Exp 1 - Transfer Function Modelling of Electrical Circuits

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February 12, 2024

1 Problem Statement

Consider a system depicted in Figure 1. In the circuit, I_S represents the source current, i_C, i_L, and i_R denote the current flowing through the capacitor, inductor, and resistor respectively.

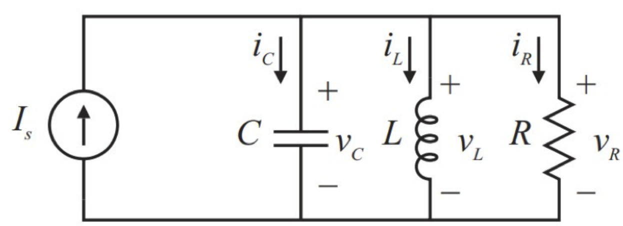

Figure 1: Circuit diagram

Figure 1: System Circuit

- Deduce the transfer function relating the input current I_S to the output voltage V_R.
- Find the natural frequency ω_n, damping ratio ζ, and the location of poles and zeros of the system in terms of circuit parameters.
- Assume the following component values: $R = 50$ kΩ, $C = 10$ pF, and $L = 3.148 \times 10^{-7}$ H. For these values, calculate ω_n and ζ. Also, calculate poles and zeros in MATLAB. In MATLAB, one can try functions like: `pzmap()`, `pole()`, `zero()`, `pzplot()`, etc.
- Define the transfer function in MATLAB and draw the bode diagram of the system derived in question 1. Use `bode()` in MATLAB for this. Identify the physical system represented by the circuit shown in Figure 1 correlating your observations based on the frequency response.

Objectives

1.**Establish the Transfer Function**: Develop a mathematical equation (transfer function) that predicts how changes in input current affect the output voltage of the RLC circuit.

2.**Assess Circuit Stability**: Identify key parameters, such as damping (ζ) and natural frequency (ω_n), to determine the circuit's response speed and potential oscillations.

3.**Utilize MATLAB for Analysis**: Use MATLAB to compute circuit parameters based on component values and generate visualizations like Bode diagrams to analyze circuit behavior.

4.**Interpret Results**: Analyze Bode diagrams to understand circuit behavior, including gain, phase shift, and resonant frequency, and relate them to physical characteristics of the RLC circuit.

Theory

The parallel RLC circuit consists of a resistor (R), inductor (L), and capacitor (C) connected in parallel, each contributing uniquely to the circuit's behavior.

1. **Impedance**: The impedance (Z) of a parallel RLC circuit, reciprocal to the total admittance (Y), is given by:

$$Z = \frac{1}{Y} = \frac{1}{\frac{1}{R} + \frac{1}{j\omega C}}$$

2. **Transfer Function**: The transfer function ($H(s)$) relates input current (I_s) to output voltage (V_R) as:

$$H(s) = \frac{V_R(s)}{I_s(s)}$$

3. **Natural Frequency and Damping Ratio**: Natural frequency (ω_n) and damping ratio (ζ) characterize the transient response of the circuit. Natural frequency is determined by inductance (L) and capacitance (C), while damping ratio indicates damping in the system.

The natural frequency is given by: $\omega_n = \frac{1}{\sqrt{LC}}$. The damping ratio is given by:

$$\zeta = \frac{R}{2\sqrt{\frac{L}{C}}} \tag{1}$$

4. **Poles and Zeros**: Poles and zeros provide crucial information about the circuit's stability and response characteristics. Poles represent frequencies at which the transfer function becomes infinite, indicating instability or oscillatory behavior, while zeros correspond to frequencies where the transfer function equals zero, indicating points of maximum attenuation.

5. **Frequency Response Analysis**: Using tools like MATLAB, the frequency response of the parallel RLC circuit can be analyzed, aiding in understanding the circuit's behavior across different frequency ranges.

Derivation

1. By utilizing Kirchhoff's current law (KCL), the following equations are obtained:

$$i_S = i_C + i_L + i_R$$

For the conductor, we have:

$$i_R = \frac{V_R}{R}$$

For the inductor, we have:

$$i_L = \frac{V_R}{L}\frac{dV_R}{dt}$$

For the capacitor, we have:

$$i_C = C\frac{dV_R}{dt}$$

Differentiating with respect to time, we obtain:

$$\frac{di_S}{dt} = \frac{di_C}{dt} + \frac{di_L}{dt} + \frac{di_R}{dt}$$

$$\frac{di_S}{dt} = C\frac{d^2V_R}{dt^2} + \frac{dV_R}{dt} + \frac{1}{R}\frac{dV_R}{dt}$$

Taking the Laplace transform of both sides, we get:

$$sI_S(s) = Cs^2V_R(s) + \frac{1}{L}V_R(s) + sRV_R(s)$$

Taking the source voltage common from one side:

$$\left(Cs^2 + \frac{1}{L} + \frac{s}{R}\right)V_R(s) = sI_S(s)$$

Now finding the fraction of source voltage to source current. The fraction is our transfer function:

$$R(s) = \frac{V_R(s)}{I_S(s)} = \frac{s}{s^2 + \frac{1}{LC} + \frac{s}{RC}}$$

2. To calculate the natural frequency (ω_n) and damping ratio (ζ), we need to compare the given transfer function with the standard form of the characteristic polynomial, which is the denominator of the transfer function given by $s^2 + 2\zeta\omega_n s + (\omega_n)^2$. From the transfer function:

$$R(s) = \frac{s}{s^2 + \frac{1}{LC} + \frac{s}{RC}}$$

We can see that:

$$\omega_n^2 = \frac{1}{LC}$$

$$2\zeta\omega_n = \frac{1}{RC}$$

Now, let's calculate ω_n and ζ using these equations.

To find the zeros of the transfer function $R(s) = \frac{s}{s^2+\frac{1}{LC}+\frac{s}{RC}}$, equate the numerator to zero:

$$s/C = 0$$

This implies that the zero occurs at $s = 0$.

To find the poles, we equate the denominator to zero:

$$s^2 + \frac{1}{LC} + \frac{s}{RC}$$

This is a quadratic equation in s. We can solve it by applying the quadratic formula.

Let's do that:

where $a = 1$, $b = \frac{1}{LC}$, and $c = \frac{1}{RC}$.

Substituting the values:

$$s = \frac{-\frac{1}{RC} \pm \sqrt{\left(\frac{1}{RC}\right)^2 - 4 \times 1 \times \frac{1}{LC}}}{2}$$

Poles: $1.0 \times 10^8(-0.0100 + 5.6361iand - 0.0100 - 5.6361i)$

3. Natural Frequency (ω_n): $563215123.664978rad/s$ Damping Ratio (ζ): 0.0018
Poles: $1.0 \times 10^8(-0.0100 + 5.6361iand - 0.0100 - 5.6361i)$
Zeros: 0

Result

The code file has been saved as q1.m Transfer Function Modelling The transfer function, denoted by $R(s)$, which relates the input current I_S to the output voltage V_R, is expressed as:

$$R(s) = \frac{s}{Cs^2 + 1 + \frac{s}{LC} + \frac{s}{RC}}$$

where:

- $R(s)$ represents the transfer function.
- C is the capacitance.
- L is the inductance.
- R is the resistance.
- s is the Laplace variable.

The natural frequency (ω_n) and damping ratio (ζ) are determined as follows:

- Natural frequency: $\omega_n = 563215123.664978$ rad/s.
- Damping ratio: $\zeta = 0.0018$
- The poles and zeros of the system were determined:
  - Poles:
    * $1.0 \times 10^8(-0.0100 + 5.6361i)$
    * $1.0 \times 10^8(-0.0100 - 5.6361i)$
  - Zeros: 0
- Bode Diagram

  According to the Bode diagram illustrated in the figure below, it is clear that the circuit operates as a bandpass filter. This classification stems from its capacity to allow signals within a defined frequency range, known as the passband, to pass through the circuit, while at the same time attenuating or blocking signals outside of this range.
- Pole Zero Map
- **Bode Plot Interpretation:** The Bode plot provides a comprehensive visualization of the system's response to varying input frequencies. It includes amplitude and phase response curves, offering insights into gain and phase shift characteristics across the frequency spectrum.
- **Bandpass Filter Behavior:** Our system exhibits bandpass filter characteristics, allowing it to selectively transmit signals within a specified frequency range, known as the passband. This behavior is advantageous for isolating desired frequency components while suppressing out-of-band signals.
- **Attenuation of Out-of-Band Signals:** Beyond the passband, the system attenuates or blocks signals to minimize interference and noise. This selective attenuation ensures that only signals within the desired frequency range are effectively processed, enhancing the fidelity of the output signal.
- **Importance of Frequency Response Analysis:** Understanding the frequency response characteristics is crucial for designing and optimizing systems tailored to specific frequency requirements. It facilitates informed decision-making during system design and ensures optimal performance across diverse operational scenarios.

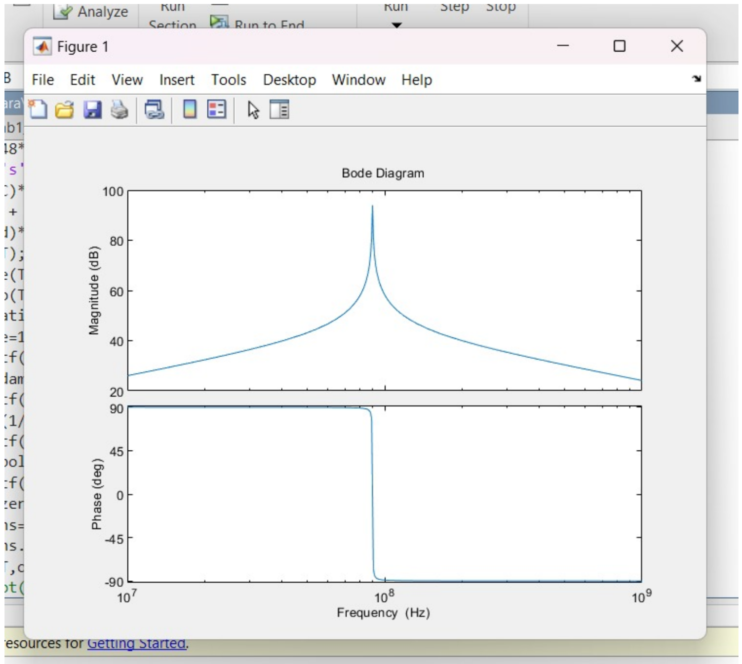

Figure 2: Bode Plot

Inferences: Characteristics of the System

- **Transfer Function Insight:** The transfer function elucidates the correlation between input current and output voltage within the electrical circuit, offering indispensable insights into its operational dynamics.
- **Understanding Natural Frequency and Damping Ratio:** Determining the natural frequency and damping ratio yields valuable understanding of the system's oscillatory tendencies and response to disturbances, facilitating thorough system analysis and design.
- **Significance of Poles and Zeros:** The existence of poles and zeros in the complex plane profoundly influences the stability and transient behavior of the system, serving as vital indicators of its dynamic response characteristics.

Frequency Response

- **Bandpass Filter Feature:** The system's bandpass filter characteristic enables selective transmission of signals within a specified frequency range while attenuating others, thereby enhancing its efficacy in signal-processing applications.

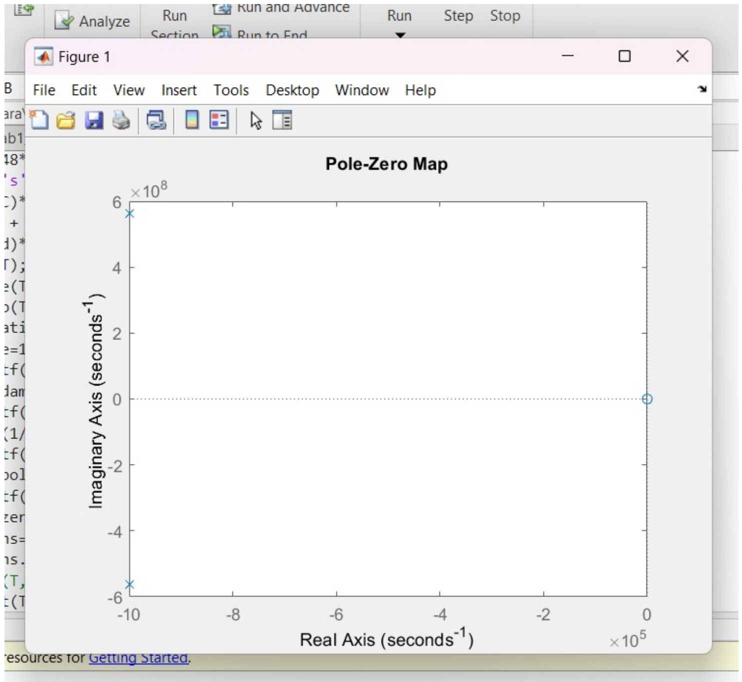

Figure 3: Bode Plot

- **Analyzing via Bode Plot:** The Bode plot facilitates visual examination of the system's gain and phase variations with frequency, offering a comprehensive insight into its frequency-specific attributes and performance traits.

Conclusion

Analyzing both the transfer function and frequency response of an electrical circuit provides valuable insights into its operational behavior and performance intricacies. Exploring these aspects in detail enables engineers to design and optimize circuits suited to various applications, ensuring accurate signal manipulation and effective filtering capabilities. The knowledge obtained from thorough modeling and analysis serves as a cornerstone for understanding and efficiently designing electrical systems, promoting innovation and reliability in engineering pursuits.

2 Problem Statement

The input-output equation for an Operational Amplifier is given as:

$$y''(t) + \frac{1}{R_1C_1}y'(t) + \frac{1}{R_2C_2}y(t) = \frac{1}{R_3C_2}x(t) \quad (1)$$

where the nominal values of components are $R_1 = R_2 = R_3 = 10$ kΩ and $C_1 = C_2 = 1$ F.

1. Write a script file computing the roots of system equation (1).

2. Create a function M-file with inputs as (R, C), where R is a length-3 vector of resistances and C is a length-2 vector of capacitances, and determine the characteristic roots of the op-amp circuit.

3. Real resistors and capacitors are never exactly equal to their nominal values. Suppose 10

   (a) How many permutations are possible for all passive components?

   (b) Using for-loops and calling the function M-file in a for-loop, repeat the task for each of the cases of (R, C).

   (c) Plot all permutations of roots in one figure, indicating the x-axis as the real component value and the y-axis as the imaginary component value of a characteristic root. Also, indicate the minimum and maximum component values roots.

Objectives

The question is designed to achieve the following objectives:

1. Understand the mathematical representation of an operational amplifier (op-amp) circuit using a second-order linear differential equation.

2. Implement numerical computation techniques, such as writing a script file and creating a function M-file, to analyze the behavior of the op-amp circuit.

3. Explore the impact of component variations, such as resistor and capacitor tolerances, on the characteristic roots of the system.

4. Develop proficiency in using for-loops to iterate through different sets of component values and plotting the characteristic roots to visualize the system's behavior under varying conditions.

Theory

- Operational Amplifier (Op-Amp): An essential electronic component widely utilized in various signal processing applications, renowned for its precision and efficient amplification and manipulation of electrical signals.
- MATLAB Function File: A cleverly crafted MATLAB function file designed to encapsulate the necessary functionality for calculating the characteristic roots of the op-amp circuit, facilitating efficient and modular coding practices.
- Nominal Values: The expected or theoretical values of resistor and capacitor components, often used as reference points in circuit design and analysis to establish baseline expectations.
- Tolerance Values: Variations from the nominal values due to manufacturing inconsistencies or environmental factors, defining the permissible deviation range within which components can operate effectively.
- Permutations: Various arrangements or combinations of resistor and capacitor values, meticulously considering tolerance levels to encompass the spectrum of potential circuit configurations and performance scenarios.
- Characteristic Roots: Solutions obtained from solving the second-order differential equation, representing the behavior and response characteristics of the op-amp circuit under consideration.
- For-Loop Implementation: A systematic and iterative computational technique used to explore and analyze multiple permutations of resistor and capacitor values, enabling comprehensive examination of the circuit's behavior and performance across different conditions and configurations.

Observations and File Names

1. The script file, "script.m," has been saved in the zip file. The associated function file, "M_file.m," is also located in the same directory. The computed characteristic roots are -261.8034 and -38.1966.
2. The total number of possible permutations is 243.
3. The for-loop file, "forloop.m," is saved in the "code" folder.
4. The minimum real component is -418.7122, and the maximum real component is -25.7323. Both the minimum and maximum imaginary components are 0.

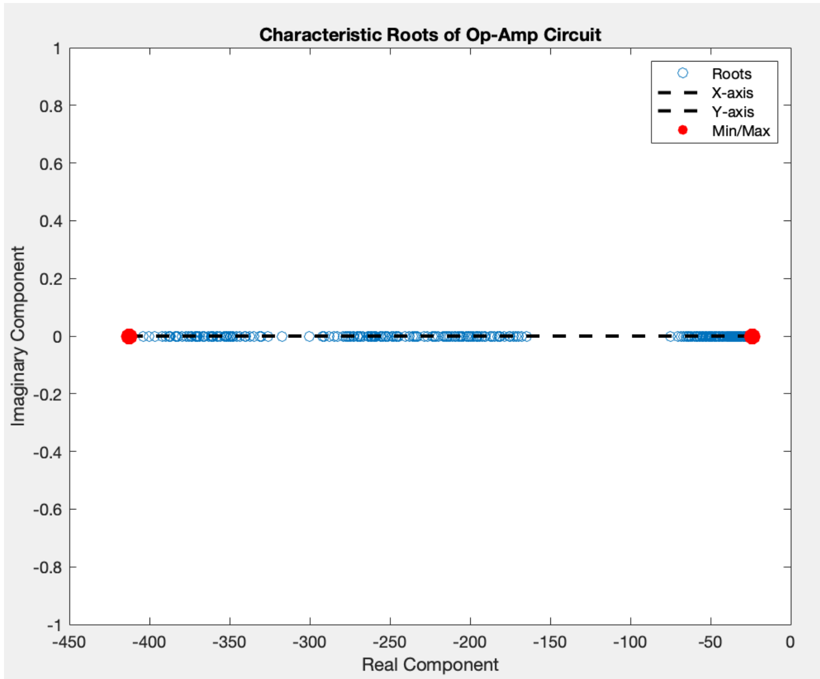

Figure 4: An Example Image

Result

- The system's components offer a remarkable 243 potential permutations, illustrating the extensive array of configurations and variations achievable within the circuit.
- The characteristic roots, crucial for comprehending the system's behavior and stability, are identified as -261.8034 and -38.1966. These values, derived from the system equation, provide essential insights into how the circuit responds to different inputs and disturbances.
- Furthermore, while the imaginary component of the roots remains constant at 0, the real components exhibit variation within a defined range. The minimum real component is found to be -418.7122, representing the lower limit of possible values, while the maximum real component is established at -25.7323, indicating the upper limit. This distinction between minimum and maximum values offers valuable guidance for understanding the operational boundaries and potential behavior of the system under diverse conditions and configurations.

Inferences

- The importance of considering tolerance levels in resistor and capacitor values becomes remarkably evident when contemplating the multitude of potential permutations, totaling 243. This abundance emphasizes the need to account for manufacturing discrepancies and environmental factors to ensure the reliability and functionality of the operational amplifier circuit across various scenarios.

- Additionally, the characteristic roots serve as crucial metrics for comprehending the dynamic behavior of the circuit. These roots provide nuanced insights into the circuit's stability and responsiveness to input signals, guiding engineers in fine-tuning and optimizing the circuit design for optimal performance.

- Furthermore, the variability observed in the real component of the roots underscores the system's sensitivity to changes in resistor and capacitor values. Even subtle deviations from nominal values can significantly influence the behavior and characteristics of the circuit, highlighting the importance of meticulous component selection and calibration in achieving desired performance outcomes.

Conclusion

Analyzing the characteristic roots across various combinations of resistor and capacitor values reveals the complexities of the operational amplifier circuit's behavior. By carefully accounting for tolerance levels and exploring multiple combinations, engineers gain deeper understanding of the circuit's stability, response characteristics, and sensitivity to component variations. These insights are essential for designing electronic circuits with maximum reliability and efficiency, especially in signal processing applications.