CS 181 EXAM #2
SPRING 2015

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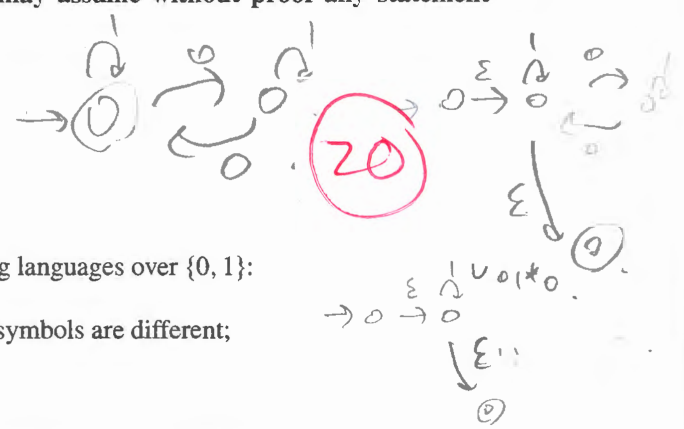
You have 90 minutes to complete this exam. You may assume without proof any statement proved in class.



20

1 Find a regular expression for each of the following languages over {0, 1}:

(1 pt) a. nonempty strings in which the first and last symbols are different;
(1 pt) b. strings in which the number of 0s is even;
(2 pts) c. strings not containing the substring 01;
(3 pts) d. strings in which the number of 0s and the number of 1s are either both even or both odd.

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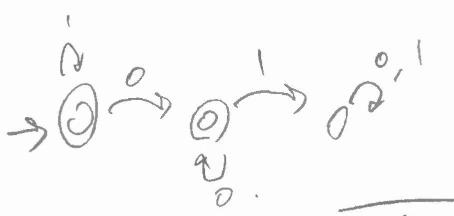
1 a) $1\Sigma^*0 \cup 0\Sigma^*1$

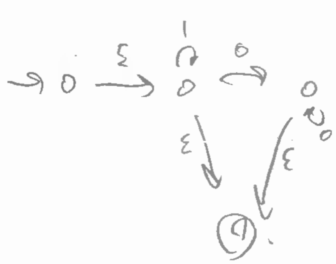
1 b) $(1 \cup 01^*0)^*$

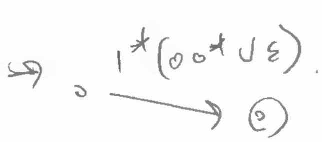
c)

2

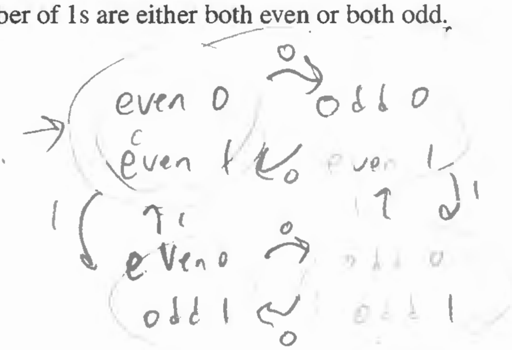


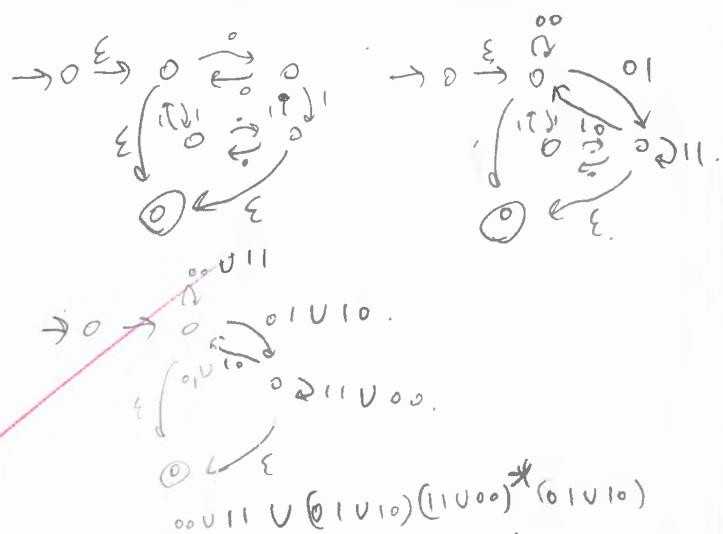
$\boxed{1^*(00^* \cup \varepsilon)}$

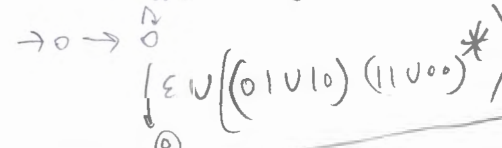




3 d)







$(00 \cup 11 \cup (01 \cup 10)(11 \cup 00)^*(01 \cup 10))^*(\varepsilon \cup ((01 \cup 10)(11 \cup 00)^*)$

$0^n 1^n$ $\quad \equiv_L$ $\quad x \equiv_L y$

$0^{n+1} 1^{n+1}$ $\quad 0^n 1^n \ldots$ $\quad y \equiv_L y$

$0^{2n} 1^{2n}$ $\quad x \equiv_L y \equiv_L z, \; y \equiv_L z$

2 Prove or disprove:

(2 pts) 2 **a.** for any regular languages $L_1 \subseteq L_2 \subseteq \ldots \subseteq L_n \subseteq \ldots$, the union $\bigcup_{n=1}^{\infty} L_n$ is regular;

(3 pts) **b.** if L_1 and L_2 are two languages such that the equivalence classes of $\equiv_{L_1}$ are exactly the same as those of $\equiv_{L_2}$, then $L_1 = L_2$. 3

a). False. 2

counter example.

$L_1 = \{\varepsilon\}$ ∈ reg.

$L_2 = \{01, \varepsilon\}$

$L_3 = \{0^2 1^2, 01, \varepsilon\}$ reg.

$L_4 = \{0^3 1^3, 0^2 1^2, 01, \varepsilon\}$

⋮

$L_n = \{0^{n-1} 1^{n-1} ; n \geq 1\}$

$L_1 \subseteq L_2 \subseteq \ldots \subseteq L_n \subseteq \ldots$, but union $\bigcup_{n=1}^{\infty} L_n$ is

$L_n = \{0^{n-1} 1^{n-1} : n \geq 1\}$.

L_n is nonregular, b/c for $i \neq j$, $i, j \geq 0$,

$0^i 1^i \in L_n$,

$0^j 1^i \notin L_n$.

$0^i \not\equiv_L 0^j$.

$\varepsilon, 0, 00, 000 \ldots$ are all distinguishable,

∴ there are ∞ many equivalence classes for L_n.

∴ L_n is nonregular

b. True. 0

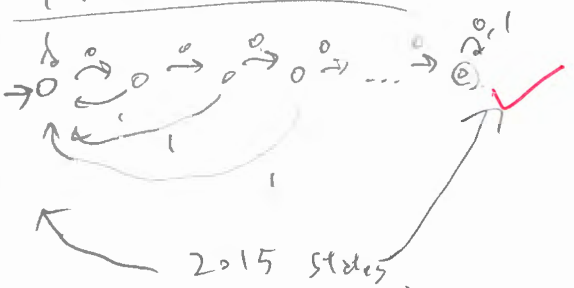
For any regular language, if equivalence classes of $\equiv L_1$ are exactly the same as those of $\equiv L_2$, then same DFA can be constructed that recognizes both L_1 & L_2

~~For any nonregular language, proof by contradiction of the contrapositive. If $L_1 \neq L_2$ and $\equiv L_1$ are exactly same as $\equiv L_2$, then L_1 & L_2 are defined by the exactly same rule that distinguishes them. Then $L_1 = L_2$, which is a contradiction~~ ∎

(3 pts) **3** Construct a language that can be recognized by a DFA with 2015 states but not with 2014 states. Prove both claims. 3

$L = \{w : w$ must contain $\underbrace{0000\ldots0}_{2014}$ as a substring$\}$.

Proof that L can be recognized by DFA with 2015 states.



This language L can be recognized by DFA with 2015 states. ∎

Proof that L can't be recognized by DFA with 2014 states.

By the proof of the pumping lemma, DFA with only 2014 states must loop itself in order to accept strings of length 2014 or greater.

Language L I constructed requires that w to contain 2014 0's as a substring.

But DFA with only 2014 states will loop with a length greater than 1, which means it will accept some strings before the string contains 2014 0's. This leads to contradiction,

∴ L can't be recognized by DFA with 2014 states ok

01011. 1000000111 1010 101110 1110 1110
(1 0000 ∪ 01000 ∪ 001000 ∪ 00100 ∪ ... ∪ 00001)*
01001
11 0010010011001 1011 0123 3210 01111 11111
4 For each of the following languages, determine whether it is regular, and prove your answer:

(2 pts) **a.** binary strings with five times as many 0s as 1s;

(2 pts) **b.** binary strings of the form uvu, where u and v are nonempty strings;

(3 pts) **c.** strings over the decimal alphabet $\{0, 1, 2, \ldots, 9\}$ with characters in sorted order;

(3 pts) **d.** binary strings such that in every suffix, the number of 0s and the number of 1s differ by at most 2.

$w\Sigma^+ w$.

Let L be language stated by a.

a) nonregular.

2

For $i \neq j$,

$1^i 0^{5i} \in L$ ✓

$1^j 0^{5i} \notin L$.

$1^i \not\equiv_L 1^j$. ✓

$\varepsilon, 1, 11, 111, 1111 \ldots$ are all pairwise distinguishable

Language stated by a. contains ∞ many equivalence classes.

$\therefore$ Language stated by a. is nonregular. ∎

b) Let L be language stated by b.

2

For $i \neq j$, $j > i$

$1^i 0 1 1 1^i 0 \in L$ ✓

$1^j 0 1 1 1^i 0 \notin L$ If $j = i+1$, $v = \varepsilon$

$1^i \not\equiv_L 1^j$. ✓

$\varepsilon, 1, 11, 111, \ldots$ are all pairwise distinguishable.

L contains ∞ many equivalence classes. $\therefore$ L is nonregular.

∎

c) $L = \{1^* 2^* 3^* \ldots 9^*$ 0^*

2.5

$\cup\ 2^* 3^* 4^* \ldots 9^*$

$\cup\ 3^* 4^* \ldots 9^*$

$\cup\ 4^* 5^* \ldots 9^*$

$\cup\ 5^* 6^* \ldots 9^*$

$\cup\ 6^* 7^* \ldots 9^*$

$\cup\ 8^* 9^*$

$\cup\ 9^* \}$.

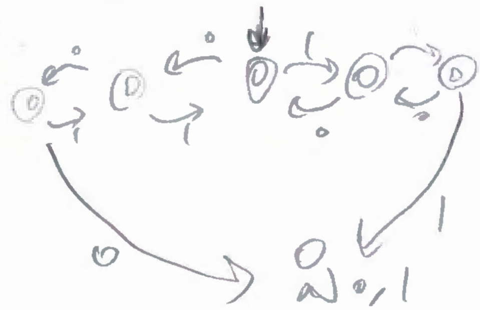
L is ~~regular~~, for regular expression exists for L.

Let L be language stated by d.

d) this DFA accepts L.

DFA exists, $\therefore$ L is regular.

1.5



7 reject