ECE 102 HW6

LIANG, NEVIN

TOTAL POINTS

82 / 85

QUESTION 1

Problem 1 32 pts

1.1 (a)(i) 6 / 6

✓ **- 0 pts** Correct

- 3 pts incorrect

- 2 pts not simplified

- 1 pts arithmetic mistake

- 6 pts no answer

1.2 (a)(ii) 6 / 6

✓ **- 0 pts** Correct

- 1 pts arithmetic error

- 2 pts incorrect H2

- 3 pts missing H2

- 6 pts no answer

1.3 (a)(iii) 5 / 6

- 0 pts Correct

✓ **- 1 pts** incorrect const coefficient

- 1 pts error in IFT

- 4 pts incorrect

- 6 pts no answer

1.4 (b) 6 / 6

✓ **- 0 pts** Correct

- 3 pts incorrect

- 1 pts partially correct

- 6 pts no answer

1.5 (c)(i) 4 / 4

✓ **- 0 pts** Correct

- 1 pts incorrect cutoff frequency

- 2 pts incomplete

- 4 pts no answer

1.6 (c)(ii) 4 / 4

✓ **- 0 pts** Correct

- 1 pts incorrect wc

- 4 pts no answer or incorrect

QUESTION 2

Problem 2 18 pts

2.1 (a) 6 / 6

✓ **- 0 pts** Correct

- 3 pts incorrect phase

- 6 pts no answer or incorrect

2.2 (b) 3 / 3

✓ **- 0 pts** Correct

- 1 pts Wrong answer

- 3 pts No answer

2.3 (c) 5 / 5

✓ **- 0 pts** Correct: $$k = \beta = 2\pi$$

- 2 pts Wrong Answer

- 5 pts No answer

2.4 (d) 4 / 4

✓ **- 0 pts** Correct

- 1 pts Wrong answer

- 4 pts No answer

QUESTION 3

Problem 3 25 pts

3.1 (a) 4 / 6

- 0 pts Correct

✓ **- 2 pts** Wrong answer

- 6 pts No answer

3.2 (b) **6 / 6**

✓ **- 0 pts** Correct

- 2 pts Wrong answer

- 6 pts No answer

3.3 (c) **6 / 6**

✓ **- 0 pts** Correct

- 2 pts wrong answer, $$\frac{T}{m}$$, for non-negative integer m

- 6 pts No answer

- 0 pts Click here to replace this description.

3.4 (d) **7 / 7**

✓ **- 0 pts** Correct

- 2 pts wrong B range

- 2 pts wrong result

- 7 pts No answer

QUESTION 4

Problem 4 10 pts

4.1 (a)(i) **3 / 3**

✓ **- 0 pts** Correct

- 0.5 pts No annotation $$1/2M(j(w - w_c))$$

- 3 pts No answer

- 2 pts No figure

4.2 (a)(ii) **3 / 3**

✓ **- 0 pts** Correct

- 2 pts wrong graph

- 3 pts No answer

- 0 pts Click here to replace this description.

4.3 (a)(iii) **4 / 4**

✓ **- 0 pts** Correct

- 2 pts wrong graph

- 4 pts No answer

- 0 pts No Answer

4.4 (b)(Cancelled) **0 / 0**

✓ **- 0 pts** Correct

Nevin Liang.

ECE 102 HW6

1. $y(t) = x(t) * h(t)$

(a) $Y(j\omega) = H(j\omega) \cdot X(j\omega)$

i. $H(j\omega) = Y(j\omega)/X(j\omega)$

$$\frac{d^2}{dt^2} y(t) + 6 \frac{d}{dt} y(t) + 8 y(t) = 3x(t)$$

$$j^2\omega^2 F(j\omega) + 6 j\omega F(j\omega) + 8 F(j\omega) = 3 X(j\omega)$$

$$\frac{Y(j\omega)}{X(j\omega)} = \boxed{\frac{3}{8+6j\omega+j^2\omega^2}}$$

ii. $H_2(j\omega) = Y(j\omega)/Y_1(j\omega)$

$$Y(j\omega) = F\left(u(t) \cdot (4e^{-t} - 4e^{-4t}\right) = 4 \cdot \frac{1}{1+j\omega} - 4 \cdot \frac{1}{+4+j\omega}$$

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$$= \boxed{\frac{6\cancel{0}}{4+j\omega}}$$

$$H_1(j\omega) = H(j\omega)/H_2(j\omega) = \frac{3(4+j\omega)}{6\cancel{0}(8+6j\omega+j\omega^2)} = \frac{1}{2} \cdot \frac{1}{2+j\omega}$$

$$= \boxed{\frac{1}{4+2j\omega}}$$

iii. $h_1(t) = \frac{1}{2} \cdot e^{-2t} u(t) \qquad h_2(t) = \frac{1}{6} \cdot e^{-4t} u(t)$

$$H(j\omega) = \frac{3}{(4+j\omega)(2+j\omega)} = \frac{+3/2}{4+j\omega} + \frac{3/4}{2+j\omega} \Rightarrow \boxed{+\frac{3}{2} \cdot e^{-4t} u(t) + \frac{3}{4} e^{-2t} u(t)}$$

(b) Since LTI, $Y(j\omega) = X(j\omega) \cdot H(j\omega)$. since $X(j\omega) = 0$ for $|\omega| > \omega_0$, Y will be like that too. since $0 \cdot H = 0$. If we process thru non-LTI, we are not guaranteed $Y = X \cdot H$, so we can't know if $Y = 0$ for $|\omega| > \omega_0$.

1.1 (a)(i) **6 / 6**

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- **- 2 pts** not simplified
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gradescope

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- **- 1 pts** error in IFT
- **- 4 pts** incorrect
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1.4 (b) **6 / 6**

- ✓ **- 0 pts** Correct
- **- 3 pts** incorrect
- **- 1 pts** partially correct
- **- 6 pts** no answer

(c)

i. $H(j\omega) = \frac{1}{1+j\omega}$ $\quad |H(j\omega)| = \frac{1-j\omega}{1+\omega^2} = \sqrt{\left(\frac{1}{1+\omega^2}\right)^2 + \frac{\omega^2}{(1+\omega^2)^2}} = \sqrt{\frac{1}{1+\omega^2}}$

$H(0) = 1$ $\quad \pm 1\% = (0.99, 1.01)$ $\quad 0.99 \le \sqrt{\frac{1}{1+\omega^2}} \le 1.01$

$-0.0197 \le \omega \le \cancel{0.02030405}$ $\boxed{0.14249}$ $/2\pi$

$\cancel{\boxed{|f| < 7.018\ Hz}}$

$f = \boxed{0.02268\ Hz}$

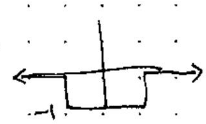
$\sqrt{\frac{1}{1+\omega^2}} = \frac{1}{\sqrt{2}} \cdot 1 = \frac{1}{\sqrt{2}}$ $\quad \frac{1}{1+\omega^2} = \frac{1}{2}$ $\quad \boxed{\omega_c = 1}$

ii. $H'(j\omega) = \frac{k}{1+j\omega}$ $\quad |H'(j\omega)| = k\sqrt{\frac{1}{1+\omega^2}}$

$= \frac{5}{1+j\omega}$ $\quad 5 \cdot H(j\omega) = 5 \cdot F(h(t)) = F(5 \cdot h(t))$

$\boxed{\text{scale } h(t) \text{ by } 5}$

2. (a) $\boxed{\alpha = -1}$ because then it becomes



phase $= \boxed{180}$ because negative

(b) ideal filters non-realizable.

$h(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} H(j\omega)\, e^{j\omega t}\, dt$

rect → sinc and sinc is not causal b/c >0 when t<0.

not causal, so not realizable.

(c) $\cancel{H_{LP_2}(j\omega) = 1 \text{ for } \omega = 0 \quad H_{LP_2} = \frac{k}{\beta}}$

$\cancel{\frac{k}{\beta+2\pi j} = \frac{1}{\sqrt{2}}\frac{k}{\beta} = \frac{k(\beta-2\pi j)}{\beta^2+4\pi^2} \quad \frac{k\beta}{\beta^2+4\pi^2} = \frac{k}{\beta\sqrt{2}}} \quad \beta^2\sqrt{2} = \beta^2+4\pi^2$

$\cancel{\beta = \sqrt{\frac{4\pi^2}{\sqrt{2}-1}} = \frac{2\pi}{\sqrt{\sqrt{2}-1}} = \boxed{9.763} \quad \frac{2\pi jk}{\beta^2+4\pi^2} = 0} \quad k=0$

1.5 (c)(i) **4 / 4**

✓ **- 0 pts** Correct

- 1 pts incorrect cutoff frequency

- 2 pts incomplete

- 4 pts no answer

(c)

i. $H(j\omega) = \frac{1}{1+j\omega}$ $\quad |H(j\omega)| = \frac{1-j\omega}{1+\omega^2} = \sqrt{\left(\frac{1}{1+\omega^2}\right)^2 + \frac{\omega^2}{(1+\omega^2)^2}} = \sqrt{\frac{1}{1+\omega^2}}$

$H(0) = 1 \quad \pm 1\% = (0.99, 1.01) \qquad 0.99 \le \sqrt{\frac{1}{1+\omega^2}} \le 1.01$

$-0.0197 \le \omega \le \cancel{0.0203040 5}$ $\boxed{0.14249}$ $/2\pi$

$\cancel{\boxed{|f| < 1.018 \text{ Hz}}}$ $\qquad f = \boxed{0.02268 \text{ Hz}}$

$\sqrt{\frac{1}{1+\omega^2}} = \frac{1}{\sqrt{2}} \cdot 1 = \frac{1}{\sqrt{2}} \qquad \frac{1}{1+\omega^2} = \frac{1}{2} \qquad \boxed{\omega_c = 1}$

ii. $H'(j\omega) = \frac{k}{1+j\omega} \qquad |H'(j\omega)| = k \sqrt{\frac{1}{1+\omega^2}}$

$= \frac{5}{1+j\omega} \qquad 5 \cdot H(j\omega) = 5 \cdot F(h(t)) = F(5 \cdot h(t))$

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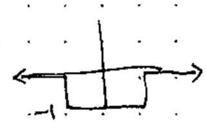

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1.6 (c)(ii) **4 / 4**

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- 1 pts incorrect wc

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i. $H(j\omega) = \frac{1}{1+j\omega}$ $\quad |H(j\omega)| = \frac{1-j\omega}{1+\omega^2} = \sqrt{\left(\frac{1}{1+\omega^2}\right)^2 + \frac{\omega^2}{(1+\omega^2)^2}} = \sqrt{\frac{1}{1+\omega^2}}$

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$-0.0197 \le w \le \cancel{0.02030405}$

$\boxed{0.14249}$ $/2\pi$

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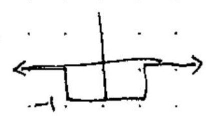

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2.1 (a) 6 / 6

- ✓ **- 0 pts** Correct
- **- 3 pts** incorrect phase
- **- 6 pts** no answer or incorrect

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i. $H(j\omega) = \frac{1}{1+j\omega}$ $\quad |H(j\omega)| = \frac{1-j\omega}{1+\omega^2} = \sqrt{\left(\frac{1}{1+\omega^2}\right)^2 + \frac{\omega^2}{(1+\omega^2)^2}} = \sqrt{\frac{1}{1+\omega^2}}$

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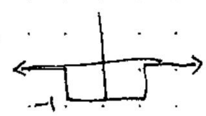

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2.2 (b) **3 / 3**

✓ **- 0 pts** Correct

- 1 pts Wrong answer

- 3 pts No answer

(c) $\omega = 0$ $\quad H_{LP,2}(j\omega) = 1 = \dfrac{k}{\beta + 0}$

$k = \beta$ $\quad \left| \dfrac{k}{\beta + 2\pi j} \right| = \dfrac{1}{\sqrt{2}} \cdot 1$

$\left| \dfrac{k}{k + 2\pi j} \right| = \dfrac{1}{\sqrt{2}}$ phase $= \dfrac{\pi}{4}$ so $\boxed{k = 2\pi}$

$\boxed{\beta = 2\pi}$

(d) $\alpha = -1$

$H_{LP,2}(j\omega) = \dfrac{2\pi}{2\pi + j\omega} + \alpha = -1 \Rightarrow \dfrac{-j\omega}{2\pi + j\omega}$

$|H_{sum}| = \dfrac{j\omega}{2\pi + j\omega}$ when ω = large, this goes to 1.

when ω = small, this is 0, so high pass.

3. (a) $H(j\omega) = \displaystyle\int_{-\infty}^{\infty} h(t)\, e^{-j\omega t}\, dt = \int_0^T \frac{1}{T} \cdot e^{-j\omega t}\, dt$

$= \dfrac{1}{T} \left(\dfrac{-1}{j\omega} \cdot e^{-j\omega t} \Big|_0^T \right)$

$= \dfrac{-1}{j\omega T} \cdot \left(e^{-j\omega T} - 1 \right)$

$= \boxed{\dfrac{1 - e^{-j\omega T}}{j\omega T}}$

(b) $\dfrac{1 - \cos(-\omega T) - i \sin(-\omega T)}{j\omega T} = \dfrac{-\sin(+\omega T) + j(1 - \cos(+\omega T))}{\omega T}$

$\dfrac{\sin^2(\omega T)}{\omega^2 T^2} + \dfrac{(1 - \cos(\omega T))^2}{\omega^2 T^2} = \dfrac{1 + 1 - 2\cos(\omega T)}{\omega^2 T^2}$

2.3 (c) **5 / 5**

- ✓ **- 0 pts** Correct: $$k = \beta = 2\pi$$
- **- 2 pts** Wrong Answer
- **- 5 pts** No answer

(c) $\omega = 0 \quad H_{LP,2}(j\omega) = 1 = \dfrac{k}{\beta + 0}$

$k = \beta \qquad \left| \dfrac{k}{\beta + 2\pi j} \right| = \dfrac{1}{\sqrt{2}} \cdot 1$

$\left| \dfrac{k}{k + 2\pi j} \right| = \dfrac{1}{\sqrt{2}}$ phase $= \dfrac{\pi}{4}$ so $\boxed{k = 2\pi}$

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$|H_{sum}| = \dfrac{j\omega}{2\pi + j\omega}$ when ω = large, this goes to 1.

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3. (a) $H(j\omega) = \displaystyle\int_{-\infty}^{\infty} h(t) e^{-j\omega t} dt = \int_0^T \frac{1}{T} \cdot e^{-j\omega t} dt$

$= \dfrac{1}{T}\left(\dfrac{-1}{j\omega} \cdot e^{-j\omega t} \Big|_0^T \right)$

$= \dfrac{-1}{j\omega T} \cdot \left(e^{-j\omega T} - 1 \right)$

$= \boxed{\dfrac{1 - e^{-j\omega T}}{j\omega T}}$

(b) $\dfrac{1 - \cos(-\omega T) - i\sin(-\omega T)}{j\omega T} = \dfrac{-\sin(+\omega T) + j(1 - \cos(+\omega T))}{\omega T}$

$\dfrac{\sin^2(\omega T)}{\omega^2 T^2} + \dfrac{(1 - \cos(\omega T))^2}{\omega^2 T^2} = \dfrac{1 + 1 - 2\cos(\omega T)}{\omega^2 T^2}$

2.4 (d) **4 / 4**

✓ **- 0 pts** Correct

- 1 pts Wrong answer

- 4 pts No answer

(c) $\omega = 0 \quad H_{LP,2}(j\omega) = 1 = \dfrac{k}{\beta + 0}$

$$k = \beta \qquad \left|\frac{k}{\beta + 2\pi j}\right| = \frac{1}{\sqrt{2}} \cdot 1$$

$$\left|\frac{k}{k + 2\pi j}\right| = \frac{1}{\sqrt{2}} \quad \text{phase} = \frac{\pi}{4} \text{ so } \boxed{k = 2\pi}$$

$$\boxed{\beta = 2\pi}$$

(d) $\alpha = -1$

$$H_{LP,2}(j\omega) = \frac{2\pi}{2\pi + j\omega} + \alpha = -1 \Rightarrow \frac{-j\omega}{2\pi + j\omega}$$

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3. (a) $H(j\omega) = \displaystyle\int_{-\infty}^{\infty} h(t) \cdot e^{-j\omega t} \, dt = \int_0^T \frac{1}{T} \cdot e^{-j\omega t} \, dt$

$$= \frac{1}{T}\left(\frac{-1}{j\omega} \cdot e^{-j\omega t} \Big|_0^T \right)$$

$$= \frac{-1}{j\omega T} \cdot \left(e^{-j\omega T} - 1 \right)$$

$$= \boxed{\frac{1 - e^{-j\omega T}}{j\omega T}}$$

(b) $\dfrac{1 - \cos(-\omega T) - i\sin(-\omega T)}{j\omega T} = \dfrac{-\sin(+\omega T) + j(1 - \cos(+\omega T))}{\omega T}$

$$\frac{\sin^2(\omega T)}{\omega^2 T^2} + \frac{(1 - \cos(\omega T))^2}{\omega^2 T^2} = \frac{1 + 1 - 2\cos(\omega T)}{\omega^2 T^2}$$

3.1 (a) **4 / 6**

- **- 0 pts** Correct
- ✓ **- 2 pts** **Wrong answer**
- **- 6 pts** No answer

(c) $\omega = 0 \quad H_{LP,2}(j\omega) = 1 = \dfrac{k}{\beta + 0}$

$k = \beta \qquad \left| \dfrac{k}{\beta + 2\pi j} \right| = \dfrac{1}{\sqrt{2}} \cdot 1$

$\left| \dfrac{k}{k + 2\pi j} \right| = \dfrac{1}{\sqrt{2}}$ phase $= \dfrac{\pi}{4}$ so $\boxed{k = 2\pi}$

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(d) $\alpha = -1$

$H_{LP,2}(j\omega) = \dfrac{2\pi}{2\pi + j\omega} + \alpha = -1 \Rightarrow \dfrac{-j\omega}{2\pi + j\omega}$

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when ω = small, this is 0, so high pass.

3. (a) $H(j\omega) = \displaystyle\int_{-\infty}^{\infty} h(t) e^{-j\omega t} dt = \int_0^T \frac{1}{T} \cdot e^{-j\omega t} dt$

$= \dfrac{1}{T} \left(\dfrac{-1}{j\omega} \cdot e^{-j\omega t} \Big|_0^T \right)$

$= \dfrac{-1}{j\omega T} \cdot \left(e^{-j\omega T} - 1 \right)$

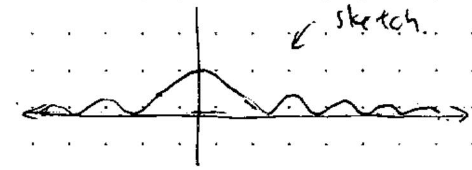
$= \boxed{\dfrac{1 - e^{-j\omega T}}{j\omega T}}$

(b) $\dfrac{1 - \cos(-\omega T) - i\sin(-\omega T)}{j\omega T} = \dfrac{-\sin(+\omega T) + j(1 - \cos(+\omega T))}{\omega T}$

$\dfrac{\sin^2(\omega T)}{\omega^2 T^2} + \dfrac{(1 - \cos(\omega T))^2}{\omega^2 T^2} = \dfrac{1 + 1 - 2\cos(\omega T)}{\omega^2 T^2}$

$$\Rightarrow \sqrt{\frac{2-2\cos(\omega T)}{\omega^2 T^2}}$$

sketch



$$= \frac{\sqrt{2-2\cos(\omega T)}}{\omega T}$$

as $\omega \to \infty$, $|H(j\omega)| \to 0$.

(c) $y(t) = C$ $\qquad$ $\cancel{Y(j\omega) = }\int_{-\infty}^{\infty}$

$\cancel{\delta(t) \Leftrightarrow 1}$

$\cancel{F(\delta(t)) = 1}$

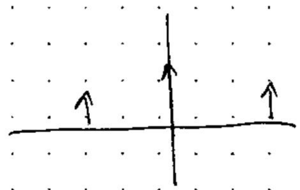
$\cancel{F(c \cdot \delta(t)) = c}$

$1 \Leftrightarrow 2\pi\, \delta(\omega)$

$C \Leftrightarrow C \cdot 2\pi\, \delta(\omega)$

if $y(t) = C$, then frequency is

in order for the amplitude response above to be constant once multiplied by $X(j\omega)$, $X(j\omega)$ must be something like this



where the right and left impulses land on 0's of $H(j\omega)$.

$\omega = 0$; $\omega =$ such that $\cos(\omega T) = 1$

$\omega T = 0, 2\pi k \quad \omega = \frac{2\pi}{T} k$

$a_1 \cos(0) + a_2 \cos\left(\frac{2\pi k}{T} t\right)$ $\qquad$ $\boxed{T' = \frac{T}{k}}$

(d) $|H(j\omega)| = \frac{\sqrt{2-2\cos(\omega T)}}{\omega T} = 0$ @ $\cos(\omega T) = 1$

$\omega T = 2\pi, -2\pi \qquad \omega = \frac{\pm 2\pi}{T}$

$\measuredangle H(j\omega) = \tan^{-1}\left(\frac{1-\cos(\omega T)}{-\sin(\omega T)}\right)$

$= \frac{1}{1 + \frac{1-2\cos(\omega T) + \cos^2(\omega T)}{\sin^2(\omega T)}} = \frac{\sin^2(\omega T)}{2-2\cos \omega T} = \frac{1+\cos(\omega T)}{2}$

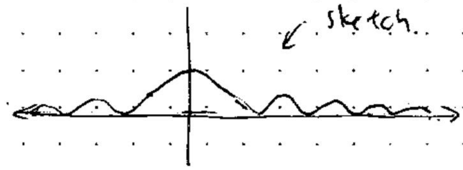
$\boxed{-\left(\frac{1+\cos(\omega T)}{2}\right)}$

3.2 (b) **6 / 6**

- ✓ **- 0 pts** Correct
- **- 2 pts** Wrong answer
- **- 6 pts** No answer

$$\Rightarrow \sqrt{\frac{2-2\cos(\omega T)}{\omega^2 T^2}}$$

sketch



$$= \frac{\sqrt{2-2\cos(\omega T)}}{\omega T}$$

as $\omega \to \infty$, $|H(j\omega)| \to 0$.

(c) $y(t) = C$ $\quad$ ~~$Y(j\omega) =$~~ $\int_{-\infty}^{\infty}$

~~$\delta(t) \Leftrightarrow 1$~~

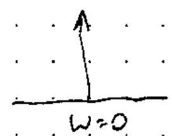
~~$F(\delta(t)) = 1$~~

~~$F(C \cdot \delta(t)) = C$~~

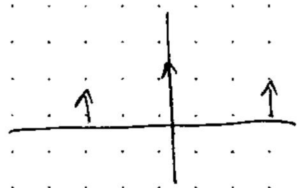
$1 \Leftrightarrow 2\pi\,\delta(\omega)$

$C \Leftrightarrow C \cdot 2\pi\,\delta(\omega)$

if $y(t) = C$, then frequency is



In order for the amplitude response above to be constant once multiplied by $X(j\omega)$, $X(j\omega)$ must be something like this



where the right and left impulses land on 0's of $H(j\omega)$.

$\omega = 0$; $\omega =$ such that $\cos(\omega T) = 1$

$\omega T = 0, 2\pi k \quad \omega = \frac{2\pi}{T}k$

$a_1 \cos(0) + a_2 \cdot \cos\left(\frac{2\pi k}{T} t\right)$ $\quad \boxed{T' = \frac{T}{k}}$

(d) $|H(j\omega)| = \frac{\sqrt{2-2\cos(\omega T)}}{\omega T} = 0$ @ $\cos(\omega T) = 1$

$\omega T = 2\pi, -2\pi \quad \omega = \frac{\pm 2\pi}{T}$

$\measuredangle H(j\omega) = \tan^{-1}\left(\frac{1-\cos(\omega T)}{-\sin(\omega T)}\right)$

$\boxed{-\left(\frac{1+\cos(\omega T)}{2}\right)}$ $= \frac{1}{\frac{1-2\cos(\omega T) + \cancel{\cos^2(\omega T)}}{\sin^2(\omega T)}} = \frac{\sin^2(\omega T)}{1-1\cos\omega T} = \frac{1+\cos(\omega T)}{2}$

3.3 (c) **6 / 6**

✓ **- 0 pts** Correct

- 2 pts wrong answer, $$\frac{T}{m}$$, for non-negative integer m

- 6 pts No answer

- 0 pts Click here to replace this description.

(d) $\left| H(j\omega) \right| = \dfrac{\sqrt{2-2\cos(\omega T)}}{\omega T} = 0$ when $\cos(\omega T) = 1$

$\omega T = 2\pi, 4\pi \ldots$

$\omega = \dfrac{2\pi}{T}$

$$\boxed{B = \frac{1}{T}}$$

$\measuredangle H(j\omega) = \dfrac{-\omega T}{2} \Rightarrow \boxed{\dfrac{T}{2}}$

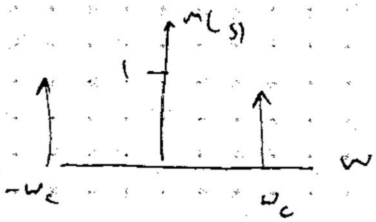
- derivative

4) i. $m(0) : \cos(\omega_c t)$

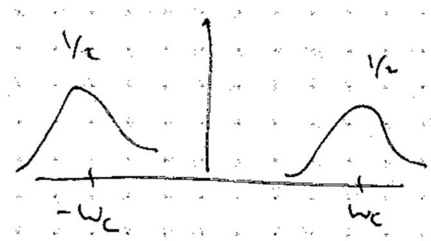
$e^{j\omega_0 t} \leftrightarrow 2\pi\delta(\omega - \omega_0)$

$\cos(\omega_c t) \leftrightarrow \pi(\delta(\omega - \omega_c) + \delta(\omega + \omega_c))$

$m(t) \Rightarrow$



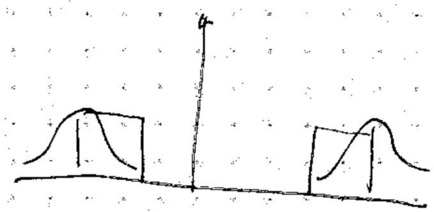
$X(j\omega)$



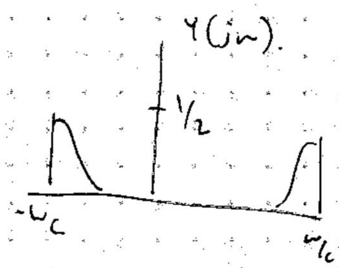
$1/2$ b/c $\dfrac{1}{2\pi}$

ii) $Y(j\omega) = X(j\omega) \cdot H(j\omega)$

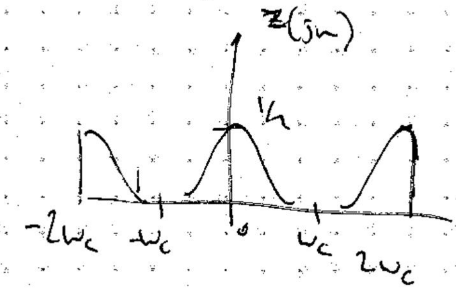
$X(j\omega) \cdot H(j\omega)$



$\Downarrow$



iii) $y(t) \cdot \cos(\omega_c t)$ recovers $m(t) \times 0.5$



scaled by 0.5

3.4 (d) **7 / 7**

✓ **- 0 pts** **Correct**

- 2 pts wrong B range

- 2 pts wrong result

- 7 pts No answer

(d) $\left|H(j\omega)\right| = \frac{\sqrt{2-2\cos(\omega T)}}{\omega T} = 0$ when $\cos(\omega T)=1$

$\omega T = 2\pi, 4\pi \ldots$

$\omega = \frac{2\pi}{T}$

$$\boxed{B = \frac{1}{T}}$$

$\angle H(j\omega) = \frac{-\omega T}{2} \Rightarrow \boxed{\frac{T}{2}}$

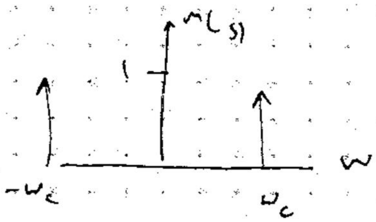
−derivative

4) i. $m(0) \cdot \cos(\omega_c t)$

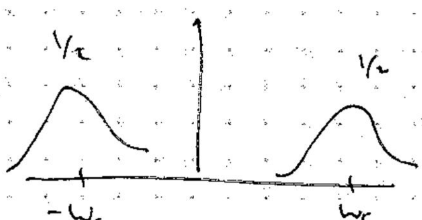
$e^{j\omega_0 t} \leftrightarrow 2\pi\delta(\omega-\omega_0)$

$\cos(\omega_c t) \leftrightarrow \pi(\delta(\omega-\omega_c)+\delta(\omega+\omega_c))$

$m(t) \to$



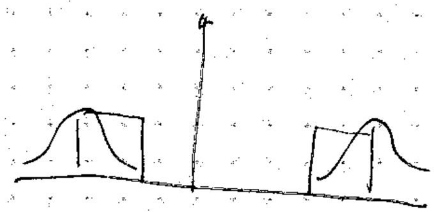
X(jω)

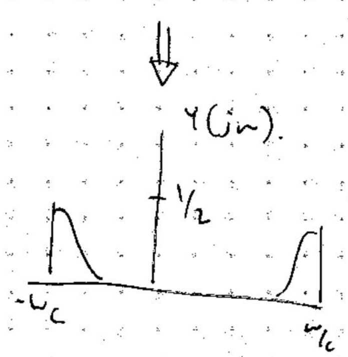


1/2 b/c $\frac{1}{2\pi}$

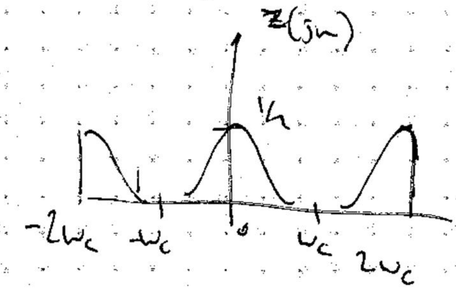
ii) $Y(j\omega) = X(j\omega)\cdot H(j\omega)$

$X(j\omega)\cdot H(j\omega)$





iii) $y(t)\cdot\cos(\omega_c t)$ recovers $m(t)\times 0.5$



$\boxed{\text{scaled by } 0.5}$

4.1 (a)(i) **3 / 3**

✓ **- 0 pts** Correct

- 0.5 pts No annotation $$1/2M(j(w - w_c))$$

- 3 pts No answer

- 2 pts No figure

(d) $\left|H(j\omega)\right| = \frac{\sqrt{2-2\cos(\omega T)}}{\omega T} = 0$ when $\cos(\omega T) = 1$

$\omega T = 2\pi, 4\pi \ldots$

$\omega = \frac{2\pi}{T}$

$$\boxed{B = \frac{1}{T}}$$

$\angle H(j\omega) = \frac{-\omega T}{2} \Rightarrow \boxed{\frac{T}{2}}$

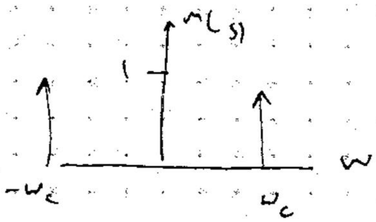
- derivative

4) i. $m(0) : \cos(\omega_c t)$

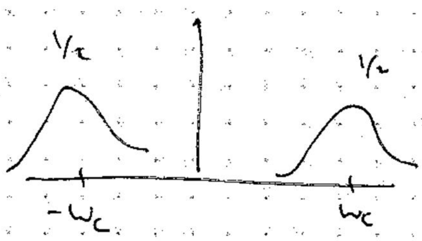
$e^{j\omega_0 t} \leftrightarrow 2\pi\delta(\omega - \omega_0)$

$\cos(\omega_c t) \longleftrightarrow \pi(\delta(\omega - \omega_c) + \delta(\omega + \omega_c))$

$m(t) \rightarrow$



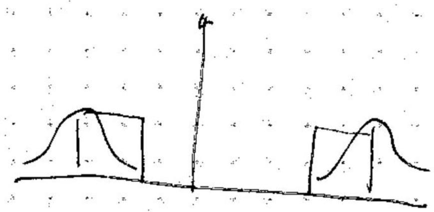
$X(j\omega)$



$1/2$ b/c $\frac{1}{2\pi}$

ii) $Y(j\omega) = X(j\omega) \cdot H(j\omega)$

$X(j\omega) \cdot H(j\omega)$



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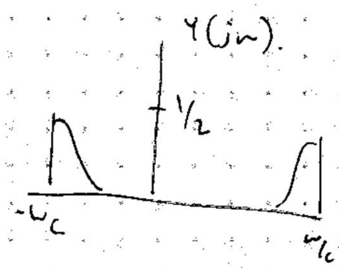
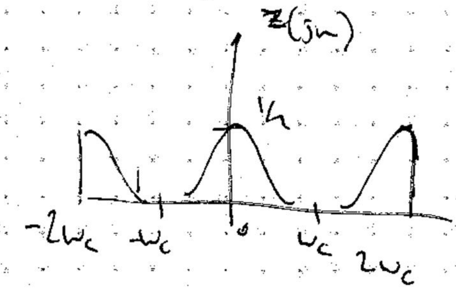


iii) $y(t) \cdot \cos(\omega_c t)$ recovers $m(t) \times 0.5$



scaled by 0.5

4.2 (a)(ii) **3 / 3**

✓ **- 0 pts** **Correct**

- 2 pts wrong graph

- 3 pts No answer

- 0 pts Click here to replace this description.

(d) $\left|H(j\omega)\right| = \dfrac{\sqrt{2-2\cos(\omega T)}}{\omega T} = 0$ when $\cos(\omega T) = 1$

$\omega T = 2\pi, 4\pi \ldots$

$\omega = \dfrac{2\pi}{T}$

$$\boxed{B = \frac{1}{T}}$$

$\measuredangle H(j\omega) = \dfrac{-\omega T}{2} \Rightarrow \boxed{\dfrac{T}{2}}$

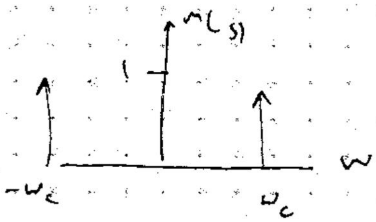
$-$derivative

4) i. $m(0) : \cos(\omega_c t)$

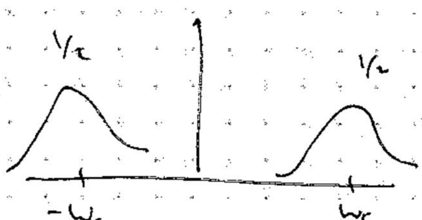
$e^{j\omega_0 t} \leftrightarrow 2\pi\delta(\omega-\omega_0)$

$\cos(\omega_c t) \longleftrightarrow \pi\left(\delta(\omega-\omega_c)+\delta(\omega+\omega_c)\right)$

$m(t) \rightarrow$



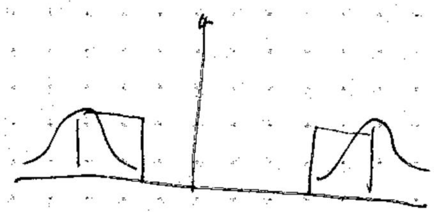
$X(j\omega)$



$1/2$ b/c $\dfrac{1}{2\pi}$

ii) $Y(j\omega) = X(j\omega)\cdot H(j\omega)$

$X(j\omega)\cdot H(j\omega)$



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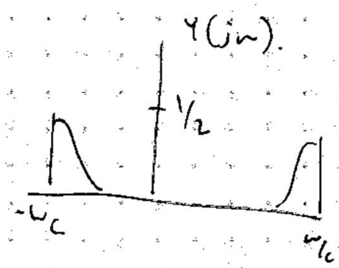
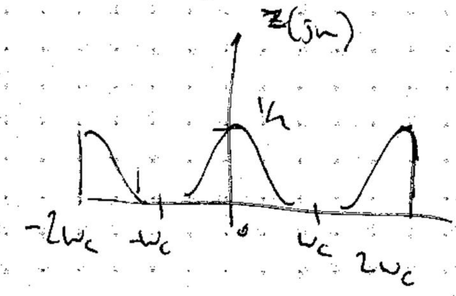


iii) $y(t)\cdot\cos(\omega_c t)$ recovers $m(t)\times 0.5$



scaled by 0.5

4.3 (a)(iii) **4 / 4**

✓ **- 0 pts** Correct

- 2 pts wrong graph

- 4 pts No answer

- 0 pts No Answer

4.4 (b)(Cancelled) 0 / 0

✓ **- 0 pts** Correct