CS161 Discussion 5
Adversarial Search: Games

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Game Search

Games:

- Require making some decision when calculating the optimal decision is infeasible
- Penalize inefficiency severely

How to choose a good move when time is limited?

Game Search

- Pruning
  - Ignore portions of the search tree that make no difference to the final choice
- Evaluation functions
  - approximate the true utility of a state without doing a complete search

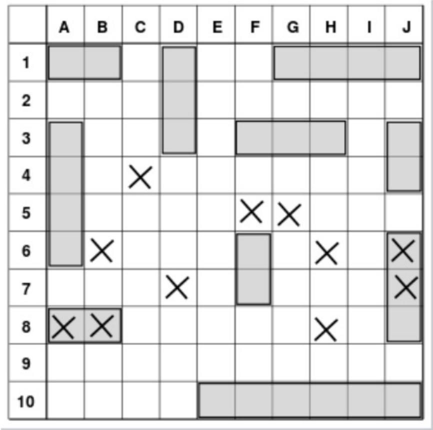
Types of Games

	deterministic	chance
perfect information	chess, checkers, go, othello	backgammon monopoly
imperfect information	battleships, blind tictactoe	bridge, poker, scrabble nuclear war

Types of Games

Go: Perfect and Deterministic

Monopoly: Perfect, Chance Introduced



Battleship: Imperfect and Deterministic

Bridge: Imperfect, Chance Introduced

Games with Two Players

- S_0: The **initial state**, which specifies how the game is set up at the start.
- $\text{PLAYER}(s)$: Defines which player has the move in a state.
- $\text{ACTIONS}(s)$: Returns the set of legal moves in a state.
- $\text{RESULT}(s, a)$: The **transition model**, which defines the result of a move.
- $\text{TERMINAL-TEST}(s)$: A **terminal test**, which is true when the game is over and false otherwise. States where the game has ended are called **terminal states**.
- $\text{UTILITY}(s, p)$: A **utility function** (also called an objective function or payoff function), defines the final numeric value for a game that ends in terminal state s for a player p. In chess, the outcome is a win, loss, or draw, with values $+1$, 0, or $\frac{1}{2}$. Some games have a wider variety of possible outcomes; the payoffs in backgammon range from 0 to $+192$. A **zero-sum game** is (confusingly) defined as one where the total payoff to all players is the same for every instance of the game. Chess is zero-sum because every game has payoff of either $0 + 1$, $1 + 0$ or $\frac{1}{2} + \frac{1}{2}$. "Constant-sum" would have been a better term, but zero-sum is traditional and makes sense if you imagine each player is charged an entry fee of $\frac{1}{2}$.

Optimal Decisions

What is an optimal solution in adversarial search?

- Normal search:
  - a sequence of actions leading to a goal state
- Adversarial search:
  - Find a contingent strategy
    - First move, moves in the states resulting from the other guy's possible moves, ...

Optimal Decisions - MINIMAX

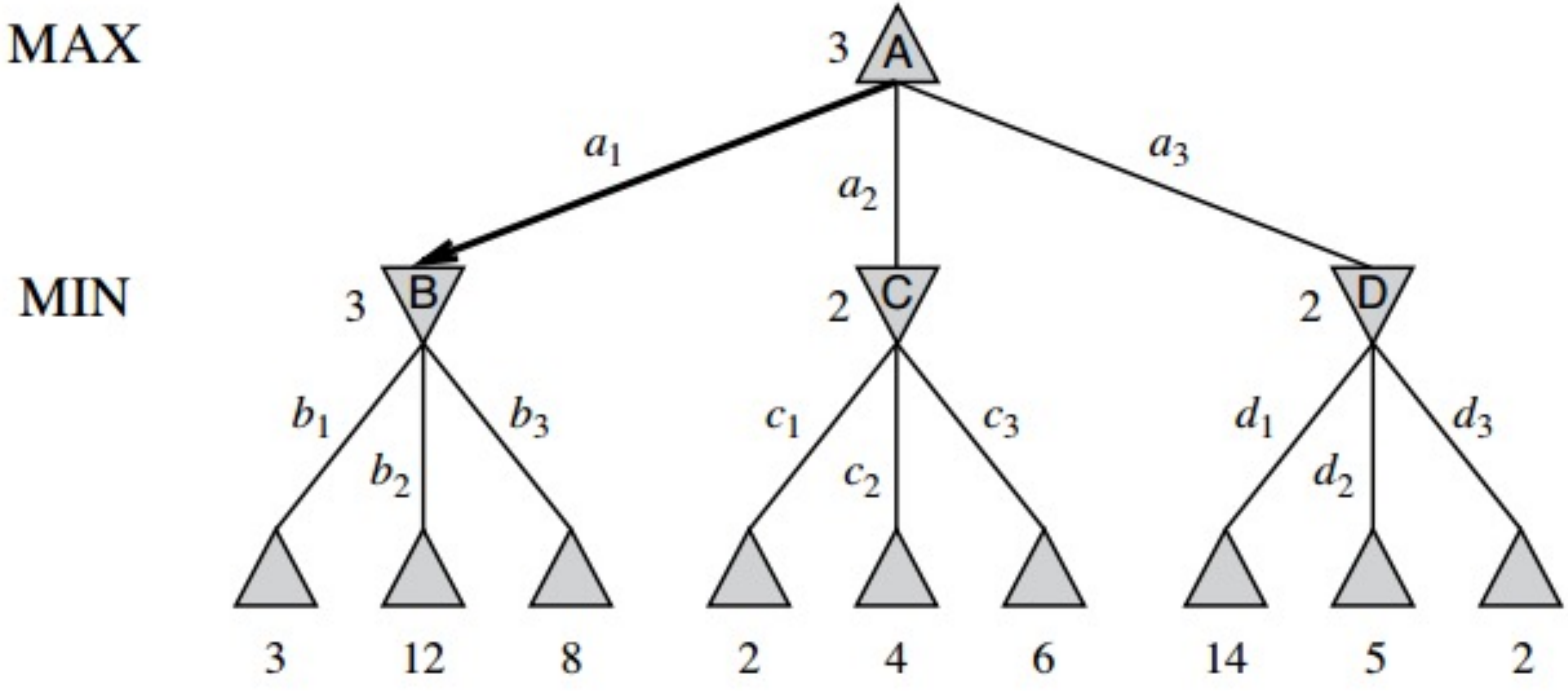
Given a game tree, how to determine the optimal strategy?

MINIMAX(n)

- The utility (for MAX)
- Assume both players play *optimally* from there to end of game
  - Given a choice, MAX prefers to move to a state of maximum value, whereas MIN prefers a state of minimum value.

$$\text{MINIMAX}(s) = \begin{cases} \text{UTILITY}(s) & \text{if TERMINAL-TEST}(s) \\ \max_{a \in Actions(s)} \text{MINIMAX}(\text{RESULT}(s, a)) & \text{if PLAYER}(s) = \text{MAX} \\ \min_{a \in Actions(s)} \text{MINIMAX}(\text{RESULT}(s, a)) & \text{if PLAYER}(s) = \text{MIN} \end{cases}$$

Optimal Decisions



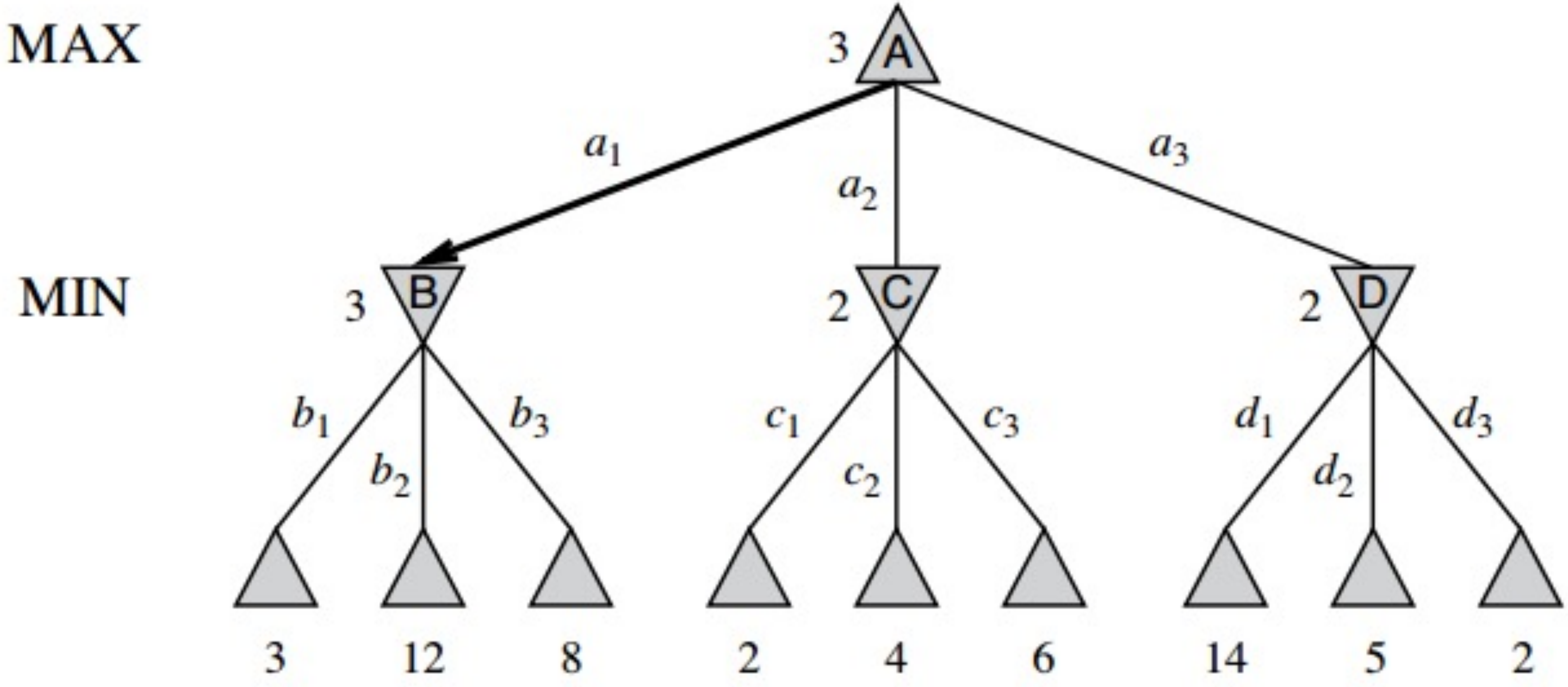
Alpha-beta pruning

- Minimax: a way of finding an optimal move in a two player game.
- Alpha-beta pruning: finding the optimal minimax solution while avoiding searching subtrees of moves which won't be selected.

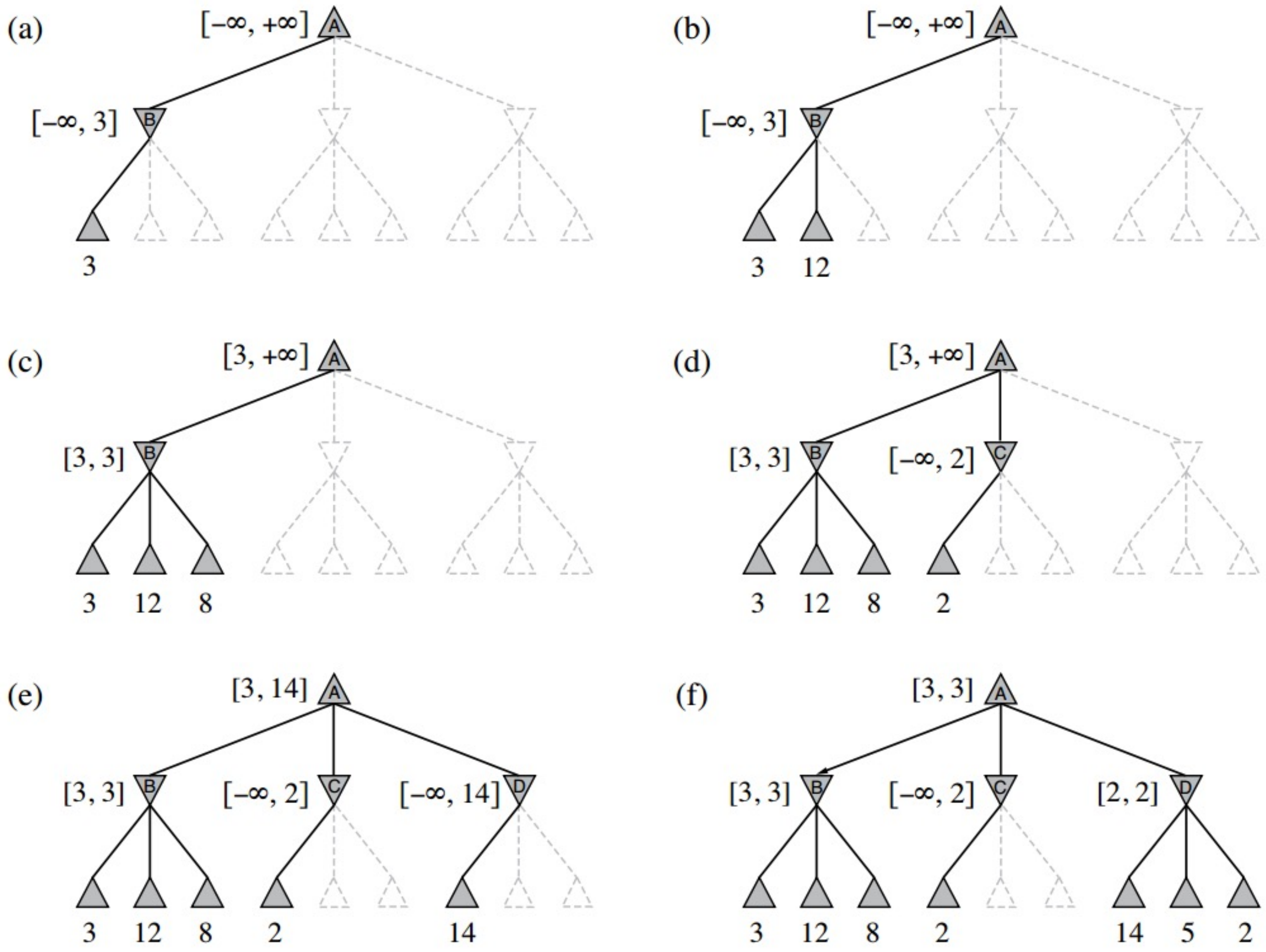
- Alpha: maximum lower bound of possible solutions
- Beta: minimum upper bound of possible solutions

$$\alpha \leq N \leq \beta$$

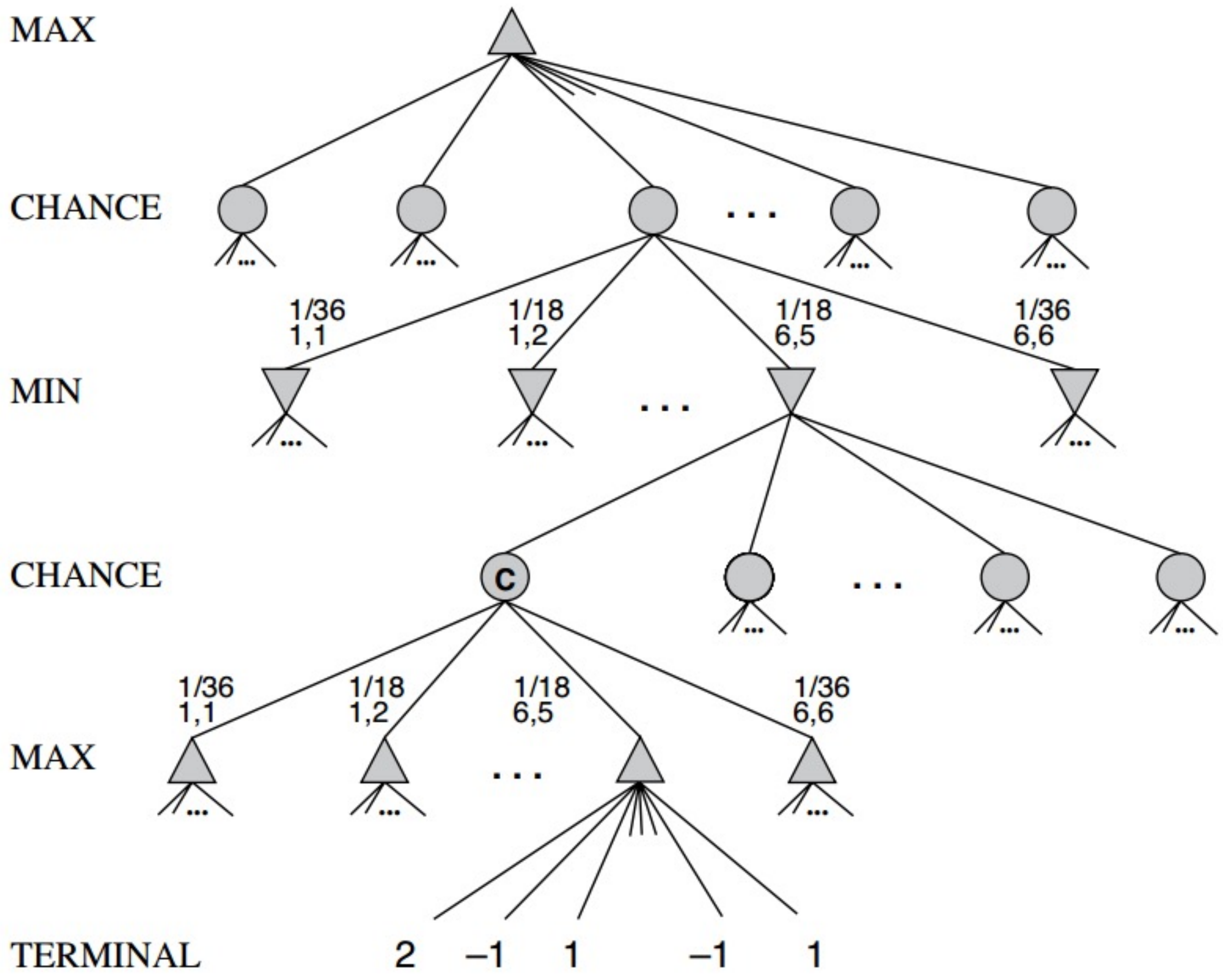
Example



Alpha-beta pruning



EXPECTMINIMAX

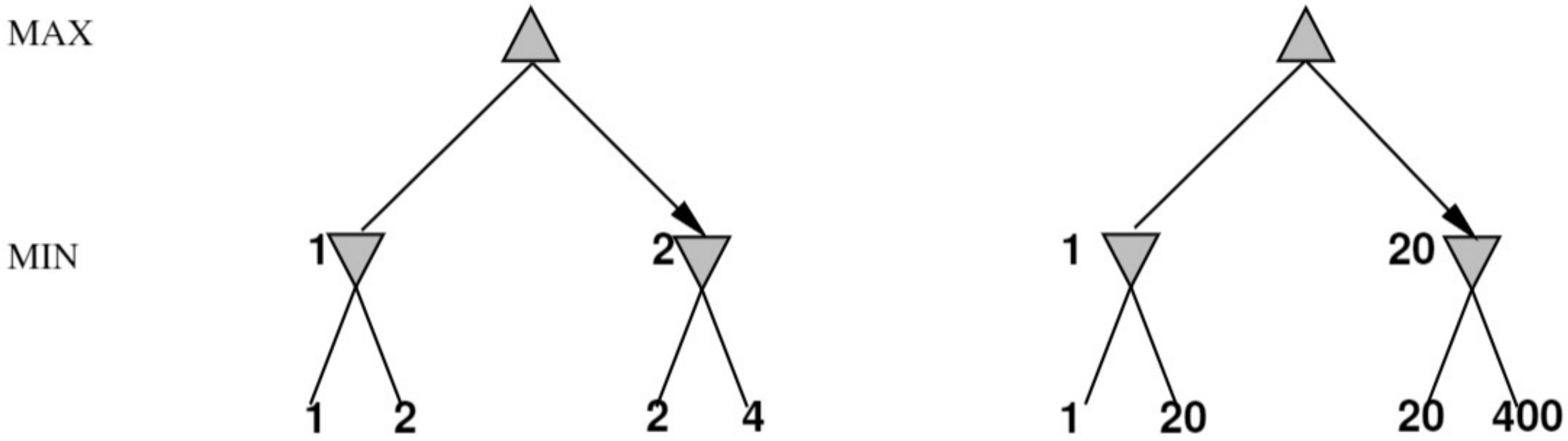


EXPECTMINIMAX

$$
\textsc{Expectiminimax}(s) =
\begin{cases}
\textsc{Utility}(s) & \text{if } \textsc{Terminal-Test}(s) \\
\max_a \textsc{Expectiminimax}(\textsc{Result}(s,a)) & \text{if } \textsc{Player}(s) = \textsc{max} \\
\min_a \textsc{Expectiminimax}(\textsc{Result}(s,a)) & \text{if } \textsc{Player}(s) = \textsc{min} \\
\sum_r P(r)\textsc{Expectiminimax}(\textsc{Result}(s,r)) & \text{if } \textsc{Player}(s) = \textsc{chance}
\end{cases}
$$

Transformation

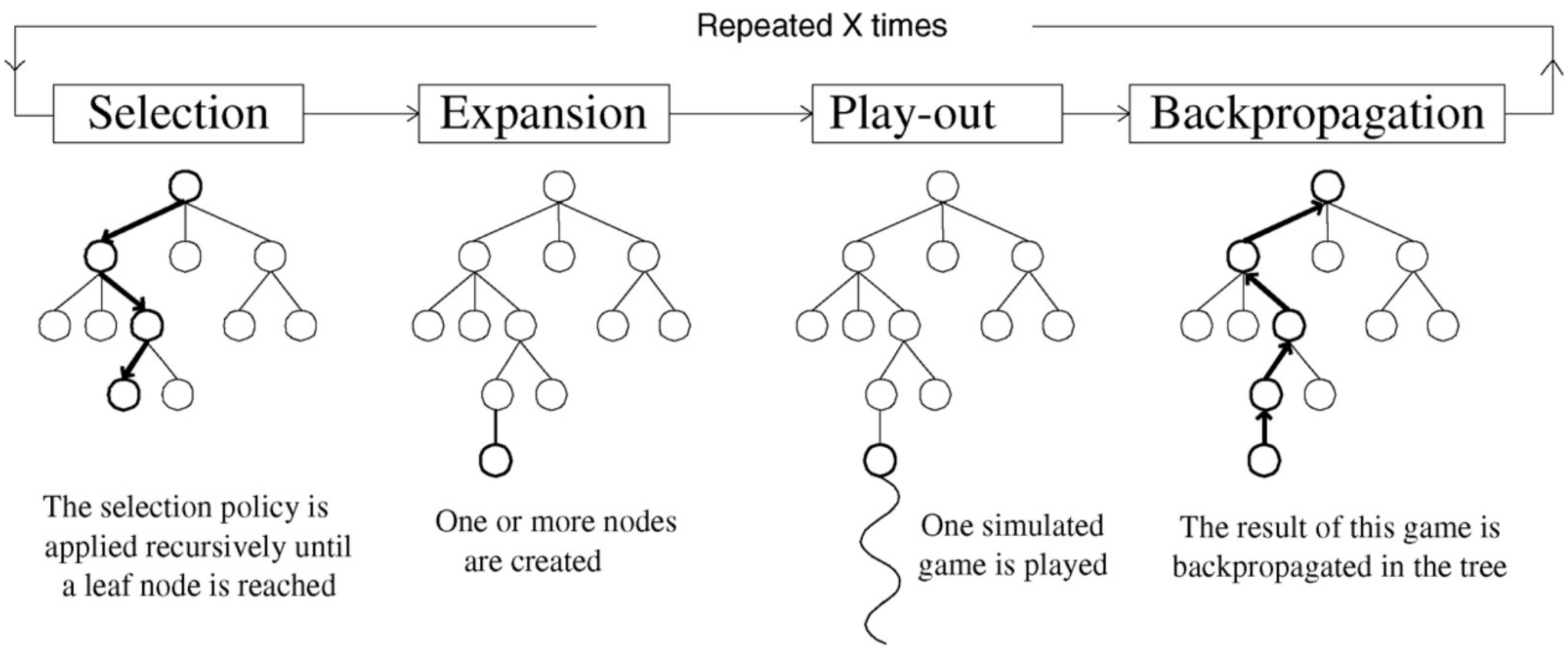
In deterministic games, exact values do not matter.



Behaviour is preserved under any **monotonic** transformation of the original EVAL

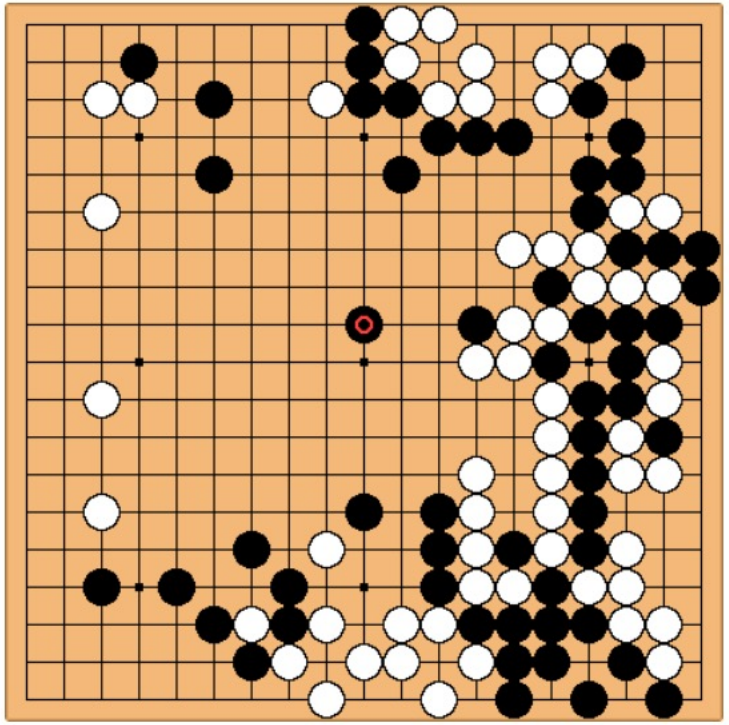
In non-deterministic games, exact values matter!!!

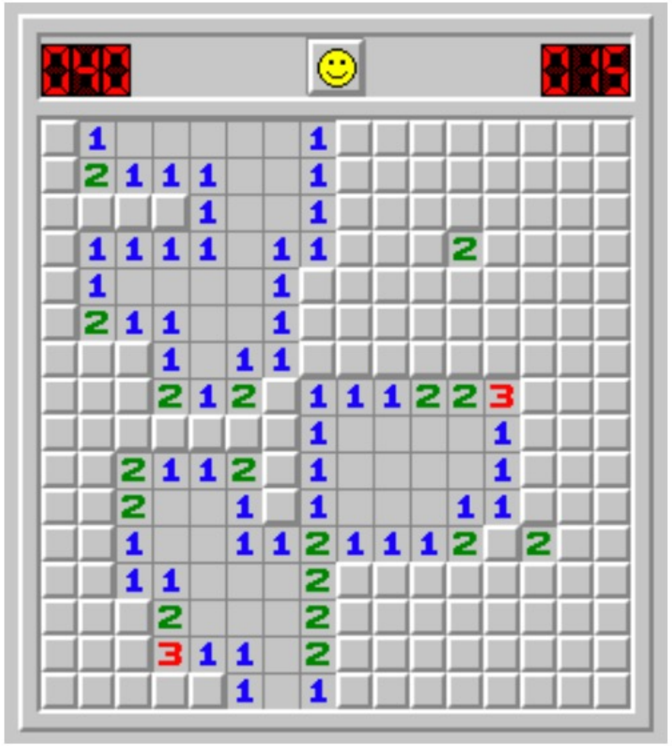
Monte Carlo Tree Search



MCTS for computer Go and MineSweeper

- Go: deterministic transitions
- MineSweeper: probabilistic transitions





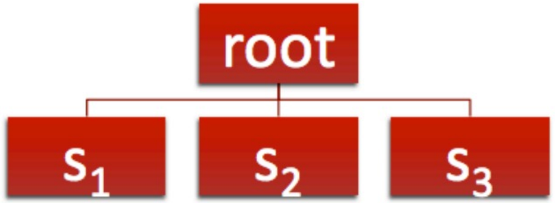
Basic Monte Carlo Simulation

- No evaluation function?
  - simulate game using random moves
  - Score game at the end, keep winning statistics
  - Play move with best winning percentage
  - Repeat
- Use this as the evaluation function, hopefully it will preserve some difference between a good position and a bad position

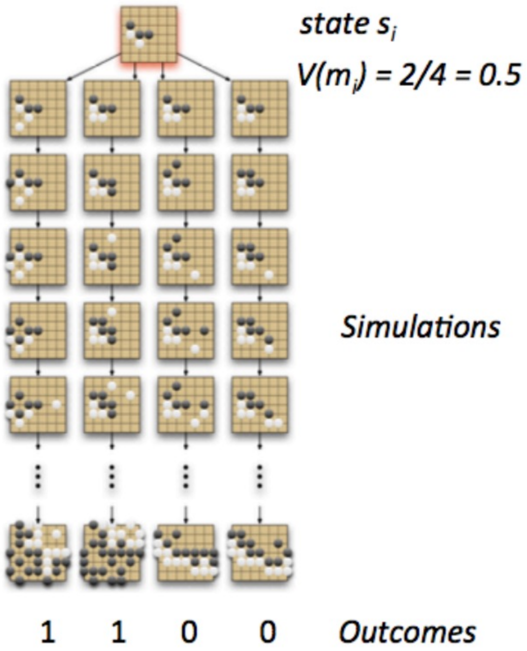
Monte Carlo Tree Search

- MCTS builds a statistics tree (detailing value of nodes) that partially maps onto the entire game tree
- Statistics tree guides the AI to focus on most interesting nodes in the game tree
- Value of nodes determined by simulations

Basic Monte Carlo Search



1 ply tree
root = current position
s_1 = state after move m_1
s_2 = …



Classical Search

- Deterministic
  - A state has fixed successor
- Designed to explore the search space systematically
- Solution: a sequence of actions
  - path from initial state to goal state

However

- Sometimes the path to the goal is not our focus. Only final configuration matters
  - n-queens, circuit design, etc.
- Sometimes the state space is continuous
  - $x^2 + y^2 = 10, x \in \mathbb{R}, y \in \mathbb{R}$

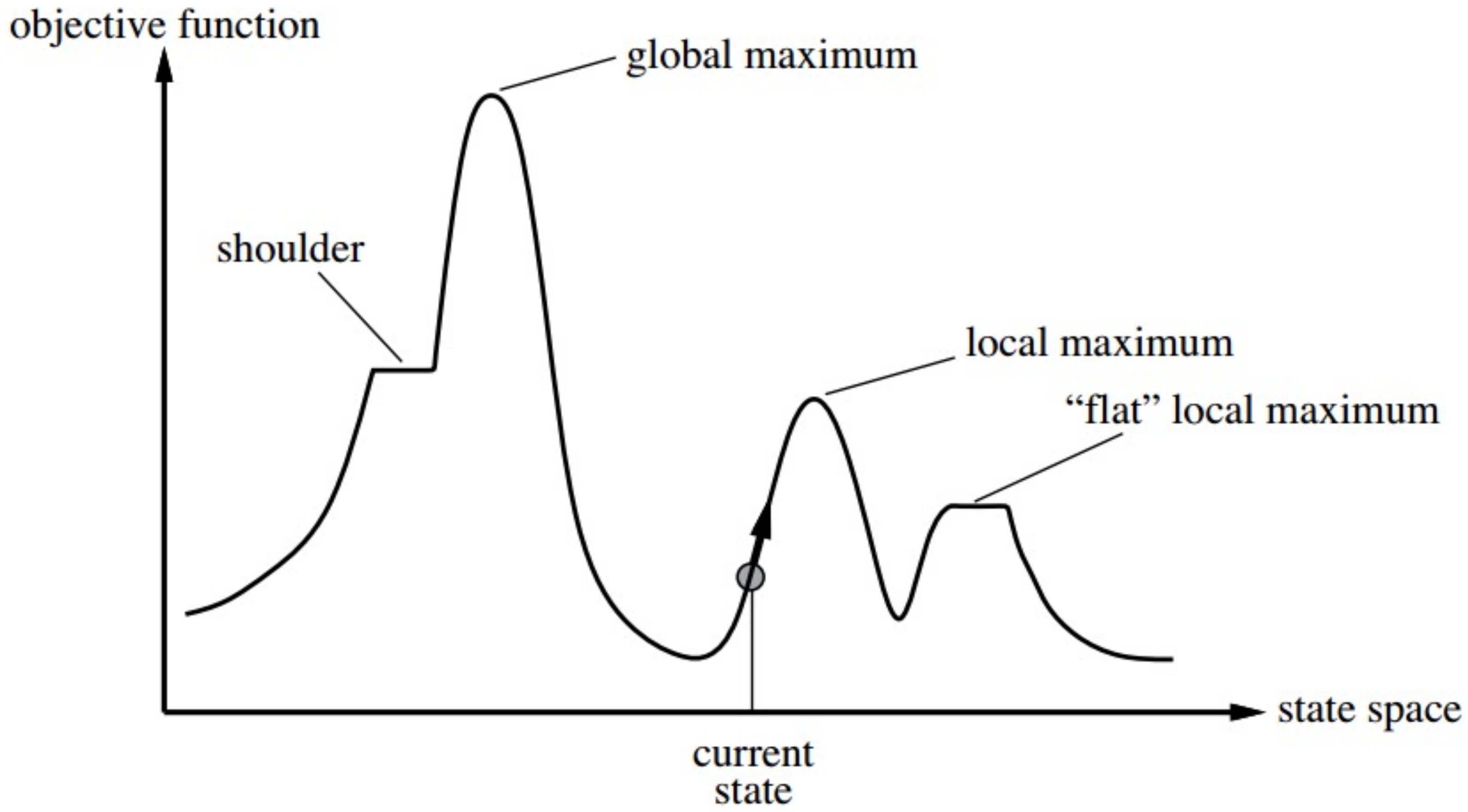
Local Search

- Keep track of a single node (current state)
- Move to neighbors
- No need to keep paths

Advantages:

- Little memory
- Find reasonable solution in large or infinite state spaces
  - Good for **optimization problems** (Find best state according to an **objective function**)

Example - State-Space Landscape

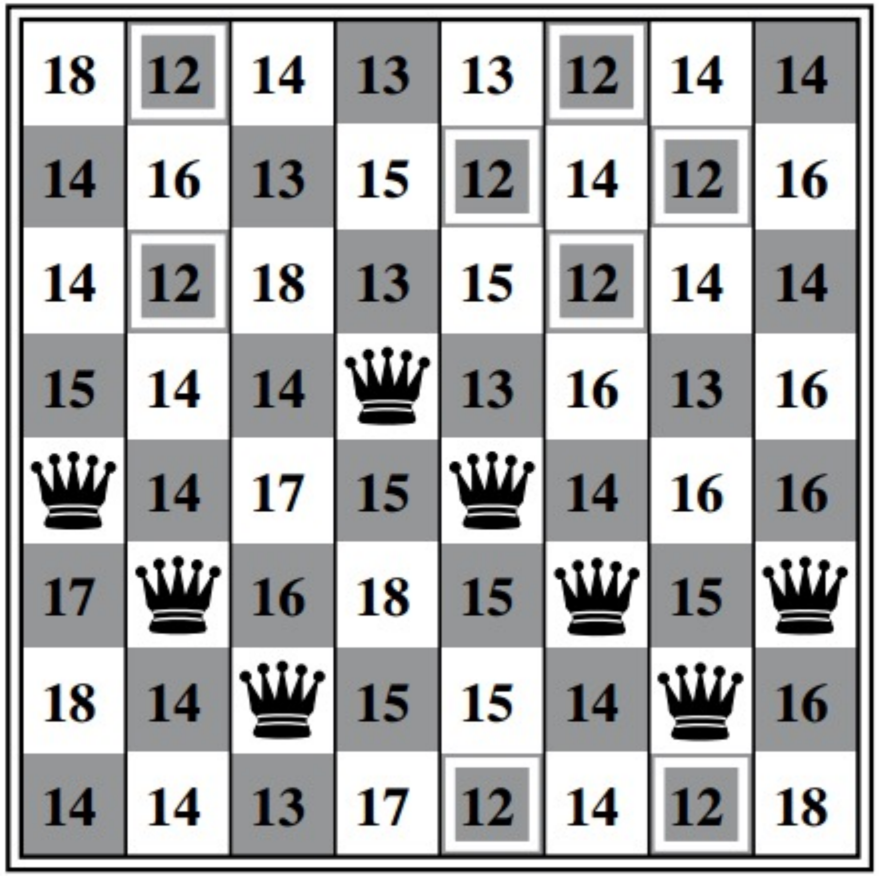


Hill-climbing search

- Greedy local search
  - Grabs a good neighbor state without thinking ahead about where to go next.

- Check all neighbors of current state
- choose the one with the highest value (lowest cost)
- Terminate when no neighbor has a higher value

Example – 8-queens

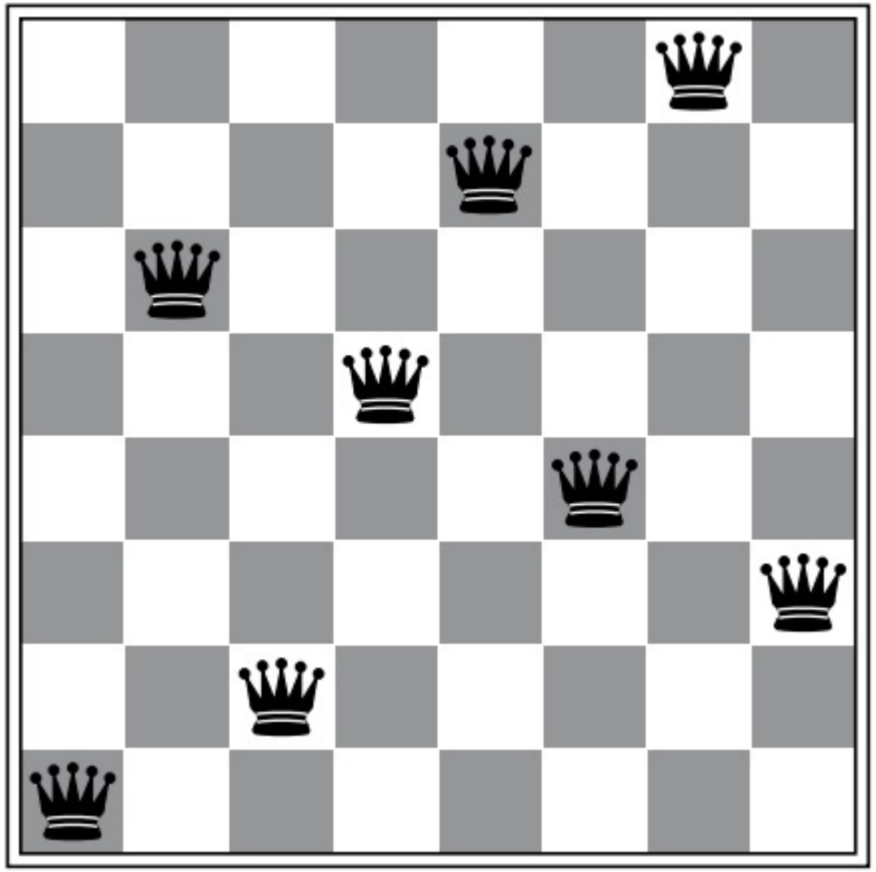


Hill-climbing search

- Advantage
  - Easy to improve a bad state (rapid progress)

- Disadvantage
  - Get stuck in
    - local optimal
    - Ridges
    - Plateau

Example – 8-queens



The state has h = 1 but every successor has a higher cost.

Hill-climbing search

- Disadvantage
  - Get stuck in
    - local optimal
    - Ridges
    - Plateau

The success of hill climbing depends very much on the shape of the state-space landscape!

NP-hard problems typically have an exponential number of local maxima to get stuck on.

Simulated Annealing

- Hill-climbing algorithms never move towards state with lower value
  - May result in local optimal

Simulated Annealing: an analogy of metropolis methods

- Randomly select candidate successor
- Go there if better
- Else go there with probability (Why?)
  function of "energy" and "temperature"

Local Beam Search and Stochastic Beam Search

Local Beam Search

- Start with k states
- Keep the best k successors

- Advantage: useful information is passed among the parallel search
- Disadvantage: lack of diversity
  - What if all the k successors get stuck at the same local optimal?

Stochastic Beam Search

(Introducing randomness)

- Randomly select k successors
- Better successor has a higher chance to be selected