EE3060A-Control Engineering Lab

Exp 2 - State Space Modelling and State Feedback Controller Design for a DC Motor

122201043(Parvat) and 122201020(Tridib Dey)

Objective :

To perform the state space modeling and pole placement through state feedback for a given DC motor system.

Theory:

If we assume the magnetic field is constant, then torque generated by a DC motor is directly proportional to armature current given by:

$$T = K_t i$$

The back emf, e, is proportional to the angular velocity of the shaft given by:

$$e = K_e \dot{\theta}$$

We are taking torque constant K_t and back emf constant K_e as K as they are equal in SI units.

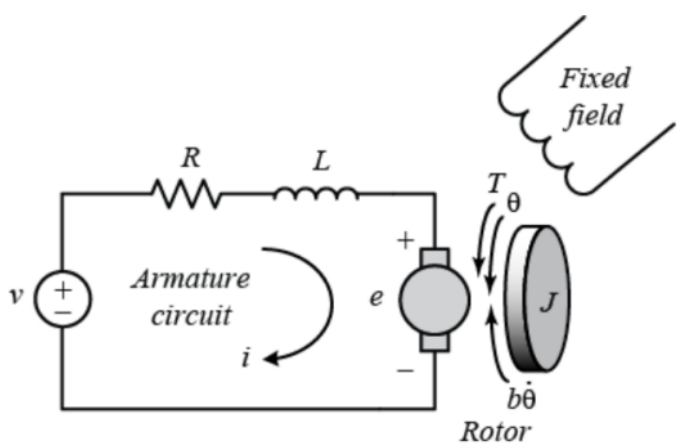


Figure 1: DC motor

From the figure above, by applying Newton's second law and Kirchhoff's voltage law, the following two equations can be derived:

$$J\ddot{\theta} + b\dot{\theta} = Ki$$

$$L\frac{di}{dt} + Ri = v - K\dot{\theta}$$

Controller design using state feedback:

Full state feedback, also known as pole placement, is a method employed in feedback control system theory to place the closed-loop poles of a plant at predetermined locations in the s-plane. For this technique, all state variables must be known to the controller. The schematic diagram of this control scheme is shown in Figure 2, where the reference pre-filter K_r is designed to eliminate the steady-state offset. It can be chosen as the reciprocal of the DC gain of the system in closed loop.

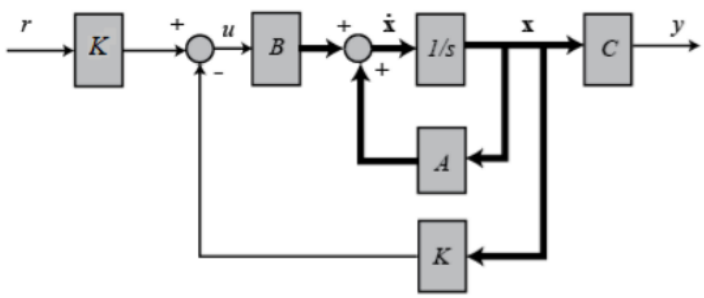


Figure 2: Pole placement technique

State-space representation:

State-space representation is a mathematical model used in control theory to describe the behavior of a dynamical system. In this representation, a system is described by a set of state variables, input variables, and output variables, which are related through a set of first-order differential equations.

The state-space representation of a linear time-invariant system can be written in matrix form as:

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

Where: x is the state vector, u is the input vector, y is the output vector, A is the state matrix, B is the input matrix, C is the output matrix, D is the feedforward matrix.

Eigen Values :

Eigenvalues of the system matrix A play a crucial role in analyzing the stability and behavior of the system. The eigenvalues represent the roots of the characteristic equation $\det(sI - A) = 0$, where I is the identity matrix. The stability of the system is determined by the real parts of the eigenvalues: if all eigenvalues have negative real parts, the system is stable; if any eigenvalue has a positive real part, the system is unstable. Eigenvalues also provide information about the modes of the system's response and its transient behavior.

Problem:

1. Make a choice of state variables as $x_1 = \omega = \dot{\theta}$, $x_2 = i$; input $u = v$ and output $y = \omega = x_1$.Obtain the state-space representation of the given system for the given choice of states,inputs, and output.
2. Using the physical parameters of the system provided in Table 1, simulate the system using ode function in MATLAB© editor.
3. Find out eigenvalues of the system both manually and through MATLAB©. Also, comment on the stability of the system based on it.
4. Using an arbitrary initial condition, design a state feedback controller $u = -K_1x_1 - K_2x_2$. Make a proper choice of feedback gains K_1 and K_2 to asymptotically stabilize the system states to the origin, i.e., $(x_1, x_2) = (0, 0)$.
5. Design a state feedback tracking controller to track unit step reference angular velocity (1 rad/s). [Note: Control law u takes the form, $u = -Kx + Kr_r$ (refer to Figure 2).]

Given Values :

Parameters	Description	Values
J	Rotor moment of inertia	0.01 kg/m^2
b	Motor viscous friction constant	0.1 N.m.s
K_e K_t	Back emf constant Motor torque constant	0.01 V/rad/sec 0.01 N.m/ Amp
R	Electric resistance	1 Ω
L	Electric inductance	0.5 H

Table 1: Parameter description and values

Solutions:

1. The system of equations is given by:

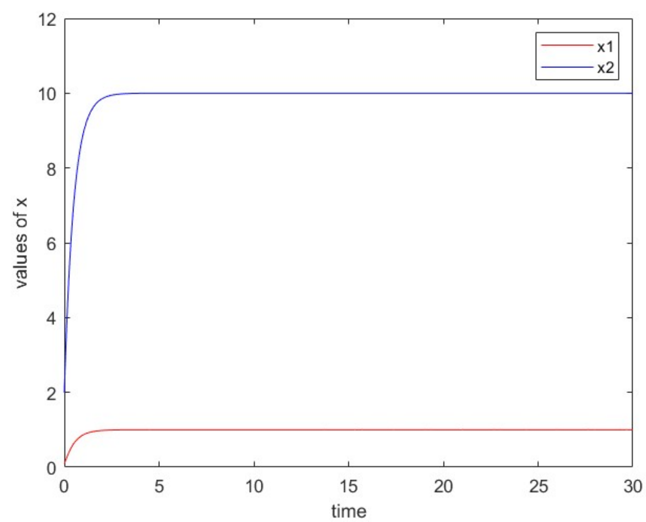
$$
\begin{aligned}
J\dot{x}_1 + bx_1 &= kx_2 \\
L\dot{x}_2 + Rx_2 &= u - Kx_1 \\
\dot{x}_1 &= \frac{K}{J}x_2 - \frac{B}{J}x_1 \\
\dot{x}_2 &= \frac{u}{L} - \frac{K}{L}x_1 - \frac{R}{L}x_2
\end{aligned}
$$

The state-state space representation of the system is:

$$
\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -\frac{b}{J} & \frac{K}{J} \\ -\frac{K}{L} & -\frac{R}{L} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} - \begin{bmatrix} 0 \\ \frac{1}{L} \end{bmatrix} u
$$

$$
y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}
$$

2. Required code is provided as "`ques_2`" and the output figure is provided as "`ques_2_fig`" in the submitted folder. The output figure is also provided below:

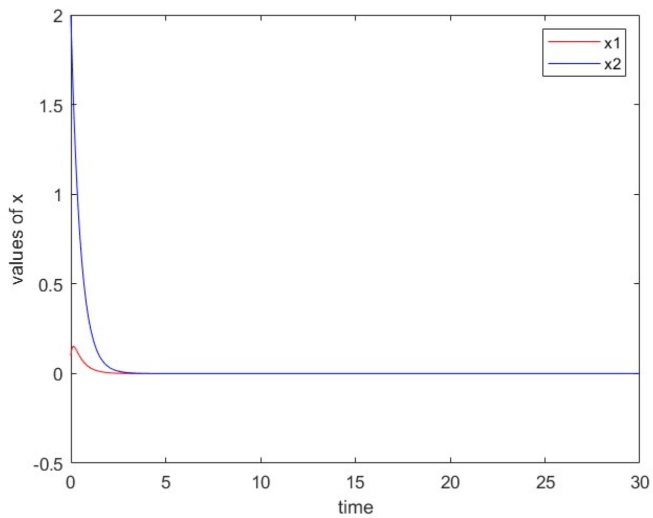


3. Required code is provided as `ques_3` in the submitted folder. As

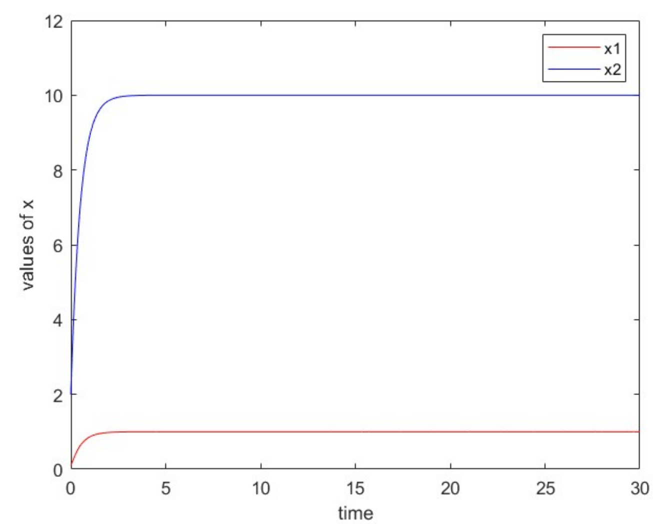
   Closed-loop system eigenvalues:
   -12.0025
   -19.9975
   As both eigenvalues are neagtive, system is stable.

4. Required code is saved as "`model_4`" and the required figure is saved as "`model_4_fig`" in the submitted folder. To run this file, open the file in

MATLAB with the "`ques_2`" code and change the model to `model_4` in "`ques_2`" and run "`ques_2`" file.



5. Required code is saved as "`model_5`" and the required figure is saved as "`model_5_fig`" in the submitted folder. To run this file, open the file in MATLAB with the "`ques_2`" code and change the model to `model_5` in "`ques_2`" and run "`ques_2`" file.



Results

In this experiment, we aimed to perform state space modeling and pole placement through state feedback for a given DC motor system. The following are the key results obtained:

1. **State-Space Representation:** By choosing the state variables as $x_1 = \omega = \dot{\theta}$ and $x_2 = i$, and the input as $u = v$, and the output as $y = \omega = x_1$, we obtained the state-space representation of the system. The state-space equations are given as:

$$\dot{x}_1 = \frac{K}{J}x_2 - \frac{B}{J}x_1$$
$$\dot{x}_2 = \frac{u}{L} - \frac{K}{L}x_1 - \frac{R}{L}x_2$$

2. **Simulation:** Using the physical parameters provided in Table 1, we simulated the system using the ode function in MATLAB.

3. **Eigenvalues Analysis:** The eigenvalues of the system matrix A were computed both manually and through MATLAB. The calculated eigenvalues were found to be -12.0025 and -19.9975. Based on these eigenvalues, we concluded that the system is stable as all eigenvalues have negative real parts.

4. **State Feedback Controller Design:** A state feedback controller was designed to asymptotically stabilize the system states to the origin ($x_1 = 0$, $x_2 = 0$) using an arbitrary initial condition. Proper choices of feedback gains K_1 and K_2 were made to achieve this stabilization.

5. **Tracking Controller Design:** Additionally, a state feedback tracking controller was designed to track a unit step reference angular velocity (1 rad/s). The control law u was formulated as $u = -Kx + Kr_r$, where K represents the feedback gains and r_r is the reference pre-filter.

Overall, the experimental results demonstrate the effectiveness of state-space modeling and state feedback control techniques in controlling the behavior of the DC motor system.

Inferences:

- State-space representation provides a concise and systematic way to model the dynamics of a dynamical system, such as the DC motor system considered in this experiment. By choosing appropriate state variables and input-output relationships, we can accurately describe the behavior of the system in terms of first-order differential equations.

- Eigenvalues analysis plays a crucial role in assessing the stability of the system. In this experiment, the computed eigenvalues indicated that the system is stable, as all eigenvalues have negative real parts. This implies that the system will converge to a steady state over time without exhibiting oscillations or instability.
- The design of state feedback controllers allows us to manipulate the system's behavior by adjusting the feedback gains. By appropriately choosing the feedback gains, we can achieve desired performance objectives, such as stabilizing the system states to the origin or tracking reference signals.
- The simulation results obtained through MATLAB provide valuable insights into the dynamic behavior of the DC motor system under different control strategies. The ability to simulate the system's response allows for thorough analysis and validation of the proposed control techniques before implementation in real-world applications.

Conclusion:

In conclusion, the experiment successfully demonstrated the effectiveness of state-space modeling and state feedback control techniques in analyzing and controlling the behavior of the DC motor system. By accurately modeling the system dynamics and designing appropriate feedback controllers, we can achieve stable and desired performance of the system, thus showcasing the importance and applicability of control theory principles in practical engineering systems.