AI353-Quiz I (Make-up Version)

Duration: 45 minutes   **Total pts:** 20

1. (a) Write the general form of a single-qubit pure state using Bloch sphere parameters θ, ϕ. Show explicitly how normalization is ensured. (*3 pts*)
   (b) For the state

   $$|\psi\rangle = \tfrac{1}{\sqrt{3}}|0\rangle + i\sqrt{\tfrac{2}{3}}\,|1\rangle,$$

   determine its Bloch sphere parameters θ, ϕ. (*3 pts*)

2. Consider the Bell state

   $$|\beta_{00}\rangle = \tfrac{1}{\sqrt{2}}(|00\rangle + |11\rangle).$$

   (a) If the first qubit is measured in the computational basis, compute the possible outcomes, their probabilities, and the post-measurement states.(*3 pts*)
   (b) Suppose instead both qubits are measured in the Hadamard basis $\{|+\rangle, |-\rangle\}$. Explain the possible correlations in outcomes.(*3 pts*)

3. (a) Describe the steps of the standard quantum teleportation protocol and explain why two classical bits are needed. (*3 pts*)
   (b) Suppose Alice omits the Hadamard and measurement on the first qubit, but still sends only the measurement result of the second qubit to Bob. Explain clearly why Bob cannot recover the original state $|+\rangle$. (*2 pts*)
   (c) Given two distinct orthonormal bases $\{|v_i\rangle\}$ and $\{|w_i\rangle\}$, explain how the matrix representations of a linear operator A in these two bases are related. (*3 pts*)

End of Paper — Answer all questions clearly.