ECE 102 HW2

LIANG, NEVIN

TOTAL POINTS

100 / 100

QUESTION 1

Question 1 22 pts

1.1 (a)i. 3 / 3

- ✓ **- 0 pts** Correct
- **- 2 pts** wrong plot
- **- 2 pts** no plot

1.2 (a)ii. 3 / 3

- ✓ **- 0 pts** Correct
- **- 2 pts** wrong plot

1.3 (a)iii. 3 / 3

- ✓ **- 0 pts** Correct
- **- 2 pts** wrong plot

1.4 (b)i. 3 / 3

- ✓ **- 0 pts** Correct
- **- 1 pts** Wrong answer

1.5 (b)ii. 3 / 3

- ✓ **- 0 pts** Correct
- **- 1 pts** wrong answer

1.6 (b)iii. 3 / 3

- ✓ **- 0 pts** Correct
- **- 1 pts** inaccurate answer

1.7 (c) 4 / 4

- ✓ **- 0 pts** Correct
- **- 1 pts** wrong answer

QUESTION 2

Question 2 23 pts

2.1 (a) 15 / 15

- ✓ **- 0 pts** Correct
- **- 1 pts** i. wrong item
- **- 1 pts** i. wrong scale
- **- 5 pts** ii. no answer
- **- 5 pts** ii. no answer
- **- 1 pts** ii. wrong item
- **- 1 pts** ii. not simplified answer
- **- 1 pts** ii. wrong scale
- **- 1 pts** ii. wrong item
- **- 1 pts** ii. wrong shift
- **- 5 pts** iii. no answer
- **- 1 pts** iii. wrong scale
- **- 2 pts** iii. wrong answer
- **- 1 pts** iii. wrong item
- **- 1 pts** iii. wrong shift
- **- 1 pts** ii. misuse of annotations.
- **- 1 pts** i. misuse of annotations.
- **- 1 pts** iii. misuse of annotations.

2.2 (b) 8 / 8

- ✓ **- 0 pts** Correct
- **- 1 pts** i. wrong shift
- **- 4 pts** i. wrong answer
- **- 1 pts** ii. wrong shift
- **- 1 pts** ii. wrong scale
- **- 1 pts** ii. wrong item
- **- 4 pts** ii. no answer
- **- 4 pts** i. not use unit-step functions
- **- 4 pts** ii. not use unit-step functions
- **- 1 pts** ii. wrong shift
- **- 4 pts** i. ii. wrong annotations.
- **- 1 pts** i. wrong item

QUESTION 3

Question 3 30 pts

3.1 (a)i. **4 / 4**

✓ **- 0 pts** **Correct (Non-linear, time-variant, not causal, stable)**

- **1 pts** wrong answer
- **1 pts** no conclusion
- **1 pts** incomplete answer

3.2 (a)ii. **4 / 4**

✓ **- 0 pts** **Correct (linear, time-invariant, not causal, stable)**

- **1 pts** wrong answer
- **1 pts** incomplete answer

3.3 (a)iii. **4 / 4**

✓ **- 0 pts** **Correct (linear, time variant, causal, unstable)**

- **1 pts** incomplete answer
- **1 pts** wrong answer
- **4 pts** no answer

3.4 (a)iv. **4 / 4**

✓ **- 0 pts** **Correct (time-variant, causal, stable)**

- **1 pts** incomplete answer
- **1 pts** wrong answer
- **4 pts** no answer

3.5 (a)v. **4 / 4**

✓ **- 0 pts** **Correct (time invariant, causal, stable)**

- **1 pts** incomplete answer
- **1 pts** wrong answer
- **4 pts** no answer

3.6 (b) **6 / 6**

✓ **- 0 pts** **Correct**

- **1 pts** wrong scale
- **6 pts** no answer
- **1 pts** wrong shift
- **1 pts** not simplified answer
- **0.5 pts** wrong annotations

3.7 (c)i. **2 / 2**

✓ **- 0 pts** **Correct**

- **1 pts** wrong answer
- **0.5 pts** no final answer
- **2 pts** no answer
- **1 pts** insufficient answer

3.8 (c)ii. **1 / 1**

✓ **- 0 pts** **Correct**

- **1 pts** wrong answer
- **1 pts** no answer
- **1 pts** insufficient answer
- **0.5 pts** no final answer

3.9 (c)iii. **1 / 1**

✓ **- 0 pts** **Correct**

- **1 pts** wrong answer
- **0.5 pts** insufficient answer
- **1 pts** no answer

QUESTION 4

Question 4 10 pts

4.1 (a) **5 / 5**

✓ **- 0 pts** **Correct**

- **1 pts** error in magnitude squared computation
- **1 pts** error in taking the limit
- **0.5 pts** no power value
- **1 pts** arithmetic error
- **5 pts** no answer

4.2 (b) **5 / 5**

✓ **- 0 pts** **Correct**

- **5 pts** no answer
- **0.5 pts** arithmetic error
- **1 pts** error in taking the limit
- **2 pts** incorrect energy value
- **0.5 pts** copied the question wrong
- **1 pts** error in the magnitude squared value
- **1 pts** not a power signal
- **2.5 pts** partially correct

QUESTION 5

Question 5 15 pts

5.1 (a) **5 / 5**

✓ **- 0 pts** Correct

- 5 pts no answer

- 2 pts incorrect sigma and plot

- 1 pts no omega and sigma values

- 0.5 pts incorrect sigma

- 0.5 pts no or incorrect interpretation of results

- 1 pts incorrect equation for y

- 1 pts incorrect plot function

- 1 pts plot is missing

5.2 (b) **5 / 5**

✓ **- 0 pts** Correct

- 5 pts no answer

- 0 pts incorrect sigma and omega resulted in incorrect plots

- 2 pts missing plots

5.3 (c) **5 / 5**

✓ **- 0 pts** Correct

- 2 pts missing plots

- 1 pts either of the plots incorrect

- 5 pts no answer

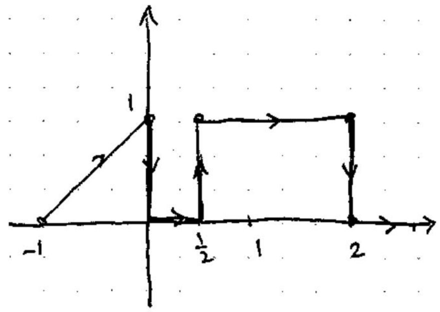
ECE 102 Fall 2020 HW #2

1\.

i) $y(t) = x(t)\left[1 - u(t) + u(2t-1)\right]$

$t: < \frac{1}{2} \quad u(2t-1) = 0$

$t \geq \frac{1}{2} \quad u(2t-1) = 1$



$t<0 \quad y(t) = x(t)$

$0 \leq t < \frac{1}{2} \quad y(t) = 0$

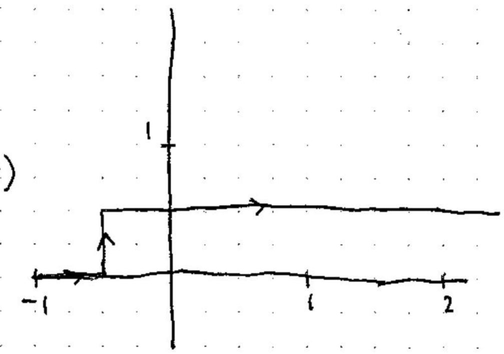
$\frac{1}{2} \leq t \quad y(t) = x(t)$

ii) $y(t) = \int_{-\infty}^{t} \delta(\tau + 0.5)\, x(\tau)\, d\tau$

if $\tau = -0.5 \quad \int = x(-0.5)$

if $t \geq -0.5 \quad y(t) = x(-0.5)$

else $\quad y(t) = 0.$

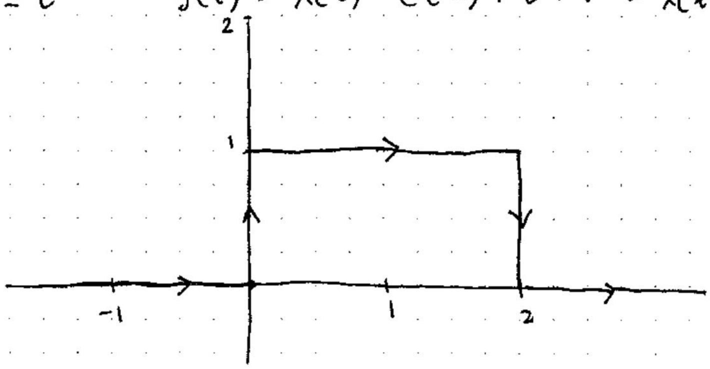


iii) $y(t) = x(t) - r(t+1) + r(t) + u(t)$

$t<-1 \quad y(t) = x(t) - 0 + 0 + 0 = x(t)$

$-1 \leq t < 0 \quad y(t) = x(t) - (t+1) + 0 + 0 = x(t) - t - 1 = t + 1 - t - 1 = 0$

$0 \leq t \quad y(t) = x(t) - (t+1) + t + 1 = x(t)$



1.1 (a)i. **3 / 3**

✓ **- 0 pts** Correct

- 2 pts wrong plot

- 2 pts no plot

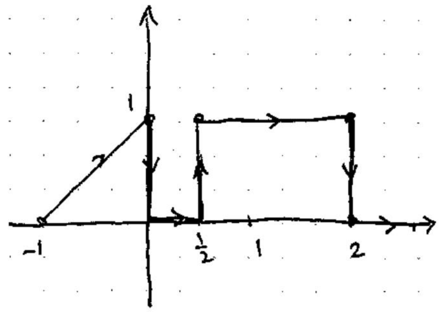
ECE 102 Fall 2020 HW #2

1\.

i) $y(t) = x(t)\left[1 - u(t) + u(2t-1)\right]$

$t < \frac{1}{2}$ $\quad u(2t-1) = 0$

$t \geq \frac{1}{2}$ $\quad u(2t-1) = 1$



$t<0 \quad y(t) = x(t)$

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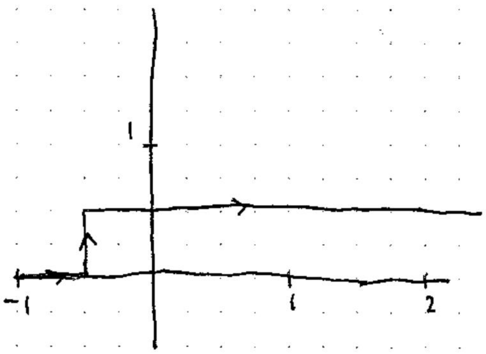
$\frac{1}{2} \leq t \quad y(t) = x(t)$

ii) $y(t) = \int_{-\infty}^{t} \delta(\tau + 0.5)\, x(\tau)\, d\tau$

if $\tau = -0.5$ $\quad \int = x(-0.5)$

if $t \geq -0.5$ $\quad y(t) = x(-0.5)$

else $\quad y(t) = 0.$

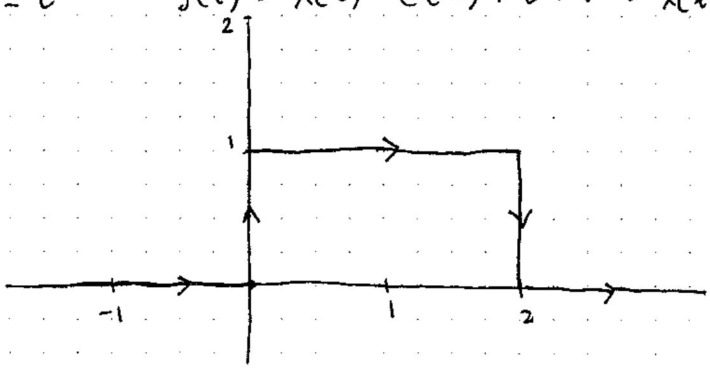


iii) $y(t) = x(t) - r(t+1) + r(t) + u(t)$

$t < -1 \quad y(t) = x(t) - 0 + 0 + 0 = x(t)$

$-1 \leq t < 0 \quad y(t) = x(t) - (t+1) + 0 + 0 = x(t) - t - 1 = t + 1 - t - 1 = 0$

$0 \leq t \quad y(t) = x(t) - (t+1) + t + 1 = x(t)$



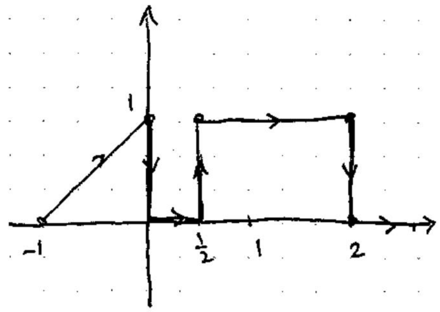
1.2 (a)ii. **3 / 3**

✓ **- 0 pts** Correct

- 2 pts wrong plot

ECE 102 Fall 2020 HW #2

1. i) $y(t) = x(t)\left[1 - u(t) + u(2t-1)\right]$



$t < \frac{1}{2}$ $\quad u(2t-1) = 0$

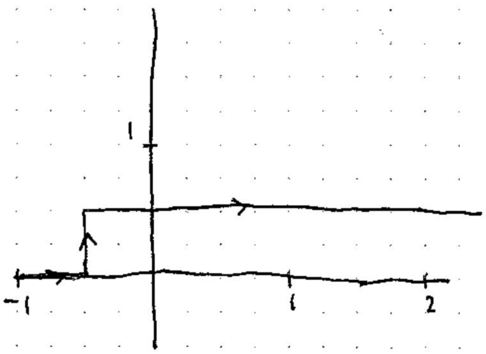
$t \geq \frac{1}{2}$ $\quad u(2t-1) = 1$

$t<0$ $\quad y(t) = x(t)$

$0 \leq t < \frac{1}{2}$ $\quad y(t) = 0$

$\frac{1}{2} \leq t$ $\quad y(t) = x(t)$

ii) $y(t) = \int_{-\infty}^{t} \delta(\tau + 0.5)\, x(\tau)\, d\tau$



if $\tau = -0.5$ $\quad \int = x(-0.5)$

if $t \geq -0.5$ $\quad y(t) = x(-0.5)$

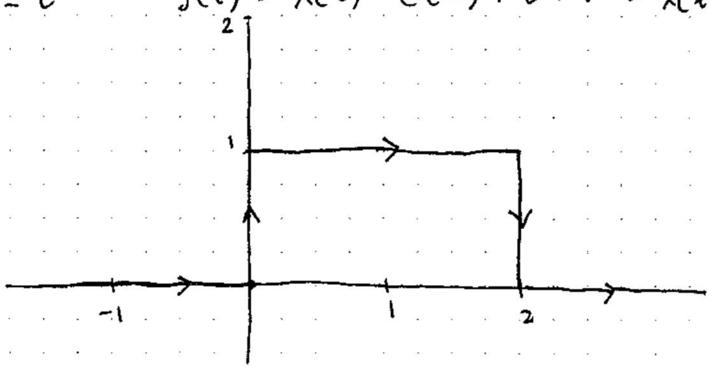
else $\quad y(t) = 0.$

iii) $y(t) = x(t) - r(t+1) + r(t) + u(t)$

$t<-1$ $\quad y(t) = x(t) - 0 + 0 + 0 = x(t)$

$-1 \leq t < 0$ $\quad y(t) = x(t) - (t+1) + 0 + 0 = x(t) - t - 1 = t+1-t-1 = 0$

$0 \leq t$ $\quad y(t) = x(t) - (t+1) + t + 1 = x(t)$



1.3 (a)iii. **3 / 3**

✓ **- 0 pts** Correct

- 2 pts wrong plot

(b) i. $\int_{-\infty}^{\infty} f(t+1)\cdot\delta(t+1)\,dt$

$= \cancel{\int_{-\infty}^{-1} f(t+1)\,\delta(t+1) + \int_{-1}^{\infty} f(t+1)\,\delta(t+1)\,dt}$

$= \boxed{f(0)}$

ii. $\int_{t}^{1} e^{-2\tau} u(\tau-1)\,d\tau + \int_{1}^{\infty} e^{-2\tau} u(\tau-1)\,d\tau$

if $t<1$ $\quad$ ii $= \int_{1}^{\infty} e^{-2\tau}\,d\tau$ $\Big\} = \int_{1}^{\infty} e^{-2\tau}\,d\tau = \frac{1}{-2}\cdot e^{-2\tau}\Big|_{1}^{\infty}$

$= -\frac{1}{2}\left(0-e^{-2}\right)$

$= \frac{1}{2}\cdot\frac{1}{e^{2}} = \boxed{\frac{1}{2e^{2}}}$

else $\quad$ ii $= \int_{t}^{\infty} e^{-2\tau}\,d\tau$

$= \frac{1}{-2} e^{-2\tau}\Big|_{t}^{\infty} = \frac{1}{-2}\left(-e^{-2t}\right)$

$= \boxed{\frac{e^{-2t}}{2}}$

iii) $\int_{0^-}^{0^+} f(t)\cdot(0+0+\delta(t))\,dt$

$+ \int_{0^+}^{\infty} f(t)\cdot(\delta(t-1)+0+0\,dt = f(0)+f(1)$

(c) $\delta(t) = \lim_{\Delta\to 0} \text{rect}_\Delta(t)$ $\quad$ ~~if Δ close to 0,~~

$\delta(bt) = \lim_{\Delta\to 0} \text{rect}_\Delta(bt)$

$\text{rect}_\Delta(bt) = \begin{cases} 1/\Delta & \text{if } |bt| \le \Delta/2 \\ 0 & \text{else} \end{cases}$

$b\cdot\text{rect}_\Delta(bt) = \begin{cases} b/\Delta & \text{if } |bt| \le \Delta/2 \\ 0 & \text{else} \end{cases}$

$= \begin{cases} b/\Delta & \text{if } |t| \le \Delta/2b \\ 0 & \text{else} \end{cases}$

if $\Delta/b = \Delta'$ $\quad = \begin{cases} 1/\Delta' & \text{if } |t| \le \Delta'/2 \\ 0 & \text{else} \end{cases}$

$= \text{rect}_\Delta(t)$

thus, $\text{rect}_\Delta(bt) = \frac{1}{b}\cdot\text{rect}_\Delta(t)$

so $\lim_{\Delta\to 0}(\;''\quad''\;) = \frac{1}{b}\lim_{\Delta\to 0}(\;''\quad''\;)$

so $\delta(bt) = \frac{1}{b}\,\delta(t)$ □

1.4 (b)i. **3 / 3**

✓ **- 0 pts** Correct

- 1 pts Wrong answer

(b) i. $\int_{-\infty}^{\infty} f(t+1)\cdot\delta(t+1)\,dt$

$= \cancel{\int_{-\infty}^{-1} f(t+1)\,\delta(t+1) + \int_{-1}^{\infty} f(t+1)\,\delta(t+1)\,dt}$

$= \boxed{f(0)}$

ii. $\int_{t}^{1} e^{-2\tau}u(\tau-1)\,d\tau + \int_{1}^{\infty} e^{-2\tau}u(\tau-1)\,d\tau$

$$\left.\begin{array}{lll} \text{if } t<1 & ii = & \int_1^\infty e^{-2\tau}\,d\tau \\ \text{else} & ii = & \int_t^\infty e^{-2\tau}\,d\tau \end{array}\right\}$$

$= \int_1^\infty e^{-2\tau}\,d\tau = \frac{1}{-2}\,e^{-2\tau}\Big|_1^\infty = -\frac{1}{2}\left(0-e^{-2}\right) = \frac{1}{2}\cdot\frac{1}{e^2} = \boxed{\frac{1}{2e^2}}$

$= \frac{1}{-2}\,e^{-2\tau}\Big|_t^\infty = \frac{1}{-2}\left(-e^{-2t}\right) = \boxed{\frac{e^{-2t}}{2}}$

iii) $\int_{0^-}^{0^+} f(t)\cdot(0+0+\delta(t))\,dt$

$+ \int_{0^+}^{\infty} f(t)\cdot(\delta(t-1)+0+0\;dt = f(0)+f(1)$

(c) $\delta(t) = \lim_{\Delta\to 0} \text{rect}_\Delta(t)$ ~~if Δ close to 0,~~

$\delta(bt) = \lim_{\Delta\to 0} \text{rect}_\Delta(bt)$

$\text{rect}_\Delta(bt) = \begin{cases} 1/\Delta & \text{if } |bt| \le \Delta/2 \\ 0 & \text{else} \end{cases}$

$b\cdot\text{rect}_\Delta(bt) = \begin{cases} b/\Delta & \text{if } |bt| \le \Delta/2 \\ 0 & \text{else} \end{cases}$

$= \begin{cases} b/\Delta & \text{if } |t| \le \Delta/2b \\ 0 & \text{else} \end{cases}$

if $\Delta/b = \Delta'$ $= \begin{cases} 1/\Delta' & \text{if } |t| \le \Delta'/2 \\ 0 & \text{else} \end{cases}$

$= \text{rect}_\Delta(t)$

thus, $\text{rect}_\Delta(bt) = \frac{1}{b}\cdot\text{rect}_\Delta(t)$

so $\lim_{\Delta\to 0}(\text{"}\ \text{"}) = \frac{1}{b}\lim_{\Delta\to 0}(\text{"}\ \text{"})$

so $\delta(bt) = \frac{1}{b}\,\delta(t)$ □

1.5 (b)ii. **3 / 3**

✓ **- 0 pts** Correct

- 1 pts wrong answer

(b) i. $\int_{-\infty}^{\infty} f(t+1) \cdot \delta(t+1)\, dt$

$= \cancel{\int_{-\infty}^{-1} f(t+1)\,\delta(t+1) + \int_{-1}^{\infty} f(t+1)\,\delta(t+1)\,dt}$

$= \boxed{f(0)}$

ii. $\int_{t}^{1} e^{-2\tau} u(\tau-1)\, d\tau + \int_{1}^{\infty} e^{-2\tau} u(\tau-1)\, d\tau$

if $t<1$ $\quad$ ii $= \int_{1}^{\infty} e^{-2\tau}\, d\tau$ $\quad = \int_{1}^{\infty} e^{-2\tau} d\tau = \frac{1}{-2} \cdot e^{-2\tau}\Big|_{1}^{\infty}$

else $\quad$ ii $= \int_{t}^{\infty} e^{-2\tau}\, d\tau$

$= \frac{1}{-2} e^{-2\tau}\Big|_{t}^{\infty} = \frac{1}{-2}\left(-e^{-2t}\right) = \boxed{\frac{e^{-2t}}{2}}$

$= -\frac{1}{2}\left(0 - e^{-2}\right) = \frac{1}{2} \cdot \frac{1}{e^2} = \boxed{\frac{1}{2e^2}}$

iii) $\int_{0^-}^{0^+} f(t) \cdot (0 + 0 + \delta(t))\, dt$

$+ \int_{0^+}^{\infty} f(t) \cdot (\delta(t-1) + 0 + 0\, dt = f(0) + f(1)$

(c) $\delta(t) = \lim_{\Delta \to 0} \text{rect}_\Delta(t)$ $\quad$ ~~if Δ close to 0,~~

$\delta(bt) = \lim_{\Delta \to 0} \text{rect}_\Delta(bt)$

$\text{rect}_\Delta(bt) = \begin{cases} 1/\Delta & \text{if } |bt| \le \Delta/2 \\ 0 & \text{else} \end{cases}$

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if $\Delta/b = \Delta'$ $\quad = \begin{cases} 1/\Delta' & \text{if } |t| \le \Delta'/2 \\ 0 & \text{else} \end{cases}$

$= \text{rect}_\Delta(t)$

thus, $\text{rect}_\Delta(bt) = \frac{1}{b} \cdot \text{rect}_\Delta(t)$

so $\lim_{\Delta\to 0}(\text{" "}) = \frac{1}{b}\lim_{\Delta \to 0}(\text{" "})$

so $\delta(bt) = \frac{1}{b}\,\delta(t)$ $\square$

1.6 (b)iii. **3 / 3**

✓ **- 0 pts** Correct

- 1 pts inaccurate answer

(b) i. $\int_{-\infty}^{\infty} f(t+1)\cdot\delta(t+1)\,dt$

$= \cancel{\int_{-\infty}^{-1} f(t+1)\,\delta(t+1) + \int_{-1}^{\infty} f(t+1)\,\delta(t+1)\,dt}$

$= \boxed{f(0)}$

ii. $\int_{t}^{1} e^{-2\tau} u(\tau-1)\,d\tau + \int_{1}^{\infty} e^{-2\tau} u(\tau-1)\,d\tau$

if $t<1$ $\quad ii = \int_{1}^{\infty} e^{-2\tau}\,d\tau$ $\;\Big\} = \int_{1}^{\infty} e^{-2\tau}\,d\tau = \frac{1}{-2}\cdot e^{-2\tau}\Big|_{1}^{\infty}$

$= -\frac{1}{2}\left(0-e^{-2}\right)$

$= \frac{1}{2}\cdot\frac{1}{e^{2}} = \boxed{\frac{1}{2e^{2}}}$

else $\quad ii = \int_{t}^{\infty} e^{-2\tau}\,d\tau$

$= \frac{1}{-2} e^{-2\tau}\Big|_{t}^{\infty} = \frac{1}{-2}\left(-e^{-2t}\right)$

$= \boxed{\frac{e^{-2t}}{2}}$

iii) $\int_{0^-}^{0^+} f(t)\cdot(0+0+\delta(t))\,dt$

$+ \int_{0^+}^{\infty} f(t)\cdot(\delta(t-1)+0+0\,dt = f(0)+f(1)$

(c) $\delta(t) = \lim_{\Delta\to 0} rect_\Delta(t)$ $\quad$ ~~if Δ close to 0,~~

$\delta(bt) = \lim_{\Delta\to 0} rect_\Delta(bt)$

$rect_\Delta(bt) = \begin{cases} 1/\Delta & \text{if } |bt|\le\Delta/2 \\ 0 & \text{else}\end{cases}$

$b\cdot rect_\Delta(bt) = \begin{cases} b/\Delta & \text{if } |bt|\le\Delta/2 \\ 0 & \text{else}\end{cases}$

$= \begin{cases} b/\Delta & \text{if } |t|\le \Delta/2b \\ 0 & \text{else}\end{cases}$

if $\Delta/b = \Delta'$ $\quad = \begin{cases} 1/\Delta' & \text{if } |t|\le\Delta'/2 \\ 0 & \text{else}\end{cases}$

$= rect_\Delta(t)$

thus, $rect_\Delta(bt) = \frac{1}{b}\cdot rect_\Delta(t)$

so $\lim_{\Delta\to 0}(\text{" "}) = \frac{1}{b}\lim_{\Delta\to 0}(\text{" "})$

so $\delta(bt) = \frac{1}{b}\,\delta(t)$ $\square$

1.7 (c) 4 / 4

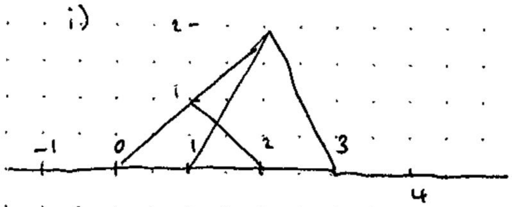
✓ **- 0 pts** Correct

- 1 pts wrong answer

m

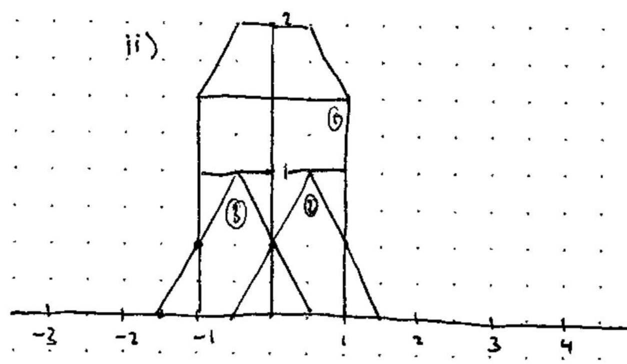
2. a)

i)



$x(t) = \Delta(t-1) + 2\Delta(t-2)$

ii)



$x(t) = \Delta(t-0.5) + \Delta(t+0.5) + \mathrm{rect}(\frac{t}{2})$

iii) $x(t) = \Delta(2t) + 2\Delta(2t-2) + 2\Delta(2t-3) + 2\Delta(2t+2) + 2\Delta(2t+3)$

(b) i) $x(t)$: $\longrightarrow$ (b) i) $x(t) = u(t+2) + u(t+1) + u(t+\frac{1}{2}) - u(t-\frac{1}{2}) - u(t-1) - u(t-2)$

ii) $x(t) = u(t+2) + u(t+1) - 3u(t-1.5) + u(t-3)$

3. a) i. $y(t) = |x(t)| + x(2t)$ $\boxed{no}$ time invariant $y(t+\tau) = |x(t+\tau)| + x(2t+2\tau)$

$= |x(t+\tau)| + x((t+\tau)2)$

$\neq x(2t+\tau)$

$\boxed{no}$ causal. each term $|x(2t)|$ ~~and~~ in future is present.

Assume $|x(t)| \leq M_x$ for all t, $x(2t) \leq |x(2t)| \leq M_x$

$y(t) \leq M_x + M_x$ Bounded, so $\boxed{yes}$

ii. $y(t) = \int_{t-\tau}^{t+\tau} x(\lambda)\, d\lambda$ $y(t+m) = \int_{t-\tau+m}^{t+\tau+m} x(\lambda)\, d\lambda = \int_{t-\tau}^{t+\tau} x(\lambda+m)\, d\lambda$

$\boxed{yes}$ time invariance

$\boxed{not}$ causal uses $x(t+\tau)$

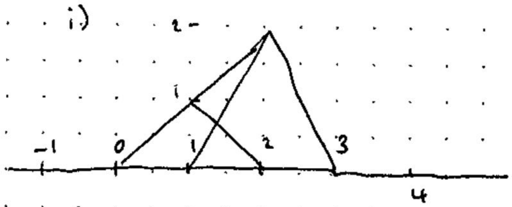
$\boxed{yes}$ stable if x bounded, area under graph bounded b/c τ finite.

2.1 (a) **15 / 15**

- ✓ **- 0 pts** Correct
- **- 1 pts** i. wrong item
- **- 1 pts** i. wrong scale
- **- 5 pts** ii. no answer
- **- 5 pts** ii. no answer
- **- 1 pts** ii. wrong item
- **- 1 pts** ii. not simplified answer
- **- 1 pts** ii. wrong scale
- **- 1 pts** ii. wrong item
- **- 1 pts** ii. wrong shift
- **- 5 pts** iii. no answer
- **- 1 pts** iii. wrong scale
- **- 2 pts** iii. wrong answer
- **- 1 pts** iii. wrong item
- **- 1 pts** iii. wrong shift
- **- 1 pts** ii. misuse of annotations.
- **- 1 pts** i. misuse of annotations.
- **- 1 pts** iii. misuse of annotations.

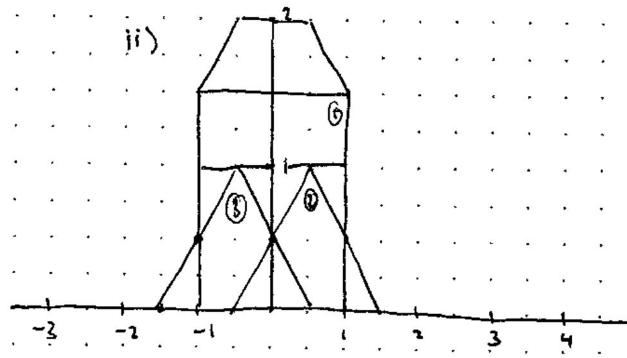
2. a)

i)



$x(t) = \Delta(t-1) + 2\Delta(t-2)$

ii)



$x(t) = \Delta(t-0.5) + \Delta(t+0.5) + \text{rect}(\frac{t}{2})$

iii) $x(t) = \Delta(2t) + 2\Delta(2t-2) + 2\Delta(2t-3) + 2\Delta(2t+2) + 2\Delta(2t+3)$

(b) i) $x(t)$: ⟶ (b) i) $x(t) = u(t+2) + 2u(t+1) + u(t+1/2) - u(t-1/2) - u(t-1) - u(t-2)$

ii) $x(t) = u(t+2) + u(t+1) - 3u(t-1.5) + u(t-3)$

3. a) i. $y(t) = |x(t)| + x(2t)$ $\boxed{no}$ time invariant $y(t+\tau) = |x(t+\tau)| + x(2t+2\tau)$ $= |x(\tau+\tau)| + x((t+\tau)2)$ $\neq x(2t+\tau)$

$\boxed{no}$ causal. each term $|x(2t)|$ ~~and~~ in future is present.

Assume $|x(t)| \leq M_x$ for all t, $x(2t) \leq |x(2t)| \leq M_x$

$y(t) \leq M_x + M_x$ Bounded, so $\boxed{yes}$

ii. $y(t) = \int_{t-\tau}^{t+\tau} x(\lambda)\, d\lambda$ $y(t+m) = \int_{t-\tau+m}^{t+\tau+m} x(\lambda)\, d\lambda = \int_{t-\tau}^{t+\tau} x(\lambda+m)\, d\lambda$

$\boxed{yes}$ time invariance

$\boxed{not}$ causal uses $x(t+\tau)$

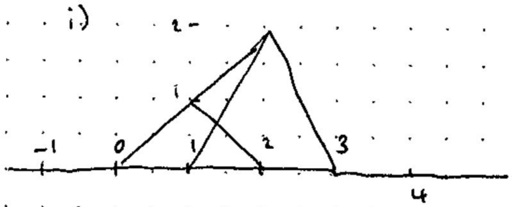
$\boxed{yes}$ stable if x bounded, area under graph bounded b/c τ finite.

2.2 (b) **8 / 8**

- ✓ **- 0 pts** Correct
- **- 1 pts** i. wrong shift
- **- 4 pts** i. wrong answer
- **- 1 pts** ii. wrong shift
- **- 1 pts** ii. wrong scale
- **- 1 pts** ii. wrong item
- **- 4 pts** ii. no answer
- **- 4 pts** i. not use unit-step functions
- **- 4 pts** ii. not use unit-step functions
- **- 1 pts** ii. wrong shift
- **- 4 pts** i. ii. wrong annotations.
- **- 1 pts** i. wrong item

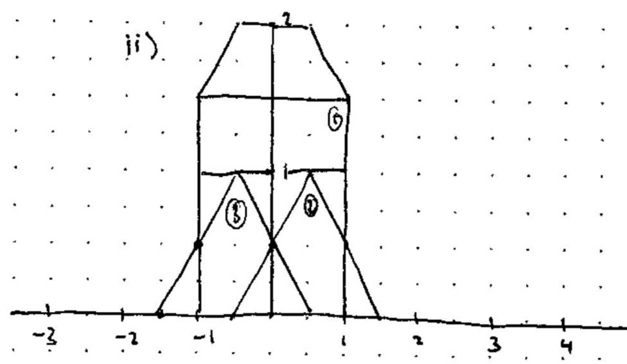
2. a)

i)



$x(t) = \Delta(t-1) + 2\Delta(t-2)$

ii)



$x(t) = \Delta(t-0.5) + \Delta(t+0.5) + \text{rect}(\frac{t}{2})$

iii) $x(t) = \Delta(2t) + 2\Delta(2t-2) + 2\Delta(2t-3) + 2\Delta(2t+2) + 2\Delta(2t+3)$

(b) i) $x(t)$: $\longrightarrow$ (b) i) $x(t) = u(t+2) + 2u(t+1) + u(t+\frac{1}{2}) - u(t-\frac{1}{2}) - u(t-1) - u(t-2)$

ii) $x(t) = u(t+2) + u(t+1) - 3u(t-1.5) + u(t-3)$

3. a) i. $y(t) = |x(t)| + x(2t)$ $\boxed{\text{no}}$ time invarient $y(t+\tau) = |x(t+\tau)| + x(2t+2\tau)$
$= |x(t+\tau)| + x((t+\tau)2)$
$\neq x(2t+\tau)$

$\boxed{\text{no}}$ causal. each term $|x(2t)|$ ~~and~~ in future is present.

Assume $|x(t)| \leq M_x$ for all t, $x(2t) \leq |x(2t)| \leq M_x$

$y(t) \leq M_x + M_x$ Bounded, so $\boxed{\text{yes}}$

ii. $y(t) = \int_{t-\tau}^{t+\tau} x(\lambda)\, d\lambda$ $\quad y(t+m) = \int_{t-\tau+m}^{t+\tau+m} x(\lambda)\, d\lambda = \int_{t-\tau}^{t+\tau} x(\lambda+m)\, d\lambda$

$\boxed{\text{yes}}$ time invariance

$\boxed{\text{not}}$ causal uses $x(t+\tau)$

$\boxed{\text{yes}}$ stable if x bounded, area under graph bounded b/c τ finite.

3.1 (a)i. **4 / 4**

✓ **- 0 pts** **Correct (Non-linear, time-variant, not causal, stable)**

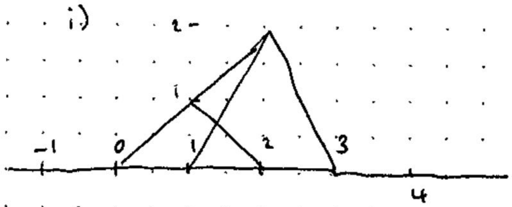
- 1 pts wrong answer

- 1 pts no conclusion

- 1 pts incomplete answer

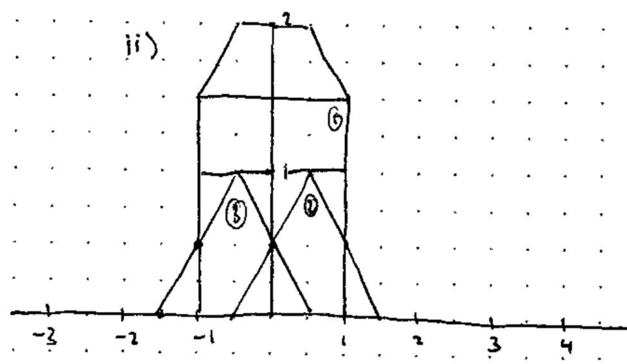
2. a)

i)



$x(t) = \Delta(t-1) + 2\Delta(t-2)$

ii)



$x(t) = \Delta(t-0.5) + \Delta(t+0.5) + \text{rect}(\frac{t}{2})$

iii) $x(t) = \Delta(2t) + 2\Delta(2t-2) + 2\Delta(2t-3) + 2\Delta(2t+2) + 2\Delta(2t+3)$

(b) i) $x(t)$:

(b) i) $x(t) = u(t+2) + 2u(t+1) + u(t+\frac{1}{2}) - u(t-\frac{1}{2}) - u(t-1) - u(t-2)$

ii) $x(t) = u(t+2) + u(t+1) - 3u(t-1.5) + u(t-3)$

3. a) i. $y(t) = |x(t)| + x(2t)$ $\boxed{\text{no}}$ time invariant $y(t+\tau) = |x(t+\tau)| + x(2t+2\tau)$
$= |x(t+\tau)| + x((t+\tau)2)$
$\neq x(2t+\tau)$

$\boxed{\text{no}}$ causal. each term $|x(2t)|$ ~~and~~ in future is present.

Assume $|x(t)| \leq M_x$ for all t, $x(2t) \leq |x(2t)| \leq M_x$

$y(t) \leq M_x + M_x$ Bounded, so $\boxed{\text{yes}}$

ii. $y(t) = \int_{t-\tau}^{t+\tau} x(\lambda)\, d\lambda$ $y(t+m) = \int_{t-\tau+m}^{t+\tau+m} x(\lambda)\, d\lambda = \int_{t-\tau}^{t+\tau} x(\lambda+m)\, d\lambda$

$\boxed{\text{yes}}$ time invariance

$\boxed{\text{not}}$ causal uses $x(t+\tau)$

$\boxed{\text{yes}}$ stable if x bounded, area under graph bounded b/c τ finite.

3.2 (a)ii. **4 / 4**

✓ **- 0 pts** **Correct (linear, time-invariant, not causal, stable)**

- 1 pts wrong answer

- 1 pts incomplete answer

iii) $y(t) = (t+1)\int_{-\infty}^{t} x(\lambda)\,d\lambda$

time-invariant: $\boxed{\text{no.}}$ b/c $y(t+\tau) = (t+\tau+1)\int_{-\infty}^{t+\tau} x(\lambda)\,d\lambda$

shift: $(t+1)\int_{-\infty}^{t} x(\lambda+\tau)\,d\lambda$

Causal: $\boxed{\text{yes}}$ b/c $\int_{-\infty}^{t}$ depends on past & present.

stable: $|x(t)| \leq M_x$ if $x(t) = 1$ then $\int = \infty$

so $\boxed{\text{no}}$

iv) $y(t) = 1 + x(t)\cos(\omega t)$

time-invariant: $\boxed{\text{no}}$ b/c $y(t+\tau)$: $1 + x(t+\tau)\cdot\cos(\omega t + \omega\tau)$

and shift: $1 + x(t+\tau)\cdot\cos(\omega t)$

Causal: $\boxed{\text{yes}}$ b/c $x(t)$ and $\cos(\omega t)$ are present.

stable: $|x(t)| \leq M_x$ $|\cos(\omega t)| \leq 1$

so $y(t) \leq 1 + M_x \cdot 1 = M_y$

$\boxed{\text{yes}}$

v) $y(t) = \dfrac{1}{1+x^2(t)}$

time-invariant: $\boxed{\text{yes}}$ $y(t) = \dfrac{1}{1+x^2(t)}$ | $y(t+\tau) = \dfrac{1}{1+x^2(t+\tau)}$

Causal: $\boxed{\text{yes}}$ $x(t)$ = present.

Stable: $\boxed{\text{yes}}$ b/c $|x(t)| \leq M_x$

min of $x(t)^2 = 0$ and $\dfrac{1}{1+0} = 1$

so y is bounded.

3.3 (a)iii. **4 / 4**

✓ **- 0 pts** **Correct (linear, time variant, causal, unstable)**

- 1 pts incomplete answer

- 1 pts wrong answer

- 4 pts no answer

iii) $y(t) = (t+1) \int_{-\infty}^{t} x(\lambda)\, d\lambda$

time-invariant: $\boxed{\text{no.}}$ b/c $y(t+\tau) = (t+\tau+1) \int_{-\infty}^{t+\tau} x(\lambda)\, d\lambda$

shift: $(t+1) \int_{-\infty}^{t} x(\lambda+\tau)\, d\lambda$

Causal: $\boxed{\text{yes}}$ b/c $\int_{-\infty}^{t}$ depends on past & present.

stable: $|x(t)| \leq M_x$ if $x(t) = 1$ then $\int = \infty$

so $\boxed{\text{no}}$

iv) $y(t) = 1 + x(t) \cos(\omega t)$

time-invariant: $\boxed{\text{no}}$ b/c $y(t+\tau): 1 + x(t+\tau) \cdot \cos(\omega t + \omega \tau)$

and shift: $1 + x(t+\tau) \cdot \cos(\omega t)$

Causal: $\boxed{\text{yes}}$ b/c $x(t)$ and $\cos(\omega t)$ are present.

stable: $|x(t)| \leq M_x$ $|\cos(\omega t)| \leq 1$

so $y(t) \leq 1 + M_x \cdot 1 = M_y$

$\boxed{\text{yes}}$

v) $y(t) = \frac{1}{1+x^2(t)}$

time-invariant: $\boxed{\text{yes}}$ $y(t) = \frac{1}{1+x^2(t)}$ | $y(t+\tau) = \frac{1}{1+x^2(t+\tau)}$

Causal: $\boxed{\text{yes}}$ $x(t)$ = present.

Stable: $\boxed{\text{yes}}$ b/c $|x(t)| \leq M_x$

min of $x(t)^2 = 0$ and $\frac{1}{1+0} = 1$

so y is bounded.

3.4 (a)iv. 4 / 4

- ✓ **- 0 pts** **Correct (time-variant, causal, stable)**
- **- 1 pts** incomplete answer
- **- 1 pts** wrong answer
- **- 4 pts** no answer

iii) $y(t) = (t+1) \int_{-\infty}^{t} x(\lambda)\, d\lambda$

time-invariant: $\boxed{\text{no.}}$ b/c $y(t+\tau) = (t+\tau+1) \int_{-\infty}^{t+\tau} x(\lambda)\, d\lambda$

shift: $(t+1) \int_{-\infty}^{t} x(\lambda+\tau)\, d\lambda$

Causal: $\boxed{\text{yes}}$ b/c $\int_{-\infty}^{t}$ depends on past & present.

stable: $|x(t)| \leq M_x$ if $x(t) = 1$ then $\int = \infty$

so $\boxed{\text{no}}$

iv) $y(t) = 1 + x(t) \cos(\omega t)$

time-invariant: $\boxed{\text{no}}$ b/c $y(t+\tau): 1 + x(t+\tau) \cdot \cos(\omega t + \omega \tau)$

and shift: $1 + x(t+\tau) \cdot \cos(\omega t)$

Causal: $\boxed{\text{yes}}$ b/c $x(t)$ and $\cos(\omega t)$ are present.

stable: $|x(t)| \leq M_x$ $|\cos(\omega t)| \leq 1$

so $y(t) \leq 1 + M_x \cdot 1 = M_y$

$\boxed{\text{yes}}$

v) $y(t) = \frac{1}{1+x^2(t)}$

time-invariant: $\boxed{\text{yes}}$ $y(t) = \frac{1}{1+x^2(t)}$ | $y(t+\tau) = \frac{1}{1+x^2(t+\tau)}$

Causal: $\boxed{\text{yes}}$ $x(t) =$ present.

Stable: $\boxed{\text{yes}}$ b/c $|x(t)| \leq M_x$

min of $x(t)^2 = 0$ and $\frac{1}{1+0} = 1$

so y is bounded.

3.5 (a)v. **4 / 4**

✓ **- 0 pts** **Correct (time invariant, causal, stable)**

- 1 pts incomplete answer

- 1 pts wrong answer

- 4 pts no answer

(b)

$$z(t) = \int_{-\infty}^{t} x(3\tau)\, d(\tau)$$

$$= \int_{-\infty}^{t} x(3\tau)\, d\tau$$

$$= 3 \int_{-\infty}^{3t} x(u)\, d\frac{u}{3}$$

$$= \frac{1}{3} \int_{-\infty}^{3t} x(u)\, du$$

$$y(t) = \boxed{3 \cdot z\left(\frac{t-4}{3}\right)}$$

(c) i. $ax_1(t) \to ay_1(t) \to ax_1(t)$

$ax_2(t) \to by_2(t) \to bx_2(t)$

$w(t) = ax_1(t) + bx_2(t)$

ii. $x(t) = x_1(t-\tau)$

$w(t) = \cancel{}\ x_1(t-\tau)$

iii From (i) it is linear
and (ii) it is time - invariant :

Thus, System B is LTI.

3.6 (b) **6 / 6**

✓ **- 0 pts** **Correct**

- 1 pts wrong scale

- 6 pts no answer

- 1 pts wrong shift

- 1 pts not simplified answer

- 0.5 pts wrong annotations

(b) $z(t) = \int_{-\infty}^{t} x(3\tau)\, d(\tau)$

$$= \int_{-\infty}^{t} x(3\tau)\, d\tau$$

$$= 3 \int_{-\infty}^{3t} x(u)\, d\frac{u}{3}$$

$$= \frac{1}{3} \int_{-\infty}^{3t} x(u)\, du$$

$$y(t) = \boxed{3 \cdot z\left(\frac{t-4}{3}\right)}$$

(c) i. $a x_1(t) \to a y_1(t) \to a x_1(t)$

$a x_2(t) \to b y_2(t) \to b x_2(t)$

$w(t) = a x_1(t) + b x_2(t)$

ii. $x(t) = x_1(t-\tau)$

$w(t) = \cancel{\text{xx}} \quad x_1(t-\tau)$

iii From (i) it is linear

and (ii) it is time - invariant :

Thus, System B is LTI.

3.7 (c)i. **2 / 2**

- ✓ **- 0 pts** Correct
- **- 1 pts** wrong answer
- **- 0.5 pts** no final answer
- **- 2 pts** no answer
- **- 1 pts** insufficient answer

(b)

$$z(t) = \int_{-\infty}^{t} x(3\tau)\, d(\tau)$$

$$= \int_{-\infty}^{t} x(3\tau)\, d\tau$$

$$= 3 \int_{-\infty}^{3t} x(u)\, d\frac{u}{3}$$

$$= \frac{1}{3}\int_{-\infty}^{3t} x(u)\, du$$

$$y(t) = \boxed{3 \cdot z\left(\frac{t-4}{3}\right)}$$

(c) i. $a x_1(t) \to a y_1(t) \to a x_1(t)$

$a x_2(t) \to b y_2(t) \to b x_2(t)$

$w(t) = a x_1(t) + b x_2(t)$

ii. $x(t) = x_1(t-\tau)$

$w(t) = x_1(t-\tau)$

iii From (i) it is linear

and (ii) it is time - invariant :

Thus, System B is LTI.

3.8 (c)ii. **1 / 1**

✓ **- 0 pts** Correct

- 1 pts wrong answer

- 1 pts no answer

- 1 pts insufficient answer

- 0.5 pts no final answer

(b)

$$z(t) = \int_{-\infty}^{t} x(3\tau)\, d(\tau)$$

$$= \int_{-\infty}^{t} x(3\tau)\, d\tau$$

$$= 3 \int_{-\infty}^{3t} x(u)\, d\frac{u}{3}$$

$$= \frac{1}{3} \int_{-\infty}^{3t} x(u)\, du$$

$$y(t) = \boxed{3 \cdot z\left(\frac{t-4}{3}\right)}$$

(c) i. $ax_1(t) \to ay_1(t) \to ax_1(t)$

$ax_2(t) \to by_2(t) \to bx_2(t)$

$$w(t) = ax_1(t) + bx_2(t)$$

ii.

$$x(t) = x_1(t-\tau)$$

$$w(t) = x_1(t-\tau)$$

iii From (i) it is linear
and (ii) it is time - invariant :

Thus, System B is LTI.

3.9 (c)iii. **1 / 1**

- ✓ **- 0 pts** Correct
- **- 1 pts** wrong answer
- **- 0.5 pts** insufficient answer
- **- 1 pts** no answer

4. (a)

$$E = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$$

$$= \int_{-\infty}^{\infty} \left| Ae^{j\omega t} + Be^{-j\omega t} \right|^2 dt$$

$$= \int_{-\infty}^{\infty} \left(Ae^{j\omega t} + Be^{-j\omega t}\right)\left(Ae^{-j\omega t} + Be^{j\omega t}\right) dt$$

$$= \int_{-\infty}^{\infty} A^2 + B^2 + ABe^{2j\omega t} + ABe^{-2j\omega t} \quad dt$$

$$= \left(A^2 + B^2\right) t \Big|_{-\infty}^{\infty} + AB\int_{-\infty}^{\infty} e^{2j\omega t} \, dt + AB\int_{-\infty}^{\infty} e^{-2j\omega t} \, dt$$

$= \infty$ NOT ENERGY

$$P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \, dt$$

$$= \lim_{T\to\infty} \frac{1}{2T} \left(\int_{-T}^{T} A^2 + B^2 + ABe^{2j\omega t} + ABe^{-2j\omega t} \, dt \right)$$

$$= \lim_{T\to\infty} \frac{1}{2T} \left(\left(A^2+B^2\right)(2T) + \cancel{AB\left[\frac{1}{2j\omega}\left(e^{2j\omega T} - e^{-2j\omega T}\right) + \frac{1}{2j\omega}\left(e^{-2j\omega T} - e^{2j\omega T}\right)\right]} \right)$$

$$= \lim_{T\to\infty} \frac{1}{\cancel{2T}} \left(A^2+B^2\right) = \text{constant}$$

$$\boxed{P = A^2 + B^2}$$ POWER :)

4.1 (a) **5 / 5**

- ✓ **- 0 pts** Correct
- **- 1 pts** error in magnitude squared computation
- **- 1 pts** error in taking the limit
- **- 0.5 pts** no power value
- **- 1 pts** arithmetic error
- **- 5 pts** no answer

(b)

$$E = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$$

$$= \int_{-\infty}^{\infty} e^{-2(1+0)t} \, u(t-1) \, dt$$

$$= \int_{1}^{\infty} e^{-2(1+0)t} \, dt$$

$$= \frac{-1}{2(1+0)} \; e^{-2(1+j\omega)t} \Big|_{1}^{\infty}$$

$$= \frac{-1}{2(1+\ )} \left[0 - e^{-2(1+j\omega)} \right]$$

$$= \boxed{\frac{e^{-2(1+0)}}{2(1+0)}} = \boxed{\frac{e^{-2}}{2}}$$

$P = 0$ $\boxed{\text{NOT A POWER SIGNAL}}$

4.2 (b) **5 / 5**

✓ **- 0 pts** Correct

- 5 pts no answer

- 0.5 pts arithmetic error

- 1 pts error in taking the limit

- 2 pts incorrect energy value

- 0.5 pts copied the question wrong

- 1 pts error in the magnitude squared value

- 1 pts not a power signal

- 2.5 pts partially correct

Problem 5:

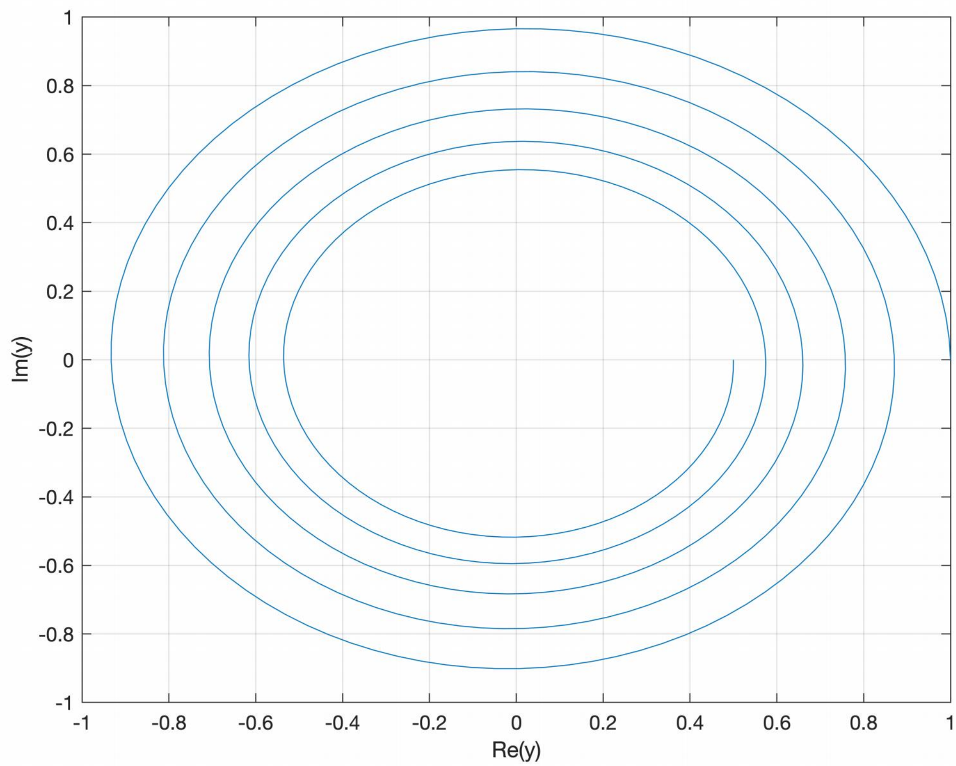
(a) Complex sinusoid has period 2.

T = 2 and T = 2pi/omega.

Thus, omega = pi.

Half-life = 10s, so sigma = 1/10 * ln(0.5)

```
sigma = 1/10 * log(0.5);
omega = pi;
t = linspace(0, 10, 500);
y = exp((sigma + j * omega) * t);
plot(y)
grid on;
xlabel("Re(y)");
ylabel("Im(y)");
```



MATLAB plots Real(y) on the x axis and Imag(y) on the y axis. Since each point represents a complex number on the coordinate plane, omega * t is the phase angle and e^(sigma * t) is the radius.

(b)

```
r = real(y);
i = imag(y);
subplot(2, 1, 1);
plot(t, r);
```

5.1 (a) **5 / 5**

- ✓ **- 0 pts** Correct
- **- 5 pts** no answer
- **- 2 pts** incorrect sigma and plot
- **- 1 pts** no omega and sigma values
- **- 0.5 pts** incorrect sigma
- **- 0.5 pts** no or incorrect interpretation of results
- **- 1 pts** incorrect equation for y
- **- 1 pts** incorrect plot function
- **- 1 pts** plot is missing

Problem 5:

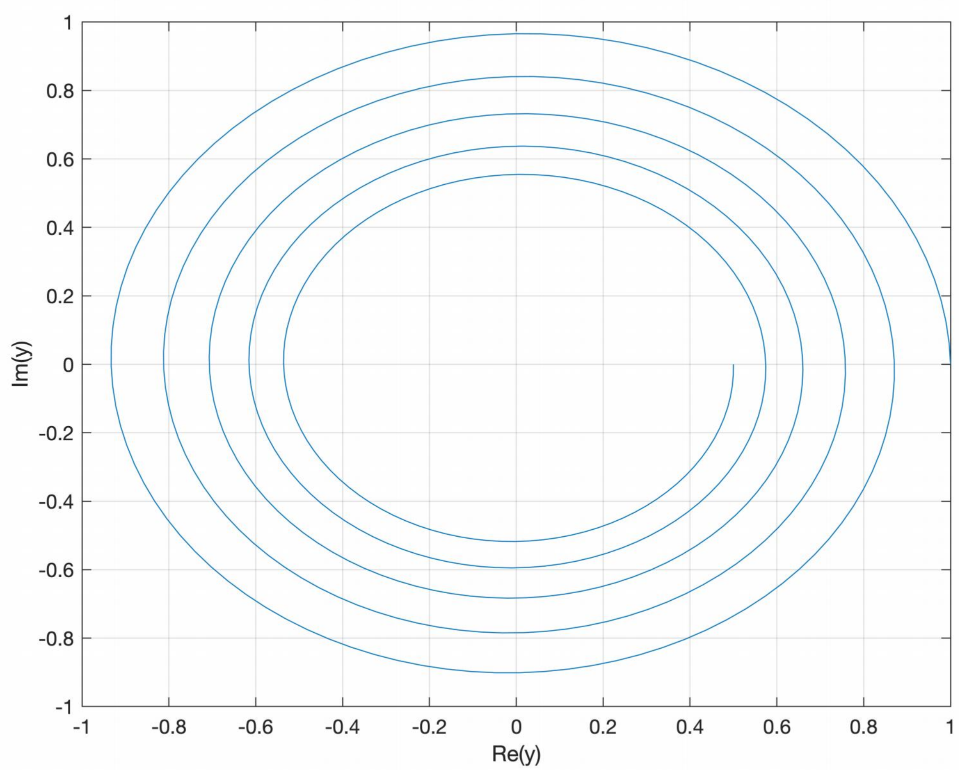
(a) Complex sinusoid has period 2.

T = 2 and T = 2pi/omega.

Thus, omega = pi.

Half-life = 10s, so sigma = 1/10 * ln(0.5)

```
sigma = 1/10 * log(0.5);
omega = pi;
t = linspace(0, 10, 500);
y = exp((sigma + j * omega) * t);
plot(y)
grid on;
xlabel("Re(y)");
ylabel("Im(y)");
```

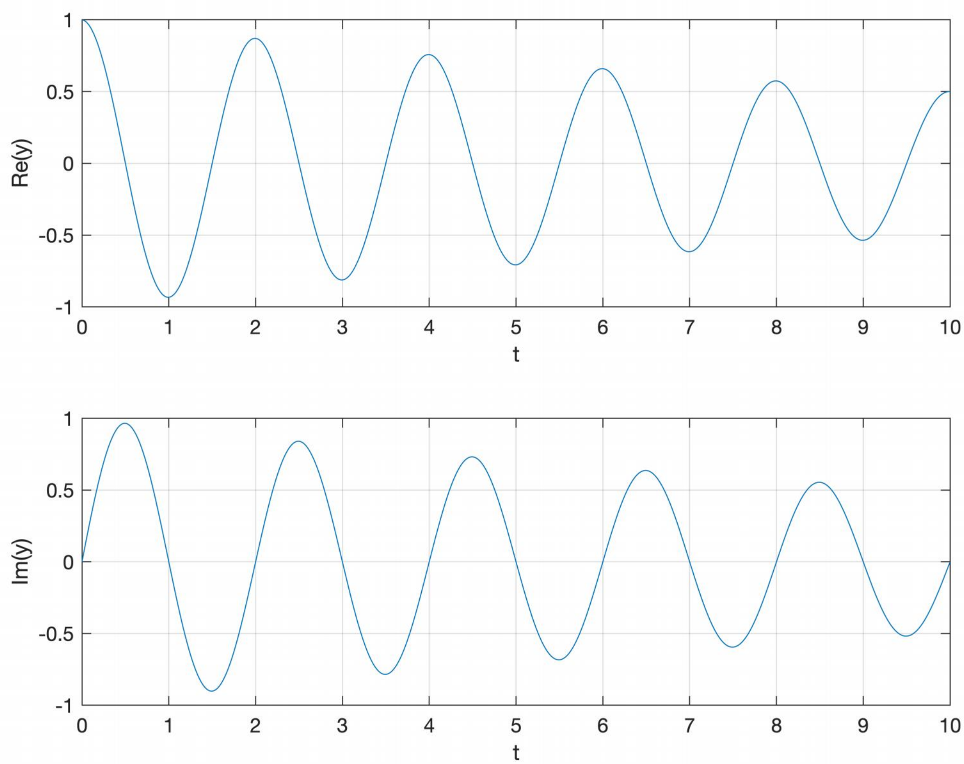


MATLAB plots Real(y) on the x axis and Imag(y) on the y axis. Since each point represents a complex number on the coordinate plane, omega * t is the phase angle and e^(sigma * t) is the radius.

(b)

```
r = real(y);
i = imag(y);
subplot(2, 1, 1);
plot(t, r);
```

```
grid on;
xlabel('t');
ylabel('Re(y)')
subplot(2, 1, 2);
plot(t, i);
grid on;
xlabel('t');
ylabel('Im(y)');
```



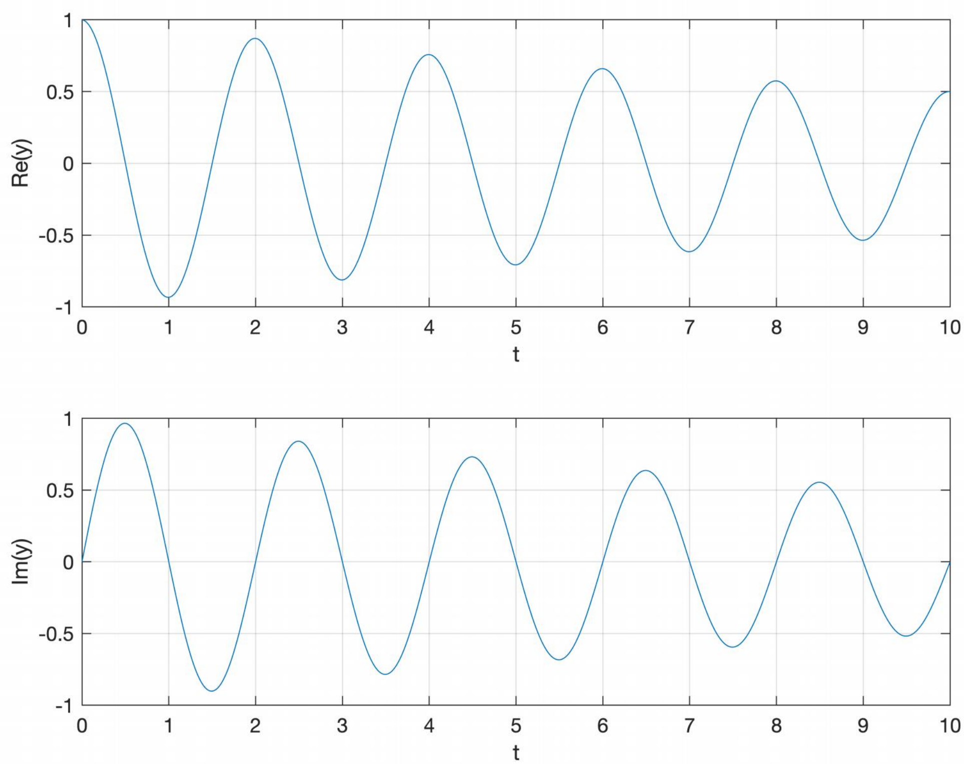
(c)

```
subplot(1, 1, 1);
magnitude = abs(y);
phase = angle(y)/(2 * pi);
plot(t, phase);
hold on;
plot(t, magnitude);
grid on;
ylabel("magnitude/phase");
xlabel("t");
```

5.2 (b) **5 / 5**

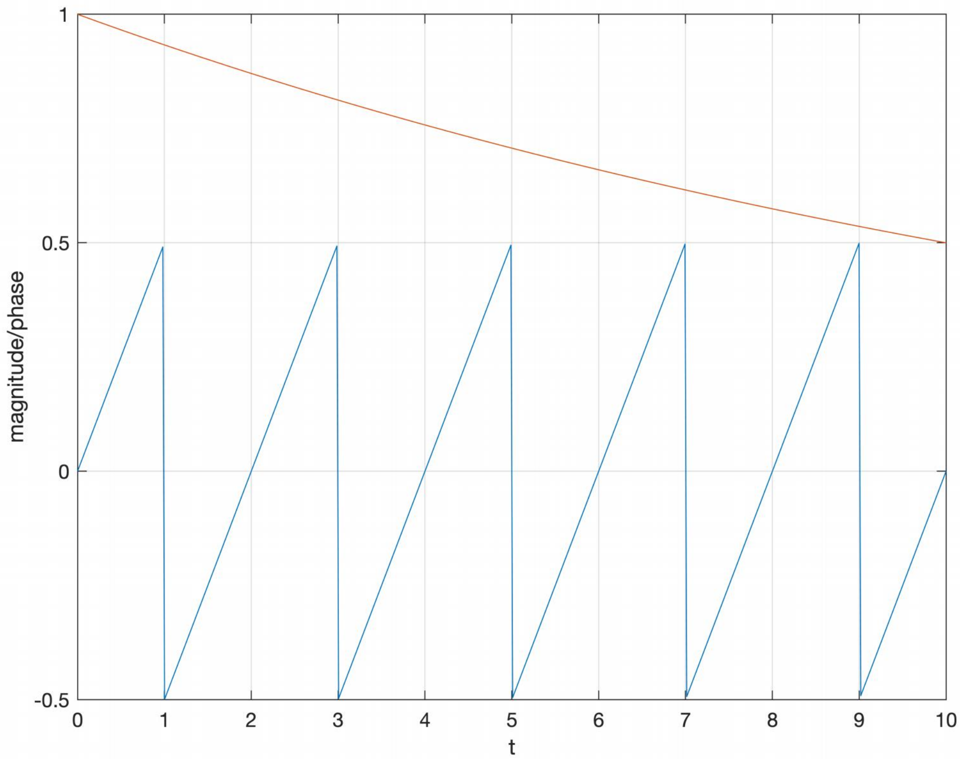
- ✓ **- 0 pts** **Correct**
- **- 5 pts** no answer
- **- 0 pts** incorrect sigma and omega resulted in incorrect plots
- **- 2 pts** missing plots

```
grid on;
xlabel('t');
ylabel('Re(y)')
subplot(2, 1, 2);
plot(t, i);
grid on;
xlabel('t');
ylabel('Im(y)');
```



(c)

```
subplot(1, 1, 1);
magnitude = abs(y);
phase = angle(y)/(2 * pi);
plot(t, phase);
hold on;
plot(t, magnitude);
grid on;
ylabel("magnitude/phase");
xlabel("t");
```


1
0.5
0
-0.5
magnitude/phase
0
1
2
3
4
5
6
7
8
9
10
t

5.3 (c) 5 / 5

- ✓ **- 0 pts** Correct
- **- 2 pts** missing plots
- **- 1 pts** either of the plots incorrect
- **- 5 pts** no answer