CS 181 A EXAM #3 **NAME** ______________________________

SPRING 2014 **UCLA ID** ______________________________

You have 90 minutes to complete this exam. You may assume without proof any statement proved in class.

1 Give regular expressions for the languages generated by the following context-free grammars:

(2 pts) **a.** $S \to aT \mid bT \mid \varepsilon$
$T \to aS \mid bS$

(2 pts) **b.** $S \to SaS \mid b$

(3 pts) **c.** $S \to \underbrace{SS\cdots S}_{2014} \mid a$

Solution.

a. $(\Sigma\Sigma)^*$

b. $b(ab)^*$

c. $a(a^{2013})^*$

2 Give context-free grammars for the following languages:

(2 pts) **a.** odd-length strings in $\{a, b\}^*$ whose first, middle, and last symbols are all the same;

(3 pts) **b.** $\{a^i b^j c^k : i \neq j + k\}$.

Solution.

a. $S \to aAa \mid bBb \mid \Sigma$
$A \to \Sigma A \Sigma \mid a$
$B \to \Sigma B \Sigma \mid b$
$\Sigma \to a \mid b$

b. $S \to aSc \mid AT \mid TB \mid TC \mid TBC$
$T \to aTb \mid \varepsilon$
$A \to Aa \mid a$
$B \to Bb \mid b$
$C \to Cc \mid c$

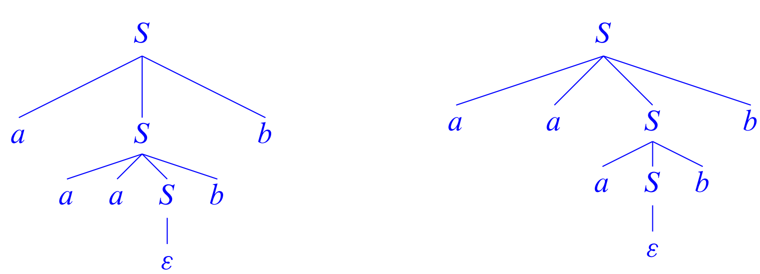
3 Consider the context-free grammar $S \to aSb \mid aaSb \mid \varepsilon$.

(2 pts) **a.** Prove that the grammar is ambiguous.

(2 pts) **b.** Find an equivalent unambiguous grammar.

Solution.

a. The string $aaabb$ has two parse trees:



b. $S \to aSb \mid T$
$T \to aaSb \mid \varepsilon$

(3 pts) **4** Is the language $\{a^{2^n} : n \geqslant 0\}$ context-free? Prove your answer.

Solution. Let L denote the above language. Fix an arbitrary integer $p \geqslant 1$ and consider the string $a^{2^p} \in L$. Consider any decomposition $a^{2^p} = uvxyz$ for some strings u, v, x, y, z with $0 < |v| + |y| \leqslant p$. Then

$$|uv^2xy^2z| > 2^p$$

and

$$\begin{aligned} |uv^2xy^2z| &\leqslant 2^p + p \\ &< 2^p + 2^p \\ &= 2^{p+1}. \end{aligned}$$

We have shown that uv^2xy^2z has length strictly between 2^p and 2^{p+1} and hence is not in L. By the pumping lemma, L is not context-free.

(3 pts) **5** Prove or disprove: the context-free grammar

$$S \to aSbScS \mid aScSbS \mid bSaScS \mid bScSaS \mid cSaSbC \mid cSbSaS \mid \varepsilon$$

generates every string in $\{a, b, c\}^*$ with equally many a's, b's, and c's.

Solution. As we proved in class, the language of strings in $\{a, b, c\}^*$ with equally many a's, b's, and c's is not context-free. Therefore, *no* context-free grammar can generate it.

(3 pts) **6** Prove that context-free languages are closed under the reverse operation.

Solution. Let L be any context-free language. To obtain a context-free grammar for L^R, take any grammar for L and reverse the right-hand side of every rule. The new grammar generates precisely those strings that are mirror images of strings in L.