System properties

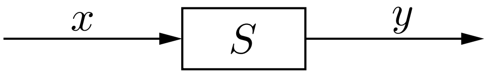
This lecture introduces systems and their properties. Topics include:

- Example systems
- Stability
- Memory
- Invertibility
- Causality
- Time-invariance
- Linearity

What is a system?

A system transforms an *input signal*, $x(t)$, into an output system, $y(t)$.

- Systems, like signals, are also *functions*. However, their inputs and outputs are signals.
- Systems can have either single or multiple inputs (SI or MI, respectively) and single or multiple outputs (SO and MO). In this class, we focus on *single input, single output* systems (SISO).
- Notation: a system S, operating on input signal x to produce output signal y, is typically denoted $y = S(x)$ in this class.
- Systems are often denoted by a *block diagram*, where lines with arrows denote signals and boxes denote systems. This is drawn below:



Example systems

The next slides go over a few examples of systems. We'll discuss more systems throughout this course.

- Scaler.
- Differentiator.
- Integrator.
- Time shifter.
- Squarer.
- Amplitude modulation
- Convolution (a substantial chunk of this class).

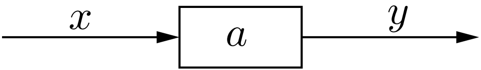
Scaling system

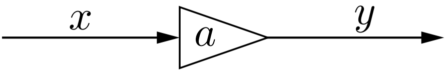
Consider an input $x(t)$ and an output $y(t)$. The scaling system is:

$$y(t) = ax(t)$$

with the following properties:

- If $|a| > 1$, the system is called an *amplifier*.
- If $|a| < 1$, the system is called an *attenuator*.
- If $a < 0$, the system is called *interventing*.
- It is common that a block diagram denotes this with a triangle.
- Block diagram below:



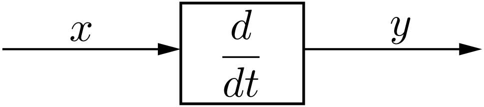


Differentiator

The differentiator is denoted:

$$\begin{aligned} y(t) &= x'(t) \\ &= \frac{d}{dt}x(t) \end{aligned}$$

Block diagram below:



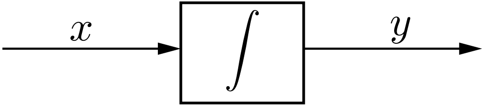
Integrator

The integrator is denoted:

$$y(t) = \int_a^t x(\tau) d\tau$$

where a is often 0 or $-\infty$.

Block diagram below:



Time shift system

The time shift system shifts a signal by T, i.e.,

$$y(t) = x(t - T)$$

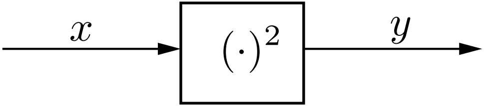
- If $T > 0$ then it is a *delay* system.
- If $T < 0$ then it is a *predictor* system.

Squarer

The squarer system squares a signal, i.e.,

$$y(t) = x^2(t)$$

Block diagram below:

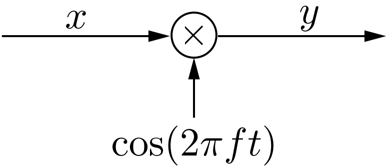


Amplitude modulation (AM radio)

Amplitude modulation takes an input "message," $x(t)$ and outputs a "transmitted signal," $y(t)$. This is denoted via:

$$y(t) = x(t)\cos(2\pi f_c t)$$

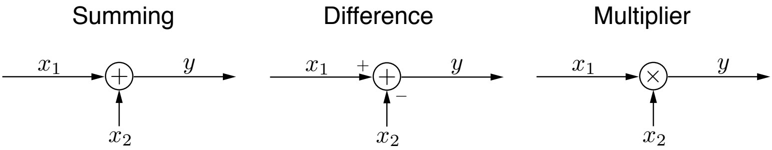
Here, f_c is called the carrier frequency. When you turn to AM radio at e.g., 880 kHz, that means the carrier frequency is $f_c = 880$ kHz. The AM block diagram is shown below.



Systems with multiple inputs

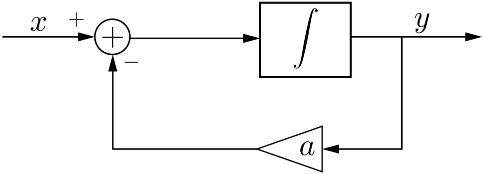
The AM system is an example of a system that takes two input signals, $x(t)$ and $\cos(\omega_c t)$, and outputs one signal, $y(t)$. This is a multiple input, single output (MISO) system. Here are a few other examples of multiple input systems.

- Summing system: $y(t) = x_1(t) + x_2(t)$.
- Difference system: $y(t) = x_1(t) - x_2(t)$.
- Multiplier system: $y(t) = x_1(t)x_2(t)$.



Example

Consider the block diagram below:



What is the equation describing the system transforming x to y?

- The input to the integrator is $x - ay$.
- Therefore, the system equation is

$$\int_{-\infty}^{t} (x(\tau) - ay(\tau))\, d\tau = y(t)$$

System properties

In these next slides, we'll discuss important system properties. These properties are:

- Stability
- Memory
- Invertibility
- Causality
- Time-invariance
- Linearity

We put particular emphasis on the last two in the class: time-invariance and linearity.

Stability

A system is *bounded-input, bounded-ouput* (BIBO) stable if every bounded input leads to a bounded output.

A bounded input is one for which the magnitude of the input is always less than a constant, M_x, i.e.,

$$|x(t)| \leq M_x < \infty$$

and similarly, a bounded output satisfies

$$|y(t)| \leq M_y < \infty$$

Mathematically, a system is BIBO stable if:

$$|x(t)| < \infty \implies |y(t)| < \infty$$

(The symbol $\implies$ means "implies" or "leads to.")

Stability examples

Are the following systems stable?

- Amplitude modulation?
- Squarer?
- The system $y(t) = 1/x(t)$?

Memory

A system has *memory* if its output depends on past or future values of the input. If the output depends only on present values of the input, the system is called *memoryless*.

Which of these systems are memoryless?

- Amplitude modulation?
- Squarer?
- Differentiator?
- Integrator?

Invertibility

A system is called *invertible* if an input can always be exactly recovered from the output. That is, a system S is invertible if there exists an S^{inv} such that

$$x = S^{\text{inv}}(S(x))$$

- Amplitude modulation?
- Squarer?
- Differentiator?
- Scaler?

Causality

A system is causal if its output only depends on past and present values of the input.

All real world systems are causal (i.e., we can't use information from the future). An example of a non-causal system is the system $x(-t) = S(x(t))$.

Time-invariance

A system is *time invariant* if a time shift in the input only produces an identical time shift of the output.

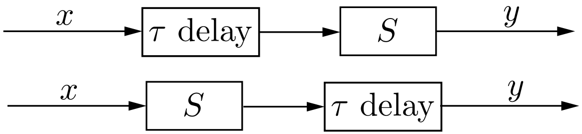
Mathematically, a system S is time-invariant if

$$y(t) = S(x(t))$$

implies that

$$y(t-\tau) = S(x(t-\tau))$$

A further implication of this is that a time-invariant system is one where delay and a system are commutable, i.e., the following block diagrams are equivalent.



Time-invariance

Are the following systems time-invariant?

- Amplitude modulation?
- Squarer?
- Differentiator?
- Integrator?

Linearity

A system is *linear* if the following two properties hold:

1. **Homogeneity**: for any signal, x, and any scalar a,

$$S(ax) = aS(x)$$

2. **Superposition**: for any two signals, x and $\tilde{x}$,

$$S(x + \tilde{x}) = S(x) + S(\tilde{x})$$

Intuitively, **homogeneity** means that scaling can occur before or after the system and **superposition** means summing can occur before or after the system.

These two can be succinctly combined, so that to check if a system is linear, one need only check that:

$$\boxed{aS(x) + bS(\tilde{x}) = S(ax + b\tilde{x})}$$

Linearity examples

Are the following systems linear?

- Amplitude modulation?
- Squarer?
- Differentiator?
- Integrator?

More linear time-invariant examples

Linearity and time-invariance are so important its worth going through more examples. For each of these systems, determine if they are linear and if they are time-invariant.

- $y(t) = \sqrt{x(t)}$
- $y(t) = x(t)z(t)$, where $z(t)$ is a known, non-zero signal.
- $y(t) = x(at)$
- $y(t) = a$
- $y(t) = x(t - \tau)$
- $y(t) = x(\tau - t)$
- Is a system S such that S^{inv} exists linear.