# Quiz I(Answer keys)

**Introduction to Quantum Computing (AI353)**

**Duration: 40 minutes**

*Date: 18/09/2025*
*Total marks: 20*

1. **(c)** General qubit form: $|\psi\rangle = \cos(\frac{\theta}{2})|0\rangle + e^{i\phi}\sin(\frac{\theta}{2})|1\rangle$. Here $|\alpha| = \frac{1}{\sqrt{3}}$, so $\theta = 2\arccos(\frac{1}{\sqrt{3}})$, and phase $\phi = \frac{\pi}{2}$.
2. **(a)** $\Pr(\text{first qubit} = 0) = |a|^2 + |b|^2 = \frac{1}{3}$. Normalization forces $|c|^2 + |d|^2 = \frac{2}{3}$.
3. **(a)** $Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$ is Hermitian and unitary, eigenvalues $\pm 1$.
4. **(a)** Spectral decomposition: $Z = |0\rangle\langle 0| - |1\rangle\langle 1|$.
5. **(a)** $U = \begin{bmatrix} 0 & e^{-i\pi/4} \\ e^{i\pi/4} & 0 \end{bmatrix}$ is unitary; maps $|0\rangle \to e^{i\pi/4}|1\rangle$, $|1\rangle \to e^{-i\pi/4}|0\rangle$.
6. **(b)** For $|\beta_{00}\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$, measuring the first qubit gives 0 or 1 (prob 1/2), leaving $|00\rangle$ or $|11\rangle$.
7. **(b)** Tensor product: $(|0\rangle + |1\rangle) \otimes (|0\rangle - |1\rangle) = |00\rangle - |01\rangle + |10\rangle - |11\rangle$.
8. **(b)** Eigenvalues of any unitary lie on the unit circle: $e^{i\theta}$ for some real $\theta$.
9. **(a)** $|\psi\rangle$ can be rewritten as: $|\psi\rangle = \frac{1}{\sqrt{2}}|\beta_{00}\rangle|+\rangle + \frac{1}{\sqrt{2}}|\beta_{01}\rangle|-\rangle$. Measuring first two qubits in Bell basis gives: with prob 1/2, $|\beta_{00}\rangle$ (third qubit $|+\rangle$), with prob 1/2, $|\beta_{01}\rangle$ (third qubit $|-\rangle$).
10. **(b)** In superdense coding, Alice applies $X$ for bits "10".
11. **(a)** Closed system evolution is unitary: $|\psi'\rangle = U|\psi\rangle$.
12. **(c)** $|\psi_1\rangle, |\psi_2\rangle$ are orthogonal in the Hadamard basis, so measure in $\{|+\rangle, |-\rangle\}$.
13. **(b)** With $A = X$, the circuit maps $|0\rangle|1\rangle \mapsto |0\rangle|0\rangle$. So a suitable $A$ exists.
14. **(d)** Without the first measurement, Bob lacks the $Z^{M_1}$ correction. One classical bit is insufficient.
15. **(a)** Matrix representations in two bases are related linearly (please check tutorial 3, question 9).
16. **(d)** False: Positive operators are Hermitian with non-negative spectrum, so they do have a spectral decomposition.
17. **(b)** $P \otimes P$ is a projector: $(P \otimes P)^2 = P \otimes P$.
18. **(a)** Hadamard operator: $H = \frac{1}{\sqrt{2}}\big[(|0\rangle + |1\rangle)\langle 0| + (|0\rangle - |1\rangle)\langle 1|\big]$.
19. **(a)** Measurement operators: $M_0 = |0\rangle\langle 0|$, $M_1 = |1\rangle\langle 1|$.
20. **(a)** Suppose $|\psi\rangle$ is an eigenstate of $M$ with eigenvalue $m$. Then, $M\,|\psi\rangle = m\,|\psi\rangle$
    - $\mathbb{E}[M] = \langle\psi|\,M\,|\psi\rangle = \langle\psi|\,m\,|\psi\rangle = m$ and $\sigma(M) = \sqrt{\langle M^2\rangle - \langle M\rangle^2} = \sqrt{m^2 - m^2} = 0$
    - Therefore, $\mathbb{E}[M] = m$ and $\sigma(M) = 0$.