APPENDIX

Appendix 1

1. Additional Constraints

1.1 Equipment Constraints

(1) Gas Turbine (GT)

The gas turbine consumes natural gas to generate electricity:

$$P_{i,t}^{\mathrm{gt}}=\lambda_{\mathrm{gas}}G_t^{\mathrm{gt}}\eta_i^{\mathrm{gt}} \tag{1}$$

P_t^{gt}、G_t^{gt}、λ_{gas}、η_t^{gt} respectively represent the power supply capacity of the GT, gas consumption, calorific value of natural gas, and power generation efficiency.

（2）Gas Boiler (GB)

Gas boilers consume natural gas and produce high-temperature steam for direct heating.

$$\begin{cases} H_{i,t}^{\mathrm{gb}}=\eta_i^{\mathrm{gb}}G_{i,t}^{\mathrm{gb}} \\ H_{i,\min}^{\mathrm{gb}} \le H_{i,t}^{\mathrm{gb}} \le H_{i,\max}^{\mathrm{gb}} \end{cases} \tag{2}$$

$H_t^{\mathrm{gb,s}}$、G_t^{gb}、η_t^{gb} respectively represent the heat of the steam produced by the gas boiler, the gas consumption, and the operational efficiency, respectively; $H_{\min}^{\mathrm{gb}}$、$H_{\max}^{\mathrm{gb}}$ are the lower and upper limits of the gas boiler's heat production.

(3) Wind Farm

The day-ahead planned output of the wind farm does not exceed the forecast.

$$0 \le P_{i,t}^{\mathrm{w}} \le P_{i,\max}^{\mathrm{wp}} \tag{3}$$

(3) Electric Boiler (EB)

EBs consume electrical energy to produce heat energy.

$$\begin{cases} H_{i,t}^{\mathrm{eb}}=\eta_i^{\mathrm{eb}}P_{i,t}^{\mathrm{eb}} \\ 0 \le H_t^{\mathrm{eb}} \le H_{\max}^{\mathrm{eb}} \end{cases} \tag{4}$$

ΔH_t^{eb}、P_t^{eb}、η_t^{eb} respectively represent the power of the medium-temperature hot water output by the EB, the consumed electric power, and the conversion coefficient; $H_{\max}^{\mathrm{eb}}$ is the maximum heating power of the heat pump.

(4) Electric Chiller (EC)

$$\begin{cases} C_{i,t}^{\mathrm{ec}}=\eta_i^{\mathrm{ec}}P_{i.t}^{\mathrm{ec}} \\ C_{i,\min}^{\mathrm{ec}} \le C_{i,t}^{\mathrm{ec}} \le C_{i,\max}^{\mathrm{ec}} \end{cases} \tag{5}$$

C_t^{ec}、P_t^{ec}、η_t^{ec} respectively represent the cooling power produced by the electric chiller, the consumed electric power, and the conversion coefficient.

(5) Ramp-up Constraint

Certain equipment should satisfy the ramp-up constraints:：

$$\begin{cases} r_{\mathrm{dn}}^i\Delta t \le P_t^i - P_{t-1}^i \le r_{\mathrm{up}}^i\Delta t \\ P_{\min}^i \le P_t^i \le P_{\max}^i \end{cases} \tag{6}$$

The equipment represented by k includes: GTs, GBs

1.2 Energy Flow Balance Constraints

(1) Electric Power Balance

$$P_{i,t}^{\mathrm{gt}} + P_{i,t}^{\mathrm{wp}} + P_{i,t}^{\mathrm{grid}} + \sum_{j=1,j\ne i}^{N_{\mathrm{os}}} P_{ji,t}^{\mathrm{tre}} = P_{i,t}^{\mathrm{el}} + P_{i,t}^{\mathrm{eb}} + P_{i,t}^{\mathrm{ec}} \tag{7}$$

(2) Heat Balance

$$H_{i,t}^{\mathrm{eb}} + H_{i,t}^{\mathrm{gb}} + \sum_{j=1,j\ne i}^{N_{\mathrm{os}}} H_{ji,t}^{\mathrm{trh}} = H_{i,t}^{\mathrm{hl}} \tag{8}$$

(3) Natural Gas Flow Balance

$$G_{i,t}^{\mathrm{grid}} = G_{i,t}^{\mathrm{gt}} + G_{i,t}^{\mathrm{gb}} \tag{9}$$

(4) Cooling Balance

$$C_{i,t}^{\mathrm{ec}} = C_{i,t}^{\mathrm{cl}} \tag{10}$$

1.3 Energy Flow Constraints between PIESs

(1) Electrical Energy

For the electrical energy supply to PIES$_j$, it satisfies:

$$\begin{cases} P_{j,t}^{\mathrm{wp}} = P_{j,t}^{\mathrm{wp\text{-}ld}} + P_{j,t}^{\mathrm{wp\text{-}eb}} + P_{j,t}^{\mathrm{wp\text{-}ec}} + \sum_{i=1,j\ne i}^{N_{\mathrm{as}}} P_{ji,t}^{\mathrm{wp\text{-}tr}} \\ P_{j,t}^{\mathrm{gt}} = P_{j,t}^{\mathrm{gt\text{-}ld}} + P_{j,t}^{\mathrm{gt\text{-}eb}} + P_{j,t}^{\mathrm{gt\text{-}ec}} + \sum_{i=1,j\ne i}^{N_{\mathrm{as}}} P_{ji,t}^{\mathrm{gt\text{-}tr}} \\ P_{j,t}^{\mathrm{gr}} = P_{j,t}^{\mathrm{gr\text{-}ld}} + P_{j,t}^{\mathrm{gr\text{-}eb}} + P_{j,t}^{\mathrm{gr\text{-}ec}} \\ P_{j,t}^{\mathrm{tre}} = \sum_{i=1,j\ne i}^{N_{\mathrm{as}}} P_{ji,t}^{\mathrm{tre}} = \sum_{i=1,j\ne i}^{N_{\mathrm{as}}} (P_{ji,t}^{\mathrm{wp\text{-}tr}} + P_{ji,t}^{\mathrm{gt\text{-}tr}}) \\ P_{j,t}^{\mathrm{trre}} = \sum_{i=1,j\ne i}^{N_{\mathrm{as}}} P_{ji,t}^{\mathrm{trre}} = \sum_{i=1,j\ne i}^{N_{\mathrm{as}}} P_{ji,t}^{\mathrm{wp\text{-}tr}} \\ P_{j,t}^{\mathrm{ld}} = P_{j,t}^{\mathrm{wp\text{-}ld}} + P_{j,t}^{\mathrm{gt\text{-}ld}} + P_{j,t}^{\mathrm{gr\text{-}ld}} \\ P_{j,t}^{\mathrm{eb}} = P_{j,t}^{\mathrm{wp\text{-}eb}} + P_{j,t}^{\mathrm{gt\text{-}eb}} + P_{j,t}^{\mathrm{gr\text{-}eb}} \\ P_{j,t}^{\mathrm{ec}} = P_{j,t}^{\mathrm{wp\text{-}ec}} + P_{j,t}^{\mathrm{gt\text{-}ec}} + P_{j,t}^{\mathrm{gr\text{-}ec}} \end{cases} \tag{11}$$

(2) Heat Energy

To reduce carbon emissions, it is assumed that the thermal loads in each area are primarily supplied by

EBs. For the thermal energy supplied to PIES$_i$, it is assumed in this paper that the outgoing thermal load is entirely produced by GBs, and its heat energy must satisfy:

$$\begin{cases} H_{j,t}^{\text{gb}} = H_{j,t}^{\text{gb-ld}} + \sum_{i=1, j\neq i}^{N_{\text{as}}} H_{ji,t}^{\text{gb-tr}} \\ H_{j,t}^{\text{trh}} = \sum_{i=1, j\neq i}^{N_{\text{as}}} H_{ji,t}^{\text{trh}} = \sum_{i=1, j\neq i}^{N_{\text{as}}} H_{ji,t}^{\text{gb-tr}} \\ H_{j,t}^{\text{load}} = H_{j,t}^{\text{eb}} + H_{i,t}^{\text{gb-ld}} \end{cases} \tag{12}$$

For the thermal energy receiving area:

$$\begin{cases} H_{i,t}^{\text{trh}} = \sum_{j=1, j\neq i}^{N_{\text{os}}} H_{ji,t}^{\text{trh}} = \sum_{j=1, j\neq i}^{N_{\text{os}}} H_{ji,t}^{\text{tr-ld}} \\ H_{i,t}^{\text{load}} = H_{i,t}^{\text{eb}} + H_{i,t}^{\text{gb}} + \sum_{j=1, j\neq i}^{N_{\text{os}}} H_{ji,t}^{\text{tr-ld}} \end{cases} \tag{13}$$

Appendix 2

2. KKT Transformation

Taking $c_i^{\text{ele}} P_{i,t}^{\text{sell,e}}$ as an example to illustrate the transformation process, after completing the formulation of the Lagrangian function for the lower-level model, the corresponding KKT (Karush-Kuhn-Tucker) conditions are obtained:

$$\begin{cases} \dfrac{\partial r}{\partial P_{i,t}^{\text{r},e}} = c_{i,t}^{\text{ele}} + e_i^{\text{lre,e}} + e_i^{\text{lca,e}} + \lambda_{i,t}^{\text{e,min}} - \lambda_{i,t}^{\text{e,max}} + \rho_{i,t}^{\text{sl,e}} - \pi_{i,t}^{\text{sl,e}} = 0 \\ \dfrac{\partial r}{\partial P_{i,t}^{\text{sl},e}} = \theta_i^e + \alpha_i^e P_{i,t}^{\text{sl,e}} - \rho_{i,t}^{\text{sl,e}} - \pi_{i,t}^{\text{sl,e}} = 0 \\ \lambda_{i,t}^{\text{e,min}} (P_{i,t}^{\text{r},e} - P_{i,t}^{\max}) = 0 \\ \lambda_{i,t}^{\text{e,max}} (P_{i,t}^{\min} - P_{i,t}^{\text{r},e}) = 0 \\ \rho_{i,t}^{\text{sl,e}} (P_{i,t}^{\text{r},e} - P_{i,t}^{\text{f},e} - P_{i,t}^{\text{sl},e}) = 0 \\ \pi_{i,t}^{\text{sl,e}} (P_{i,t}^{\text{f},e} - P_{i,t}^{\text{r},e} \le P_{i,t}^{\text{sl},e}) = 0 \end{cases} \tag{14}$$

Based on the KKT conditions in the transformation process, the objective function is converted into a convex function:

$$\begin{aligned} &(c_{i,t}^{\text{ele}} + e_{i,t}^{\text{lre,e}} + e_{i,t}^{\text{lca,e}}) P_{i,t}^{\text{r},e} \\ &= (\lambda_{i,t}^{\text{e,min}} - \lambda_{i,t}^{\text{e,max}} + \rho_{i,t}^{\text{sl,e}} - \pi_{i,t}^{\text{sl,e}}) P_{i,t}^{\text{r},e} \\ &= \lambda_{i,t}^{\text{e,min}} P_{i,t}^{\min} - \lambda_{i,t}^{\text{e,max}} P_{i,t}^{\max} + (\rho_{i,t}^{\text{sl,e}} - \pi_{i,t}^{\text{sl,e}}) P_{i,t}^{\text{f},e} - (\theta_i^e + \alpha_i^e P_{i,t}^{\text{sl,e}}) P_{i,t}^{\text{sl,e}} \end{aligned} \tag{15}$$

Appendix 3

3. Nash Negotiation Model and its Transformation

In collaboration with other PIESPs, the PIESP establishes the transaction volume and pricing for energy resources, encompassing renewable electricity. The pursuit of optimal operational revenue among PIESs, facilitated by Nash bargaining, can be conceptualized as a cooperative game model, succinctly expressed as:

$$\begin{cases} \max \prod_{i=1}^{N} (F_i^{\text{C}} - F_i^{\text{N},0}) \\ \text{s.t. } F_i^{\text{C}} - F_i^{\text{N},0} \ge 0 \end{cases} \tag{16}$$

Herein, F_i^{C} signifies each participant's utility function in the cooperative game, while $F_i^{\text{N},0}$ corresponds to the operational revenue under non-cooperative conditions, or the Nash bargaining impasse. The difference between these values reflects the benefits derived from cooperative gaming. The mathematical expression implies that, relative to independent operation, engaging in bargaining transactions maximizes the benefits for all PIESPs entities.

We transform (16) into subproblems of maximizing cooperative benifit and payment utility, which are sequentially solved. Based on the mean inequality:

$$\prod_{i=1}^{N} (F_i^{\text{C}} - F_i^{\text{N},0}) \le \left[\frac{1}{N} \sum_{i=1}^{N} \left(F_i^{\text{C}} - F_i^{\text{N},0} \right) \right]^N \tag{17}$$

The maximization of (16) requires satisfying the “one positive, two constants, and three equal” conditions. “One positive” implies that $F_i^{\text{c}} - F_i^{\text{N},0}$ are all positive values; this paper assumes that all PIES entities can achieve a benefit increase through cooperation, thus validating this condition. “Two constants” refers to the maximum value of $\sum_{i=1}^{N} (F_i^{\text{c}} - F_i^{\text{N},0})$ is constant, where $F_i^{\text{N},0}$ is a constant; after optimizing the value of F_i^{c}, $\sum_{i=1}^{N} (F_i^{\text{c}} - F_i^{\text{N},0})$ can be considered constant. Lastly, “three equal” signifies that when $F_i^{\text{c}} - F_i^{\text{N},0} = \frac{1}{N} \sum_{i=1}^{N} (F_i^{\text{c}} - F_i^{\text{N},0})$, the maximum value of (16) is attained. Building on the analysis, the larger the derived value of $\sum_{i=1}^{N} (F_i^{\text{c}} - F_i^{\text{N},0})$, the higher the objective function's upper bound in (16); the transformation process for the solution objective is:

$$\max\prod_{i=1}^{N}(F_i^{\mathrm{C}}-F_i^{\mathrm{N},0})\Leftrightarrow\max\sum_{i=1}^{N}\left(F_i^{\mathrm{C}}-F_i^{\mathrm{N},0}\right)\Leftrightarrow\max\sum_{i=1}^{N}F_i^{\mathrm{C}}$$

As transaction costs f_i^{tr} between PIESs counterbalance each other during system cost accumulation, the problem can be expressed as:

$$\max\sum_{i=1}^{N}f_i^{\mathrm{rc}} \tag{18}$$

This represents the cooperative benefit maximization subproblem, resulting in the optimal value $f_i^{\mathrm{rc},0}$ and energy transaction volume among PIESPs. Substituting the optimized results into (16) and taking the natural logarithm of the objective, the original maximization problem is equivalent to solving (19) for the minimum value, ultimately determining energy transaction prices between systems.

$$\max\prod_{i=1}^{N}(F_i^{\mathrm{C}}-F_i^{\mathrm{N},0})\Leftrightarrow\min\sum_{i=1}^{N}-\ln(f_i^{\mathrm{rc},0}+f_i^{\mathrm{tr}}-F_i^{\mathrm{N},0}) \tag{19}$$

From the above analysis, once the value of the maximized cooperative benefit is determined, the equilibrium solution can be obtained by solving the energy trading negotiation sub-problem.

Appendix 4

4. Analysis of PIESP Energy Trading Results

(1) Basic Scheduling Results

Figure 1 and Figure 2 show the optimized results for electrical and thermal energy in the two parks

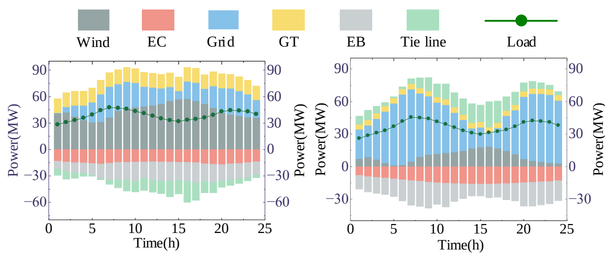

Figure 1 Optimization Results for PIES's Electrical Energy

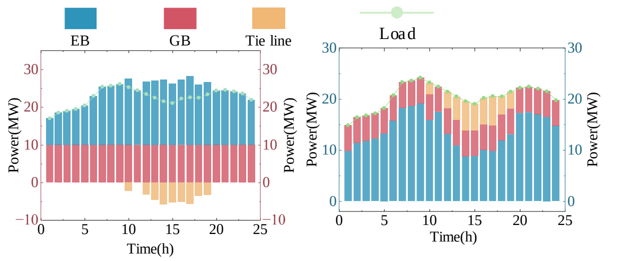

Figure 2 Optimization Results for PIES's Heat Energy

(2) Analysis of Energy Trading and Pricing Among PIESPs

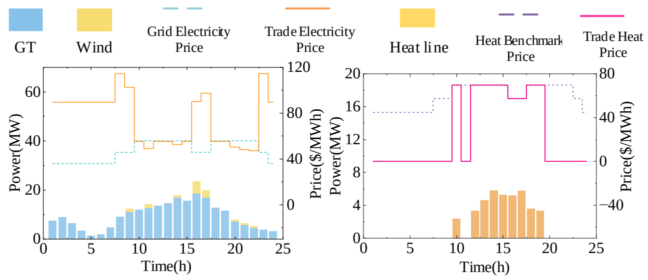

Figure 3 Inter-PIESP trading energy and price

From Figure 3, it is evident that the electric power between PIESPs is primarily wind power. Since renewable energy simultaneously fulfills the multiple roles of meeting the proportion assessment of renewable energy and reducing carbon emissions, its transactional price is relatively higher. In terms of heat energy trading, only when the trading price is not zero and there is a trading volume, does the trading price attain its predetermined upper limit.