

ECE 102 Midterm

LIANG, NEVIN

TOTAL POINTS

103 / 106

QUESTION 1

Problem 1 30 pts

1.1 (a) 6 / 6

- ✓ - **0 pts** Correct
- **3 pts** Amp incorrect
- **3 pts** Phase incorrect.
- **6 pts** Neither amp nor phase is correct

1.2 (b) 6 / 6

- ✓ - **0 pts** Correct
- **4 pts** Conclusion correct while reasoning is incorrect, incomplete or not rigorous.
- **6 pts** Incorrect

1.3 (c) 6 / 6

- ✓ - **0 pts** Correct
- **6 pts** Incorrect
- **4 pts** correctly apply the sifting property but cannot get result right

1.4 (d) 12 / 12

- ✓ - **0 pts** Correct
- **3 pts** causal
- **3 pts** linear
- **3 pts** time variant
- **3 pts** unstable

QUESTION 2

Problem 2 40 pts

2.1 (a)(i) 5 / 5

- ✓ - **0 pts** Correct
- **4 pts** Didn't use the sifting property
- **5 pts** Incorrect
- **2 pts** Missing the step function

2.2 (a)(ii) 10 / 10

- ✓ - **0 pts** Correct
- **10 pts** Incorrect
- **5 pts** Missing the constant exponential term
- **3 pts** Missing the step function
- **3 pts** Missing the time shifts in the exponent

2.3 (a)(iii) 10 / 10

- ✓ - **0 pts** Correct
- **10 pts** Incorrect
- **5 pts** Missing the time shift in the exponent
- **5 pts** Missing the time shift in the step function
- **5 pts** Missing the exponential terms
- **5 pts** Incomplete

2.4 (a)(iv) 2 / 5

- **0 pts** Correct
- **5 pts** Incorrect
- **3 pts** Missing the exponential terms
- ✓ - **3 pts** Missing the step function terms
- **1 pts** Wrong sign
- **2 pts** Wrong exponents
- **4 pts** Incomplete answer
- **2 pts** Wrong coefficients

2.5 (b)(i) 5 / 5

- ✓ - **0 pts** Correct
- **5 pts** Incorrect
- **3 pts** Missing the interchange of differentiation and integration.
- **2 pts** Showing the validity for a specific example only

2.6 (b)(ii) 5 / 5

- ✓ - **0 pts** Correct
- **5 pts** Incorrect

- **2 pts** Showing the validity for a specific example.
- **3 pts** Missing the interchange of integration and differentiation.

QUESTION 3

Problem 3 30 pts

3.1 (a) 15 / 15

- ✓ - **0 pts** Correct
- **5 pts** Algebra error, correct formula
- **10 pts** Incomplete work. Just formula.
- **5 pts** Give fourier series in terms of complex exponential
- **15 pts** Blank answer

3.2 (b) 15 / 15

- ✓ - **0 pts** Correct
- **15 pts** Blank answer
- **5 pts** part i incorrect
- **5 pts** part ii incorrect
- **5 pts** part iii incorrect
- **10 pts** Incorrect answers, formulas written correctly

QUESTION 4

4 Bonus problem 6 / 6

- ✓ + **3 pts** $y(0.5)$ fully correct with appropriate work + justification.
- ✓ + **3 pts** $y(1)$ fully correct, or almost correct (just miscalculated area, usually missed a factor of 2), with appropriate work + justification
- + **4 pts** $y(0.5)$ correct, with partial credit for $y(1)$
- + **4 pts** Did shifts correctly, but not scaling of $x(t)$, hence getting a shifted answer. To show correct shifts, the x-axis values of shifted $h(t)$ must be shown. Otherwise you got the rubric item +2. I also gave this rubric item if the shifts appeared correct but the dragged signal started off in what appeared to be the wrong place.
- + **2 pts** A correct answer on one or both of $y(0.5)$ and $y(1)$, but no sufficient work to demonstrate how they got there. For example, not writing the bounds of

when the rects start / stop.

+ **2 pts** Partial credit for reasonable scaling or shifting operation to compute $y(0.5)$ or $y(1)$, including a shift/scale without taking into account the factor of 2; or convolved $x(t)$ and $h(t)$ with no factor of 2; or related work.

+ **1 pts** Partial credit for scaling the impulse response and input correctly, or related work

+ **0 pts** No answer or incorrect work.

1. Signal and System Properties (30 points).

(a) (6 points) What are the phase and amplitude of the following number?

$$x = (1+j)e^{3j} \quad (1)$$

$$\begin{aligned} x &= (1+j) \cdot e^{3j} = \sqrt{2} \left(\frac{1}{\sqrt{2}} + \frac{j}{\sqrt{2}} \right) \cdot e^{3j} \\ &= \sqrt{2} (e^{j\pi/4} \cdot e^{j \cdot 3}) \\ &= \sqrt{2} \cdot e^{j(3 + \pi/4)} \end{aligned}$$

$$\text{amplitude} = \sqrt{2}$$

$$\text{phase} = 3 + \pi/4$$

(b) (6 points) If $x(t)$ is an even function, and $x(t-1)$ is also even, is $x(t)$ periodic? Explain your reasoning.

$$\text{If } x(t) \text{ is even, } x(t) = x(-t)$$

$$\text{If } x(t-1) \text{ is even, } x(t-1) = x(-t-1)$$

$$\begin{aligned} x(t-1) &= x(-t-1), \text{ but } x(t-1) = x(-(t-1)) \\ &= x(-t+1) \end{aligned}$$

$$\begin{aligned} \text{Thus, } x(-t-1) &= x(-t+1) \Rightarrow x(t-1) = x(t+1) \\ &\text{for all } t. \end{aligned}$$

$$x(t) = x(t+2) \text{ for all } t.$$

Yes, it is periodic. \square

1.1(a) 6 / 6

✓ - 0 pts Correct

- 3 pts Amp incorrect

- 3 pts Phase incorrect.

- 6 pts Neither amp nor phase is correct

1. Signal and System Properties (30 points).

(a) (6 points) What are the phase and amplitude of the following number?

$$x = (1+j)e^{3j} \quad (1)$$

$$\begin{aligned} x &= (1+j) \cdot e^{3j} = \sqrt{2} \left(\frac{1}{\sqrt{2}} + \frac{j}{\sqrt{2}} \right) \cdot e^{3j} \\ &= \sqrt{2} (e^{j\pi/4} \cdot e^{j \cdot 3}) \\ &= \sqrt{2} \cdot e^{j(3 + \pi/4)} \end{aligned}$$

$$\text{amplitude} = \sqrt{2}$$

$$\text{phase} = 3 + \pi/4$$

(b) (6 points) If $x(t)$ is an even function, and $x(t-1)$ is also even, is $x(t)$ periodic? Explain your reasoning.

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$$x(t) = x(t+2) \text{ for all } t.$$

Yes, it is periodic. \square

1.2 (b) 6 / 6

✓ - 0 pts Correct

- 4 pts Conclusion correct while reasoning is incorrect, incomplete or not rigorous.

- 6 pts Incorrect

(c) (6 points) Evaluate the expression

$$\int_0^{\infty} \delta(t-1)(t-2)^2 dt.$$

$$\delta(t-1) = 1 \text{ @ } t=1 \text{ and } 0 \text{ @ } t \neq 1$$

$$\begin{aligned} \text{So, } \int_0^{\infty} \delta(t-1) \cdot (t-2)^2 dt \\ = (1-2)^2 \int_{1^-}^{1^+} \delta(t-1) dt \\ = (1-2)^2 \cdot 1 = \boxed{1} \end{aligned}$$

(d) (12 points) Consider the system with input $x(t)$ and output $y(t)$:

$$y(t) = \int_{-\infty}^t x(\lambda) u(t+1) d\lambda.$$

Determine if the system is:

- i. Linear
- ii. Time-invariant
- iii. Causal
- iv. Stable

You must justify your answer to receive full credit. (The next page is intentionally left blank as additional space to show work for this question.)

i. Linear : $x(t) = ax_1(t) + bx_2(t)$ and $ay(x_1) + by(x_2)$ equal.

$$\begin{aligned} \text{Case 1: } y(t) &= \int_{-\infty}^t (ax_1(t) + bx_2(t)) u(t+1) d\lambda \\ &= \int_{-\infty}^t ax_1(t) \cdot u(t+1) d\lambda + \int_{-\infty}^t bx_2(t) u(t+1) d\lambda \end{aligned}$$

1.3 (c) 6 / 6

✓ - 0 pts Correct

- 6 pts Incorrect

- 4 pts correctly apply the sifting property but cannot get result right

(c) (6 points) Evaluate the expression

$$\int_0^{\infty} \delta(t-1)(t-2)^2 dt.$$

$$\delta(t-1) = 1 \text{ @ } t=1 \text{ and } 0 \text{ @ } t \neq 1$$

$$\begin{aligned} \text{So, } \int_0^{\infty} \delta(t-1) \cdot (t-2)^2 dt \\ = (1-2)^2 \int_{1^-}^{1^+} \delta(t-1) dt \\ = (1-2)^2 \cdot 1 = \boxed{1} \end{aligned}$$

(d) (12 points) Consider the system with input $x(t)$ and output $y(t)$:

$$y(t) = \int_{-\infty}^t x(\lambda) u(t+1) d\lambda.$$

Determine if the system is:

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- iv. Stable

You must justify your answer to receive full credit. (The next page is intentionally left blank as additional space to show work for this question.)

i. Linear : $x(t) = ax_1(t) + bx_2(t)$ and $ay(x_1) + by(x_2)$ equal.

$$\begin{aligned} \text{Case 1: } y(t) &= \int_{-\infty}^t (ax_1(t) + bx_2(t)) u(t+1) d\lambda \\ &= \int_{-\infty}^t ax_1(t) \cdot u(t+1) d\lambda + \int_{-\infty}^t bx_2(t) u(t+1) d\lambda \end{aligned}$$

Additional space for question 1d.

$$= ay(x_1) + by(x_2) \quad \boxed{\text{Yes, linear}}$$

ii. Time - Invariant:

shift $x(t)$ left t_0 should equal shift $y(t)$ left t_0 .

$$x(t) \leftarrow t_0: \quad y(t) = \int_{-\infty}^t x(\lambda - t_0) \cdot u(t+1) d\lambda$$

$$\begin{aligned} y(t) \leftarrow t_0: \quad y(t - t_0) &= \int_{-\infty}^{t - t_0} x(\lambda) \cdot u(t+1 - t_0) d\lambda \\ &= \int_{-\infty}^t x(\lambda - t_0) \cdot u(t+1 - t_0) d\lambda \end{aligned}$$

$\boxed{\text{No, not the same.}} \rightarrow \boxed{\text{Not TI}}$

$$\begin{aligned} \text{iii. Let } x(t) = \delta(t) \Rightarrow h(t) &= \int_{-\infty}^t \delta(\lambda) \cdot u(t+1) d\lambda = u(t+1) \int_{-\infty}^t \delta(\lambda) d\lambda \\ &= u(t+1) \cdot u(t) \end{aligned}$$

CAUSAL $\forall t \quad h(t) = 0$ for $t < 0$

$$\text{iv. } y(t) = \int_{-\infty}^t x(\tau) \cdot u(t+1) d\tau = u(t+1) \cdot \int_{-\infty}^t x(\tau) d\tau.$$

the term $\int_{-\infty}^t x(\tau) d\tau$ is not bounded even

if $x(t)$ is bounded. Ex: $x(t) = 1$. Thus, NO

$\boxed{\text{not stable}}$

1.4 (d) 12 / 12

✓ - 0 pts Correct

- 3 pts causal
- 3 pts linear
- 3 pts time variant
- 3 pts unstable

2. **Impulse response, LTI systems and Convolution** (40 points).

Note: Parts (a) and (b) of this problem are independent. In part (a) of the problem, you might find the following convolution result useful:

$$e^{at}u(t) * e^{bt}u(t) = \frac{1}{a-b}e^{at}u(t) + \frac{1}{b-a}e^{bt}u(t)$$

(a) Consider the LTI system characterized by the Input/Output relationship:

$$\text{System 1 : } y(t) = \int_{-\infty}^t e^{-2(t-\tau)}x(\tau)d\tau$$

i. (5 points) Write the impulse response of the system, $h_1(t)$.

$$\begin{aligned} h_1(t) &= \int_{-\infty}^t e^{-2(t-\tau)} \cdot \delta(\tau) d\tau \\ &= \int_{-\infty}^{\infty} e^{-2(t-\tau)} \cdot \delta(\tau) \cdot u(t-\tau) d\tau \\ &= e^{-2t} \cdot u(t) \end{aligned}$$

2.1 (a)(i) 5 / 5

✓ - 0 pts Correct

- 4 pts Didn't use the sifting property

- 5 pts Incorrect

- 2 pts Missing the step function

- ii. (10 points) Determine a closed-form expression for the output ($y(t)$) of System 1 for the input

$$x(t) = 3e^{-4t}u(t-2)$$

$$h_1(t) = e^{-2t} \cdot u(t)$$

$$y(t) = x(t) * h(t)$$

$$= (3 \cdot e^{-4t} u(t-2)) * (e^{-2t} u(t))$$

$$3 \cdot e^{-4(t+2)} \cdot u(t) * e^{-2t} \cdot u(t)$$

$$= 3 \cdot e^{-8} \left[\frac{1}{-2} \cdot e^{-4t} u(t) + \frac{1}{2} \cdot e^{-2t} u(t) \right]$$

$$= \frac{3 \cdot e^{-8}}{2} \left[-e^{-4t} u(t) + e^{-2t} u(t) \right]$$

since time-invariant:

$$y(t) = \frac{3 \cdot e^{-8}}{2} \left[e^{-2(t-2)} u(t-2) - e^{-4(t-2)} u(t-2) \right]$$

2.2 (a)(ii) 10 / 10

✓ - 0 pts Correct

- 10 pts Incorrect

- 5 pts Missing the constant exponential term

- 3 pts Missing the step function

- 3 pts Missing the time shifts in the exponent

- iii. (10 points) Determine a closed-form expression for the output ($y(t)$) of System 1 for the input

$$x(t) = u(t) - u(t-3)$$

$$\begin{aligned} y(t) &= x(t) * h(t) \\ &= [u(t) - u(t-3)] * [e^{-2t} \cdot u(t)] \end{aligned}$$

$$\begin{aligned} \textcircled{1} \quad u(t) * [e^{-2t} \cdot u(t)] &= \frac{1}{0+2} e^{0t} \cdot u(t) + \frac{1}{-2} \cdot e^{-2t} \cdot u(t) \\ &= \frac{1}{2} u(t) - \frac{1}{2} e^{-2t} u(t) \end{aligned}$$

time-invariance:

$$\textcircled{2} \quad u(t-3) * [e^{-2t} \cdot u(t)] = \frac{1}{2} u(t-3) - \frac{1}{2} e^{-2(t-3)} \cdot u(t-3)$$

$$y(t) = \textcircled{1} - \textcircled{2} \text{ by distributive prop.}$$

$$= \frac{1}{2} u(t) - \frac{1}{2} e^{-2t} u(t) - \frac{1}{2} u(t-3) + \frac{1}{2} e^{-2(t-3)} u(t-3)$$

2.3 (a)(iii) 10 / 10

✓ - 0 pts Correct

- 10 pts Incorrect

- 5 pts Missing the time shift in the exponent

- 5 pts Missing the time shift in the step function

- 5 pts Missing the exponential terms

- 5 pts Incomplete

iv. (5 points) Determine the output of System 1, $y(t)$, for the input

$$x(t) = \delta(t) - \delta(t-3)$$

$$y(t) = x(t) * h(t)$$

$$= (\delta(t) - \delta(t-3)) * (e^{-2t} \cdot u(t))$$

$$= \delta(t) * [e^{-2t} \cdot u(t)] - \delta(t-3) * [e^{-2t} \cdot u(t)]$$

$$= e^0 \cdot u(0) - e^{-6} \cdot u(3)$$

$$= \boxed{u(0) - e^{-6}}$$

$$(u(0) = 1 \text{ ? } \frac{1}{2} \text{ ? } 0 \text{ ?})$$

$$= \boxed{1 - e^{-6}}$$

2.4 (a)(iv) 2 / 5

- 0 pts Correct
- 5 pts Incorrect
- 3 pts Missing the exponential terms
- ✓ - 3 pts Missing the step function terms
- 1 pts Wrong sign
- 2 pts Wrong exponents
- 4 pts Incomplete answer
- 2 pts Wrong coefficients

- (b) If $y(t) = x(t) * h(t)$ is the output of an LTI system with input $x(t)$ and impulse response $h(t)$, then show the following properties. Note, you will not receive credit if you simply state that differentiation and convolution are LTI systems and can be interchanged. Although this is true, we want you to show your work with the convolution integral.

Hint: You may interchange the order of integration and differentiation.

- i. (5 points) $\frac{d}{dt}y(t) = x(t) * (\frac{d}{dt}h(t))$

$$y(t) = x(t) * h(t)$$

$$= \int_{-\infty}^{\infty} x(\tau) \cdot h(t-\tau) d\tau$$

$$\begin{aligned} \text{LHS} = \frac{d}{dt} y(t) &= \frac{d}{dt} \int_{-\infty}^{\infty} x(\tau) \cdot h(t-\tau) d\tau \\ &= \int_{-\infty}^{\infty} \frac{d}{dt} (x(\tau) \cdot h(t-\tau)) d\tau \end{aligned}$$

$x(\tau)$ is a constant w.r.t t , so

$$= \int_{-\infty}^{\infty} x(\tau) \cdot \frac{d}{dt} h(t-\tau) d\tau \quad (1)$$

$$\text{RHS} = x(t) * \frac{d}{dt} (h(t)) = \int_{-\infty}^{\infty} x(\tau) \cdot \frac{d}{dt} (h(t-\tau)) d\tau \quad (2)$$

$$(1) = (2) \quad \square$$

2.5 (b)(i) 5 / 5

✓ - 0 pts Correct

- 5 pts Incorrect

- 3 pts Missing the interchange of differentiation and integration.

- 2 pts Showing the validity for a specific example only

ii. (5 points) $\frac{d}{dt}y(t) = (\frac{d}{dt}x(t)) * h(t)$

LHS:

$$y(t) = x(t) * h(t) = h(t) * x(t)$$

$$= \int_{-\infty}^{\infty} h(\tau) \cdot x(t-\tau) d\tau$$

$$\frac{d}{dt} y(t) = \frac{d}{dt} \int_{-\infty}^{\infty} h(\tau) \cdot x(t-\tau) d\tau$$

$$= \int_{-\infty}^{\infty} \frac{d}{dt} (h(\tau) \cdot x(t-\tau)) d\tau$$

$$h(\tau) = \text{constant w.r.t } t,$$

$$= \int_{-\infty}^{\infty} h(\tau) \cdot \frac{d}{dt} (x(t-\tau)) d\tau \quad (1)$$

$$\text{RHS: } \left(\frac{d}{dt} x(t) \right) * h(t) = h(t) * \left(\frac{d}{dt} x(t) \right)$$

$$= \int_{-\infty}^{\infty} h(\tau) \cdot \frac{d}{dt} (x(t-\tau)) d\tau \quad (2)$$

$$(1) = (2) \quad \text{so } \square$$

2.6 (b)(ii) 5 / 5

✓ - 0 pts Correct

- 5 pts Incorrect

- 2 pts Showing the validity for a specific example.

- 3 pts Missing the interchange of integration and differentiation.

3. Fourier Series (30 points).

- (a) (15 points) Find the Fourier series of the function: $f(x) = \begin{cases} -1 & \text{if } -\pi \leq x \leq 0 \\ 1 & \text{if } 0 < x \leq \pi \end{cases}$. To receive full credit, you must simplify your answer, including complex exponentials if possible. For this part of the problem, you may assume that $f(x)$ has a fundamental period of 2π .

$$\begin{aligned}
 f_k &= \frac{1}{2\pi} \int_{-\pi}^{\pi} f(t) \cdot e^{-jk\omega_0 t} dt \quad \boxed{T=2\pi, \omega_0=1} \\
 &= \frac{1}{2\pi} \left[\int_{-\pi}^0 -1 \cdot e^{-jk\omega_0 t} dt + \int_0^{\pi} 1 \cdot e^{-jk\omega_0 t} dt \right] \\
 &= \frac{1}{2\pi} \left[\left. \frac{-1}{-jk\omega_0} \cdot e^{-jk\omega_0 t} \right|_{-\pi}^0 + \left. \frac{1}{-jk\omega_0} \cdot e^{-jk\omega_0 t} \right|_0^{\pi} \right] \\
 &= \frac{1}{2\pi} \cdot \frac{1}{jk\omega_0} \left[1 - e^{jk\omega_0 \pi} - (e^{-jk\omega_0 \pi} - 1) \right] \\
 &= \frac{1}{2\pi j k \omega_0} \left[2 - e^{j\pi \cdot k \omega_0} - e^{-j\pi \cdot k \omega_0} \right] \\
 &= \frac{1}{2\pi j k} \left[2 - e^{j\pi k} - e^{-j\pi k} \right] \\
 &= \frac{1}{\pi j k} - \left[\frac{e^{j\pi k} + e^{-j\pi k}}{2} \cdot \frac{1}{\pi j k} \right] \\
 &= \frac{1}{\pi j k} (1 - \cos(\pi k)) \\
 f(x) &= \sum_{k=-\infty}^{\infty} \frac{1}{\pi j k} (1 - \cos(\pi k)) \cdot e^{j k t}
 \end{aligned}$$

3.1 (a) 15 / 15

✓ - 0 pts Correct

- 5 pts Algebra error, correct formula
- 10 pts Incomplete work. Just formula.
- 5 pts Give fourier series in terms of complex exponential
- 15 pts Blank answer

(b) (15 points) Let $x(t)$ be a periodic signal whose Fourier series coefficients are:

$$c_k = \begin{cases} 2 & k=0 \\ j(\frac{1}{2})^{|k|} & \text{otherwise} \end{cases}$$

Use Fourier series properties to answer the following questions:

i. Is $x(t)$ real?

if $x(t)$ is real,

$$|c_k| = |c_{-k}| \text{ and } \angle c_k = -\angle c_{-k}^*$$

$$\text{or just: } c_k^* = c_{-k}$$

$$c_{-k} = \begin{cases} 2 & k=0 \\ j(\frac{1}{2})^{|k|} & \text{otherwise} \end{cases} = c_k$$

$$c_k^* = \begin{cases} 2 & k=0 \\ -j(\frac{1}{2})^{|k|} & \text{otherwise} \end{cases} \neq c_k. \quad \boxed{\text{No}}$$

ii. Is $x(t)$ even?

$$x(t) \text{ even} \quad \longleftrightarrow \quad C_k = C_{-k}$$

$$C_{-k} = \begin{cases} 2 & k=0 \\ j\left(\frac{1}{2}\right)^{1-|k|} & k \neq 0 \end{cases}$$

$$= \begin{cases} 2 & k=0 \\ j\left(\frac{1}{2}\right)^{|k|} & k \neq 0 \end{cases} = C_k \quad \boxed{\text{YES}}$$

iii. Is $\frac{dx(t)}{dt}$ even?

let C'_k be the coefficients of $\frac{dx}{dt}$.

$$C'_k = C_k \cdot j\omega_0 k$$

$$= \begin{cases} 2j\omega_0 k & k=0 \\ -\omega_0 k \cdot \left(\frac{1}{2}\right)^{|k|} & k \neq 0 \end{cases} = \begin{cases} 0 & k=0 \\ -\omega_0 k \left(\frac{1}{2}\right)^{|k|} & k \neq 0 \end{cases}$$

$$C'_{-k} = \begin{cases} -2j\omega_0 k & k=0 \\ \omega_0 k \cdot \left(\frac{1}{2}\right)^{|k|} & k \neq 0 \end{cases} = \begin{cases} 0 & k=0 \\ \omega_0 k \left(\frac{1}{2}\right)^{|k|} & k \neq 0 \end{cases}$$

No, the $k \neq 0$ case doesn't equal.

therefore, not even.

3.2 (b) 15 / 15

✓ - 0 pts Correct

- 15 pts Blank answer
- 5 pts part i incorrect
- 5 pts part ii incorrect
- 5 pts part iii incorrect
- 10 pts Incorrect answers, formulas written correctly

Bonus Question (6 points)

Consider two signals, $x(t) = \text{rect}(t - 2.5)$ and $h(t) = \text{rect}(t)$. We convolve $x(2t)$ and $h(2t)$, i.e.,

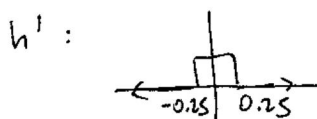
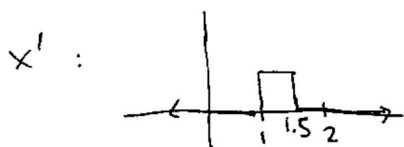
$$y(t) = x(2t) * h(2t)$$

Using the flip and drag technique, compute the values of:

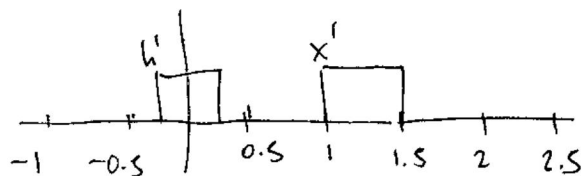
- $y(0.5)$
- $y(1)$

To receive full credit, you must draw a sketch of the flip and drag convolution computation you use to compute $y(0.5)$ and $y(1)$. You will not receive any partial credit for solving this analytically, although you may of course use an analytical answer to check your work. (This is because the purpose of this question is to test your understanding of signal operations and computing convolution with flip and drag.)

$$\begin{aligned} x(2t) &= \text{rect}(2t - 2.5) = x' \\ h(2t) &= \text{rect}(2t) = h' \end{aligned}$$



flip h' is the same haha.



$$\begin{aligned} t < 0.75 & \quad y(t) = 0 \\ 0.75 \leq t < 1.25 & \quad y(t) = t - 0.75 \\ 1.25 \leq t < 1.75 & \quad y(t) = 0.5 - (t - 1.25) = 1.75 - t \\ t > 1.75 & \quad y(t) = 0 \end{aligned}$$

$$y(0.5) = \boxed{0}; \quad y(1) = \boxed{0.25}$$

4 Bonus problem 6 / 6

✓ + 3 pts $y(0.5)$ fully correct with appropriate work + justification.

✓ + 3 pts $y(1)$ fully correct, or almost correct (just miscalculated area, usually missed a factor of 2), with appropriate work + justification

+ 4 pts $y(0.5)$ correct, with partial credit for $y(1)$

+ 4 pts Did shifts correctly, but not scaling of $x(t)$, hence getting a shifted answer. To show correct shifts, the x-axis values of shifted $h(t)$ must be shown. Otherwise you got the rubric item +2. I also gave this rubric item if the shifts appeared correct but the dragged signal started off in what appeared to be the wrong place.

+ 2 pts A correct answer on one or both of $y(0.5)$ and $y(1)$, but no sufficient work to demonstrate how they got there. For example, not writing the bounds of when the rects start / stop.

+ 2 pts Partial credit for reasonable scaling or shifting operation to compute $y(0.5)$ or $y(1)$, including a shift/scale without taking into account the factor of 2; or convolved $x(t)$ and $h(t)$ with no factor of 2; or related work.

+ 1 pts Partial credit for scaling the impulse response and input correctly, or related work

+ 0 pts No answer or incorrect work.