Homework 7

2.9 Give a context-free grammar that generates the language

$$A = \{a^i b^j c^k \mid i = j \text{ or } j = k \text{ where } i, j, k \geq 0\}.$$

Is your grammar ambiguous? Why or why not?

2.20 Let $A/B = \{w \mid wx \in A \text{ for some } x \in B\}$. Show that if A is context free and B is regular, then A/B is context free.

***2.24** Let $E = \{a^i b^j \mid i \neq j \text{ and } 2i \neq j\}$. Show that E is a context-free language.

2.30 Use the pumping lemma to show that the following languages are not context free.

- **a.** $\{0^n 1^n 0^n 1^n \mid n \geq 0\}$
- [A] **c.** $\{w\#t \mid w \text{ is a substring of } t, \text{ where } w, t \in \{a, b\}^*\}$
- **d.** $\{t_1\#t_2\# \cdots \#t_k \mid k \geq 2, \text{ each } t_i \in \{a, b\}^*, \text{ and } t_i = t_j \text{ for some } i \neq j\}$

2.31 Let B be the language of all palindromes over $\{0,1\}$ containing equal numbers of 0s and 1s. Show that B is not context free.

***2.45** Let $A = \{wtw^{\mathcal{R}} \mid w, t \in \{0,1\}^* \text{ and } |w| = |t|\}$. Prove that A is not a CFL.

2.47 Let $\Sigma = \{0,1\}$ and let B be the collection of strings that contain at least one 1 in their second half. In other words, $B = \{uv \mid u \in \Sigma^*, v \in \Sigma^* 1 \Sigma^* \text{ and } |u| \geq |v|\}$.

- **a.** Give a PDA that recognizes B.
- **b.** Give a CFG that generates B.

8.

Give PDAs for the following languages:

- **a.** binary strings in which every prefix contains at least as many 0s as 1s;
- **b.** binary strings that contain at least as many 0s as 1s;
- **c.** binary strings that contain equally many 0s and 1s;
- **d.** odd-length binary strings with middle symbol 0;
- **e.** strings of the form $v\#w$, where v and w are binary strings and w contains v^R.

$a^i b^j$ $i \neq j$ $2i \neq j$

$i > j$ $\quad j < i$

$i < j < 2i$

$j > 2i$

$S_1 = aS_1b \mid aS_1 \mid \varepsilon$

~~$S_2 = aS_2b$~~ $\mid$

$= aS_1bb \mid S_1b \mid b$

$T_2 = aT_2bb \mid abb$

$S_2 = aS_2b \mid T_2$

aT_2b

$a \; bb$

$aa \; bbbb$

$\underbrace{000..00}_{p}\underbrace{111..11}_{p}\underbrace{000..000}_{p}\underbrace{111..111}_{p}\underbrace{000..000}_{p}$

$0^n 1^m 0^n$

000..00 1111..111 111... 111000..00