ECE102, Fall 2020 **Discussion 10**
Signals & Systems Prof. J.C. Kao
University of California, Los Angeles; Department of ECETAs: A. Ghosh, T. Monsoor & G. Zhao

1. **Laplace transform**
   Determine the Laplace transform and associated ROC for each of the following time functions.

   (a) $\sum_{k=0}^{\infty} a^k \delta(t - kT), \;\; a > 0$

   (b) $\sin(\omega_0 t + b) e^{-at} u(t), \;\; a > 0$

2. **Inverse Laplace transform**
   Determine the time function $x(t)$ for each Laplace transform $X(s)$ given below.

   (a) $\frac{s+1}{s^2+5s+6}, \;\; ROC : \sigma > -2$

   (b) $\frac{s+1}{(s+1)^2+4}, \;\; ROC : \sigma > -1$

3. **Stability and Causality of LTI systems**
   Consider a continuous-time LTI system for which the input $x(t)$ and output $y(t)$ are related by the differential equation

$$\frac{d^2y(t)}{dt^2} - \frac{dy(t)}{dt} - 2y(t) = x(t)$$

   Let $X(s)$ and $Y(s)$ denote the Laplace transforms of $x(t)$ and $y(t)$, and let $H(s)$ denote the Laplace transform of the impulse response $h(t)$ of the preceding system. Assume all initial conditions are zero.

   (a) Determine $H(s)$. Sketch the pole-zero plot.

   (b) Sketch the ROC for each of the following cases:

      i. The system is stable.

      ii. The system is causal.

      iii. The system is neither stable nor causal.

   (c) Determine $h(t)$ when the system is causal.