Sapienza University of Rome

Master in Engineering in Computer Science

Machine Learning

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13. Multiple learners

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with contributions from Valsamis Ntouskos

Overview

- Combining multiple learners
- Voting
- Bagging
- Boosting
- AdaBoost

Reference
E. Alpaydin. Introduction to Machine Learning. Chapter 17.
C. Bishop. Pattern Recognition and Machine Learning. Chapter 14.

Multiple learners / Ensemble learning

General idea: instead of training a complex learner/model, train many different learners/models and then combine their results.

Committees: set of models trained on a dataset.

Models can be trained in parallel (*voting* or *bagging*) or in sequence (*boosting*).

Voting

Given a dataset D

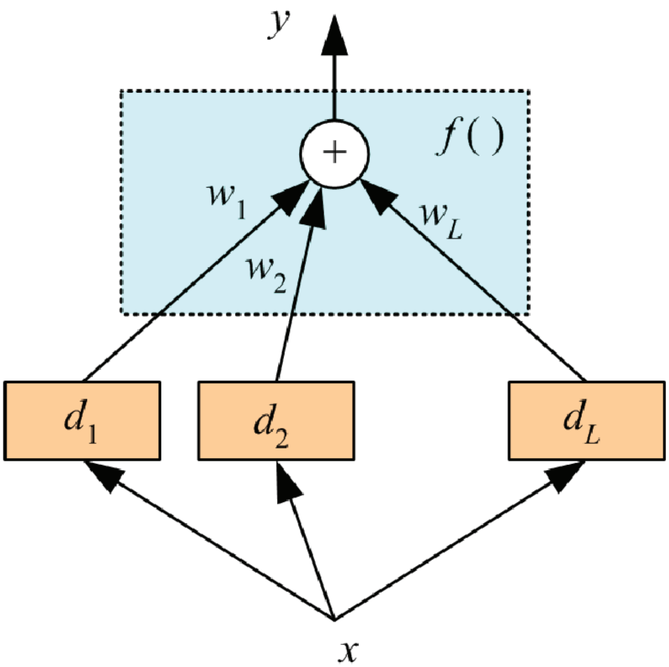
1. use D to train a set of models $y_m(\mathbf{x})$, for $m = 1, \ldots, M$
2. make predictions with

$$y_{voting}(\mathbf{x}) = \sum_{m=1}^{M} w_m y_m(\mathbf{x}) \qquad \text{(regression)}$$

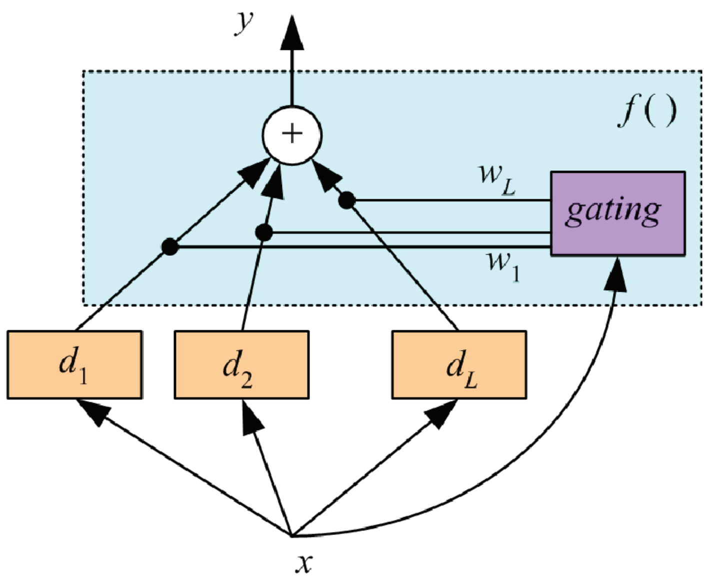
$$y_{voting}(\mathbf{x}) = \operatorname{argmax}_{c} \sum_{m=1}^{M} w_m I(y_m(\mathbf{x}) = c) \qquad \text{weighted majority (classification)}$$

with $w_m \geq 0$, $\sum_m w_m = 1$ (prior probability of each model),
$I(e) = 1$ if e is true, 0 otherwise.

Voting

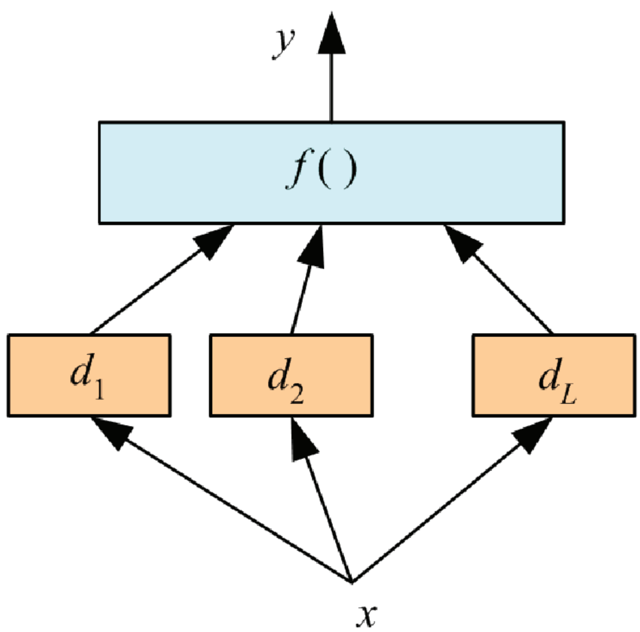


Mixture of experts



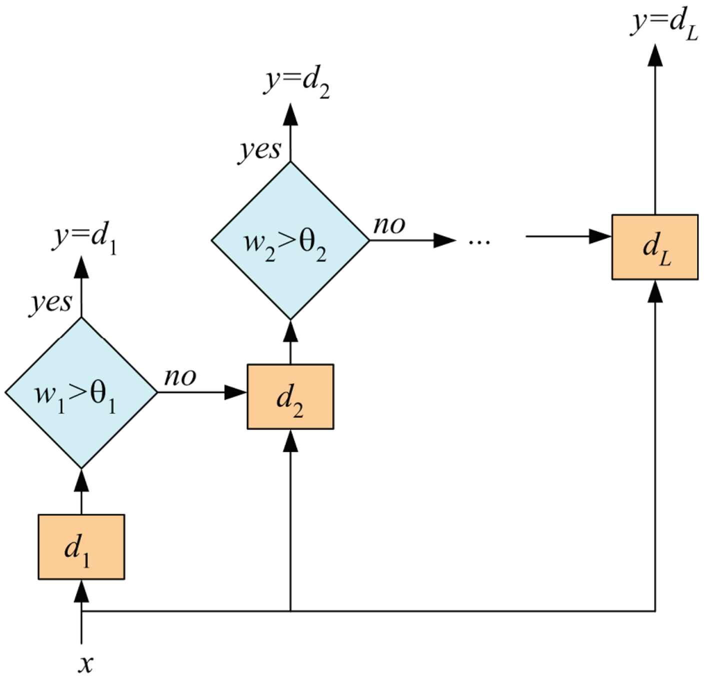
Non linear gating function f depending on input

Stacking



Combination function f is also learned

Cascading



Cascade learners based on confidence thresholds

Bagging

Given a dataset D,

1. generate M bootstrap data sets $D_1, \ldots, D_M$, with $D_i \subset D$
2. use each bootstrap data set D_m to train a model $y_m(\mathbf{x})$, for $m = 1, \ldots, M$
3. make predictions with a voting scheme

$$y_{bagging}(\mathbf{x}) = \frac{1}{M} \sum_{m=1}^{M} y_m(\mathbf{x})$$

In general, this is better than training any individual model.

Bootstrap data sets chosen with *random sampling with replacement*

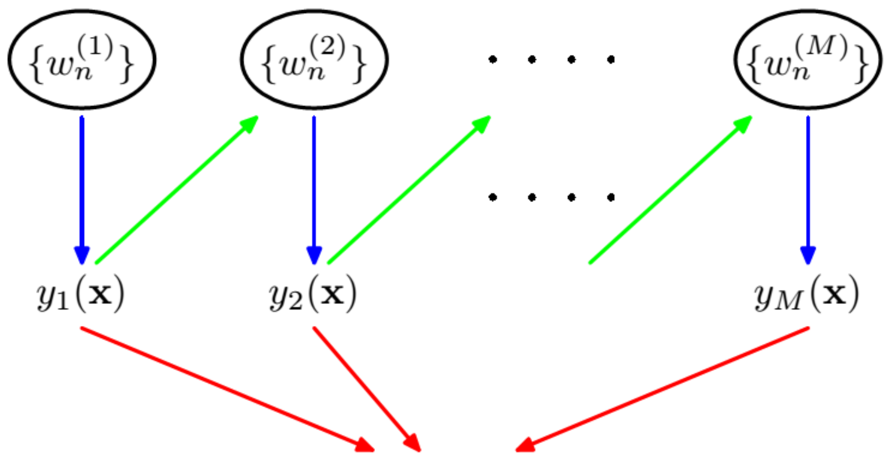
Boosting: general approach

Main points:

- Base classifiers (*weak learners*) trained sequentially
- Each classifier trained on weighted data
- Weights depend on performance of previous classifiers
- Points misclassified by previous classifiers are given greater weight
- Predictions based on weighted majority of votes

Boosting: general approach

Base classifiers are trained in sequence using a weighted data set where weights are based on performance of previous classifiers.



$$Y_M(\mathbf{x}) = \text{sign}\left(\sum_{m=1}^{M} \alpha_m y_m(\mathbf{x})\right)$$

AdaBoost

Given $D = \{(\mathbf{x}_1, t_1), \ldots, (\mathbf{x}_N, t_N)\}$, where $\mathbf{x}_n \in \mathbf{X}$, $t_n \in \{-1, +1\}$

1. Initialize $w_n^{(1)} = 1/N$, $n = 1, \ldots, N$.
2. For $m = 1, \ldots, M$:

- Train a weak learner $y_m(\mathbf{x})$ by minimizing the weighted error function:

$$J_m = \sum_{n=1}^{N} w_n^{(m)} I(y_m(\mathbf{x}_n) \neq t_n), \text{ with } I(e) = \begin{cases} 1 & \text{if } e \text{ is true} \\ 0 & \text{otherwise} \end{cases}$$

- Evaluate: $\epsilon_m = \dfrac{\sum_{n=1}^{N} w_n^{(m)} I(y_m(\mathbf{x}_n) \neq t_n)}{\sum_{n=1}^{N} w_n^{(m)}}$ and $\alpha_m = \ln\left[\dfrac{1-\epsilon_m}{\epsilon_m}\right]$
- Update the data weighting coefficients:

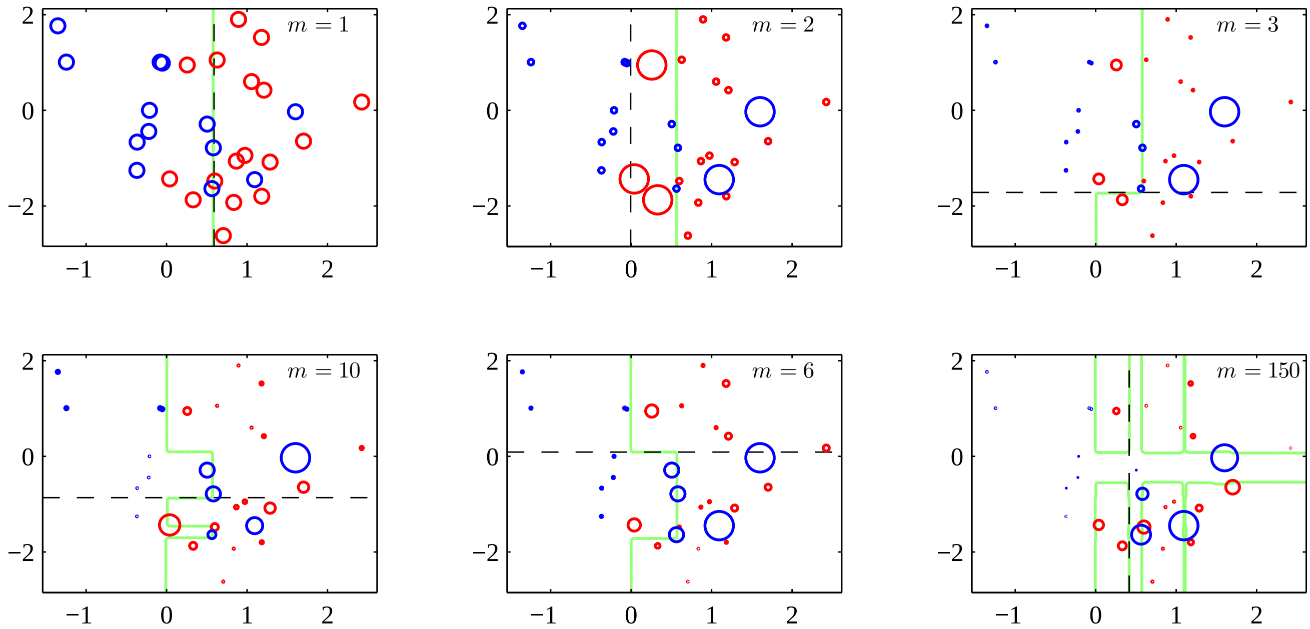
$$w_n^{(m+1)} = w_n^{(m)} \exp[\alpha_m I(y_m(\mathbf{x}_n) \neq t_n)]$$

AdaBoost

3. Output the final classifier

$$Y_M(\mathbf{x}) = \text{sign}\left(\sum_{m=1}^{M} \alpha_m y_m(\mathbf{x})\right)$$

AdaBoost



Exponential error minimization

AdaBoost can be explained as the sequential minimization of an exponential error function.

Consider the error function

$$E = \sum_{n=1}^{N} \exp[-t_n f_M(\mathbf{x}_n)],$$

where

$$f_M(\mathbf{x}) = \frac{1}{2} \sum_{m=1}^{M} \alpha_m y_m(\mathbf{x}), \quad t_n \in \{-1, +1\}$$

Goal:

minimize E w.r.t. $\alpha_m, y_m(\mathbf{x}),\ m = 1, \ldots, M$

Exponential error minimization

Sequential minimization. Instead of minimizing E globally

- assume $y_1(\mathbf{x}), \ldots, y_{M-1}(\mathbf{x})$ and $\alpha_1, \ldots, \alpha_{M-1}$ fixed;
- minimize w.r.t. $y_M(\mathbf{x})$ and α_M.

Making $y_M(\mathbf{x})$ and α_M explicit we have:

$$E = \sum_{n=1}^{N} \exp\left[-t_n f_{M-1}(\mathbf{x}_n) - \frac{1}{2} t_n \alpha_M y_M(\mathbf{x}_n)\right]$$
$$= \sum_{n=1}^{N} w_n^{(M)} \exp\left[-\frac{1}{2} t_n \alpha_M y_M(\mathbf{x}_n)\right],$$

with $w_n^{(M)} = \exp[-t_n f_{M-1}(\mathbf{x}_n)]$ constant as we are optimizing w.r.t. α_M and $y_M(\mathbf{x})$.

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Exponential error minimization

From sequential minimization of E, we obtain

$$w_n^{(m+1)} = w_n^{(m)} \exp[\alpha_m I(y_m(\mathbf{x}_n) \neq t_n)] \text{ and } \alpha_m = \ln\left[\frac{1-\epsilon_m}{\epsilon_m}\right]$$

predictions are made with

$$\text{sign}(f_M(\mathbf{x})) = \text{sign}\left(\frac{1}{2}\sum_{m=1}^{M} \alpha_m y_m(\mathbf{x})\right)$$

which is equivalent to

$$Y_M(\mathbf{x}) = \text{sign}\left(\sum_{m=1}^{M} \alpha_m y_m(\mathbf{x})\right)$$

thus proving that AdaBoost minimizes such error function.

AdaBoost Remarks

Advantages:

- fast, simple and easy to program
- no prior knowledge about base learner is required
- no parameters to tune (except for M)
- can be combined with any method for finding base learners
- theoretical guarantees given sufficient data and base learners with moderate accuracy

Issues:

- Performance depends on data and the base learners
  (can fail with insufficient data or when base learners are too weak)
- Sensitive to noise

Summary

- Instead of designing a learning algorithm that is accurate over the entire space one can focus on finding base learning algorithms that only need to be better than random
- Combined learners theoretically outperforms any individual learner
- AdaBoost practically outperforms many other base learners in many problems
- Ensembles of small DNN outperform very deep NN in some cases