ECE102, Fall 2020 **Final exam review**

Signals & Systems Prof. J.C. Kao

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1. **Signals and systems basics**

   (a) A continuous time signal $x(t)$ is shown below

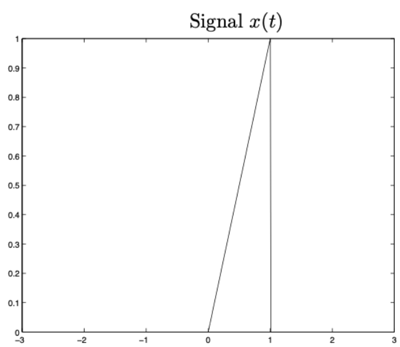


Figure 1: $x(t)$

i. What is the energy in $x(t)$ over the infinite interval, that is, what is E_∞?

ii. What is the power in $x(t)$ over the infinite interval, that is, what is P_∞?

iii. Eight signals labeled "Signal A" through "Signal H" are shown below. Match these signals with the following eight signals. Each entry must be a letter from "A" through "H".

A. The even part of $x(t)$, that is, $x_{even}(t)$.

B. The odd part of $x(t)$, that is, $x_{odd}(t)$.

C. $x(t+1)$.

D. $x(t-1)$.

E. $x(2t+1)$.

F. $x(2t+2)$.

G. $x(1-2t)$.

H. $x(2-2t)$.

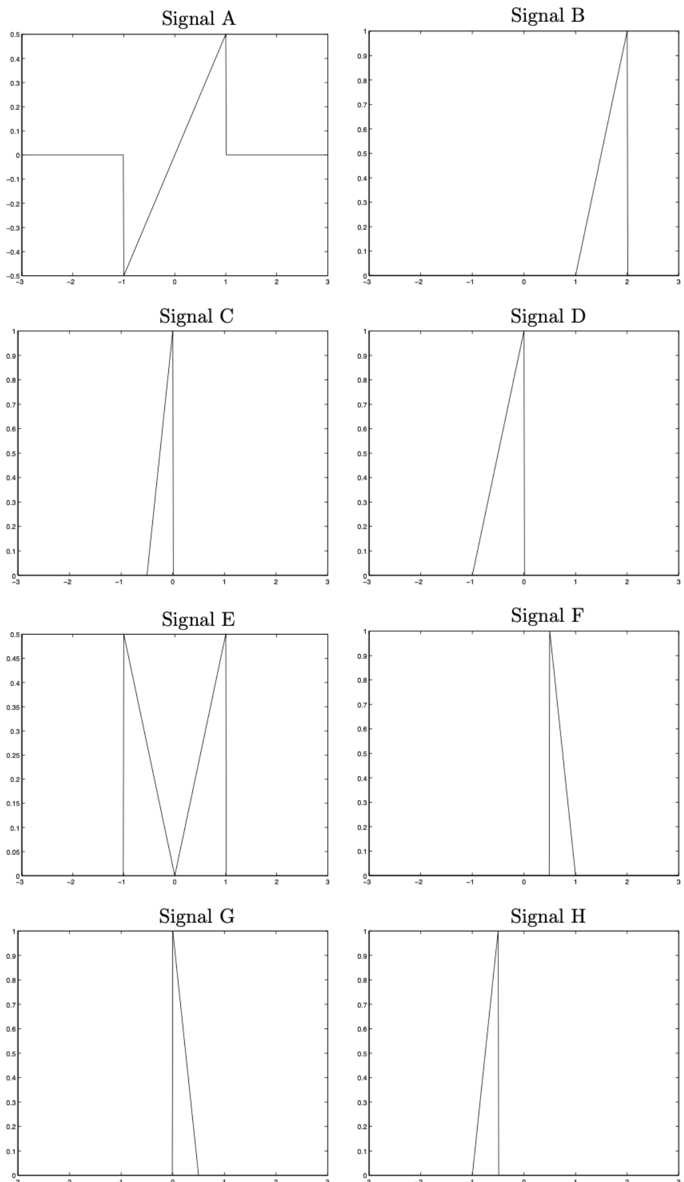


Figure 2: $A - H$

(b) Justify whether the following statements are true or false:

i. Series cascade of two LTI system results in an equivalent system that is not LTI.

ii. Parallel cascade of two LTI system results in an equivalent system that is LTI.

2. **Fourier transform basics**

Consider the time-domain signal $x(t)$ plotted below. Using the properties of the Fourier Transform, you are able to compute all of the quantities required in each of the parts below without having to determine the corresponding Fourier Transform, $X(j\omega)$. Indicate which properties you are using and clearly label your final answer for each part.

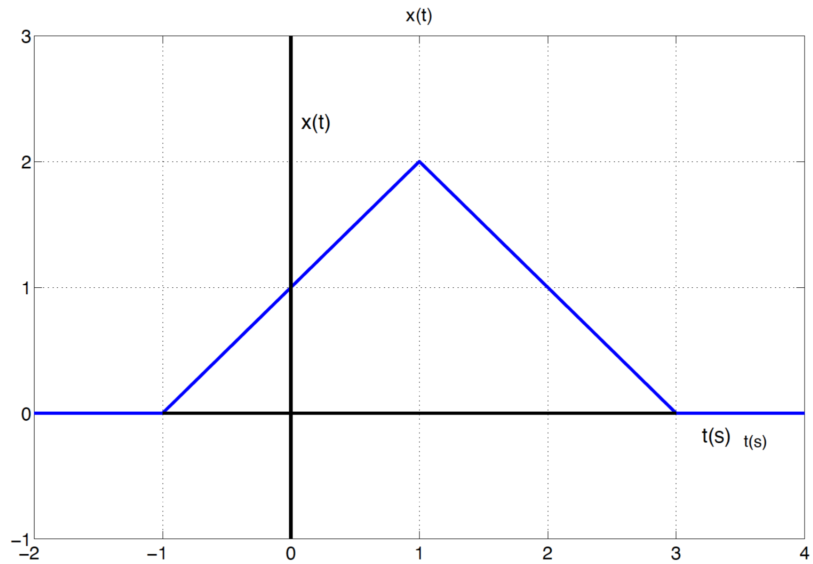


Figure 3: $x(t)$

(a) Find $X(0)$

(b) Evaluate $\int_{-\infty}^{\infty} X(j\omega)d\omega$

(c) Evaluate $\int_{-\infty}^{\infty} |X(j\omega)|^2 d\omega$

(d) Plot the phase of the Fourier Transform

(e) Find the value of the integral below

$$\int_{-\infty}^{\infty} X(j\omega)H(j\omega)e^{j2\omega}d\omega$$

where $H(j\omega) = 2\frac{\sin(\omega)}{\omega}$

3. **Properties of Fourier Transform**

You are given the Fourier Transform pair below

$$\{1+\cos(\pi t)\}rect(\frac{t}{2}) \overset{\mathcal{F}}{\longleftrightarrow} 2\frac{\sin\omega}{\omega}\frac{\pi^2}{\pi^2-\omega^2}$$

(a) Determine the Fourier Transform, $X_a(j\omega)$, of the signal below. Indicate which properties you are using.

$$x_a(t) = \frac{\sin t}{\pi t}\frac{\pi^2}{\pi^2 - t^2}$$

(b) Determine and plot the Fourier Transform, $X_b(j\omega)$, of the signal below. Indicate which properties you are using.

$$x_b(t) = W\frac{\sin Wt}{\pi Wt}\frac{\pi^2}{\pi^2 - (Wt)^2}$$

(c) Determine and plot the Fourier Transform, $X_c(j\omega)$, of the signal below. Indicate which properties you are using.

$$x_c(t) = \frac{d}{dt}x_b(t)$$

4. **Frequency response and output of LTI systems**

(a) You are given that the impulse response of an ideal Hilbert Transformer has the frequency response given below

$$h(t) = \frac{1}{\pi t} \overset{\mathcal{F}}{\longleftrightarrow} H(j\omega) = \begin{cases} j, & \omega < 0 \\ -j, & \omega > 0 \end{cases}$$

Just view the above as a Fourier transform pair, and use one or more of the Fourier transform properties to determine the Fourier Transform of

$$x(t) = sgn(t) = \begin{cases} -1, & t < 0 \\ 1, & t > 0 \end{cases}$$

(b) Given the Fourier Transform pair below

$$x(t) = \cos(\pi t)rect(t) \overset{\mathcal{F}}{\longleftrightarrow} X(j\omega) = \frac{2\pi\cos(\frac{\omega}{2})}{\pi^2 - \omega^2}$$

Determine the Fourier Transform of the signal below in terms of T.

$$y(t) = \cos(\pi\frac{t}{T})rect(\frac{t}{T})$$

(c) Consider an LTI system with impulse response

$$h(t) = \frac{\pi}{2}\frac{\sin(2t)}{\pi t}\frac{\sin(10t)}{\pi t}$$

Determine the output $y(t)$ for the input $x(t)$ given below, which is the Fourier series expansion for a periodic sawtooth waveform with period $T = \pi$

$$x(t) = \sum_{k=-\infty}^{-1}\frac{j(-1)^k}{k\pi}e^{jk2t} + \sum_{k=1}^{\infty}\frac{j(-1)^k}{k\pi}e^{jk2t}$$

Show work and write your expression for $y(t)$.

5. **Sampling and modulation**

(a) In lecture, we defined the impulse train with period T as

$$\delta_T(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)$$

Find the fourier coefficients, c_k, of $\delta_T(t)$.

(b) In lecture we also derived an equation for the fourier transform of a periodic signal. Use that result to derive the fourier transform of $\delta_T(t)$.

(c) Now lets consider the shifted impulse train

$$p(t) = \delta_T(t - \frac{T}{2})$$

Use the properties of the Fourier transform to compute the Fourier transform of $p(t)$.

(d) We define a signal $r(t)$ as follows

$$r(t) = m(t)(cos(\omega_s(t)) + sin(\omega_s(t)))$$

where $m(t)$ has real spectrum defined by:

$$M(j\omega) = \Delta(\frac{\omega}{\omega_m}), \omega_s > 2\omega_m$$

Find and plot the spectrum of $r(t)$.

6. **Sampling**

(a) The maximum frequency for the signal $x_a(t)$ is ω_{max}, such that $X_a(j\omega) = 0$ for $|\omega| > \omega_{max}$. What is the maximum frequency for the signal $y_a(t) = x_a^2(t)$ in terms of ω_{max}?

(b) If you sample the sinewave $x_a(t) = \cos(8t)$ at a rate of $\omega_s = 14 radians/sec$, which is below the Nyquist rate, what frequency (in radians/sec) does the under-sampled sinewave get aliased to?

(c) Consider a continuous time signal $x_a(t)$ with bandwidth (maximum frequency) W in rads/sec. The sampling rate is chosen to be above the Nyquist rate at $\omega_s = 3.5W$, where $\omega_s = \frac{2\pi}{T_s}$. $x_a(t)$ is reconstructed perfectly according to the formula below. Let $H(j\omega)$ be the Fourier transform of $h(t)$. Determine the respective values of ω_1 and ω_2, both in terms of W, so that $H(j\omega)$ is flat up to the bandwidth W and then rolls off to zero at $\omega_s - W$.

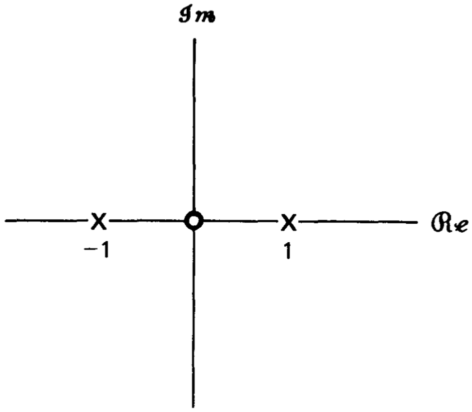
$$x_a(t) = \sum_{n=-\infty}^{\infty} x_a(nT_s)h(t - nT_s)$$

where,

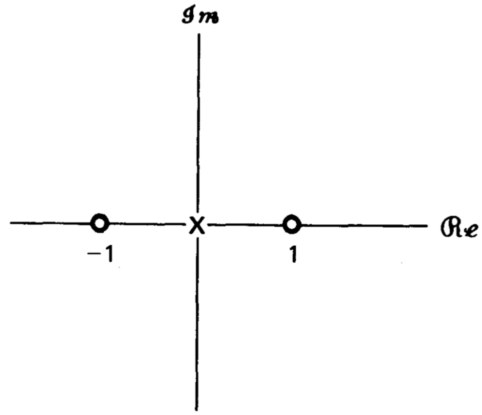
$$h(t) = T_s \frac{\pi}{\omega_1} \frac{\sin(\omega_1 t)}{\pi t} \frac{\sin(\omega_2 t)}{\pi t}$$

and

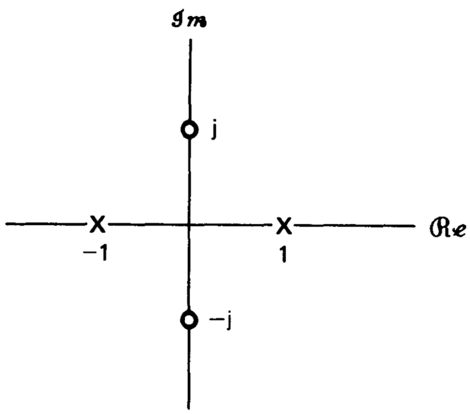
$$\omega_s = \frac{7}{2} W$$



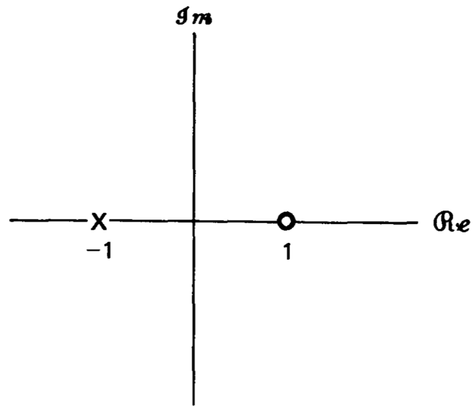
(a) Pole-zero plot 1



(b) Pole-zero plot 2



(c) Pole-zero plot 3



(d) Pole-zero plot 4

Figure 4: Four pole-zero plots

7. **Laplace transform**

   (a) Find the Laplace transforms of the following signals and determine their region of convergence.

      i. $f(t) = t\sin(t)$
      ii. $f(t) = \int_{0^-}^{t} x\sin(x)dx, \;\; t \geq 0$
      iii. $f(t) = r(t-2) - 2r(t-3) + r(t-4)$

   (b) We are given four pole-zero plots in Figure 4.

      i. Determine which, if any, of the pole-zero plots in the figure below could correspond to an even time function. For those that could, indicate the required ROC.
      ii. Determine which, if any, of the pole-zero plots in the figure below could correspond to an odd time function. For those that could, indicate the required ROC.