Correction des exercices 2;3;4;5;6 et 7
Série ANALYSE II

Exercice 2:

1)
$$\int \cos(x)^3\, dx = \int \cos(x)\cdot(1-\sin(x)^2)\, dx = \int \cos(x) - \cos(x)\cdot\sin(x)^2 dx = -\frac{1}{3}\sin(x)^3 + \sin(x) + c:$$

(la forme $u'\cdot u^n$ avec $u = \sin(x)$:)

deuxième méthode (Linéarisation)

$$\cos(x) = \frac{(\exp(ix) + \exp(-ix))}{2}:$$

$expand((a+b)^3)$

$$a^3 + 3\, a^2\, b + 3\, a\, b^2 + b^3 \qquad \textbf{(1)}$$

$$\left(\frac{e^{i\cdot x} + e^{-i\cdot x}}{2}\right)^3 = \frac{1}{8}\left[\, (e^{ix})^3 + 3\cdot(e^{ix})^2\cdot e^{-i\cdot x} + 3\cdot e^{ix}\cdot(e^{-ix})^2 + (e^{-i\cdot x})^3\right]$$
$$= \frac{1}{8}\left[\, (e^{ix})^3 + 3\cdot e^{ix} + 3\cdot e^{-ix} + (e^{-i\cdot x})^3\right]$$
$$= \frac{1}{8}\, e^{i\cdot 3\cdot x} + \frac{3}{8}\, e^{ix} + \frac{3}{8}\cdot e^{-ix} + \frac{1}{8}\cdot e^{-i\cdot 3\cdot x}$$
$$= \frac{1}{8}\cdot(e^{i\cdot 3\cdot x} + e^{-i\cdot 3\cdot x}) + \frac{3}{8}\cdot(e^{i\cdot x} + e^{-i\cdot x})$$
$$= \frac{1}{8}\cdot 2\cdot\cos(3\cdot x) + \frac{3}{8}\cdot 2\cdot\cos(x)$$
$$= \frac{1}{4}\cdot\cos(3\cdot x) + \frac{3}{4}\cdot\cos(x):$$

$$\cos(x)^3 = \frac{1}{4}\cos(3\, x) + \frac{3}{4}\cos(x)$$
$$\int \cos(x)^3\, dx = \int \frac{1}{4}\cos(3\, x) + \frac{3}{4}\cos(x)\, dx = \frac{1}{12}\sin(3\, x) + \frac{3}{4}\sin(x) : :$$

vérifions les deux résultats (avec Maple)

$combine\left(-\frac{1}{3}\sin(x)^3 + \sin(x),\ sincos\right)$

$$\frac{1}{12}\sin(3\, x) + \frac{3}{4}\sin(x) \qquad \textbf{(2)}$$

2)

$$\int \sin(x)^4\,dx = \int (\sin(x)^2)^2\,dx = \int \left(\frac{1-\cos(2\cdot x)}{2}\right)^2 dx = \int \frac{1}{4}(1-2\cdot\cos(2\cdot x)+\cos(2\cdot x)^2)\,dx$$

$$= \int \frac{1}{4} - \frac{1}{2}\cdot\cos(2\cdot x) + \frac{1}{4}\cdot\cos(2\cdot x)^2\,dx$$

$$= \int \frac{1}{4} - \frac{1}{2}\cdot\cos(2\cdot x) + \frac{1}{4}\cdot\frac{1+\cos(4\cdot x)}{2}\,dx$$

$$= \int \frac{1}{4} - \frac{1}{2}\cdot\cos(2\cdot x) + \frac{1}{8} + \frac{\cos(4\cdot x)}{8}\,dx$$

$$= \int \frac{3}{8} - \frac{1}{2}\cdot\cos(2\cdot x) + \frac{\cos(4\cdot x)}{8}\,dx$$

$$= \frac{3}{8}\cdot x - \frac{1}{8}\cdot\sin(2\cdot x) + \frac{1}{32}\cdot\sin(4\cdot x) + c:$$

3)

$$\int \cosh(x)^3\,dx = \int \cosh(x)\cdot(1+\sinh(x)^2)\,dx = \int \cosh(x) + \cosh(x)\cdot\sinh(x)^2 dx = \frac{1}{3}\sinh(x)^3 + \sinh(x) + c:$$

4) $\int \sin(x)\cdot\cos(x)^2\,dx = -\frac{1}{3}\cos(x)^3 + c:$ (la forme $u'\cdot u^n$ avec $u=\cos(x)$:)

5) $$\int \sin(x)^2\cdot\cos(x)^3\,dx = \int \sin(x)^2\cdot\cos(x)^2\cdot\cos(x)\,dx = \int \sin(x)^2\cdot(1-\sin(x)^2)\cdot\cos(x)\,dx$$

$$= \int (\sin(x)^2 - \sin(x)^4)\cdot\cos(x)\,dx$$

$$= \int \sin(x)^2\cdot\cos(x) - \sin(x)^4\cdot\cos(x)\,dx$$

$$= \frac{1}{3}\cdot\sin(x)^3 - \frac{1}{5}\cdot\sin(x)^5 + c:$$

6) $$\int \sin(x)^3\cdot\cos(x)^3\,dx = \int (\sin(x)\cdot\cos(x))^3 dx$$

$$= \int \left(\frac{1}{2}\cdot\sin(2\cdot x)\right)^3 dx$$

$$= \int \frac{1}{8}\cdot\sin(2\cdot x)^3 dx$$

$$= \int \frac{1}{8}\cdot\sin(2\cdot x)^2\cdot\sin(2\cdot x)\,dx$$

$$= \int \frac{1}{8}\cdot(1-\cos(2\cdot x)^2)\cdot\sin(2\cdot x)\,dx$$

$$=\int \frac{1}{8}\cdot\sin(2\cdot x)-\frac{1}{8}\cos(2\cdot x)^2\cdot\sin(2\cdot x)\,\mathrm{d}x:$$

, on pose u=cos(2*x) d'où du=-2* sin(2*x)dx

$$\int \sin(x)^3\cdot\cos(x)^3\,\mathrm{d}x=-\frac{1}{16}\cdot\cos(2\cdot x)+\int\frac{1}{16}\,u^2\mathrm{d}u$$

$$=-\frac{1}{16}\cdot\cos(2\cdot x)+\frac{1}{48}\cdot\cos(2\cdot x)^3+c:$$

$$combine\left(-\frac{1}{16}\cdot\cos(2\cdot x)+\frac{1}{48}\cdot\cos(2\cdot x)^3, sincos\right)$$

$$-\frac{3}{64}\cos(2\,x)+\frac{1}{192}\cos(6\,x) \qquad \textbf{(3)}$$

7) $\int_0^x t^2\cdot\exp(t)\,\mathrm{d}t=e^x x^2-2\,e^x x+2\,e^x-2:$

8) $\int_0^x t^2\cdot\ln(t)\,\mathrm{d}t=\frac{1}{3}\,x^3\ln(x)-\frac{1}{9}\,x^3:$

9) $\int_0^x t^2\cdot\sin(t)\,\mathrm{d}t=-2-x^2\cos(x)+2\cos(x)+2\,x\sin(x):$

$with(IntegrationTools):$

Exercice 3:

1) $I1:=\int\arcsin(t)\,\mathrm{d}t:$

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\> $Parts(I1, \arcsin(t))$

$$t\arcsin(t)+\sqrt{-t^2+1} \qquad \textbf{(4)}$$

2) $I2:=\int\arctan(t)\,\mathrm{d}t=t\arctan(t)-\frac{1}{2}\ln\left(t^2+1\right)+c:$

$Parts(I2, \arctan(t))$

$$t\arctan(t)-\frac{1}{2}\ln(t^2+1) \tag{5}$$

3) $I3 := \int t\cdot\arctan(t)\,\mathrm{d}t = \frac{1}{2}\arctan(t)\,t^2-\frac{1}{2}\,t+\frac{1}{2}\arctan(t)+c:$

$Parts(I3,\ t)$

$$\frac{1}{2}\arctan(t)\,t^2-\frac{1}{2}\,t+\frac{1}{2}\arctan(t) \tag{6}$$

4) $I4 := \int \frac{\mathrm{t}}{\cos(t)^2}\,\mathrm{d}t = \tan(t)+\ln(|\cos(t)|)+c:$

$Parts(I4,\ \mathrm{t})$

$$t\tan(t)+\ln(\cos(t)) \tag{7}$$

5) $I5 := \int \frac{\ln(t)}{\mathrm{t}^n}\,\mathrm{d}t:$

$simplify(Parts(I5,\ \mathrm{t}^{-n}))$

$$-\frac{t^{-n+1}\,(\ln(t)\,n-\ln(t)+1)}{(n-1)^2} \tag{8}$$

Exercice 4 :

1) $\int \frac{1}{x\,(x-1)}\,\mathrm{d}x = \int -\frac{1}{x}+\frac{1}{x-1}\,\mathrm{d}x = -\ln(|x|)+\ln\left(\left|x-1\right|\right)+c:$

2) $\frac{1}{x\,(x-1)^2} = \frac{1}{(x-1)^2}+\frac{1}{x}-\frac{1}{x-1}:$

$\int \frac{1}{x\,(x-1)^2}\,\mathrm{d}x = \ln(|x|)-\frac{1}{x-1}-\ln\left(\left|x-1\right|\right)+c:$

3) $\frac{1}{x\cdot(x^2+1)} = \frac{1}{x}-\frac{x}{x^2+1}:$

$\int \frac{1}{x\cdot(x^2+1)}\,\mathrm{d}x = \int \frac{1}{x}-\frac{x}{x^2+1}\,\mathrm{d}x = \ln(|x|)-\frac{1}{2}\ln(x^2+1)+c:$

4) $\displaystyle\int \frac{x}{x^2+2}\,\mathrm{d}x = \frac{1}{2}\int \frac{2\cdot x}{x^2+2}\,\mathrm{d}x \frac{1}{2}\ln(x^2+2)+c:$

5) $$\int \frac{x^2}{x^2+2}\,\mathrm{d}x = \int 1-\frac{2}{x^2+2}\,\mathrm{d}x = x-2\cdot\int \frac{1}{2\cdot\left(\frac{x^2}{2}+1\right)}\,\mathrm{d}x = x-\int \frac{1}{\left(\frac{x}{\sqrt{2}}\right)^2+1}\,\mathrm{d}x:$$

on pose $u=\dfrac{x}{\sqrt{2}}$: $du=\dfrac{1}{\sqrt{2}}dx$: *d'où* $dx=\sqrt{2}\,du$:

$$\int \frac{x^2}{x^2+2}\,\mathrm{d}x = x-\int \frac{\sqrt{2}}{u^2+1}\,\mathrm{d}u = x-\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)+c:$$

6) $$\int \frac{1}{x^2\cdot(x^2-1)^2}\,\mathrm{d}x = \int \frac{3}{4\,(x+1)}+\frac{1}{4\,(x-1)^2}+\frac{1}{x^2}-\frac{3}{4\,(x-1)}+\frac{1}{4\,(x+1)^2}\,\mathrm{d}x$$
$$=\frac{3}{4}\ln\left(\left|x+1\right|\right)-\frac{1}{4\,(x-1)}-\frac{1}{x}-\frac{3}{4}\ln(|x-1|)-\frac{1}{4\,(x+1)}+c:$$

7) $$\int \frac{x+1}{(x^2+1)^2}\,\mathrm{d}x = \int \frac{x}{(x^2+1)^2}\,\mathrm{d}x+\int \frac{1}{(x^2+1)^2}\,\mathrm{d}x$$
$$=\frac{1}{2}\int \frac{2\cdot x}{(x^2+1)^2}\,\mathrm{d}x+J \ \ avec\ J=\int \frac{1}{(x^2+1)^2}\,\mathrm{d}x$$
$$=-\frac{1}{2}\cdot\frac{1}{x^2+1}+J:$$

Calcul de J:

une primitive de $\displaystyle\int \frac{1}{1+x^2}\,\mathrm{d}x=\arctan(x)$:

on pose $u(x)=\dfrac{1}{1+x^2}$: $u'(x)=-\dfrac{2\,x}{(1+x^2)}$:

$v'(x)=1$: $v(x)=x$:

$$\arctan(x)=\left[\frac{x}{1+x^2}\right]-\int -\frac{2\,x^2}{(1+x^2)^2}\,\mathrm{d}x=\frac{x}{1+x^2}+\int \frac{2(x^2+1)-2}{(1+x^2)^2}\,\mathrm{d}x$$
$$=\frac{x}{1+x^2}+2\int \frac{1}{1+x^2}\,\mathrm{d}x-2\int \frac{1}{(1+x^2)^2}\,\mathrm{d}x$$
$$=\frac{x}{1+x^2}+2\cdot\arctan(x)-2\cdot J:$$

d'où:

$$2\cdot J=\frac{x}{1+x^2}+2\cdot\arctan(x)-\arctan(x):$$

une primitive de J est donnée par :

$$J=\frac{1}{2}\cdot\frac{x}{1+x^2}+\frac{1}{2}\arctan(x):$$

conclusion :

$$\int\frac{x+1}{(x^2+1)^2}\,dx=-\frac{1}{2}\cdot\frac{1}{x^2+1}+\frac{1}{2}\cdot\frac{x}{1+x^2}+\frac{1}{2}\arctan(x)+c:$$

8) $$\int\frac{1}{(x+2)\cdot(x^2+2\cdot x+5)}\,dx=\int\frac{\frac{1}{5}}{(x+2)}-\frac{\frac{1}{5}\cdot x}{x^2+2\,x+5}\,dx$$

$$=\frac{1}{5}\ln(|x+2|)-\frac{1}{5}\int\frac{\mathrm{x}}{x^2+2\,x+5}dx$$

$$=\frac{1}{5}\ln(|x+2|)-\frac{1}{5}\int\frac{\frac{1}{2}\cdot(2\cdot x+2\ )-1}{x^2+2\,x+5}dx$$

$$=\frac{1}{5}\ln(|x+2|)-\frac{1}{10}\int\frac{2\cdot x+2}{x^2+2\,x+5}\,dx+\frac{1}{5}\int\frac{1}{x^2+2\,x+5}\,dx$$

$$=\frac{1}{5}\ln(|x+2|)-\frac{1}{10}\ln(x^2+2\cdot x+5)+\frac{1}{5}\int\frac{1}{x^2+2\,x+5}\,dx$$

$$=\frac{1}{5}\ln(|x+2|)-\frac{1}{10}\ln(x^2+2\cdot x+5)+\frac{1}{5}\int\frac{1}{(x+1)^2+4}\,dx$$

$$=\frac{1}{5}\ln(|x+2|)-\frac{1}{10}\ln(x^2+2\cdot x+5)+\frac{1}{5}$$

$$\int\frac{1}{4\left(\frac{(x+1)^2}{4}+1\right)}\,dx$$

$$=\frac{1}{5}\ln(|x+2|)-\frac{1}{10}\ln(x^2+2\cdot x+5)+\frac{1}{20}\int\frac{1}{\left(\frac{x+1}{2}\right)^2+1}\,dx$$

$on\ pose\ u = \frac{x+1}{2}$: $d'où\ du = \frac{1}{2}dx$: $c'est\ à\ dire\ dx = 2 \cdot du$:

$$\int \frac{1}{(x+2)\cdot(x^2+2\cdot x+5)}\,dx = \frac{1}{5}\ln(|x+2|) - \frac{1}{10}\ln(x^2+2\cdot x+5) + \frac{1}{20}\int \frac{2}{1+u^2}\,du$$

$$= \frac{1}{5}\ln(|x+2|) - \frac{1}{10}\ln(x^2+2\cdot x+5) + \frac{1}{10}\arctan\left(\frac{x+1}{2}\right) + c:$$

EXERCICE 5

1) $I := \int \frac{\left(x+\sqrt{x^2+1}\right)}{\sqrt{x^2+1}}\,dx$

on prendera le changement de variable $t = x + \sqrt{x^2+1}$ d'où

$dt = 1 + \frac{x}{\sqrt{x^2+1}}dx = \frac{\sqrt{x^2+1}+x}{\sqrt{x^2+1}}dx$:

c à d

$dx = \frac{\sqrt{x^2+1}}{\sqrt{x^2+1}+x}dt = \frac{\sqrt{x^2+1}}{t}dt$

d'où $I = \int \frac{t}{\sqrt{x^2+1}}\,\frac{\sqrt{x^2+1}}{t}\,dt = \int 1\,dt = x + \sqrt{x^2+1} + c$:

> $int\left(\frac{x+\mathrm{sqrt}(x^2+1)}{\mathrm{sqrt}(x^2+1)}, x\right)$

$$\sqrt{x^2+1} + x \qquad (9)$$

2) $J = \int \frac{1}{x\cdot(x^2+3)}\,dx$: on poste $x = \frac{1}{t}$ d'où $dx = -\frac{1}{t^2}dt$:

$$J = \int \frac{1}{x\cdot(x^2+3)}\,dx = \int \frac{1}{\frac{1}{t}\cdot\left(\frac{1}{t^2}+3\right)}\cdot\left(-\frac{1}{t^2}\right)dt = \int -\frac{t}{3\,t^2+1}\,dt = -\frac{1}{6}\cdot\int\frac{6\cdot t}{3\,t^2+1}\,dt =$$

$$-\frac{1}{6}\ln(3\,t^2+1) + c:$$

3)

$$K=\int \frac{1}{1+\cosh(x)}\,\mathrm{d}x=\int \frac{1}{1+\frac{\exp(x)+\exp(-x)}{2}}\,\mathrm{d}x=\int \frac{1}{\frac{2+\exp(x)+\exp(-x)}{2}}\,\mathrm{d}x=$$
$$\int \frac{2}{2+\exp(x)+\exp(-x)}\,\mathrm{d}x:$$

On prendra le changement de variable u=exp(x) ; du= exp(x)dx=udx

$$K=\int \frac{1}{1+\cosh(x)}\,\mathrm{d}x=\int \frac{1}{1+\frac{u+\frac{1}{u}}{2}}\cdot\frac{1}{u}\,\mathrm{d}u=\int \frac{2}{u^2+2u+1}\,\mathrm{d}u=\int \frac{2}{(1+u)^2}\,\mathrm{d}u=-\frac{2}{1+\mathrm{u}}$$
$$+c=-\frac{2}{1+\exp(x)}+c:$$

5) $R:=\int \frac{2}{\sinh(x)^3\cosh(x)}\,\mathrm{d}x:$

on utilisera le changement $u=\cosh(x)^2:$

$du=2\sinh(x)\cdot\cosh(x)dx:$

$$R:=\int \frac{2}{\sinh(x)^3\cosh(x)}\,\mathrm{d}x=\int \frac{2}{\sinh(x)^2\cdot\sinh(x)\cdot\cosh(x)}\,\mathrm{d}x=$$
$$\int \frac{2}{\left(\cosh(x)^2-1\right)\cdot\sinh(x)\cdot\cosh(x)}\,\mathrm{d}x=\int \frac{1}{(\mathrm{u}-1)\cdot\sinh(x)^2\cdot\cosh(x)^2}\,\mathrm{d}u:$$

$$R=\int \frac{1}{(\mathrm{u}-1)\cdot(u-1)\cdot u}\,\mathrm{d}u=\int \frac{1}{(\mathrm{u}-1)^2\cdot u}\,\mathrm{d}u=\int -\frac{1}{u-1}+\frac{1}{(u-1)^2}+\frac{1}{u}\,\mathrm{d}u=$$
$$-\ln\left(\left|\cosh(x)^2-1\right|\right)-\frac{1}{\cosh(x)^2-1}+\frac{1}{\cosh(x)^2}+c:$$

$$R:=-\ln\left(\cosh(x)^2-1\right)-\frac{1}{\cosh(x)^2-1}+\frac{1}{\cosh(x)^2}+c:$$

EXERCICE 7

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> $L := Int\left(\dfrac{1}{x+\mathrm{sqrt}(x-1)}, x\right)$

$$L := \int \frac{1}{x+\sqrt{-1+x}}\,\mathrm{d}x \tag{10}$$

> $with(IntegrationTools):$

> $Change(L, u=\mathrm{sqrt}(x-1))$

$$\int \frac{2\,u}{u^2+u+1}\,\mathrm{d}u \tag{11}$$

on pose le changement de variable $u=\sqrt{x-1}$; $u^2=|x-1|$;

On calculera une primitive dans l'intervalle $[1\ ;\ +\infty[$

sur cette intervalle $u^2 = x-1$ *donc* $x = u^2+1$

$$du = \frac{1}{2\cdot\sqrt{x-1}}\,dx = \frac{1}{2\cdot u}\cdot dx$$

d'où $dx = 2\cdot u\cdot du$

$$\int \frac{1}{x+\sqrt{x-1}}\,\mathrm{d}x = \int \frac{2\cdot u}{u+u^2+1}\,\mathrm{d}u = \int \frac{2\cdot u+1-1}{u^2+u+1}\,\mathrm{d}u = \int \frac{2\cdot u+1}{u^2+u+1}\,\mathrm{d}u - \int \frac{1}{u^2+u+1}\,du$$

$$= \ln(u^2+u+1) - \int \frac{1}{\left(u+\frac{1}{2}\right)^2+\frac{3}{4}}\,\mathrm{d}u$$

$$= \ln(u^2+u+1) - \int \frac{1}{\frac{3}{4}\left[\frac{\left(u+\frac{1}{2}\right)^2}{\frac{3}{4}}+1\right]}\,\mathrm{d}u$$

$$= \ln(u^2+u+1) - \frac{4}{3}\int \frac{1}{\left[\left(\frac{u+\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right)^2+1\right]}\,du$$

on pose $v = \frac{u+\frac{1}{2}}{\frac{\sqrt{3}}{2}}$, $dv = \frac{2}{\sqrt{3}}du$, $d'où\ du = \frac{\sqrt{3}}{2}dv.$

$$\int \frac{1}{x+\sqrt{x-1}}\,dx = \ln(u^2+u+1) - \frac{4}{3}\int \frac{\frac{\sqrt{3}}{2}}{[v^2+1]}\,dv$$

$$= \ln(u^2+u+1) - \frac{2}{\sqrt{3}}\arctan(v) + c$$

$$= \ln(x-1+\mathrm{sqrt}(x-1)+1) - \frac{2}{\sqrt{3}}\arctan\left(\frac{\mathrm{sqrt}(x-1)+\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right) + c$$

$$= \ln(x+\mathrm{sqrt}(x-1)) - \frac{2}{\sqrt{3}}\arctan\left(\frac{2}{\sqrt{3}}\mathrm{sqrt}(x-1) + \frac{1}{\sqrt{3}}\right) + c$$

>

> $Int\left(\frac{1}{x+\sqrt{x-1}}, x = 1..2\right) = int\left(\frac{1}{x+\sqrt{x-1}}, x = 1..2\right)$

$$\int_1^2 \frac{1}{x+\sqrt{-1+x}}\,dx = -\frac{1}{9}\pi\sqrt{3} + \ln(3) \qquad \textbf{(12)}$$

$$\int \frac{1}{\sqrt{4x^2+9}}\,dx = \int \frac{1}{\sqrt{4\left(x^2+\frac{9}{4}\right)}}\,dx = \frac{1}{2}\int \frac{1}{\sqrt{x^2+\frac{9}{4}}}\,dx = \frac{1}{2}\int \frac{1}{\sqrt{\frac{9}{4}\left(\frac{x^2}{\frac{9}{4}}+1\right)}}\,dx$$

$$= \frac{\frac{1}{2}}{\frac{3}{2}} \int \frac{1}{\sqrt{\left(\frac{x}{\frac{3}{2}}\right)^2 + 1}} dx$$

$$= \frac{1}{3} \int \frac{1}{\sqrt{\left(\frac{2}{3}x\right)^2 + 1}} dx$$

on pose $u = \frac{2}{3}x$ *d'où* $du = \frac{2}{3} dx$ *c'est à dire* $dx = \frac{3}{2} \cdot du$

$$\int \frac{1}{\sqrt{4x^2 + 9}} \, \mathrm{d}x = \frac{1}{3} \int \frac{\frac{3}{2}}{\sqrt{u^2 + 1}} du = \frac{1}{2} \operatorname{arcsinh}(u) + c = \frac{1}{3} \operatorname{arcsinh}\left(\frac{2}{3}x\right) + c$$

on cherchera une primitive dans l'intervalle $\left]-\infty, 1\right]$

on pose $u = \sqrt{1-x}$, $du = -\frac{1}{2\sqrt{1-x}} dx = -\frac{1}{2u} dx$ *d'où* $dx = -2 \cdot u \cdot du$

$$\int \frac{1}{1 + \sqrt{1-x}} \, \mathrm{d}x = \int \frac{-2 \cdot u}{1 + u} \, \mathrm{d}u = \int -2 + \frac{2}{1+u} \, \mathrm{d}u = -2\sqrt{1-x} + 2 \ln\left(1 + \sqrt{1-x}\right) + c$$

\> $Int\left(\frac{1}{1+\sqrt{1-x}}, x = 0..1\right) = int\left(\frac{1}{1+\sqrt{1-x}}, x = 0..1\right)$

$$\int_0^1 \frac{1}{1 + \sqrt{1-x}} \, \mathrm{d}x = 2 - 2\ln(2) \tag{13}$$

\>

pour x dans l'intervalle $]-1; 1[$

$$\frac{1}{1-x} \cdot \sqrt{\frac{1-x}{1+x}} = \frac{1}{\sqrt{1-x^2}}$$

d'où $\int \frac{1}{1-x} \cdot \sqrt{\frac{1-x}{1+x}} \, \mathrm{d}x = \int \frac{1}{\sqrt{1-x^2}} \, \mathrm{d}x = \arcsin(x) + c$

$\int \sqrt{1-x^2}\, dx = \frac{1}{2}\, x\sqrt{1-x^2} + \frac{1}{2} \arcsin(x) + c$: on pose x=sin(t), pour

x *dans l'intervalle* $[-1;1]$ *on a* $t = \arcsin(x)$, $t \in \left[-\frac{\pi}{2}; \frac{\pi}{2}\right]$

en effet

$\int \sqrt{1-x^2}\, dx =$

$\int \sqrt{1-\sin(t)^2}\cos(t)\,dt = \int \sqrt{\cos(t)^2}\cos(t)\,dt = \int |\cos(t)|\cos(t)\,dt = \int \cos(t)^2 dt$, *car* $\cos(t)$

≥ 0 *pour tout* $t \in \left[-\frac{\pi}{2}; \frac{\pi}{2}\right]$.

$\int \sqrt{1-x^2}\, dx =$

$\int \frac{1+\cos(2\,t)}{2}\, dt = \frac{1}{2}t + \frac{1}{4}\sin(2\,t) + c = \frac{1}{2}\mathrm{t} + \frac{1}{4}\cdot 2\sin(t)\cos(t) + c = \frac{1}{2}\arcsin(x) + \frac{1}{2}x$

$\cdot\sqrt{1-x^2} + c$

$\int \sqrt{x^2-1}\, dx = \frac{1}{2}\, x\sqrt{x^2-1} - \frac{1}{2}\ln\left(x+\sqrt{x^2-1}\right) + c$:

(*Utiliser une intégration par partie*)

> $\int x^2\sqrt{x^2-1}\, dx = -\frac{1}{4}\, x\,(-x^2+1)^{3/2} + \frac{1}{8}\, x\sqrt{-x^2+1} + \frac{1}{8}\arcsin(x) + c$:

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> $\int x\cdot\sqrt{\frac{1-x}{1+x}}\, dx = \frac{1}{2}\, x\sqrt{1-x^2} - \frac{1}{2}\arcsin(x) - \sqrt{1-x^2} + c$:

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