Distributed Robust Dispatch for Integrated Transmission and Urban Energy Systems with Ensured and Expedited Convergence

I. APPENDIX

A. Specific Expression of the (6)

$$\begin{cases} f_t^{\mathrm{th},0} = \sum_{i \in I^{\mathrm{ts}}} a_i (P_{i,t}^{\mathrm{th}0})^2 + b_i P_{i,t}^{\mathrm{th}0} + c_i \\ f_t^{\mathrm{wp},0} = \sum_{w \in W^{\mathrm{ts}}} a^{\mathrm{wp}} \Delta P_{w,t}^{\mathrm{wp},0} \\ f_t^{\mathrm{el},0} = \sum_{d \in D^{\mathrm{ts}}} a^{\mathrm{el}} \Delta P_{d,t}^{\mathrm{el},0} \\ \Delta f_t^{\mathrm{th},r} = \sum_{i \in I^{\mathrm{ts}}} b_i^{\mathrm{th}+} \Delta P_{i,t}^{\mathrm{th}+} + b_i^{\mathrm{th}-} \Delta P_{i,t}^{\mathrm{th}-} \\ f_t^{\mathrm{wp,r}} = \sum_{wt \in W^{\mathrm{ts}}} b^{\mathrm{wp}} \Delta P_{wt,t}^{\mathrm{wp,r}} \\ f_t^{\mathrm{el,r}} = \sum_{d \in D^{\mathrm{ts}}} b^{\mathrm{el}} \Delta P_{d,t}^{\mathrm{el},r} \end{cases} \tag{A-1}$$

$a^{(\cdot)}/b^{(\cdot)}$ is the day-ahead/intraday corresponding penalty coefficient of each item.

In the optimization model, inspired by the linearization of the absolute value method [1], we decompose conventional unit power changes into positive and negative parts($\Delta P_{i,t}^{\mathrm{th}+}$ & $\Delta P_{i,t}^{\mathrm{th}-}$). The intra-day power can thus be represented as: $P_{i,t}^{\mathrm{th},r} = P_{i,t}^{\mathrm{th},0} + \Delta P_{i,t}^{\mathrm{th}+} - \Delta P_{i,t}^{\mathrm{th}-}$. At the same time, the objective function contains the sum of positive and negative power ($b_i^{\mathrm{th}+} \Delta P_{i,t}^{\mathrm{th}+} + b_i^{\mathrm{th}-} \Delta P_{i,t}^{\mathrm{th}-}$). Therefore, the optimization objective value is minimized only when at least one of the positive and negative powers is zero. This choice avoids the introduction of 0-1 variables, ensuring the continuity and theoretical convergence of the model.

[1] J. Bisschop, AIMMS optimization modeling: Lulu. com, 2006.

B. Day-ahead Constraints of TS

TS's day-ahead constraints include power balance constraint (A-2), power generation correlation constraint (A-3), wind farm constraint (A-4), load planning constraint (A-5), DC current security constraint (A-6), TS and UIES tie-line constraint (A-7).

$$\sum_{i \in I^{\mathrm{ts}}} P_{i,t}^{\mathrm{th},0} + \sum_{wt \in W^{\mathrm{ts}}} P_{wt,t}^{\mathrm{wp},0} = \sum_{d \in D^{\mathrm{ts}}} P_{d,t}^{\mathrm{el},0} + \sum_{k \in K} P_{k,t}^{\mathrm{tu},0} \tag{A-2}$$

$$\begin{cases} P_i^{\mathrm{th,min}} \le P_{i,t}^{\mathrm{th},0} \le P_i^{\mathrm{th,max}} \\ -r_i^{\mathrm{us}} \Delta t \le P_{i,t}^{\mathrm{th},0} - P_{i,t-1}^{\mathrm{th},0} \le r_i^{\mathrm{up}} \Delta t \\ R_{i,t}^{\mathrm{gu}} = U_i^{\mathrm{th}} \Delta t, R_{i,t}^{\mathrm{gd}} = D_i^{\mathrm{th}} \Delta t \end{cases} \tag{A-3}$$

$$P_{wt,t}^{\mathrm{wp},0} = P_{wt,t}^{\mathrm{wp,p}} - \Delta P_{wt,t}^{\mathrm{wp},0} \tag{A-4}$$

$$\Delta P_{d,t}^{\mathrm{el},0} = P_{d,t}^{\mathrm{el,p}} - P_{d,t}^{\mathrm{el},0} \tag{A-5}$$

$$\begin{aligned} &\Big| \sum_{i \in I^{\mathrm{ts}}} G_{l-i} P_{i,t}^{\mathrm{th},0} + \sum_{w \in W^{\mathrm{ts}}} G_{l-w} P_{wt,t}^{\mathrm{wp},0} - \\ &\sum_{d \in D^{\mathrm{ts}}} G_{l-d} P_{d,t}^{\mathrm{el},0} - \sum_{k \in K} G_{l-k} P_{k,t}^{\mathrm{tu},0} \Big| < P_l^{\max} \end{aligned} \tag{A-6}$$

$$-P_{k,\max}^{\mathrm{tu}} \le P_{k,t}^{\mathrm{tu},0} \le P_{k,\max}^{\mathrm{tu}} \tag{A-7}$$

G_{l-i} is generation shift distribution factors, and $P_l^{\max}$ is power flow limit of line *l*.

C. Specific Expression of the (9)

$$\begin{cases} f_t^{\mathrm{buy},0} = \sum_{m \in B_k} c^{\mathrm{buy}} Q_{bu,t}^{\mathrm{buy},0} \\ f_t^{\mathrm{load},0} = \sum_{d \in D_k} c^{\mathrm{el}} \Delta P_{d,t}^{\mathrm{el},0} + \sum_{h \in H_k} c^{\mathrm{hl}} \Delta H_{h,t}^{\mathrm{hl},0} + \sum_{m \in M_k} c^{\mathrm{gl}} \Delta Q_{m,t}^{\mathrm{gl},0} \\ f_t^{\mathrm{wp},0} = \sum_{w \in W_k} c^{\mathrm{wp}} \Delta P_{w,t}^{\mathrm{wp},0} \\ \Delta f_t^{\mathrm{eq},r} = \sum_{g \in E_k^{\mathrm{eq}}} d_g^{\mathrm{eq}-} \Delta P_{g,t}^{\mathrm{eq}-} + d_g^{\mathrm{eq}+} \Delta P_{g,t}^{\mathrm{eq}+} \\ \Delta f_t^{\mathrm{buy},r} = \sum_{m \in B_k} c^{\mathrm{buy}+} \Delta Q_{m,t}^{\mathrm{buy}+} + c^{\mathrm{buy}-} \Delta Q_{bu,t}^{\mathrm{buy}-} \\ f_t^{\mathrm{load},r} = \sum_{d \in D_k} d^{\mathrm{el}} \Delta P_{d,t}^{\mathrm{el},r} + \sum_{h \in H_k} d^{\mathrm{hs}} \Delta H_{h,t}^{\mathrm{s},r} + \sum_{m \in M_k} d^{\mathrm{gl}} \Delta Q_{m,t}^{\mathrm{gl},r} \\ f_t^{\mathrm{wp},r} = \sum_{w \in W_k} d^{\mathrm{wp}} \Delta P_{w,t}^{\mathrm{wp},r} \end{cases} \tag{A-8}$$

'eq' represents the general notation for all types of equipment symbols in the system, with $eq \in \{\mathrm{GT,GB,EB}\}$. $c^{(\cdot)}/d^{(\cdot)}$ denotes the day-ahead/intraday corresponding penalty coefficient for each item.

D. UIES$_k$ Day-Ahead Distribution Network Constraints

The constraints mainly consist of distribution network constraints (A-9) and load-related constraints (A-10).

$$\begin{cases} \begin{cases} P_{j,t}^{\mathrm{in},0} = \sum_{i \in fr(j)} P_{ij,t}^{\mathrm{us},0} - \sum_{k \in to(j)} P_{jk,t}^{\mathrm{us},0} \\ P_{j,t}^{\mathrm{in},0} = \sum_{g \in j} P_{g,t}^{\mathrm{gt},0} + \sum_{h \in j} P_{wu,t}^{\mathrm{wp},0} - \sum_{z \in j} P_{e,t}^{\mathrm{eb},0} \\ \qquad - \sum_{k \in j} P_{k,t}^{\mathrm{ut},0} - \sum_{u \in j} P_{ku,k,t}^{\mathrm{uu},0} - P_{j,t}^{\mathrm{el},0} \end{cases} \\ U_{i,t}^0 - (r_{ij} P_{ij,t}^0 + x_{ij} Q_{ij,t}) / U_{\mathrm{r}} = U_{j,t}^0 \\ U_{j,\min} \le U_{j,t}^0 \le U_{j,\max} \\ 0 \le P_{ij,t}^0 \le P_{ij,\max} \end{cases} \tag{A-9}$$

$$\begin{cases} \Delta P_{j,t}^{\mathrm{elmin}} \le \Delta P_{j,t}^{\mathrm{el},0} \le \Delta P_{j,t}^{\mathrm{elmax}} \\ P_{j,t}^{\mathrm{el},0} = P_{j,t}^{\mathrm{el,p}} - \Delta P_{j,t}^{\mathrm{el},0} \end{cases} \tag{A-10}$$

$m \in j$ indicates that gas turbine m is connected to node j. $to(j)$ represents the set of starting nodes with node j as the end node branch, while $fr(j)$ denotes the set of terminal nodes with node j as the starting node branch.

E. UIES$_k$ Day-Ahead Equipment Constraints

This part includes constraints on conversion equipment (A-11) and production equipment (A-12). The equipment that converts primary energy into secondary energy is referred to as energy production equipment (GT and GB), which satisfies upper and lower limits of output, ascent and descent slopes. Equipment then used convert secondary energy to secondary energy is categorized as energy conversion equipment (WHB and EB).

$$\begin{cases} H_{e,t}^{\mathrm{eb},0} = \eta_e^{\mathrm{eb}} P_{e,t}^{\mathrm{eb},0} \\ H_{g,t}^{\mathrm{whb,in},0} = \dfrac{P_{g,t}^{\mathrm{gt},0}(1-\eta_g^{\mathrm{gt}})}{\eta_g^{\mathrm{gt}}} \eta_g^{\mathrm{whb}} \\ P_{w,t}^{\mathrm{tra,min}} \le P_{w,t}^{\mathrm{tra},0} \le P_{w,t}^{\mathrm{tra,max}} \end{cases} \tag{A-11}$$

$$\begin{cases} P_{v,t}^{\mathrm{pro,min}} \le P_{v,t}^{\mathrm{pro},0} \le P_{v,t}^{\mathrm{pro,max}} \\ -r_{v,t}^{\mathrm{pro,up}} \le P_{v,t}^{\mathrm{pro},0} - P_{v,t-1}^{\mathrm{pro},0} \le r_{v,t}^{\mathrm{pro,up}} \\ R_{v,t}^{\mathrm{prou},0} = U_{v,t}^{\mathrm{pro}} \Delta t \\ R_{v,t}^{\mathrm{prod},0} = D_{v,t}^{\mathrm{pro}} \Delta t \\ \begin{cases} P_{g,t}^{\mathrm{gt},0} = \eta_g^{\mathrm{gt}} L_{\mathrm{NG}} Q_{g,t}^{\mathrm{gt},0} \\ H_{b,t}^{\mathrm{gb},0} = \eta_b^{\mathrm{gb}} L_{\mathrm{NG}} Q_{b,t}^{\mathrm{gb},0} \end{cases} \end{cases} \tag{A-12}$$

'tra' is the unified representation of energy conversion equipment. 'pro' is the unified representation of energy production equipment, 'v' is the corresponding equipment number. 'LNG' is the calorific value of natural gas. Specifically, the WHB's input heat power generates heat for GT operation.

F. MP and SP of the (30)

MP:

$$\begin{cases} \min\limits_{x^{\mathrm{ts}},x_k^{\mathrm{us}},y^{\mathrm{ts}},y_k^{\mathrm{us}},\eta^{\mathrm{ts}},\eta_k^{\mathrm{us}}} f^{\mathrm{ts}1}(\boldsymbol{x}^{\mathrm{ts}}) + \sum_k f_k^{\mathrm{us}1}(\boldsymbol{x}_k^{\mathrm{us}}) + \eta^{\mathrm{ts}} + \sum_k \eta_k^{\mathrm{us}} \\ \text{s.t.} \quad \eta^{\mathrm{ts}} \ge f^{\mathrm{ts}2}(\boldsymbol{y}^{\mathrm{ts}}, \boldsymbol{\mu}^{\mathrm{ts}}, \boldsymbol{p}^{\mathrm{ts}*}) \\ \qquad \eta_k^{\mathrm{us}} \ge f_k^{\mathrm{us}2}(\boldsymbol{y}_k^{\mathrm{us}}, \boldsymbol{\mu}_k^{\mathrm{us}}, \boldsymbol{p}_k^{\mathrm{us}*}) \quad \forall k \\ \qquad G(\boldsymbol{x}^{\mathrm{ts}}, \boldsymbol{y}^{\mathrm{ts}}, \boldsymbol{\mu}^{\mathrm{ts}}) \le \boldsymbol{g} \\ \qquad H(\boldsymbol{x}_k^{\mathrm{us}}, \boldsymbol{y}_k^{\mathrm{us}}, \boldsymbol{\mu}_k^{\mathrm{us}}) \le \boldsymbol{h} \quad \forall k \end{cases} \tag{A-13}$$

SP:

$$\begin{cases} \max\limits_{p^{\mathrm{ts}}} \min\limits_{y^{\mathrm{ts}}} f^{\mathrm{ts}2}(\boldsymbol{y}^{\mathrm{ts}}, \boldsymbol{\mu}^{\mathrm{ts}}, \boldsymbol{p}^{\mathrm{ts}}) + \sum_k \max\limits_{p_k^{\mathrm{us}}} \min\limits_{y_k^{\mathrm{us}}} f_k^{\mathrm{us}2}(\boldsymbol{y}_k^{\mathrm{us}}, \boldsymbol{\mu}_k^{\mathrm{us}}, \boldsymbol{p}_k^{\mathrm{us}}) \\ \text{s.t.} \quad L(\boldsymbol{x}^{\mathrm{ts}*}, \boldsymbol{y}^{\mathrm{ts}}, \boldsymbol{\mu}^{\mathrm{ts}}) \le \boldsymbol{l} \\ \qquad M(\boldsymbol{x}_k^{\mathrm{us}*}, \boldsymbol{y}_k^{\mathrm{us}}, \boldsymbol{\mu}_k^{\mathrm{us}}) \le \boldsymbol{m} \quad \forall k \\ \qquad \boldsymbol{p}^{\mathrm{ts}} \in \Omega^{\mathrm{ts}}, \quad \boldsymbol{p}_k^{\mathrm{us}} \in \Omega_k^{\mathrm{us}} \end{cases} \tag{A-14}$$

Here, $\boldsymbol{L}$ and $\boldsymbol{l}$ represent the constraints related to the TS' SP. $\boldsymbol{M}$ and $\boldsymbol{m}$ stand for the constraints related to the UIES$_k$' SP. The SPs are independent of each other without coupled variable constraints, so they can be solved in parallel by each ESO.

Using TS as an example, the MP simultaneously optimizes variables such as the thermal power unit plan and wind power plan for both day-ahead and intraday scheduling phases. These optimized values are passed to the SP. The SP revolves solely around the day-ahead plan given in the MP and optimizes it for various scenarios during the intraday phase. Once optimized, the adverse distribution and its corresponding constraints are added back to the MP.

G. Comparison of Different Robust methods

Table I Comparison of three methods

Cost($)	Approach		
	DRO	RO	SO
Phase I	67983.7	67995.0	67982.2
Phase II	8467.4	10452.4	8395.1
Total	76451.1	78447.4	76377.3

H. Flowchart of solution methodology

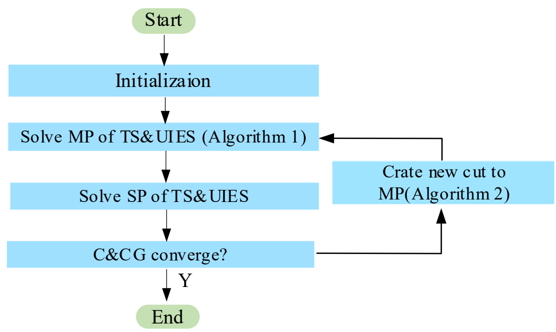

Fig. A-1 Flowchart of solution methodology