

Knowledge, action, and the frame problem

Reasoning about Actions

Francesco Petri, Charlotte Primiceri and Flavio Maiorana

May 29, 2024



SAPIENZA
UNIVERSITÀ DI ROMA



Table of Contents

- ▶ Scherl and Levesque's Approach
 - Introduction
 - Knowledge as a fluent
 - Defining a successor state axiom for K
- ▶ Theorems and Regression
 - Theorems
 - Regression
- ▶ Example: The Gardening Robot
 - Axiomatization
 - Instantiation



Situation calculus provides a framework for reasoning about actions.

This work presents an expansion to handle the *knowledge* possessed or acquired by the agent, and allow it to shape the agent's decisions.

- Knowledge is represented by one additional fluent
- Uniform axiomatization with the rest of sitcalc
- Ordinary actions and knowledge-producing ones are strictly separated
- Easy expansion of regression as defined in [Reiter2001]
- Desirable properties are direct consequences of the axiomatization (e.g. knowledge persistence / memory)

The K fluent



$$K(s', s)$$

Defines an accessibility relation between situations.

(Informal) definition

$K(s', s)$ is true if an agent in situation s could mistake the current situation for the other s' , given its current knowledge.



Definition of knowledge

A fluent is known to be true (false) in a situation s if and only if it is true (false) in all situations accessible from s .

Shorthand notation: $\mathbf{Knows}(\phi, s) \stackrel{\text{def}}{=} \forall s' K(s', s) \rightarrow \phi(s')$



Knowledge-producing actions

Actions that have an effect on the agent's knowledge

SENSE actions

Learn the truth value of a formula. Example: check if a door is open or closed.

$$\mathbf{Knows}(P, \text{DO}(\text{SENSE}_P, s)) \vee \mathbf{Knows}(\neg P, \text{DO}(\text{SENSE}_P, s))$$

READ actions

Learn the value of a term. Example: read a number on a sheet of paper.

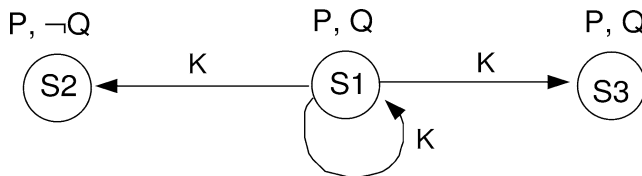
$$\exists x \mathbf{Knows}(\tau = x, \text{DO}(\text{READ}_\tau, s))$$



Knowledge effects

In order to complete the specification of the K fluent, we need to define its successor state axiom, determining how ordinary actions and knowledge-producing actions affect it.

Consider this case study with three accessible situations. The agent is in S1.



$$\mathbf{Knows}(P, S1) \wedge \neg \mathbf{Knows}(Q, S1)$$



Ordinary actions

Assume the agent performs a DROP action.

Informal idea

The agent cannot distinguish the current situation s from all the other s' accessible from it. Therefore, after executing the action, the agent may believe to be in any situation resulting from any s' after executing DROP.

Axiomatization

$$K(s'', \text{DO}(\text{DROP}, s)) \equiv \exists s' (\text{POSS}(\text{DROP}, s') \wedge K(s', s) \wedge s'' = \text{DO}(\text{DROP}, s'))$$

Ordinary actions



The **only knowledge gained is that the DROP action has been performed**, as well as anything that can be derived from the action effects.

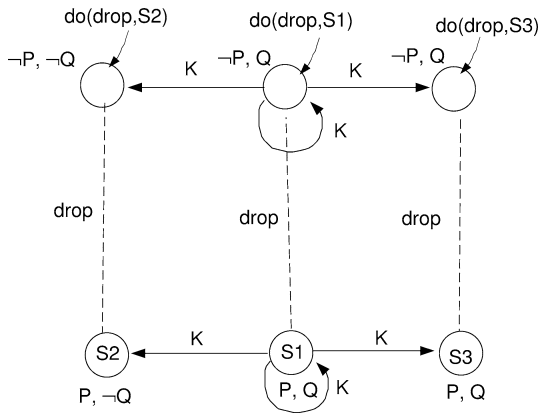
For example, if DROP makes P false:

$$P(\text{DO}(a, s)) \equiv a \neq \text{DROP} \wedge P(s)$$

then

$$\mathbf{Knows}(\neg P, \text{DO}(\text{DROP}, S1))$$

but no extra knowledge is gained about Q .





Knowledge-producing actions

Consider an action SENSE_Q that provides information on whether Q is true or false. We define a **sensing result function** to represent the signal received by the agent in response:

Sensing result function

$$\text{SR}(\text{SENSE}_Q, s) = r \equiv (r = \text{"YES"} \wedge Q(s)) \vee (r = \text{"NO"} \wedge \neg Q(s))$$



Knowledge-producing actions

When the agent executes SENSE_Q , what are the accessible situations afterwards?

Informal definition

Initially, the agent cannot distinguish the current situation s from all the other s' accessible from it. Therefore, after executing SENSE_Q , the agent may believe to be in any situation that:

- results from any s' after executing the action,
- **AND** would yield the same sensing result as the one that has been observed.

Axiomatization

$$\begin{aligned} K(s'', \text{DO}(\text{SENSE}_Q, s)) &\equiv \exists s' (\text{POSS}(\text{SENSE}_Q, s') \wedge K(s', s) \wedge \\ &\quad s'' = \text{DO}(\text{SENSE}_Q, s') \wedge \text{SR}(\text{SENSE}_Q, s) = \text{SR}(\text{SENSE}_Q, s')) \end{aligned}$$

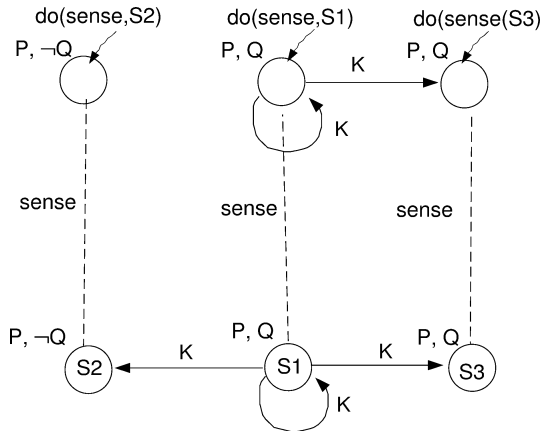
Knowledge-producing actions



After executing $SENSE_Q$, **only situations with the same truth value for Q are accessible.**

Thus, in addition to knowing that $SENSE_Q$ has been performed, the agent now knows the truth value of Q as well, by definition.

$$\text{Knows}(Q, \text{DO}(\text{SENSE}_Q, S1))$$





Sensing results in general

The concept of sensing result extends to all types of action, allowing for a uniform axiomatization.

Ordinary actions

$$\text{SR}(\text{DROP}, s) = r \equiv r = \text{"OK"}$$

SENSE-type knowledge-producing actions

$$\text{SR}(\text{SENSE}_Q, s) = r \equiv (r = \text{"YES"} \wedge Q(s)) \vee (r = \text{"NO"} \wedge \neg Q(s))$$

READ-type knowledge-producing actions

$$\text{SR}(\text{SENSE}_\tau, s) = r \equiv r = \tau(s)$$



The successor state axiom for K

Putting it all together, the definitive form is as follows:

Successor state axiom for the K fluent

$$K(s'', \text{DO}(a, s)) \equiv \exists s' (\text{POSS}(a, s') \wedge K(s', s) \wedge \\ s'' = \text{DO}(a, s') \wedge \text{SR}(a, s) = \text{SR}(a, s'))$$



What about...

- ...mixing ordinary and knowledge effects?

We assume that ordinary and knowledge actions are disjoint: each action is going to be axiomatized as either affecting *only* the K fluent or as not affecting it at all. This does not cause loss of generality.

- ...knowledge of arbitrary formulae?

They already work within this system.

Example: **Knows**($\forall x(\text{MAN}(x) \rightarrow \text{MORTAL}(x)) \wedge \text{MAN}(\text{Socrates})$)



Table of Contents

- ▶ Scherl and Levesque's Approach
 - Introduction
 - Knowledge as a fluent
 - Defining a successor state axiom for K
- ▶ Theorems and Regression
 - Theorems
 - Regression
- ▶ Example: The Gardening Robot
 - Axiomatization
 - Instantiation



Theorem 1 (Knowledge-producing effects). *For all situations s , all fluents P (other than K) and knowledge-producing action terms α , if $P(s)$ then $P(\text{DO}(\alpha, s))$.*

Theorem 2 (Default persistence of ignorance). *For an action α and a situation s , if $\neg\mathbf{Knows}(P, s)$ holds and the axiomatization entails*

$$\forall s \, P(s) \equiv P(\text{DO}(\alpha, s))$$

and

$$\forall y \, \neg\mathbf{Knows}((\text{POSS}(\alpha) \wedge \text{SR}(\alpha) = y) \rightarrow P, s)$$

then

$$\neg\mathbf{Knows}(P, \text{DO}(\alpha, s))$$

holds as well.



Theorem 3 (Knowledge incorporation). *For a knowledge-producing action α , a fluent or the negation of a fluent F , a fluent or the negation of a fluent P , and a situation s , if the axiomatization entails*

$$\exists y \text{ **Knows**}(F \equiv \text{SR}(\alpha) = y, s)$$

and also

$$F(s), \quad \text{POSS}(\alpha, s),$$

and

$$\text{Knows}(F \rightarrow P, s)$$

hold, then

$$\text{Knows}(P, \text{DO}(\alpha, s))$$

holds as well.



Theorem 4 (Memory). *For all fluents P and situations s , if $\mathbf{Knows}(P, s)$ holds then $\mathbf{Knows}(P, \text{DO}(\alpha, s))$ holds as long as the axiomatization entails*

$$\forall s \, P(s) \equiv P(\text{DO}(\alpha, s))$$

Theorem 5 (Knowledge of effects of actions). *If α is an ordinary (not a knowledge-producing) action, and if the axiomatization entails*

$$\forall s \, \phi[s] \rightarrow P(\text{DO}(\alpha, s))$$

where ϕ is an arbitrary formula with situation terms suppressed⁸ and P is a fluent or its negation, then the following is also entailed:

$$\mathbf{Knows}((\text{POSS}(\alpha) \wedge \phi), s) \rightarrow \mathbf{Knows}(P, \text{DO}(\alpha, s))$$



Reasoning Tasks

Projection Task

Determining if a sentence G is true in the situation resulting from the execution of an action sequence is represented by the query:

$$F \models G(\text{DO}([\alpha_1, \dots, \alpha_n], S_0))$$



Reasoning Tasks

Projection Task

Determining if a sentence G is true in the situation resulting from the execution of an action sequence is represented by the query:

$$F \models G(\text{DO}([\alpha_1, \dots, \alpha_n], S_0))$$

Legality Task

The situation term

$$\text{DO}(\alpha_m, \text{DO}(\alpha_{m-1} \dots \text{DO}(\alpha_1, S_0) \dots))$$

is a legal situation iff

$$[\alpha_1, \dots, \alpha_m]$$

is a legal action sequence

Regression



- Reduce Reasoning about future situations to reasoning about the initial situation;
- Sound and Complete Reasoner.

Regression



Definition of Regressor Operator through:

- Ordinary Actions
- Knowledge-Producing Actions



Regression Theorem

Theorem

For any ground situation term

$$F \models G(s_{gr}) \quad \text{iff} \quad F - F_{SS} \models R_{\theta}^*[G(s_{gr})]$$

Proof.

It suffices to show the preservation of logical equivalence

$$F \models G(s_{gr}) \equiv R_{\theta}^*[G(s_{gr})]$$



Regression Theorem



Theorem (Regression-Ordinary Actions)

$$R_{\theta}^*[\mathbf{Knows}(W, DO(a, s))] \equiv$$

$$\mathbf{Knows}(POSS(a) \rightarrow R_{\theta}^*[W[DO(a, s')]], s)$$



Regression-Ordinary Actions

Proof.

1. **Knows**(W, DO(a, s))
2. by the definition of **Knows**:

$$\forall s'' K(s'', DO(a, s)) \equiv \rightarrow W[s'']$$





Regression-Ordinary Actions

Proof.

3. by Successor State Axiom:

$$K(s'', DO(a, s)) \equiv \exists s' (K(s', s) \wedge POSS(a, s) \wedge s'' = DO(a, s'))$$

rephrase as:

$$\forall s'' (\exists s' (K(s', s) \wedge POSS(a, s) \wedge s'' = DO(a, s')) \rightarrow W[s''])$$

4. the axiomatization entails:

$$\forall s, s' SR(a, s) = SR(a, s')$$

66:



Regression-Ordinary Actions

Proof.

5. Inductive Hypothesis:

$$\forall s' (K(s', s) \wedge (\text{POSS}(a, s'))) \rightarrow R_{\theta}^*[W[\text{DO}(a, s')]]$$

$$\forall s' (K(s', s) \wedge (\text{POSS}(a, s'))) \rightarrow R_{\theta}^*[W[\text{DO}(a, s')]]^{-1}$$

6. again by definition of **Knows**:

$$\mathbf{Knows}(\text{POSS}(a) \rightarrow R_{\theta}^*[W[\text{DO}(a, s')]]^{-1}, s)$$





Regression Theorem

Theorem (Regression- Knowledge Producing Actions)

$$R_{\theta}^*[\mathbf{Knows}(W, DO(SENSE_i, s))] \equiv$$

$$\exists y SR(SENSE_i, s) = y \wedge$$

$$\mathbf{Knows}((POSS(a) \wedge SR(SENSE_i) = y) \rightarrow R_{\theta}^*[W[DO(SENSE_i)]]^{-1}, s)$$



Regression- Knowledge Producing Actions

(vi) **Knows**($W, \text{DO}(\text{SENSE}_i, s)$)

by the definition of **Knows**

$$\forall s'' \text{ K}(s'', \text{DO}(\text{SENSE}_i, s)) \rightarrow W[s'']$$

by the successor state axiom for K (sentence (18)), and also and the inductive hypothesis

$$\begin{aligned} \forall s' \left(\text{K}(s', s) \wedge \right. \\ \left. \text{POSS}(\text{SENSE}_i, s') \wedge \text{SR}(\text{SENSE}_i, s) = \text{SR}(\text{SENSE}_i, s') \right) \rightarrow \\ \mathcal{R}_\Theta[W[\text{DO}(\text{SENSE}_i, s')]] \end{aligned}$$

by the definition of equality and the existential quantifier

$$\begin{aligned} \forall s' \left(\text{K}(s', s) \wedge \right. \\ \left. \text{POSS}(\text{SENSE}_i, s') \wedge \exists y \text{ SR}(\text{SENSE}_i, s) = y \right. \\ \left. \wedge \text{SR}(\text{SENSE}_i, s') = y \right) \rightarrow \mathcal{R}_\Theta[W[\text{DO}(\text{SENSE}_i, s')]] \end{aligned}$$



Regression- Knowledge Producing Actions

by the definition of the connectives and quantifiers

$$\begin{aligned} \forall y \text{ SR}(\text{SENSE}_i, s) = y \rightarrow \\ \forall s' \left(\text{K}(s', s) \wedge \text{POSS}(\text{SENSE}_i, s') \wedge \text{SR}(\text{SENSE}_i, s') = y \right) \rightarrow \\ \mathcal{R}_\Theta[W[\text{DO}(\text{SENSE}_i, s')]] \end{aligned}$$

by the definition of the connectives, quantifiers, and the fact that there can only be one denotation of $\text{SR}(\text{SENSE}_i, s)$

$$\begin{aligned} \exists y \text{ SR}(\text{SENSE}_i, s) = y \wedge \forall s' \left(\text{K}(s', s) \rightarrow \right. \\ \left. \text{POSS}(\text{SENSE}_i, s') \wedge \text{SR}(\text{SENSE}_i, s') = y \right) \rightarrow \\ \mathcal{R}_\Theta[W[\text{DO}(\text{SENSE}_i, s')]] \end{aligned}$$

by the definitions of **Knows**

$$\begin{aligned} \exists y \text{ SR}(\text{SENSE}_i, s) = y \wedge \\ \mathbf{Knows}((\text{POSS}(\text{SENSE}_i) \wedge \text{SR}(\text{SENSE}_i) = y) \\ \rightarrow \mathcal{R}_\Theta[W[\text{DO}(\text{SENSE}_i, s')]]^{-1}, s) \quad \square \end{aligned}$$



Legality Testing

Method

Given n actions, the action precondition axioms are:

$$\forall x_n \text{ POSS}(A_n(x_n), s) \equiv \Pi A_n(x_n, s)$$

A sequence is a legal action sequence iff:

$$F \models \Pi(\alpha_1)[s \rightarrow S_0] \wedge \Pi(\alpha_2)[s \rightarrow \text{DO}(\alpha_1, S_0)] \wedge \dots \\ \wedge \Pi(\alpha_m)[s \rightarrow \text{DO}([\alpha_1, \dots, \alpha_{m-1}], S_0)]$$

So iff, by Regression:

$$F - F_{SS} \models \Pi(\alpha_1)[s \rightarrow S_0] \wedge R_{\theta}^*[\Pi(\alpha_2)[s \rightarrow \text{DO}(\alpha_1, S_0)]] \wedge \dots \\ \wedge R_{\theta}^*[\Pi(\alpha_m)[s \rightarrow \text{DO}([\alpha_1, \dots, \alpha_{m-1}], S_0)]]$$



Table of Contents

- ▶ Scherl and Levesque's Approach
 - Introduction
 - Knowledge as a fluent
 - Defining a successor state axiom for K
- ▶ Theorems and Regression
 - Theorems
 - Regression
- ▶ Example: The Gardening Robot
 - Axiomatization
 - Instantiation



The Problem

- A robot has to manage **n plants** in a garden.
- The robot performs an action on one plant at a time.
- **A plant can be watered only if it is dry and the temperature is known.**
- The robot has access to a watering can with unlimited capacity.
- The robot performs action on an object only if it is near it.
- The robot can hold only one object at a time.



(Non)Fluents

Fluents

- $\text{NEAR}(x, s) \rightarrow$ Robot is near object x in situation s
- $\text{HOLDING}(x, s) \rightarrow$ Robot is holding object x in situation s
- $\text{MOIST}(p, s) \rightarrow$ Plant p is moist in situation s
- $\text{TEMPERATURE}(p, s) \rightarrow$ Temperature value of the spot near plant p
- $\text{MOISTPLANTS}(s) \rightarrow$ Number of moist plants

Non-Fluents

- $\text{WATERINGCAN}(x) \rightarrow$ Object x is a watering can
- $\text{THERMOMETER}(x) \rightarrow$ Object x is a thermometer
- $\text{MOISTUREMETER}(x) \rightarrow$ Object x is a moisturemeter



Actions

All actions are to be axiomatized as affecting only either the K fluent or other fluents.

Normal

- $\text{GOTO}(x) \rightarrow \text{Go to object } x$
- $\text{WATER}(p) \rightarrow \text{Water plant } p$
- $\text{PICKUP}(x) \rightarrow \text{Pick up object } x$
- $\text{PUTDOWN}(x) \rightarrow \text{Put down object } x$

Knowledge

- $\text{CHECKMOISTURE}(p) \rightarrow \text{Check moisture of plant } p$
- $\text{CHECKTEMPERATURE}(p) \rightarrow \text{Check the temperature of the spot near plant } p$



Sensing Result Axioms

- $\text{SR}(\text{GOTO}(x), s) = r \equiv r = \text{"OK"}$
- $\text{SR}(\text{CHECKMOISTURE}(p), s) = r \equiv (r = \text{"YES"} \wedge \text{MOIST}(p, s)) \vee (r = \text{"NO"} \wedge \neg \text{MOIST}(p, s))$
- $\text{SR}(\text{CHECKTEMPERATURE}(p), s) = r \equiv r = \text{TEMPERATURE}(p, s)$

Knowledge Action Effects

- **Kwhether**($\text{MOIST}(p), \text{DO}(\text{CHECKMOISTURE}(p), s)$)
- **Kref**($\text{TEMPERATURE}(p), \text{DO}(\text{CHECKTEMPERATURE}(p), s)$)



Preconditions

- $\text{POSS}(\text{WATER}(p), s) \equiv \text{NEAR}(p, s) \wedge \text{HOLDING}(x, s) \wedge \text{WATERINGCAN}(x) \wedge \mathbf{Knows}(\neg \text{MOIST}(p), s) \wedge \mathbf{Kref}(\text{TEMPERATURE}(p), s)$
- $\text{POSS}(\text{PICKUP}(x), s) \equiv \text{NEAR}(x, s) \wedge \neg \exists y. \text{HOLDING}(y, s)$
- $\text{POSS}(\text{PUTDOWN}(x), s) \equiv \text{HOLDING}(x, s)$
- $\text{POSS}(\text{CHECKMOISTURE}(p), s) \equiv \text{NEAR}(p, s) \wedge \text{HOLDING}(x, s) \wedge \text{MOISTUREMETER}(x)$
- $\text{POSS}(\text{CHECKTEMPERATURE}(p), s) \equiv \text{NEAR}(p, s) \wedge \text{HOLDING}(x, s) \wedge \text{THERMOMETER}(x)$



Successor State Axioms

In general $F(x, \text{DO}(\alpha, s)) \equiv \Phi_F^+(x, a, s) \vee (F(x, s) \wedge \neg \Phi_F^-(x, a, s))$

- $\text{NEAR}(x, \text{DO}(\alpha, s)) \equiv \alpha = \text{GOTO}(x) \vee (\text{NEAR}(x, s) \wedge \neg \exists y. \alpha = \text{GOTO}(y))$
- $\text{HOLDING}(x, \text{DO}(\alpha, s)) \equiv \alpha = \text{PICKUP}(x) \vee (\text{Holding}(x, s) \wedge \neg \exists r. \alpha = \text{PUTDOWN}(x))$
- $\text{MOIST}(p, \text{DO}(\alpha, s)) \equiv \alpha = \text{WATER}(p) \vee (\text{MOIST}(p, s) \wedge \neg \exists r. \alpha = \text{WATER}(p))$
- $\text{TEMPERATURE}(p, \text{DO}(\alpha, s)) \equiv \text{TEMPERATURE}(p, s)$
- $\text{MOISTPLANTS}(p, \text{DO}(\alpha, s)) = n \equiv (\text{MOISTPLANTS}(p, s) = n - 1 \wedge \alpha = \text{WATER}(p)) \vee (\text{MOISTPLANTS}(p, s) = n)$



1. Knowledge-producing actions do not change the state of the world. If $P(s)$ then $P(\text{DO}(\alpha, s))$, given that α is knowledge-producing and P is not K
2. Nothing is learned about P by doing action α , as long as α does not affect P .
3. Agents know the consequences of knowledge acquired through knowledge-producing actions.
4. If the agent knows P at s , then P is also known at $\text{DO}(\alpha, s)$ as long as the effect of α is not to make P false.
5. Agents know the effects of (ordinary) actions.

Initial Situation



- WATERINGCAN(c)
- THERMOMETER(t)
- MOISTUREMETER(m)
- PLANT(p_1)
- \vdots
- PLANT(p_n)
- NEAR(c)

Goal

Have n moist plants, namely $\text{MOISTPLANTS}(p, s) = n$



For every plant p , with the following situations

$$S_1 =$$
$$\text{DO}(\text{PUTDOWN}(m), \text{DO}(\text{CHECKMOISTURE}(p), \text{DO}(\text{GOTO}(p), \text{DO}(\text{PICKUP}(m), \text{DO}(\text{GOTO}(m), S_0))))))$$
$$S_2 =$$
$$\text{DO}(\text{PUTDOWN}(t), \text{DO}(\text{CHECKTEMPERATURE}(p), \text{DO}(\text{PICKUP}(t), \text{DO}(\text{GOTO}(t), S_1))))$$
$$S_3 =$$
$$\text{DO}(\text{PUTDOWN}(c), \text{DO}(\text{WATER}(p), \text{DO}(\text{PICKUP}(c), \text{DO}(\text{GOTO}(c), S_2))))$$

In the end, we need to entail that $\text{MOISTPLANTS}(p, s) = n$

We can use the successor state axioms for that



Legality testing (with a different S_0)

$$\text{DO}(\text{WATER}(p), \text{DO}(\text{PICKUP}(c), \text{DO}(\text{GOTO}(c), S_0)))$$


We need to entail

$$\text{NEAR}(p, S_0) \wedge \text{HOLDING}(t, S_0) \wedge \text{THERMOMETER}(t, S_0) \wedge \mathbf{Knows}(\text{MOIST}(p, S_0)) \wedge$$
$$\mathbf{Kref}(\text{TEMPERATURE}(p), \text{DO}(\text{CHECKTEMPERATURE}(p), S_0))$$

$$\exists y. y = \text{TEMPERATURE}(p, s)$$

Golog



```
1: while  $\neg$ Knows(HEALTHYPLANTS =  $n$ ) do
2:   ( $\Pi p$ ) GOTO( $p$ );
3:     CHECKMOISTURE( $p$ );
4:     if Knows( $\neg$ MOIST( $p$ ))
5:       GOTO( $t$ );
6:       PICKUP( $t$ );
7:       GOTO( $p$ );
8:       CHECKTEMPERATURE( $p$ );
9:       PUTDOWN( $t$ );
10:    GOTO( $c$ );
11:    PICKUP( $c$ );
12:    GOTO( $p$ );
13:    WATER( $p$ );
14:    PUTDOWN( $c$ );
15: end while
```

Knowledge, action, and the frame problem

Thank you for listening!
Any questions?