ECE 102 HW7

LIANG, NEVIN

TOTAL POINTS

97 / 100

QUESTION 1

Problem 1 13 pts

1.1 1(a) 4 / 4

✓ - **0 pts** Correct

- **4 pts** incorrect or no answer

1.2 1(b) 9 / 9

✓ - **0 pts** Correct

- **1 pts** error in plots
- **9 pts** no answer

QUESTION 2

Problem 2 15 pts

2.1 2(a) 3 / 3

✓ - **0 pts** Correct

- **1.5 pts** wrong answer
- **0.5 pts** not finished
- **0.5 pts** wrong shift
- **3 pts** Not attempt

2.2 2(b) 2.5 / 3

- **0 pts** Correct
- **0.5 pts** wrong value over $$e$$
- **1.5 pts** wrong answer
- **0.5 pts** wrong $$\sum$$ position
- **0.5 pts** wrong scale

✓ - **0.5 pts** **not simplified**

- **3 pts** Not attempt

2.3 2(c) 2.5 / 3

- **0 pts** Correct

✓ - **0.5 pts** **Not finished**

- **1.5 pts** wrong answer
- **3 pts** Not attempt

2.4 2(d) 3 / 3

✓ - **0 pts** Correct

- **1 pts** No graph
- **1 pts** wrong graph
- **3 pts** no answer

2.5 2(e) 3 / 3

✓ - **0 pts** Correct

- **1 pts** wrong answer
- **3 pts** No answer

QUESTION 3

Problem 3 18 pts

3.1 3(a)(i) 3 / 4

- **0 pts** Correct

✓ - **1 pts** **Wrong spectrum.**

- **4 pts** No answer

3.2 3(a)(ii) 3 / 4

- **0 pts** Correct

✓ - **1 pts** **Wrong spectrum**

- **4 pts** No answer

3.3 3(a)(iii) 4 / 4

✓ - **0 pts** Correct

- **1 pts** Wrong spectrum
- **4 pts** No answer

3.4 3(a)(iv) 4 / 4

✓ - **0 pts** Correct

- **1 pts** Wrong spectrum
- **4 pts** No answer

3.5 3(b) 2 / 2

✓ - **0 pts** Correct

- **1 pts** Wrong answer
- **2 pts** No answer

QUESTION 4

Problem 4 20 pts

4.1 4(a)(i) 5 / 5

✓ - **0 pts** **Effort**

- **5 pts** no effort

4.2 4(a)(ii) 5 / 5

✓ - **0 pts** **Effort**

- **5 pts** No effort

4.3 4(b)(i) 5 / 5

✓ - **0 pts** **Effort**

- **5 pts** No effort

4.4 4(b)(ii) 5 / 5

✓ - **0 pts** **Effort**

- **5 pts** No effort

QUESTION 5

Problem 5 18 pts

5.1 5(a) 6 / 6

✓ - **0 pts** **effort**

- **6 pts** Not attempt

5.2 5(b) 6 / 6

✓ - **0 pts** **effort**

- **6 pts** not attempt

5.3 5(c) 6 / 6

✓ - **0 pts** **effort**

- **6 pts** not attempt

QUESTION 6

Problem 6 16 pts

6.1 6(a) 5 / 5

✓ - **0 pts** **Effort**

- **5 pts** No answer

6.2 6(b) 5 / 5

✓ - **0 pts** **Effort**

- **5 pts** No answer

6.3 6(c) 6 / 6

✓ - **0 pts** **Effort**

- **6 pts** No answer

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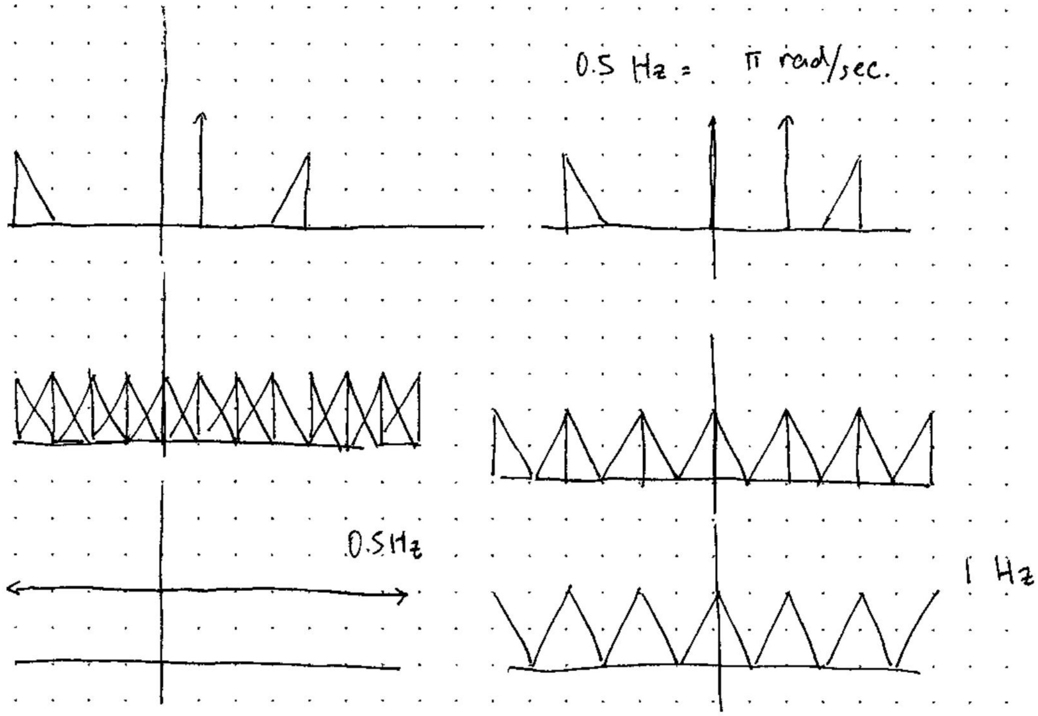
Homework #7

Nevin Liang

1. (a) Nyquist rate: $2B = 2 \cdot 2\pi \cdot \cancel{4\pi = \boxed{16\pi^2}}$ oops

$$= 2 \cdot \frac{\omega}{2\pi} = \frac{4\pi}{\pi} = \boxed{4} \text{ Hz}$$

(b)



$F_s = 1\text{Hz}$ makes the signal still recoverable.

2. (a) $f(t) = \sum \delta(t-2k) + \sum \delta(t-2k+\tau+1)$

$$= \sum_{k=-\infty}^{\infty} \delta(t-2k) + \sum_{k=-\infty}^{\infty} \delta(t-2k+\tau+1) = \delta_2(t) + \delta_2(t-\tau-1)$$

(b) $\delta_T(t) \leftrightarrow \omega_0 \delta_{\omega_0}(\omega) \qquad 2\pi/T = \omega_0$

$$\delta_2(t) \leftrightarrow \pi \delta_\pi(\omega)$$

$$\delta_2(t-\tau-1) \leftrightarrow \pi \delta_\pi(\omega) \cdot e^{-(\tau+1)j\omega}$$

(i) $\tau = 0 \Rightarrow \pi \delta_\pi(\omega) + \pi \delta_\pi(\omega) \cdot e^{-j\omega}$

1.1 1(a) **4 / 4**

✓ **- 0 pts** Correct

- 4 pts incorrect or no answer

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Homework #7

1. (a) Nyquist rate: $2B = 2 \cdot 2\pi \cdot \cancel{4\pi} = \cancel{\boxed{16\pi^2}}$ oops

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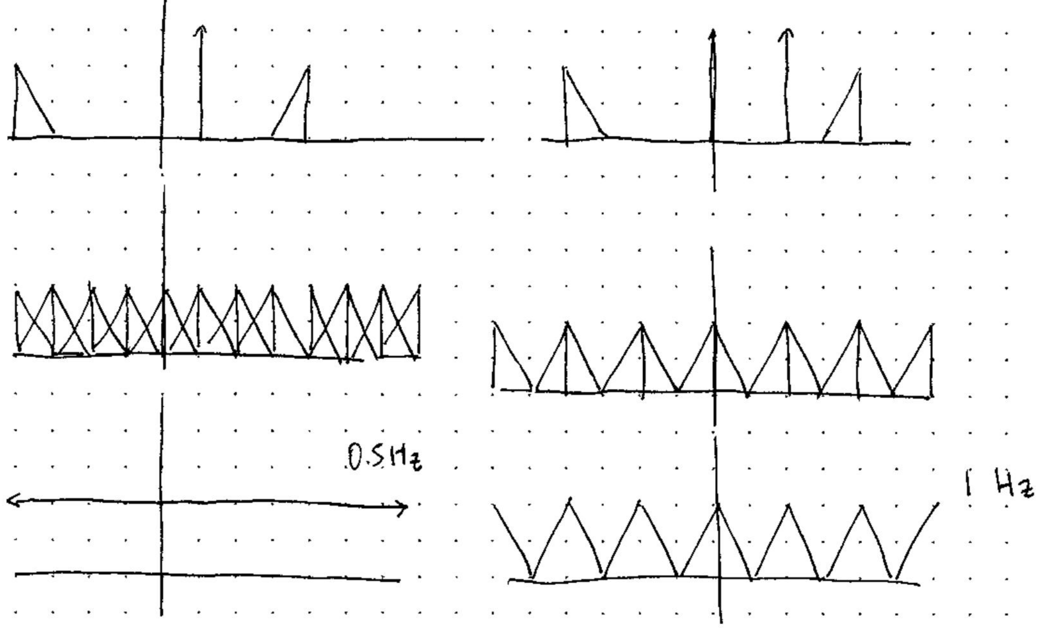
(b) 0.5 Hz = π rad/sec.



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(i) $\tau = 0 \Rightarrow \pi\,\delta_\pi(\omega) + \pi\,\delta_\pi(\omega) \cdot e^{-j\omega}$

1.2 1(b) **9 / 9**

✓ **- 0 pts** Correct

- 1 pts error in plots

- 9 pts no answer

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Homework #7

Nevin Liang

1. (a) Nyquist rate: $2B = 2 \cdot 2\pi \cdot \cancel{4\pi} = \cancel{\boxed{16\pi^2}}$ oops

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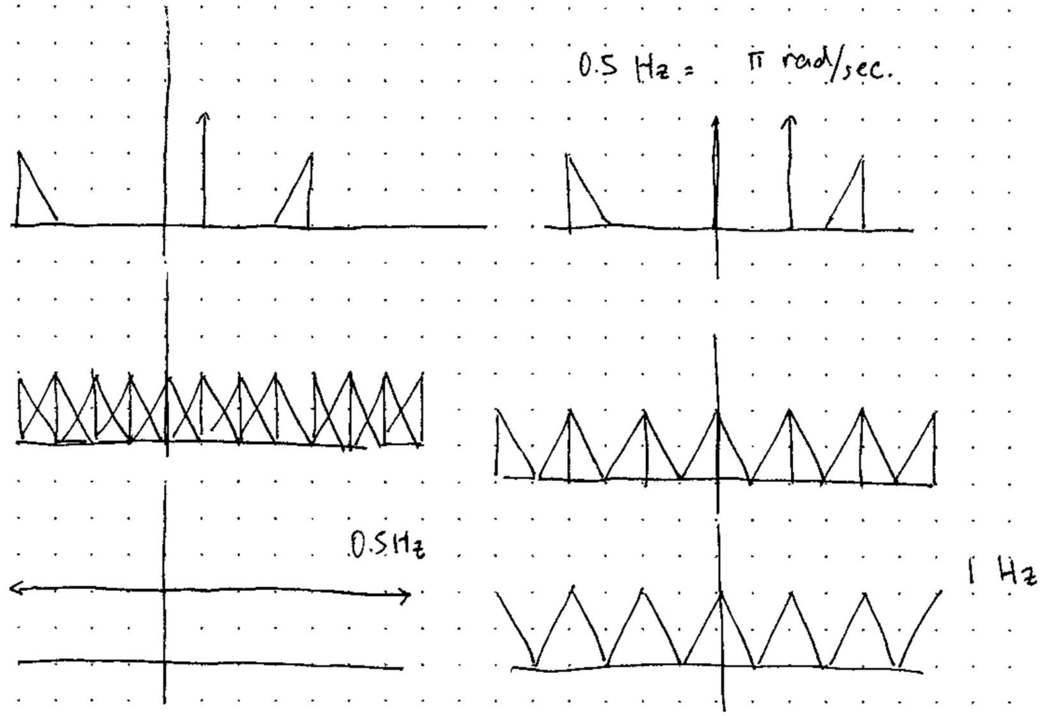

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(i) $\tau = 0 \Rightarrow \pi\, \delta_\pi(\omega) + \pi\, \delta_\pi(\omega) \cdot e^{-j\omega}$

2.1 2(a) 3 / 3

- ✓ **- 0 pts** Correct
- **- 1.5 pts** wrong answer
- **- 0.5 pts** not finished
- **- 0.5 pts** wrong shift
- **- 3 pts** Not attempt

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Homework #7

1. (a) Nyquist rate: $2B = 2 \cdot 2\pi \cdot \cancel{4\pi} = \cancel{\boxed{16\pi^2}}$ oops

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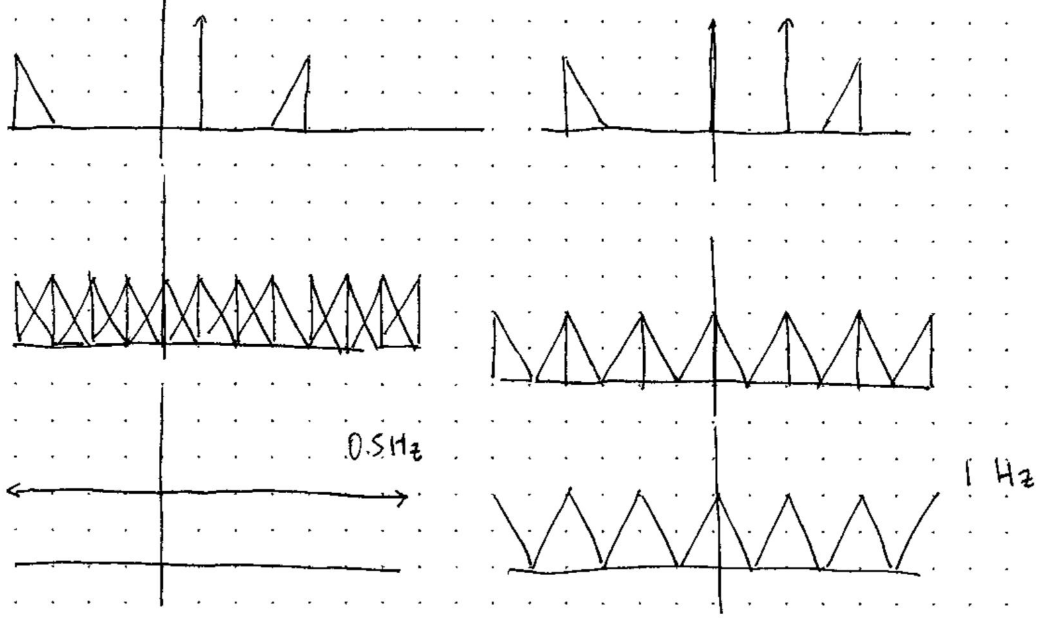

$F_s = 1\text{Hz}$ makes the signal still recoverable.

2. (a) $f(t) = \sum \delta(t-2k) + \sum \delta(t-2k+\tau+1)$

$$= \sum_{k=-\infty}^{\infty} \delta(t-2k) + \sum_{k=-\infty}^{\infty} \delta(t-2k+\tau+1) = \delta_2(t) + \delta_2(t-\tau-1)$$

(b) $\delta_T(t) \longleftrightarrow \omega_0 \delta_{\omega_0}(\omega) \qquad 2\pi/T = \omega_0$

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(i) $\tau = 0 \Rightarrow \pi \delta_\pi(\omega) + \pi \delta_\pi(\omega) \cdot e^{-j\omega}$

2.2 2(b) **2.5 / 3**

- **- 0 pts** Correct
- **- 0.5 pts** wrong value over $$e$$
- **- 1.5 pts** wrong answer
- **- 0.5 pts** wrong $$\sum$$ position
- **- 0.5 pts** wrong scale
- ✓ **- 0.5 pts** **not simplified**
- **- 3 pts** Not attempt

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Homework #7.

1. (a) Nyquist rate: $2B = 2 \cdot 2\pi \cdot \cancel{4\pi = \boxed{16\pi^2}}$ oops

$= 2 \cdot \frac{\omega}{2\pi} = \frac{4\pi}{\pi} = \boxed{4}$ Hz

(b)

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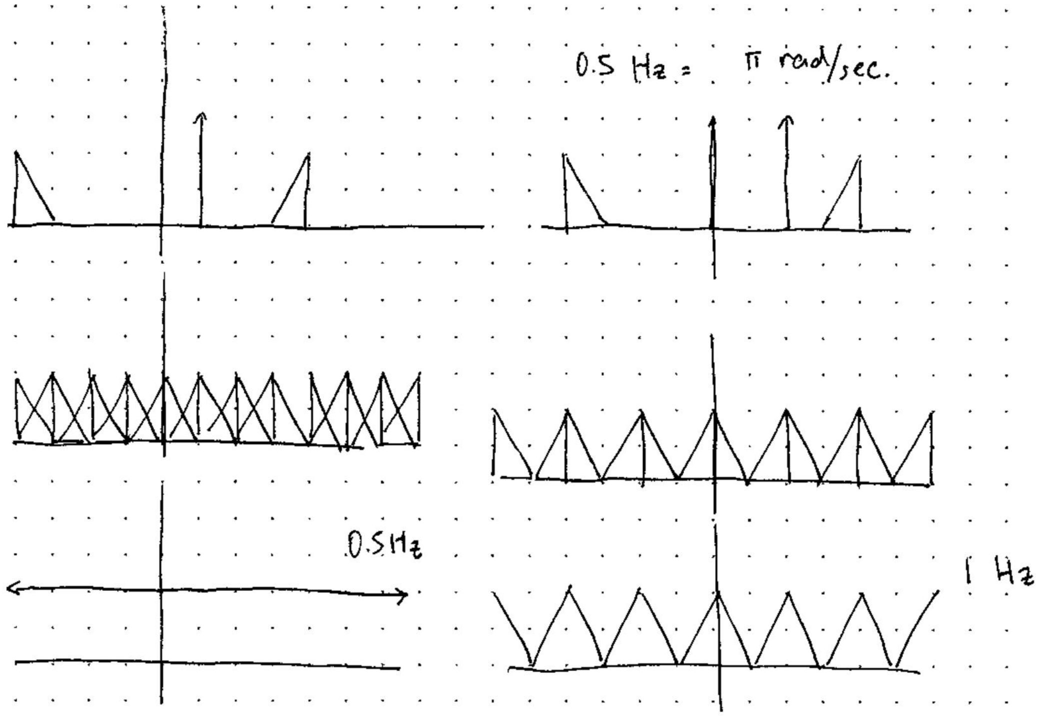


$F_s =$ 1Hz makes the signal still recoverable.

2. (a) $f(t) = \Sigma \delta(t - 2k) + \Sigma \delta(t - 2k + \tau + 1)$

$= \sum_{k=-\infty}^{\infty} \delta(t - 2k) + \sum_{k=-\infty}^{\infty} \delta(t - 2k + \tau + 1) = \delta_2(t) + \delta_2(t - \tau - 1)$

(b) $\delta_T(t) \leftrightarrow \omega_0 \delta_{\omega_0}(\omega) \qquad 2\pi/T = \omega_0$

$\delta_2(t) \leftrightarrow \pi \delta_\pi(\omega)$

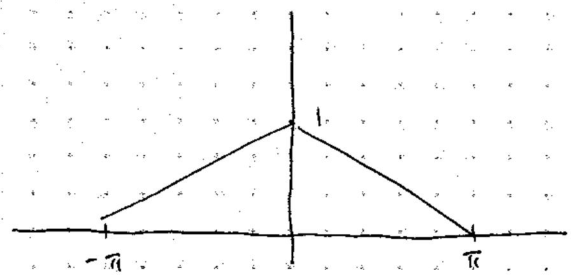
$\delta_2(t - \tau - 1) \leftrightarrow \pi \delta_\pi(\omega) \cdot e^{-(\tau+1)j\omega}$

(i) $\tau = 0 \Rightarrow \pi \delta_\pi(\omega) + \pi \delta_\pi(\omega) \cdot e^{-j\omega}$

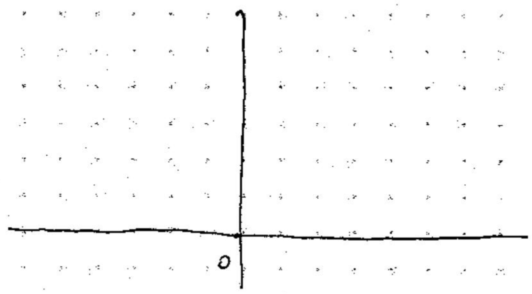
2.3 2(c) **2.5 / 3**

- **- 0 pts** Correct
- ✓ **- 0.5 pts** **Not finished**
- **- 1.5 pts** wrong answer
- **- 3 pts** Not attempt

(d)



① Draw FT of sampling sig.



(c) continued.

$$\delta_T(t) \leftrightarrow \omega_0 \delta_{\omega_0}(\omega)$$

thus,

$$\pi \delta_\pi(\omega) + \pi \delta_\pi(\omega) \cdot e^{-j\omega}$$

$$\leftrightarrow \delta_2(t) + \delta_2(t-1)$$

$$= \delta_1(t) \checkmark$$

$$e^{j\omega\tau} = 1 + j\omega\tau$$

$$\pi \cdot \delta_\pi(\omega) + \pi \cdot \delta_\pi(\omega) \cdot e^{-(\tau+1)j\omega} = \pi \delta_\pi(\omega) \cdot \left[1 + e^{-(\tau+1)\cdot j\omega}\right]$$

$$= \cancel{\pi \cdot \delta_\pi(\omega) + \pi \cdot \delta_\pi(\omega) \cdot \left[e^{-j\omega} \cdot (1 - j\omega\tau)\right]}$$

$$= \cancel{\pi \delta_\pi(\omega) + \pi \delta_\pi(\omega) e^{-j\omega} - \pi \cdot \delta_\pi(\omega) \, j\omega\tau}$$

$$= \pi \cdot \delta_\pi(\omega) \cdot \left[1 + e^{-j\omega} \cdot (1 - j\omega\tau)\right]$$

$\omega = \pm\pi, 0$ so $k = \pm 1, 0$ where $k = \omega/\pi$

$$= \pi \cdot \delta_\pi(k\pi) \cdot \left[1 + (-1)^k \cdot (1 - jk\pi\tau)\right]$$

$k = 0$

$$\pi \cdot \delta(0) \cdot [2],$$

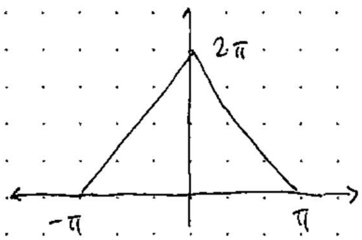
$k = 1$

$$\pi \cdot \delta(\pi) \cdot [j\pi\tau]$$

$k = -1$

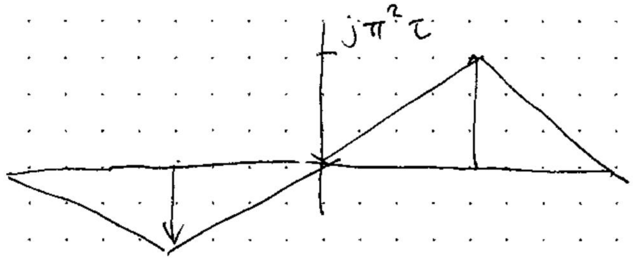
$$\pi \cdot \delta(-\pi) \cdot [-j\pi\tau]$$

$2\pi\,\delta(0)$:



$j\pi^2\tau \cdot \delta(\pi)$

$-j\pi^2\tau : \delta(-\pi)$



(e) $g(t)$ is real and even, $G(j\omega)$ is real.

We can recover $g(t)$ from $g(t) \cdot f(t)$ s/c no aliasing.

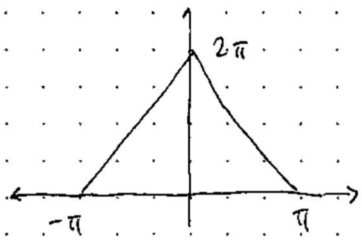
in the sample of g, there is aliasing in imaginary

so g needs to be real.

2.4 2(d) **3 / 3**

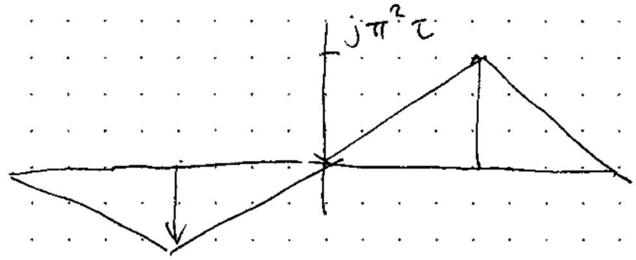
- ✓ **- 0 pts** Correct
- **- 1 pts** No graph
- **- 1 pts** wrong graph
- **- 3 pts** no answer

$2\pi\,\delta(0)$:



$j\pi^2\tau \cdot \delta(\pi)$

$-j\pi^2\tau \cdot \delta(-\pi)$



(e) $g(t)$ is real and even, $G(jw)$ is real.

We can recover $g(t)$ from $g(t) \cdot f(t)$ s/c no aliasing.

in the sample of g, there is aliasing in imaginary

so g needs to be real.

2.5 2(e) **3 / 3**

✓ **- 0 pts** Correct

- 1 pts wrong answer

- 3 pts No answer

(3) (a) $x_p(t) = \cos w_0 t \cdot \delta_{1/3}(t)$

$$X_p(jw) = F(\cos w_0 t) * F(\delta_{1/3}(t))$$

$$= \int_{-\infty}^{\infty} \cos(w_0 t) \cdot e^{-jwt}\, dt \; * \; 6\pi\, \delta_{6\pi}(w)$$

$$= \int_{-\infty}^{\infty} \frac{e^{w_0 t j} + e^{-w_0 j t}}{2} \cdot e^{-jwt}\, dt \; * \; 6\pi\, \delta_{6\pi}(w)$$

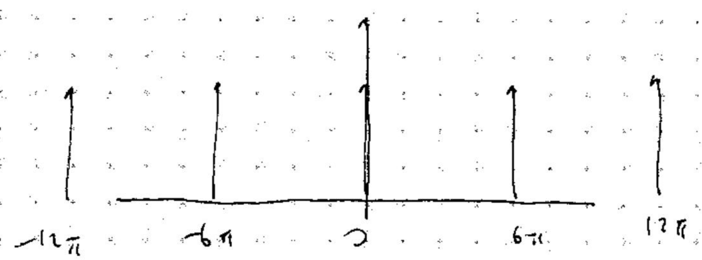
$$= \cancel{\tfrac{1}{2}((\delta} \quad \pi\left[\delta(w-w_0) + \delta(w+w_0)\right] * 6\pi\, \delta_{6\pi}(w)$$

$$6\pi^2 \left(\delta(w-w_0) * \delta_{6\pi}(w) + \delta(w+w_0) * \delta_{6\pi}(w) \right)$$

$$\downarrow \qquad\qquad\qquad \downarrow$$

$0 \quad w < w_0$ $\qquad\qquad$ $0 \quad w < -w_0$

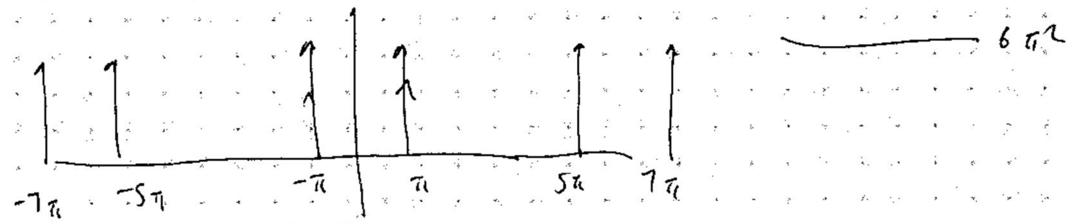
$\delta_{6\pi}(w-w_0) \quad w > w_0$ $\qquad\qquad$ $\delta_{6\pi}(w+w_0) \quad w > -w_0$



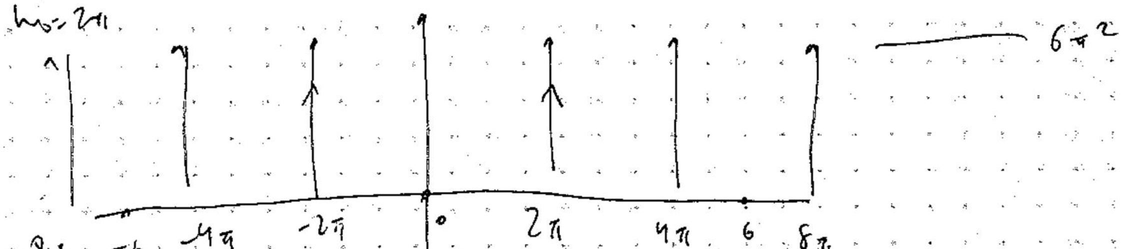
$6\pi \cdot \delta_{6\pi}(w)$

i. $w_0 = \pi$

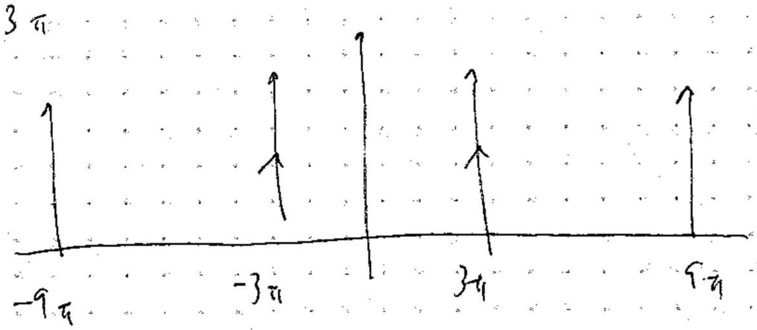
$$\pi \left[\delta(w - \pi) + \delta(w + \pi) \right]$$



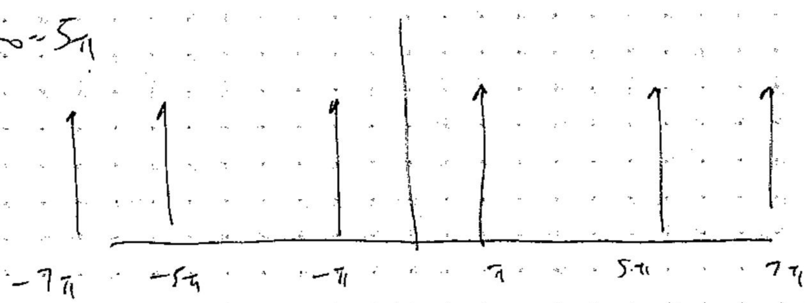
ii $w_0 = 2\pi$



iii $w_0 = 3\pi$



iv $w_0 = 5\pi$



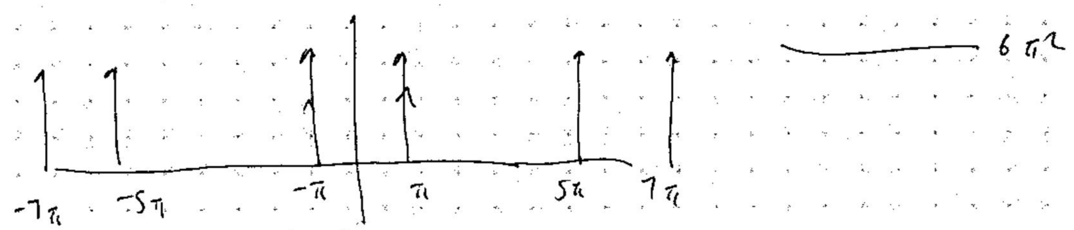
(b) same $\boxed{w_0 = \pi \text{ and } 5\pi}$

3.1 3(a)(i) **3 / 4**

- **- 0 pts** Correct
- ✓ **- 1 pts** **Wrong spectrum.**
- **- 4 pts** No answer

i. $\omega_0 = \pi$

$$\pi \left[\delta(\omega - \pi) + \delta(\omega + \pi) \right]$$



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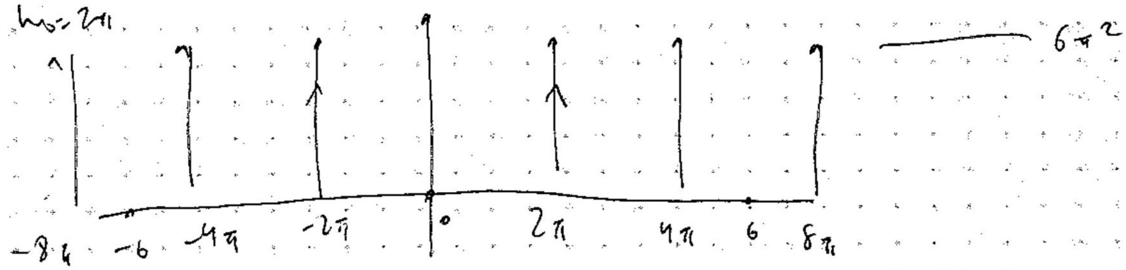

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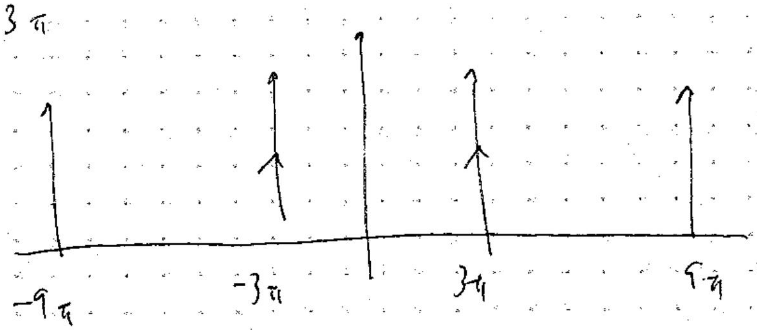

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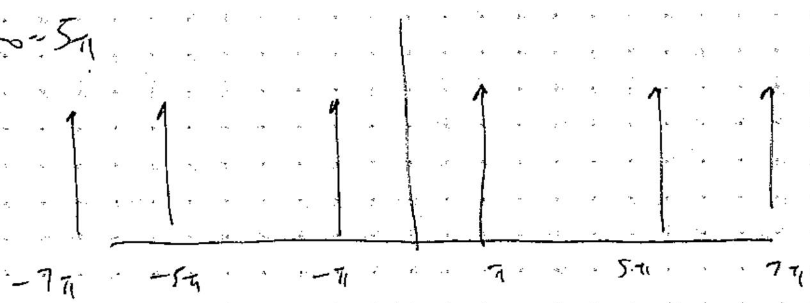


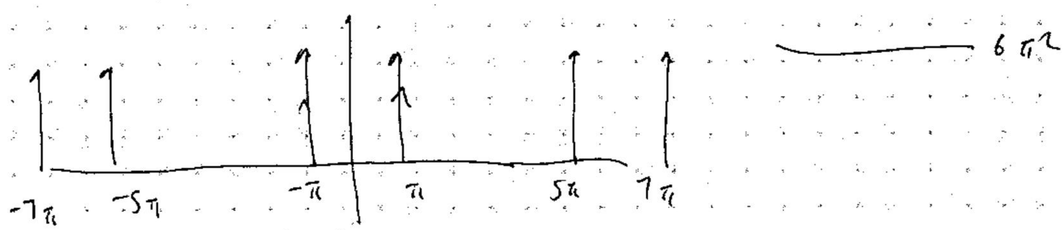
(b) same $\boxed{\omega_0 = \pi \text{ and } 5\pi}$

3.2 3(a)(ii) **3 / 4**

- **- 0 pts** Correct
- ✓ **- 1 pts** **Wrong spectrum**
- **- 4 pts** No answer

i. $w_0 = \pi$

$$\pi\left[\delta(\omega-\pi)+\delta(\omega+\pi)\right]$$



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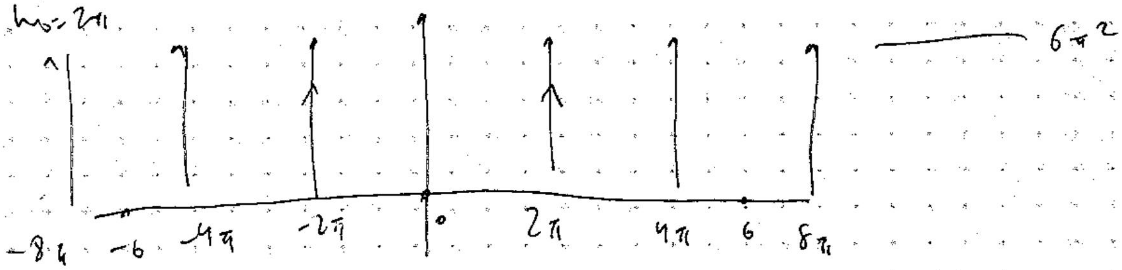

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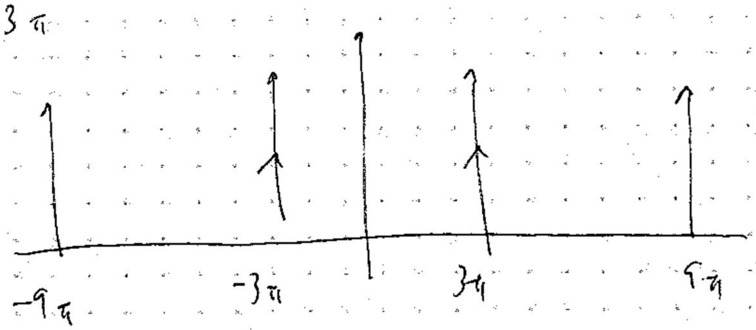

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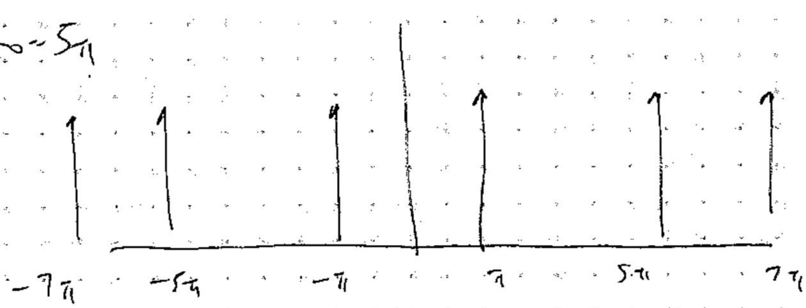


(b) same $\boxed{w_0 = \pi \text{ and } 5\pi}$

3.3 3(a)(iii) 4 / 4

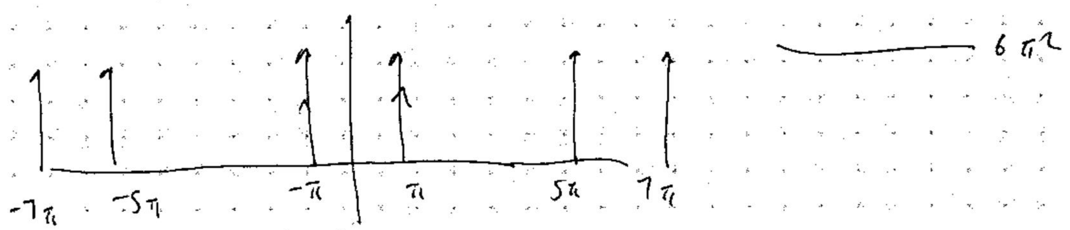
✓ **- 0 pts** **Correct**

- 1 pts Wrong spectrum

- 4 pts No answer

i. $\omega_0 = \pi$

$$\pi\left[\delta(\omega-\pi)+\delta(\omega+\pi)\right]$$



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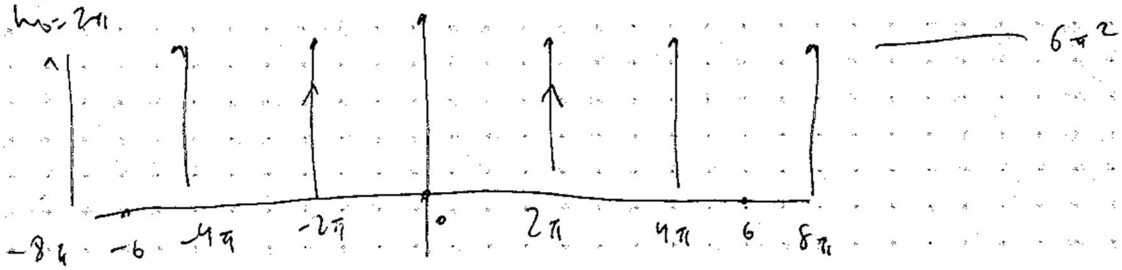

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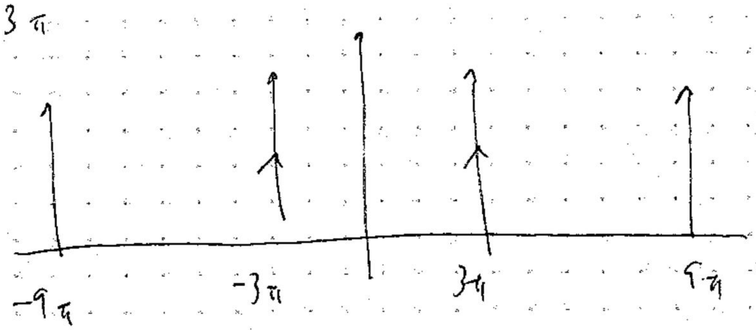

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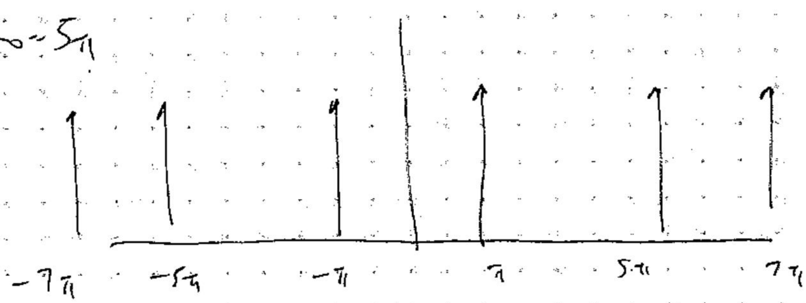

(b) same $\boxed{\omega_0 = \pi \text{ and } 5\pi}$

3.4 3(a)(iv) **4 / 4**

- ✓ **- 0 pts** Correct
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- **- 4 pts** No answer

i. $w_0 = \pi$

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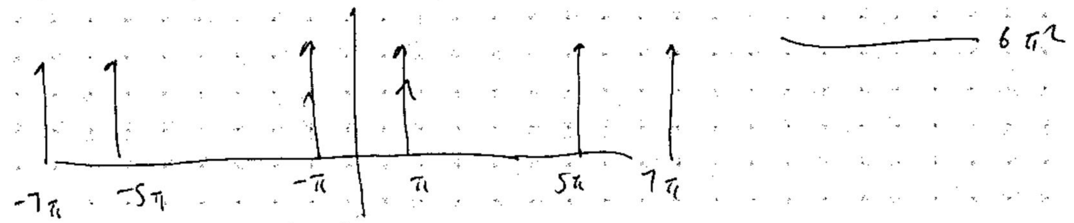
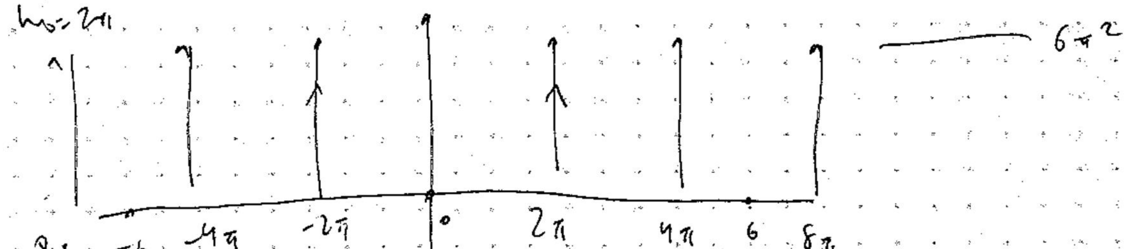
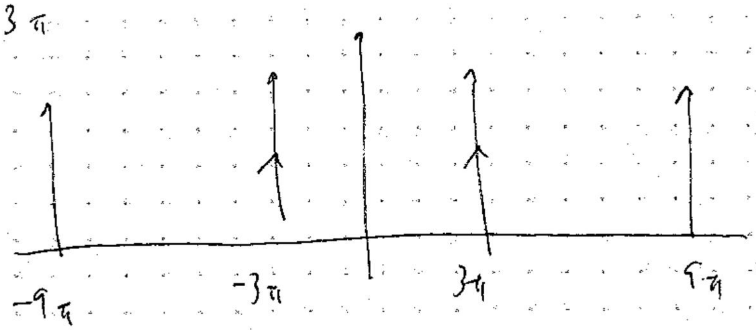


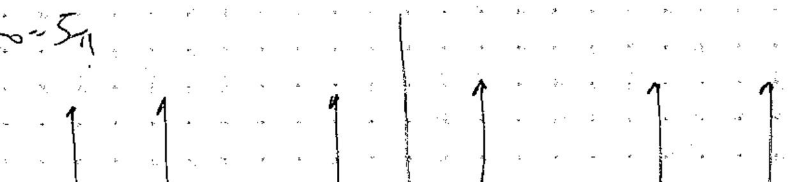
ii $w_0 = 2\pi$



iii $w_0 = 3\pi$



iv $w_0 = 5\pi$



(b) same $\boxed{w_0 = \pi \text{ and } 5\pi}$

3.5 3(b) **2 / 2**

- ✓ **- 0 pts** Correct
- **- 1 pts** Wrong answer
- **- 2 pts** No answer

4. (a) i. $f(t) = t\cdot e^{-at} \cdot (\sin w_0 t)^2\, u(t)$

$= t\cdot e^{-at}\, u(t) \cdot \left[\frac{1}{2j}\left(e^{jw_0 t} - e^{-jw_0 t}\right)\right]^2$

$= t\cdot e^{-at}\, u(t) \cdot \frac{-1}{4}\left(e^{2jw_0 t} + e^{-2jw_0 t} - 2\right)$

$= \underbrace{\frac{1}{2} t e^{-at} u(t)}_{①} - \underbrace{\frac{t}{4} u(t)\cdot e^{(2jw_0 - a)t}}_{②} - \underbrace{\frac{t}{4} u(t)\cdot e^{(-2jw_0 - a)t}}_{③}$

$-a-s > 0 \Rightarrow s > -a$: ROC

$\mathcal{L}[①] = \frac{1}{2(a+s)^2}$ $\quad \mathcal{L}[2] = \frac{-1}{4[s+a-2jw_0]^2}$ $\quad \mathcal{L}[3] = \frac{-1}{4[s+a+2jw_0]^2}$

$\mathcal{L}[f(t)] = \frac{1}{2(s+a)^2} - \frac{1}{4(s+a-2jw_0)^2} - \frac{1}{4(s+a+2jw_0)^2}$

ii. $F(s) = \int_0^1 e^{-st}\cdot 0\, dt + \int_1^2 e^{-st}\, dt + \int_2^3 e^{-st}\cdot e^{-2(t-3)}\, dt + \int_3^\infty e^{-st}\, dt$

$= \left(-\frac{1}{2}e^{-2} + e^{-1}\right) - \frac{e^6}{s}\cdot e^{-3(2+s)} + \frac{e^6}{s+2} e^{-2(s+2)} + \frac{e^{-3s}}{s}$

(b) $X(s) = \int_0^\infty e^{-st}\, x(t)\cdot dt \quad \sigma > -1$ $\qquad FT = \frac{1}{(jw)^2 + 2jw + 5} = \frac{1}{5 - w^2 + 2jw}$

i. $\mathcal{L}\{x(t)\cdot e^{-t}\} = \int_0^\infty e^{-(s+1)t}\, x(t)\, dt$ $\qquad \sigma + 1 > -1 \rightarrow \sigma > -2$

ii. $\mathcal{L}\{x(t)\cdot e^{2t}\} = \int_0^\infty e^{-(s-2)t}\, x(t)\, dt \quad R_e(s-2) > 1 \quad \sigma - 2 > -1 \quad \sigma > +1$

not defined

5. (a) $F(s) = \frac{s^2}{(s+2)^2} = \left[\frac{A}{s+2} + \frac{B}{(s+2)^2}\right] s^2 = 1 - \frac{4}{s+2} + \frac{4}{(s+2)^2}$

$\delta(t) - 4e^{-2t} u(t) + 4te^{-2t}$

(b) $\frac{s+1}{(s-2)^2(s-3)} = e^{-s}\left[\frac{4}{s-3} - \frac{4}{s-2} - \frac{3}{(s-2)^2}\right] = 4e^{3(t-1)} u(t-1) - 4e^{2(t-1)} u(t-1) - 3(t-1)\, e^{2(t-1)}$

(c) $\frac{s+4}{s^3+4s} = \frac{}{s} + \frac{}{s^2+4}$ $\quad A = 1 \quad B + C = 0 \quad B = -1 \quad C = 1 \quad \frac{1}{s} + \frac{1-s}{s^2+4}$

$u(t) + \frac{1}{2}\sin 2t - \cos 2t \quad$ ROC $\sigma > 0$.

6. (a) $\frac{1}{2}e^t + 4\cdot\frac{1}{2}e^t + 3\cdot\frac{1}{2}e^t = a\cdot e^t \quad a = 4 \quad 4x(t)$.

$H_1(s) = \frac{4}{s^2+4s+3} = \boxed{\frac{4}{(s+3)(s+1)}}$

(b) $\frac{4}{s(s+3)(s+1)} = \frac{A}{s} \frac{B}{s+1} \frac{C}{s+3}$. $\quad A = \frac{4}{3} \quad B = -2 \quad C = \frac{2}{3}$.

$y(t) = \frac{4}{3}u(t) + 2e^{-t}u(t) + \frac{2}{3}e^{-3t}u(t)$

4.1 4(a)(i) **5 / 5**

✓ **- 0 pts** **Effort**

- 5 pts no effort

4. (a) i. $f(t) = t\cdot e^{-at}\cdot(\sin w_0 t)^2\, u(t)$

$$= t\cdot e^{-at}\, u(t)\cdot\left[\tfrac{1}{2j}(e^{jw_0 t} - e^{-jw_0 t})\right]^2$$

$$= t\cdot e^{-at}\, u(t)\cdot \tfrac{-1}{4}(e^{2jw_0 t} + e^{-2jw_0 t} - 2)$$

$$= \underbrace{\tfrac{1}{2} t e^{-at} u(t)}_{①} - \underbrace{\tfrac{t}{4} u(t)\cdot e^{(2jw_0 - a)t}}_{②} - \underbrace{\tfrac{t}{4} u(t)\cdot e^{(-2jw_0 - a)t}}_{③}$$

$-a-s > 0 \Rightarrow s > -a$: ROC

$\mathcal{L}[①] = \frac{1}{2(a+s)^2}$ $\quad \mathcal{L}[2] = \frac{-1}{4[s+a-2jw_0]^2}$ $\quad \mathcal{L}[3] = \frac{-1}{4[s+a+2jw_0]^2}$

$$\mathcal{L}[f(t)] = \frac{1}{2(s+a)^2} - \frac{1}{4(s+a-2jw_0)^2} - \frac{1}{4(s+a+2jw_0)^2}$$

ii. $F(s) = \int_0^1 e^{-st}\cdot 0\, dt + \int_1^2 e^{-st}\, dt + \int_2^3 e^{-st}\cdot e^{-2(t-3)}\, dt + \int_3^\infty e^{-st}\, dt$

$$= \left(-\tfrac{1}{2}e^{-2} + e^{-1}\right) - \frac{e^6}{s}\cdot e^{-3(2+s)} + \frac{e^6}{s+2} e^{-2(s+2)} + \frac{e^{-3s}}{s}$$

(b) $X(s) = \int_0^\infty e^{-st}\, x(t)\cdot dt \quad \sigma > -1$ $\qquad FT = \frac{1}{(jw)^2 + 2jw + 5} = \frac{1}{5 - w^2 + 2jw}$

i. $\mathcal{L}\{x(t)\cdot e^{-t}\} = \int_0^\infty e^{-(s+1)t}\, x(t)\, dt$ $\qquad \sigma + 1 > -1 \rightarrow \sigma > -2$

ii. $\mathcal{L}\{x(t)\cdot e^{2t}\} = \int_0^\infty e^{-(s-2)t}\, x(t)\, dt$ $\quad R_e(s-2) > -1 \quad \sigma - 2 > -1 \quad \sigma > +1$

not defined

5. (a) $F(s) = \frac{s^2}{(s+2)^2} = \left[\frac{A}{s+2} + \frac{B}{(s+2)^2}\right] s^2 = 1 - \frac{4}{s+2} + \frac{4}{(s+2)^2}$

$$\delta(t) - 4e^{-2t} u(t) + 4te^{-2t}$$

(b) $\frac{s+1}{(s-2)^2(s-3)} = e^{-s}\left[\frac{4}{s-3} - \frac{4}{s-2} - \frac{3}{(s-2)^2}\right] = 4e^{3(t-1)} u(t-1) - 4e^{2(t-1)} u(t-1) - 3(t-1)\, e^{2(t-1)}$

(c) $\frac{s+4}{s^3+4s} = \frac{}{s} + \frac{}{s^2+4}$ $\quad A = 1 \quad B+C = 0 \quad B = -1 \quad C = 1$ $\quad \frac{1}{s} + \frac{1-s}{s^2+4}$

$$u(t) + \tfrac{1}{2}\sin 2t - \cos 2t \quad \text{ROC } \sigma > 0.$$

6. (a) $\tfrac{1}{2}e^t + 4\cdot\{e^t\}\cdot\{e^t\} = a\cdot e^2 \quad a = 4 \quad 4x(t).$

$$H_1(s) = \frac{4}{s^2+4s+3} = \boxed{\frac{4}{(s+3)(s+1)}}$$

(b) $\frac{4}{s(s+3)(s+1)} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s+3}$ $\quad A = \frac{4}{3} \quad B = -2 \quad C = \frac{2}{3}.$

$$y(t) = \tfrac{4}{3}u(t) + 2e^{-t}u(t) + \tfrac{2}{3}e^{-3t}u(t)$$

4.2 4(a)(ii) **5 / 5**

✓ **- 0 pts** **Effort**

- 5 pts No effort

4. (a) i. $f(t) = t\cdot e^{-at}\cdot(\sin w_0 t)^2\, u(t)$

$= t\cdot e^{-at}\cdot u(t)\cdot\left[\frac{1}{2j}(e^{jw_0 t} - e^{-jw_0 t})\right]^2$

$= t\cdot e^{-at}\cdot u(t)\cdot\frac{-1}{4}(e^{2jw_0 t} + e^{-2jw_0 t} - 2)$

$= \underbrace{\frac{1}{2}te^{-at}u(t)}_{①} - \underbrace{\frac{t}{4}u(t)\cdot e^{(2jw_0 - a)t}}_{②} - \underbrace{\frac{t}{4}u(t)\cdot e^{(-2jw_0 - a)t}}_{③}$

$-a-s > 0 \Rightarrow s > -a$: ROC

$\mathcal{L}[①] = \frac{1}{2(a+s)^2}$ $\quad \mathcal{L}[2] = \frac{-1}{4[s+a-2jw_0]^2}$ $\quad \mathcal{L}[3] = \frac{-1}{4[s+a+2jw_0]^2}$

$\mathcal{L}[f(t)] = \frac{1}{2(s+a)^2} - \frac{1}{4(s+a-2jw_0)^2} - \frac{1}{4(s+a+2jw_0)^2}$

ii. $F(s) = \int_0^1 e^{-st}\cdot 0\, dt + \int_1^2 e^{-st}\, dt + \int_2^3 e^{-st}\cdot e^{-2(t-3)}\, dt + \int_3^\infty e^{-st}\, dt$

$= (-\frac{1}{2}e^{-2} + e^{-1}) - \frac{e^6}{s}\cdot e^{-3(2+s)} + \frac{e^6}{s+2}e^{-2(s+2)} + \frac{e^{-3s}}{s}$

(b) $X(s) = \int_0^\infty e^{-st}\, x(t)\cdot dt \quad \sigma > -1$ $\qquad FT = \frac{1}{(jw)^2 + 2jw + 5} = \frac{1}{5 - w^2 + 2jw}$

i. $\mathcal{L}\{x(t)\cdot e^{-t}\} = \int_0^\infty e^{-(s+1)t}\, x(t)\, dt$ $\qquad \sigma + 1 > -1 \rightarrow \sigma > -2$

ii. $\mathcal{L}\{x(t)\cdot e^{2t}\} = \int_0^\infty e^{-(s-2)t}\, x(t)\, dt \quad R_e(s-2) > -1 \quad \sigma - 2 > -1 \quad \sigma > +1$

not defined

5. (a) $F(s) = \frac{s^2}{(s+2)^2} = \left[\frac{A}{s+2} + \frac{B}{(s+2)^2}\right]s^2 = 1 - \frac{4}{s+2} + \frac{4}{(s+2)^2}$

$\delta(t) - 4e^{-2t}u(t) + 4te^{-2t}$

(b) $\frac{s+1}{(s-2)^2(s-3)} = e^{-s}\left[\frac{4}{s-3} - \frac{4}{s-2} - \frac{3}{(s-2)^2}\right] = 4e^{3(t-1)}u(t-1) - 4e^{2(t-1)}u(t-1) - 3(t-1)e^{2(t-1)}$

(c) $\frac{s+4}{s^3+4s} = \frac{}{s} + \frac{}{s^2+4}$ $\quad A = 1 \quad B + C = 0 \quad B = -1 \quad C = 1 \quad \frac{1}{s} + \frac{1-s}{s^2+4}$

$u(t) + \frac{1}{2}\sin 2t - \cos 2t$ ROC $\sigma > 0$.

6. (a) $\frac{1}{2}e^t + 4\cdot\frac{1}{2}e^t + 3\cdot\frac{1}{2}e^t = a\cdot e^t \quad a = 4 \quad 4x(t)$.

$H_1(s) = \frac{4}{s^2+4s+3} = \boxed{\frac{4}{(s+3)(s+1)}}$

(b) $\frac{4}{s(s+3)(s+1)} = \frac{A}{s} \quad \frac{B}{s+1} \quad \frac{C}{s+3}$. $\quad A = \frac{4}{3} \quad B = -2 \quad C = \frac{2}{3}$.

$y(t) = \frac{4}{3}u(t) + 2e^{-t}u(t) + \frac{2}{3}e^{-3t}u(t)$

4.3 4(b)(i) **5 / 5**

✓ **- 0 pts** **Effort**

- 5 pts No effort

4. (a) i. $f(t) = t \cdot e^{-at} \cdot (\sin \omega_0 t)^2 u(t)$

$= t \cdot e^{-at} u(t) \cdot \left[\frac{1}{2j} (e^{j\omega_0 t} - e^{-j\omega_0 t}) \right]^2$

$= t \cdot e^{-at} u(t) \cdot \frac{-1}{4} (e^{2j\omega_0 t} + e^{-2j\omega_0 t} - 2)$

$= \underbrace{\frac{1}{2} t e^{-at} u(t)}_{①} - \underbrace{\frac{t}{4} u(t) \cdot e^{(2j\omega_0 - a)t}}_{②} - \underbrace{\frac{t}{4} u(t) \cdot e^{(-2j\omega_0 - a)t}}_{③}$

$-a-s > 0 \Rightarrow s > -a$; ROC

$\mathcal{L}[①] = \frac{1}{2(a+s)^2}$ $\quad \mathcal{L}[2] = \frac{-1}{4[s+a-2j\omega_0]^2}$ $\quad \mathcal{L}[3] = \frac{-1}{4[s+a+2j\omega_0]^2}$

$\mathcal{L}[f(t)] = \frac{1}{2(s+a)^2} - \frac{1}{4(s+a-2j\omega_0)^2} - \frac{1}{4(s+a+2j\omega_0)^2}$

ii. $F(s) = \int_0^1 e^{-st} \cdot 0 \, dt + \int_1^2 e^{-st} dt + \int_2^3 e^{-st} \cdot e^{-2(t-3)} dt + \int_3^\infty e^{-st} dt$

$= \left(-\frac{1}{2} e^{-2} + e^{-1}\right) - \frac{e^6}{s} \cdot e^{-3(2+s)} + \frac{e^6}{s+2} e^{-2(s+2)} + \frac{e^{-3s}}{s}$

(b) $X(s) = \int_0^\infty e^{-st} x(t) \cdot dt \quad \sigma > -1$ $\qquad FT = \frac{1}{(j\omega)^2 + 2j\omega + 5} = \frac{1}{5 - \omega^2 + 2j\omega}$

i. $\mathcal{L}\{x(t) \cdot e^{-t}\} = \int_0^\infty e^{-(s+1)t} x(t) \, dt$ $\qquad \sigma + 1 > -1 \rightarrow \sigma > -2$

ii. $\mathcal{L}\{x(t) \cdot e^{2t}\} = \int_0^\infty e^{-(s-2)t} x(t) \, dt \quad R_e(s-2) > 1 \quad \sigma - 2 > -1 \quad \sigma > +1$

not defined

5. (a) $F(s) = \frac{s^2}{(s+2)^2} = \left[\frac{A}{s+2} + \frac{B}{(s+2)^2} \right] s^2 = 1 - \frac{4}{s+2} + \frac{4}{(s+2)^2}$

$\delta(t) - 4e^{-2t} u(t) + 4te^{-2t}$

(b) $\frac{s+1}{(s-2)^2(s-3)} = e^{-s} \left[\frac{4}{s-3} - \frac{4}{s-2} - \frac{3}{(s-2)^2} \right] = 4e^{3(t-1)} u(t-1) - 4e^{2(t-1)} u(t-1) - 3(t-1) e^{2(t-1)}$

(c) $\frac{s+4}{s^3+4s} = \frac{}{s} + \frac{}{s^2+4}$ $\quad A = 1 \quad B+C=0 \quad B = -1 \quad C = 1 \quad \frac{1}{s} + \frac{1-s}{s^2+4}$

$u(t) + \frac{1}{2} \sin 2t - \cos 2t$ ROC $\sigma > 0$.

6. (a) $\frac{1}{2} e^t + 4 \cdot \frac{1}{6} e^t \cdot \frac{1}{3} e^t = a \cdot e^t \quad a = 4 \qquad 4x(t)$.

$H_1(s) = \frac{4}{s^2+4s+3} = \boxed{\frac{4}{(s+3)(s+1)}}$

(b) $\frac{4}{s(s+3)(s+1)} = \frac{A}{s} \quad \frac{B}{s+1} \quad \frac{C}{s+3}$. $\quad A = \frac{4}{3} \quad B = -2 \quad C = \frac{2}{3}$.

$y(t) = \frac{4}{3} u(t) + 2e^{-t} u(t) + \frac{2}{3} e^{-3t} u(t)$

6

4.4 4(b)(ii) **5 / 5**

✓ **- 0 pts** **Effort**

- 5 pts No effort

4. (a) i. $f(t) = t\cdot e^{-at}\cdot(\sin\omega_0 t)^2\, u(t)$

$$= t\cdot e^{-at}\, u(t)\cdot\left[\frac{1}{2j}\left(e^{j\omega_0 t} - e^{-j\omega_0 t}\right)\right]^2$$

$$= t\cdot e^{-at}\, u(t)\cdot \frac{-1}{4}\left(e^{2j\omega_0 t} + e^{-2j\omega_0 t} - 2\right)$$

$$= \underbrace{\frac{1}{2} t e^{-at} u(t)}_{①} - \underbrace{\frac{t}{4} u(t)\cdot e^{(2j\omega_0 - a)t}}_{②} - \underbrace{\frac{t}{4} u(t)\cdot e^{(-2j\omega_0 - a)t}}_{③}$$

$-a-s>0 \Rightarrow s>-a$: ROC

$\mathcal{L}[①] = \frac{1}{2(a+s)^2}$ $\quad \mathcal{L}[2] = \frac{-1}{4[s+a-2j\omega_0]^2}$ $\quad \mathcal{L}[3] = \frac{-1}{4[s+a+2j\omega_0]^2}$

$$\mathcal{L}[f(t)] = \frac{1}{2(s+a)^2} - \frac{1}{4(s+a-2j\omega_0)^2} - \frac{1}{4(s+a+2j\omega_0)^2}$$

ii. $F(s) = \int_0^1 e^{-st}\cdot 0\, dt + \int_1^2 e^{-st}\, dt + \int_2^3 e^{-st}\cdot e^{-2(t-3)}\, dt + \int_3^\infty e^{-st}\, dt$

$$= \left(-\frac{1}{2}e^{-2} + e^{-1}\right) - \frac{e^6}{s}\cdot e^{-3(2+s)} + \frac{e^6}{s+2} e^{-2(s+2)} + \frac{e^{-3s}}{s}$$

(b) $X(s) = \int_0^\infty e^{-st}\, x(t)\cdot dt \quad \sigma>-1$ $\qquad FT = \frac{1}{(j\omega)^2 + 2j\omega + 5} = \frac{1}{5-\omega^2+2j\omega}$

i. $\mathcal{L}\{x(t)\cdot e^{-t}\} = \int_0^\infty e^{-(s+1)t}\, x(t)\, dt$ $\qquad \sigma+1>-1 \rightarrow \sigma>-2$

ii. $\mathcal{L}\{x(t)\cdot e^{2t}\} = \int_0^\infty e^{-(s-2)t}\, x(t)\, dt$ $\quad R_e(s-2)>-1 \quad \sigma-2>-1 \quad \sigma>+1$

not defined

5. (a) $F(s) = \frac{s^2}{(s+2)^2} = \left[\frac{A}{s+2} + \frac{B}{(s+2)^2}\right] s^2 = 1 - \frac{4}{s+2} + \frac{4}{(s+2)^2}$

$\delta(t) - 4e^{-2t}u(t) + 4te^{-2t}$

(b) $\frac{s+1}{(s-2)^2(s-3)} = e^{-s}\left[\frac{4}{s-3} - \frac{4}{s-2} - \frac{3}{(s-2)^2}\right] = 4e^{3(t-1)}u(t-1) - 4e^{2(t-1)}u(t-1) - 3(t-1)e^{2(t-1)}$

(c) $\frac{s+4}{s^3+4s} = \frac{\;}{s} + \frac{\;}{s^2+4}$ $\quad A=1 \quad B+C=0 \quad B=-1 \quad C=1$ $\quad \frac{1}{s} + \frac{1-s}{s^2+4}$

$u(t) + \frac{1}{2}\sin 2t - \cos 2t$ $\quad$ ROC $\sigma>0$.

6. (a) $\frac{1}{2}e^{t} + 4\cdot\frac{1}{2}e^{t}(1)\cdot\frac{1}{2}e^{t} = a\cdot e^{t}$ $\quad a=4 \quad 4x(t)$.

$H_1(s) = \frac{4}{s^2+4s+3} = \boxed{\frac{4}{(s+3)(s+1)}}$

(b) $\frac{4}{s(s+3)(s+1)} = \frac{A}{s}\ \frac{B}{s+1}\ \frac{C}{s+3}$. $\quad A=\frac{4}{3} \quad B=-2 \quad C=\frac{2}{3}$.

$y(t) = \frac{4}{3}u(t) + 2e^{-t}u(t) + \frac{2}{3}e^{-3t}u(t)$

5.1 5(a) **6 / 6**

✓ **- 0 pts** effort

- 6 pts Not attempt

4. (a) i. $f(t) = t\cdot e^{-at}\cdot(\sin\omega_0 t)^2\, u(t)$

$$= t\cdot e^{-at}\, u(t)\cdot\left[\frac{1}{2j}\left(e^{j\omega_0 t} - e^{-j\omega_0 t}\right)\right]^2$$

$$= t\cdot e^{-at}\, u(t)\cdot \frac{-1}{4}\left(e^{2j\omega_0 t} + e^{-2j\omega_0 t} - 2\right)$$

$$= \underbrace{\frac{1}{2} t e^{-at} u(t)}_{①} - \underbrace{\frac{t}{4} u(t)\cdot e^{(2j\omega_0 - a)t}}_{②} - \underbrace{\frac{t}{4} u(t)\cdot e^{(-2j\omega_0 - a)t}}_{③}$$

$-a-s > 0 \Rightarrow s > -a$: ROC

$\mathcal{L}[①] = \frac{1}{2(a+s)^2}$ $\quad \mathcal{L}[2] = \frac{-1}{4[s+a-2j\omega_0]^2}$ $\quad \mathcal{L}[3] = \frac{-1}{4[s+a+2j\omega_0]^2}$

$$\mathcal{L}[f(t)] = \frac{1}{2(s+a)^2} - \frac{1}{4(s+a-2j\omega_0)^2} - \frac{1}{4(s+a+2j\omega_0)^2}$$

ii. $F(s) = \int_0^1 e^{-st}\cdot 0\, dt + \int_1^2 e^{-st}\, dt + \int_2^3 e^{-st}\cdot e^{-2(t-3)}\, dt + \int_3^\infty e^{-st}\, dt$

$$= \left(-\frac{1}{2}e^{-2} + e^{-1}\right) - \frac{e^6}{s}\cdot e^{-3(2+s)} + \frac{e^6}{s+2} e^{-2(s+2)} + \frac{e^{-3s}}{s}$$

(b) $X(s) = \int_0^\infty e^{-st} x(t)\cdot dt \quad \sigma > -1$ $\qquad FT = \frac{1}{(j\omega)^2 + 2j\omega + 5} = \frac{1}{5 - \omega^2 + 2j\omega}$

i. $\mathcal{L}\{x(t)\cdot e^{-t}\} = \int_0^\infty e^{-(s+1)t} x(t)\, dt$ $\qquad \sigma + 1 > -1 \rightarrow \sigma > -2$

ii. $\mathcal{L}\{x(t)\cdot e^{2t}\} = \int_0^\infty e^{-(s-2)t} x(t)\, dt$ $\quad R_e(s-2) > -1 \quad \sigma - 2 > -1 \quad \sigma > +1$

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$\delta(t) - 4e^{-2t} u(t) + 4te^{-2t}$

(b) $\frac{s+1}{(s-2)^2(s-3)} = e^{-s}\left[\frac{4}{s-3} - \frac{4}{s-2} - \frac{3}{(s-2)^2}\right] = 4e^{3(t-1)} u(t-1) - 4e^{2(t-1)} u(t-1) - 3(t-1)e^{2(t-1)}$

(c) $\frac{s+4}{s^3+4s} = \frac{A}{s} + \frac{Bs+C}{s^2+4}$ $\quad A = 1 \quad B+C=0 \quad B=-1 \quad C=1$ $\quad \frac{1}{s} + \frac{1-s}{s^2+4}$.

$u(t) + \frac{1}{2}\sin 2t - \cos 2t$ $\quad$ ROC $\sigma > 0$.

6. (a) $\frac{1}{2}e^{t} + 4\cdot\frac{1}{2}e^{t} + 3\cdot\frac{1}{2}e^{t} = a\cdot e^{t}$ $\quad a = 4$ $\quad 4x(t)$.

$$H_1(s) = \frac{4}{s^2+4s+3} = \boxed{\frac{4}{(s+3)(s+1)}}$$

(b) $\frac{4}{s(s+3)(s+1)} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s+3}$. $\quad A = \frac{4}{3} \quad B = -2 \quad C = \frac{2}{3}$.

$$y(t) = \frac{4}{3}u(t) + 2e^{-t}u(t) + \frac{2}{3}e^{-3t}u(t)$$

5.2 5(b) **6 / 6**

✓ **- 0 pts** effort

- 6 pts not attempt

4. (a) i. $f(t) = t \cdot e^{-at} \cdot (\sin w_0 t)^2 \, u(t)$

$$= t \cdot e^{-at} \, u(t) \cdot \left[\frac{1}{2j} (e^{jw_0 t} - e^{-jw_0 t}) \right]^2$$

$$= t \cdot e^{-at} \, u(t) \cdot \frac{-1}{4} (e^{2jw_0 t} + e^{-2jw_0 t} - 2)$$

$$= \underbrace{\frac{1}{2} t e^{-at} u(t)}_{①} - \underbrace{\frac{t}{4} u(t) \cdot e^{(2jw_0 - a)t}}_{②} - \underbrace{\frac{t}{4} u(t) \cdot e^{(-2jw_0 - a)t}}_{③}$$

$-a-s > 0 \Rightarrow s > -a$: ROC

$\mathcal{L}[①] = \frac{1}{2(a+s)^2}$ $\quad \mathcal{L}[2] = \frac{-1}{4[s+a-2jw_0]^2}$ $\quad \mathcal{L}[3] = \frac{-1}{4[s+a+2jw_0]^2}$

$$\mathcal{L}[f(t)] = \frac{1}{2(s+a)^2} - \frac{1}{4(s+a-2jw_0)^2} - \frac{1}{4(s+a+2jw_0)^2}$$

ii. $F(s) = \int_0^1 e^{-st} \cdot 0 \, dt + \int_1^2 e^{-st} \, dt + \int_2^3 e^{-st} \cdot e^{-2(t-3)} \, dt + \int_3^\infty e^{-st} \, dt$

$$= \left(-\frac{1}{2} e^{-2} + e^{-1}\right) - \frac{e^6}{s} \cdot e^{-3(2+s)} + \frac{e^6}{s+2} e^{-2(s+2)} + \frac{e^{-3s}}{s}$$

(b) $X(s) = \int_0^\infty e^{-st} x(t) \cdot dt \quad \sigma > -1$ $\qquad FT = \frac{1}{(jw)^2 + 2jw + 5} = \frac{1}{5 - w^2 + 2jw}$

i. $\mathcal{L}\{x(t) \cdot e^{-t}\} = \int_0^\infty e^{-(s+1)t} x(t) \, dt$ $\qquad \sigma + 1 > -1 \rightarrow \sigma > -2$

ii. $\mathcal{L}\{x(t) \cdot e^{2t}\} = \int_0^\infty e^{-(s-2)t} x(t) \, dt \quad R_e(s-2) > 1 \quad \sigma - 2 > -1 \quad \sigma > +1$

not defined

5. (a) $F(s) = \frac{s^2}{(s+2)^2} = \left[\frac{A}{s+2} + \frac{B}{(s+2)^2} \right] s^2 = 1 - \frac{4}{s+2} + \frac{4}{(s+2)^2}$

$$\delta(t) - 4e^{-2t} u(t) + 4te^{-2t}$$

(b) $\frac{s+1}{(s-2)^2(s-3)} = e^{-s}\left[\frac{4}{s-3} - \frac{4}{s-2} - \frac{3}{(s-2)^2} \right] = 4e^{3(t-1)} u(t-1) - 4e^{2(t-1)} u(t-1) - 3(t-1) e^{2(t-1)}$

(c) $\frac{s+4}{s^3+4s} = \frac{A}{s} + \frac{Bs+C}{s^2+4} \quad A = 1 \quad B + C = 0 \quad B = -1 \quad C = 1 \quad \frac{1}{s} + \frac{1-s}{s^2+4}.$

$$u(t) + \frac{1}{2} \sin 2t - \cos 2t \quad \text{ROC } \sigma > 0.$$

6. (a) $\frac{1}{2} e^t + 4 \cdot \frac{1}{2} e^t + 3 \cdot \frac{1}{2} e^t = a \cdot e^t \quad a = 4 \quad 4x(t).$

$$H_1(s) = \frac{4}{s^2 + 4s + 3} = \boxed{\frac{4}{(s+3)(s+1)}}$$

(b) $\frac{4}{s(s+3)(s+1)} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s+3}. \quad A = \frac{4}{3} \quad B = -2 \quad C = \frac{2}{3}.$

$$y(t) = \frac{4}{3} u(t) + 2e^{-t} u(t) + \frac{2}{3} e^{-3t} u(t)$$

5.3 5(c) **6 / 6**

✓ **- 0 pts** effort

- 6 pts not attempt

4. (a) i. $f(t) = t \cdot e^{-at} \cdot (\sin w_0 t)^2 \, u(t)$

$= t \cdot e^{-at} \cdot u(t) \cdot \left[\frac{1}{2j} (e^{jw_0 t} - e^{-jw_0 t}) \right]^2$

$= t \cdot e^{-at} \cdot u(t) \cdot \frac{-1}{4} (e^{2jw_0 t} + e^{-2jw_0 t} - 2)$

$= \underbrace{\frac{1}{2} t e^{-at} u(t)}_{①} - \underbrace{\frac{t}{4} u(t) \cdot e^{(2jw_0 - a)t}}_{②} - \underbrace{\frac{t}{4} u(t) \cdot e^{(-2jw_0 - a)t}}_{③}$

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$\mathcal{L}[①] = \frac{1}{2(a+s)^2}$ $\quad \mathcal{L}[2] = \frac{-1}{4[s+a-2jw_0]^2}$ $\quad \mathcal{L}[3] = \frac{-1}{4[s+a+2jw_0]^2}$

$\mathcal{L}[f(t)] = \frac{1}{2(s+a)^2} - \frac{1}{4(s+a-2jw_0)^2} - \frac{1}{4(s+a+2jw_0)^2}$

ii. $F(s) = \int_0^1 e^{-st} \cdot 0 \, dt + \int_1^2 e^{-st} dt + \int_2^3 e^{-st} \cdot e^{-2(t-3)} dt + \int_3^\infty e^{-st} dt$

$= \left(-\frac{1}{2} e^{-2} + e^{-1}\right) - \frac{e^6}{s} \cdot e^{-3(2+s)} + \frac{e^6}{s+2} e^{-2(s+2)} + \frac{e^{-3s}}{s}$

(b) $X(s) = \int_0^\infty e^{-st} x(t) \cdot dt \quad \sigma > -1$ $\qquad FT = \frac{1}{(jw)^2 + 2jw + 5} = \frac{1}{5 - w^2 + 2jw}$

i. $\mathcal{L}\{x(t) \cdot e^{-t}\} = \int_0^\infty e^{-(s+1)t} x(t) \, dt$ $\qquad \sigma + 1 > -1 \rightarrow \sigma > -2$

ii. $\mathcal{L}\{x(t) \cdot e^{2t}\} = \int_0^\infty e^{-(s-2)t} x(t) \, dt$ $\quad R_e(s-2) > -1 \quad \sigma - 2 > -1 \quad \sigma > +1$

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$\delta(t) - 4e^{-2t} u(t) + 4te^{-2t}$

(b) $\frac{s+1}{(s-2)^2(s-3)} = e^{-s} \left[\frac{4}{s-3} - \frac{4}{s-2} - \frac{3}{(s-2)^2} \right] = 4e^{3(t-1)} u(t-1) - 4e^{2(t-1)} u(t-1) - 3(t-1) e^{2(t-1)}$

(c) $\frac{s+4}{s^3 + 4s} = \frac{}{s} + \frac{}{s^2+4}$ $\quad A = 1 \quad B + C = 0 \quad B = -1 \quad C = 1$ $\quad \frac{1}{s} + \frac{1-s}{s^2+4}$

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(b) $\frac{4}{s(s+3)(s+1)} = \frac{A}{s} \; \frac{B}{s+1} \; \frac{C}{s+3}$. $\quad A = \frac{4}{3} \quad B = -2 \quad C = \frac{2}{3}$.

$y(t) = \frac{4}{3} u(t) + 2e^{-t} u(t) + \frac{2}{3} e^{-3t} u(t)$

6

6.1 6(a) **5 / 5**

✓ **- 0 pts** Effort

- 5 pts No answer

4. (a) i. $f(t) = t\cdot e^{-at}\cdot(\sin w_0 t)^2\, u(t)$

$= t\cdot e^{-at}\, u(t)\cdot\left[\frac{1}{2j}\left(e^{jw_0 t} - e^{-jw_0 t}\right)\right]^2$

$= t\cdot e^{-at}\, u(t)\cdot \frac{-1}{4}\left(e^{2jw_0 t} + e^{-2jw_0 t} - 2\right)$

$= \underbrace{\frac{1}{2} t e^{-at} u(t)}_{①} - \underbrace{\frac{t}{4} u(t)\cdot e^{(2jw_0 - a)t}}_{②} - \underbrace{\frac{t}{4} u(t)\cdot e^{(-2jw_0 - a)t}}_{③}$

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$\mathcal{L}[f(t)] = \frac{1}{2(s+a)^2} - \frac{1}{4(s+a-2jw_0)^2} - \frac{1}{4(s+a+2jw_0)^2}$

ii. $F(s) = \int_0^1 e^{-st}\cdot 0\, dt + \int_1^2 e^{-st}\, dt + \int_2^3 e^{-st}\cdot e^{-2(t-3)}\, dt + \int_3^\infty e^{-st}\, dt$

$= \left(-\frac{1}{2}e^{-2} + e^{-1}\right) - \frac{e^6}{s}\cdot e^{-3(2+s)} + \frac{e^6}{s+2} e^{-2(s+2)} + \frac{e^{-3s}}{s}$

(b) $X(s) = \int_0^\infty e^{-st}\, x(t)\cdot dt \quad \sigma > -1$ $\qquad FT = \frac{1}{(jw)^2 + 2jw + 5} = \frac{1}{5 - w^2 + 2jw}$

i. $\mathcal{L}\{x(t)\cdot e^{-t}\} = \int_0^\infty e^{-(s+1)t}\, x(t)\, dt$ $\qquad \sigma + 1 > -1 \rightarrow \sigma > -2$

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(c) $\frac{s+4}{s^3+4s} = \frac{}{s} + \frac{}{s^2+4}$ $\quad A = 1 \quad B + C = 0 \quad B = -1 \quad C = 1 \quad \frac{1}{s} + \frac{1-s}{s^2+4}$

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6. (a) $\frac{1}{2}e^{t} + 4\cdot\frac{1}{2}e^{t}\ldots\; \frac{1}{2}e^{t} = a\cdot e^{t} \quad a = 4 \quad 4x(t)$.

$H_1(s) = \frac{4}{s^2 + 4s + 3} = \boxed{\frac{4}{(s+3)(s+1)}}$

(b) $\frac{4}{s(s+3)(s+1)} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s+3}$. $\quad A = \frac{4}{3} \quad B = -2 \quad C = \frac{2}{3}$.

$y(t) = \frac{4}{3}u(t) + 2e^{-t}u(t) + \frac{2}{3}e^{-3t}u(t)$

6.2 6(b) **5 / 5**

✓ **- 0 pts** Effort

- 5 pts No answer

(c) $Y(s) = \frac{1}{3} \mathcal{L}\left[x(t) - x(t-1)\right]$ $H(s) = \frac{Y(s)}{X(s)} = \frac{e^{s}-1}{3e^{s}}$

$Y(s) = \frac{1}{3}\left[X(s) - e^{-s}X(s)\right]$

$$\frac{4}{s^2(3s+1)(s+1)}(1-e^{-s}) = \frac{A}{s^2} + \frac{B}{s} + \frac{C}{s+\frac{1}{3}} + \frac{D}{s+1}$$

$A = 4/3$; $C = -2/9$; $D = 2$; $B = -16/9$

$$y(t) = \frac{4}{3}t - \frac{16}{9}u(t) - \frac{2}{9}e^{-t/3}u(t) + 2e^{-t}u(t)$$
$$-\frac{4}{3}(t-1) + \frac{16}{9}u(t-1) + \frac{2}{9}e^{-\frac{1}{3}(t-1)}u(t-1) - 2e^{-(t-1)}u(t-1).$$

6.3 6(c) **6 / 6**

✓ **- 0 pts** Effort

- 6 pts No answer