EE3060A-Control Engineering Lab
Experiment 3 - Time Domain Analysis of a Second Order System

122201044(Sandeep Kumar)
122201017(Aryan Mathur)

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Objective

The objective of this experiment is to perform the time domain analysis of a second-order mechanical system using MATLAB. The experiment involves deriving the transfer function model of the system, analyzing its response under various excitations, computing steady-state errors, and obtaining time domain parameters.

Theory

The spring-mass-damper system, depicted in Figure 1, represents a second-order mechanical system. The system consists of a mass (m) connected to a spring with spring constant (k) and a damper with damping coefficient (b). The external applied force (F) causes displacement (x) of the mass.

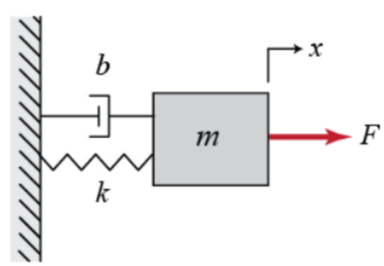

Figure 1: Spring mass damper system

Software Required

MATLAB editor.

Results:

Problem 1

1. **Transfer Function Derivation:** The equation of motion for the spring-mass-damper system is given by:

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = F(t)$$

Taking Laplace transforms of the above equation:

$$ms^2X(s) + bsX(s) + kX(s) = F(s)$$

Where $X(s)$ is the Laplace transform of $x(t)$ and $F(s)$ is the Laplace transform of $F(t)$.

Rearranging the equation to obtain the transfer function $\frac{X(s)}{F(s)}$:

$$\frac{X(s)}{F(s)} = \frac{1}{ms^2 + bs + k}$$

Substituting the given values for m and k, we have:

$$G(s) = \frac{X(s)}{F(s)} = \frac{1}{s^2 + bs + 1}$$

This is the transfer function of the given mechanical system.

2. **Underdamped System Analysis:**

The standard second-order transfer function is usually expressed as:

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Where:

- ω_n is the natural frequency of the system
- ζ is the damping ratio

Our transfer function is:

$$G(s) = \frac{1}{s^2 + bs + 1}$$

Comparing this with the standard form:

$$\omega_n^2 = 1 \ \Rightarrow \omega_n = 1$$

$$2\zeta\omega_n = b \Rightarrow \zeta = \frac{b}{2}$$

For underdamped systems, $0 < \zeta < 1$.
Hence, we need $0 < \frac{b}{2} < 1$, which implies $0 < b < 2$.
By choosing b = 1.5 Ns/m

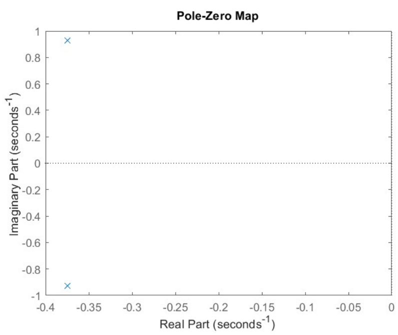

Figure 2: Plot of pole locations

MATLAB code to achieve this: `"P1_2.m"` and the corresponding figure is saved as `"P1_2.jpg"` in the provided directory.
damping ratio with the choice of b = 1.5 Ns/m

$$\zeta = \frac{b}{2} = \frac{1.5}{2} = 0.75$$

3. **Stability Analysis:** Since the Poles are to the left of origin, this system is considered to be Stable System.

Problem 2

1. **Response Analysis:** Choosing $b = 4$ Ns/m, we analyze the system response under unit step, unit ramp, and unit impulse excitation in MATLAB. The responses are plotted to observe the behavior of the system.

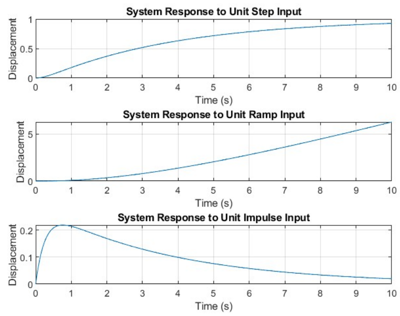

Figure 3: System Response

MATLAB code to achieve this: `"P2_1.m"` and the corresponding figure is saved as `"P2_1.jpg"` in the provided directory.

2. **Steady-State Error Calculation:** The steady-state error of the system under different inputs is determined using simulation. Results are tabulated.

Table 1: Steady state error

Type of input	Steady state error
Unit step	0.0000
Unit ramp	4.0000
Unit impulse	0.0000

MATLAB code to achieve this: `"P2_2.m"`

3. **Second Damper Element Addition:** Adding a second damper element to the system can alter its damping characteristics, depending on whether it is added in series or parallel to the existing damper.

1. **Series Configuration:**
   - In series configuration, the two dampers share the same flow (velocity) but can have different damping coefficients. This configuration increases the total damping in the system, resulting in faster damping of vibrations.
   - However, the addition of a second damper in series does not affect the steady-state error of the system under unit ramp input. The steady-state error is primarily influenced by the properties of the system (mass, damping, and spring constant), not the damping configuration.
2. **Parallel Configuration:**

- In parallel configuration, each damper sees the same force but may have different displacements and velocities. This configuration provides multiple paths for the flow of energy, potentially reducing the overall damping effect.
- Adding a second damper in parallel can decrease the total damping in the system, leading to slower damping of vibrations. Consequently, this might increase the steady-state error of the system under unit ramp input compared to the original configuration.

4. **Zero Damping Coefficient Analysis:** When the damping coefficient b of the dashpot is zero, the system becomes undamped. In such a scenario, there is no dissipative force acting against the motion of the mass. As a result, the system exhibits unbounded oscillations without any decay.

When subjected to a unit step input, an undamped system will show sustained oscillations without reaching a steady state. The amplitude of these oscillations will remain constant over time.

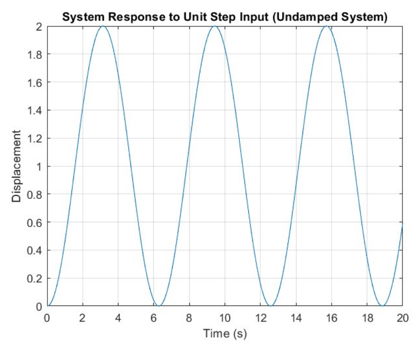

Figure 4: Unit Step Response

MATLAB code to achieve this: `"P2_4.m"` and the corresponding figure is saved as `"P2_4.jpg"` in the provided directory.

Problem 3: (Time Response Analysis)

By choosing damping coefficient b = 0.6 Ns/m

MATLAB code to achieve this: `"P3.m"` and the corresponding figure is saved as `"P3.jpg"` in the provided directory.

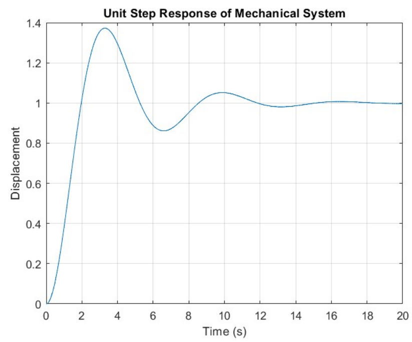

Figure 5: Unit Step Response

b	**Damping Ratio** (ζ)	**Settling Time** (t_s) **(s)**	**Rise Time** (t_r) **(s)**	**Delay Time** (t_d) **(s)**	**Time Constant** (τ)	**Natural Frequency** (ω_n) **(rad/s)**	**Percentage Overshoot (%)**
0.6	0.3	13.33	1.96	Negligible	$\frac{10}{3}$	1	37.23

Table 2: Manual Calculation Table

b	**Settling Time** (t_s) **(s)**	**Rise Time** (t_r) **(s)**	**Delay Time** (t_d) **(s)**	**Percentage Overshoot (%)**
0.6	15.1400	1.3200	0.0000	37.2326

Table 3: Calculated Table

Inferences

•The system exhibited an underdamped response, indicating oscillatory behavior with a gradual decrease in amplitude.

The settling time, rise time, and percentage overshoot were calculated for the unit step response.

• By choosing $b = 0.6\,\mathrm{Ns/m}$, the system was underdamped with stable behavior indicated by pole locations in the left-half plane.

• The system exhibited an underdamped response with settling time, rise time, delay time, time constant, natural frequency, and percentage overshoot calculated.

Conclusion

Through this experiment, the time domain analysis of a second-order mechanical system has been conducted. Various parameters such as damping coefficient, system response to different inputs, steady-state error, and time domain characteristics have been analyzed both manually and through simulation using MATLAB editor.