AI353: Introduction to Quantum Computing

Tutorial Sheet - Week 2

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Instructions

This tutorial focuses on the topic **Linear Algebra**, as covered in lectures. You are encouraged to work in pairs for solving these problems. Active participation during the tutorial session is rewarded: volunteers who solve and present a problem on the whiteboard will receive points.

Points Distribution:

- Attendance: 15 points
- Active participation: 10 points
- **Total: 25 points**

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1. Show that the set $\{(1,1),(1,-1)\}$ is linearly independent in $\mathbb{R}^2$.

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2. Write down the 2×2 identity matrix and compute its action on the vector $(2,3)$.

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3. Suppose $\{|v_i\rangle\}$ is an orthonormal basis for an inner product space V. What is the matrix representation for the operator $|v_j\rangle\langle v_k|$, with respect to the $\{|v_i\rangle\}$ basis?

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4. Verify that the Pauli matrices X and Y anticommute, i.e., $XY = -YX$.

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5. Show that all eigenvalues of a unitary matrix have modulus 1, i.e., they can be written in the form $e^{i\theta}$ for some real θ.

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6. Show that the Pauli matrices are Hermitian and unitary.

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7. Let

$$B = \begin{bmatrix} 0 & 1+i \\ 1-i & 0 \end{bmatrix}.$$

Show that B is Hermitian.

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8. Let V be the subspace of $\mathbb{R}^3$ spanned by $\{(1,0,1),(0,1,1),(1,1,2)\}$. Find a basis for V and compute its dimension.

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9. Let $|\psi\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$. Write out $|\psi\rangle^{\otimes 2}$ and $|\psi\rangle^{\otimes 3}$ explicitly, both in terms of tensor products like $|0\rangle|1\rangle$, and using the Kronecker product.

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10. Calculate the matrix representation of the tensor products of the Pauli operators:

    (a) X and Z

    (b) I and X

    (c) X and I

    Is the tensor product commutative?

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11. Show that the transpose, complex conjugation, and adjoint operations distribute over the tensor product:

$$(A \otimes B)^* = A^* \otimes B^*; \quad (A \otimes B)^T = A^T \otimes B^T; \quad (A \otimes B)^\dagger = A^\dagger \otimes B^\dagger.$$