ECE102, Fall 2020 **Homework #3**

Signals & Systems Prof. J.C. Kao

University of California, Los Angeles; Department of ECETAs: A. Ghosh, T. Monsoor & G. Zhao

Due Friday, 30 Oct 2019, by 11:59pm to Gradescope.
Covers material up to Lecture 6.
100 points total.

1. (20 points) **Linear systems** Determine whether each of the following systems is linear or not. Explain your answer.
   (a) $y(t) = \sin(t)x(t)$
   (b) $y(t) = \frac{d}{dt}(\frac{1}{3}x(t)^3)$
   (c) $y(t) = e^{x(t)}$
   (d) $y(t) = x(t) + u(t+1)$

2. (13 points) **LTI systems**
   (a) The input $x(t)$ and the corresponding output $y(t)$ of a linear time-invariant (LTI) system are

   $$x(t) = u(t) - u(t-1) \qquad \longrightarrow y(t) = r(t) - 2r(t-1) + r(t-2)$$

   where $r(t)$ is the ramp signal defined in lecture. Determine the outputs $y_i(t), i = 1, 2, 3$ corresponding to the following inputs
      i. (2 points) $x_1(t) = u(t) - u(t-1) - u(t-2) + u(t-3)$
      ii. (2 points) $x_2(t) = u(t+1) - 2u(t) + u(t-1)$
      iii. (3 points) $x_3(t) = \delta(t) - \delta(t-1)$

   (b) (6 points) Assume we have a linear system with the following input-output pairs:
      - the output is $y_1(t) = \cos(t)u(t)$ when the input is $x_1(t) = u(t)$;
      - the output is $y_2(t) = \cos(t)(u(t+1) - u(t))$ when the input is $x_2(t) = \mathrm{rect}(t + \frac{1}{2})$.

   Is the system time-invariant?

3. (38 points) **Convolution**
   (a) (10 points) For each pair of the signals given below, compute their convolution using the flip-and-drag technique. Please provide a piecewise formula for $y(t)$.
      i. $f(t) = \delta(t+1) + 2\delta(t-2), \qquad g(t) = e^{-t}u(t)$
      ii. $f(t) = 2\,\mathrm{rect}(t - \frac{3}{2}), \qquad g(t) = 2\,r(t-1)\mathrm{rect}(t - \frac{3}{2})$

   (b) (10 points) For each of the following, find a function $h(t)$ such that $y(t) = x(t) * h(t)$.
      i. $y(t) = \int_{t-T}^{t} x(\tau)d\tau$
      ii. $y(t) = \sum_{n=-\infty}^{\infty} x(t - nT_s)$
      Note: this last operation creates a periodic extension of $x(t)$ where the period is T_s.

(c) (10 points) Use the properties of convolution to simplify the following expressions:

i. $[\delta(t-3)+\delta(t+2)] * \left[e^{3t}u(-t)+\delta(t+2)+2\right]$

ii. $\frac{d}{dt}\left[(u(t)-u(t-1)) * u(t-2)\right]$, *Hint: Show first that* $u(t) * u(t) = r(t)$ *where* $r(t)$ *is the ramp function.*

(d) (8 points) Explain whether each of the following statements is true or false.

i. If $x(t)$ and $h(t)$ are both odd functions, and $y(t) = x(t) * h(t)$, then $y(t)$ is an even function.

ii. If $y(t) = x(t) * h(t)$, then $y(2t) = h(2t) * x(2t)$.

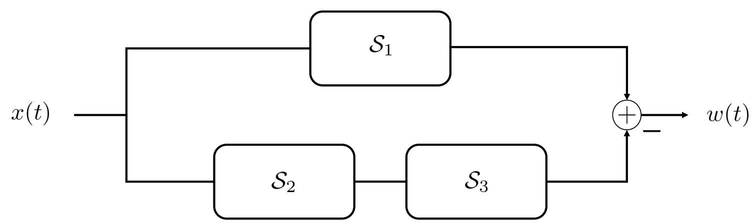
4. (12 points) **Impulse response and LTI systems**

Consider the following three LTI systems:

- The first system $\mathcal{S}_1$ is given by its input-output relationship: $y(t) = \int_{-\infty}^{t-t_0} x(\tau-2)d\tau$;
- The second system $\mathcal{S}_2$ is given by its impulse response: $h_2(t) = u(t+2)$;
- The third system $\mathcal{S}_3$ is given by its impulse response: $h_3(t) = \delta(t-4)$.

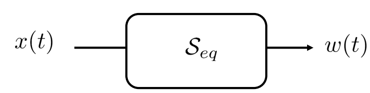
(a) (4 points) Compute the impulse responses $h_1(t)$ of system $\mathcal{S}_1$.

(b) (4 points) The three systems are interconnected as shown below.



,

Determine the impulse response $h_{eq}(t)$ of the equivalent system.



(c) (4 points) Determine the response of the overall system to the input $x(t) = 0.5 * \delta(t-2) + \delta(t-3)$.

5. (17 points) **MATLAB**

To complete the following MATLAB tasks, we will provide you with a MATLAB function (`nconv()`), which numerically evaluates the convolution of two continuous-time functions. Make sure to download it from CCLE and save it in your working directory in order to use it.

The function syntax is as follows:

```
[y, ty] = nconv(x,tx,h,th)
```

where the inputs are:

`x` : input signal vector
`tx`: times over which `x` is defined
`h` : impulse response vector
`th`: times over which `h` is defined

and the outputs are:

`y` : output signal vector
`ty`: times over which `y` is defined.

The function is implemented with the MATLAB's `conv()` function. You are encouraged to look at the implementation of the function provided (the explanations are included as comments in the code).

(a) (5 points) **Task 1**

Using the `nconv()` function, perform the convolution of two unit rect functions: $\text{rect(t)} * \text{rect(t)}$. Plot and label the result.

(b) (5 points) **Task 2**

Using the result of task 1 and the same MATLAB function, calculate $y(t) = \text{rect(t)} * \text{rect(t)} * \text{rect(t)}$. Plot and label the result.

(c) (7 pointss) **Task 3**

Now, what happens if we consider $\text{rect(t)} * \text{rect(t)} * \cdots * \text{rect(t)} = \text{rect}^{(N)}(t)$? Using for loop, calculate the result of convolving N rect(t) functions together. Plot and label the result (use $N = 100$).

Side note in case you have taken any probability course before: Convolution is an operator that is also useful in statistics. We use it to compute the pdf (probability density function) of the sum of N independent random variables. So if we have $Y = X_1 + X_2 + X_3$, the pdf of Y is the convolution of the pdfs of X_1, X_2 and X_3. In task 4, we are computing the pdf of the sum of N uniform random variables (the pdf of a uniform random variable is a rect function), by convolving N times the rect function. The resulting curve will have a bell-shape. This is related to a theorem in statistics called 'The Central Limit Theorem'.