

## ECE 100 HW 3

$$1. \quad \textcircled{1} \quad \frac{V_1}{10} + \frac{V_1 - V_2}{5} = 1 \quad \textcircled{2} \quad \frac{V_2}{20} + 0.5 i_x = i_x$$

$$\textcircled{3} \quad \frac{V_1 - V_2}{5} = i_x \quad \frac{V_2}{20} = 0.5 i_x$$

$$\frac{V_1 - V_2}{5} = \frac{V_2}{10}$$

$$2V_1 - 2V_2 = V_2$$

$$2V_1 = 3V_2$$

$$V_1 + 2V_1 - 2V_2 = 10$$

$$3V_1 - 2V_2 = 10$$

$$\frac{9}{2} V_1 - 2V_2 = 10 \rightarrow V_2 = 4, V_1 = 6, i_x = \frac{2}{5} = 0.4 \text{ A}$$

$$V_1 = 6 \text{ V} \quad V_2 = 4 \text{ V} \quad i_x = 0.4 \text{ A}$$

2.

$$\frac{V_2 - V_3}{3k} + \frac{V_2}{2k} + \frac{V_2 - V_1}{4k} = 0$$

$$-\frac{1}{4k} V_1 + \left(\frac{1}{4k} + \frac{1}{2k} + \frac{1}{3k}\right) V_2 - \frac{1}{3k} V_3 = 0$$

$$\frac{V_3 - V_2}{3k} + \frac{V_3 - V_1}{1k} + \frac{V_3 - V_4}{2k} - 5\text{m} = 0$$

$$-\frac{1}{1k} V_1 - \frac{1}{3k} V_2 + \left(\frac{1}{3k} + \frac{1}{1k} + \frac{1}{2k}\right) V_3 - \frac{1}{2k} V_4 = 5\text{m}$$

$$\frac{V_1 - V_3}{1k} + \frac{V_1 - V_2}{4k} - I = 0$$

$$\rightarrow \frac{V_1 - V_3}{1k} + \frac{V_1 - V_2}{4k} + \frac{V_4}{5k} - \frac{V_1 - V_3}{2k} = 0$$

$$\frac{V_4}{5k} + \frac{V_4 - V_3}{2k} + I = 0$$

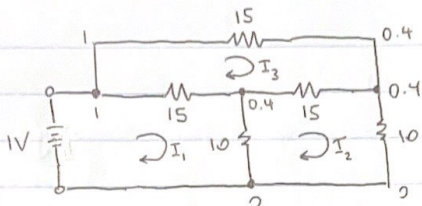
$$\left(\frac{1}{1k} + \frac{1}{4k}\right) V_1 - \frac{1}{4k} V_2 - \left(\frac{1}{1k} + \frac{1}{2k}\right) V_3 + \left(\frac{1}{2k} + \frac{1}{5k}\right) V_4 = 0$$

$$V_1 - V_4 = 20$$

$$A = \begin{bmatrix} -\frac{1}{4k} & \frac{1}{4k} + \frac{1}{2k} + \frac{1}{3k} & -\frac{1}{3k} & 0 \\ -\frac{1}{1k} & -\frac{1}{3k} & \frac{1}{3k} + \frac{1}{1k} + \frac{1}{2k} & -\frac{1}{2k} \\ \frac{1}{1k} + \frac{1}{4k} & -\frac{1}{4k} & -\frac{1}{1k} - \frac{1}{2k} & \frac{1}{2k} + \frac{1}{5k} \end{bmatrix} \quad b = \begin{bmatrix} 0 \\ 5\text{m} \\ 0 \\ 20 \end{bmatrix}$$

$$V = A \setminus b = \begin{bmatrix} 20.73 & 9.71 & 16 & 0.73 \\ \text{V} & \text{V} & \text{V} & \text{V} \end{bmatrix} \quad \text{matlab :}$$

3.



$$1 - 15(I_1 - I_3) - 10(I_1 - I_2) = 0 \Rightarrow 25I_1 - 10I_2 - 15I_3 = 1$$

$$1 - 15I_3 - 15(I_3 - I_2) - 15(I_3 - I_1) = 1 \Rightarrow 15I_1 + 15I_2 - 45I_3 = 0$$

$$0 - 10(I_2 - I_1) - 15(I_2 - I_3) - 10I_2 = 0 \Rightarrow 10I_1 - 35I_2 + 15I_3 = 0$$

$$A = \begin{bmatrix} 25 & -10 & -15 \\ 15 & 15 & -45 \\ 10 & -35 & 15 \end{bmatrix} \quad b = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad A \setminus b = \begin{bmatrix} 0.08 \\ 0.04 \\ 0.04 \end{bmatrix}$$

matlab ↴

$$I_1 = 0.08 \Rightarrow R_{eq} = \frac{V}{I_1} = \frac{1}{0.08} = 12.5 \Omega$$

4.

$$20 - 5i_1 - 15(i_1 - i_2) = 0 \Rightarrow 20i_1 - 15i_2 = 20$$

$$10 - 15(i_2 - i_1) - 10i_2 = 0 \Rightarrow 15i_1 - 25i_2 = -10$$

$$A = \begin{bmatrix} 20 & -15 \\ 15 & -25 \end{bmatrix} \quad b = \begin{bmatrix} 20 \\ -10 \end{bmatrix} \quad A \setminus b = \begin{bmatrix} 2.36 \\ 1.82 \end{bmatrix}$$

$$i_1 = 2.36 \quad i_2 = 1.82$$

$$P = I^2 R = (2.36 - 1.82)^2 \cdot 15 = 4.374 \text{ W}$$

5.

$$10 - 2(i_2 - i_1) - 2(i_2 - i_1) - 6i_3 = 0$$

$$i_1 = -1$$

$$i_3 - i_2 = 2$$

$$4i_1 - 2i_2 - 6i_3 = -10$$

$$2i_1 - i_2 - 4i_3 = -5$$

$$i_2 + 4i_3 = 3$$

$$i_3 = 1A \quad i_2 = -1A \quad i_1 = -1A$$



6.

$$0 - 20(i_1 - 1) - 10i_1 - 5(i_1 + 2) = 0$$

$$-20i_1 + 20 - 10i_1 - 5i_1 - 10 = 0$$

$$35i_1 = 10 \rightarrow i_1 = \frac{10}{35}$$

$$\boxed{0.2857 \text{ A}}$$

7.

$$-30i_1 - 20(i_1 - i_3) = 0$$

$$50i_1 = 20i_3$$

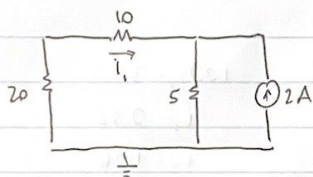
$$-12i_2 - 6(i_2 - i_3) = 0$$

$$18i_2 = 6i_3$$

$$i_3 = 3$$

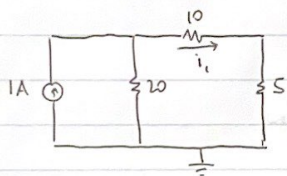
$$\rightarrow i_2 = 1 \text{ A} \rightarrow i_1 = 1.2 \text{ A}$$

8.



$$-20i_1 - 10i_1 - 5(i_1 + 2) = 0$$

$$i_1 = \frac{-10}{35} \text{ A}$$



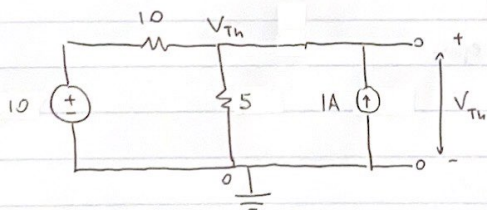
$$-10i_1 - 5i_1 - 20(i_1 - 1) = 0$$

$$20 = 35i_1$$

$$i_1 = \frac{20}{35}$$

$$\sum_{j=1}^2 i_j = \frac{20-10}{35} = \frac{10}{35} = \boxed{0.2857 \text{ A}}$$

9.

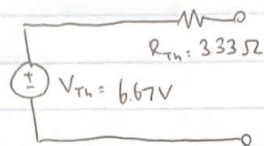


$$\frac{V_{Th}}{5} = 1 \text{ A} + \frac{10 - V_{Th}}{10} \rightarrow 2V_{Th} = 10 + 10 - V_{Th} \rightarrow V_{Th} = \frac{20}{3} = 6.67 \text{ V}$$

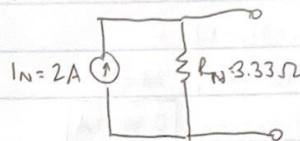
$$R_{Th} = 10 \parallel 5 = \frac{50}{15} = 3.33 \Omega = R_N$$

$$I_N = \frac{V_{Th}}{R_{Th}} = 2 \text{ A}$$

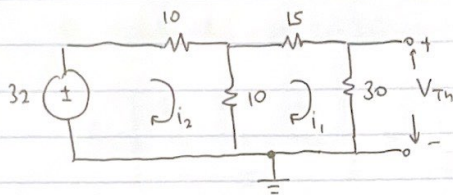
Theremin:



Norton:



10.



$$32 - 10i_2 - 10(i_2 - i_1) = 0$$

$$10i_1 - 20i_2 = -32$$

$$100i_1 = 32$$

$$-15i_1 - 30i_1 - 10(i_1 - i_2) = 0$$

$$-55i_1 + 10i_2 = 0$$

$$i_1 = 0.32$$

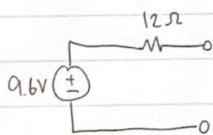
$$-110i_1 + 20i_2 = 0$$

$$i_2 = 1.76$$

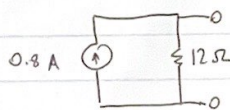
$$V_{Th} = 30i_1 = 9.6V$$

$$R_{Th} = (10 \parallel 10 - 15) \parallel 30 = 20 \parallel 30 = 12\Omega = R_N$$

$$I_N = 9.6/12 = 0.8A$$



Theremin eq.



Norton eq.