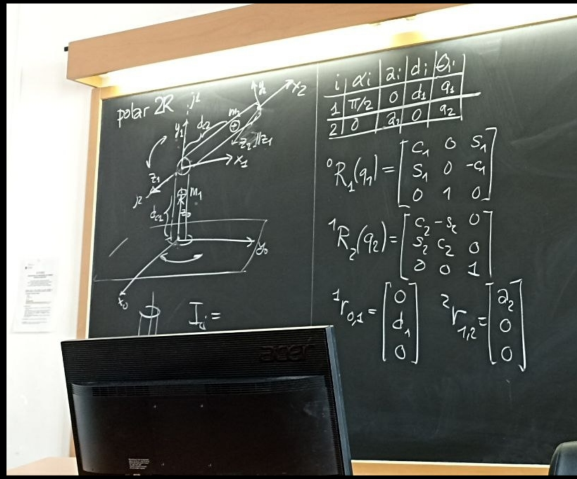
29|03



$${}^iI_{c_i} = \begin{bmatrix} I_{xx} & & \\ & I_{yy} & \\ & & I_{zz} \end{bmatrix} \qquad i=1,2$$

$I_{1xx} = I_{1zz}$ First link

$I_{2yy} = I_{2zz}$ Second link

$${}^1r_{c_1} = \begin{bmatrix} 0 \\ -d_1 + dc_1 \\ 0 \end{bmatrix} \qquad {}^2r_{c_2} = \begin{bmatrix} -a_2 + dc_2 \\ 0 \\ 0 \end{bmatrix}$$

Inizialization

${}^0\omega_0 = 0$ ${}^0v_0 = 0$

we need ${}^i\omega_i$ ${}^iv_i$ ${}^iv_{c_i}$

$\boxed{i=1}$

$${}^1\omega_1 = {}^0R_1^T(q_1)\overbrace{\left[\underbrace{0}_{{}^0\omega_0} + \dot q_1 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\right]}^{{}^0\omega_1} = {}^0R_1^T(q_1)\begin{bmatrix} 0 \\ 0 \\ \dot q_1 \end{bmatrix} = \begin{bmatrix} 0 \\ \dot q_1 \\ 0 \end{bmatrix}$$

min 9.30

$${}^1v_1 = {}^0R_1^T(q_1)\overbrace{\left[\underbrace{0}_{{}^0v_0} + {}^0\omega_1 \times {}^0r_{0,1}\right]}^{{}^0v_1} \xrightarrow{\text{Bringing in } {}^0R_1^T} \begin{bmatrix} 0 \\ \dot q_1 \\ 0 \end{bmatrix} \times \underbrace{\begin{bmatrix} 0 \\ d_1 \\ 0 \end{bmatrix}}_{{}^1r_{0,1}} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

min 13.00

$${}^1v_{c_1} = {}^1v_1 + {}^1\omega_1 \times {}^1r_{c_1} = 0$$

min 14.00

König $\rightarrow$ $T_1 = \frac{1}{2} m \cancel{\|v_{c_1}\|^2}^{\to 0} + \frac{1}{2}\, {}^1\omega_1^T\, {}^1I_{c_1}\, {}^1\omega_1 = \frac{1}{2} I_{1yy} \dot q_1^2$

$\boxed{i=2}$

$$ {}^2\omega_2 = {}^1R_2^T(q_2)\Big[\overbrace{\begin{bmatrix}0\\ \dot q_1\\ 0\end{bmatrix} + \dot q_2\begin{bmatrix}0\\0\\1\end{bmatrix}}^{{}^1\omega_2}\Big] = $$

$$ = {}^1R_2^T(q_2)\begin{bmatrix}0\\ \dot q_1\\ \dot q_2\end{bmatrix} = \begin{bmatrix}s_2\dot q_1\\ c_2\dot q_1\\ \dot q_2\end{bmatrix} $$

$$ {}^2v_2 = {}^1R_2^T(q_2)\Big[\overbrace{\underset{{}^1v_1}{0} + \underset{{}^1\omega_2}{\begin{bmatrix}0\\ \dot q_1\\ \dot q_2\end{bmatrix}} \times {}^1r_{1,2}}^{{}^1v_2}\Big] = \begin{bmatrix}s_2\dot q_1\\ c_2\dot q_1\\ \dot q_2\end{bmatrix} \times \begin{bmatrix}q_2\\0\\0\end{bmatrix} $$

$$ {}^2v_{c2} = {}^2v_2 + {}^2\omega_2 \times {}^2r_{c2} = \begin{bmatrix}s_2\dot q_1\\ c_2\dot q_1\\ \dot q_2\end{bmatrix} \times \begin{bmatrix}dc_2\\0\\0\end{bmatrix} = \begin{bmatrix}0\\ \dot q_2\, dc_2\\ -c_2\dot q_1 dc_2\end{bmatrix} $$

$$ T_2 = \frac{1}{2} m_2 dc_2^2\left(\dot q_2^2 + c_2^2\dot q_1^2\right) + $$

$$ \frac{1}{2} I_{2,zz}\dot q_2^2 + \frac{1}{2}\left(I_{2,xx}s_2^2 + I_{2,yy}c_2^2\right)\dot q_1^2 $$

$$ M(q) = \begin{bmatrix} I_{1,yy} + m_2 dc_2^2 c_2^2 + I_{2xx}s_2^2 + I_{2yy}c_2^2 & 0 \\ 0 & \underset{\text{Baricentric inertia}}{I_{2,zz}} + \underset{\text{parallel axis}}{m_2 dc_2^2} \end{bmatrix} $$

Inertia matrix

$$M(q) = \begin{bmatrix} I_{1yy} + I_{2xx} + \left(I_{2yy} + m_2 dc_2^2 - I_{2xx}\right) c_2^2 & // \\ // & // \end{bmatrix}$$

$$M(q) = \begin{bmatrix} a_1 + a_2 c_2^2 & 0 \\ 0 & a_3 \end{bmatrix}$$

a_1, a_2, a_3 are not DH params

CORIOLIS and centrifugal terms

$$c_k(q, \dot{q}) = \dot{q}^T C_k(q) \dot{q} \qquad C_k(q) = \frac{1}{2}\left(\frac{\partial M_k}{\partial q} + \left(\frac{\partial M_k}{\partial q}\right)^T - \frac{\partial M}{\partial q_k}\right)$$

$$C_1(q) = \frac{1}{2}\begin{bmatrix} 0 & -2a_2 S_2 C_2 \\ -2a_2 S_2 C_2 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -a_2 S_2 C_2 \\ -a_2 S_2 C_2 & 0 \end{bmatrix}$$

$$c_1(q) = -2 a_2 S_2 C_2 \dot{q}_1 \dot{q}_2$$

First joint has only a coriolis term, namely when both joints are moving, there's an extra torque on joint 1

$$C_2(q) = \frac{1}{2}\left(0 + 0 - \begin{bmatrix} -2a_2 C_2 S_2 & 0 \\ 0 & 0 \end{bmatrix}\right) = \begin{bmatrix} a_2 S_2 C_2 & 0 \\ 0 & 0 \end{bmatrix}$$

$$c_2(q) = a_2 S_2 C_2 \dot{q}_1^2$$

min. 41.30

Second link has only a centrifugal term, which means that for this link the torque needs to contrast the centrifugal force coming from the first link, namely when the first link rotates, the second would go away to the outside, unless some torque is applied

Gravity terms

$U_1 = 0$ $\qquad$ $U_2 = -m_2 g^T r_{0,c_2} =$ $\qquad$ ${}^0g = \begin{bmatrix} 0 \\ 0 \\ -g_0 \end{bmatrix}$

$= m_2 g_0 (d_1 + d_{c_2} s_2)$ $\qquad$ $g_0 \to -9.81$

$U = U_1 + U_2$ $\qquad$ ${}^0r_{0,c_2} = \begin{bmatrix} * \\ * \\ d_1 + d_{c_2} s_2 \end{bmatrix}$

column vector

$$g(q) = \left(\frac{dU}{dq}\right)^T = \begin{bmatrix} 0 \\ m_2 g_0 d_{c_2} c_2 \end{bmatrix}$$