**ECE102, Fall 2020** — Signals & Systems — UCLA; Department of ECE

**Homework #5** — Prof. J.C. Kao — TAs: A. Ghosh, T. Monsoor, G. Zhao

Due Friday, 20 Nov 2020, by 11:59pm to Gradescope.
Covers material up to Lecture 10.
100 points total.
This homework covers questions relate to Fourier series and Fourier transform.

1. (18 points) **Fourier Series**

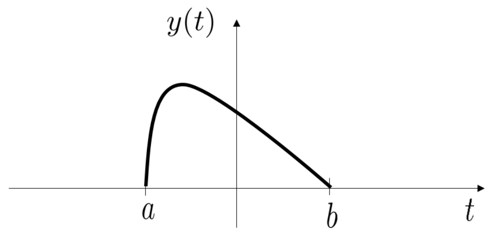
   (a) (7 points) When the periodic signal $f(t)$ is real, you have seen in class some properties of symmetry for the Fourier series coefficients of $f(t)$ (handout 8, slide 41). How do these properties of symmetry change when $f(t)$ is pure imaginary?

   (b) Suppose we are given the following information about a signal $x(t)$:

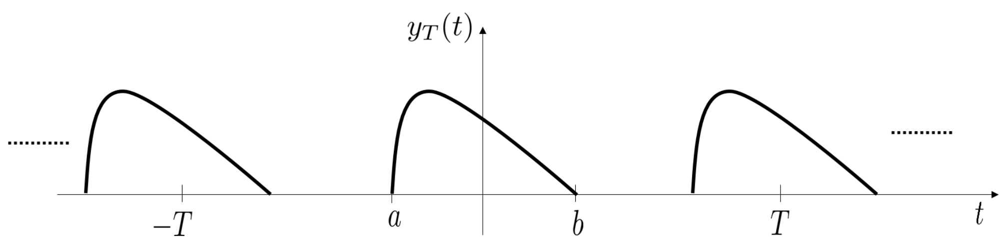
   - $x(t)$ is real and odd.
   - $x(t)$ is periodic with period $T = 2$ and has Fourier coefficients $a_k$.
   - $a_k = 0$ for $|k| > 1$.
   - $\frac{1}{2}\int_0^2 |x(t)|^2 dt = 1$

   Find $x(t)$

   (c) (4 points) Consider the signal $y(t)$ shown below and let $Y(j\omega)$ denote its Fourier transform.




   Let $Y_T(t)$ denote its periodic extension:




   How can the Fourier series coefficients of $y_T(t)$ be obtained from the Fourier transform $Y(j\omega)$ of $y(t)$? (Note that the figures given in this problem are for illustrative purposes, the question is for any arbitrary $y(t)$).

2. (32 points) **Symmetry properties of Fourier transform**

(a) (16 points) Determine which of the signals, whose Fourier transforms are depicted in Fig. 1, satisfy each of the following:

i. $x(t)$ is even
ii. $x(t)$ is odd
iii. $x(t)$ is real
iv. $x(t)$ is complex (neither real, nor pure imaginary)
v. $x(t)$ is real and even
vi. $x(t)$ is imaginary and odd
vii. $x(t)$ is imaginary and even
viii. There exists a non-zero $\omega_0$ such that $e^{j\omega_0 t}x(t)$ is real and even

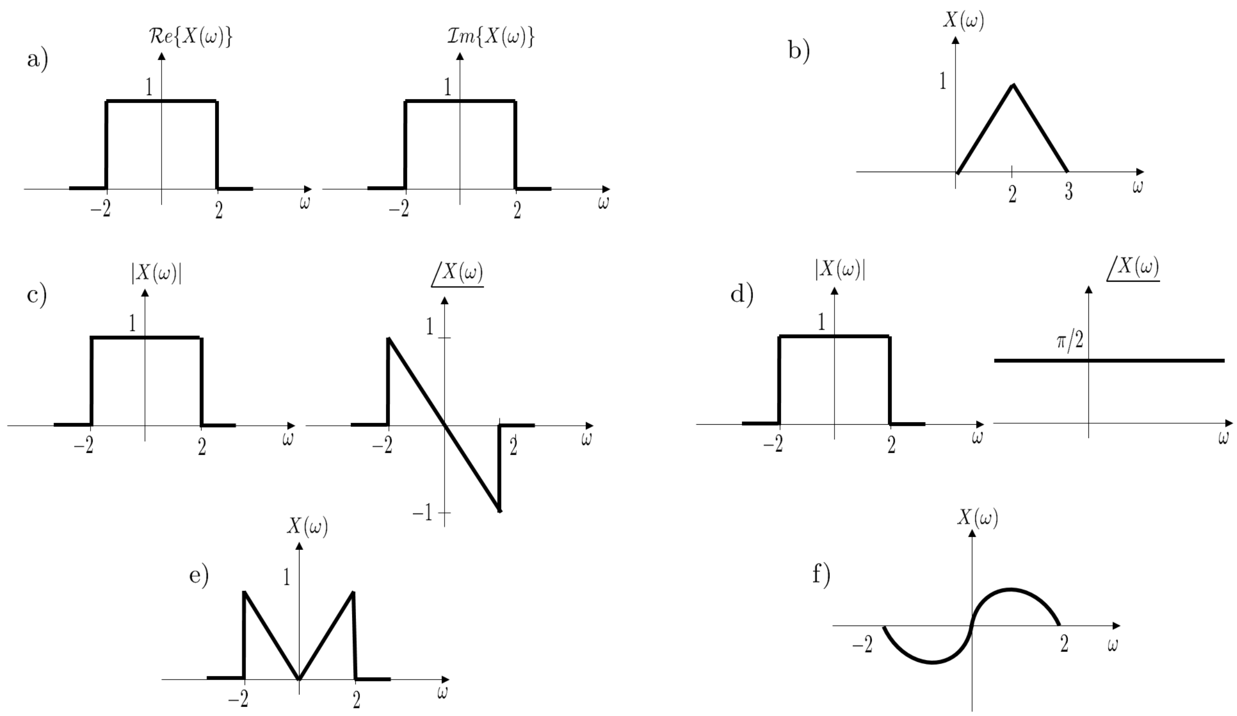



Figure 1: P2.a

(b) (8 points) Using the properties of Fourier transform, determine whether the assertions are true or false.

i. The convolution of a real and even signal and a real and odd signal is odd.
ii. The convolution of a signal and the same signal reversed is an even signal.

(c) (8 points) Show the following statements:

i. If $x(t) = x^*(-t)$, then $X(j\omega)$ is real.
ii. If $x(t)$ is a real signal with $X(j\omega)$ its Fourier transform, then the Fourier transforms $X_e(j\omega)$ and $X_o(j\omega)$ of the even and odd components of $x(t)$ satisfy the following:

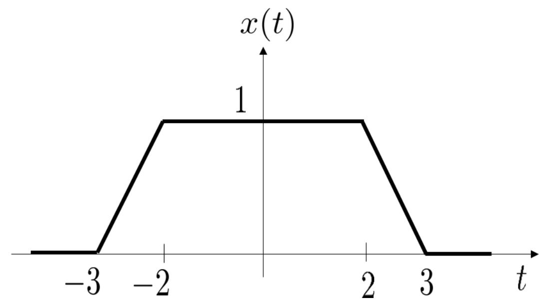
$$X_e(j\omega) = Re\{X(j\omega)\}$$

and

$$X_o(j\omega) = jIm\{X(j\omega)\}$$

3. (15 points) **Fourier transform properties**

(a) (10 points) Let $X(j\omega)$ denote the Fourier transform of the signal $x(t)$ sketched below:




Evaluate the following quantities without explicitly finding $X(j\omega)$:

i. $\int_0^\infty X(j\omega)d\omega$

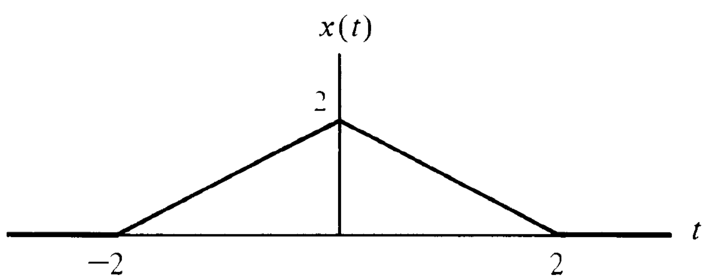
ii. $X(j\omega)|_{\omega=0}$

iii. $\angle X(j\omega)$

iv. $\int_{-\infty}^{\infty} e^{-j\omega} X(j\omega)d\omega$

v. Plot the inverse Fourier transform of $\mathcal{R}e\{e^{-3j\omega}X(j\omega)\}$

(b) (5 points) By first expressing the triangular signal $x(t)$ shown below as the convolution of a rectangular pulse with itself, determine the Fourier transform of $x(t)$.



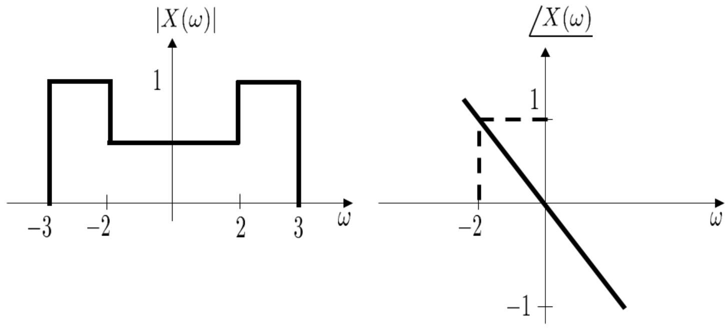

4. (35 points) **Fourier transform and its inverse**

(a) (21 points) Find the Fourier transform of each of the signals given below:
Hint: you may use Fourier Transforms derived in class.

i. (**optional**) $x_1(t) = 2\text{rect}\left(\frac{-t-3}{2}\right)\cos(10\pi t)$

ii. $x_2(t) = e^{(2+3j)t}u(-t+1)$

iii. $x_3(t) = \begin{cases} 1+\cos(\pi t), & |t| < 1 \\ 0, & \text{otherwise} \end{cases}$




iv. $x_4(t) = te^{-2t}u(t)$

(b) (6 points) Find the inverse Fourier transform of the signal shown below:

(c) (8 points) Two signals $f_1(t)$ and $f_2(t)$ are defined as

$$f_1(t) = \text{sinc}(2t)$$
$$f_2(t) = \text{sinc}(t)\cos(3\pi t)$$

Let the convolution of the two signals be

$$f(t) = (f_1 * f_2)(t)$$

i. Find $F(j\omega)$, the Fourier transform of $f(t)$.

ii. Find $f(t)$.