters (2003, 2007), and Christiano, Eichenbaum, and Evans (2005). Those models also allow for some degree of wage indexation to prices. An analysis of the optimal implementable rules in a medium-scale model can be found in Schmitt-Grohé and Uribe (2006).

Appendix

6.1 A Second-Order Approximation to Welfare Losses with Price and Wage Stickiness

A second-order Taylor expansion to the representative household's period t utility around the steady state, combined with a goods market clearing condition, yields:

$$
\begin{aligned}
U_t - U \simeq\ & U_c C \left(\frac{C_t - C}{C}\right) + U_n N \int_0^1 \left(\frac{N_t(j) - N}{N}\right) dj \\
& + \frac{1}{2} U_{cc} C^2 \left(\frac{C_t - C}{C}\right)^2 + \frac{1}{2} U_{nn} N^2 \int_0^1 \left(\frac{N_t(j) - N}{N}\right)^2 dj \\
& + U_c C \left(\frac{C_t - C}{C}\right)\left(\frac{Z_t - Z}{Z}\right) \\
& + U_n N \left(\frac{Z_t - Z}{Z}\right) \int_0^1 \left(\frac{N_t(j) - N}{N}\right) dj + t.i.p.
\end{aligned}
$$

where $t.i.p.$ stands for *terms independent of policy*. Equivalently,

$$
\begin{aligned}
U_t - U \simeq\ & U_c C \left((1 + z_t)\widehat{y}_t + \frac{1-\sigma}{2}\widehat{y}_t^2\right) \\
& - U_n N \left((1 + z_t)\int_0^1 \widehat{n}_t(j) dj + \frac{1+\varphi}{2}\int_0^1 \widehat{n}_t(j)^2 dj\right)
\end{aligned}
$$

where $\sigma \equiv -\frac{U_{cc}}{U_c} C$ and $\varphi \equiv \frac{U_{nn}}{U_n} N$, and where use of the market clearing condition $\widehat{c}_t = \widehat{y}_t$ was made.

Define aggregate employment as $N_t \equiv \int_0^1 \mathcal{N}_t(j) dj$, or, in terms of log deviations from steady state and up to a second-order approximation

$$\widehat{n}_t + \frac{1}{2}\widehat{n}_t^2 \simeq \int_0^1 \widehat{n}_t(j) dj + \frac{1}{2}\int_0^1 \widehat{n}_t(j)^2 dj$$

Note also that

$$\begin{aligned}
\int_0^1 \widehat{n}_t(j)^2 dj &= \int_0^1 (\widehat{n}_t(j) - \widehat{n}_t + \widehat{n}_t)^2 dj \\
&= \widehat{n}_t^2 - 2\widehat{n}_t \epsilon_w \int_0^1 \widehat{w}_t(j) dj + \epsilon_w^2 \int_0^1 \widehat{w}_t(j)^2 dj \\
&\simeq \widehat{n}_t^2 + \epsilon_w^2 \ var_j\{w_t(j)\}
\end{aligned}$$

where use of the labor demand equation $\widehat{n}_t(j) - \widehat{n}_t = -\epsilon_w \widehat{w}_t(j)$ has been made and the fact that $\int_0^1 \widehat{w}_t(j) dj = \frac{(\epsilon_w - 1)}{2} var_i\{w_t(i)\}$ is of second order, a result analogous to that obtained for prices.

Thus, period utility can be written up to a second order approximation as

$$\begin{aligned}
U_t - U \simeq U_c C &\left((1 + z_t)\widehat{y}_t + \frac{1-\sigma}{2}\ \widehat{y}_t^2 \right) \\
&- U_n N \left((1 + z_t)\widehat{n}_t + \frac{1+\varphi}{2}\ \widehat{n}_t^2 + \frac{\epsilon_w^2 \varphi}{2}\ var_j\{w_t(j)\} \right)
\end{aligned}$$

Next, derive a relationship between aggregate employment and output

$$\begin{aligned}
N_t &= \int_0^1 \int_0^1 N_t(i,j) dj di \\
&= \int_0^1 N_t(i) \int_0^1 \frac{N_t(i,j)}{N_t(i)} dj di \\
&= \Delta_{w,t} \int_0^1 N_t(i) di
\end{aligned}$$

$$= \Delta_{w,t} \left(\frac{Y_t}{A_t}\right)^{\frac{1}{1-\alpha}} \int_0^1 \left(\frac{Y_t(i)}{Y_t}\right)^{\frac{1}{1-\alpha}} di$$

$$= \Delta_{w,t} \Delta_{p,t} \left(\frac{Y_t}{A_t}\right)^{\frac{1}{1-\alpha}}$$

where $\Delta_{w,t} \equiv \int_0^1 \left(\frac{W_t(j)}{W_t}\right)^{-\epsilon_w} dj$ and $\Delta_{p,t} \equiv \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{\frac{-\epsilon_p}{1-\alpha}} di$. Thus, the following relation between (log) aggregate output and (log) aggregate employment holds

$$(1-\alpha)\widehat{n}_t = \widehat{y}_t - a_t + d_{w,t} + d_{p,t}$$

where $d_{w,t} \equiv (1-\alpha) \log \int_0^1 \left(\frac{W_t(j)}{W_t}\right)^{-\epsilon_w} dj$ and $d_{p,t} \equiv (1-\alpha) \times \log \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{\frac{-\epsilon_p}{1-\alpha}} di$.

As shown in appendix 4.1, $d_{p,t} \simeq \frac{\epsilon_p}{2\Theta}\, var_i\{p_t(i)\}$. Using an analogous derivation, one can show $d_{w,t} \simeq \frac{(1-\alpha)\epsilon_w}{2}\, var_j\{w_t(j)\}$.

Hence, deviations of period utility from steady state can be rewritten as

$$U_t - U \simeq U_c C \left((1+z_t)\widehat{y}_t + \frac{1-\sigma}{2}\widehat{y}_t^2 \right)$$
$$-\frac{U_n N}{(1-\alpha)} \left((1+z_t)\widehat{y}_t + \frac{\epsilon_p}{2\Theta}\, var_i\{p_t(i)\} + \frac{\Upsilon}{2} var_j\{w_t(j)\} \right.$$
$$\left. + \frac{1+\varphi}{2(1-\alpha)} \int_0^1 (\widehat{y}_t - a_t)^2 \, dj \right) + t.i.p.$$

where $\Upsilon \equiv \epsilon_w(1-\alpha)(1+\epsilon_w\varphi)$ and where $t.i.p.$ stands for *terms independent of policy*.

Let Φ denote the size of the steady state distortion, implicitly defined by $-\frac{U_n}{U_c} = MPN(1-\Phi)$. Using the fact that $MPN = (1-\alpha)(Y/N)$,

$$\frac{U_t - U}{U_c C} = (1+z_t)\widehat{y}_t + \frac{1-\sigma}{2}\widehat{y}_t^2$$
$$-(1-\Phi) \left((1+z_t)\widehat{y}_t + \frac{\epsilon_p}{2\Theta} var_i\{p_t(i)\} + \frac{\Upsilon}{2}\, var_j\{w_t(j)\} \right.$$
$$\left. + \frac{1+\varphi}{2(1-\alpha)} (\widehat{y}_t - a_t)^2 \right) + t.i.p.$$

Under the "small distortion" assumption (so that the product of Φ with a second-order term is taken to be negligible) and ignoring $t.i.p.$ terms,

$$
\begin{aligned}
\frac{U_t - U}{U_c C}\, dj &= \Phi \widehat{y}_t - \frac{1}{2}\left(\frac{\epsilon_p}{\Theta}\, var_i\{p_t(i)\} + \Upsilon\, var_j\{w_t(j)\} - (1-\sigma)\widehat{y}_t^2 \right.\\
&\quad \left. + \frac{1+\varphi}{1-\alpha}(\widehat{y}_t - a_t)^2\right)\\
&= \Phi \widehat{y}_t - \frac{1}{2}\ \left(\frac{\epsilon_p}{\Theta} var_i\{p_t(i)\} + \Upsilon\ var_j\{w_t(j)\}\right.\\
&\quad \left. + \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)\widehat{y}_t^2 - 2\left(\frac{1+\varphi}{1-\alpha}\right)\widehat{y}_t a_t\right)\\
&= \Phi \widehat{y}_t - \frac{1}{2}\ \left(\frac{\epsilon_p}{\Theta} var_i\{p_t(i)\} + \Upsilon\ var_j\{w_t(j)\}\right.\\
&\quad \left. + \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)(\widehat{y}_t^2 - 2\widehat{y}_t \widehat{y}_t^e)\right)\\
&= \Phi \widehat{y}_t - \frac{1}{2}\ \left(\frac{\epsilon_p}{\Theta}\ var_i\{p_t(i)\} + \Upsilon\ var_j\{w_t(j)\}\right.\\
&\quad \left. + \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)(\widehat{y}_t - \widehat{y}_t^e)^2\right)
\end{aligned}
$$

where $\widehat{y}_t^e \equiv y_t^e - y^e$, and where the fact has been used that $\widehat{y}_t^e = \frac{1+\varphi}{\sigma(1-\alpha)+\varphi+\alpha}\ a_t$.

Accordingly, a second-order approximation to the consumer's welfare losses can be written as follows (expressed as a fraction of steady state consumption and up to additive terms independent of policy):

$$
\begin{aligned}
\mathbb{W} &= -E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{U_t - U}{U_c C}\right)\\
&= -E_0 \sum_{t=0}^{\infty} \beta^t \left[\Phi \widehat{y}_t - \frac{1}{2}\left(\frac{\epsilon_p}{\Theta} var_i\{p_t(i)\} + \Upsilon\ var_j\{w_t(j)\}\right.\right.\\
&\quad \left.\left. + \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)(\widehat{y}_t - \widehat{y}_t^e)^2\right)\right]
\end{aligned}
$$

Using lemma 2 in appendix 4.1, rewrite the welfare losses as

$$\mathbb{W} = -E_0 \sum_{t=0}^{\infty} \beta^t \left[\Phi \widehat{y}_t - \frac{1}{2} \left(\frac{\epsilon_p}{\lambda_p} (\pi_t^p)^2 + \frac{\epsilon_w (1-\alpha)}{\lambda_w} (\pi_t^w)^2 \right. \right.$$

$$\left. \left. + \left(\sigma + \frac{\varphi + \alpha}{1-\alpha} \right) (\widehat{y}_t - \widehat{y}_t^e)^2 \right) \right]$$

Note that in the particular case of an efficient steady state $\Phi = 0$. In addition, the model satisfies $y_t^n = y_t^e$ for all t, thus implying $\widehat{y}_t - \widehat{y}_t^e = \widetilde{y}_t$.

Definition of $\mathbf{A}_0^*$, $\mathbf{A}_1^*$ and $\mathbf{B}^*$

$$\mathbf{A}_0^* \equiv \begin{bmatrix} -\varkappa_p & 1 & 0 & 0 & 0 & 0 & 0 \\ -\varkappa_w & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 1 & 0 & 0 & 0 \\ \sigma + \frac{\varphi+\alpha}{1-\alpha} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{\epsilon_p}{\lambda_p} & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & \frac{\epsilon_w(1-\alpha)}{\lambda_w} & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{A}_1^* \equiv \begin{bmatrix} 0 & \beta & 0 & \lambda_p & 0 & 0 & 0 \\ 0 & 0 & \beta & -\lambda_w & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\varkappa_p & -\varkappa_w & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -\lambda_p & \lambda_w & \beta \end{bmatrix}$$

$$\mathbf{B}_0^* \equiv [0, 0, \psi_{wa}, 0, 0, 0, 0]'$$

References

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Exercises

6.1. **Optimal Monetary Policy in a Sticky Wage Economy**
Assume a representative firm that is perfectly competitive and has access to a technology described by

$$y_t = a_t + n_t$$

where y_t, n_t, and a_t denote the logs of output, employment, and productivity, respectively. Prices are flexible. Assume

$$a_t = \rho a_{t-1} + \varepsilon_t$$

The representative household's optimal labor supply is given by

$$w_t - p_t = \varphi n_t$$

where w_t and p_t denote the log of the wage and price levels, respectively.

a. Derive the equilibrium behavior of employment and output under the assumption of flexible wages and prices.
b. Next, sticky wages are introduced. For each period, half the workers set the (log) nominal wage, which remains constant for two periods, according to

$$w_t^* = \frac{1}{2}(p_t + E_t\{p_{t+1}\}) + \frac{\varphi}{2}(n_t + E_t\{n_{t+1}\})$$

The average effective (log) wage paid by the firm in period t is thus

$$w_t = \frac{1}{2}(w_t^* + w_{t-1}^*)$$

Show that inflation evolves according to

$$\pi_t = E_t\{\pi_{t+1}\} + \varphi \overline{n}_t + u_t$$

where $\overline{n}_t \equiv n_{t-1} + E_{t-1}\{n_t\} + n_t + E_t\{n_{t+1}\}$ and $u_t \equiv -4a_t - (p_t - E_{t-1}\{p_t\})$.

c. Suppose that aggregate demand is given by by the IS equation

$$y_t = -\frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} - \rho) + E_t\{y_{t+1}\}$$

and assume that the optimal policy requires that the flexible wage allocation be replicated. Describe the equilibrium behavior of the interest rate, wage inflation, and price inflation under the optimal policy.

6.2. **Optimal Monetary Policy with Wages Set in Advance**
The representative firm is perfectly competitive and has access to a technology described by

$$y_t = a_t + n_t$$

where y, n, and a denote the logs of output, employment, and productivity, respectively. Prices are flexible. Assume

$$a_t = \rho_a a_{t-1} + \varepsilon_t^a$$

The optimal labor supply satisfies

$$w_t - p_t = \varphi n_t$$

where w and p denote the log of the (nominal) wage and price levels, respectively.

Aggregate demand is given by the dynamic IS equation

$$y_t = -\frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} - \rho) + E_t\{y_{t+1}\}$$

where i_t denotes the nominal interest rate and $\pi_t \equiv p_t - p_{t-1}$ is the inflation rate.

a. Derive the equilibrium behavior of employment, output, and the real interest rate under the assumption of flexible wages and prices. Can one determine the corresponding equilibrium values for the nominal rate and inflation? Explain.

b. Next, wage stickiness is introduced by assuming that nominal wages are set in advance (i.e., at the end of the previous period), according to the rule

$$w_t = E_{t-1}\{p_t\} + \varphi E_{t-1}\{n_t\}$$

c. Characterize the equilibrium behavior of output, employment, inflation, and the real wage under the assumption that the central bank follows the simple rule

$$i_t = \rho + \phi_\pi \pi_t$$

d. Characterize the optimal policy and its associated equilibrium in the presence of sticky wages, and suggest an interest rate rule that would implement it. (Note: Assume efficiency of the equilibrium allocation in the absence of sticky wages.)

6.3. **Labor Market Institutions as a Source of Long-Run Money Non-neutrality**

As shown in Blanchard and Summers (1986), a perfectly competitive representative firm maximizes profits each period

$$P_t Y_t - W_t N_t$$

subject to a technology $Y_t = N_t^{1-\alpha}$. Assume that the desired labor supply is inelastic and equal to one. Equilibrium in the goods market is given by

$$Y_t = \frac{M_t}{P_t}$$

with the nominal money supply following an AR(1) process (in logs)

$$m_t = \rho_m m_{t-1} + \varepsilon_t$$

Derive the equilibrium process for (the log) of output y_t, employment n_t, prices p_t, and real wages $w_t - p_t$ under each of the alternative assumptions on the wage setting process:

a. Nominal wages are fully flexible and determined competitively.
b. Nominal wages are set in advance, so that the labor market clears in expectation (i.e., $E_{t-1}\{n_t\} = 0$).

c. Nominal wages are set in advance by a union, so that in expectation only currently employed workers are employed (i.e., $E_{t-1}\{n_t\} = n_{t-1}$).
d. Discuss the empirical relevance of the three preceding scenarios in light of their implied properties (comovements, persistence) for real wages, employment, and output.

6.4. **Monetary Policy and Real Wage Rigidities**
Assume that the representative household's utility is given by $E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t)$ with $U(C_t, N_t) = C_t - \frac{1}{2} N_t^2$, where C_t denotes consumption and N_t denotes hours worked. Let firms' technology be given by the production function $Y_t = A_t N_t$, where Y_t denotes output and A_t is an exogenous technology parameter. All output is consumed.

Firms set prices in a staggered fashion à la Calvo, which results in the inflation dynamics equation

$$\pi_t = \beta E_t\{\pi_{t+1}\} - \lambda \widehat{\mu}_t$$

where $\widehat{\mu}_t \equiv \mu_t - \mu$ represents the deviation of average price markup from its constant desired level.

a. Derive an expression for the (log) of the *efficient* level of output (which is denoted by y_t^e) as a function of (log) productivity a_t (i.e., the level of output that a benevolent social planner would choose, given preferences and constraints).
b. Assume that the (log) nominal wage w_t is set each period according to the schedule $w_t = p_t + \frac{1}{1+\delta} n_t$, where $\delta > 0$ (the same assumption is maintained for parts (c), (d), and (e) below). Compare the behavior of the equilibrium real wage under that schedule with the one that would be observed under competitive labor markets. In what sense can the condition $\delta > 0$ be interpreted as a "real rigidity"?
c. Derive the implied (log) *natural* level of output (denoted by y_t^n) defined as the equilibrium level of output under flexible prices (when all firms keep a constant (log) markup μ).
d. Derive an expression for the markup gap $\widehat{\mu}_t$ as a function of the output gap $\widetilde{y}_t \equiv y_t - y_t^n$.
e. Derive the inflation equation in terms of the welfare-relevant output gap $y_t - y_t^e$. Show how the presence of real wage rigidities ($\delta > 0$) generates a tradeoff between stabilization of inflation and stabilization of the welfare-relevant employment gap.

f. Suppose that the monetary authority has a loss function given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\alpha_y (y_t - y_t^*)^2 + \pi_t^2\right)$$

Solve for the equilibrium process for inflation and output under the optimal monetary policy under discretion (time-consistent solution), under the assumption of an i.i.d. technology process a_t. Explain the difference with the case of perfect competition in the labor market. (Note: For simplicity, assume that the frictionless markup μ is infinitesimally small when answering this question.)

Chapter 7

UNEMPLOYMENT IN THE NEW KEYNESIAN MODEL

The New Keynesian model with staggered price and wage setting developed in chapter 6 remains a framework of reference for the analysis of fluctuations and stabilization policies[1]. Many central banks and policy institutions have adopted medium-scale versions of that model as part of their toolkit for the purpose of forecasting and simulation of alternative policies.[2]

But like any other macroeconomic framework, the New Keynesian model has many shortcomings. Among the latter, the lack of any reference to unemployment is often pointed to as one of the model's main weaknesses. This is not surprising, given the central role of that variable in the policy debate and in public perceptions of the costs associated with business cycles. Furthermore, the conspicuous absence of unemployment from the standard New Keynesian model could also be interpreted as suggesting that there is no reason why central banks should monitor or respond to it in a systematic way. It would seem as if, through the lens of the New Keynesian model, unemployment and the frictions underlying it were not essential for understanding fluctuations in nominal and real variables, nor a key ingredient in the design of monetary policy.

Over the past few years a growing number of researchers have sought to rectify that anomaly by developing frameworks that combine the nominal rigidities and consequent monetary non-neutralities of the New Keynesian model with labor market imperfections that give rise to unemployment. Those frictions are generally introduced by embedding a labor market with search and matching, in the tradition of Mortensen and Pissarides (1994), into some version of the New Keynesian model.

[1] The present chapter draws heavily from my Zeuthen lectures, published in Galí (2011b).

[2] See, e.g., Smets and Wouters (2003, 2007) and Christiano, Eichenbaum, and Evans (2005) for examples of medium-scale versions of those models. For descriptions of versions of those models developed at policy institutions, see Christoffel, Coenen, and Warne (2008), Edge, Kiley, and Laforte (2007), and Erceg, Guerrieri, and Gust (2006), among others.

The resulting framework has been used in both positive and normative applications, with and without the assumption of wage rigidities.[3]

In the present chapter a different approach is adopted to introducing unemployment in the New Keynesian framework. The proposed approach, based on Galí (2011a, 2011b), involves a *reinterpretation* of the labor market in the standard New Keynesian model with staggered wage setting, as developed in chapter 6, rather than a modification or an extension of that model. The resulting framework preserves the convenience of the representative household paradigm, and allows one to determine the equilibrium levels of employment, the labor force, and, hence, the unemployment rate (as well as other macro variables of interest) conditional on the monetary policy rule in place. Unemployment in the model results from the presence of market power in labor markets, reflected in a positive average wage markup, that is, a positive gap between the prevailing wage and the disutility of work (expressed in terms of consumption) for the marginal worker employed. On the other hand, fluctuations in the unemployment rate are associated with variations in that average wage markup due to the presence of nominal wage rigidities.

One important advantage of the proposed approach lies in its compatibility with a variety of assumptions regarding aspects of the model unrelated to unemployment, including the specific form of price and wage rigidities, the specification of the household utility, and the possible presence of time-varying desired markups. Needless to say, the proposed framework also has limitations of its own. In particular, it abstracts from potential sources of unemployment other than noncompetitive wages, including those associated with the costly reallocation of labor across firms or sectors (in terms of time and other resources) which may give rise to frictional unemployment. It is important to stress, however, that the findings of much of the recent literature on labor market frictions suggest that the latter are not enough in order to generate unemployment fluctuations of size and persistence similar to those observed in the data, pointing to the need for some kind of wage rigidity.[4]

The chapter is organized as follows. First, the basic model of unemployment is introduced and embedded in the standard New Keynesian framework of chapter 6. Using a calibrated version of the model, its predictions regarding the properties of unemployment in response to technology, demand, and monetary policy shocks are derived, under the assumption that the central bank follows a conventional Taylor rule. The analysis puts special emphasis on the role played by nominal

[3] See section 7.4 for some of the key contributions to that literature.

[4] See, e.g., Hall (2005), Gertler and Trigari (2009), Galí (2011a), and Shimer (2005).

wage rigidities in accounting for the volatility and persistence of unemployment. The chapter then turns its focus to the relation between unemployment and the design of monetary policy, by analyzing the behavior of unemployment and several other macro variables under the optimal monetary policy, and comparing it to that prevailing under a standard Taylor rule. That analysis suggests the presence of likely welfare gains from stabilizing the unemployment rate beyond what is implied by the Taylor rule. This is confirmed by the study of the properties of a generalized Taylor rule, one that allows for a systematic response to the unemployment rate, in addition to output and price inflation. The chapter concludes with a note on the related literature, with references to alternative approaches to the one presented here.

7.1 Introducing Unemployment in the New Keynesian Model

A reformulation of the standard New Keynesian model with staggered wage and price setting developed in chapter 6 is described next. The variant presented here, based on Galí (2011a, 2011b), assumes that labor is indivisible, that is, in each period any given individual either works a fixed number of hours or does not work at all. As a result, all variations in labor input take place at the extensive margin (i.e., in the form of variations in employment). Since that margin is the one that dominates observed variations in total hours of work, the assumption of indivisible labor remains a good first approximation. Most importantly, however, that assumption leads to a definition of unemployment consistent with its empirical counterpart.

As discussed below, the model's equilibrium includes the same set of equations as the model in chapter 6, to which an additional equation describing the evolution of unemployment is added. Thus, the reader is referred to chapter 6 for the details of many of the derivations.

7.1.1 Households, Wage Setting, and Unemployment

The economy is assumed to have a large number of identical households. Each household has a continuum of members represented by the unit square and indexed by a pair $(j, s) \in [0, 1] \times [0, 1]$. The first index, $j \in [0, 1]$, represents the type of labor service that a given household member is specialized in. The second index, $s \in [0, 1]$, determines the disutility from work. The latter is given by χs^{φ} if he is employed and zero otherwise, where $\chi > 0$ and $\varphi > 0$ are exogenous parameters.

As in Merz (1995) and much of the subsequent literature, full risk sharing within the household is assumed. Given the separability of preferences, this implies the same level of consumption for all household members, independently of their work status. This is not an innocuous assumption, especially from a welfare viewpoint, but one that I stick to in order to preserve the model's tractability. [5]

The household's period utility is given by the integral of its members' utilities and can thus be written as follows

$$U(C_t, \{N_t(j)\}; Z_t) \equiv \left(\frac{C_t^{1-\sigma} - 1}{1-\sigma} - \chi \int_0^1 \int_0^{\mathcal{N}_t(j)} s^{\varphi} ds dj \right) Z_t$$
$$= \left(\frac{C_t^{1-\sigma} - 1}{1-\sigma} - \chi \int_0^1 \frac{\mathcal{N}_t(j)^{1+\varphi}}{1+\varphi} dj \right) Z_t$$

where $C_t \equiv \left(\int_0^1 C_t(i)^{1-\frac{1}{\epsilon_p}} di \right)^{\frac{\epsilon_p}{\epsilon_p - 1}}$ is a consumption index, $C_t(i)$ is the quantity consumed of good i, for $i \in [0, 1]$, and $\mathcal{N}_t(j) \in [0, 1]$ is the fraction of members specialized in type j labor who are employed in period t. As in previous chapters below the preference shifter $z_t \equiv \log Z_t$ is assumed to follow the AR(1) process:

$$z_t = \rho_z z_{t-1} + \varepsilon_t^z$$

where $\rho_z \in [0, 1]$ and ε_t^z is a white noise process with zero mean and variance σ_z^2.

Each household seeks to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, \{\mathcal{N}_t(j)\}; Z_t)$$

subject to a sequence of flow budget constraints given by

$$\int_0^1 P_t(i) C_t(i) di + Q_t B_t \leq B_{t-1} + \int_0^1 W_t(j) \mathcal{N}_t(j) dj + D_t \qquad (1)$$

where $P_t(i)$ is the price of good i, $W_t(j)$ is the nominal wage for type j labor, B_t represents purchases of a nominally riskless one-period discount

[5] See Christiano, Trabandt, and Walentin (2010) for an unemployment model with separable preferences but different levels of consumption for employed and un-employed household members.

bond paying one monetary unit, Q_t is the price of that bond, and D_t is a lump-sum component of income (which may include, among other items, dividends from the ownership of firms). As in previous chapters, the above sequence of period budget constraints is supplemented with a solvency condition that prevents the household from engaging in Ponzi schemes.

Optimal demand for each good resulting from utility maximization takes the familiar form:

$$C_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon_p} C_t \tag{2}$$

where $P_t \equiv \left(\int_0^1 P_t(i)^{1-\epsilon_p} di\right)^{\frac{1}{1-\epsilon_p}}$ denotes the price index for final goods. Note also that (2) implies that total consumption expenditures can be written as $\int_0^1 P_t(i)C_t(i)di = P_t C_t$.

The household's intertemporal optimality condition is given by

$$Q_t = \beta E_t \left\{ \left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} \left(\frac{Z_{t+1}}{Z_t}\right) \left(\frac{P_t}{P_{t+1}}\right) \right\} \tag{3}$$

As in the model of chapter 6, the wage for each type of labor is set by a union representing all the workers specialized in it, whereas the corresponding employment level $\mathcal{N}_t(j)$ is determined by the aggregation of firms' labor demand, assumed to be allocated uniformly across households. Thus, both $W_t(j)$ and $\mathcal{N}_t(j)$ are taken as given by each household, for all $j \in [0,1]$. More specifically, and following Calvo's formalism (Calvo (1983)), it is assumed that workers specialized in a given type of labor (or the union representing them) reset their *nominal* wage with probability $1 - \theta_w$ each period. That probability is independent of the time elapsed since they last reset their wage, in addition to being independent across labor types. Thus, a fraction of workers θ_w keep their wage unchanged in any given period, making that parameter a natural index of nominal wage rigidities.

Optimal wage setting proceeds as in chapter 6, to which the reader is referred. As in that chapter, the optimal wage setting decision can be combined with the law of motion for the average nominal wage to derive the following equation describing wage inflation dynamics:

$$\pi_t^w = \beta E_t\{\pi_{t+1}^w\} - \lambda_w \widehat{\mu}_t^w \tag{4}$$

where $\pi_t^w \equiv w_t - w_{t-1}$ is wage inflation, $\mu_t^w \equiv w_t - p_t - mrs_t$ denotes the (log) *average* wage markup, where $mrs_t \equiv \sigma c_t + \varphi n_t + \xi$ defines the economy's *average* marginal rate of substitution (with $\xi \equiv \log \chi$), $\widehat{\mu}_t^w \equiv \mu_t^w - \mu^w$ (with $\mu^w \equiv \log \mathcal{M}^w$), and $\lambda_w \equiv \frac{(1-\theta_w)(1-\beta\theta_w)}{\theta_w(1+\epsilon_w\varphi)} > 0$. In words, wage inflation depends positively on expected one-period-ahead wage inflation and negatively on the deviation of the average wage markup from its desired value.

Next, and following Galí (2011a, 2011b), unemployment is introduced in the framework above, allowing the wage inflation equation (4) to be reformulated in terms of the unemployment rate.

7.1.2 *Introducing Unemployment*

Consider individual (j,s) specialized in type j labor and with disutility of work χs^φ. *Using the welfare of its household as a criterion*, and *taking as given* current labor market conditions (as summarized by the wage prevailing in his trade) that individual will be willing to work (and thus be part of the labor force) in period t if and only if

$$\frac{W_t(j)}{P_t} \geq \chi C_t^\sigma s^\varphi$$

that is, if and only if the relevant real wage exceeds the disutility from work, where the latter is expressed in terms of consumption by dividing the disutility term χs^φ by the household's marginal utility of consumption $C_t^{-\sigma}$.

Thus, the *marginal* supplier of type j labor, denoted by $L_t(j)$, is given by

$$\frac{W_t(j)}{P_t} = \chi C_t^\sigma L_t(j)^\varphi \tag{5}$$

Define the aggregate labor force (or participation rate) as $L_t \equiv \int_0^1 L_t(j)dj$. Taking logs and integrating over j one can derive the following approximate relation:

$$w_t - p_t = \sigma c_t + \varphi l_t + \xi \tag{6}$$

where use is made of the first order approximations around the symmetric steady state $w_t \simeq \int_0^1 w_t(j)dj$ and $l_t \simeq \int_0^1 l_t(j)dj$. Equation (6) can be thought of as an aggregate labor supply or participation equation.

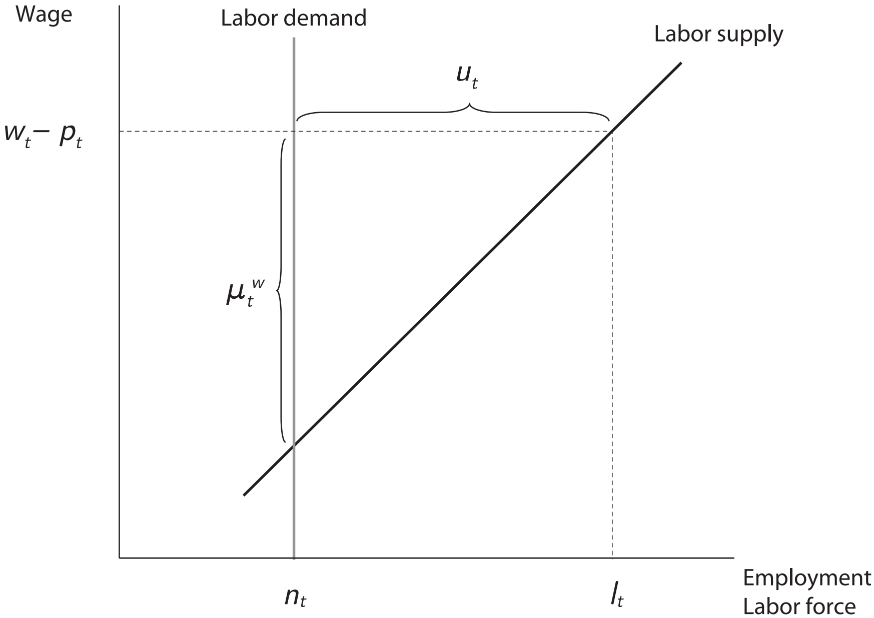

Figure 7.1. The Wage Markup and the Unemployment Rate.

Following Galí (2011a, 2011b), the unemployment rate u_t is defined as the log difference between the labor force and employment:

$$u_t \equiv l_t - n_t \tag{7}$$

This definition of the unemployment rate is, for low values of that variable, very close to the conventional one, that is, $1 - \frac{N_t}{L_t}$.[6] Combining the definition of the average wage markup $\mu_t^w \equiv (w_t - p_t) - (\sigma c_t + \varphi n_t + \xi)$ with (6) and (7), one can obtain the following simple linear relation between the wage markup and the unemployment rate

$$\mu_t^w = \varphi u_t \tag{8}$$

Figure 7.1 represents graphically the above relationship between the average wage markup and the unemployment rate, using a conventional labor market diagram. The figure emphasizes the idea that employment is

[6] Note that $1 - \frac{N_t}{L_t} = 1 - \exp\{-u_t\} \simeq u_t$ for unemployment rates near zero.

demand determined, with labor demand given by the inverse production function (in logs)

$$n_t = \frac{1}{1-\alpha}(y_t - a_t) \tag{9}$$

and, hence, is not directly influenced by the wage.

The labor force, on the other hand, is determined by the labor supply equation (6). The unemployment rate corresponds to the horizontal gap between the labor supply and labor demand schedules, at the level of the prevailing average real wage. The wage markup μ_t^w, on the other hand, is represented in the figure by the vertical gap between the wage and the labor supply, at the level of current employment n_t. Given the assumed linearity, the ratio between "the two gaps" is constant and given by φ, the slope of the labor supply schedule, as implied by (8).

I define the *natural* rate of unemployment, u_t^n, as the unemployment rate that would prevail in the absence of nominal wage rigidities. It follows from the assumption of a constant desired wage markup that such a natural rate is constant and given by

$$u^n = \frac{\mu^w}{\varphi} \tag{10}$$

Equations (8) and (10) reveal the nature of unemployment in the present model. In particular, (10) shows that the presence of market power in the labor market, reflected in the wage markup $\mu^w > 0$, accounts for the existence of positive unemployment, even in the absence of wage rigidities. On the other hand, (8) implies that fluctuations in unemployment are associated with variations in the wage markup. In the New Keynesian model of chapter 6, wage markup variations are the result of nominal wage rigidities. Such rigidities are, accordingly, a necessary ingredient for the model to display unemployment fluctuations.

Note, however, that the above conclusion hinges critically on the assumption of a constant desired wage markup μ^w, an assumption maintained below. But it should be clear that the analysis above can be easily generalized to an environment in which the desired wage markup itself varies over time. In that case the natural rate of unemployment will fluctuate in response to variations in the desired markup, whereas fluctuations in actual unemployment will have two components: a first one associated with changes in the natural rate (driven by changes in the desired wage markup), and a second one linked to deviations of wage markups from their desired levels resulting from nominal wage rigidities.[7]

Finally, note that by combining (4), (8), and (10) one can derive a simple relation between wage inflation and the unemployment gap, $u_t - u^n$:

$$\pi_t^w = \beta E_t\{\pi_{t+1}^w\} - \lambda_w \varphi (u_t - u^n) \tag{11}$$

Galí (2011a) refers to the previous equation as the *New Keynesian Wage Phillips Curve*, and provides some evidence in its support using postwar U.S. data. Note that in contrast with Phillips's original curve (Phillips (1958)), which involved a simple static relation between wage inflation and unemployment, (11) is a forward-looking relation, which can be rewritten to express wage inflation as a function of current and expected future unemployment gaps. Furthermore, while the original Phillips curve was a purely empirical relation, without any theoretical justification, (11) is derived from first principles, with its coefficients being a function of structural parameters.

7.2 Equilibrium

The remaining equilibrium conditions correspond to those of the New Keynesian framework developed in chapter 6. Since they were derived and analyzed in detail therein, they are only listed below, together with (11), for convenience:

$$\widetilde{y}_t = -\frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}^p\} - r_t^n) + E_t\{\widetilde{y}_{t+1}\} \tag{12}$$

$$\pi_t^p = \beta E_t\{\pi_{t+1}^p\} + \varkappa_p \widetilde{y}_t + \lambda_p \widetilde{\omega}_t \tag{13}$$

$$\pi_t^w = \beta E_t\{\pi_{t+1}^w\} - \lambda_w \varphi \widehat{u}_t \tag{14}$$

$$\widetilde{\omega}_t \equiv \widetilde{\omega}_{t-1} + \pi_t^w - \pi_t^p - \Delta\omega_t^n \tag{15}$$

[7] The interested reader can find an analysis of a medium-scale version of the present model allowing for variations in the desired wage markup in Galí , Smets, and Wouters (2012).

to which one must add an equation relating the unemployment gap $\widehat{u}_t \equiv u_t - u^n$ to the output and real wage gaps, namely:[8]

$$\begin{aligned}\varphi\widehat{u}_t &= \widehat{\mu}_t^w \\ &= \widetilde{\omega}_t - (\sigma\widetilde{c}_t + \varphi\widetilde{n}_t) \\ &= \widetilde{\omega}_t - \left(\sigma + \frac{\varphi}{1-\alpha}\right)\widetilde{y}_t \end{aligned} \tag{16}$$

The natural values of the real interest rate and the real wage are now given by (ignoring constant terms):

$$\begin{aligned} r_t^n &= \rho - \sigma(1-\rho_a)\psi_{ya}a_t + (1-\rho_z)z_t \\ \widehat{\omega}_t^n &= \psi_{wa}a_t \end{aligned}$$

where, as in chapter 6, $\psi_{ya} \equiv \frac{1+\varphi}{\sigma(1-\alpha)+\varphi+\alpha}$ and $\psi_{wa} \equiv \frac{1-\alpha\psi_{ya}}{1-\alpha} > 0$.

In addition, equations (7) and (9), together with the fact that $y_t^n = y + \psi_{ya}a_t$ can be used to determine the equilibrium behavior of employment, n_t, and participation, l_t.

Finally, and in order to close the model, one or more equations describing monetary policy must be added. Below, the optimal policy under commitment as well as some simple interest rate rules are analyzed.

7.3 Monetary Policy Design and Unemployment Fluctuations

In this section the properties of the unemployment rate in the face of different shocks are determined for a calibrated version of the model developed above. One goal of the exercise is to assess the ability of the New Keynesian model to account for the empirical properties of unemployment fluctuations. In doing so, it is important to recognize the model's inherent limitations to provide a full account of the observed behavior of macro variables, since it lacks many of the bells and whistles found in medium-scale DSGE models (habit formation, capital

[8] Note that (14) and (16) can be combined to yield the wage inflation equation used in chapter 6, namely:

$$\pi_t^w = \beta E_t\{\pi_{t+1}^w\} + \varkappa_w\widetilde{y}_t - \lambda_w\widetilde{\omega}_t$$

where $\varkappa_w \equiv \lambda_w\left(\sigma + \frac{\varphi}{1-\alpha}\right)$. Thus the extension described here introduces a new variable $(\widehat{u}_t)$ and an extra equation (16).

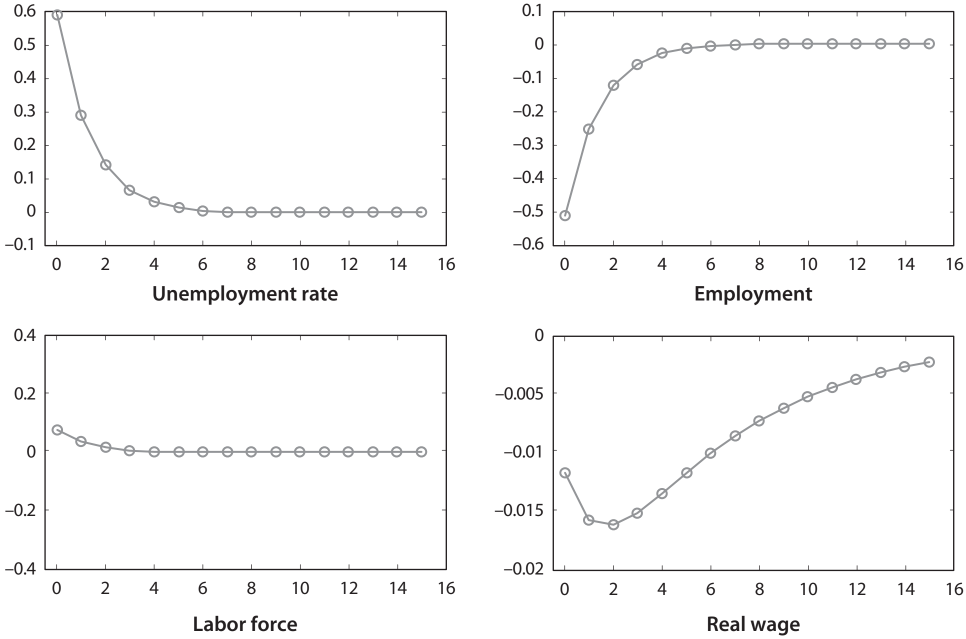

Figure 7.2. Response of Labor Market Variables to a Monetary Policy Shock.

accumulation, indexation, etc.).[9] Its advantage, on the other hand, lies in its simplicity and transparency.

The baseline calibration underlying the quantitative results reported below is identical to that used in chapter 6, so the reader is referred to the description therein. An advantage of the framework above lies in the discipline it provides in the choice of a setting for ϵ_w, the elasticity of substitution among labor types in production. Thus, note that the average wage markup is tied to the natural rate of unemployment through the relation $\mathcal{M}^w \equiv \frac{\epsilon_w}{\epsilon_w - 1} = \exp\{\varphi u^n\}$. The assumptions of $\varphi = 5$ used earlier and $u^n = 0.05$ (consistent with an average unemployment rate of 5 percent), implies $\epsilon_w = 4.5$, which is in turn is associated with an average wage markup of 28 percent.

7.3.1 *The Effects of a Monetary Policy Shock on the Labor Market*

Figure 7.2 plots the dynamic response of employment, n_t, the unemployment rate, u_t, the labor force, l_t, and the real wage $w_t - p_t$ to an

[9] See Galí, Smets, and Wouters (2012) for an analysis of the sources of unemployment fluctuations in an estimated medium-scale version of the present model.

exogenous monetary policy shock, under the assumption that the central bank follows a conventional Taylor rule of the form

$$i_t = 0.01 + 1.5\pi_t^p + 0.125\widehat{y}_t + v_t \tag{17}$$

As in earlier chapters, it is assumed an autoregressive coefficient $\rho_v = 0.5$ in the AR(1) process followed by the monetary policy shifter v_t, with the size of the initial exogenous impulse being 0.25 (i.e., corresponding to an increase in the nominal interest rate of 100 basis points, in annualized terms). The responses are expressed in percentage deviations from the initial steady state.

As the figure makes clear, the unemployment rate rises substantially, and persistently, in response to the tightening of monetary policy. The increase in the unemployment rate is largely due to the decline in employment and only marginally to an increase in the labor force. The latter is, in turn, a consequence of the negative wealth effect, as reflected in the consumption decline (not shown), combined with a nearly unchanged real wage.[10] Finally, the real wage is shown to decline, albeit by a small amount, due the downward pressure of unemployment on nominal wages, which appears to dominate the effect of the decline in price inflation.

The connection between wage rigidities and the properties of unemployment fluctuations is further explored next. For the sake of concreteness, focus is restricted to fluctuations driven by monetary shocks. One may want to view those shocks as a stand-in for other aggregate demand shocks.[11]

Table 7.1 reports measures of volatility, persistence, and cyclicality of unemployment, conditioned on monetary policy shocks, for alternative configurations of θ_w, the degree of wage stickiness, and ρ_v, the autoregressive coefficient of the shock process. The standard deviation of the shock σ_v is set to 0.25 percent (i.e., 1 percent in annualized terms). The question posed here can be stated as follows: How does the degree of nominal rigidities, indexed by θ_w , influence the volatility, persistence and cyclicality of the unemployment rate?

[10] Though small, the countercyclical response of the labor force to a monetary shock appears to be at odds with the empirical evidence reported in Christiano, Trabandt, and Walentin (2010). Galí, Smets, and Wouters (2012) modify the household's utility function along the lines of Jaimovich and Rebelo (2009), and show that a specification of the latter consistent with small wealth effects implies a procyclical (and small) response of the labor force to a monetary shock, in a way consistent with the evidence.

[11] In particular, shocks to the discount rate would have an identical effect on labor market variables.

TABLE 7.1
Wage Rigidities and Unemployment Fluctuations

	Volatility			*Persistence*			*Cyclicality*		
θ_w :	0.1	0.5	0.75	0.1	0.5	0.75	0.1	0.5	0.75
$\rho_v = 0.0$	0.25	0.32	0.33	−0.14	−0.02	−0.01	−0.99	−0.99	−0.99
$\rho_v = 0.5$	0.36	0.60	0.67	0.24	0.44	0.48	−0.96	−0.99	−0.99
$\rho_v = 0.9$	0.31	1.24	2.47	0.51	0.80	0.87	−0.77	−0.98	−0.99

The statistics reported in table 7.1 seek to answer that question. First, note that unemployment *volatility*, measured by the standard deviation of the unemployment rate, increases with the degree of nominal wage rigidities θ_w, for any given degree of persistence of the shock. Thus, other things equal, more rigid wages imply more volatile unemployment. That effect is particularly strong when the shock is more persistent.[12]

The second panel of table 7.1 displays the first-order autocorrelation of the unemployment rate, as a measure of persistence. Persistence is also increasing in the degree of nominal wage rigidities (and, less surprisingly, in the persistence of the shock). Note also that the autocorrelation of unemployment always remains below the autocorrelation of the shock process itself. Under the baseline calibration the persistence of the unemployment rate is substantial (0.48), though still below the persistence observed in the data.[13] The latter is approximated, however, when assuming $\rho_v = 0.9$. It is also interesting to note that for sufficiently low values of ρ_v, the unemployment rate becomes negatively serially correlated, a property inherited from employment.

Finally, the third panel of table 7.1 shows the correlation between the unemployment rate and output, a measure of cyclicality. Like in the data, that correlation is strongly negative for all calibrations. Thus, the strong countercyclicality of unemployment in response of monetary shocks seems to be a robust feature of the model, independently of the degree of wage stickiness and the persistence of those shocks.

The simplicity of the underlying model notwithstanding, the above findings suggest that a New Keynesian model with realistic levels of nominal wage rigidities and persistence of monetary (or other demand

[12] As discussed above, as $\theta_w \to 0$ the volatility of unemployment converges to zero.

[13] The autocorrelation of the (HP-detrended) unemployment rate is as high 0.90 in the United States and 0.92 in the euro area. See Galí (2011b) for these and other labor market statistics.

shocks) can potentially generate unemployment fluctuations with properties comparable to those observed in actual economies.

7.3.2 *Optimal Monetary Policy*

The economy's behavior under the optimal policy can be represented by equations (13), (14), (15), and (16) above, combined with the three optimality conditions derived in chapter 6 and which are reproduced here for convenience:

$$\left(\sigma + \frac{\varphi + \alpha}{1 - \alpha}\right) \widetilde{y}_t + \varkappa_p \zeta_{1,t} + \varkappa_w \zeta_{2,t} = 0 \tag{18}$$

$$\frac{\epsilon_p}{\lambda_p} \pi_t^p - \Delta \zeta_{1,t} + \zeta_{3,t} = 0 \tag{19}$$

$$\frac{\epsilon_w (1 - \alpha)}{\lambda_w} \pi_t^w - \Delta \zeta_{2,t} - \zeta_{3,t} = 0 \tag{20}$$

$$\lambda_p \zeta_{1,t} - \lambda_w \zeta_{2,t} + \zeta_{3,t} - \beta E_t\{\zeta_{3,t+1}\} = 0 \tag{21}$$

for $t = 0, 1, 2, \ldots$ where $\{\zeta_{1,t}\}$, $\{\zeta_{2,t}\}$, and $\{\zeta_{3,t}\}$ denote the Lagrange multipliers associated with the constraints of the optimal policy problem (given by (13), (14), and (15) and (16) combined, respectively). Note that the optimality conditions are identical to those derived in chapter 3 since the slight reformulation of the households' problem does not affect the second order approximation to the welfare losses or the constraints of the optimal policy problem.[14]

Figure 7.3 plots the responses of six macro variables (including the unemployment rate) to a one percent positive technology shock under the optimal policy (shown as a line with diamonds). The figure also shows the corresponding responses under Taylor rule (17) (line with circles). Both sets of responses assume that nonpolicy parameters are at their baseline values.

Note that the optimal policy is more accommodative of the productivity improvement than the Taylor rule, with output increasing more, and employment remaining largely unchanged (as opposed to declining,

[14] See Galí (2011b) for details.

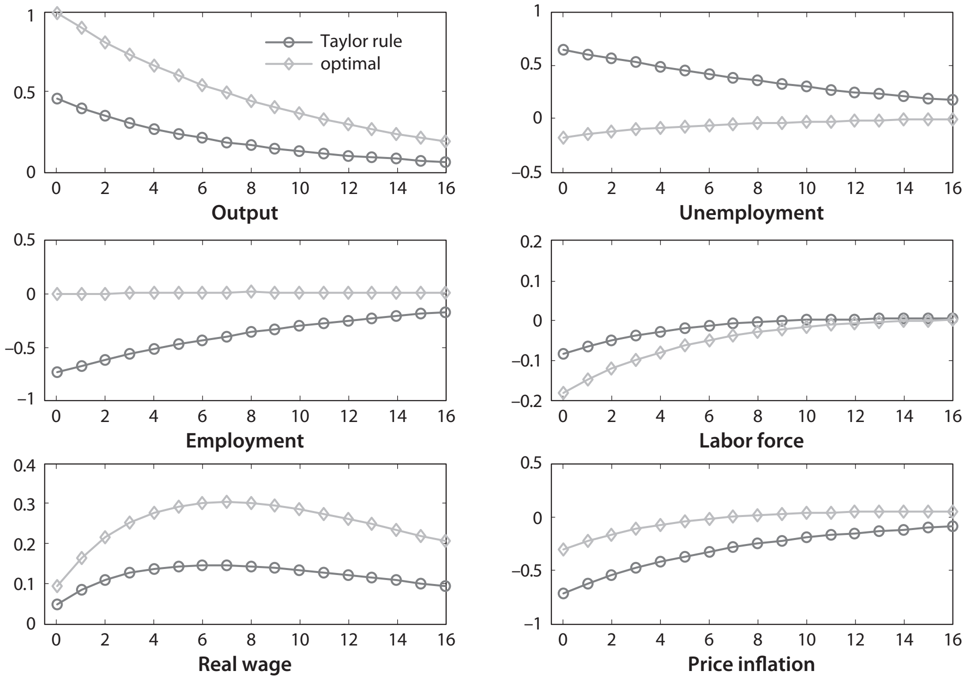

Figure 7.3. Optimal Policy vs. Taylor Rule: Technology Shocks.

as implied by the Taylor rule) under the optimal policy. Note also that the response of those variables is very close to the efficient one, which involves a zero response of employment and participation and a one-for-one adjustment of output to the change in technology under the assumption made here that $\sigma = 1$.

The response of the unemployment rate is substantially different across the two policies: it declines somewhat under the optimal policy, which contrasts with the strong rise observed under the Taylor rule. Due to the presence of rigidities in both prices and wages, the adjustment of the real wage under the optimal policy is considerably muted relative to its natural counterpart (which moves one-for-one with technology), but is stronger than under the Taylor rule. This is due to a larger upward adjustment of the nominal wage (facilitated by the lower unemployment rate), despite a more muted deflation (associated with higher activity and, hence, higher marginal costs, relative to the Taylor rule case).

Figure 7.4 displays the corresponding responses to a one percent increase in the exogenous demand shifter, z_t, under the optimal policy and the Taylor rule. As discussed above, output, inflation, and all

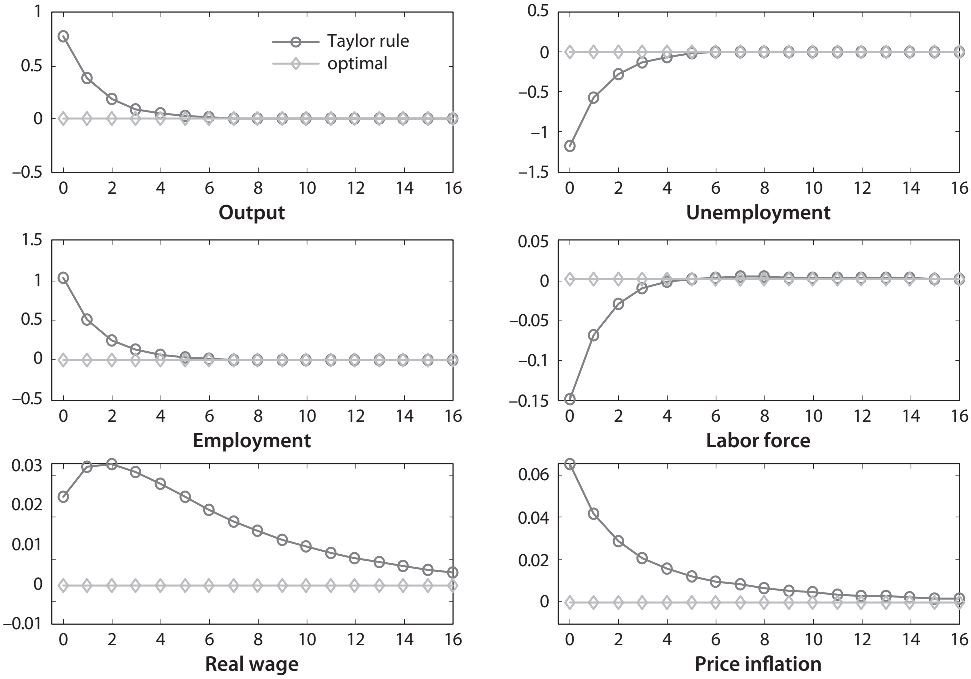

Figure 7.4. Optimal Policy vs. Taylor Rule: Demand Shocks.

labor market variables are fully insulated from the shock under the optimal policy. Only the nominal and real interest rates (not shown) vary, responding one-for-one to the change in the natural rate. On the other hand, under a Taylor rule the demand shock generates a persistent increase in output and employment, a reduction in unemployment, and positive inflation. In other words, the Taylor rule is not sufficiently stabilizing in the face of demand shocks.

7.3.3 *A Simple Interest Rate Rule with Unemployment*

The explicit introduction of unemployment in the model makes it possible to study the properties of simple interest rate rules that have that variable as an argument. To illustrate that point, the equilibrium of the New Keynesian economy developed above is analyzed under the assumption that monetary policy is described by the simple rule:

$$i_t = 0.01 + 1.5\pi_t^p - 0.5\widehat{u}_t \tag{22}$$

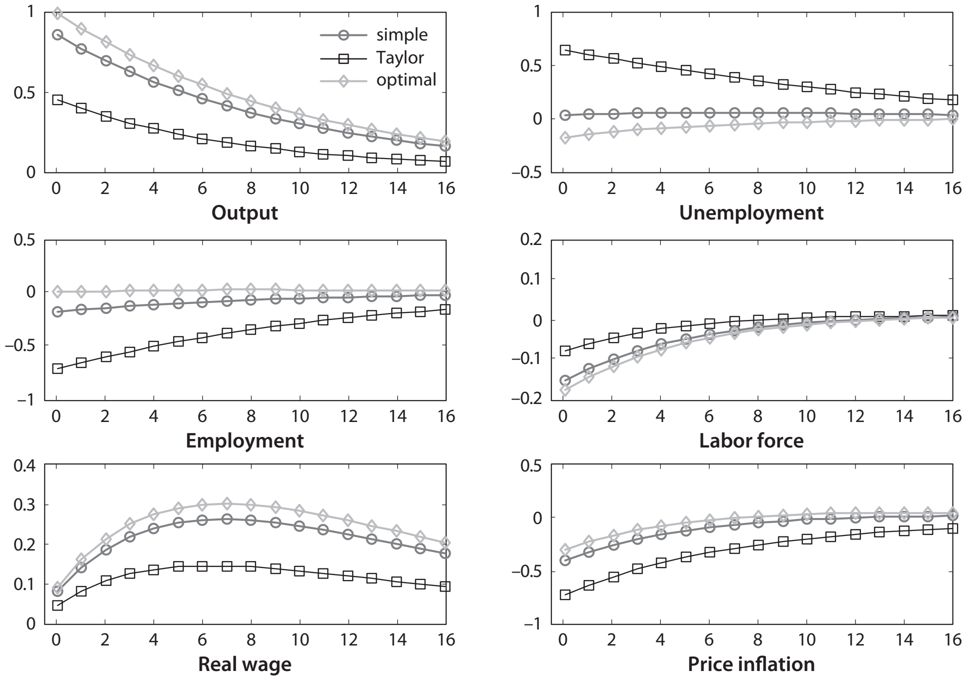

Figure 7.5. Optimal Policy vs. Simple Rule: Technology Shocks.

Galí (2011b) argues that such a rule provides a good account of the Fed's interest rate decisions during the 1987 to 2008 period, before the federal funds rate hit the zero lower bound. In particular, the empirical fit of such a rule seems to improve on the standard Taylor rule.

Figure 7.5 displays the responses of different macro variables to a one percent positive innovation in technology under the above simple rule (line with circles). The figure also show, for the sake of comparison, the responses under both the optimal policy (line with diamonds) and the Taylor rule (line with squares). Note that the responses to a technology shock under simple rule (22) approximate pretty well the responses under the optimal policy. For all variables considered the simple rule tracks the optimal policy much better than the Taylor rule. A similar result holds when demand shocks are the source of fluctuations, as illustrated in figure 7.6, which displays the responses to a positive demand shock under the three rules.

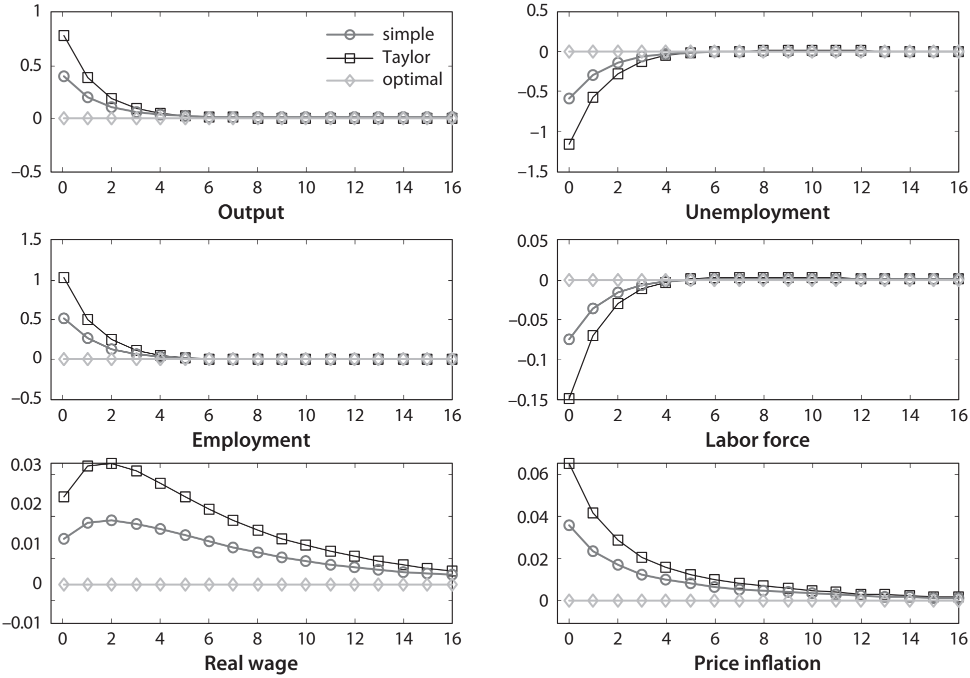

Figure 7.6. Optimal Policy vs. Simple Rule: Demand Shocks.

7.4 Notes on the Literature

The general approach to introducing unemployment in a macro model underlying this chapter can be found in Galí (1996). The specific application to the New Keynesian model was developed in Galí (2011a, 2011b) and, independently, in Casares (2010). Galí, Smets, and Wouters (2012) use the same approach in order to introduce unemployment in a version of Smets and Wouters' (2003, 2007) medium-scale New Keynesian model. Among other results, they show how the use of unemployment data allows the separate identification of wage markup shocks and labor supply shocks, thus overcoming an earlier critique of such models (Chari, Kehoe, and McGrattan (2009)).

Much of the literature on unemployment in the New Keynesian model has followed an alternative approach, one that combines the nominal rigidities that are the hallmark of the New Keynesian framework with the labor market frictions of search and matching models (and which are usually studied in the context of real models; see, e.g., Pissarides (2000)). Walsh (2003, 2005) and Trigari (2009), which were among the earliest contributions to that literature, analyzed the impact of embedding labor market frictions into the basic New Keynesian model with sticky prices but flexible wages, with a focus on the size and persistence of the effects of monetary policy shocks. Later contributions have extended that work in two dimensions. Some have relaxed the assumption of flexible wages, and introduced different forms of nominal and real wage rigidity. The work of Trigari (2006), Christoffel and Linzert (2005), and Sveen and Weinke (2008) falls into that category. Perhaps more interestingly, the focus of attention has gradually turned to normative issues, and more specifically, to the implications of labor market frictions and unemployment for the design of monetary policy. See, For example the work of Blanchard and Galí (2010), Faia (2008, 2009), Ravenna and Walsh (2011), and Thomas (2008).[15] Applications using that approach to interpret U.S. postwar data include Gertler, Sala, and Trigari (2008), who estimate a medium-scale New Keynesian model with search and matching frictions, and Barnichon (2010), who examines the model's ability to account for the changing cyclical properties of labor productivity.

Most of the work combining search and matching frictions with nominal rigidities assumes some form of Nash bargaining of wages. Wage stickiness is generally introduced by assuming that each period only a fraction of firms engage in that bargaining with their workers (Thomas (2008)), following the standard Calvo formalism. An alternative approach can be found in Christiano, Eichenbaum, and Trabandt (2013), who embed the "alternating offers model" of Hall and Milgrom (2008) in a New Keynesian model with search and matching frictions, demonstrating its ability to generate endogenously a sluggish response of aggregate nominal wages and a persistent response of unemployment to various macro shocks.

Alternatives approaches include the work of Christiano, Trabandt, and Walentin (2010), who have modified the New Keynesian model by embedding in it an alternative model of unemployment, where the probability of finding a job is increasing in search effort, and where imperfect risk sharing among individuals is a consequence of the unobservability of effort.

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[15] Galí (2010) provides a survey of that literature.

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EXERCISES

7.1. **Unemployment and monetary policy in a model with flexible prices**

Consider a variation of the model developed in chapters 6 and 7, with firms' technology now being given by

$$Y_t = N_t$$

where N_t is a CES function of the quantities hired of the different types of labor. The elasticity of substitution among the latter is ϵ_w. All firms reset their prices optimally each period ("flexible prices"). An optimal employment subsidy is assumed which exactly offsets the monopolistic competition distortion in the goods market.

a. Show that the dynamics of price inflation can be represented by equation

$$\pi_t^p = \beta E_t\{\pi_{t+1}^p\} - \varphi\lambda_w(u_t - u^n)$$

b. Taking the welfare loss function derived in chapter 6 as a starting point, show that the welfare loss for the present economy can be written (up to a multiplicative scalar) as

$$E_0 \sum_{t=0}^{\infty} \beta^k [\alpha_u u_t^2 + (\pi_t^p)^2]$$

where u_t is the unemployment rate and $\pi_t^p \equiv p_t - p_{t-1}$ is price inflation, and derive an expression for the weight α_u as a function of exogenous parameters.

c. Derive the optimal discretionary monetary policy and characterize the implied equilibrium processes for unemployment and price inflation. Show, in particular, that average price inflation will be positive. Can you think of a way to eliminate that inflationary *bias*?

d. Derive an interest rate rule that will guarantee the implementation of the optimal discretionary policy outcome.

e. Derive the optimal monetary policy under commitment and characterize the implied equilibrium processes for unemployment and price inflation. Describe its main differences relative to the optimal policy under discretion.

7.2. **Wage inflation dynamics with wage markup and labor supply shocks**
Consider an extension of the model developed in the present chapter in which labor disutility for household member (j, s) is given by $\chi_t s^{\varphi}$, for all $(j, s) \in [0, 1] \times [0, 1]$, where $\xi_t \equiv \log \chi_t$ follows an exogenous stationary stochastic process. In addition, a time-varying elasticity of substitution between labor types is assumed, $\{\epsilon_{w,t}\}$, implying a variable desired wage markup $\mu_t^w \equiv \log \frac{\epsilon_{w,t}}{\epsilon_{w,t}-1}$.

a. Derive the equation describing wage inflation dynamics as a function of the real wage, consumption, and employment, and show how the two disturbances ξ_t and μ_t^w enter its error term in a symmetric way thus creating an identification problem in attempts to estimate the model (remark: this is the basis for the Chari, Kehoe, and McGrattan (2009) critique of New Keynesian models).
b. Discuss how the reformulation of the wage equation in terms of the unemployment rate overcomes the previous shortcoming.

7.3. **Taming the wealth effect on labor supply**
Consider a modification of the model developed in the present chapter in which individual utility for household member $(j, s) \in [0, 1] \times [0, 1]$ is assumed to be given by:

$$\log C_t(j, s) - \mathbf{1}_t(j, s)\chi \Theta_t s^{\varphi}$$

where $\mathbf{1}_t(j, s)$ is an indicator function taking a value equal to one if individual (j, s) is employed in period t, and zero otherwise, and

$$\Theta_t \equiv \frac{Z_t}{\overline{C}_t}$$

where $\overline{C}_t$ denotes aggregate consumption, and Z_t evolves over time according to the difference equation

$$Z_t = Z_{t-1}^{1-\upsilon}\overline{C}_t^{\,\upsilon}$$

where $\upsilon \in [0, 1]$. Full risk sharing of consumption among household members is assumed.

a. Show that the household's period utility, defined as the integral over its members' utilities, is given by:

$$\log C_t - \chi_t \Theta_t \int_0^1 \frac{N_t(j)^{1+\varphi}}{1+\varphi} dj$$

where $N_t(j) \in [0, 1]$ denotes the employment rate in period t among workers specialized in type j labor.

b. Derive the aggregate participation equation, and discuss how υ will affect the response of the labor force to different shocks.

Chapter 8

MONETARY POLICY IN THE OPEN ECONOMY

All the models analyzed in earlier chapters assumed a closed economy: households and firms were not able to trade in goods or financial assets with agents located in other economies. This chapter relaxes that assumption by developing an open economy extension of the basic New Keynesian model analyzed in chapter 3. The framework introduces explicitly the exchange rate, the terms of trade, exports and imports, as well as international financial markets. Such a framework can in principle be used to assess the implications of alternative monetary policy rules in an open economy. Because the framework nests as a limiting case the closed economy model of chapter 3, it allows one to assess the extent to which the opening of the economy affects some of the conclusions regarding monetary policy obtained for the closed economy and, in particular, the desirability of a policy that seeks to stabilize inflation (see chapter 4). It is also worth analyzing what role, if any, the exchange rate plays in the optimal design of monetary policy. Finally, the framework can be used to determine the implications of alternative simple rules, as was done in chapter 4 for the closed economy.

The analysis of a monetary open economy raises a number of issues that a modeler needs to confront. First, a choice needs to be made between the modeling of a "large" or a "small" economy, that is, between allowing or not, respectively, for repercussions in the rest of the world of developments (including policy decisions) in the economy being modeled. Second, the existence of two or more economies subject to imperfectly correlated shocks makes room for cross-border trade in assets, for risk sharing purposes and/or in order to smooth consumption over time. Hence, a decision must be made regarding the nature of international asset markets and, more specifically, the set of securities that can be traded in those markets, with possible assumptions ranging from financial autarky to complete markets. Third, one needs to make some assumption about firms' ability to discriminate across countries in the price they charge for the goods they produce ("pricing to market" versus "law of one price"). Furthermore, whenever discrimination is possible and prices are not readjusted continuously, an assumption must be made regarding the currency in which the prices of exported goods are set ("local currency pricing," i.e., prices are set in the currency of the importing economy versus "producer currency pricing," i.e., prices are set in the currency of the producer's country). Other dimensions of open economy modeling that require some choices include the allowance or not for nontradable goods, the existence of trading costs, the possibility of international policy coordination, and so on.

A comprehensive analysis of the range of possible modeling dimensions and how alternative assumptions may affect the design of monetary policy would require a book of its own, thus it is clearly beyond the scope of this chapter. The modest objective here is to present an example of a monetary open economy model to illustrate some of the issues that emerge in the analysis of such economies and which are absent in their closed economy counterparts. In particular, the small open economy model developed below assumes complete international financial markets as well as the law of one price. Then, in the discussion of the model's policy implications and in the notes on the literature at the end of the chapter, reference is made to a number of papers that adopt different assumptions, and the extent to which their findings differ from those obtained here is briefly discussed.

The framework developed below is a simplified version of the one in Galí and Monacelli (2005). In order to focus on the issues brought about by the openness of the economy, the presence of either cost-push shocks or nominal wage rigidities is ignored. The assumptions on preferences and technology, combined with a Calvo price-setting structure and the assumption of complete financial markets, give rise to a highly tractable model and to simple and intuitive log-linearized equilibrium conditions. The latter can be reduced to a two-equation dynamical system consisting of a New Keynesian Phillips curve and a dynamic IS-type equation, whose form is identical to the system derived in chapter 3 for the closed economy, though its coefficients depend on parameters that are specific to the open economy while the driving forces are a function of world variables (that are taken as exogenous to the small open economy). As in its closed economy counterpart, the two equations must be complemented with a description of how monetary policy is conducted.

After describing the model and deriving a simple representation of its equilibrium dynamics, section 8.3 analyzes the transmission of monetary policy shocks, emphasizing the role played by openness in that transmission. Section 8.4 turns to the issue of optimal monetary policy design, focusing on a particular case for which the flexible price allocation is efficient. Under the same assumptions one can derive a second-order approximation to the consumer's utility, which can be used to evaluate alternative policy rules. Section 8.5 assesses the merits of alternative simple monetary policy rules, including an exchange rate peg.

Section 8.6 concludes with a brief note on the related literature.

8.1 A Small Open Economy Model

The economy is modeled as being infinitesimally small (relative to the world economy), with its performance not having any impact on the rest of the world. Next, the problem facing households and firms located in one such economy is described in detail.

8.1.1 Households

The small open economy is inhabited by a representative household seeking to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t; Z_t) \tag{1}$$

where N_t denotes employment (or work hours), C_t is a composite consumption index defined by[1]

$$C_t \equiv \left((1-\upsilon)^{\frac{1}{\eta}} {C_{H,t}}^{1-\frac{1}{\eta}} + \upsilon^{\frac{1}{\eta}} {C_{F,t}}^{1-\frac{1}{\eta}} \right)^{\frac{\eta}{\eta-1}} \tag{2}$$

and $C_{H,t}$ is an index of consumption of domestic goods given by the constant elasticity of consumption (CES) function

$$C_{H,t} \equiv \left(\int_0^1 C_{H,t}(i)^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}}$$

where $i \in [0,1]$ denotes the good variety. $C_{F,t}$ denotes the quantity of imported goods consumed. Z_t is an exogenous preference shifter. Parameter $\epsilon > 1$ denotes the elasticity of substitution between varieties produced domestically. Parameter $\upsilon \in [0,1]$ can be interpreted as a measure of openness. Parameter $\eta > 0$ measures the substitutability between domestic and foreign goods.

[1] As $\eta \to 1$ the consumption index is given by

$$C_t = \frac{1}{(1-\upsilon)^{(1-\upsilon)} \upsilon^{\upsilon}} {C_{H,t}}^{1-\upsilon} {C_{F,t}}^{\upsilon}$$

Maximization of (1) is subject to a sequence of budget constraints of the form

$$\int_0^1 P_{H,t}(i)C_{H,t}(i)di + P_{F,t}C_{F,t} + E_t\{Q_{t,t+1}D_{t+1}\} \leq D_t + W_t N_t \tag{3}$$

for $t = 0, 1, 2, \ldots$ $P_{H,t}(i)$ is the price of domestic variety $i \in [0, 1]$. $P_{F,t}$ is the price of imported goods. D_{t+1} is the nominal payoff in period $t + 1$ of the portfolio held at the end of period t (possibly including shares in firms), W_t is the nominal wage. The previous variables are all expressed in units of domestic currency. $Q_{t,t+1}$ is the stochastic discount factor for one-period-ahead nominal payoffs (to be derived below). Households have access to a complete set of contingent claims, traded internationally.

As in the closed economy, the optimal allocation across varieties of any given expenditure on domestic goods yields the demand functions

$$C_{H,t}(i) = \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\epsilon} C_{H,t} \tag{4}$$

for all $i \in [0, 1]$, where $P_{H,t} \equiv \left(\int_0^1 P_{H,t}(i)^{1-\epsilon}di\right)^{\frac{1}{1-\epsilon}}$ is the *domestic* price index (i.e., an index of prices of domestically produced goods). Combining (4) with the definitions of $P_{H,t}$ and $C_{H,t}$ yields the convenient result $\int_0^1 P_{H,t}(i)C_{H,t}(i)\, di = P_{H,t}C_{H,t}$.

On the other hand, the optimal allocation of expenditures between domestic and imported goods is given by

$$C_{H,t} = (1 - \upsilon)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t \quad ; \quad C_{F,t} = \upsilon\left(\frac{P_{F,t}}{P_t}\right)^{-\eta} C_t \tag{5}$$

where $P_t \equiv \left[(1-\upsilon)(P_{H,t})^{1-\eta} + \upsilon(P_{F,t})^{1-\eta}\right]^{\frac{1}{1-\eta}}$ is the *consumer* price index (CPI).[2] Note that under the assumption of $\eta = 1$ or, alternatively, when the price indexes for domestic and foreign goods are equal (as in the steady state described below), parameter υ corresponds to the share of consumption expenditures allocated to imported goods.

Using (5) and the definitions of indexes C_t and P_t it can be shown that total consumption expenditures by domestic households can be

[2] In the particular case of $\eta = 1$, the CPI takes the form

$$P_t = (P_{H,t})^{1-\upsilon}(P_{F,t})^{\upsilon}$$

.

conveniently expressed as $P_{H,t}C_{H,t} + P_{F,t}C_{F,t} = P_tC_t$. Thus, the period budget constraint simplifies to

$$P_tC_t + E_t\{Q_{t,t+1}\ D_{t+1}\} \leq D_t + W_tN_t \tag{6}$$

As in chapter 3, the period utility function is specialized to be of the form

$$U(C_t, N_t; Z_t) = \begin{cases} \left(\dfrac{C_t^{1-\sigma} - 1}{1-\sigma} - \dfrac{N_t^{1+\varphi}}{1+\varphi}\right) Z_t & \text{for } \sigma \neq 1 \\ \left(\log C_t - \dfrac{N_t^{1+\varphi}}{1+\varphi}\right) Z_t & \text{for } \sigma = 1 \end{cases}$$

with $z_t \equiv \log Z_t$ following an exogenous $AR(1)$ process

$$z_t = \rho_z z_{t-1} + \varepsilon_t^z$$

where $\rho_z \in [0, 1)$.

Under the assumption of perfectly competitive labor markets (which is maintained throughout this chapter), the intratemporal optimality condition for the household's problem takes the same form as in the closed economy model of chapter 3, namely:

$$C_t^\sigma N_t^\varphi = \frac{W_t}{P_t} \tag{7}$$

In order to derive the relevant intertemporal optimality condition note that the following relation must hold for the optimizing household in the small open economy

$$\frac{V_{t,t+1}}{P_t}\ Z_tC_t^{-\sigma} = \xi_{t,t+1}\beta Z_{t+1}C_{t+1}^{-\sigma}\ \frac{1}{P_{t+1}} \tag{8}$$

where $V_{t,t+1}$ is the period t price (in domestic currency) of an Arrow security, that is, a one-period security that yields one unit of domestic currency if a specific state of nature is realized in period $t + 1$, and nothing otherwise, and where $\xi_{t,t+1}$ is the probability of that state of nature being realized in $t+1$ (conditional on the state of nature at t). C_{t+1}, Z_{t+1}, and P_{t+1} on the right side should be interpreted as representing the values taken by those variables at $t + 1$ conditional on the state of nature in which the Arrow security generates a payoff. Thus, the left side captures the utility loss resulting from the purchase of an Arrow security considered (with the corresponding reduction in consumption), whereas the right side measures the expected one-period-ahead utility gain from the additional consumption made possible by the (eventual) security payoff. If the consumer is optimizing the expected utility gain, it must exactly offset the current utility loss.

The price of the nominal Arrow security introduced above must satisfy $V_{t,t+1} = \xi_{t,t+1} Q_{t,t+1}$. Thus, (8) can be rewritten as[3]

$$\beta \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \left(\frac{Z_{t+1}}{Z_t} \right) \left(\frac{P_t}{P_{t+1}} \right) = Q_{t,t+1} \tag{9}$$

which is assumed to be satisfied for all possible states of nature at t and $t+1$.

Taking conditional expectations on both sides of (9) and rearranging terms, a conventional stochastic Euler equation can be derived

$$Q_t = \beta E_t \left\{ \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \left(\frac{Z_{t+1}}{Z_t} \right) \left(\frac{P_t}{P_{t+1}} \right) \right\} \tag{10}$$

where $Q_t \equiv E_t\{Q_{t,t+1}\}$ denotes the price of a one-period discount bond paying off one unit of domestic currency in all states at $t+1$.

Equations (7) and (9) can be respectively written in log-linearized form as

$$w_t - p_t = \sigma c_t + \varphi n_t$$

$$c_t = E_t\{c_{t+1}\} - \frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} - \rho) + \frac{1}{\sigma}(1 - \rho_z) z_t \tag{11}$$

where lowercase letters denote the logs of the respective variables, $i_t \equiv -\log Q_t$ is the short-term nominal rate, $\rho \equiv -\log \beta$ is the time discount rate, and $\pi_t \equiv p_t - p_{t-1}$ is CPI inflation (with $p_t \equiv \log P_t$).

[3] Note that under complete markets a simple no room for arbitrage argument implies that the price of a one-period asset (or portfolio) yielding a random payoff D_{t+1} must be given by $\sum V_{t,t+1} D_{t+1}$ where the sum is over all possible states at $t+1$. Equivalently, that price can be written as $E_t \left\{ \frac{V_{t,t+1}}{\xi_{t,t+1}} D_{t+1} \right\}$. Thus, the one-period stochastic discount factor can be defined as $Q_{t,t+1} \equiv \frac{V_{t,t+1}}{\xi_{t,t+1}}$.

8.1.1.1 DOMESTIC INFLATION, CPI INFLATION, THE REAL EXCHANGE RATE, AND THE TERMS OF TRADE: SOME IDENTITIES

Next, several assumptions and definitions are introduced, and a number of identities are derived that are extensively used below. The *terms of trade, denoted by* $\mathcal{S}_t$, is defined as the price of foreign goods relative to domestic goods. Formally,

$$\mathcal{S}_t \equiv \frac{P_{F,t}}{P_{H,t}}$$

or, in logs, $s_t \equiv \log \mathcal{S}_t = p_{F,t} - p_{H,t}$.

Log-linearization of the CPI index around a symmetric steady state with $\mathcal{S} = 1$ yields the following approximate relation between consumer prices, domestic prices and the terms of trade.[4]

$$\begin{aligned} p_t &= (1-\upsilon)p_{H,t} + \upsilon p_{F,t} \\ &= p_{H,t} + \upsilon s_t \end{aligned} \tag{12}$$

It follows that *domestic inflation,* defined as the rate of change in the index of domestic goods prices, that is, $\pi_{H,t} \equiv p_{H,t} - p_{H,t-1}$, and *CPI inflation,* are linked according to the relation

$$\pi_t = \pi_{H,t} + \upsilon \Delta s_t \tag{13}$$

that is, the gap between the two measures of inflation is proportional to the percentage change in the terms of trade, with the coefficient of proportionality given by the openness index υ.

It is assumed that the *law of one price* holds for individual goods at all times (both for import and export prices). In particular, $P_{F,t} = \mathcal{E}_t P_t^*$, where $\mathcal{E}_t$ is the nominal exchange rate (defined as the price of foreign currency in terms of domestic currency), and P_t^* is the price of foreign goods expressed in foreign currency. P_t^* can also be interpreted as a world price index, given that the size of the small open economy is assumed to be negligible relative to the rest of the world. On the same grounds, for the world as a whole there is no distinction between CPI and domestic price level.

[4] It is useful to note, for future reference, that the approximation (12) holds *exactly* when $\eta = 1$.

The assumed law of one price, combined with the definition of the terms of trade, implies

$$s_t = e_t + p_t^* - p_{H,t} \tag{14}$$

where $e_t \equiv \mathcal{E}_t$.

The *real exchange rate* is defined as the ratio of world and domestic CPI's, both expressed in domestic currency

$$\mathcal{Q}_t \equiv \frac{P_{F,t}}{P_t}$$

Thus, in logs:

$$\begin{aligned} q_t &= p_{F,t} - p_t \\ &= s_t + p_{H,t} - p_t \\ &= (1 - \upsilon)s_t \end{aligned}$$

where the last equality holds exactly if $\eta = 1$, but only up to a first-order approximation when $\eta \neq 1$.

8.1.1.2 INTERNATIONAL RISK SHARING

Under the assumption of a complete set of state-contingent securities traded internationally, a condition analogous to (8) must also hold for households in the rest of the world. Assuming a utility function identical to that of domestic households this condition takes the form:

$$\frac{V_{t,t+1}}{\mathcal{E}_t P_t^*}(C_t^*)^{-\sigma} = \xi_{t,t+1}\beta(C_{t+1}^*)^{-\sigma}\frac{1}{\mathcal{E}_{t+1} P_{t+1}^*}$$

where C_t^* denotes (per capita) consumption by foreign households. The presence of the exchange rate terms reflects the fact that the Arrow security considered has a price $V_{t,t+1}$ and a unit payoff expressed in the currency of the small open economy. For simplicity, it is assumed that foreign households are not subject to preference shocks.

The previous relation can be written in terms of our small open economy's stochastic discount factor as

$$\beta\left(\frac{C_{t+1}^*}{C_t^*}\right)^{-\sigma}\left(\frac{P_t^*}{P_{t+1}^*}\right)\left(\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}\right) = Q_{t,t+1} \tag{15}$$

Combining (9) and (15), together with the definition of the real exchange rate and the law of one price, yields

$$C_t = \vartheta\, C_t^* Z_t^{\frac{1}{\sigma}} \mathcal{Q}_t^{\frac{1}{\sigma}} \tag{16}$$

for all t, and where ϑ is a constant which will generally depend on initial conditions regarding relative net asset positions. Henceforth, and without loss of generality, symmetric initial conditions are assumed (i.e., zero net foreign asset holdings and an ex-ante identical environment), implying $\vartheta = 1$.

Under the assumption that the small economy has an infinitesimal size relative to the rest of the world, $C_t^* = Y_t^*$ for all t, where Y_t^* denotes world output (expressed in per capita terms). Imposing that condition on (16) and taking logs,

$$\begin{aligned} c_t &= y_t^* + \frac{1}{\sigma}(z_t + q_t) \qquad (17) \\ &= y_t^* + \frac{1}{\sigma} z_t + \left(\frac{1-\upsilon}{\sigma}\right) s_t \end{aligned}$$

where the second equality holds only up to a first-order approximation when $\eta \neq 1$. Thus, the assumption of a complete market for state-contingent securities at the international level leads to a simple relationship linking domestic consumption with world output and the terms of trade.

8.1.1.3 A BRIEF DETOUR: UNCOVERED INTEREST PARITY AND THE TERMS OF TRADE

Under the assumption of complete international financial markets, the equilibrium price (in terms of the small open economy's domestic currency) of a bond which pays one unit of foreign currency is given by $\mathcal{E}_t Q_t^* = E_t\{Q_{t,t+1}\mathcal{E}_{t+1}\}$, where Q_t^* is the price of the bond in terms of foreign currency. The previous pricing equation can be combined with the domestic bond pricing equation, $Q_t = E_t\{Q_{t,t+1}\}$ to obtain a version of the *uncovered interest parity* condition

$$E_t\{Q_{t,t+1}[\exp\{i_t\} - \exp\{i_t^*\}(\mathcal{E}_{t+1}/\mathcal{E}_t)]\} = 0$$

Log-linearizing around a perfect foresight steady state yields the familiar uncovered interest parity condition:

$$i_t = i_t^* + E_t\{\Delta e_{t+1}\} \tag{18}$$

Combining (14) and (18) yields the stochastic difference equation

$$s_t = (i_t^* - E_t\{\pi_{t+1}^*\}) - (i_t - E_t\{\pi_{H,t+1}\}) + E_t\{s_{t+1}\} \tag{19}$$

In a symmetric steady state the terms of trade take a unit value, that is, $\mathcal{S} = 1$. That fact, combined with the assumption of stationarity in the model's driving forces, implies that $\lim_{T\to\infty} E_t\{s_T\} = 0$.[5] Hence, (19) can be solved forward to obtain

$$s_t = E_t\left\{\sum_{k=0}^{\infty}[(i_{t+k}^* - \pi_{t+k+1}^*) - (i_{t+k} - \pi_{H,t+k+1})]\right\} \tag{20}$$

that is, the terms of trade are a function of current and anticipated real interest rate differentials.

It must be pointed out that while equations (19) and (20) provide a convenient (and intuitive) way of representing the connection between terms of trade and interest rate differentials, they do not constitute an additional independent equilibrium condition. In particular, it is easy to check that (19) can be derived by combining the consumption Euler equations for both the domestic and world economies with the risk sharing condition (17) and equation (13).

Next, attention is turned to the supply side of the economy.

8.1.2 Firms

8.1.2.1 TECHNOLOGY

A typical firm in the home economy produces a differentiated good with a technology represented by the production function

$$Y_t(i) = A_t N_t(i)^{1-\alpha}$$

[5] The assumption regarding the steady state implies that the real interest rate differential will revert to a zero mean. More generally, the real interest rate differential will revert to a constant mean, as long as the terms of trade are stationary in first differences. That would be the case if, say, the technology parameter had a different average rate of growth relative to the rest of the world.

where $a_t \equiv \log A_t$ is a technology shifter common to all domestic firms following the AR(1) process $a_t = \rho_a a_{t-1} + \varepsilon_t^a$.

8.1.2.2 PRICE SETTING

As in the basic model of chapter 3, it is assumed that domestic firms set prices in a staggered fashion. In particular, a measure $1-\theta$ of (randomly selected) firms sets new prices each period, with an individual firm's probability of reoptimizing in any given period being independent of the time elapsed since it last reset its price. As shown in chapter 3, the optimal price-setting strategy for the typical firm resetting its price in period t can be approximated by the (log-linear) rule

$$\overline{p}_{H,t} = \mu + (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\{\psi_{t+k|t}\} \tag{21}$$

where $\overline{p}_{H,t}$ denotes the price newly set by domestic firms (in logs), $\psi_{t+k|t}$ is the (log) nominal marginal cost in period $t+k$ for a firm last resetting its price in period t, and $\mu \equiv \log \frac{\epsilon}{\epsilon-1}$ is the log of the (gross) steady state markup.[6]

As shown in chapter 3, the previous optimal price setting condition can be combined with an equation describing the evolution of the domestic price level, together with a relation linking the conditional marginal cost $\psi_{t+k|t}$ to the average marginal cost ψ_{t+k}, to yield the following domestic inflation equation:

$$\pi_{H,t} = \beta E_t\{\pi_{H,t+1}\} - \lambda\widehat{\mu}_t \tag{22}$$

where $\widehat{\mu}_t \equiv p_{H,t} - \psi_{t+k} - \mu$ is the markup gap, and

$$\lambda \equiv \left(\frac{(1-\theta)(1-\beta\theta)}{\theta}\right)\Theta$$

where $\Theta \equiv \frac{1-\alpha}{1-\alpha+\alpha\epsilon} \in [0,1]$.

Thus, it can be seen that in the open economy the relation between domestic price inflation and the markup gap is not affected by the degree of openness on the substitutability between domestic and foreign goods.

[6] $\overline{p}_{H,t}$ is used to denote newly set prices instead of p_t^* (used in chapter 3), because in this chapter letters with an asterisk refer to world economy variables.

8.1.3 Exports

The demand for exports of good $i \in [0, 1]$ is assumed to be given by

$$X_t(i) = \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\epsilon} X_t$$

where $X_t \equiv \left(\int_0^1 X_t(i)^{\frac{\epsilon-1}{\epsilon}} di\right)^{\frac{\epsilon}{\epsilon-1}}$ is an index of aggregate exports from the small open economy to the rest of the world. Aggregate exports are, in turn, assumed to be given by

$$X_t = \upsilon \left(\frac{P_{H,t}}{\mathcal{E}_t P_t^*}\right)^{-\eta} Y_t^*$$
$$= \upsilon \mathcal{S}^{\eta} Y_t^*$$

The previous results hold under the assumption that the preferences of households in the rest of the world are identical to those of domestic households, combined with the fact that global goods market clearing implies $C_t^* = Y_t^*$.[7] Thus, in the symmetric steady state with $\mathcal{S} = 1$, $X(i) = X = \upsilon Y^*$, for all $i \in [0, 1]$. Moreover, given that in that steady state $C_F = \upsilon C$, and that $C = Y^*$ (from the risk sharing condition) it follows that trade is balanced at the symmetric steady state.

8.2 Equilibrium

8.2.1 Aggregate Demand and Output Determination

8.2.1.1 Consumption and Output in the Small Open Economy

Goods market clearing in the home economy requires

$$Y_t(i) = C_{H,t}(i) + X_t(i) \tag{23}$$
$$= \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\epsilon} \left[(1-\upsilon)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t + \upsilon \mathcal{S}^{\eta} Y_t^*\right]$$

for all $i \in [0, 1]$ and all t.

[7] See Galí and Monacelli (2005) for a more careful derivation, under the assumption that the world is made up of a continuum of small economies like the one analyzed here.

Plugging (23) into the definition of aggregate domestic output $Y_t \equiv \left(\int_0^1 Y_t(i)^{1-\frac{1}{\epsilon}} di\right)^{\frac{\epsilon}{\epsilon-1}}$ yields

$$Y_t = (1-\upsilon)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t + \upsilon \mathcal{S}_t^{\eta} Y_t^* \tag{24}$$

which can be approximated around the symmetric steady state as

$$y_t = (1-\upsilon)c_t + \upsilon(2-\upsilon)\eta s_t + \upsilon y_t^* \tag{25}$$

Finally, combining (13) with the Euler equation (11) gives

$$c_t = E_t\{c_{t+1}\} - \frac{1}{\sigma}(i_t - E_t\{\pi_{H,t+1}\} - \rho) + \frac{\upsilon}{\sigma}E_t\{\Delta s_{t+1}\} + \frac{1}{\sigma}(1-\rho_z)z_t \tag{26}$$

Note that (17) and (25) can be combined to derive the following expression for the terms of trade:

$$s_t = \sigma_\upsilon(y_t - y_t^*) - (1-\upsilon)\Phi z_t \tag{27}$$

where $\sigma_\upsilon \equiv \sigma\Phi > 0$ with $\Phi \equiv \frac{1}{1+\upsilon(\varpi-1)} > 0$ and where $\varpi \equiv \sigma\eta + (1-\upsilon)(\sigma\eta - 1)$.

Finally, combining (25), (26), and (27) yields (after some algebra) a version of the dynamic IS equation for the small open economy:

$$\begin{aligned} y_t = {} & E_t\{y_{t+1}\} - \frac{1}{\sigma_\upsilon}(i_t - E_t\{\pi_{H,t+1}\} - \rho) + \upsilon(\varpi - 1)E_t\{\Delta y_{t+1}^*\} \\ & + \frac{1-\upsilon}{\sigma}(1-\rho_z)z_t \end{aligned} \tag{28}$$

Note that, in general, the degree of openness influences the sensitivity of output to a change in the domestic real interest rate $i_t - E_t\{\pi_{H,t+1}\}$, as measured by $1/\sigma_\upsilon$. In particular, if $\sigma\eta > 1$ an increase in openness raises that sensitivity, since σ_υ is decreasing in υ under that assumption.

As its closed economy counterpart, equation (28) can be rewritten in terms of the output and real interest rate gaps as:

$$\widetilde{y}_t = E_t\{\widetilde{y}_{t+1}\} - \frac{1}{\sigma_\upsilon}(i_t - E_t\{\pi_{H,t+1}\} - r_t^n) \tag{29}$$

where

$$r_t^n = \rho + \sigma_\upsilon E_t\{\Delta y_{t+1}^n\} + \sigma_\upsilon \upsilon(\varpi - 1)E_t\{\Delta y_{t+1}^*\} + \Phi(1-\upsilon)(1-\rho_z)z_t \tag{30}$$

is the natural interest rate, with y_t^n denoting the natural level of output, determined below.

Thus, it is seen that the small open economy's equilibrium is characterized by a forward-looking dynamic IS-type equation similar to that found in the closed economy. Nevertheless, two differences are worth pointing out. First, the degree of openness influences the sensitivity of the output gap to interest rate changes. Second, openness generally makes the natural interest rate depend on expected world output growth, in addition to domestic shocks (also because, as shown below, the natural level of output will generally depend on world output).

8.2.1.2 THE TRADE BALANCE

Let $nx_t \equiv \left(\frac{1}{Y}\right)\left(Y_t - \frac{P_t}{P_{H,t}}C_t\right)$ denote net exports in terms of domestic output, expressed as a fraction of steady state output Y. Note that under the assumptions made above trade is balanced in the steady state. A first-order approximation around that steady state yields $nx_t = y_t - c_t - \upsilon s_t$, which combined with (17) and (25) implies a simple relation between net exports and the terms of trade

$$nx_t = \upsilon\left(\frac{\varpi}{\sigma} - 1\right)s_t - \frac{\upsilon}{\sigma}z_t \tag{31}$$

Note that, given the terms of trade, a positive preference shock induces a trade deficit in the small open economy. In the special case of $\sigma = \eta = 1$, net exports are invariant to changes in the terms of trade: the latter's price effects on the trade balance exactly offset its quantity effects in that case. More generally, the sign of the relationship between the terms of trade and net exports is ambiguous, depending on the relative sizes of υ, σ, and η. If υ and/or η are sufficiently large, a term of trade depreciation (i.e., an increase in s_t) will improve the trade balance.

8.2.2 *The Supply Side: Marginal Cost and Inflation Dynamics*

8.2.2.1 AGGREGATE OUTPUT AND EMPLOYMENT

As in chapter 3, one can derive an approximate aggregate production function relating aggregate domestic output to aggregate employment. Notice that equilibrium in the labor market requires

$$N_t \equiv \int_0^1 N_t(i)di = \left(\frac{Y_t}{A_t}\right)^{\frac{1}{1-\alpha}} \int_0^1 \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\frac{\epsilon}{1-\alpha}} di$$

As shown in chapter 3, however, variations in $d_t \equiv \log \int_0^1 \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\epsilon} di$ around the perfect foresight steady state are of second order. Thus, and up to a first-order approximation, the following relationship between (log) aggregate output and (log) aggregate employment holds

$$y_t = a_t + (1-\alpha)n_t \tag{32}$$

8.2.2.2 MARKUPS AND INFLATION DYNAMICS IN THE SMALL OPEN ECONOMY

Next a relation between the average price markup and domestic output is derived. That relation differs somewhat from that in the closed economy, due to the existence of a wedge between output and consumption, and between domestic and consumer prices. Thus, in the present model (and ignoring constant terms),

$$\begin{aligned}
\mu_t &= p_{H,t} - (w_t - a_t + \alpha n_t) \\
&= -(w_t - p_t) - (p_t - p_{H,t}) + a_t - \alpha n_t \\
&= -(\sigma c_t + \varphi n_t) - \upsilon s_t + a_t - \alpha n_t \\
&= -\left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right) y_t + \upsilon(\varpi - 1)s_t + \left(1 + \frac{\varphi+\alpha}{1-\alpha}\right) a_t - \upsilon z_t
\end{aligned} \tag{33}$$

where the last equality makes use of (17), (25), and (32). Note that, in addition to domestic output and technology, the average markup is a function of the terms of trade and the preference shifter z_t. This is due to the influence of both variables on the real wage through an income effect on labor supply resulting from their impact on domestic consumption. In addition, changes in the terms of trade have a direct effect, captured by the term υs_t, on the product wage, for any given consumption wage. The influence of technology (through its direct effect on labor productivity) and of domestic output (through its effect on employment and, hence, on the real wage for a given output level) is analogous to that observed in the closed economy.

Combining (33) with (22) (and ignoring constant terms) yields a version of the New Keynesian Phillips curve for the small open economy:

$$\pi_{H,t} = \beta E_t\{\pi_{H,t+1}\} + \kappa y_t - \lambda\upsilon(\varpi - 1)s_t - \lambda\left(1 + \frac{\varphi+\alpha}{1-\alpha}\right) a_t + \lambda\upsilon z_t \tag{34}$$

where $\kappa \equiv \lambda\left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)$, as in the closed economy model of chapter 3.

Next it is shown how a version of the New Keynesian Phillips curve linking domestic inflation and the output gap can be derived. Evaluating (33) at the flexible price equilibrium and subtracting the resulting expression yields:

$$\widehat{\mu}_t = -\left(\sigma + \frac{\varphi + \alpha}{1 - \alpha}\right) \widetilde{y}_t + \upsilon(\varpi - 1)\widetilde{s}_t$$

where $\widetilde{y}_t \equiv y_t - y_t^n$ and $\widetilde{s}_t \equiv s_t - s_t^n$ denote the output gap and the terms of trade gap, respectively, where the natural level of output is given by

$$y_t^n = \Gamma_a a_t + \Gamma_z z_t + \Gamma_* y_t^* \tag{35}$$

with $\Gamma_a \equiv \frac{1+\varphi}{\sigma_\upsilon(1-\alpha)+\varphi+\alpha} > 0$, $\Gamma_* \equiv -\frac{\upsilon(\varpi-1)\sigma_\upsilon(1-\alpha)}{\sigma_\upsilon(1-\alpha)+\varphi+\alpha}$ and $\Gamma_z \equiv -\frac{\upsilon\varpi\Phi(1-\alpha)}{\sigma_\upsilon(1-\alpha)+\varphi+\alpha}$ and where the natural terms of trade can be determined using (27):

$$s_t^n = \sigma_\upsilon(y_t^n - y_t^*) - (1 - \upsilon)\Phi z_t$$

Note that the sign of the effect on the domestic natural output of changes in world output and preference shock is ambiguous. In the particular case of $\sigma = \eta = 1$, however, world output has no effect on the domestic natural output ($\Gamma_* = 0$), while preference shocks z_t have a negative effect ($\Gamma_z < 0$). The latter effect of preference shocks was absent in the closed economy,[8] and is a consequence of the increase in consumption implied by risk sharing with the rest of the world.

Using the fact that $\widetilde{s}_t = \sigma_\upsilon \widetilde{y}_t$ (as implied by (27)), the following simple relation between the average markup and the output gap can be derived:

$$\widehat{\mu}_t = -\left(\sigma_\upsilon + \frac{\varphi + \alpha}{1 - \alpha}\right) \widetilde{y}_t \tag{36}$$

where $\widetilde{y}_t \equiv y_t - y_t^n$. Plugging (36) into (22) yields a version of a New Keynesian Phillips curve of identical form as its closed economy counterpart derived in chapter 3:

$$\pi_{H,t} = \beta E_t\{\pi_{H,t+1}\} + \kappa_\upsilon \widetilde{y}_t \tag{37}$$

where $\kappa_\upsilon \equiv \lambda\left(\sigma_\upsilon + \frac{\varphi+\alpha}{1-\alpha}\right)$. For $\upsilon = 0$ (or, alternatively, $\sigma = \eta = 1$) the slope coefficient is given by $\lambda\left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right) \equiv \kappa$, as in the basic closed

[8] Note that $\Gamma_z = 0$ when $\upsilon = 0$.

economy model of chapter 3. More generally, the degree of openness, υ, and the substitutability between domestic and foreign goods, η, affect the dynamics of inflation through their influence on the slope coefficient κ_υ, which determines the inflation response to variations in (current and future) output gaps. In particular, if $\sigma\eta > 1$ (and, hence, $\varpi > 1$), an increase in openness lowers σ_υ, reducing the sensitivity of domestic inflation to the output gap due to the smaller effect of the latter on the average markup, as discussed above.

Finally, note that by combining (30) with (35) a closed form expression for the natural rate of interest can now be derived

$$r_t^n \equiv \rho - \sigma_\upsilon \Gamma_a (1 - \rho_a) a_t + \Psi_* E_t\{\Delta y_{t+1}^*\} + \Psi_z (1 - \rho_z) z_t \tag{38}$$

is the small open economy's natural rate of interest, with $\Psi_* \equiv \sigma_\upsilon\,(\upsilon(\varpi - 1) + \Gamma_*)$ and $\Psi_z \equiv (1 - \upsilon)\Phi - \sigma_\upsilon \Gamma_z$. Note that $\lim_{\upsilon \to 0} \Psi_* = 0$ and $\lim_{\upsilon \to 0} \Psi_z = 1$.

8.3 Equilibrium Dynamics under an Interest Rate Rule

In the previous two sections the equilibrium conditions making up the nonpolicy block of the small open economy model have been derived. Those conditions take the form of a set of equations involving all the relevant endogenous variables, with technology and preference shocks as driving forces. As in previous chapters, in order to close the model, that set of equations has to be supplemented with a description of monetary policy.

As an illustration, the present section characterizes the model's equilibrium under the assumption that the monetary authority follows a Taylor-type interest rate rule similar to the one introduced in chapter 3, namely

$$i_t = \rho + \phi_\pi \pi_{H,t} + \phi_y \widehat{y}_t + v_t \tag{39}$$

where ϕ_π and ϕ_y are non-negative coefficients chosen by the monetary authority, and where v_t is an exogenous monetary policy shock that evolves according to the $AR(1)$ process:

$$v_t = \rho_v v_{t-1} + \varepsilon_t^v$$

where $\rho_v \in [0, 1)$.

Combining (29), (37), and (39), the equilibrium dynamics for the output gap and domestic inflation can be represented by means of the system of difference equations

$$\begin{bmatrix} \widetilde{y}_t \\ \pi_{H,t} \end{bmatrix} = \mathbf{A}_\upsilon \begin{bmatrix} E_t\{\widetilde{y}_{t+1}\} \\ E_t\{\pi_{H,t+1}\} \end{bmatrix} + \mathbf{B}_\upsilon u_t \tag{40}$$

where

$$u_t \equiv \widehat{r}_t^n - \phi_y \widehat{y}_t^n - v_t$$

and

$$\mathbf{A}_\upsilon \equiv \Omega_\upsilon \begin{bmatrix} \sigma_\upsilon & 1 - \beta\phi_\pi \\ \sigma_\upsilon\kappa_\upsilon & \kappa_\upsilon + \beta(\sigma_\upsilon + \phi_y) \end{bmatrix} \quad ; \quad \mathbf{B}_\upsilon \equiv \Omega_\upsilon \begin{bmatrix} 1 \\ \kappa_\upsilon \end{bmatrix}$$

with $\Omega_\upsilon \equiv \frac{1}{\sigma_\upsilon + \phi_y + \kappa_\upsilon \phi_\pi}$. The previous system has the same form as the one analyzed in chapter 3 for the closed economy, with the only difference lying in the fact that some of the coefficients are a function of the "open economy parameters" υ and η, and that $\widehat{y}_t^n$ and $\widehat{r}_t^n$ are now given by (35) and (38), respectively. In particular, the condition for a locally unique stationary equilibrium under rule (39) takes the same form as the one introduced in chapter 3, namely

$$\kappa_\upsilon(\phi_\pi - 1) + (1 - \beta)\phi_y > 0 \tag{41}$$

which is assumed to hold for the remainder of this section. Once the equilibrium path for the output gap and inflation has been determined, one can use the various equilibrium relations derived in the previous two sections to determine the corresponding equilibrium values for the remaining variables of interest.

The previous framework is used below to examine the economy's response to an exogenous monetary policy shock, that is, an exogenous change in v_t. Given the isomorphism with the closed economy model of chapter 3, many of the results derived there can be exploited. The analysis of the effects of a technology shock, a preference shock, or a change in world output is not pursued here, but an identical approach can be used: First, one should determine the implications of the shock considered for $\widehat{r}_t^n$ and $\widehat{y}_t^n$, and then proceed to solve for the equilibrium response of the output gap and domestic inflation exactly as done next for the case of a monetary policy shock.[9]

[9] Of course, as in chapter 3, it must be taken into account that a technology shock or a shock to world output also leads to a variation in the natural output level, thus breaking the identity between output and the output gap.

8.3.1 The Effects of a Monetary Policy Shock

Neither the natural rate of interest nor the natural level of output are affected by a monetary policy shock, so $\widehat{r}_t^n = \widehat{y}_t^n = 0$ for all t for the purposes of this exercise. As in chapter 3, let us guess that the solution takes the form $\widetilde{y}_t = \psi_{yv} v_t$ and $\pi_t = \psi_{\pi v} v_t$, where ψ_{yv} and $\psi_{\pi v}$ are coefficients to be determined. Imposing the guessed solution on (40) and using the method of undetermined coefficients,

$$\widetilde{y}_t = -(1 - \beta\rho_v)\Lambda_v v_t$$

and

$$\pi_{H,t} = -\kappa_v \Lambda_v v_t$$

where $\Lambda_v \equiv \frac{1}{(1-\beta\rho_v)[\sigma_v(1-\rho_v)+\phi_y]+\kappa_v(\phi_\pi-\rho_v)}$. It can be easily shown that as long as (41) is satisfied, $\Lambda_v > 0$. Hence, as in the closed economy, an exogenous increase in the interest rate leads to a persistent decline in output and domestic inflation. The size of the effect of the shock relative to the closed economy benchmark depends on the values taken by a number of parameters. More specifically, if $\sigma\eta > 1$ then Λ_v can be shown to be increasing in the degree of openness, thus implying that a given monetary policy shock will have a larger impact in the small open economy than in its closed economy counterpart.

Using interest rate rule (39), the response of the nominal rate can be determined, taking into account the central bank's endogenous reaction to changes in inflation and the output gap

$$i_t = \rho + \left[1 - \Lambda_v(\phi_\pi \kappa_v + \phi_y(1 - \beta\rho_v))\right] v_t$$

Note that as in the closed economy model, the full response of the nominal rate may be positive or negative, depending on parameter values. The response of the real interest rate (expressed in terms of domestic goods) is given by

$$\begin{aligned} r_t &= i_t - E_t\{\pi_{H,t+1}\} \\ &= \rho + \left[1 - \Lambda_v((\phi_\pi - \rho_v)\kappa_v + \phi_y(1 - \beta\rho_v))\right] v_t \end{aligned}$$

Thus the real rate increases unambiguously when v_t rises, because the term in square brackets is always positive.

Using (27), the response of the terms of trade to the monetary policy shock can then be uncovered

$$\begin{aligned} s_t &= \sigma_v y_t \\ &= -\sigma_v(1 - \beta\rho_v)\Lambda_v v_t \end{aligned}$$

while the change in the nominal exchange rate is given in turn by

$$\begin{aligned}\Delta e_t &= \Delta s_t + \pi_{H,t} \\ &= -\Lambda_v[\sigma_v(1-\beta\rho_v)+\kappa_v]v_t + \sigma_v(1-\beta\rho_v)\Lambda_v v_{t-1}\end{aligned}$$

Thus, a tightening of monetary policy leads to a persistent improvement in the terms of trade (i.e., a decrease in the relative price of foreign goods) as well as a nominal exchange rate appreciation, both in the short run and in the long run. To understand the long-run effect, note that the terms of trade revert back to their original level in response to the monetary policy shock, which requires that the nominal exchange rate appreciate by the same amount as the permanent decrease in domestic price level, which is given by $\frac{\kappa_v \Lambda_v}{1-\rho_v}$ (given an initial shock of size normalized to unity). Hence, the exchange rate will overshoot its long-run level in response to the monetary policy shock, if and only if,

$$\sigma_v(1-\beta\rho_v)(1-\rho_v) > \kappa_v \rho_v$$

which requires that the shock is not too persistent. It can be easily checked that the previous condition corresponds to that for an increase in the nominal interest rate in response to a positive v_t shock. Note that, in that case, the subsequent exchange rate depreciation required by the interest parity condition (18) leads to an initial overshooting.

Figure 8.1 displays the dynamic responses of several macro variables to an exogenous tightening of monetary policy. For the parameters common with the closed economy, the baseline calibration from chapter 3 is used here. Regarding the additional, open-economy-related, parameters, it is assumed a unit elasticity of substitution between domestic and foreign goods ($\eta = 1$) and a openness index υ of 0.4, following Galí and Monacelli (2005).

As shown in the figure, both output (which equals the output gap in this case) and domestic inflation decline persistently in response to a tightening of monetary policy, due to the rise in both nominal and (most importantly) real interest rates. The latter is also responsible for the short-run appreciation of the terms of trade and the nominal exchange rate, which leads to a large decline of CPI inflation on impact. The subsequent depreciation of the exchange rate helps bring CPI inflation back close to its initial level despite the persistent response of domestic inflation. In the long run, the responses of the CPI, the domestic price level, and the nominal exchange rate converge to the same negative values, though their trajectories during the adjustment are rather

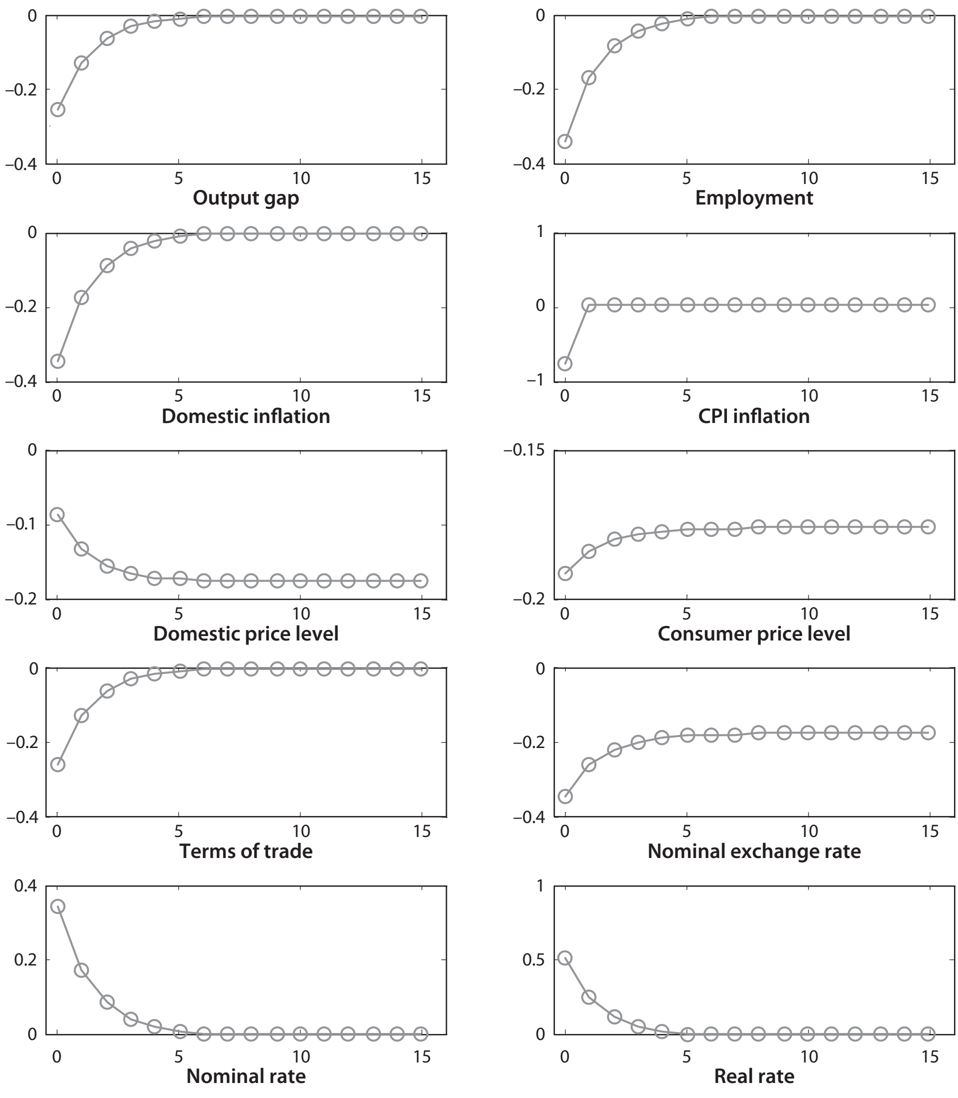

Figure 8.1. Dynamic Responses to a Monetary Policy Shock.

different. Note also that the analysis above implies that for the special case considered here the size and pattern of the responses of output and domestic inflation to the monetary policy shocks do not differ from those in the closed economy.

8.4 Optimal Monetary Policy: A Special Case

This section derives and characterizes the optimal monetary policy for the small open economy described above, as well as the implications of that policy for a number of macroeconomic variables. The analysis, which follows closely that of Galí and Monacelli (2005), is restricted to the special case for which a second-order approximation to the welfare of the representative consumer can be easily derived analytically. Its conclusions should thus not be taken as applying to a more general environment. Instead, this exercise is presented as an illustration of the approach to optimal monetary policy design in an open economy rather than the basis for a robust policy prescription.

The basic New Keynesian model of a closed economy provides a useful benchmark for the analysis here. As discussed in chapter 4, under the assumption of a constant employment subsidy τ that neutralizes the distortion associated with firms' market power, the optimal monetary policy in that model is the one that replicates the flexible price equilibrium allocation. The intuition for that result is straightforward: With the subsidy in place, there is only one distortion left at work in the economy, namely, sticky prices. By stabilizing markups at their frictionless level, nominal rigidities cease to be binding, since firms do not feel any desire to adjust prices. By construction, the resulting equilibrium allocation is efficient, and the price level remains constant.

In an open economy—and as noted, among others, by Corsetti and Pesenti (2001)—an additional factor beyond the presence of market power distorts the incentives of the monetary authority to deviate from the flexible price equilibrium allocation: the possibility of influencing the terms of trade in a way beneficial to domestic consumers. This possibility is a consequence of the imperfect substitutability between domestic and foreign goods (which makes the terms of trade a function of relative supplies), combined with sticky prices (that render monetary policy non-neutral). Furthermore, the strength of the incentive to deviate from the flexible price equilibrium allocation is a function of the preference shock. As a result, and as shown below, the introduction of an employment subsidy that exactly offsets firms' market power distortion is *in general* not enough to render the flexible price equilibrium allocation optimal for, at the margin, the monetary authority has an incentive to deviate from that allocation in order to improve the terms of trade.[10]

However, as shown below, for the special parameter configuration $\sigma = \eta = 1$, and in the absence of preference shocks (i.e., assuming

[10] See also the discussion in Benigno and Benigno (2003) in the context of a two-country model.

$Z_t = 1$, for all t), there is a constant employment subsidy that exactly offsets the combined effects of market power and the terms of trade distortions. Under the assumption that such a subsidy is in place, the flexible price equilibrium allocation becomes optimal. The policy that maximizes welfare in that case requires that *domestic* inflation be fully stabilized, while allowing the nominal exchange rate (and, as a result, CPI inflation) to adjust as needed in order to replicate the response of the terms of trade that would be observed under flexible prices.

One may wonder to what extent the desirability of strict domestic inflation targeting is specific to the special case considered here or whether it constitutes a good approximation for a more general case of the model developed above. The optimal policy analysis undertaken in Faia and Monacelli (2008), using a model nearly identical to the one considered here, suggests that while the optimal policy involves some variation in the domestic price level, the latter is almost negligible from a quantitative point of view, thus making strict domestic inflation targeting a good approximation to the optimal policy (or at least conditional on the productivity shocks considered here). Using a different approach, de Paoli (2009a) reaches a similar conclusion, except when an (implausibly) high elasticity of substitution is assumed.[11] But even in the latter case, the losses that arise from following a domestic inflation targeting policy are negligible.[12] More generally, it is clear that there are several channels in the open economy that may potentially render a strict domestic inflation policy suboptimal, including a nonunitary elasticity of substitution, local currency pricing, incomplete financial markets, and so on, all of which are unrelated to the sources of policy tradeoffs that may potentially arise in the closed economy. Furthermore, the presence of sources of fluctuations other than technology (e.g., preference shocks) may also lead to an optimal policy that deviates from strict domestic inflation targeting. The analysis of the quantitative significance of the effects of those channels (individually or jointly) is beyond the scope of this chapter.

With that consideration in mind, let us next turn to the analysis of the optimal policy in the special case mentioned above (i.e., $\sigma = \eta = 1$, and no preference shocks).

[11] Those results are conditional on productivity shocks being the driving force. Not surprisingly, in the presence of cost-push shocks of the kind considered in chapter 5, stabilizing domestic inflation is not optimal (as in the closed economy).

[12] In solving the optimal policy problem for the general case, de Paoli (2009a) adopts the linear-quadratic approach originally developed in Benigno and Woodford (2005), which replaces the linear terms in the approximation to the households' welfare losses using a second-order approximation to the equilibrium conditions. Faia and Monacelli (2008) solve for the Ramsey policy using the original nonlinear equilibrium conditions as constraints of the policy problem.

8.4.1 The Efficient Allocation and Its Decentralization

Let us first characterize the optimal allocation from the viewpoint of a social planner facing the same resource constraints to which the small open economy is subject in equilibrium (in relation to the rest of the world), given the assumption of complete markets. In that case, the optimal allocation must maximize $U(C_t, N_t; Z_t)$ subject to (i) the technological constraint $Y_t = A_t N_t^{1-\alpha}$, (ii) a consumption/output possibilities set implicit in the international risk sharing condition (16), and (iii) the market clearing condition (24).

Under the special case of $\sigma = \eta = 1$, and $Z_t = 1$ for all t considered here, (16) and (24) can be used to derive an exact consumption/output possibilities set under complete markets, which takes the form:

$$C_t = Y_t^{1-\upsilon}(Y_t^*)^{\upsilon}$$

Note that, in contrast with the closed economy, the elasticity of consumption with respect to domestic output is not one, but $1-\upsilon$ instead. The optimal allocation (from the viewpoint of the small open economy, which takes world output as given) must satisfy the optimality conditions

$$-\frac{U_n(C_t, N_t)}{U_c(C_t, N_t)} = (1-\upsilon)(1-\alpha)\frac{C_t}{N_t}$$

which, under the assumed preferences and given $\sigma = 1$, can be written as

$$C_t N_t^{\varphi} = (1-\upsilon)(1-\alpha)\frac{C_t}{N_t}$$

thus implying a constant optimal level of employment $N = [(1-\upsilon) \times (1-\alpha)]^{\frac{1}{1+\varphi}}$.

Consider, on the other hand, the flexible price equilibrium allocation. In the latter the gross price markup is constant and satisfies

$$\begin{aligned}
\mathcal{M} &= \frac{P_{H,t}}{W_t/MPN_t} \\
&= \frac{(1-\alpha)(Y_t/N_t)}{(1-\tau)C_t N_t^{\varphi}\mathcal{S}_t^{\upsilon}} \\
&= \frac{(1-\alpha)}{(1-\tau)N_t^{1+\varphi}}
\end{aligned}$$

where the third equality holds in the special case considered here.

Hence, by setting τ such that $(1-\tau)(1-\upsilon)\mathcal{M} = 1$, the optimality of the flexible price equilibrium allocation is guaranteed. Note that the required subsidy is smaller than necessary to offset firms' market power distortion, in order to exploit the gains from a more favorable terms of trade brought about by a somewhat lower domestic output.

With the optimal subsidy in place, and as in the closed economy of chapter 3, the optimal monetary policy requires stabilizing the output gap (i.e., $\widetilde{y}_t = 0$, for all t). Equation (37) then implies that domestic prices are also stabilized under that optimal policy (i.e., $\pi_{H,t} = 0$ for all t). Thus, in the special case under consideration, (strict) *domestic inflation targeting* (SDIT) is indeed the optimal policy.

8.4.2 Implementation and Macroeconomic Implications

This section discusses the implementation of a *strict domestic inflation targeting* policy and characterizes some of its equilibrium implications. While that policy has been shown to be optimal only for the special case considered above, its implications for the general case can also be considered.

8.4.2.1 IMPLEMENTATION

As discussed above, full stabilization of domestic prices implies

$$\widetilde{y}_t = \pi_{H,t} = 0$$

for all t. This in turn implies that $y_t = y_t^n$ and $i_t = r_t^n$ will hold in equilibrium for all t, with all the remaining variables matching their natural levels at all times.

For the reasons discussed in chapter 4, an interest rate rule of the form $i_t = r_t^n$ is associated with an indeterminate equilibrium, and hence, does not guarantee that the outcome of full price stability be attained. That result follows from the equivalence between the dynamical system describing the equilibrium of the small open economy and that of the closed economy, as discussed in chapter 4. As shown there, the indeterminacy problem can be avoided, and the uniqueness of the price stability outcome restored by having the central bank follow a rule that makes the interest rate respond with sufficient strength to deviations of domestic inflation and/or the output gap from target. More precisely, the central bank can guarantee that the desired outcome is attained by committing to a rule of the form

$$i_t = r_t^n + \phi_\pi \pi_{H,t} \tag{42}$$

where $\phi_\pi > 1$. In equilibrium, the term $\phi_\pi \pi_{H,t}$ vanishes (because $\pi_{H,t} = 0$ for all t), implying that $i_t = r_t^n$, for all t.

8.4.2.2 MACROECONOMIC IMPLICATIONS

Under a strict domestic inflation targeting policy, the behavior of real variables in the small open economy corresponds to the one that would be observed in the absence of nominal rigidities. Hence, it is seen from the inspection of equation (35) that domestic output always increases in response to a positive technology shock at home. As discussed earlier, the sign of the response to a rise in world output is ambiguous, however, and depends on the sign of Γ_*, which in turn depends on the elasticity of substitution η and the risk aversion parameter σ.

As shown above the natural level of the terms of trade is given by

$$\begin{aligned} s_t^n &= \sigma_\upsilon (y_t^n - y_t^*) - (1-\upsilon)\Phi z_t \\ &= \sigma_\upsilon [\Gamma_a a_t + \Gamma_z z_t + (\Gamma_* - 1) y_t^*] - (1-\upsilon)\Phi z_t \\ &= \sigma_\upsilon \Gamma_a a_t + \sigma_\upsilon (\Gamma_* - 1) y_t^* + [\sigma_\upsilon \Gamma_z - (1-\upsilon)\Phi] z_t \end{aligned}$$

Thus, given world output, an improvement in domestic technology always leads to a real depreciation, through its expansionary effect on domestic output. Positive preference shocks z_t generate a real appreciation, since $\sigma_\upsilon \Gamma_z - (1-\upsilon)\Phi < 0$. On the other hand, changes in world output have an effect on the terms of trade of ambiguous sign.

Given that domestic prices are fully stabilized under DIT, it follows that $e_t^{DIT} = s_t^n - p_t^*$, that is, the nominal exchange rate moves one-for-one with the (natural) terms of trade and (inversely) with the world price level. Assuming constant world prices, the nominal exchange rate will inherit all the statistical properties of the natural terms of trade. In particular, in response to technology shocks, the nominal exchange rate under DIT will be proportional to the gap between the natural level of domestic output (in turn related to productivity) and world output. Thus, the volatility of the nominal exchange rate will tend to be high (low) when domestic natural output displays a weak (strong) positive comovement with world output.

The implied equilibrium process for the CPI can also be derived. Given the constancy of domestic prices it is given by

$$\begin{aligned} p_t^{DIT} &= \upsilon s_t^n \\ &= \upsilon (e_t^{DIT} + p_t^*) \end{aligned}$$

Thus, it is seen that under the DIT regime, the CPI level will also vary with the (natural) terms of trade and will inherit its statistical properties. If the economy is very open, and if domestic productivity (and hence, the natural level of domestic output) is not much synchronized with world output, CPI prices could potentially be highly volatile, even if the domestic price level is constant.

An important lesson emerges from the previous analysis: Potentially large and persistent fluctuations in the nominal exchange rate, as well as in some inflation measures (like the CPI), are not necessarily undesirable, nor do they require a policy response aimed at dampening such fluctuations. Instead, and especially for an economy that is very open and subject to large idiosyncratic shocks, those fluctuations may be an equilibrium consequence of the adoption of an optimal policy, as illustrated by the model above.

8.4.3 The Welfare Costs of Deviations from the Optimal Policy

Under the particular assumptions under which strict domestic inflation targeting has been shown to be optimal (i.e., log utility, a unit elasticity of substitution between domestic and foreign goods and no preference shocks), it is relatively straightforward to derive a second-order approximation to the discounted utility losses of the domestic representative consumer associated with deviations from the optimal policy. As shown in appendix 8.1, those losses, expressed as a fraction of steady state consumption, can be written as (ignoring terms independent of policy):

$$\mathbb{W} = \frac{(1-\upsilon)}{2} \sum_{t=0}^{\infty} \beta^t \left(\left(\frac{1+\varphi}{1-\alpha} \right) \widetilde{y}_t^2 + \frac{\epsilon}{\lambda} \pi_{H,t}^2 \right) \tag{43}$$

The expected period welfare losses of any policy that deviates from strict inflation targeting can be written in terms of the variances of inflation and the output gap

$$\mathbb{V} = \frac{(1-\upsilon)}{2} \left[\left(\frac{1+\varphi}{1-\alpha} \right) var(\widetilde{y}_t) + \frac{\epsilon}{\lambda}\, var(\pi_{H,t}) \right] \tag{44}$$

Note that the previous expressions for the welfare losses are, up to the multiplicative term $(1-\upsilon)$, identical to the ones derived for the closed economy in chapter 4, with domestic inflation being the relevant inflation variable here. Below, (44) is used to assess the welfare implications of alternative monetary policy rules and to rank those rules on welfare grounds.

8.5 Simple Monetary Policy Rules for the Small Open Economy

This section analyzes the macroeconomic implications of alternative monetary policy rules for the small open economy. The first rule, already analyzed in the previous section, is *strict domestic inflation targeting* (SDIT, for short), which requires

$$\pi_{H,t} = 0$$

for all t. Under the second rule considered, referred to as *strict CPI inflation targeting* (SCIT, for short), consumer price inflation is fully stabilized:

$$\pi_t = 0$$

for all t. The third rule is an *exchange rate peg* (PEG, for short), which can be formally represented as

$$e_t = 0$$

for all t (which in turn requires that $i_t = i_t^*$, also for all t).

In addition to the previous rules, two stylized Taylor-type rules are considered, which can be thought of as flexible versions of the inflation targeting rules specified above, allowing for some concern for output fluctuations. The first has the domestic interest rate respond systematically to domestic inflation and output, whereas the second assumes that CPI inflation is the variable the domestic central bank reacts to (in addition to output). Formally, the *flexible domestic inflation targeting* (FDIT, for short) is specified as

$$i_t = 0.01 + 1.5\pi_{H,t} + 0.125\widehat{y}_t$$

whereas the *flexible CPI inflation targeting* rule (FCIT, for short) is given by

$$i_t = 0.01 + 1.5\pi_t + 0.125\widehat{y}_t$$

for all t, where the size of the coefficients is based on the original Taylor (1993) rule.

Below, a comparison is provided of the equilibrium properties of several macroeconomic variables under the above simple rules for a calibrated version of the small open economy model developed above.[13]

[13] See Galí and Monacelli (2005) for a similar analysis (though with a slightly different calibration).

TABLE 8.1
Properties of Simple Policy Rules

	SDIT	SCIT	PEG	FDIT	FCIT
$\sigma(y)$	2.29	1.92	1.72	1.85	1.76
$\sigma(\widetilde{y})$	0	0.66	0.92	0.44	0.62
$\sigma(\pi_H)$	0	0.19	0.35	0.69	0.58
$\sigma(\pi)$	0.41	0	0.21	0.70	0.52
$\sigma(s)$	2.29	1.92	1.72	1.85	1.76
$\sigma(\Delta e)$	1.02	0.29	0	0.95	0.59
$\mathbb{L}$	0	0.05	0.17	0.61	0.43

8.5.1 A Quantitative Analysis of Alternative Rules

8.5.1.1 CALIBRATION

This section presents some quantitative results based on a calibrated version of the small open economy. The baseline calibration assumes $\sigma = \eta = 1$ in a way consistent with the special case considered above. A baseline value for υ (the degree of openness) is set at 0.4. The latter corresponds roughly to the import/GDP ratio in Canada, which is taken as a prototype small open economy. The remaining parameters are set at the values found in the baseline calibration of chapter 3. The analysis is restricted to technology shocks.[14]

8.5.1.2 MACROECONOMIC VOLATILITY AND WELFARE LOSSES

Table 8.1 reports the standard deviations of several key variables under the alternative monetary policy rules introduced above. A critical element that distinguishes each simple rule relative to the optimal policy is the excess smoothness of both the terms of trade and the (first-differenced) nominal exchange rate.[15] This in turn is often reflected in too high a volatility of the output gap and domestic inflation. In particular, the PEG regime is the one that amplifies both output gap and domestic inflation volatility to the largest extent, with the CITR regime lying somewhere in between. Note also that the terms of trade are more stable under an exchange rate peg than under any other policy regime, but this is not reflected in smaller welfare losses. That finding, which is consistent with the evidence of Mussa (1986), points to the existence of "excess smoothness" in the real exchange rate under a nominal exchange rate

[14] Note that under the assumed calibration, changes in world output do not have any effect on the domestic economy.

[15] Statistics are reported for the nominal *depreciation* rate, as opposed to the level, given the unit root in the nominal exchange rate.

peg. That feature is a consequence of the inability of prices (which are sticky) to compensate for the constancy of the nominal exchange rate.[16]

The last row in table 8.1 reports the welfare losses associated with the alternative policy rules introduced above, expressed as a percentage of steady state consumption. Under the baseline calibration all rules other than SDIT are suboptimal because they involve nontrivial deviations from the efficient/natural allocation. Note that both Taylor rules imply substantially larger welfare losses than the other three rules considered, largely due to their implied higher volatility in domestic inflation. However, and as is usually the case in welfare exercises of this sort found in the literature, the implied welfare losses are quantitatively small under all policy regimes.

8.6 Notes on the Literature

Earlier work on optimizing open economy models with nominal rigidities focused on the transmission of monetary policy shocks, typically represented as disturbances to an exogenous stochastic process for the money supply.[17] A key contribution in that area is Obstfeld and Rogoff (1995), who develop a two-country model where monopolistically competitive firms set prices before the realization of the shocks (i.e., one period in advance). The framework is used to analyze the dynamics of the exchange rate and other variables in response to a change in the money supply (and government spending) and the welfare effects resulting from that intervention. An earlier paper, by Svensson and van Wijnbergen (1989) contains a related analysis under the assumption of full risk-sharing among consumers from different countries.

Corsetti and Pesenti (2001) develop a version of the Obstfeld-Rogoff model that allows for home bias in preferences, leading to terms of trade effects that are argued to have potentially important welfare effects. Betts and Devereux (2000) revisit the analysis in Obstfeld and Rogoff (1995) while departing from the assumption of the law of one price found in the latter paper. In particular, they allow firms to price discriminate across markets assuming they set prices (in advance) in terms of the currency of the importing country ("pricing to market").

The effects of money supply shocks on the persistence and volatility of nominal and real exchange rates are analyzed under the assumption of staggered price setting in Kollmann (2001) and Chari, Kehoe,

[16] See Monacelli (2004) for a detailed analysis of the implications of fixed exchange rates.
[17] See Lane (1999) for an excellent survey of that literature.

and McGrattan (2002).[18] The assumption of staggered price setting (and staggered wage setting in Kollmann's case) induces much richer and more realistic dynamics than that of price setting one period in advance.

A more recent strand of the literature has attempted to go beyond the analysis of the transmission of exogenous monetary policy shocks, and has focused instead on the implications of sticky price open economy models for the design of optimal monetary policy, using a welfare theoretic approach.[19] Early examples of papers analyzing the properties of alternative monetary policy arrangements in a two-country setting assumed that prices are set one period in advance. They include the work of Obstfeld and Rogoff (2002) and Benigno and Benigno (2003), both using the assumption of producer currency pricing. Bacchetta and van Wincoop (2000), Sutherland (2006), Devereux and Engel (2003), and Corsetti and Pesenti (2005) use the same assumption in the context of economies with local currency pricing.

More recent frameworks have instead adopted the staggered price-setting structure à la Calvo. Galí and Monacelli (2005), on which much the analysis of the present chapter is built, is an illustration of work along those lines for a small open economy. Their analysis uncovers the assumptions under which replicating the flexible price equilibrium allocation through full stabilization of domestic prices is optimal, in what amounts to an open economy version of the "divine coincidence" result discussed in chapter 4. Those assumptions include producer currency pricing, complete asset markets, log utility of consumption, and a unit elasticity of substitution between domestic and foreign goods. An extension of that framework, incorporating cost-push shocks and featuring tradeoffs similar to those analyzed in chapter 5 can be found in Clarida, Galí, and Gertler (2001). Kollmann (2002) considers a more general model of a small open economy with several sources of shocks, and carries out a numerical analysis of the welfare implications of alternative rules. Erceg, Gust, and López-Salido (2009) analyze the role of openness in the transmission of shocks using a version of the Galí-Monacelli model that incorporates staggered wage setting in a way analogous to the model in chapter 6. Galí and Monacelli (2014) study the gains from increased wage flexibility in such a framework, and their dependence on the exchange rate policy.

[18] Kollmann (2001) assumes prices and wages are set à la Calvo—as in the model of the present chapter—whereas Chari, Kehoe, and McGrattan (2002) assume price setting à la Taylor, i.e., with deterministic price durations.

[19] Ball (1999) and Svensson (2000) carry out an analysis similar in spirit, but in the context of nonoptimizing models.

Several papers have examined the consequences on optimal monetary policy of departures from the benchmark assumptions of the Galí-Monacelli model in closely related models. Faia and Monacelli (2008) and de Paoli (2009a) study an environment with more general preferences, and show how the size of the elasticity of substitution between domestic and foreign goods affects the extent to which the central bank wants to stabilize the exchange rate (thus departing from strict domestic inflation targeting), in order to improve welfare. De Paoli (2009b) shows how the conclusions from that analysis need to be modified when the assumption of complete markets is relaxed. Using a similar framework as a starting point, Monacelli (2005) shows that the introduction of imperfect pass-through generates a tradeoff between stabilization of domestic inflation and the output gap, leading to gains from commitment similar to those analyzed in chapter 5 for the closed economy. Campolmi (2014) introduces staggered wage setting in a small open economy (coexisting with staggered price setting, as in the closed economy of chapter 6). She shows that the presence of sticky wages generally makes CPI inflation targeting more desirable than domestic inflation targeting.

In contrast with the previous references, which study monetary policy in a small open economy, a number of papers have framed their analysis of monetary policy design in the context of two-country models with staggered price setting à la Calvo. The papers by Clarida, Galí, and Gertler (2002), Pappa (2004), and Benigno and Benigno (2006) provide examples of that literature, with a special focus on the gains from cooperation, and under the assumption of producer currency pricing. Engel (2011) studies the implications for optimal monetary policy of assuming local currency pricing instead in an otherwise similar framework, showing how that modification warrants a focus on CPI—rather than domestic price—stabilization. Benigno (2009) studies the implications of incomplete asset markets and financial imbalances in a similar environment, showing that those factors may justify a deviation from a strict domestic inflation targeting policy. Woodford (2009) uses a similar framework to study the challenges posed by trade and financial integration on domestic monetary policy. Corsetti, Dedola, and Leduc (2010) provide a unified treatment of the consequences of deviations from the assumptions underlying the divine coincidence result.

Finally, Galí and Gertler (2009) contains a number of contributions that use related frameworks to understand different aspects of the international dimension of monetary policy.

Appendix

8.1 A Second-Order Approximation to Welfare Losses in the Small Open Economy

This appendix derives a second-order approximation to the utility of the representative household in the small open economy analyzed above for the special case of $\sigma = \eta = 1$, when the economy remains in a neighborhood of an efficient steady state (i.e., when the optimal constant subsidy derived in the main text is in place).

Using the same approach as in appendix 4.1, the second-order Taylor expansion of $U_t \equiv U(C_t, N_t; Z_t)$ around a steady state $(C, N; 1)$ can be written for the special case at hand as

$$U_t - U \simeq U_c C\, \widehat{c}_t(1 + z_t) + U_n N \left(\widehat{n}_t(1 + z_t) + \frac{1 + \varphi}{2}\, \widehat{n}_t^2 \right) + t.i.p.$$

$$= U_c C\, (1 - \upsilon)\widehat{y}_t(1 + z_t) + \frac{U_n N}{1 - \alpha} \left(\widehat{y}_t(1 + z_t) + \frac{\epsilon}{2\Theta}\, var_i\{p_{H,t}(i)\} \right.$$

$$\left. + \frac{1}{2}(1 + \varphi)(1 - \alpha)\, \widehat{n}_t^2 \right) + t.i.p.$$

where the fact has been used that $\widehat{c}_t = (1 - \upsilon)\widehat{y}_t + \upsilon \widehat{y}_t^* + \varsigma z_t$ holds for the assumed special case.

As shown in the main text, efficiency of the steady state implies $-\frac{U_n}{U_c} = (1 - \upsilon)(1 - \alpha)\frac{C_t}{N_t}$. Hence one can write:

$$\frac{U_t - U}{U_c C} \simeq -\frac{1 - \upsilon}{2} \left[\frac{\epsilon}{\Theta}\, var_i\{p_{H,t}(i)\} + (1 + \varphi)(1 - \alpha)\, \widehat{n}_t^2 \right] + t.i.p.$$

Accordingly, a second-order approximation to the consumer's welfare losses can be written and expressed as a fraction of steady state consumption (and up to additive terms independent of policy) as

$$\mathbb{W} = -E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{U_t - U}{U_c C} \right)$$

$$= -\frac{1 - \upsilon}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{\epsilon}{\Theta}\, var_i\{p_{H,t}(i)\} + (1 + \varphi)\,(1 - \alpha)\widehat{n}_t^2 \right)$$

The final step consists in rewriting the terms involving the price dispersion variable as a function of inflation. In order to do so, make use of the following lemma, which carries over from the closed economy case

Lemma 2: $\sum_{t=0}^{\infty} \beta^t \; var_i\{p_{H,t}(i)\} = \frac{\theta}{(1-\beta\theta)(1-\theta)} \sum_{t=0}^{\infty} \beta^t \pi_{H,t}^2$

Proof: Woodford (2003, chap. 6)

Using the fact that $\lambda \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta} \Theta$, the previous lemma can be combined with the expression for the welfare losses above to obtain

$$\mathbb{W} = -\frac{1-\upsilon}{2} \; E_0 \sum_{t=0}^{\infty} \beta^t \left[\left(\frac{\epsilon}{\lambda} \right) \pi_t^2 + (1+\varphi)\,(1-\alpha)\widehat{n}_t^2 \right]$$

Note that in the absence of preference shocks, $y_t^n = a_t$ and $n_t^n = n$ for all t in the special case considered, thus implying $\widetilde{y}_t = (1-\alpha)\widehat{n}_t$, which allows one to rewrite the loss function in terms of domestic inflation and the output gap as:

$$\mathbb{W} = \frac{1-\upsilon}{2} \; E_0 \sum_{t=0}^{\infty} \beta^t \left[\left(\frac{\epsilon}{\lambda} \right) \pi_t^2 + \left(\frac{1+\varphi}{1-\alpha} \right) \widetilde{y}_t^2 \right]$$

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Exercises

8.1. **A small open economy model**
Consider a small open economy where no international trade in assets is allowed (implying that trade is always balanced). Hence,

$$p_t + c_t = p_{H,t} + y_t$$

where c_t denotes consumption, y_t is output, $p_{H,t}$ is the domestic price level, and p_t is the CPI (all in logs). Assuming a constant price level in the rest of the world ($p_t^* = 0$),

$$p_t = (1 - \upsilon) p_{H,t} + \upsilon e_t$$

where e_t is the nominal exchange rate.

Let $s_t \equiv e_t - p_{H,t}$ denote the terms of trade. Under the assumption of a unit elasticity of substitution between foreign and domestic goods,

$$s_t = y_t - y_t^*$$

where y_t^* is (log) output in the rest of the world (assumed to evolve exogenously). The domestic aggregate technology can be written as

$$y_t = a_t + n_t$$

where a_t is an exogenous technology process. Assume perfect competition in both goods and labor markets with flexible prices and wages. The labor supply takes the form

$$w_t - p_t = \sigma c_t + \varphi n_t$$

Finally, assume a money demand function of the form $m_t - p_t = c_t$.

a. Determine the equilibrium processes for output, consumption, the terms of trade, and the nominal exchange rate in the small open economy, as a function of productivity a_t, foreign output y_t^*, and the money supply under the assumption that the latter evolves exogenously. Discuss the implications of assuming $\sigma = 1$.
b. How would your answer have to be modified if a fixed nominal exchange rate regime were in place?
c. Discuss, in words, how some of the results in (a) and (b) would change qualitatively in the presence of imperfect competition and sticky prices.

8.2. **The effects of technology shocks in the open economy**
Consider the small open economy model described in this chapter. The equilibrium dynamics for domestic inflation $\pi_{H,t}$ and the output gap $\widetilde{y}_t$ are described by the equations

$$\pi_{H,t} = \beta \; E_t\{\pi_{H,t+1}\} + \kappa_\upsilon \widetilde{y}_t$$

$$\widetilde{y}_t = E_t\{\widetilde{y}_{t+1}\} - \frac{1}{\sigma_\upsilon}(i_t - E_t\{\pi_{H,t+1}\} - r_t^n)$$

and where r_t^n is given by

$$r_t^n = \rho - \psi_{ra} a_t$$

Natural output is in turn given by

$$y_t^n = \psi_{ya} a_t$$

The technology parameter follows a stationary AR(1) process

$$a_t = \rho_a a_{t-1} + \varepsilon_t^a$$

where $\rho_a \in [0, 1)$.

Assume that the monetary authority follows the simple interest rate rule

$$i_t = \rho + \phi_\pi \pi_{H,t}$$

where $\phi_\pi > 1$.

a. Determine the response of output, domestic inflation, the terms of trade, and the nominal exchange rate to a positive domestic technology shock (note: for the purposes of this exercise assume $y_t^* = p_t^* = 0$ for all t).
b. Suppose that the central bank pegs the nominal exchange rate so that $e_t = 0$ for all t. Characterize the economy's response to a technology shock in that case.

Chapter 9

LESSONS, EXTENSIONS, AND NEW DIRECTIONS

The previous chapters have provided an introduction to the New Keynesian model and its use for monetary policy evaluation. Throughout the analysis has been restricted to relatively simple versions of that framework, in order to preserve tractability. In recent years, however, larger versions of the model have been developed incorporating many features in order to provide a better fit to the data (e.g., Christiano, Eichenbaum, and Evans (2005), Smets and Wouters (2003, 2007), Galí, Smets, and Wouters (2012)). Thus, and in addition to the staggered price and wage setting analyzed in chapter 6, the resulting frameworks allow for habit formation, capital accumulation with investment adjustment costs, backward-looking indexation of wages and prices, and a variety of structural shocks, including markup shocks, neutral and investment specific technology shocks, preference shocks, and so on. Many central banks have already started using versions of those models in simulation and forecasting exercises, a development that can only add further discipline to their decision-making and communication processes.[1]

Their simplicity notwithstanding, the models discussed in the present book suffice to convey many of the main policy insights generated by the new vintage of monetary models. Some of those insights represent important differences with the traditional macroeconometric models that preceded the new vintage. In that respect, the New Keynesian research program has gone beyond the mere provision of microfoundations to the traditional macro models.[2]

In particular, there are two key implications of the new framework that are worth emphasizing in this concluding chapter

1. The importance of expectations. The transmission of monetary policy depends critically on private sector expectations on the future path of the central bank's policy instrument, that is, the short-term interest rate. This dependence is central to the New Keynesian model. Thus, as we have seen in chapter 3 and subsequent chapters, aggregate demand and output depend at any point in time on expectations about

[1] See, e.g., Smets et al. (2010) for a discussion of the uses of DSGE models at the ECB.

[2] See Galí and Gertler (2007) for an extensive discussion of the differences between the two vintages of models. The following discussion draws heavily on that paper.

future short-term interest rates and inflation. Current inflation, in turn, is a function of current and expected levels of economic activity. As a consequence, the current values of aggregate output and inflation depend not only on the central bank's current choice of the short-term interest rate, but also on the anticipated future path of this instrument. Thus, the central bank's management of private sector expectations about its future policy settings is an important factor in determining the overall effectiveness of monetary policy. In other words, the policy process is as much, if not more, about communicating the future intentions of policy, as it is about choosing the current policy instrument. In this respect, the new framework provides a clear rationale for the trend toward greater transparency pursued by central banks around the globe. In particular, the inflation targeting framework adopted by a large number of central banks places a large weight to the publication of a quantitative objective for inflation, supplemented with an active communications policy (press conferences, inflation reports, speeches, etc.) aimed at explaining how the central bank intends to attain the inflation target. The regular publication by some central banks of their own projections regarding the future path of the policy rate provides a clear example of the importance attached by policymakers to the correct public understanding of their intended policy actions. A most dramatic illustration of the role given to expectations about future short-term interest can be found in the so-called "forward guidance" strategy adopted by the Fed and other central banks in the wake of the recent financial crisis, and which involved a "promise" to keep the interest rate at zero or near zero levels for a longer period than would be warranted by economic developments in normal circumstances, intended as a way to compensate for the inability to lower the short-term rate below that zero lower bound.

2. *The importance of the natural levels of output and the interest rate*, that is, the values for those variables that would arise in the equilibrium without nominal rigidities. As argued in earlier chapters, those variables are important reference points for monetary policy, in part because they reflect the constrained efficient level of economic activity, but also because monetary policy cannot create persistent departures from those natural values without inducing either inflationary or deflationary pressures. Within traditional frameworks, the natural levels of output and the real interest were typically modeled by means of smooth trends. Within the new framework they are instead determined by economic factors, and correspond, roughly speaking, to the values of output and the real interest rate that a frictionless Real Business Cycle (RBC) model would generate, given the assumed preferences and technology. As RBC theory suggests, further, they can vary considerably, given that the

economy is continually buffeted by "real" shocks including oil price shocks, shifts in the pace of technological change, tax changes, and so on. Thus, these new models identify tracking the natural equilibrium of the economy, which is not directly observable, as an important challenge for central banks. The development and use of estimated DSGE models may play a key role in meeting that challenge.

9.1 Extensions

The remainder of this concluding chapter briefly lists a number of extensions of the basic New Keynesian model that have been the focus of much research over the past few years, but which we have been not covered in the previous chapters. For each extension a list of selected readings is provided, with no attempt to be exhaustive.

State dependent pricing. In the models analyzed in the previous chapters, the timing of price readjustments for any given firm is exogenous and, hence, independent of the gap between its current and desired prices. In such models, which are known as time-dependent models, the fraction of firms adjusting prices in any given period is independent of the state of the economy (e.g., the rate of inflation). In a seminal paper, Caplin and Spulber (1987) alerted to the potentially misleading implications of time-dependent models, by developing an example of an economy in which each firm chooses optimally the timing of each adjustment, incurring a menu cost whenever it changes its price. Despite that stickiness at the micro level, in the Caplin-Spulber model the aggregate price level varies in proportion to the money supply, rendering changes in the latter fully neutral. Its simplicity and strong assumptions notwithstanding, the Caplin-Spulber model yields an important insight: when firms choose optimally the timing of their price adjustments, a selection effect emerges—firms whose prices are more out of line with their target prices are more likely to adjust their price, and to do so by a larger amount. As a result, the response of the aggregate price level to shocks is likely to be larger than under the assumption that the adjusting firms are chosen randomly.

Recently, there has been a renewed effort to develop models with state dependent pricing, and in which the latter is fully integrated into a general equilibrium framework. In contrast with the earlier literature, the new vintage of state dependent models are more amenable to a quantitative analysis, that is, to a calibration and evaluation of their quantitative predictions in light of the existing evidence, both micro and macro. Influential examples of this recent literature are Danziger (1999), Dotsey, King, and Wolman (1999), Dotsey and King (2005), Golosov and Lucas (2007), Midrigan (2011), Klenow and Kryvstov (2008), Gertler and Leahy (2008), Nakamura and Steinsson (2008), Costain and Nakov (2011a, 2011b), Alvarez and Lippi (2014), and Alvarez, Lippi, and Le Bihan (2014), all of which have developed tractable quantitative models, and assessed their ability to match different dimensions of the data. Gertler and Leahy (2008), in particular, show how it is possible to derive an inflation equation in a model with state dependent pricing and infrequent firm-specific productivity shocks, that is very similar in form to the New Keynesian Phillips curve derived in chapter 3 in the context of a model with time-dependent pricing.

Imperfect information and learning. Underlying the monetary policy analysis contained in the previous chapters are the assumptions of perfect information and rational expectations, that is, that both private agents and the central bank know the structure of the economy (model specification and parameter values), are able to observe the shocks impinging on the latter, and form expectations in a way consistent with that (correct) model. A great deal of research in macroeconomics over the past decade has sought to relax some of those assumptions, which are widely regarded as unrealistically strong. Much of that work has focused on monetary applications, and has adopted a normative perspective, exploring the implications of imperfect information and learning for the optimal design of monetary policy, with many of those applications being cast in the context of the New Keynesian model developed in previous chapters.[3]

Some papers in this literature have focused on imperfect information and learning by private agents, studying the implications for monetary policy design of having private sector expectations being formed with some adaptive learning algorithm (e.g., recursive least squares). In particular some authors have studied the conditions that an interest rate rule has to satisfy in that case for the economy to converge to the rational expectations equilibrium (see, e.g., Bullard and Mitra (2002) and Evans and Honkapohja (2003)). Other authors have characterized the optimal monetary policy in such an environment, and shown how that policy tries to "influence" the learning process in order to improve the tradeoff facing the central bank, typically by anchoring inflation expectations

[3] A smaller but highly influential literature has adopted instead a positive perspective, seeking to interpret some features of the data (e.g., the rise and fall of inflation in the postwar period) as a consequence of policymakers' learning about the structure of the economy. See Sargent (1999) for a prominent example in that tradition.

through an aggressive response to any surge in inflation (e.g., Gaspar, Smets, and Vestin (2006)).[4] Within the same class of models, Woodford (2010) investigates the nature of the optimal robust monetary policy when the central bank does not know with certainty the mechanism used by the private sector to form expectations, but only that the latter do not differ "too much" from their rational counterpart (an assumption that he terms near-rational expectations), and finds that many of the qualitative features of the optimal policy under rational expectations carry over to this environment (including the importance of commitment and history dependence).

Other authors have focused instead on the implications of central bank's imperfect knowledge of the structure of the economy or limited observability of shocks or endogenous variables by the central bank (e.g., Aoki (2003), Svensson and Woodford (2003, 2004)). Other work has sought to characterize the optimal policy rules when the policymaker faces uncertainty regarding the model's parameters, and seeks to minimize its expected losses given a prior on the parameters' distribution or, alternatively, under a worst-case parameter configuration (e.g., Giannoni (2006)).

A branch of the literature on monetary policy and imperfect information has focused on the hypothesis of rational inattention, originally developed by Sims (1998, 2003), that formalizes the idea that agents face some constraints on the amount of information they can process. Mackowiak and Wiederholt (2009) develop a model with rational inattention applied to the price setting decisions by monopolistic competitors facing both (small) aggregate and (large) idiosyncratic shocks, and show how they will optimally choose to adjust prices little in response to monetary policy shocks, leading to large effects on real variables even in the absence of any (exogenous) nominal rigidities. Paciello and Wiederholt (2014) analyze the implications of rational inattention in price setting for the design of optimal monetary policy, and show that in such an environment, a strict inflation targeting policy may be optimal even in the face of shocks that generate inefficient fluctuations, in contrast with the analysis (with perfect information) in chapter 5.

Endogenous capital accumulation. For the sake of simplicity, all the models analyzed in the previous chapters have abstracted from capital and its accumulation. The introduction of endogenous capital accumulation in New Keynesian models poses no major difficulty if

[4] See Orphanides and Williams (2005) for another key reference in that literature, though in a framework with a supply side specification that differs from the one associated with the standard New Keynesian model emphasized here.

one is willing to assume the existence of a competitive rental market where capital services can be purchased by firms, as found in many versions of the New Keynesian model (e.g., Yun (1996), Christiano, Eichenbaum, and Evans (2005)). Further complications arise if capital is assumed to be firm-specific, with investment decisions being made by the same firms that adjust prices infrequently, for in that case the price set by any firm depends on its own current and expected capital stock, which will generally differ across firms, given differences in price-setting history (e.g., Woodford (2005)). In that case, the conditions that a Taylor-type interest rate rule needs to satisfy in order to guarantee a unique equilibrium must be modified, with the Taylor principle no longer offering a reliable criterion (e.g., Sveen and Weinke (2005, 2007)).

Financial market imperfections. The baseline New Keynesian model developed in the previous chapters assumes that capital markets are perfect. In many instances, this approximation may be reasonable. However, and as the recent financial crisis has made clear, there are periods when frictions and imperfections in the workings of financial markets cannot be ignored. Over the past few years, a great deal of effort has been made to incorporate financial factors within the New Keynesian framework, with the aim of better understanding the appropriate role of monetary policy in mitigating the effects of financial crises.

A reference model combining nominal rigidities and credit frictions has been developed in Bernanke, Gertler, and Gilchrist (1999). That model features a "financial accelerator" property, whereby any shocks affecting the net worth of borrowers see their effects on aggregate demand and output amplified through their impact on the "external finance premium" paid by borrowing firms, which is inversely related to the net worth. Other recent papers have explored the policy implications of the coexistence of nominal rigidities with different types of credit frictions, including collateral-based borrowing constraints (e.g., Iacoviello (2005) and Monacelli (2006)), balance-sheet constraints on financial intermediaries (e.g., Gertler and Karadi (2011)), endogenous, time-varying spreads between policy and loan rates (e.g., Curdia and Woodford (2010)), time-varying volatility of idiosyncratic shocks (Christiano, Motto, and Rostagno (2014)), as well as the presence of a fraction of households with no access to financial markets (e.g., Galí, López-Salido, and Vallés (2004, 2007)).

The above-mentioned papers generally assume a stationary linear environment, with financial frictions of different sort providing a mechanism for the amplification of the aggregate effects of shocks. While those amplification mechanisms are likely to be relevant and even quantitatively significant, they fall short of providing an endogenous explanation for the occurrence of financial crises, which are generally viewed as the consequence of the gradual building up of financial imbalances that eventually become unsustainable. Introducing those features in the class of models developed in the present book, and analyzing its policy implications, will likely be the focus of much research effort in the years to come.[5]

It is still too early to tell which, if any, of the previous features will be permanently incorporated in empirical, larger-scale versions of the New Keynesian model. Most likely, those models will continue to evolve as we accumulate more data and experience more economic shocks. It may very well be the case that important new features are introduced and that ones that are central for performance today are less so in the future. At the same time, while models are expected to change, the general approach is not likely to: Quantitative macroeconomic modeling along with its role in the policy-making process is probably here to stay.

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[5] See Woodford (2012) for a sketch of such a model.

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