ECE 102, Fall 2018 **Midterm**
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UCLA True Bruin academic integrity principles apply.
Open: Two pages of cheat sheet allowed.
Closed: Book, computer, internet.
2:00-3:50pm.
Wednesday, 14 Nov 2018.

State your assumptions and reasoning.
No credit without reasoning.
Show all work on these pages.

Name: ____________________

Signature: ____________________

ID#: _________________

Problem 1	_____	/	19
Problem 2	_____	/	17
Problem 3	_____	/	16
Problem 4	_____	/	20
Problem 5	_____	/	28
BONUS	_____	/	6 bonus points
Total	_____	/	100 points + 6 bonus points

Problem 1 (19 points)

(a) (9 points) Determine if each of the following statements is true or false. Briefly explain your answer to receive full credit.

i. (3 points) If $x(t)$ is an energy signal, then $y(t) = x(t) + 1$ is also an energy signal.

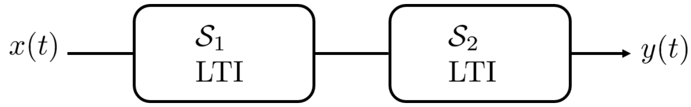
ii. (3 points) If $x(t)$ is an even signal, then $y(t) = x(t-1)$ is also an even signal.

iii. (3 points) If the input to an LTI system is periodic, then its output is also periodic.

(b) (10 points) Is the following system linear? Is it time invariant? (Check both properties). Explain your answer.

$$y(t) = \begin{cases} x(t-1), & t \geq 1 \\ 0, & \text{otherwise} \end{cases}$$

Problem 2 (17 points) Consider the series cascade of the following two systems:



The system $\mathcal{S}_1$ is LTI with impulse response

$$h_1(t) = \int_{-\infty}^{t} u(\tau)\delta(\tau - 2)d\tau$$

The system $\mathcal{S}_2$ is also LTI, with unknown impulse response $h_2(t)$ that we need to find. We are also given that, when the input $x(t)$ is $\delta(t)$, the output $y(t)$ is $r(t-3) + u(t-2)$.

Note: $r(t-3)$ *is the ramp function delayed by* 3*.*

This question continues on the next page.

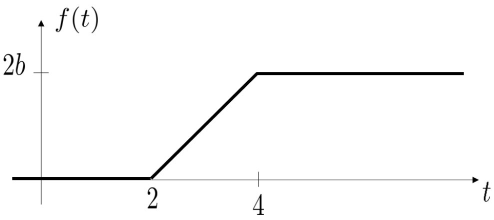
(a) (11 points) Find the impulse response $h_2(t)$ of the system $\mathcal{S}_2$ **and** determine if the system $\mathcal{S}_2$ is causal.

(b) (6 points) Find the output $y(t)$ to the following input:

$$x(t) = (1 + e^{-t})\delta(t + 1)$$

Problem 3 (16 points)

(a) (8 points) Consider the signal $f(t)$ shown below:

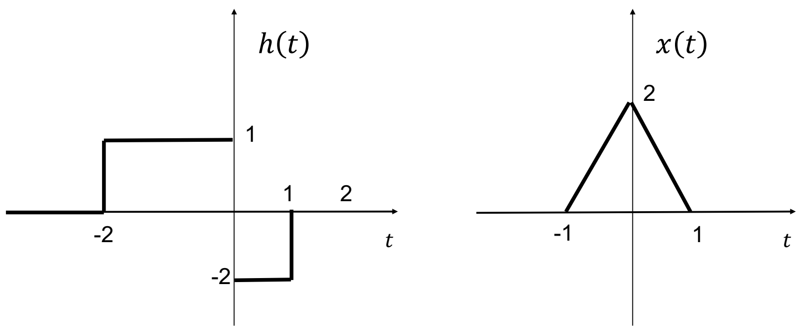


This signal can be written as

$$u(t-a) * \text{rect}\left(\frac{t}{2b}\right)$$

where $a > 0$ and $b > 0$. Find a and b. (Hint: use the flip and drag technique.)

(b) (8 points) An input, $x(t)$, is given to an LTI system with impulse response $h(t)$. Both $x(t)$ and $h(t)$ are shown below.



Let $y(t)$ denote the output of the system, i.e., $y(t) = x(t) * h(t)$. Find the value of t at which the output $y(t)$ reaches its maximum value. Determine this maximum value.

*Note: to answer this question, you do **not** need to find $y(t)$ for all t.*

Problem 4 (20 points)

Consider the following two periodic signals $f(t)$ and $g(t)$. They both have the same period T_0. Let f_k and g_k respectively denote the Fourier series coefficients of $f(t)$ and $g(t)$.

(a) (6 points) If $f(t) = -g\left(t + \frac{T_0}{2}\right)$, how is f_k related to g_k?

(b) (6 points) If $f(t) = -f\left(t + \frac{T_0}{2}\right)$, for what k are the coefficients f_k zero?

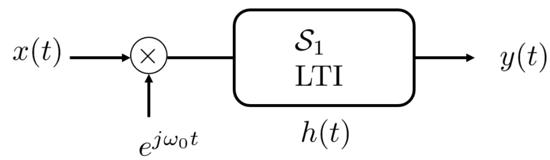
(c) (8 points) This question has two parts. *Note: part (c) is independent of parts (a) and (b).*

i. (4 points) Let $f_e(t)$ denote the even part of $f(t)$. Express the Fourier series coefficients of $f_e(t)$ in terms of f_k.

ii. (4 points) Determine the DC component of $f_o(t)$, the odd part of $f(t)$.

Problem 5 (28 points)

Consider the following system ($\omega_0 > 0$):



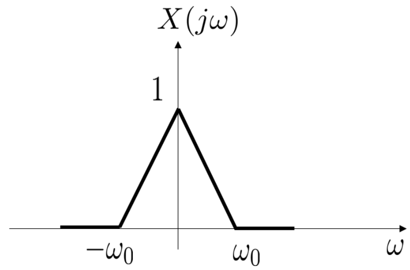
The system $\mathcal{S}_1$ is LTI and $h(t)$ represents its impulse response.

(a) (10 points) Show that the overall system, with input $x(t)$ and output $y(t)$, is not time-invariant.

(b) (12 points) Consider the following impulse response for system $\mathcal{S}_1$:

$$h(t) = e^{j\frac{\omega_0}{2}t}\text{sinc}\left(\frac{\omega_0}{2\pi}t\right)$$

We give the system an input $x(t)$, where $x(t)$ has the following Fourier transform $X(j\omega)$:



Find and sketch the Fourier transform $Y(j\omega)$ of the corresponding output $y(t)$. After this, determine (i) if $y(t)$ is real and (ii) if $y(t)$ is even. *Note: you do not need to give an expression for $Y(j\omega)$, a sketch of it is enough. There is space on the next page if needed.*

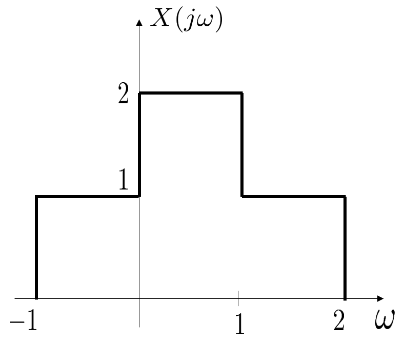
(c) (6 points) Suppose

$$z(t) = y(3t - 2)$$

Express $Z(j\omega)$ in terms of $Y(j\omega)$. *Note: part (c) is independent of parts (a) and (b).*

BONUS (6 points)

(a) (4 points) The Fourier transform $X(j\omega)$ of a signal $x(t)$ is given as follows:



Find the phase of $x^2(t)$.

(b) (2 points) If a signal $x(t)$ is causal with $x(0) = 0$, how can we retrieve $x(t)$ from its even component $x_e(t)$?

Fourier Transform Tables

Property	Signal	Transform
Linearity	$\alpha\, x_1\,(t) + \beta\, x_2\,(t)$	$\alpha\, X_1\,(j\omega) + \beta \quad X_2\,(j\omega)$
Duality	$X\,(t)$	$2\pi\, x\,(-\omega)$
Conjugate symmetry	$x\,(t)$ real	$X^*\,(j\omega) = \quad X\,(-j\omega)$ Magnitude: $\lvert X\,(-j\omega)\rvert = \lvert X\,(j\omega)\rvert$ Phase: $\Theta\,(-\omega) = \quad -\Theta\ \ (\omega)$ Real part: $X_r\,(-j\omega) = \quad X_r\,(j\omega)$ Imaginary part: $X_i\,(-j\omega) = \quad -X_i\,(j\omega)$
Conjugate antisymmetry	$x\,(t)$ imaginary	$X^*\,(j\omega) = -X\,(-j\omega)$ Magnitude: $\lvert X\,(-j\omega)\rvert = \lvert X\,(j\omega)\rvert$ Phase: $\Theta\,(-\omega) = \quad -\Theta\ \ (\omega) \mp \pi$ Real part: $X_r\,(-j\omega) = \quad -X_r\,(j\omega)$ Imaginary part: $X_i\,(-j\omega) = \quad X_i\,(j\omega)$
Even signal	$x\,(-t) = x\,(t)$	$X\,(j\omega)$: even
Odd signal	$x\,(-t) = -x\,(t)$	$X\,(j\omega)$: odd
Time shifting	$x(t-\tau)$	$X\,(j\omega)\; e^{-j\omega\tau}$
Frequency shifting	$x\,(t)\; e^{j\omega_0 t}$	$X\,(j(\omega-\omega_0))$
Modulation property	$x\,(t)\; \cos(\omega_0 t)$	$\frac{1}{2}\;[X\,(j(\omega-\omega_0)\,) + \;X\,(j(\omega+\omega_0))]$
Time and frequency scaling	$x\,(at)$	$\dfrac{1}{\lvert a\rvert}\, X\left(\dfrac{j\omega}{a}\right)$
Differentiation in time	$\dfrac{d^n}{dt^n}\,[x\,(t)]$	$(j\omega)^n\, X\,(j\omega)$
Differentiation in frequency	$(-jt)^n\; x\,(t)$	$\dfrac{d^n}{d\omega^n}\,[X\,(j\omega)]$
Convolution	$x_1\,(t) * x_2\,(t)$	$X_1\,(j\omega)\; X_2\,(j\omega)$
Multiplication	$x_1\,(t)\;\; x_2\,(t)$	$\dfrac{1}{2\pi}\, X_1\,(j\omega) * X_2\,(j\omega)$
Integration	$\displaystyle\int_{-\infty}^{t} x(\lambda)\, d\lambda$	$\dfrac{X\,(j\omega)}{j\omega} + \pi\, X(0)\, \delta\,(\omega)$
Parseval's theorem	$\displaystyle\int_{-\infty}^{\infty} \lvert x\,(t)\rvert^2\; dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} \lvert X\,(j\omega)\rvert^2\, d\omega$	

Table 4.4 – Fourier transform properties.

Additional properties:		
	x(t): even and real	$X\,(j\omega)$: even and real
	x(t): odd and real	$X\,(j\omega)$: odd and imaginary
	x(t): even and imaginary	$X\,(j\omega)$: even and imaginary
	x(t): odd and imaginary	$X\,(j\omega)$: odd and real

Name	Signal	Transform
Rectangular pulse	$x(t) = A\, rect(t/\tau)$	$X(j\omega) = A\tau \operatorname{sinc}\left(\frac{\omega\tau}{2\pi}\right)$
Triangular pulse	$x(t) = A\,\Lambda(t/\tau)$	$X(j\omega) = A\tau \operatorname{sinc}^2\left(\frac{\omega\tau}{2\pi}\right)$
Right-sided exponential	$x(t) = e^{-at}\,u(t)$	$X(j\omega) = \frac{1}{a + j\omega}$
Two-sided exponential	$x(t) = e^{-a\lvert t\rvert}$	$X(j\omega) = \frac{2a}{a^2 + \omega^2}$
Signum function	$x(t) = \operatorname{sgn}(t)$	$X(j\omega) = \frac{2}{j\omega}$
Unit impulse	$x(t) = \delta(t)$	$X(j\omega) = 1$
Sinc function	$x(t) = \operatorname{sinc}(t)$	$X(j\omega) = rect\left(\frac{\omega}{2\pi}\right)$
Constant-amplitude signal	$x(t) = 1,\ \text{all } t$	$X(j\omega) = 2\pi\,\delta(\omega)$
	$x(t) = \frac{1}{\pi t}$	$X(j\omega) = -j \operatorname{sgn}(\omega)$
Unit-step function	$x(t) = u(t)$	$X(j\omega) = \pi\,\delta(\omega) + \frac{1}{j\omega}$
Modulated pulse	$x(t) = rect\left(\frac{t}{\tau}\right)\cos(\omega_0 t)$	$X(j\omega) = \frac{\tau}{2}\operatorname{sinc}\left(\frac{(\omega-\omega_0)\tau}{2\pi}\right) + \frac{\tau}{2}\operatorname{sinc}\left(\frac{(\omega+\omega_0)\tau}{2\pi}\right)$

Table 4.5 – Some Fourier transform pairs.

Note:

$$\operatorname{sinc}(\alpha) = \frac{\sin(\pi\alpha)}{\pi\alpha}$$

$$\operatorname{rect}(t/\tau) = u(t + \tau/2) - u(t - \tau/2)$$