CS 181 EXAM #3 UCLA [redacted]
FALL 2019

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You have 90 minutes to complete this exam. You may state without proof any fact taught in class or assigned as homework.

1 Describe the languages generated by the following context-free grammars with alphabet $\{a, b\}$. You may provide a verbal description or a regular expression, as appropriate.

2 (2 pts) a. $S \to XY$
$X \to Xa \mid Xb \mid a$
$Y \to aY \mid bY \mid b$

a. $a(a \cup b)^* b$ ✓

2 (2 pts) b. $S \to aT \mid bT \mid \varepsilon$
$T \to aS$

b. $\cup ((a \cup b)a)^*$ → b. $(aa \cup ba)^*$ ✓

2 (2 pts) c. $S \to \Sigma S \mid aS_1$
$S_1 \to \Sigma S_1 \mid aS_2$
$S_2 \to \Sigma S_2 \mid aS_3$
$S_3 \to \Sigma S_3 \mid \varepsilon$
$\Sigma \to a \mid b$

c. Strings with at least 3 a's ✓

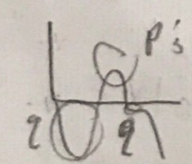
2 Give context-free grammars for the following languages:

2 (2 pts) a. strings of the form $a^n\#a^m$, where n and m are nonnegative integers with $n \neq m$;

2 (2 pts) b. strings over the alphabet $\{a, b\}$ in which some prefix contains more b's than a's.

a. $S \to aSa \mid X\# \mid \#X$
$X \to aX \mid a$ ✓

b.



$P \to ab \mid aPb \mid abP \mid aPbP$
$Q \to ba \mid bQa \mid baQ \mid bQaQ$

$(P \cup Q)^* bQ$

$(P \cup Q)^* b \Sigma^*$

$S \to PS \mid bX$
$P \to ab \mid aPb \mid abP \mid aPbP$
$X \to aX \mid bX \mid \varepsilon$ ✓

$S \to aSb \mid A \mid B$
$A \to aA \mid a$
$B \to bB \mid b$

$b^+ \cup (a \cup b)^* ba (a \cup b)^*$

3 Draw a pushdown automaton for the complement of the language $\{a^n b^n : n \geq 0\}$ over the alphabet $\{a, b\}$.

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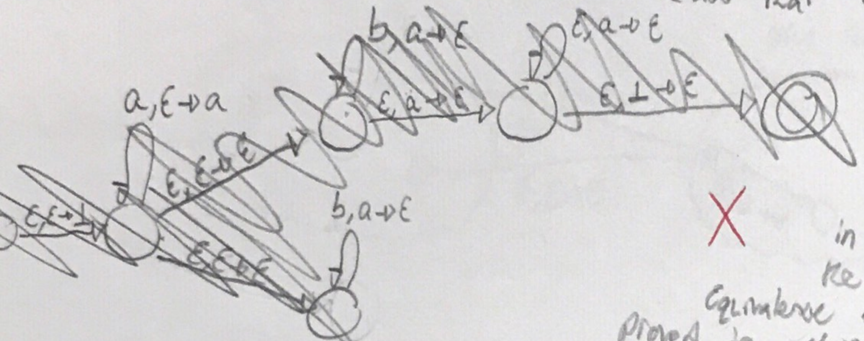
Shown in class that $\{a^n b^n : n \geq 0\}$ is context free but not regular. Since context free languages are not closed under the complement; $\overline{\{a^n b^n : n \geq 0\}}$ is not context free and doesn't have a PDA. Proved in class that context free languages aren't closed under complement. We know L is context free and thus not closed under the complement as in can be represented by the grammar $S \to aSb \mid \varepsilon$. Equivalence between PDA's and grammars proved in class. $\overline{L}$ has no CFG thus it has no PDA



X

4 Consider the context-free grammar

$$S \to \Sigma S \Sigma \mid \Sigma T a$$
$$T \to \Sigma \Sigma T \mid \varepsilon$$
$$\Sigma \to a \mid b.$$

$\Sigma T a$
$\Sigma \Sigma \Sigma T a$

$\Sigma S \Sigma$
$\Sigma \Sigma S \Sigma \Sigma$
$\Sigma \Sigma \Sigma T a \Sigma \Sigma$
$\Sigma \Sigma \Sigma \Sigma \Sigma T a \Sigma \Sigma$
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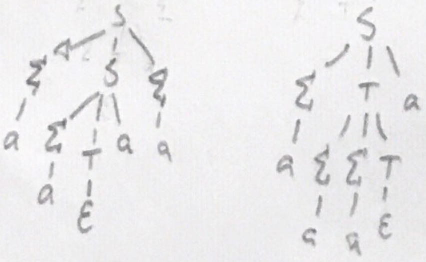
(1 pt) a. Describe the language generated by this grammar.

(1.5 pt) b. Prove that this grammar is ambiguous.

(1.5 pt) c. Give an equivalent unambiguous grammar.

a. Strings of even length with an "a" in the second half

b. The string aaaa has at least two parse trees



c. $S \to aTa \mid bSb \mid aSb \mid bTa$

$T \to aTa \mid bTb \mid aTb \mid bTa \mid \varepsilon$

5 For languages A and B, define $A \diamond B = \{uv : u \in A, v \in B, |u| = |v|\}$. Explain how to construct a PDA for $A \diamond B$ from DFAs for A and B.

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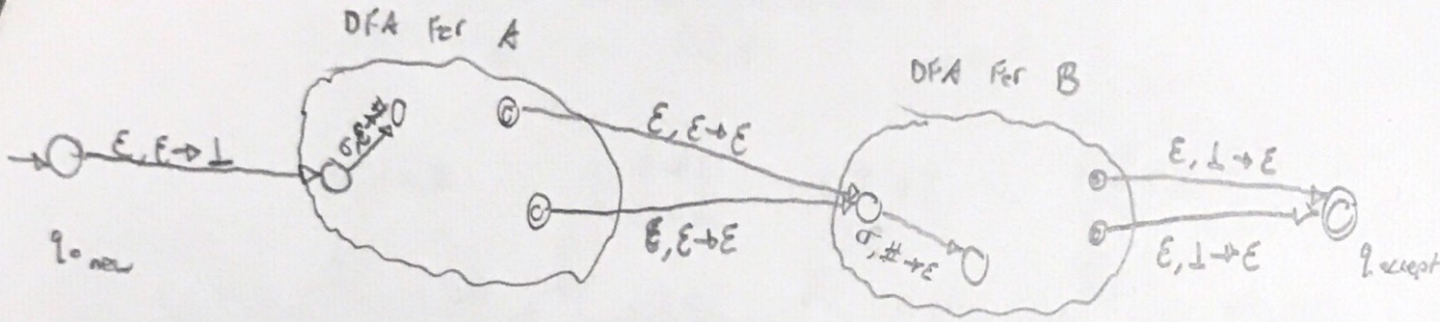


Add a new start state q_{new} which transitions to the start state of A's DFA using label $\varepsilon, \varepsilon \to \perp$. Change all labels of transitions in A's DFA from σ where $\sigma \in \Sigma_A$ to $\sigma, \varepsilon \to \#$. From all accept states in A's DFA add transitions $\varepsilon, \varepsilon \to \varepsilon$ to the start state of B's DFA. Change all labels of transitions in B's DFA from σ where $\sigma \in \Sigma_B$ to $\sigma, \# \to \varepsilon$. To all accept states in B's DFA, add the transition $\varepsilon, \perp \to \varepsilon$ to our PDA's single accept state q_{accept}

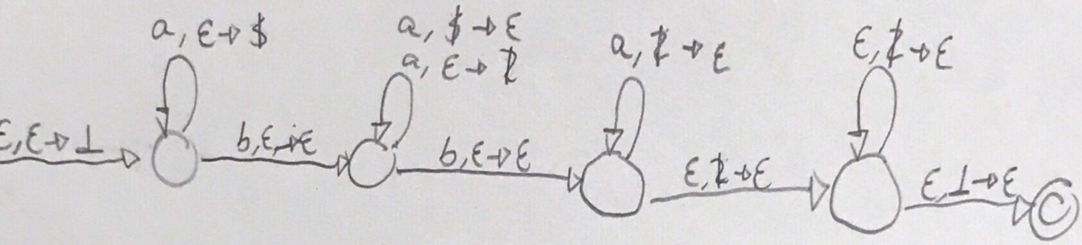
6 For each of the following languages L over the alphabet $\{a, b\}$, determine whether it is context-free and prove your answer:

2 a. $\{a^n b a^m b a^k : m > n + k\}$;

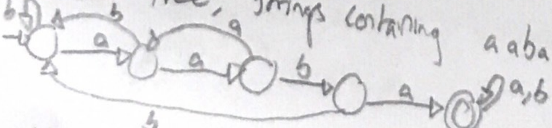
2 b. strings that do not contain $aaba$;

1 c. strings that begin with $a^n b^n a^n$ for some $n \geq 1$.

a. Context free via PDA



b. Context free; strings containing aaba is regular by DFA



thus its complement is also regular. Therefore strings not containing aaba is regular and as learned in class, the language is context free because it is Regular

c. Not context free. Examine string $s = a^{2p} b^{2p} a^{2p} \in L$; as $|vxy| \leq p$; vxy must fall entirely within a set of a's, within all b's; or spanning from the first set of a's to the set of b's or from the set of b's into the second set of a's. In the cases where vxy contains only a's or only b's; pumping down shows that $uxz \notin L$ as the equality $a^n b^n a^n$ will not hold. In the cases where vxy contains a's and b's; $vxy \in a^+b^+$ or $vxy \in b^+a^+$; and as a result of pumping in these cases our sets of a's at the start and end of the string will be of different lengthes so $uxz \notin L$ as $a^n b^n a^n$ doesn't hold. Therefore L is not context free