☑ VARD
☑ pmf loi ... ?! **
☑ E, V ←

## Exercice

Soit $X$ une variable aléatoire discrète non usuelle de loi:

$$\rightarrow P(X=k) = \frac{c}{k(k+1)(k+2)}, \quad k = 1, 2, 3, \ldots \qquad k \in \mathbb{N}^*$$

1. Déterminer la constante $c$ pour que ce soit une loi de probabilité
2. Calculer $E[X]$ et $V(X)$

| $k$ | $1$ | $2$ | $\cdots$ | $k$ | $\cdots$ |
|---|---|---|---|---|---|
| $\mathbb{P}(X=k)$ | $\frac{c}{6}$ | . | . | $\frac{c}{k(k+1)(k+2)}$ | |

$$\mathbb{P}(X=1) = \frac{c}{1(1+1)(1+2)} = \frac{c}{6}$$

$$\sum_{k=1}^{\infty} \mathbb{P}(X=k) = 1 \iff \sum_{k=1}^{\infty} \frac{c}{k(k+1)(k+2)} = 1$$

$$\iff c \sum_{k=1}^{\infty} \frac{1}{k(k+1)(k+2)} = 1 \iff \frac{c}{4} = 1$$

$$\iff \boxed{c = 4}$$

step1 $\frac{1}{k(k+1)(k+2)} = \frac{\alpha}{\boxed{k}} + \frac{\beta}{k+1} + \frac{\gamma}{k+2}$ ? $\alpha, \beta, \gamma$ ?

$$\alpha = \cancel{k} \times \frac{1}{\cancel{k}(k+1)(k+2)} \Big|_{k=0} = \frac{1}{2} \quad ☑$$

↑ 0 : nul $k$

$$\beta = \cancel{(k+1)} \frac{1}{k\cancel{(k+1)}(k+2)} \Big|_{k=-1} = \frac{1}{-1} = -1$$

$-1 \times 1$

$$\gamma = \cancel{(k+2)} \frac{1}{k(k+1)\cancel{(k+2)}} \Big|_{k=-2} = \frac{1}{2}$$

$-2 \times -1$

$\alpha = \frac{1}{2} \qquad \beta = -1 \qquad \gamma = \frac{1}{2}$

$$\frac{1}{k(k+1)(k+2)} = \frac{1/2}{k} + \frac{-1}{k+1} + \frac{\frac{1}{2}}{k+2}$$

$$\sum_{k=1}^{+\infty} \frac{1}{k(k+1)(k+2)} = ??$$

$$S_n = \sum_{k=1}^{n} \frac{1}{k(k+1)(k+2)} = \sum_{k=1}^{n} \frac{\frac{1}{2}}{k} + \frac{-1}{k+1} + \frac{\frac{1}{2}}{k+2} =$$

$k=1$ $\qquad$ $k=2$ $\qquad$ $k=3$

$$\cancel{\frac{\frac{1}{2}}{1}} + \cancel{\frac{-1}{2}} + \cancel{\frac{\frac{1}{2}}{3}} + \cancel{\frac{\frac{1}{2}}{2}} + \cancel{\frac{-1}{3}} + \boxed{\frac{\frac{1}{2}}{4}} + \cancel{\frac{\frac{1}{2}}{3}} + \cancel{\frac{-1}{4}} + \cancel{\frac{\frac{1}{2}}{5}}$$

$k=4$ $\qquad$ $k=5$ $\qquad$ $k=6$

$$\boxed{\frac{\frac{1}{2}}{4}} + \cancel{\frac{-1}{5}} + \cancel{\frac{\frac{1}{2}}{6}} + \cancel{\frac{\frac{1}{2}}{5}} + \cancel{\frac{-1}{6}} + \frac{\frac{1}{2}}{7} + \cancel{\frac{\frac{1}{2}}{6}} + \frac{-1}{7} + \frac{\frac{1}{2}}{8}$$

$k=n-2$ $\qquad$ $k=n-1$ $\qquad$ $k=n$

$$\frac{1/2}{n-2} + \frac{-1}{n-1} + \frac{1/2}{n} + \frac{1/2}{n-1} + \frac{-1}{n} + \underbrace{\frac{\frac{1}{2}}{n+1}} + \frac{1/2}{n} + \underbrace{\frac{-1}{n+1}} + \frac{1/2}{n+2}$$

$$\frac{-\frac{1}{2}}{n+1} + \frac{\frac{1}{2}}{n+2}$$

$$S_n = \frac{1}{4} + \frac{-\frac{1}{2}}{n+1} + \frac{1/2}{n+2} \xrightarrow{n \to +\infty} \frac{1}{4}$$

$$\frac{-\frac{1}{2}}{n+1} \to 0 \qquad \frac{1/2}{n+2} \to 0$$

$$\sum_{k=1}^{+\infty} \frac{1}{k(k+1)(k+2)} = \frac{1}{4}$$

$$S_n = \sum_{k=1}^{n} \frac{\frac{1}{2}}{k} + \frac{-1}{k+1} + \frac{\frac{1}{2}}{k+2}$$

$$= \cancel{\frac{\frac{1}{2}}{1}} + \boxed{\frac{\frac{1}{2}}{2}} + \cancel{\frac{\frac{1}{2}}{3}} + \cancel{\phantom{x}} \cdots + \cancel{\frac{\frac{1}{2}}{n-1}} + \cancel{\frac{\frac{1}{2}}{n}} + \cdot$$

$$+ \cancel{\frac{-1}{2}} + \cancel{\frac{-1}{3}} + \cancel{\phantom{x}} \cdots + \cancel{\frac{-1}{n-1}} + \cancel{\frac{-1}{n}} + \frac{-1}{n+1}$$

$$+ \cancel{\frac{\frac{1}{2}}{3}} + \cancel{\phantom{x}} \cdots + \cancel{\frac{\frac{1}{2}}{n-1}} + \cancel{\frac{\frac{1}{2}}{n}} + \frac{\frac{1}{2}}{n+1} + \frac{\frac{1}{2}}{n+2}$$

$$= \frac{1}{4} - \frac{1/2}{n+1} + \frac{\frac{1}{2}}{n+2}$$

$$\mathbb{E}(X) = \sum_{k=1}^{+\infty} k.\mathbb{P}(X=k) = \sum_{k=1}^{+\infty} \cancel{k} \frac{4}{\cancel{k}(k+1)(k+2)}$$

$$= 4 \sum_{k=1}^{+\infty} \frac{1}{(k+1)(k+2)}.$$

$$\frac{1}{(k+1)(k+2)} = \frac{\boxed{1}}{k+1} + \frac{\boxed{-1}}{k+2}$$

$$S'_n = \sum_{k=1}^{n} \frac{1}{(k+1)(k+2)} = \sum_{k=1}^{n} \boxed{\frac{1}{k+1}} + \boxed{\frac{-1}{k+2}}$$

$$= \left[\boxed{\frac{1}{2}} + \frac{1}{3} + \dots + \frac{1}{n} + \frac{1}{n+1}\right] +$$

$$+ \quad + \frac{-1}{3} \quad + \frac{-1}{n} + \frac{-1}{n+1} + \boxed{\frac{-1}{n+2}}$$

$$S'_n = \frac{1}{2} - \frac{1}{n+2} \xrightarrow{n \to +\infty} \boxed{\frac{1}{2}}$$

$$\mathbb{E}(X) = 4 \sum_{k=1}^{\infty} \frac{1}{(k+1)(k+2)} = 4 \times \frac{1}{2} = \boxed{2}$$

$$\mathbb{V}(X) = \mathbb{E}(X^2) - \mathbb{E}(X)^2$$

$$\mathbb{E}(X^2) = \sum_{k=1}^{+\infty} k^2 \cdot \mathbb{P}(X=k)$$

$$= \sum_{k=1}^{+\infty} \frac{k^2 \times 4}{k \times (k+1)(k+2)}$$

$$= \sum_{k=1}^{\infty} \frac{4k}{(k+1)(k+2)} \quad \underline{\underline{\text{div}}}$$

Car $\frac{4k}{(k+1)(k+2)} \underset{+\infty}{\sim} \frac{4}{k^{1}}$ $\quad \sum \frac{1}{n^\alpha}$ conv $\alpha > 1$

$\boxed{\checkmark}$ $\underline{\underline{cl}}$ X : n'admet pas de variance.