**Appendix 1: KKT Transformation**

Taking as an example to illustrate the transformation process. The objective function and constraints related to heat load in MELA*i* include the -.





Based on the objective function and constraints mentioned above, write the corresponding Lagrangian function 𝑟. After completing the formulation of the Lagrangian function for the lower-level model, the corresponding KKT (Karush-Kuhn-Tucker) conditions are obtained:



Based on the KKT conditions in the transformation process, the objective function is converted into a convex function:



**Appendix 2: Nash Negotiation Model and its Transformation**

In collaboration with other PIESPs, the PIESP establishes the transaction volume and pricing for energy resources, encompassing renewable electricity. The pursuit of optimal operational revenue among PIESs, facilitated by Nash bargaining, can be conceptualized as a cooperative game model, succinctly expressed as:



Herein, signifies each participant's utility function in the cooperative game, while corresponds to the operational revenue under non-cooperative conditions, or the Nash bargaining impasse. The difference between these values reflects the benefits derived from cooperative gaming. The mathematical expression implies that, relative to independent operation, engaging in bargaining transactions maximizes the benefits for all PIESPs entities.

We transform into subproblems of maximizing cooperative benefits and payment utility, which are sequentially solved. Based on the mean inequality:



The maximization of requires satisfying the “one positive, two constants, and three equals” conditions. “One positive” implies that  are all positive values; this paper assumes that all PIES entities can achieve a benefit increase through cooperation, thus validating this condition. “Two constants” refers to the maximum value of is constant, where is a constant; after optimizing the value of , can be considered constant. Lastly, “three equals” signifies that when , the maximum value of is attained. Building on the analysis, the larger the derived value of, the higher the objective function's upper bound in ; the transformation process for the solution objective is:



As transaction costs between PIESs counterbalance each other during system cost accumulation, the problem can be expressed as:



This represents the cooperative benefit maximization subproblem, resulting in the optimal value and energy transaction volume among PIESPs. Substituting the optimized results into and taking the natural logarithm of the objective, the original maximization problem is equivalent to solving for the minimum value, ultimately determining energy transaction prices between systems.



From the above analysis, once the value of the maximized cooperative benefit is determined, the equilibrium solution can be obtained by solving the energy trading negotiation sub-problem.