Formulaire - Loi Normale et Table de la Lois Normale Centrée Réduite

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Utilisation de la Table de la Lois Normale Centrée Réduite

Principe de base

Soit $Z \sim \mathcal{N}(0,1)$, la table donne $\Phi(z) = P(Z \leq z)$.
Propriété fondamentale : $\Phi(-z) = 1 - \Phi(z)$

Comment lire la table

z	0.00	0.01	0.02	0.03	0.04
1.2	0.8849	0.8869	0.8888	0.8907	0.8925
1.3	0.9032	0.9049	0.9066	0.9082	0.9099
1.4	0.9192	0.9207	0.9222	0.9236	0.9251

$z = 1.21 = 1.2 + 0.01$

$\mathbb{P}(Z < 1{,}21) =$ $\Phi(z) = \Phi(1.21) = 0.8869$

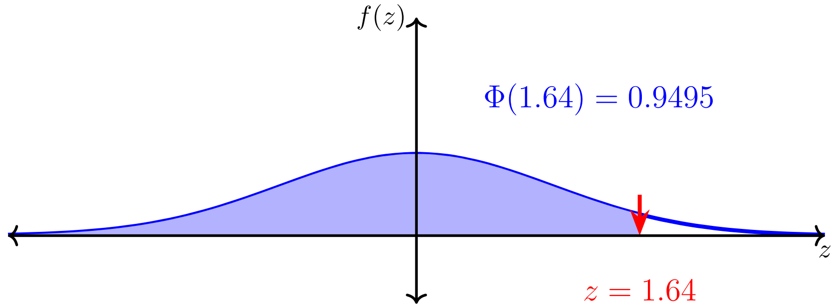
Pour trouver $\Phi(1.21)$:

Ligne 1.2 + Colonne 0.01 = 0.8869

Scénarios de Calcul avec la Loi Normale

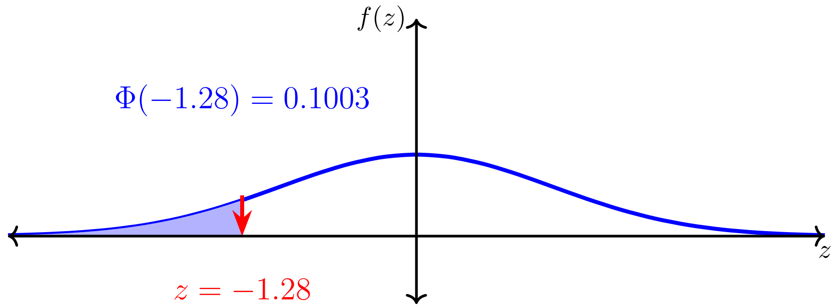
Scénario 1 : $P(Z \leq z)$ avec $z > 0$

- **Direct :** $P(Z \leq z) = \Phi(z)$
- **Exemple :** $P(Z \leq 1.64) = \Phi(1.64) = 0.9495$



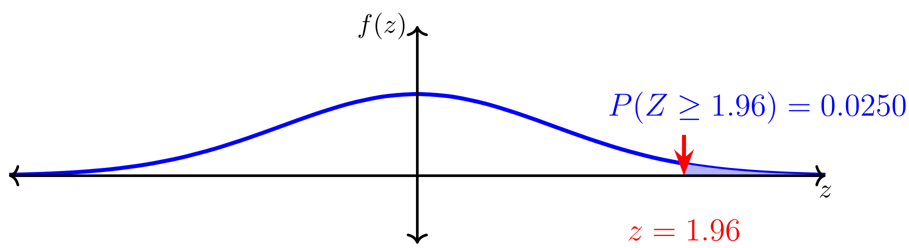
Scénario 2 : $P(Z \leq z)$ avec $z < 0$

- **Utiliser :** $P(Z \leq z) = \Phi(z) = 1 - \Phi(-z)$
- **Exemple :** $P(Z \leq -1.28) = \Phi(-1.28) = 1 - \Phi(1.28) = 1 - 0.8997 = 0.1003$



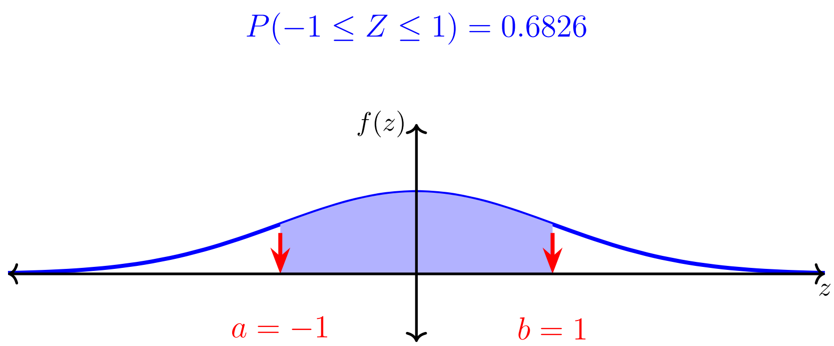
Scénario 3 : $P(Z \geq z)$

- **Formule :** $P(Z \geq z) = 1 - \Phi(z)$
- **Exemple :** $P(Z \geq 1.96) = 1 - \Phi(1.96) = 1 - 0.9750 = 0.0250$



Scénario 4 : $P(a \leq Z \leq b)$

- **Formule :** $P(a \leq Z \leq b) = \Phi(b) - \Phi(a)$
- **Exemple :** $P(-1 \leq Z \leq 1) = \Phi(1) - \Phi(-1) = 0.8413 - 0.1587 = 0.6826$



Transformation vers la Lois Normale Centrée Réduite

Formule de transformation

Si $X \sim \mathcal{N}(\mu, \sigma^2)$, alors :

$$Z = \frac{X - \mu}{\sigma} \sim \mathcal{N}(0, 1)$$

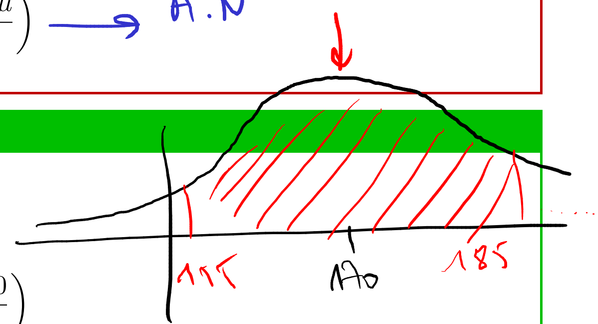
Calcul de probabilités pour la loi normale $X \sim N(m, \sigma^2)$

$$\rightarrow P(X \leq x) = P\left(Z \leq \frac{x - \mu}{\sigma}\right) = \Phi\left(\frac{x - \mu}{\sigma}\right) \longrightarrow A.N$$

$$\rightarrow P(X \geq x) = 1 - \Phi\left(\frac{x - \mu}{\sigma}\right) \longrightarrow A.N$$

$$\rightarrow P(a \leq X \leq b) = \Phi\left(\frac{b - \mu}{\sigma}\right) - \Phi\left(\frac{a - \mu}{\sigma}\right) \longrightarrow A.N$$

Exemple concret 1 taille :



Soit $X \sim \mathcal{N}(170, 15^2)$ (taille en cm). Calculer $P(X \leq 185)$.

$$\begin{aligned} P(X \leq 185) &= P\left(Z \leq \frac{185 - 170}{15}\right) \\ &= P(Z \leq 1) \\ &= \Phi(1) = 0.8413 \end{aligned}$$

Interprétation : 84.13% des personnes mesurent moins de 185 cm.

Exemple concret 2

Soit $X \sim \mathcal{N}(100, 10^2)$ (QI). Calculer $P(90 \leq X \leq 115)$.

$\phi(-1) = 1 - \phi(1)$

$$\begin{aligned} P(\overset{a}{90} \leq X \leq \overset{b}{115}) &= \Phi\left(\overset{b}{\frac{115 - 100}{10}}\right) - \Phi\left(\overset{a}{\frac{90 - 100}{10}}\right) \\ &= \Phi(1.5) - \Phi(-1) \\ &= 0.9332 - 0.1587 = 0.7745 \end{aligned}$$

Interprétation : 77.45% des personnes ont un QI entre 90 et 115.

$$\mathbb{P}(X \leq a) \Longrightarrow \mathbb{P}\left(Z \leq \frac{a-m}{\sigma}\right)$$

$$\mathbb{P}(X \leq 185) = \mathbb{P}\left(Z \leq \frac{185-170}{15}\right) = \underbrace{\mathbb{P}(Z \leq 1)}_{\simeq 0.8413}$$

$\mathbb{P}(Z \leq t)$

Chiffre des centièmes

Chiffre des unités et des dixièmes

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7703	0.7734	0.7764	0.7793	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8906	0.8925	0.8943	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9986	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

$$\mathbb{P}(X \leq 165) = \mathbb{P}\left(Z \leq \frac{165-170}{15}\right)$$

$-\frac{1}{3} = -0{,}33$

$$= \mathbb{P}\left(Z \leq \frac{-5}{15}\right) = \mathbb{P}\left(Z \leq -\frac{1}{3}\right)$$

$$\mathbb{P}(Z \leq -a) = 1 - \mathbb{P}(Z \leq a)$$

$$\mathbb{P}\left(Z \leq -\frac{1}{3}\right) = 1 - \mathbb{P}\left(Z \leq \frac{1}{3}\right)$$

$\frac{1}{3} = 0{,}33$

$$= 1 - 0{,}6293$$

$$= \ldots\ldots$$

Valeurs Remarquables et Intervalles de Confiance

Valeurs usuelles

Probabilité	z_α	Intervalle	Valeur $\Phi(z)$
90%	1.645	$\mu \pm 1.645\sigma$	0.95
95%	1.960	$\mu \pm 1.960\sigma$	0.975
99%	2.576	$\mu \pm 2.576\sigma$	0.995
99.9%	3.291	$\mu \pm 3.291\sigma$	0.9995

Exemple : Pour $X \sim \mathcal{N}(\mu, \sigma^2)$:

$$P(\mu - 1.96\sigma \leq X \leq \mu + 1.96\sigma) = 0.95$$

Recherche de quantiles

Pour trouver z_α tel que $P(Z \leq z_\alpha) = \alpha$:

- Si $\alpha > 0.5$, lire directement dans la table
- Si $\alpha < 0.5$, utiliser $z_\alpha = -z_{1-\alpha}$

Exemple : Trouver $z_{0.95}$:

$$P(Z \leq z_{0.95}) = 0.95 \Rightarrow z_{0.95} = 1.645$$

Problème complet

Le poids des nouveau-nés suit $\mathcal{N}(3.2, 0.5^2)$ kg.

1. Quelle proportion pèse plus de 4 kg ?
2. Entre quels poids se situent 90% des nouveau-nés ?

Solution : $1 - \mathbb{P}(X \leq 4)$ Table

1. $P(X > 4) = 1 - \Phi\left(\frac{4-3.2}{0.5}\right) = 1 - \Phi(1.6) = 1 - 0.9452 = 0.0548$
2. On cherche a et b tels que $P(a \leq X \leq b) = 0.90$
   $z_{0.95} = 1.645$ (car 5% dans chaque queue)
   $a = 3.2 - 1.645 \times 0.5 = 2.3775$ kg
   $b = 3.2 + 1.645 \times 0.5 = 4.0225$ kg