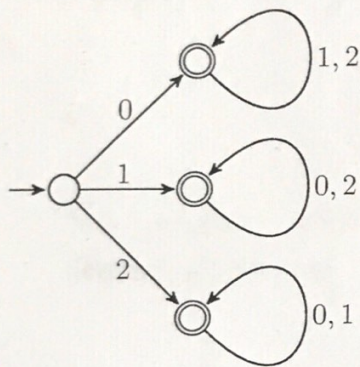


18.5

You have 90 minutes to complete this exam. You may state without proof any fact taught in class or assigned as homework.

3 (3 pts)

- 1 Give a simple verbal description of the language recognized by the following NFA with alphabet $\{0, 1, 2\}$:



A non-empty string over the given alphabet
where the first symbol seen is
not seen again, it only appears in the string
as the first symbol. ✓

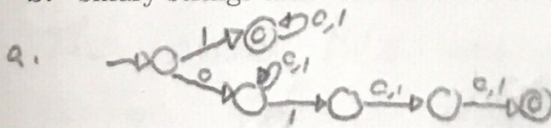
- 2 Draw NFAs for the following languages, taking full advantage of nondeterminism:

2 (2 pts)

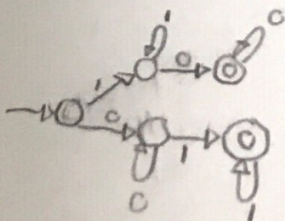
a. binary strings that start with a 1 or have a 1 in the third position from the end;

2 (2 pts)

b. binary strings that contain 01 or 10 but not both.

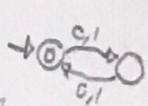
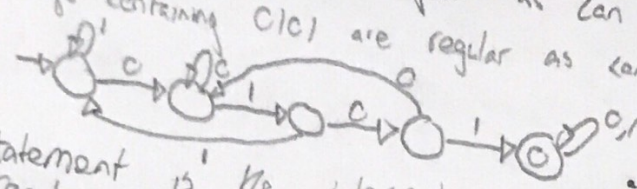


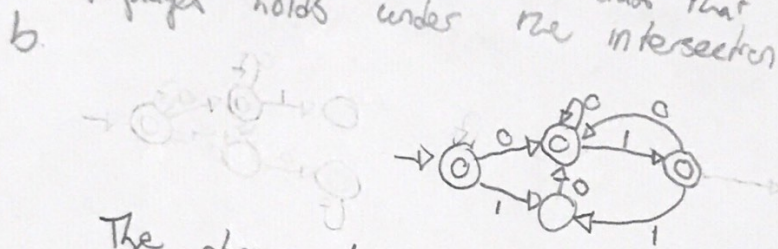
b.



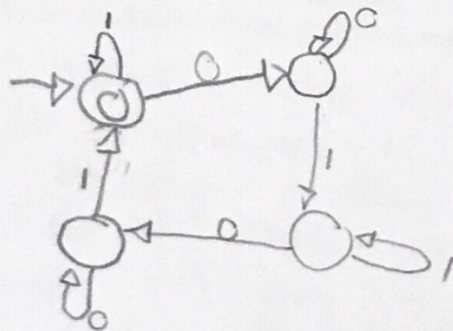
3 Prove that the following languages over the binary alphabet are regular:

- even-length strings that contain 0101;
- strings in which every 1 is adjacent to a 0;
- strings in which the substring 01 occurs an even number of times.

a. Even length strings are regular as can be seen in DFA  strings containing 0101 are regular as can be seen in DFA . Thus the language in the problem statement is the intersection of these two languages and must be regular as we learned in class that the closure of regular languages holds under the intersection operation. ✓



The above NFA holds for the language of the problem statement, showing it is a regular language. ✓



The above DFA describes the language of the problem statement, showing it is a regular language. ✓

shown in
as providing
DFA or
FA for a
language is
sufficient proof
regularity

(3 pts)

- 6 Describe an algorithm that takes as input an NFA N and outputs the minimum length of a string rejected by N . If no such string exists, the algorithm should output " ∞ ." Your algorithm must run in finite time.

1. If the start state of N is rejecting; output 0
2. Else your current state is accepting and your counter remains at 0
3. Starting at q_0 with your current state generalized to q ; counter starts at 0
3. Perform DFS using only Epsilon transitions from q until you either reach a reject state in which case you output your counter, or have no more ϵ -transitions to follow in which case you pop that reached accept state onto a queue. If you reach a state that has already been processed don't reprocess it. Increment counter 1
- X 4. Add q to your queue, increment counter
5. Repeat 3-4 for all states in your queue; performing Breadth first search on the NFA after using Depth-First search on each state popped off the queue to hunt for a reject state only using epsilon transitions.
6. Return ∞ if no reject states found

No state should be visited twice and all available epsilon transitions should be used before the cost of a single symbol is accepted to search a new state. Always explore ϵ first

- 7 Let D be a given DFA. Let W be the set of all strings w such that every state of D is visited while processing w . Prove that W is regular.

$$D = (Q, \Sigma, \delta, q_0, F)$$

We can construct an NFA by taking DFA D and adding a new q_{start} and q_{end} where q_{end} is the only accept state in the NFA. Epsilon transitions are added from q_{start} to q_0 . Remove all transitions from D which self-loop or revisit an already visited state. Then add epsilon transitions to q_{end} only from old states of D which are $|Q|-1$ transitions from q_0 once

