Quiz I

Introduction to Quantum Computing (AI353)

Duration: 40 minutes

Date: 18/09/2025
Total marks: 20

Name: ____________________________
Roll Number: ____________________________

Instructions:

1. Attemp all the 20 questions. Each question has one correct option. You get +1 mark per correct answer.
2. Show your answers clearly by circling or ticking the right option in this sheet.
3. Use of **mobile phones, smart watches, or any other electronic gadget is strictly prohibited**.
4. If you are caught using unfair means, you will be disqualified from this quiz.

Q1. Bloch-sphere parameterization: A qubit is in the state

$$|\psi\rangle = \frac{1}{\sqrt{3}}|0\rangle + i\sqrt{\frac{2}{3}}\,|1\rangle.$$

Expressing $|\psi\rangle$ as $\cos(\theta/2)|0\rangle + e^{i\phi}\sin(\theta/2)|1\rangle$ (global phase ignored) gives which of the following (θ, ϕ) pairs?

(a) $\theta = \arccos\sqrt{\frac{1}{3}},\ \phi = \frac{\pi}{2}$

(b) $\theta = 2\arccos\sqrt{\frac{2}{3}},\ \phi = \pi$

(c) $\theta = 2\arccos\sqrt{\frac{1}{3}},\ \phi = \frac{\pi}{2}$

(d) $\theta = 2\arccos\sqrt{\frac{2}{3}},\ \phi = -\frac{\pi}{2}$

Q2. For $|\phi\rangle = a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle$, suppose measuring the first qubit yields outcome 0 with probability 1/3. Which constraint ensures normalization?

(a) $|a|^2 + |b|^2 = \frac{1}{3}$ and $|c|^2 + |d|^2 = \frac{2}{3}$

(b) $|a|^2 + |b|^2 + |c|^2 + |d|^2 = \frac{1}{3}$

(c) $|a| + |b| = \frac{1}{\sqrt{3}},\ |c| + |d| = \sqrt{\frac{2}{3}}$

(d) $|a|^2 + |b|^2 - |c|^2 - |d|^2 = \frac{1}{3}$

Q3. Let $Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$, which is a Pauli matrix. Which is correct?

(a) Eigenvalues ± 1, unitary and Hermitian

(b) Eigenvalues $\pm i$, unitary but not Hermitian

(c) Eigenvalues ± 1, Hermitian but not unitary

(d) Eigenvalues $\pm i$, neither unitary nor Hermitian

Q4. Compute the spectral decomposition of Pauli Z and select the correct option below.

(a) $Z = |0\rangle\langle 0| - |1\rangle\langle 1|$

(b) $Z = |0\rangle\langle 0| + |1\rangle\langle 1|$

(c) $Z = |0\rangle\langle 1| + |1\rangle\langle 0|$

(d) $Z = -|0\rangle\langle 0| + |1\rangle\langle 1|$

Q5. Define $U = \begin{bmatrix} 0 & e^{-i\pi/4} \\ e^{i\pi/4} & 0 \end{bmatrix}$. Which is correct?

(a) Unitary, maps $|0\rangle \mapsto e^{i\pi/4}|1\rangle$, $|1\rangle \mapsto e^{-i\pi/4}|0\rangle$

(b) Hermitian and unitary, equal to Pauli X

(c) Not unitary

(d) Equal to Hadamard

Q6. For $|\beta_{00}\rangle = (|00\rangle + |11\rangle)/\sqrt{2}$, measure first qubit in computational basis:

(a) Outcome 0, prob 1/2, leaving $|00\rangle$; outcome 1, prob 1/2, leaving $|10\rangle$

(b) Outcome 0, prob 1/2, leaving $|00\rangle$; outcome 1, prob 1/2, leaving $|11\rangle$

(c) Outcome 0, prob 1, leaving $|00\rangle$

(d) Outcome 1, prob 1/2, leaving $|10\rangle$; outcome 0, prob 1/2, leaving $|01\rangle$

Q7. For $|\psi\rangle = |0\rangle + |1\rangle$ and $|\varphi\rangle = |0\rangle - |1\rangle$, what is $|\psi\rangle \otimes |\varphi\rangle$?

(a) $|00\rangle + |01\rangle + |10\rangle + |11\rangle$

(b) $|00\rangle - |01\rangle + |10\rangle - |11\rangle$

(c) $|00\rangle + |11\rangle$

(d) $|01\rangle - |10\rangle + |11\rangle - |00\rangle$

Q8. For any unitary matrix U, which of the following statements is true about its eigenvalues?

(a) Eigenvalues can always be written as $\lambda e^{i\theta}$ for some real θ and λ such that $\lambda > 1$.

(b) Eigenvalues can always be written as $e^{i\theta}$ for some real θ.

(c) Eigenvalues cannot be complex numbers.

(d) None of the above.

Q9. For $|\psi\rangle = (|000\rangle + |111\rangle)/\sqrt{2}$, what is the outcome of measuring the first two qubits in Bell basis:

(a) Outcome $|\beta_{00}\rangle$, prob 1/2, third qubit $|+\rangle$; outcome $|\beta_{01}\rangle$, prob 1/2, third qubit $|-\rangle$.

(b) Outcome $|\beta_{10}\rangle$, prob 1/2, third qubit $|0\rangle$; outcome $|\beta_{11}\rangle$, prob 1/2, third qubit $|1\rangle$

(c) Outcome $|\beta_{00}\rangle$, prob 1, third qubit $|+\rangle$

(d) Outcome $|\beta_{01}\rangle$, prob 1, third qubit $|-\rangle$

Q10. Recall superdense coding, where the Shared Bell state was $(|00\rangle + |11\rangle)/\sqrt{2}$. To encode bits "10", Alice applies?

(a) No gate

(b) Pauli X

(c) Pauli Z

(d) Hadamard

Q11. Recall postulate 2: Evolution. Which is correct amongst the following?

(a) Closed system evolves unitarily

(b) Any linear transformation

(c) Described by measurement operators

(d) Maps pure to mixtures

Q12. Consider the qubit states

$$|\psi_1\rangle = \frac{|0\rangle+|1\rangle}{\sqrt{2}}, \qquad |\psi_2\rangle = \frac{|0\rangle-|1\rangle}{\sqrt{2}}.$$

Alice sends Bob either $|\psi_1\rangle$ or $|\psi_2\rangle$. Which measurement allows Bob to distinguish these states with certainty?

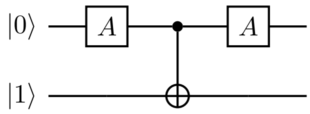
(a) No measurement can reliably distinguish the two states in $\mathbb{C}^2$.

(b) Measurement in the computational basis $\{|0\rangle, |1\rangle\}$.

(c) Measurement in the Hadamard basis $\{|+\rangle, |-\rangle\}$.

(d) Measurement in any orthonormal basis will always suffice.

Q13. Observe the circuit and select the correct option.



It is claimed that the output equals $|0\rangle \otimes |0\rangle$ for the given input $|0\rangle \otimes |1\rangle$.

(a) Such a circuit is impossible due to the no-cloning theorem.

(b) There exists a unitary A for which the claim holds.

(c) The claim holds precisely when $A = H$ (Hadamard).

(d) No unitary A can make the claim true.

Q14. In the quantum teleportation protocol, Alice wishes to send the state $|\psi\rangle = |+\rangle$ to Bob. She modifies the standard protocol by removing the Hadamard gate and the measurement on the topmost wire, while keeping everything else the same. She measures only the second qubit, obtaining outcome M_2, and sends this classical bit to Bob. Bob then applies the correction X^{M_2} to his half of the shared Bell state $|\beta_{00}\rangle$.

Which of the following statements is correct?

(a) Sending only M_2 suffices: Bob always recovers $|+\rangle$ after applying X^{M_2}.

(b) Alice should send the two-bit string $0M_2$ so that Bob can recover $|+\rangle$.

(c) Alice should send the two-bit string $1M_2$ so that Bob can recover $|+\rangle$.

(d) No classical message derived only from M_2 allows Bob to recover $|+\rangle$; the missing Hadamard gate and the top measurement are essential.

Q15. Given a vector space V with two distinct orthonormal bases $\{|v_i\rangle\}$ and $\{|w_i\rangle\}$, let A' and A'' be the matrix representations of a linear operator A in these bases. Which of the following is true?

(a) A'_{ij} can be represented as a linear combination of all entries of the matrix A''.

(b) A' and A'' are the same.

(c) A' and A'' are always related as: $A'_{ij} = \sum_k c_k A''_{kk}$ for some $c_k \in \mathbb{C}$

(d) There is no relationship between the two matrix representations of the linear operator A.

Q16. Let A be a positive operator. Which of the following statements is **false**?

(a) A is Hermitian.

(b) $A^\dagger A$ is a positive operator.

(c) A can be written as $A = B + iC$, where B and C are Hermitian.

(d) A does not admit a spectral decomposition.

Q17. Given a projector P on the subspace spanned by $|v_j\rangle \in \{|v_i\rangle\}$, where $\{|v_i\rangle\}$ is an orthonormal basis of the vector space V. Select the correct option below.

(a) P can be written as $\sum_i |v_i\rangle\langle v_i|$.

(b) $P \otimes P$ is also a projector.

(c) $P^2 = I$.

(d) $\forall k \neq j$, we have that $\langle v_k|P|v_k\rangle > 0$.

Q18. Which of the following is the correct expression for Hadamard operator for one qubit?

(a) $H = \frac{1}{\sqrt{2}}\left[(|0\rangle + |1\rangle)\langle 0| + (|0\rangle - |1\rangle)\langle 1|\right]$.

(b) $H = \frac{1}{\sqrt{2}}\left[(|0\rangle + |1\rangle)\langle 1| + (|0\rangle - |1\rangle)\langle 0|\right]$.

(c) $H = \frac{1}{2}|0\rangle\langle 0| + \frac{1}{2}|1\rangle\langle 1|$

(d) $H = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) + \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$

Q19. In the computational basis, what best describes the measurement operator for a single qubit $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$

(a) For outcomes, $m = 0, 1$; $M_0 = |0\rangle\langle 0|$ and $M_1 = |1\rangle\langle 1|$.

(b) For outcomes, $m = 0, 1$; $M_0 = |1\rangle\langle 1|$ and $M_1 = |0\rangle\langle 0|$.

(c) For outcomes, $m = 0, 1$; $M_0 = |0\rangle\langle 0| + |1\rangle\langle 1|$ and $M_1 = |0\rangle\langle 0| - |1\rangle\langle 1|$.

(d) Any of the above is good for measuring $|\psi\rangle$.

Q20. Suppose we prepare a quantum system in an eigenstate $|\psi\rangle$ of some observable M, with corresponding eigenvalue m. What is the average observed value of M, and the standard deviation?

(a) Average observed value of M is m and the standard deviation is 0.

(b) Average observed value of M is m and the standard deviation is m.

(c) Average observed value of M is m^2 and the standard deviation is 1.

(d) Average observed value and the standard deviation of M can't be computed.

End of Paper — Check your answers carefully