Overview of this class

- Motivation for studying signals and systems
- Learning goals
- Foundations for future work

What is a signal?

A signal is a *function* of one or more variables.

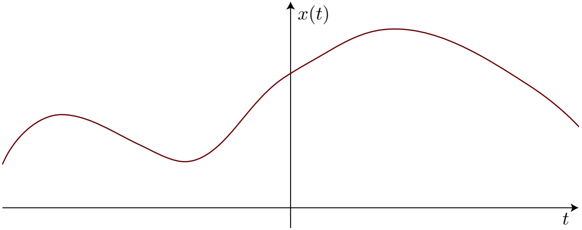
What is a function?

- We ought be familiar with functions from mathematics: denoted by $f(\cdot)$, it typically accepts some input, x and return some output, y. We write this as:
$$y = f(x)$$
- We usually denote this function as: $f : \mathbb{R} \to \mathbb{R}$, indicating that f is a function mapping a real number (the first $\mathbb{R}$) to another real number (the second $\mathbb{R}$).

The time domain

Signals usually have to do with *time* domain representations.

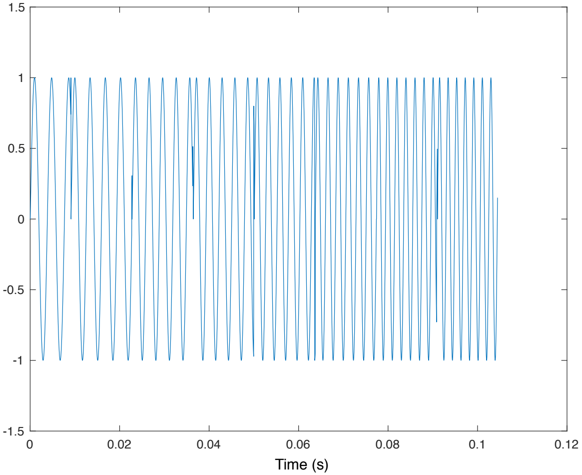
- That is, signals are usually functions that accept an input time, t, and return the value of the signal at that time. For example, a signal could be represented $x(t)$, which denotes the value of the signal at time t.
- We think of this as a time series, or a sequence of values through time.



- With this information, we can say what value the signal takes happening at any given time. But what about other characteristics of the signal?
- How can we intuit things about this signal?

A motivating example

- Consider the signal I generated a signal below.
- It looks fairly structured. Any guesses as to what this is?



A motivating example

- We can "see" structure.
- But what is this structure, and what's one way we can quantify it?

How music works

- Music is something most of us are familiar with.
- Music is fundamentally sine wave signal, $x(t) = \sin(2\pi f t)$. The tone of the note is determined by its *frequency*, denoted f in the equation.
- Every note (or tone) has a unique frequency associated with it. As the notes go higher, the frequencies also go higher. E.g.,
  - C: 261.6 Hz
  - D: 293.7 Hz
  - E: 329.6 Hz
  - F: 349.2 Hz
  - G: 392 Hz
  - A: 440 Hz
  - B: 493.9 Hz
  - C: 523.2 Hz
- Whenever you hear an off-pitch note, it's in part because it's in between these frequencies.
- Notice also structure in music: the C that is an *octave* higher is exactly $2\times$ the frequency.

An aside: why do these frequencies matter?

At a high-level:

- Consider a guitar, violin, piano, or other stringed instrument.
- When we play a certain note, what happens is that string *vibrates* at this frequency. If you could zoom in on the string, you could see that it looks like a sine wave.
- This in turn vibrates the air around it. These vibrations emanate in all directions at the speed of sound (≈ 343 m/s).
- Eventually, these sine waves reach your ear. Your ear has machinery to turn these vibrations into internal vibrations in the bones, and subsequently to your inner ear fluid.
- The cochlea then sends the frequency information to your brain, so you perceive that note.

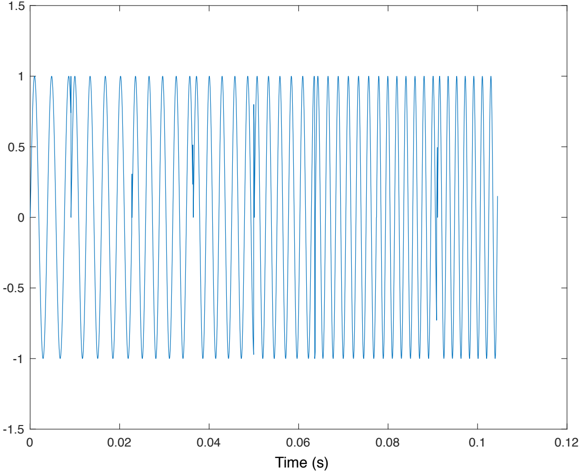
One of the great secrets of the Universe

Every signal has a spectrum and is determined by its spectrum. You can analyze the signal either in the time (or spatial) domain or in the frequency domain. I think this qualifies as a Major Secret of the Universe.

- Prof. Brad Osgood, Stanford University

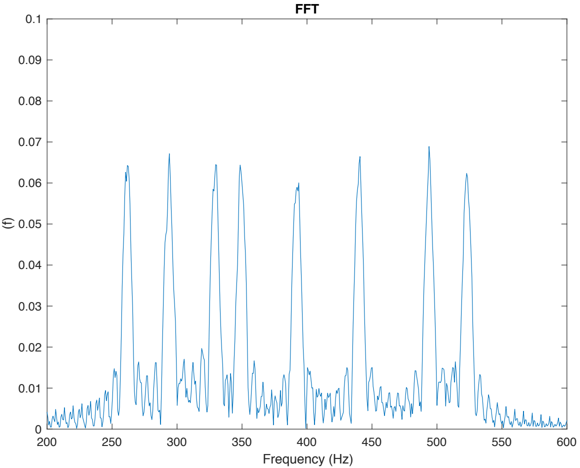
The frequency domain

- A key take-home message of this class is that every signal in time can be represented in a frequency domain.
- In this domain, the x-axis is frequency, and y-axis tells us how much of that frequency composes the signal.



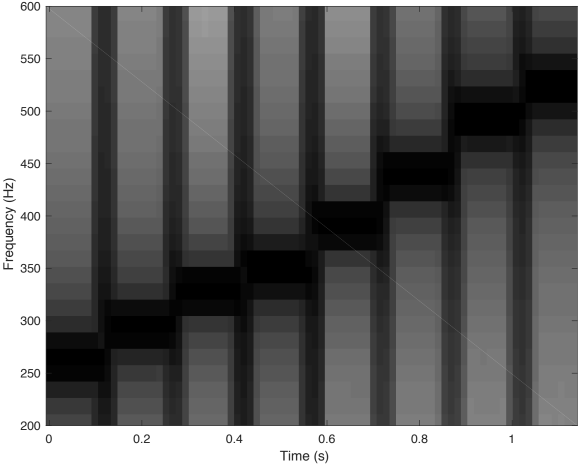
The frequency domain

- Below is the frequency domain signal (or *spectrum*) for the C-major scale.
- You can see there are 8 peaks, corresponding to the eight notes of the C-major scale.



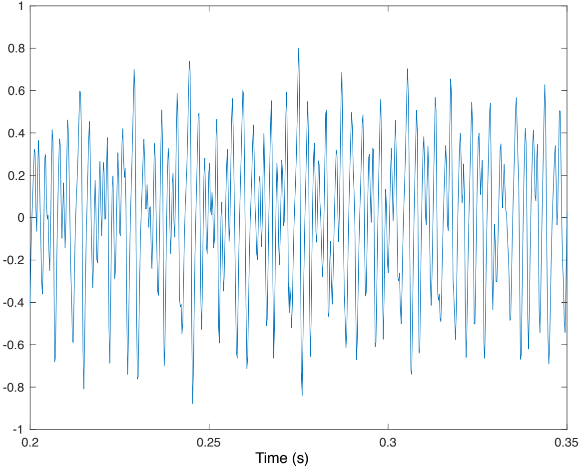
Plotting the spectrum through time

- If we plot a heatmap (where black is the largest amplitude) of the frequency power through time, we see that the frequency increases at discrete jumps, as in a scale.



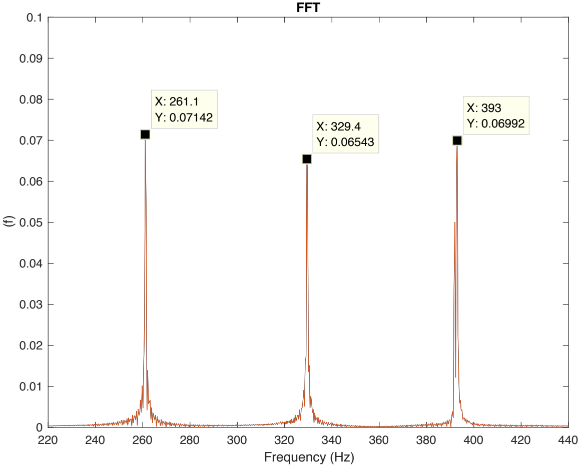
A more complicated signal

- In our prior example, it was easy to see clear sine waves.
- What about when it's not as clear?



A more complex signal

- Sometimes the frequency domain reveals clear structure not apparent in the time domain.

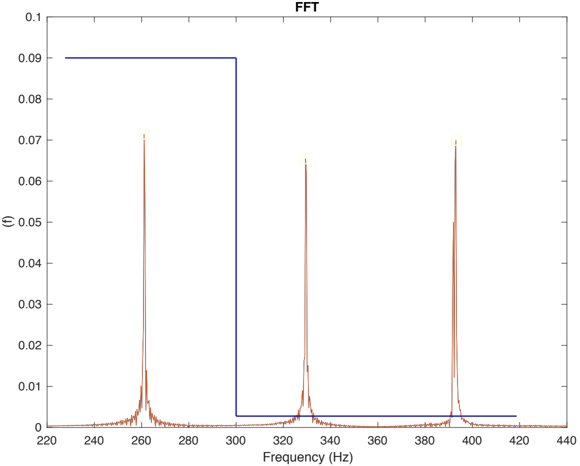


The Fourier Transform

- One of the key things you will learn in this class is how to perform a Fourier transform, which takes a signal from the time domain to the frequency domain.
- But what use is it to go to the frequency domain, if all we get to see is structure?
- It turns out we can also do *operations* in the frequency domain. This is the basis for analog circuits, communications, and even things like the amplifiers in your car.

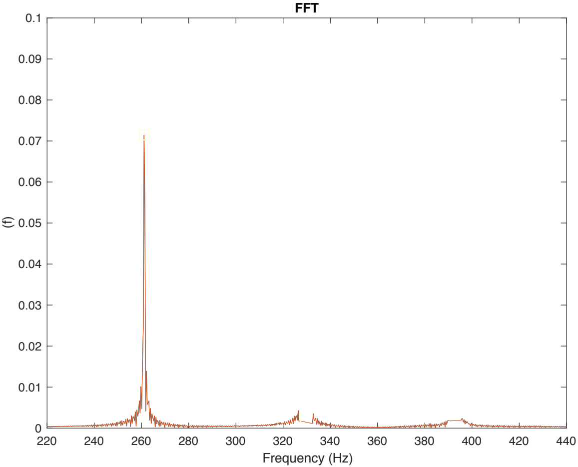
The Fourier Transform (cont.)

- Consider taking our spectrum of the C-chord (in orange) and multiplying it by the blue trace. What happens?



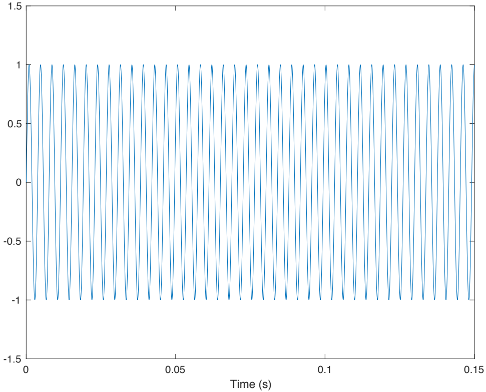
The Fourier Transform (cont.)

- The *systems* part of this class encompasses performing such operations to manipulate signals.
- Here, our *system* attenuated the E and G note of the chord.



The inverse Fourier transform

- The Fourier transform uniquely maps a signal from the time to frequency domain.
- The *inverse* Fourier transform does the opposite: it takes a signal from its frequency domain representation back to the time-domain.
- Our above spectrum back in the frequency domain is now predominantly a sine wave at the frequency of the note C.



Where are sine waves outside of music?

- This sine wave example is clear for music.
- But a lot of signals in real life don't look like sine waves.
- So how is this relevant?

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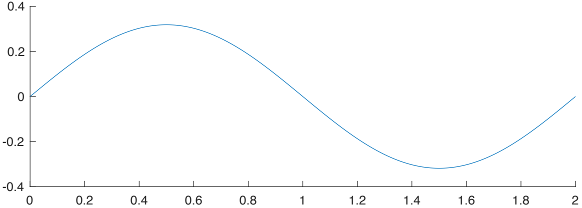
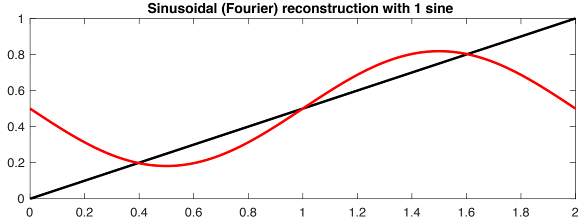
An illustration of where sine waves fit in

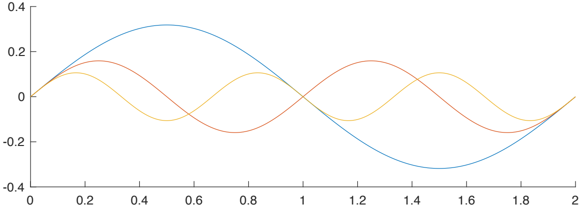
- One thing we do constantly run into are straight lines.
- Can we make a straight line out of sines?
- Let's try below. If we get one sine wave (red) to reconstruct the straight line (black), we obviously don't do a good job.

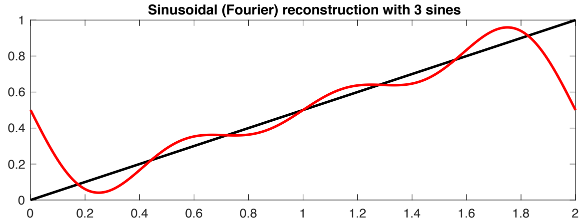




An illustration of where sine waves fit in

- What if we get to add together multiple sine waves? Say 3?





- We can see we're already starting to do better.

An illustration of where sine waves fit in

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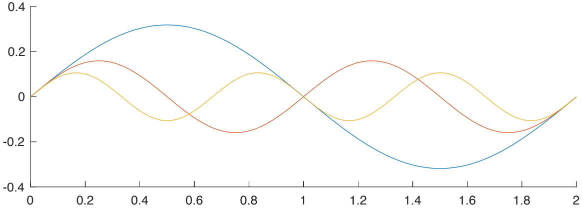
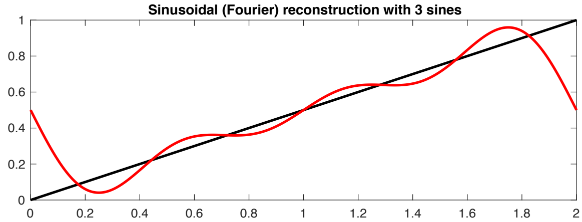
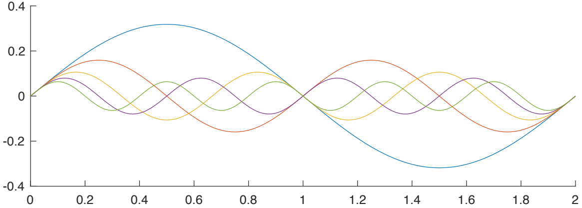


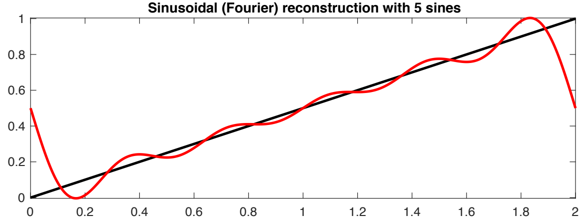


- We can see we're already starting to do better.

An illustration of where sine waves fit in

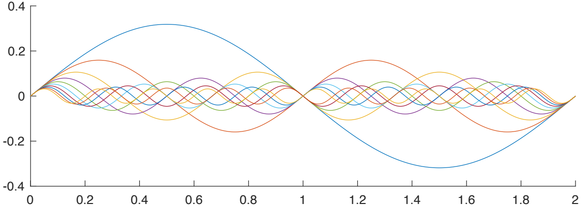
- Reconstruction with 5 sine waves.

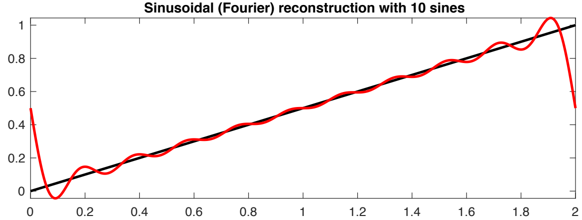




An illustration of where sine waves fit in

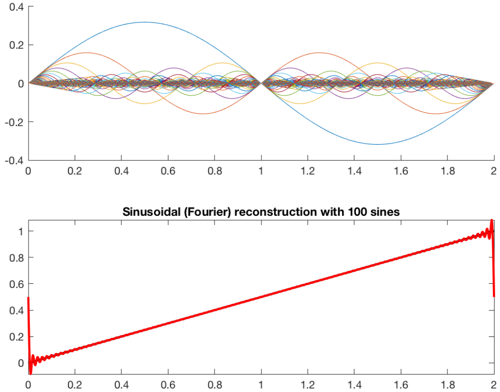
- Reconstruction with 10 sine waves.





An illustration of where sine waves fit in

- Reconstruction with 100 sine waves.



Bottom line

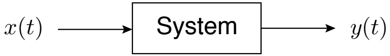
- Once we understand the mathematics of how to create things with sine waves (frequency domain or spectrum), we can do very powerful operations. This is the basis for many technologies that we may (sometimes) take for granted.
- AM / FM / Satellite radio is all possible because of this mathematics.
- So are your cell phone communications.
- Your WiFi / modem too!
- Audio amplifiers for music.
- Most circuit design is facilitated by these analyses.
- This also lays down the foundation for work in other domains, such as controls.
- In general, one way to elucidate structure in signals is to look at its spectrum, no matter your field. I do this when looking at signals from the brain.
- Thus, this is an extremely general approach to analyzing signals.

What is a system?

We established that a signal is a *function* of one or more variables and that in this class, we'll typically care about functions that vary through time, $x(t)$. This signal typically carries information that can be stored or processed.

This class is called signals **and systems**. So then what is a system?

- A system is something that operates on a signal.
- e.g., a system may take some input $x(t)$ and transform it into a different signal $y(t)$, as shown below.



- An example of a system is extracting the C note from the C major chord we earlier showed.

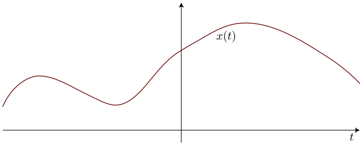
Examples of signals and systems

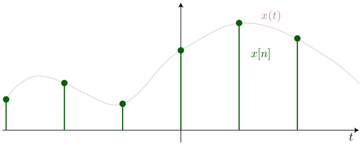
- Your speech is a signal, your brain is a system.
- Voltage and currents are signals, and a circuit that manipulates these is a system.
- Your speech is a signal, and your cell phone is a system that processes that signal to be sent to a cell tower and sent to whoever you're talking to.
- An electric guitar generates a signal, and an audio amplifier that adjusts the sound is a system.
- The federal deficit is a signal, and national / global economies, federal laws, etc. are systems.
- An example from my own research.

Continuous and discrete signals

Signals can be continuous or discrete.

- A *continuous time* signal has values for all points in time over some (possibly infinite) interval. This class primarily deals with continuous signals.
- A *discrete time* signal has values for only discrete points in time. ECE 113 primarily covers this class of signals, although we will talk about them briefly in this class.
- An illustration of this is below, where we have a continuous signal, $x(t)$, and a discrete signal, $x[n]$, defined by taking the value of $x(t)$ at certain times.





Discrete time signals

- Often, a discrete signal is composed of samples of a continuous time signal (e.g., it's impractical to store the value of every single time point; there are infinite time points, and we can't store infinite values).
- One way to get a discrete signal is to sample a continuous one at intervals of length T, so that
$$x[n] = x_{\text{cts}}(nT)$$
where n is an integer. e.g., if $T = 2$, then $x[0] = x_{\text{cts}}(0)$, $x[1] = x_{\text{cts}}(2)$, $x[2] = x_{\text{cts}}(4)$, and so on...
- Discrete time signals do not have to be sampled at uniform times. For example, maybe your discrete time signal is the price of a given stock; there are gaps when the stock exchange closes.

Signals can get more complicated

The following topics are beyond the scope of this class, but these are still worth mentioning.

Noisy signals:

- In this class, we will work with *deterministic* signals, which means that at every point in time, we know the *exact* value of the signal.
- But in real life, signals have noise. Thus, they have a *random* component, and their values at a given time cannot be exactly known. This is described by probability theory, e.g., ECE 131A.

Multivariate signals:

- In this class, we will work with signals that are *univariate*, which means the value of the signal at every point in time is a scalar (or real value).
- In real life, signals are sometimes *vectors*, meaning that the signal is multi-dimensional. This is described by linear algebra.