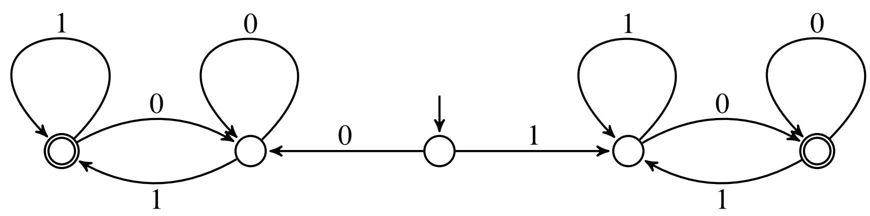
CS 181 EXAM #1 **NAME** ______________________

FALL 2016 **UCLA ID** ______________________

You have 90 minutes to complete this exam. You may state without proof any fact taught in class or assigned as homework.

(3 pts) **1** Give a simple verbal description of the language recognized by the following DFA.



Solution. Binary strings in which the first and last symbols differ.

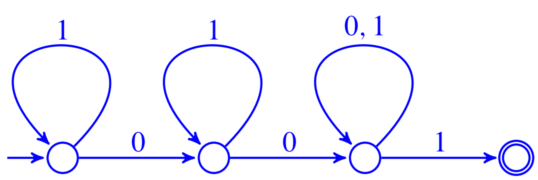
2 Draw NFAs for the following languages, taking full advantage of nondeterminism:

(2 pts) **a.** binary strings that contain at least two 0s and end with a 1;

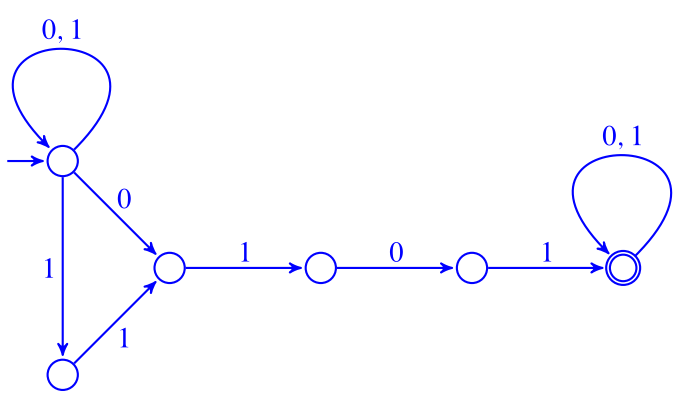
(2 pts) **b.** binary strings that contain 0101 or 11101 or both as a substring.

Solution.

a.



b.



3 Prove that the following languages are regular:

(2 pts) **a.** binary strings in which the number of 0s and the number of 1s are equal modulo 2016;

(2 pts) **b.** binary strings in which every 0 is immediately preceded and immediately followed by a 1;

(2 pts) **c.** binary strings of length 8 that read the same backward as forward.

Solution.

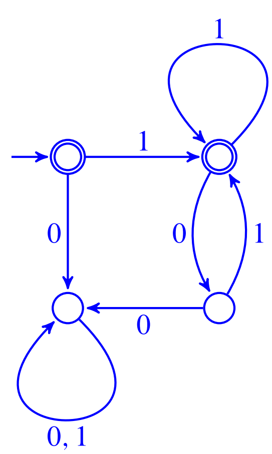
a. A DFA for this language just counts the number of 0s and the number of 1s modulo 2016, accepting if the numbers match. Formally, the DFA is

$$(\{0, 1, \ldots, 2015\} \times \{0, 1, \ldots, 2015\}, \{0, 1\}, \delta, (0, 0), \{(0, 0), (1, 1), \ldots, (2015, 2015)\}),$$

where

$$\delta((i, j), \sigma) = \begin{cases} ((i + 1) \bmod 2016, j) & \text{if } \sigma = 0, \\ (i, (j + 1) \bmod 2016) & \text{if } \sigma = 1. \end{cases}$$

b. This language is recognized by the following DFA:



c. This language contains a finite number of strings and is therefore regular.

(3 pts) **4** Describe an algorithm that takes as input two DFAs and determines whether there is a string that they both accept. Your algorithm must run in finite time.

Solution. Let D_1 and D_2 be the input DFAs. Use the Cartesian product construction from class to obtain a DFA D that recognizes the intersection of the languages of D_1 and D_2. Then, check to see if D has a path from the start state to an accept state. This second step can be done either by trying out all candidate paths in a brute force manner, or by using an efficient graph algorithm that you have encountered in CS 180, such as depth- or breadth-first search.

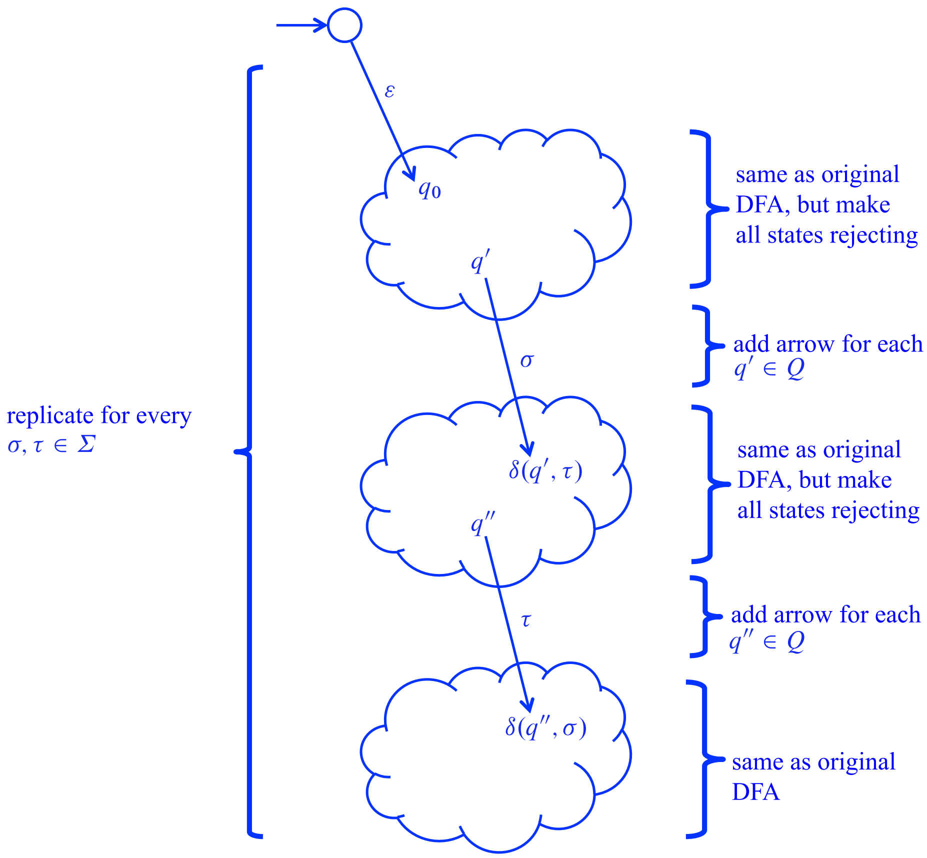
(3 pts) **5** For a language L, let $\text{trap}(L)$ denote the set of strings v such that no string in L starts with v. Prove that $\text{trap}(L)$ is regular whenever L is regular.

Solution. Let $(Q, \Sigma, \delta, q_0, F)$ be a DFA for L. Then $\text{trap}(L)$ is recognized by $(Q, \Sigma, \delta, q_0, F')$, where $F' \subseteq Q$ is the set of all states from which it is impossible to reach any state in F.

Alternate solution. Observe that $\text{trap}(L) = \overline{\text{prefix}(L)}$. Since regular languages are closed under the prefix and complement operations, it follows that $\text{trap}(L)$ is regular.

(3 pts) **6** For a language L over a given alphabet Σ, define swap(L) to be the set of strings that can be obtained from a string in L by swapping two characters in it. Formally, swap(L) is the set of all strings of the form $u\sigma v\tau w$ such that $u\tau v\sigma w \in L$, where u, v, w denote arbitrary strings and σ, τ denote alphabet symbols. Prove that regular languages are closed under the swap operation.

Solution. Let $(Q, \Sigma, \delta, q_0, F)$ be a DFA for the original language L. An NFA for swap(L) is as follows.



(3 pts) **7** For a string v, a *subsequence* of v is either v itself or any string obtained by deleting one or more characters in v. For example, the subsequences of 123 are ε, 1, 2, 3, 12, 13, 23, 123. Prove that for any regular language L, the set of all subsequences of strings in L is regular.

Solution. Starting with a DFA for L, supplement the label of every arrow to contain "ε". The resulting NFA accepts exactly those strings that are subsequences of strings in L.