Suite de l'ex 5.

ω

5. $Y'' + 2Y' = \cos(2x) + x\sin(2x)$
6. $Y'' + Y' = \cos(2x) + \sin(x)$
7. $Y'' - 2Y' + Y = xe^{-x} + \sin(x)$

(5) $y'' + 2y' = \cos 2x + x \sin 2x$

- Equation Homogène $y'' + 2y' = 0$ (E_h)

$E_c : r^2 + 2r = 0 \iff r(r+2) = 0$

$r_1 = 0$

$r_2 = -2$

Les solutions de (E_h) : $y_h(x) = Ae^{0.x} + Be^{-2x}$ $A, B \in \mathbb{R}$

$[y_h(x) = A + Be^{-2x}]$ $A, B \in \mathbb{R}$

- Solution particulière : (E) $y'' + 2y' = \cos 2x + x \sin 2x$

$= P_1 \cos 2x + P_2 \sin 2x$

avec $\begin{cases} P_1(x) = 1 \\ P_2(x) = x \end{cases}$ $\quad \omega = 2$ $\quad \phi = 0$

$i\omega = i \times 2 = 2i$ n'est pas solution de E_c donc on prendra

$y_p(x) = \overline{Q_1(x)} \cos 2x + \overline{Q_2(x)} \sin 2x$

avec Q_1, Q_2 deux polynômes tels que

$\deg Q_1 = \deg Q_2 = \max\{\underbrace{\deg P_1}_{0}, \underbrace{\deg P_2}_{1}\} = 1$

D'où $y_p(x) = \overbrace{(\alpha_1 x + \beta_1)}^{Q_1} \cos 2x + \underbrace{(\alpha_2 x + \beta_2)}_{Q_2} \sin 2x$

? α_1, β_1 ? α_2, β_2

$$y_p'(x) = \alpha_1 \cos 2x - 2\sin 2x\,(\alpha_1 x + \beta_1) + \alpha_2 \sin 2x + 2\cos 2x\,(\alpha_2 x + \beta_2)$$

(2) $$y_p'(x) = (\alpha_1 + 2\beta_2 + 2\alpha_2 x)\cos 2x + (\alpha_2 - 2\beta_1 - 2\alpha_1 x)\sin 2x$$

$$y_p''(x) = 2\alpha_2 \cos 2x - 2\sin 2x\,(\alpha_1 + 2\beta_2 + 2\alpha_2 x) + $$
$$-2\alpha_1 \sin 2x + 2\cos 2x\,(\alpha_2 - 2\beta_1 - 2\alpha_1 x)$$

$$y_p''(x) = (4\alpha_2 - 4\beta_1 - 4\alpha_1 \cdot x)\cos 2x + (-4\alpha_1 - 4\beta_2 - 4\alpha_2 x)\sin 2x$$

y_p est une solution de (E) $\Leftrightarrow$

$$y_p''(x) + 2y_p'(x) = \cos 2x + x\sin 2x$$

$$(2\alpha_1 + 4\beta_2 + 4\alpha_2 x + 4\alpha_2 - 4\beta_1 - 4\alpha_1 x)\cos 2x + (2\alpha_2 - 4\beta_1 - 4\alpha_1 x + -4\alpha_1 - 4\beta_2 - 4\alpha_2 x)\sin 2x = \cos 2x + x\sin 2x$$

$$\Leftrightarrow \begin{cases} (1)\ 4\alpha_2 - 4\alpha_1 = 0 \\ (2)\ 2\alpha_1 + 4\beta_2 + 4\alpha_2 - 4\beta_1 = 1 \\ (3)\ -4\alpha_1 - 4\alpha_2 = 1 \\ (4)\ 2\alpha_2 - 4\beta_1 - 4\alpha_1 - 4\beta_2 = 0 \end{cases} \Leftrightarrow \begin{cases} \alpha_1 = -\frac{1}{8} \\ \alpha_2 = -\frac{1}{8} \\ -8\beta_1 = 1 + 2\alpha_1 - 6\alpha_2 = \frac{3}{2} \end{cases}$$

$$+4\beta_2 = 2\alpha_2 - 4\beta_1 - 4\alpha_1$$
$$= -\frac{2}{8} + \frac{6}{8} + \frac{4}{8} = 1$$

$$\Leftrightarrow \begin{cases} \alpha_1 = -\frac{1}{8} \\ \alpha_2 = -\frac{1}{8} \\ \beta_1 = -\frac{3}{16} \\ \beta_2 = \frac{1}{4} \end{cases}$$

$\underline{\underline{d}}$ $$y_p(x) = \left(-\frac{1}{8}x - \frac{3}{16}\right)\cos 2x + \left(-\frac{1}{8}x + \frac{1}{4}\right)\sin 2x \ \boxtimes$$

$\underline{\text{cl}}$ Les solutions de (E) sont :

$$y(x) = A + Be^{-2x} + \left(-\frac{1}{8}x - \frac{3}{16}\right)\cos 2x + \left(-\frac{1}{8}x + \frac{1}{4}\right)\sin 2x$$

$A, B \in \mathbb{R}$

avec Maple.

Rq $\left(B = -\frac{C_1}{2}\ \checkmark\right)$

$$y(x) = \left(-\frac{3}{16} - \frac{1}{8}x\right)\cos(2x) + \left(-\frac{1}{8}x + \frac{1}{4}\right)\sin(2x) - \frac{1}{2}e^{-2x}\_C1 + \_C2$$

6) $y'' + y' = \cos 2x + \sin x$ $\quad$ $\underline{\text{Rq}}$

- E_H : $y'' + y' = 0$; E_c : $r^2 + r = 0 \Rightarrow \begin{matrix} r_1 = 0 \\ r_2 = -1 \end{matrix}$

Les solutions de E_H : sont : $y_h(x) = A + Be^{-x}$

- Solutions particulière :

$$\begin{cases} E_1 : y'' + y' = \cos 2x & ; P_1(x) = 1 \quad ; P_2(x) = 0 \quad \omega_1 = 2 \\ E_2 : y'' + y' = \sin x & ; R_1(x) = 0 \quad ; R_2(x) = 1 \quad \omega_2 = 1 \end{cases}$$

$\begin{cases} i\omega_1 = 2i \\ i\omega_2 = i \end{cases}$ ne sont pas solutions de E_c : $r^2 + r = 0$

on prendra $\begin{cases} y_{p_1}(x) = Q_1(x)\cos 2x + Q_2(x)\sin 2x & (\text{sol de } E_1) \\ \deg Q_1 = \deg Q_2 = 0 & \\ y_{p_2}(x) = S_1(x)\cos x + S_2(x)\sin x & (\text{sol de } E_2) \\ \deg S_1 = \deg S_2 = 0 & \end{cases}$

D'où $\begin{cases} y_{p_1}(x) = \alpha_1 \cos 2x + \beta_1 \sin 2x & (\text{sol de } E_1) \\ y_{p_2}(x) = \alpha_2 \cos x + \beta_2 \sin x & (\text{sol de } E_2) \end{cases}$

- $y'_{p_1}(x) = 2\beta_1 \cos 2x - 2\alpha_1 \sin(2x)$
- $y''_{p_1}(x) = -4\alpha_1 \cos 2x - 4\beta_1 \sin 2x$

y_{p_1} est sol de (E_1) $\Leftrightarrow$ $y''_{p_1} + y'_{p_1} = \cos 2x$

$\Leftrightarrow (2\beta_1 - 4\alpha_1)\cos 2x + (-2\alpha_1 - 4\beta_1)\sin 2x = \cos 2x$

$\Leftrightarrow \begin{cases} 2\beta_1 - 4\alpha_1 = 1 \\ -2\alpha_1 - 4\beta_1 = 0 \end{cases} \Leftrightarrow \begin{cases} -10\alpha_1 = 2 \\ \beta_1 = -\frac{1}{2}\alpha_1 \end{cases}$

$\Leftrightarrow \begin{cases} \alpha_1 = -\frac{1}{5} \\ \beta_1 = \frac{1}{10} \end{cases}$ cl$_1$: $y_{p_1}(x) = -\frac{1}{5}\cos 2x + \frac{1}{10}\sin 2x$

- $y'_{p_2}(x) = \beta_2 \cos x - \alpha_2 \sin x$

$y''_{p_2}(x) = -\alpha_2 \cos x - \beta_2 \sin x$

$y_{p_2}(x)$ est une sol de (E_2) $y''_{p_2}(x) + y'_{p_2}(x) = \sin x$

$\Leftrightarrow (\beta_2 - \alpha_2)\cos x + (-\alpha_2 - \beta_2)\sin x = \sin x$

$\Leftrightarrow \begin{cases} \beta_2 - \alpha_2 = 0 \\ -\alpha_2 - \beta_2 = 1 \end{cases} \Leftrightarrow \begin{cases} \alpha_2 = -\frac{1}{2} \\ \beta_2 = -\frac{1}{2} \end{cases}$

D'où $y_{p_2}(x) = -\frac{1}{2}\cos x - \frac{1}{2}\sin x$ $\boxtimes$

d Les solutions de (E)

$$y(x) = A + Be^{-x} - \frac{1}{5}\cos 2x + \frac{1}{10}\sin 2x - \frac{1}{2}\cos x - \frac{1}{2}\sin x$$

$A, B \in \mathbb{R}$.

$-C_1 = B$ $\quad$ A

$$y(x) = -\frac{1}{2}\sin(x) - \frac{1}{2}\cos(x) + \frac{1}{10}\sin(2x) - \frac{1}{5}\cos(2x) \underbrace{- e^{-x}\_C1 + \_C2}$$