Q 1)

a) Since the tree is a complete one levels before the bottom one will always be full.

So $\sum_{n=0}^{n=h-1} 2^n + 1$ is for levels excluding the bottom level. for the bottom level

leaf count . height is sufficent.

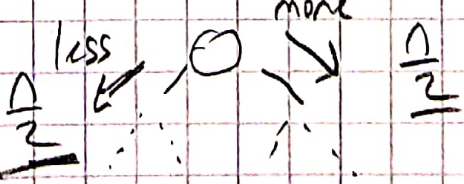
Finally summing the two:

h: height of tree
l: leaf count

$$\left(\sum_{n=0}^{n=h-1} 2^n + 1 \right) + l \cdot h$$

b) Since after every comparison we are left with roughly half of the elements the average number of comparisons ends up with:

$\rightarrow O(\log n)$



c) Yes, there is.

if tree is full one its node count is limited.

for internal nodes we should subtract 1 from number of leaves.

$i = l - 1$

internal and leaf nodes are connected in full binary tree.

proof by induction:

$L(n)$: number of leaves in a non-empty, full tree of n internal nodes.

Base: $L(0) \overset{!}{=} 1 = n+1$

Inductive: assume $L(i) = i+1$ for $i < n$

Given T with n nodes, remove two siblings.

T' has $n-1$ internal nodes, and by induction hypothesis $L(n-1) = n$ leaves

Replace Removed leaves to return to tree T.

Turns a leaf into an internal node, adds two new leaves.

thus $L'(n) = n+2-1 = n+1$.