ECE102, Fall 2020 **Discussion 8**
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1. **Frequency response of RC circuits**

   Consider the first-order RC circuit shown in Figure 1

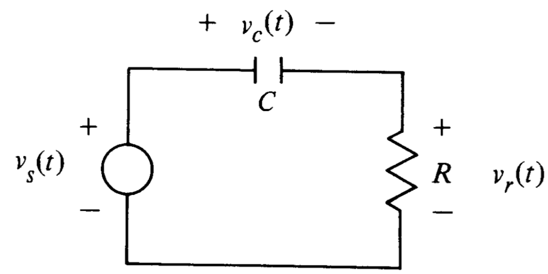

Figure 1: First order RC circuit

   (a) Determine, $H_1(j\omega)$, the transfer function from $v_s(t)$ to $v_c(t)$. Compute the magnitude and phase of $H_1(j\omega)$.

   Hint: The differential equation relating $v_s(t)$ and $v_c(t)$ is given by

$$RC\frac{dv_c(t)}{dt} + v_c(t) = v_s(t)$$

   (b) Determine, $H_2(j\omega)$, the transfer function from $v_s(t)$ to $v_r(t)$. Compute the magnitude and phase of $H_2(j\omega)$.

   Hint: The differential equation relating $v_s(t)$ and $v_r(t)$ is given by

$$RC\frac{d(v_s(t) - v_r(t))}{dt} + v_s(t) - v_r(t) = v_s(t)$$

   (c) What are the cutoff frequencies for $H_1(j\omega), H_2(j\omega)$? (For this problem, the cutoff frequency is defined as the frequency at which the magnitude of the frequency response is $\frac{1}{\sqrt{2}}$ times its maximum value.)

   (d) Suppose we now consider the system shown in Figure 2
   Compute $\frac{V(j\omega)}{V_s(j\omega)}$. What is the corresponding cutoff frequency?

2. **Parameterized filter**

   Consider the system in Figure 3. Let $G(j\omega)$ be real and have the form shown in Figure 4. Plot the resulting frequency response magnitude and phase for the following cases.

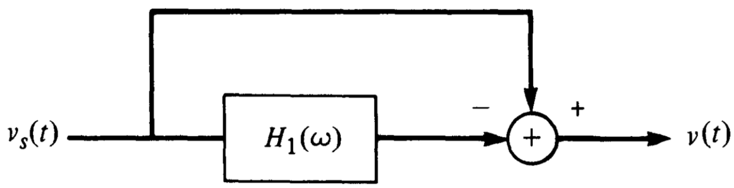

Figure 2: System for 1(d)

(a) $\alpha > 1$

(b) $1 > \alpha > 0$

(c) $\alpha < 0$

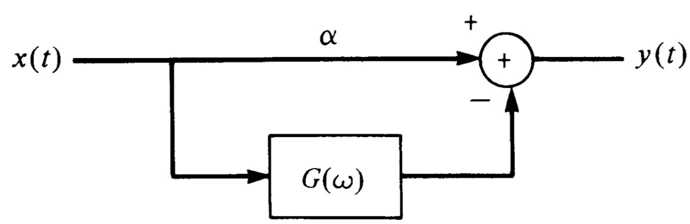

Figure 3: System for 2

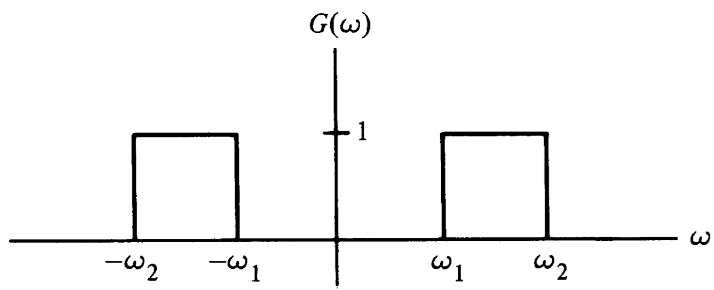

Figure 4: $G(j\omega)$

3. **Signal recovery**

Consider the modulated signal $z(t) = A(t)\cos(\omega_c + \theta_c)$, where ω_c is known but θ_c is unknown. We would like to recover A(t) from z(t).

(a) Show that $z(t) = x(t)\cos\omega_c t + y(t)\sin\omega_c t$ and express $x(t)$ and $y(t)$ in terms of $A(t)$ and θ_c.

(b) Show how to recover $x(t)$ from $z(t)$ by modulation followed by filtering.

(c) Show how to recover $y(t)$ from $z(t)$ by modulation followed by filtering.

(d) Express $A(t)$ in terms of $x(t)$ and $y(t)$ with no reference to θ_c and show in a block diagram how to recover $A(t)$ from $z(t)$. The following trigonometric identities may be useful:

$$
\begin{aligned}
\cos(A+B) &= \cos(A)\cos(B) - \sin(A)\sin(B) \\
\cos^2(A) &= \frac{1}{2}(1+\cos(2A)) \\
\sin^2(A) &= \frac{1}{2}(1-\cos(2A)) \\
\sin(2A) &= 2\sin(A)\cos(A)
\end{aligned}
$$