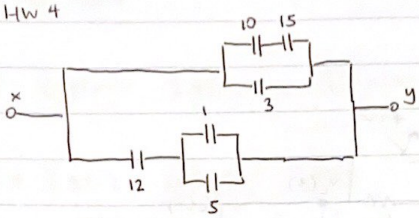
ECE HW 4

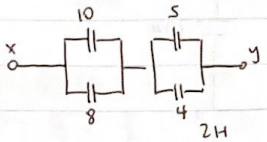
Q1. a)



$((10-15)\|3)\,\|\,(12-5\|1))$

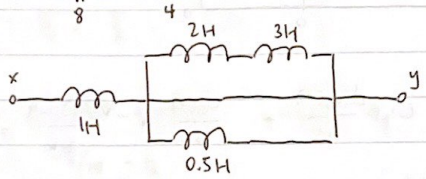
$= (6\|3)\,\|\,(12-6) = 9\,\|\,4 = \boxed{13\ \mu F}$

b)



$(10\|8)-(5\|4) = 18-9 = \boxed{6\ \mu F}$

Q2. a)

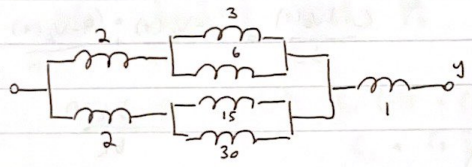


$1-((2-3)\|0\|0.5)$

$= 1-(5\|0\|0.5)$

$= 1-0 = \boxed{1\ H}$

b)



$(2-(3\|6))\,\|\,(2-(15\|30))-1$

$= (2-2)\,\|\,(2-10)-1$

$= 4\|12-1 = 3-1 = \boxed{4\ H}$

Q3. $I_{src} = 2mA \quad C = 10\mu F \quad \rightarrow Q = CV$

$\frac{dQ}{dt} = C\cdot\frac{dV}{dt} = I$

$\frac{dV}{dt} = \frac{I}{C} \rightarrow V = \int \frac{I}{C}\,dt$

$V = \frac{1}{10\mu F}\cdot 2mA\cdot t + C$

$= 0.2\ kV_s\cdot t + C = 0.2t\cdot 1000 + c$

$V(0)=0 \rightarrow V = 0.2t\cdot 1000 = 200t \rightarrow t = 20\cdot 10^{-3} \rightarrow V = 4000\cdot 10^{-3} = \boxed{4V}$

$P = IV = 2mA\cdot 4V = \boxed{8mW}$

$U = \frac{1}{2}CV^2 = \frac{1}{2}\cdot 10mF\cdot 4^2 = 80\mu F \rightarrow \boxed{80\mu J}$

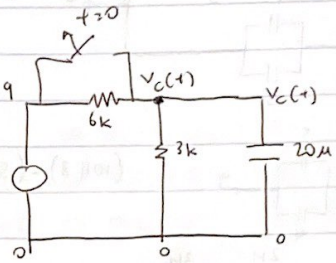
Q4. $V = L\,dI/dt \qquad 5 = 3\cdot dI/dt \qquad I = 5/3\,t + C$

$I(0)=0 \rightarrow I = 5/3\,t$

$t=2 \rightarrow \boxed{I = 10/3\ A} \qquad P = IV = 10/3\cdot 5 = \boxed{50/3\ W} \qquad U = \frac{1}{2}LI^2 = \frac{1}{2}\cdot 3\cdot \frac{100}{9} = \boxed{\frac{50}{3}\ J}$

5. $V_c(t)$ when $t=0$ is $9V$

$V_c(t)$ @ $t=\infty$ is $9 - \frac{9}{6+3} \cdot 6 = 3V$



$V_c(t) = 9 - (9-3)\, e^{-t/\tau}$

$\tau = RC =$

$Q = CV$

~~$\frac{dQ}{dt} = C\frac{dV}{dt} = I$~~

~~$V = \frac{I}{C}$~~

C. $\frac{dV_c(t)}{dt} + \frac{V_c(t)}{3k} + \frac{V_c(t) - 9}{6k} = 0$

$\frac{C \cdot dV_c(t)}{dt} = -\frac{V_c(t)}{3k} + \frac{9 - V_c(t)}{6k}$

$= \frac{9 - 3V_c(t)}{6k}$

$= \frac{3 - V_c(t)}{2k}$

$\int \frac{dv}{3-v} = \int \frac{1}{0.04}\, dt$

$-\ln(3-v) = \frac{1}{0.04} t + C$

$3 - v = e^{-\frac{1}{0.04}t} \cdot A$

$V(t) = 3 - e^{-\frac{1}{0.04}t} \cdot A$

$t=0 \rightarrow v(0) = 3 - A = 9$

$A = -6$

$V_c = 3 + 6 \cdot e^{-\frac{1}{0.04}t}$

$= 3 + 6 \cdot e^{-25t}$

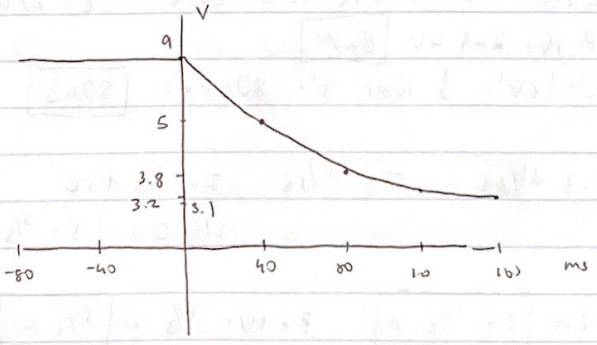
$20 \cdot 10^{-3} \cdot \frac{dV}{dt} = \frac{3-V}{2000}$

$40 \cdot 10^{-3}\, dV = (3-v)\, dt$

$\frac{4 \cdot 10^{-2}}{3-V}\, dV = dt$

$V = V_0 + (V_{inf} - V_0)\, e^{-t/RC}$

discharging a capacitor

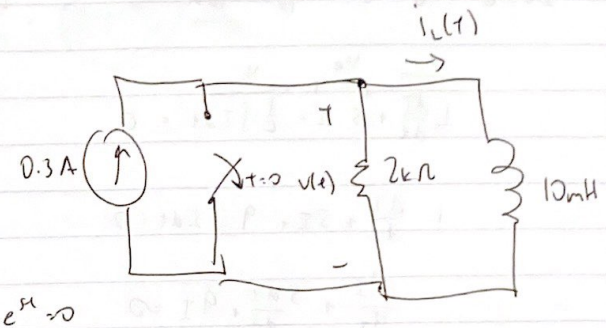


V

6.

$$-0.3 + \frac{V(t)}{R} + i_L(t) = 0$$

$$-0.3 + L \cdot \frac{dI}{dt} \cdot \frac{1}{R} + I = 0$$



$$I = K_1 + K_2 e^{st}$$

$$-0.3 + L \cdot (K_2 s e^{st}) \cdot \frac{1}{R} + K_1 + K_2 e^{st} = 0$$

$$\frac{1}{R} L K_2 s = -K_2 \qquad K_1 = 0.3$$

$$-0.3 + K_1 = 0$$

$$s = -R/L$$

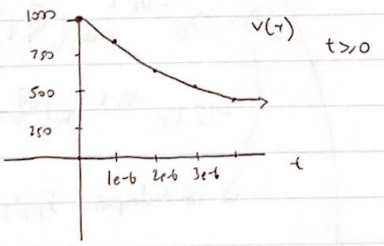
$$I = 0.3 + K_2 e^{-Rt/L}$$

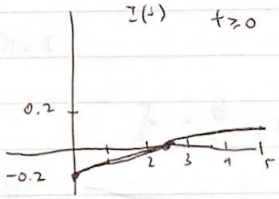
$$= 0.3 + K_2 e^{-200000t}$$

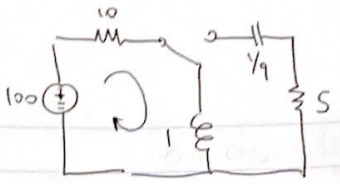
$$= 0.3 - 0.5 e^{-200000t}$$

$$V = L \cdot \frac{dI}{dt} = 10e{-}3 \cdot \left(100000 e^{-200000t}\right)$$

$$= 1000 e^{-200000t}$$







7. $I(t)$ @ $t=0^-$ $= \frac{100}{10} = \underline{10A}$

$$\overbrace{L\frac{dI}{dt}}^{V_L} + \overbrace{5\cdot I}^{V_R} + \overbrace{\frac{1}{C}\int I\,dt}^{V_C} = 0$$

$$1\cdot\frac{dI}{dt} + 5I + 9\cdot\int I\,dt = 0$$

$$\frac{d^2I}{dt^2} + 5\frac{dI}{dt} + 9I = 0$$

$I = Ae^{st}$

$$As^2e^{st} + \cancel{5Ae^{st}} + 9Aes^t = 0$$
$$5Ase^{st}$$

$$e^{st}(5As + 9A + As^2) = 0$$

$$Ae^{st}(As^2 + 5As + 9A) = 0$$

$$s = \frac{-5 \pm \sqrt{25-36}}{2}$$

$$s = \frac{-5 \pm i\sqrt{11}}{2}$$

$t=0 \quad I=10$

$I = 10e^{st}$

$\alpha = \frac{b}{2a} = \frac{5}{2}$

$\omega = \frac{1}{2}\sqrt{36-25} = \frac{\sqrt{11}}{2}$

$L\cdot\frac{dI}{dt} + 5I = 0 \quad @\ t=0$

$\frac{dI}{dt} = -50$

$-25e^{-5/2t}\cos\left(\frac{\sqrt{11}}{2}t\right)$

$+ A_2e^{-5/2t}\cdot\frac{\sqrt{11}}{2}\cdot\cos\left(\frac{\sqrt{11}}{2}t\right) = -50$

$\frac{A_2\sqrt{11}}{2} - 25 = -50$

-25

-50

$A_1 = \frac{-50}{\sqrt{11}} = -15.1$

$I(t) = 10e^{-5/2t}\cos\left(\frac{\sqrt{11}}{2}t\right)$

$-15.1e^{-5/2t}\sin\left(\frac{\sqrt{11}}{2}t\right)$

↓ underdamped s_1, s_2 complex,

$$I(t) = \cancel{A_1}\,10e^{-5/2t}\cos\left(\frac{\sqrt{11}}{2}t\right) + A_2e^{-5/2t}\sin\left(\frac{\sqrt{11}}{2}t\right)$$

$f = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{1 \cdot \frac{1}{4}}} = \frac{1}{2\pi} = 0.477$ Hz.