15.25

You have 90 minutes to complete this exam. You may state without proof any fact taught in class or assigned as homework.

1 Prove the following assertions, where $\Sigma = \{0, 1\}$.

1 (1 pts) a. $\overline{\Sigma^* \setminus (0^* \cup 1^*)} = 0^* \cup 1^*$

1 (2 pts) b. $0^*(10^+)^*(\varepsilon \cup 1) = (\varepsilon \cup 1)(01 \cup 0)^*$

a. $\overline{0^* \cup 1^*} = \Sigma^* 01 \Sigma^* \cup \Sigma^* 10 \Sigma^*$

$\overline{\Sigma^* \setminus (0^* \cup 1^*)} = 0^* \cup 1^*$

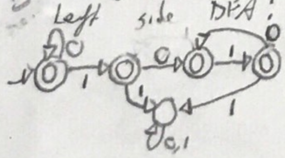
$\overline{\overline{(0^* \cup 1^*)}} = 0^* \cup 1^*$

Because Σ^* contains all strings $\Sigma^* \setminus L = \overline{L}$, where L represents a regular expression. Then the complement of L's complement is L meaning that:

$\overline{\Sigma^* \setminus (0^* \cup 1^*)} = \overline{\overline{(0^* \cup 1^*)}} = 0^* \cup 1^*$ ✓

b. $0^*(10^+)^* = (10 \cup 0)^*$ ✓

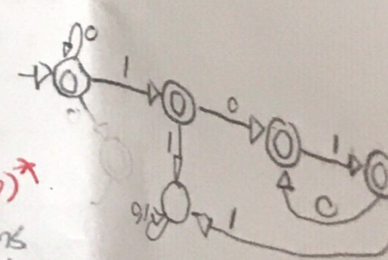
Left side DFA:



then?

$(10 \cup 0)^* (\varepsilon \cup 1) \overset{\text{prove}}{=} (\varepsilon \cup 1)(01 \cup 0)^*$

Right Side DFA:



The left and right sides are both regular expressions meaning they are both regular languages; since they are regular languages and share a DFA as shown above they must be equivalent.

2 Give a regular expression for each of the following languages over $\Sigma = \{0, 1\}$:

a. even-length strings that contain 01;

b. strings in which every 1 is adjacent to a 0.

1.5 (2 pts)

1.5 (2 pts)

a. $(\Sigma\Sigma)^* 01 (\Sigma\Sigma)^*$

lack odd - 01 - odd

b. ~~$(0^+1 \cup 10^+ \cup 0^-)^*$~~

$\boxed{(01 \cup 10 \cup 0)^*}$

not all strings

ε

10

01

0101

101

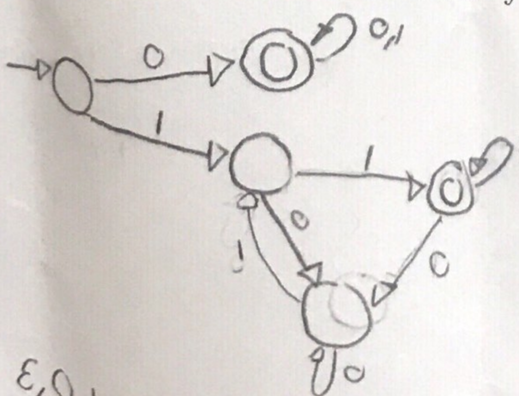
$0^+1 \cup 10^+$

2.5 (3 pts) 3 Let L be a nonempty finite language in which the longest string has length n. Prove that any DFA for L must have at least $n+1$ states.

L is finite → L must have a DFA, DFA for L cannot have a cycle ~~X~~ of the language would accept infinite strings and we know that L is finite not infinite. Since it takes n transitions from the start state to process an n characters string in a DFA, and we know there are no cycles in the DFA for L, then n distinct states of the DFA must be reached from the start state in processing the longest string in L; this does not include the start state, so including the start state we know L's DFA has at least $n+1$ states

1 small mistake

3 (3 pts) 4 Construct a DFA for the language $0\Sigma^*\cup\Sigma^*11$ over the binary alphabet, using the smallest possible number of states. Prove that your DFA is the smallest possible.



	ε	0	1	11	10
ε					
0	ε				
1	1	ε			
11	ε	0	ε		
10	0	ε	1	1	

✓

ε, 0, 1, 11, and 10 are in different equivalence classes and as a result there are at least 5 equivalence classes meaning the shown DFA of 5 states is minimal size for the given language

0 (2 pts)
0 (2 pts)
0. ~~15~~ 2 pts)

5 Prove or disprove:

a. if L is a nonregular language and w a string, then the concatenation wL is nonregular;
b. if L is a nonregular language, then prefix(L) is also nonregular;
c. if L is a regular language, then the language L' of even-length strings whose first half is in L is also regular.

a. False

b. True

c. False

$0^{10} 1 0^{5} 1$ 12

6 For each of the following languages L over the binary alphabet, determine whether it is regular and prove your answer:

2 (2 pts) a. even-length strings whose first half contains as many 0s as the second half;

2 (2 pts) b. strings w such that every prefix of w is equal to some suffix of w;

0 (2 pts) c. strings whose length, when expressed as a decimal integer, uses no digits other than 0 and 1.

0, 1, 10, 11, 100, 101

10101

* ∪ 0*

a. False; For $i \neq j \Rightarrow 0^{2i} 1 0^{2i} 1 \in L$

$0^{2j} 1 0^{2i} 1 \notin L$ $(j > i)$

Thus $\epsilon, 00, 0000, \ldots$ are in different equivalence classes and as there are infinite equivalence classes in L, the language is not regular

b. True; regex is $1^* \cup 0^*$, existence of a regular expression implies regularity

✓

c. False; ~~only accepted when $|w| = 10^n$ or $10^n + 1$ for $n \geq 0$~~

~~$L = (\bigcup_{i=0}^{\infty} |w| \,\%\, 10^i = 0) \cup |w| \,\%$~~

lengths accepted are 0, 1, 10, 11, 100, 101, 110, 111; 10^n for $n \geq 0$ plus all powers of 10: 10^i where $10^i = \sum_{j=0}^{j=n} 10^j$. No such string can be pumped for $p \geq 1$

✗