

## Homework 7

2.9 Give a context-free grammar that generates the language

$$A = \{a^i b^j c^k \mid i = j \text{ or } j = k \text{ where } i, j, k \geq 0\}.$$

Is your grammar ambiguous? Why or why not?

2.20 Let  $A/B = \{wx \mid wx \in A \text{ for some } x \in B\}$ . Show that if  $A$  is context free and  $B$  is regular, then  $A/B$  is context free.

\*2.24 Let  $E = \{a^i b^j \mid i \neq j \text{ and } 2i \neq j\}$ . Show that  $E$  is a context-free language.

2.30 Use the pumping lemma to show that the following languages are not context free.

a.  $\{0^n 1^n 0^n 1^n \mid n \geq 0\}$

<sup>A</sup>c.  $\{w\#t \mid w \text{ is a substring of } t, \text{ where } w, t \in \{a, b\}^*\}$

d.  $\{t_1 \# t_2 \# \dots \# t_k \mid k \geq 2, \text{ each } t_i \in \{a, b\}^*, \text{ and } t_i = t_j \text{ for some } i \neq j\}$

2.31 Let  $B$  be the language of all palindromes over  $\{0,1\}$  containing equal numbers of 0s and 1s. Show that  $B$  is not context free.

\*2.45 Let  $A = \{wtw^R \mid w, t \in \{0,1\}^* \text{ and } |w| = |t|\}$ . Prove that  $A$  is not a CFL.

2.47 Let  $\Sigma = \{0,1\}$  and let  $B$  be the collection of strings that contain at least one 1 in their second half. In other words,  $B = \{uv \mid u \in \Sigma^*, v \in \Sigma^* 1 \Sigma^* \text{ and } |u| \geq |v|\}$ .

a. Give a PDA that recognizes  $B$ .

b. Give a CFG that generates  $B$ .

8.

Give PDAs for the following languages:

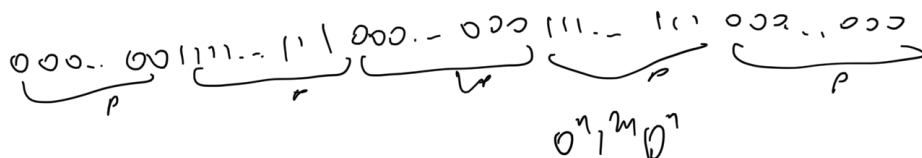
a. binary strings in which every prefix contains at least as many 0s as 1s;

b. binary strings that contain at least as many 0s as 1s;

c. binary strings that contain equally many 0s and 1s;

d. odd-length binary strings with middle symbol 0;

e. strings of the form  $v\#w$ , where  $v$  and  $w$  are binary strings and  $w$  contains  $v^R$ .



000, 001, 0011, 00111, 001111, 0011111, 00111111, 001111111

$$a^i b^j \quad i \neq j \quad 2i \neq j$$

$$\begin{aligned} i &\neq j \\ i &< j < 2i \\ j &> 2i \end{aligned}$$

$$S_1 = a^i b^j \mid a^i b^j \mid \epsilon$$

$$S_2 = a^i b^j \mid a^i b^j \mid \epsilon$$

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$$T_2 = a^i b^j \mid a^i b^j \mid \epsilon$$

$$S_2 = a^i b^j \mid a^i b^j \mid \epsilon$$

$$a^i b^j$$

$$a^i b^j$$

$$a^i b^j$$