Distributed Robust Dispatch for Integrated Transmission and Urban Energy Systems with Ensured and Expedited Convergence

Appendix A

A. Specific Expression of the (6)

$$\begin{cases} f_t^{\text{th},0} = \sum\limits_{i\in I^{\text{ts}}} a_i \left(P_{i,t}^{\text{th0}}\right)^2 + b_i P_{i,t}^{\text{th0}} + c_i \\ f_t^{\text{wp},0} = \sum\limits_{w\in W^{\text{ts}}} a^{\text{wp}} \Delta P_{w,t}^{\text{wp},0} \\ f_t^{\text{el},0} = \sum\limits_{d\in D^{\text{ts}}} a^{\text{el}} \Delta P_{d,t}^{\text{el},0} \\ \Delta f_t^{\text{th},r} = \sum\limits_{i\in I^{\text{ts}}} b_i^{\text{th}+} \Delta P_{i,t}^{\text{th}+} + b_i^{\text{th}-} \Delta P_{i,t}^{\text{th}-} \\ f_t^{\text{wp,r}} = \sum\limits_{wt\in W^{\text{ts}}} b^{\text{wp}} \Delta P_{wt,t}^{\text{wp,r}} \\ f_t^{\text{el,r}} = \sum\limits_{d\in D^{\text{ts}}} b^{\text{el}} \Delta P_{d,t}^{\text{el},r} \end{cases} \tag{1}$$

$a^{(\cdot)}/b^{(\cdot)}$ is the day-ahead/intraday corresponding penalty coefficient of each item.

B. Day-ahead Constraints of TS

TS's day-ahead constraints include power balance constraint (2), power generation correlation constraint (3), wind farm constraint (4), load planning constraint (5), DC current security constraint (6), TS and UIES tie-line constraint (7).

$$\sum_{i\in I^{\text{ts}}} P_{i,t}^{\text{th},0} + \sum_{wt\in W^{\text{ts}}} P_{wt,t}^{\text{wp},0} = \sum_{d\in D^{\text{ts}}} P_{d,t}^{\text{el},0} + \sum_{k\in K} P_{k,t}^{\text{tu},0} \tag{2}$$

$$\begin{cases} P_i^{\text{th,min}} \le P_{i,t}^{\text{th},0} \le P_i^{\text{th,max}} \\ -r_i^{\text{us}} \Delta t \le P_{i,t}^{\text{th},0} - P_{i,t-1}^{\text{th},0} \le r_i^{\text{up}} \Delta t \\ R_{i,t}^{\text{gu}} = U_i^{\text{th}} \Delta t, R_{i,t}^{\text{gd}} = D_i^{\text{th}} \Delta t \end{cases} \tag{3}$$

$$P_{wt,t}^{\text{wp},0} = P_{wt,t}^{\text{wp,p}} - \Delta P_{wt,t}^{\text{wp},0} \tag{4}$$

$$\Delta P_{d,t}^{\text{el},0} = P_{d,t}^{\text{el,p}} - P_{d,t}^{\text{el},0} \tag{5}$$

$$\begin{aligned} &|\sum_{i\in I^{\text{ts}}} G_{l-i} P_{i,t}^{\text{th},0} + \sum_{w\in W^{\text{ts}}} G_{l-w} P_{wt,t}^{\text{wp},0} - \\ &\sum_{d\in D^{\text{ts}}} G_{l-d} P_{d,t}^{\text{el},0} + \sum_{k\in K} G_{l-k} P_{k,t}^{\text{ts},0}| < P_l^{\max} \end{aligned} \tag{6}$$

$$-P_{k,\max}^{\text{tu}} \le P_{k,t}^{\text{tu},0} \le P_{k,\max}^{\text{tu}} \tag{7}$$

G_{l-i} is generation shift distribution factors, and $P_l^{\max}$ is power flow limit of line l.

C. Specific Expression of the (9)

$$\begin{cases} f_t^{\text{buy},0} = \sum\limits_{m\in B_k} c^{\text{buy}} Q_{bu,t}^{\text{buy},0} \\ f_t^{\text{load},0} = \sum\limits_{d\in D_k} c^{\text{el}} \Delta P_{d,t}^{\text{el},0} + \sum\limits_{h\in H_k} c^{\text{hl}} \Delta H_{h,t}^{\text{hl},0} + \sum\limits_{m\in M_k} c^{\text{gl}} \Delta Q_{m,t}^{\text{gl},0} \\ f_t^{\text{wp},0} = \sum\limits_{w\in W_k} c^{\text{wp}} \Delta P_{w,t}^{\text{wp},0} \\ \Delta f_t^{\text{eq},r} = \sum\limits_{g\in E_k^{\text{eq}}} d_g^{\text{eq}-} \Delta P_{g,t}^{\text{eq}-} + d_g^{\text{eq}+} \Delta P_{g,t}^{\text{eq}+} \\ \Delta f_t^{\text{buy},r} = \sum\limits_{m\in B_k} c^{\text{buy}+} \Delta Q_{m,t}^{\text{buy}+} + c^{\text{buy}-} \Delta Q_{bu,t}^{\text{buy}-} \\ f_t^{\text{load},r} = \sum\limits_{d\in D_k} d^{\text{el}} \Delta P_{d,t}^{\text{el},r} + \sum\limits_{h\in H_k} d^{\text{hs}} \Delta H_{h,t}^{\text{s},r} + \sum\limits_{m\in M_k} d^{\text{gl}} \Delta Q_{m,t}^{\text{gl},r} \\ f_t^{\text{wp},r} = \sum\limits_{w\in W_k} d^{\text{wp}} \Delta P_{w,t}^{\text{wp},r} \end{cases} \tag{8}$$

'eq' represents the general notation for all types of equipment symbols in the system, with $eq \in \{\text{GT}, \text{GB}, \text{EB}\}$. $c^{(\cdot)}/d^{(\cdot)}$ denotes the day-ahead/intraday corresponding penalty coefficient for each item.

D. UIES_k Day-Ahead Distribution Network Constraints

The constraints mainly consist of distribution network constraints (9) and load-related constraints (10).

$$\begin{cases} \begin{cases} P_{j,t}^{\text{in},0} = \sum\limits_{i\in fr(j)} P_{ij,t}^{\text{us},0} - \sum\limits_{k\in to(j)} P_{jk,t}^{\text{us},0} \\ P_{j,t}^{\text{in},0} = \sum\limits_{g\in j} P_{g,t}^{\text{gt},0} + \sum\limits_{h\in j} P_{wu,t}^{\text{wp},0} - \sum\limits_{z\in j} P_{e,t}^{\text{eb},0} \\ \qquad - \sum\limits_{k\in j} P_{k,t}^{\text{ut},0} - \sum\limits_{u\in j} P_{ku,k,t}^{\text{uu},0} - P_{j,t}^{\text{el},0} \end{cases} \\ U_{i,t}^0 - \left(r_{ij} P_{ij,t}^0 + x_{ij} Q_{ij,t}\right) \Big/ U_{\text{r}} = U_{j,t}^0 \\ U_{j,\min} \le U_{j,t}^0 \le U_{j,\max} \\ 0 \le P_{ij,t}^0 \le P_{ij,\max} \end{cases} \tag{9}$$

$$\begin{cases} \Delta P_{j,t}^{\text{el min}} \le \Delta P_{j,t}^{\text{el},0} \le \Delta P_{j,t}^{\text{el max}} \\ P_{j,t}^{\text{el},0} = P_{j,t}^{\text{el,p}} - \Delta P_{j,t}^{\text{el},0} \end{cases} \tag{10}$$

$m \in j$ indicates that gas turbine m is connected to node j. $to(j)$ represents the set of starting nodes with node j as the end node branch, while $fr(j)$ denotes the set of terminal nodes with node j as the starting node branch.

E. $UIES_k$ Day-Ahead Equipment Constraints

This part includes constraints on conversion equipment (11) and production equipment (12). The equipment that converts primary energy into secondary energy is referred to as production equipment, which satisfies upper and lower limits of output, ascent and descent slopes. Equipment then used convert secondary energy to secondary energy is categorized as conversion equipment.

$$\begin{cases} H_{e,t}^{\mathrm{eb},0} = \eta_e^{\mathrm{eb}} P_{e,t}^{\mathrm{eb},0} \\ H_{g,t}^{\mathrm{rb,in},0} = \dfrac{P_{g,t}^{\mathrm{gt},0}(1-\eta_g^{\mathrm{gt}})}{\eta_g^{\mathrm{gt}}} \eta_g^{\mathrm{rb}} \\ P_{g,t}^{\mathrm{tra,min}} \le P_{g,t}^{\mathrm{tra},0} \le P_{g,t}^{\mathrm{tra,max}} \end{cases} \tag{11}$$

$$\begin{cases} P_{\mathrm{pro},v,t}^{\min} \le P_{v,t}^{\mathrm{pro},0} \le P_{\mathrm{pro},v,t}^{\max} \\ -r_{v,t}^{\mathrm{pro,us}} \le P_{v,t}^{\mathrm{pro},0} - P_{v,t-1}^{\mathrm{pro},0} \le r_{v,t}^{\mathrm{pro,up}} \\ R_{v,t}^{\mathrm{prou},0} = U_{v,t}^{\mathrm{pro}}\Delta t, R_{v,t}^{\mathrm{pro},0} = D_{v,t}^{\mathrm{pro}}\Delta t \\ \begin{cases} P_{g,t}^{\mathrm{gt},0} = \eta_{\mathrm{g}}^{\mathrm{gt}} L_{\mathrm{NG}} Q_{g,t}^{\mathrm{gt},0} \\ H_{b,t}^{\mathrm{gb},0} = \eta_b^{\mathrm{gb}} L_{\mathrm{NG}} Q_{b,t}^{\mathrm{gb},0} \end{cases} \end{cases} \tag{12}$$

'pro' is the unified representation of energy production equipment, such as GTs and GBs; 'v' is the corresponding equipment number; 'tra' is the unified representation of conversion equipment, such as waste heat boilers and EBs; and 'LNG' is the calorific value of natural gas. Specifically, the WHB's input heat power generates heat for GT operation.

Appendix B

A. MP and SP of the (30)

$$(\mathrm{MP}) \begin{cases} \min\limits_{\boldsymbol{x}^{\mathrm{us}},\boldsymbol{y}^{\mathrm{us}},\eta} \boldsymbol{e}^{\mathrm{T}}\boldsymbol{x}^{\mathrm{ts}} + \eta \\ \text{s.t. } \eta \ge \sum\limits_{p_r \in \Omega^{\mathrm{us}}} p_r \boldsymbol{H}\left(\boldsymbol{y}^{\mathrm{us}}, \boldsymbol{\mu}^{\mathrm{us}}\right) \\ \qquad \boldsymbol{I}\boldsymbol{x}^{\mathrm{us}} \le \boldsymbol{f} \\ \qquad \boldsymbol{J}\boldsymbol{x}^{\mathrm{us}} + \boldsymbol{K}\boldsymbol{y}^{\mathrm{us}} \le \boldsymbol{g} \\ \qquad \boldsymbol{L}\boldsymbol{x}^{\mathrm{us}} + \boldsymbol{M}\boldsymbol{\mu}^{\mathrm{us}} \le \boldsymbol{h} \end{cases} \tag{13}$$

$$(\mathrm{SP}) \begin{cases} \max\limits_{p_z \in \Omega^{\mathrm{us}}} \sum\limits_{z \in Z} p_z \min\limits_{y^{\mathrm{us}}} \boldsymbol{H}\left(\boldsymbol{y}^{\mathrm{us}}, \boldsymbol{\mu}^{\mathrm{us}}\right) \\ \text{s.t.} \quad \boldsymbol{J}\boldsymbol{x}^{\mathrm{us}} + \boldsymbol{K}\boldsymbol{y}^{\mathrm{us}} \le \boldsymbol{g} \\ \qquad \boldsymbol{L}\boldsymbol{x}^{\mathrm{us}} + \boldsymbol{M}\boldsymbol{\mu}^{\mathrm{us}} \le \boldsymbol{h} \end{cases} \tag{14}$$

B. Comparison of Different Robust methods

TABLE I: Comparison of three methods

Cost($)	Approach		
	DRO	RO	SO
Phase I cost	67983.7	67995.0	67982.2
Phase II cost	8467.4	10452.4	8395.1
Total cost	76451.1	78447.4	76377.3

C. Diagram of T&2U

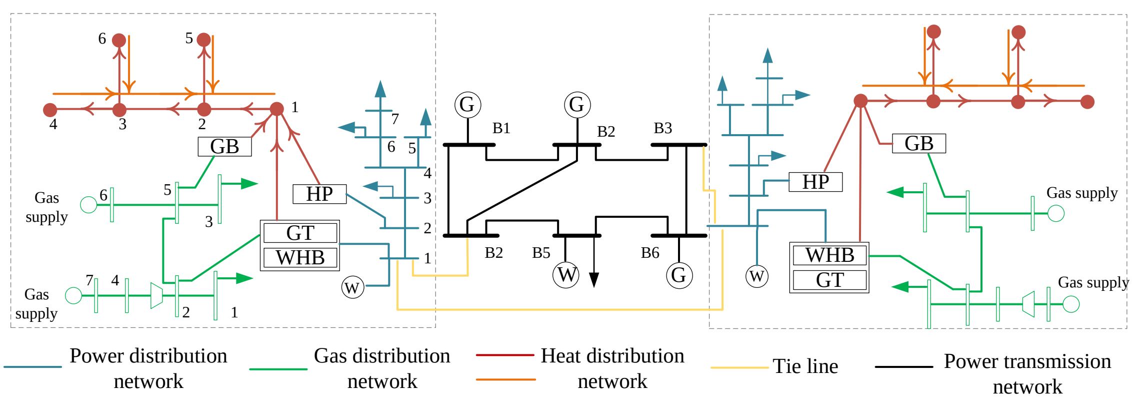

Fig. 1: Diagram of test system