Exercice 1. Justifier l'existence des intégrales suivantes puis les calculer :

$$I_1 = \int_{-1}^{1} (t+1)(t+2)^2 dt, \quad I_2 = \int_0^4 \sqrt{x}\,(x - 2\sqrt{x})\,dx, \quad I_3 = \int_1^2 3^u du, \quad I_4 = \int_1^4 \frac{1}{y\sqrt{y}} dy,$$

$$I_5 = \int_0^1 (2z-1)\exp(z^2 - z) dz, \quad I_6 = \int_1^2 \frac{a^2}{\sqrt{1+a^3}} da, \quad I_7 = \int_1^2 \frac{2\sqrt{c}}{2 + 3c^{3/2}} dc,$$

$$I_8 = \int_1^e \frac{(\ln b)^5}{b} db, \quad I_9 = \int_0^{(\ln 2)/2} \frac{e^{2t}}{e^{2t}+2} dt, \quad I_{10} = \int_e^{e^3} \frac{\ln(3\gamma)}{\gamma} d\gamma, \quad I_{11} = \int_0^2 \beta^4 \exp(-\beta^5) d\beta,$$

$$I_{12} = \int_{1/2}^{3/2} \frac{1-\alpha}{(\alpha^2 - 2\alpha)^4} d\alpha, \quad I_{13} = \int_0^2 x^2(x^3+1)^{3/2} dx, \quad I_{14} = \int_0^1 \frac{s^{2004}}{(1+s^{2005})^{2006}} ds,$$

$$I_{15} = \int_1^4 \frac{e^{-\sqrt{\zeta}}}{\sqrt{\zeta}} d\zeta, \quad I_{16} = \int_{-4}^4 \frac{e^u - e^{-u}}{e^u + e^{-u}} du, \quad I_{17} = \int_1^{\sqrt{3}} \frac{\exp(-3/v^2)}{v^3} dv, \quad I_{18} = \int_0^1 \frac{y^4}{\sqrt[3]{1+7y^5}} dy$$

Exercice 2. Calculer :

1) $\displaystyle\int_0^x \cos^3(t)dt$ 2) $\displaystyle\int_0^x \sin^4(t)dt$ 3) $\displaystyle\int_0^x \cosh^3(t)dt$

4) $\displaystyle\int_0^x \sin(t)\cos^2(t)dt$ 5) $\displaystyle\int_0^x \sin^2(t)\cos^3(t)dt$ 6) $\displaystyle\int_0^x \sin^3(t)\cos^3(t)dt$

7) $\displaystyle\int_0^x t^2\exp(t)dt$ 8) $\displaystyle\int_1^x t^2\ln(t)dt$ 9) $\displaystyle\int_0^x t^2\sin(t)dt$

Exercice 3. Calculer les primitives :

1) $\displaystyle\int \arcsin(t)dt$ 2) $\arctan(t)dt$ 3) $\displaystyle\int t\arctan(t)dt$

4) $\displaystyle\int \frac{t}{\cos^2(t)}dt$ 5) $\displaystyle\int \frac{\ln(t)}{t^n}dt$ pour $n \neq 1$.

Exercice 4. Calculer les primitives :

1) $\displaystyle\int \frac{dx}{x(x+1)}$ 2) $\displaystyle\int \frac{dx}{x(x-1)^2}$ 3) $\displaystyle\int \frac{1}{x(x^2+1)}dx$ 4) $\displaystyle\int \frac{x}{x^2+2}dx$

5) $\displaystyle\int \frac{x^2}{x^2+2}$ 6) $\displaystyle\int \frac{dx}{x^2(x^2-1)^2}$ 7) $\displaystyle\int \frac{x+1}{(x^2+1)^2}dx$ 8) $\displaystyle\int \frac{dx}{(x+2)(x^2+2x+5)}$

Exercice 5. Calculer les primitives en utilisant un changement de variable :

1) $\displaystyle\int \frac{(x+\sqrt{x^2+1})^2}{\sqrt{x^2+1}}\,dx.$ $(t = x + \sqrt{x^2+1})$ 2) $\displaystyle\int \frac{dx}{x(x^2+3)}$, $(x = \frac{1}{t})$

3) $\displaystyle\int \frac{dx}{1+\cosh(x)}$ 4) $\displaystyle\int \frac{2}{\cosh(x)\sinh^3(x)}dx$; $(\,t = \cosh^2(x))$

Exercice 6. Calculer les primitives sur un domaine convenable :

1) $\displaystyle\int \frac{dx}{x+\sqrt{x-1}}$ 2) $\displaystyle\int \frac{x}{\sqrt{9+4x^2}}dx$ 3) $\displaystyle\int \frac{dx}{1+\sqrt{1-x}}$ 4) $\displaystyle\int \frac{1}{1-x}\sqrt{\frac{1-x}{1+x}}dx$

5) $\displaystyle\int \sqrt{1-x^2}dx$ 6) $\displaystyle\int \sqrt{x^2-1}dx$ 7) $\displaystyle\int x^2\sqrt{1-x^2}dx$ 8) $\displaystyle\int x\sqrt{\frac{1-x}{1+x}}dt$