Numerical Computation of Advection Equations using the Lax-Friedrichs Scheme

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The purpose of this assignment is for you

- warm up to coding at the beginning of the semester.
- understand that programming languages have different intrinsic speeds.

1 The Lax-Friedrichs Scheme for the Advection Equation

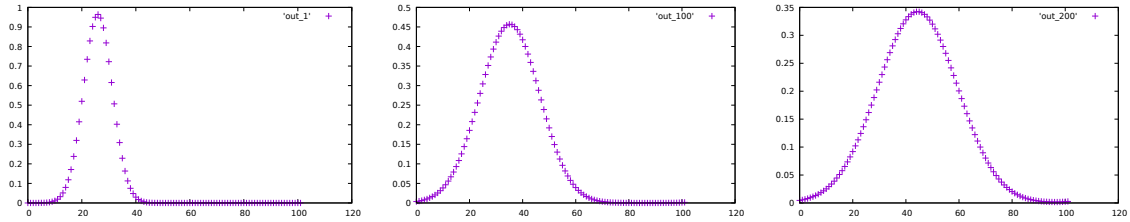
Consider the following algorithm.

Data: $N \in \mathbb{N}$, $dt \in \mathbb{R}$
// Simulation parameters
$t_{max} = 2.0$;
$x_{min} = 0.0$;
$x_{max} = 1.0$;
$v = 1.0$;
$xc = 0.25$;
$dx = \frac{x_{max} - x_{min}}{2}$;
$nbstep = t_{max}/dt$;
$\alpha = v * dt/(2 * dx)$;
// Simulation domain
$x[i] = x_{min} + (i-1) * dx, \forall i \in \{0 \dots N+2\}$;
// Initial condition
$u_0[i] = e^{-200*(x[i]-xc)^2}, \forall i \in \{0 \dots N+2\}$;
$u = u_0$;
$u_{new} = u_0$;
for $timestamp \in \{1, \dots, nbsteps\}$ **do**
  $currenttime = timestamp * dt$;
  // The Lax-Friedrichs scheme
  for $j \in \{1, \dots, N+1\}$ **do**
    $u_{new}[j] = u[j] - \alpha(u[j+1] - u[j-1]) + \frac{1}{2}(u[j+1] - 2 * u[j] + u[j-1])$;
  end
  $u = u_{new}$;
  // Enforcing Periodic Boundary Conditions
  $u[0] = u[N+1]$;
  $u[N+2] = u[1]$;
end

For context, this algorithm solves the advection equation for the advection equation $U_t + vU_x = 0$ over the spatial domain of $0 \leq x \leq 1$ that is discretized into N nodes using the Lax-Friedrichs scheme for an initial profile of a Gaussian curve, defined by $U(x,t) = exp(-200 * (x - xc - v * t).^2)$ where xc is the center of the curve at t=0. **If you don't understand what that means, it is not important.**

Question: Implement the algorithm in any language.

If you want to check it is correct, you can dump the values of u at different time step and it should look like a pattern moving to the right, like:



2 Timing experiment

2.1 On your machine

Question: How much time does the algorithm take on your machine for $N = 103$ and $dt = 0.0009$?
Question: How much time does the algorithm take on your machine for $N = 1003$ and $dt = 0.00009$?

2.2 On Centaurus (if you have access to it by then)

Question: How much time does the algorithm take on a mamba compute node for $N = 103$ and $dt = 0.0009$?
Question: How much time does the algorithm take on a mamba compute node for $N = 1003$ and $dt = 0.00009$?