ECE 102 HW5

LIANG, NEVIN

TOTAL POINTS

93.5 / 100

QUESTION 1

Problem 1 18 pts

1.1 (a) 7 / 7

✓ **- 0 pts** **$$Correct\\Re(c_{k}) = -Re(c_{-k}))\\Im(c_{k}) = Im(c_{-k})\\c_{k}^{*} = -c_{-k}\\|c_{k}| = |c_{-k}|\\\angle c_{k} = -\angle c_{k}^{*}\\\angle c_{k} = -\angle c_{-k}\pm \pi$$**

- 0 pts $$phase(c_{k}) = -phase(c_{-k})$$, though not correct, we don't deduct points.

- 2 pts Partially correct.

- 7 pts No Answer

1.2 (b) 5 / 7

- 0 pts Correct

✓ **- 2 pts** **Wrong answer.**

- 7 pts No answer

1.3 (c) 4 / 4

✓ **- 0 pts** **Correct**

- 2 pts Wrong answer

- 4 pts No answer

QUESTION 2

Problem 2 32 pts

2.1 (a) 16 / 16

✓ **- 0 pts** **$$Correct\\i. a,d,e\\ii. f\\iii. c, e\\iv. a, b\\v. e\\vi. f\\vii. d\\viii. None\;or\;b$$**

- 2 pts Partially correct

- 16 pts No answer

2.2 (b)(i) 4 / 4

✓ **- 0 pts** **Correct: True**

- 2 pts Wrong Answer

- 4 pts No answer

2.3 (b)(ii) 4 / 4

✓ **- 0 pts** **Correct: True**

- 2 pts Wrong answer

- 4 pts No answer

2.4 (c)(i) 4 / 4

✓ **- 0 pts** **Correct**

- 0.5 pts No conclusion

- 4 pts No answer

2.5 (c)(ii) 2 / 4

- 0 pts Correct

- 0.5 pts the imaginary term should multiply with 'j'

- 4 pts No answer

- 1 pts No answer for$$X_o$$

✓ **- 2 pts** **wrong answer for $$X_e, X_o$$**

QUESTION 3

Problem 3 15 pts

3.1 (a)(i) 2 / 2

✓ **- 0 pts** **Correct**

- 1 pts wrong answer

- 0.5 pts Not finished

- 2 pts No answer

3.2 (a)(ii) 2 / 2

✓ **- 0 pts** **Correct**

- 1 pts wrong answer

- 2 pts No answer

3.3 (a)(iii) 1.5 / 2

- 0 pts Correct

- 1 pts wrong answer

- 2 pts No answer

- 0.5 pts Not finished

✓ **- 0.5 pts** **$$\pi$$ if X(jw) < 0**

- 0.5 pts 0 if X(jw) > 0

3.4 (a)(iv) **2 / 2**

✓ **- 0 pts** **Correct**

- 2 pts No answer

- 0.5 pts Not finished

- 1 pts wrong answer

3.5 (a)(v) **2 / 2**

✓ **- 0 pts** **Correct**

- 0.5 pts not have x < 0 part / not have x > 0 part

- 2 pts No answer

- 1 pts wrong answer / no graph

3.6 (b) **5 / 5**

✓ **- 0 pts** **Correct**

- 1 pts wrong scale

- 1 pts wrong coefficient in sinc

- 2 pts wrong answer

- 5 pts no answer

- 0.5 pts Not finished

QUESTION 4

Problem 4 35 pts

4.1 (a)(i)(Optional) **0 / 0**

✓ **- 0 pts** **Correct**

4.2 (a)(ii) **7 / 7**

✓ **- 0 pts** **Correct**

- 1 pts extra factor of -1 in the denominator

- 1 pts missing +2 in the denominator

- 2 pts missing exp(-jw) in the numerator

- 2.5 pts partially correct

- 2 pts missing a factor of exp(3j)

- 5 pts incorrect use of step function and incorrect FT of shifts

- 7 pts no answer

4.3 (a)(iii) **5 / 7**

- 0 pts Correct

- 1 pts extra factor of 2pi

- 1 pts added extra t

- 1 pts incorrect simplification

- 0.5 pts arithmetic error

✓ **- 2 pts** **incorrect integration or convolution**

- 2 pts final answer not shown

- 1 pts off by a factor

- 7 pts no answer

4.4 (a)(iv) **7 / 7**

✓ **- 0 pts** **Correct**

- 1 pts extra factor

- 1 pts copied question wrong

- 2 pts incorrect FT transform

- 1 pts not simplified

- 3 pts partially correct

- 1 pts incorrect integration

- 1 pts arithmetic error

- 3 pts Ft of *t missing

- 7 pts no answer

4.5 (b) **6 / 6**

✓ **- 0 pts** **Correct**

- 1 pts incorrect compression (w/2)

- 1 pts arithmetic error

- 2 pts partially correct

- 1 pts factor of 1/2 missing for the bound -2,2

- 1 pts factor of 1/(2*pi) missing

- 2 pts incorrect integration

- 3 pts incorrect setup

- 6 pts no answer

4.6 (c)(i) **4 / 4**

✓ **- 0 pts** **Correct**

- 1 pts error in F1(jw)

- 1 pts error in F2(jw)

- 0.5 pts F(jw)=0 (no overlap)

- 4 pts no answer

4.7 (c)(ii) **4 / 4**

✓ **- 0 pts** **Correct**

- 4 pts no or incorrect answer

EE102

1. (a) when $f(t)$ is pure imaginary, $R(C_k) = -R(C_{-k})$

$I(C_k) = I(C_{-k})$

$a+bi \rightarrow -a+bi$

$C_k^* = -C_{-k}$

$|C_k| = |C_{-k}|$

$\angle C_k = -\angle C_{-k} + \pi$

(b) $x(t)$ is real + odd + periodic $T=2$ $\quad a_k = 0$ for $|k|>1$

$\cancel{\frac{1}{2}\int_0^2 |x(t)|^2 dt = 1}$

real: $Re(C_k) = Re(C_{-k})$, $Im(C_k) = -Im(C_{-k})$

odd: $C_k = -C_{-k} \rightarrow a+bi = -(a-bi) \Rightarrow a=0$

$C_1 = bj \quad C_{-1} = -bj \quad C_0 = 0$

$x(t) = \sum_{-\infty}^{\infty} C_k e^{jw_0kt} = C_1 e^{jw_0t} + C_{-1}e^{-jw_0t}$

$\frac{1}{2}\int_0^2 x(t)^2 dt = 1 = |a_0|^2 + |a_1|^2 + |a_{-1}|^2 = \cancel{bj\cdot(\cos(w_0t) + j\sin(w_0t))}$

$2|a_1|^2 = 1 \quad = 1 \quad \cancel{+(-bj)(\cos(w_0t) - j\sin(w_0t))}$

$= -c^2 - c^2 = 1 \quad \cancel{= -b\sin(w_0t) - b\sin(w_0t)}$

$a_1 = \pm\frac{\sqrt{2}}{2}, \cancel{\frac{1}{2}\int_0^2 |x(t)|^2 dt} \quad \cancel{= -2b\sin(w_0t)}$

$\cancel{= \frac{1}{2}\int_0^2 4b^2\sin^2(w_0t)\,dt = 2b^2\int_0^2 \sin^2(w_0t)\,dt = 1}$

$\boxed{\sqrt{2}\sin(\pi t)}$

$\cancel{= 2b^2\left(1 - \frac{\sin(4w_0)}{4w_0}\right) = 1}$

(c) $Y(wj) = \int_{-\infty}^{\infty} y(t)\cdot e^{-jwt}dt \qquad C_k = \frac{1}{T}\int_0^T y_T(t)\cdot e^{-jkw_0t}dt$

$\frac{Y(jkw_0)}{T} = \frac{1}{T}\cdot\int_{-\infty}^{\infty} y(t)\cdot e^{-jkw_0t}dt$

$C_k = \frac{1}{T}Y(jw)$ @ $w = kw_0$

$\frac{1}{T}\int_0^T y_T(t)\cdot e^{-jkw_0t}dt$

1.1 (a) **7 / 7**

✓ **- 0 pts** **$$Correct\\Re(c_{k}) = -Re(c_{-k}))\\Im(c_{k}) = Im(c_{-k})\\c_{k}^{*} = -c_{-k}\\|c_{k}| = |c_{-k}|\\\angle c_{k} = -\angle c_{k}^{*}\\\angle c_{k} = -\angle c_{-k}\pm \pi$$**

- **- 0 pts** $$phase(c_{k}) = -phase(c_{-k})$$, though not correct, we don't deduct points.
- **- 2 pts** Partially correct.
- **- 7 pts** No Answer

EE102

1. (a) when $f(t)$ is pure imaginary, $R(C_k) = -R(C_{-k})$

$I(C_k) = I(C_{-k})$

$a+bi \rightarrow -a+bi$

$C_k^* = -C_{-k}$

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real: $Re(C_k) = Re(C_{-k})$, $Im(C_k) = -Im(C_{-k})$

odd: $C_k = -C_{-k} \rightarrow a+bi = -(a-bi) \Rightarrow a = 0$

$C_1 = bj \quad C_{-1} = -bj \quad C_0 = 0$

$x(t) = \sum_{-\infty}^{\infty} C_k e^{jw_0kt} = C_1 e^{jw_0t} + C_{-1} e^{-jw_0t}$

$\frac{1}{2}\int_0^2 x(t)^2 dt = 1 = |a_0|^2 + |a_1|^2 + |a_{-1}|^2 = \cancel{bj\cdot(\cos(w_0t) + j\sin(w_0t))}$

$2|a_1|^2 = 1 \quad = 1 \quad \cancel{+(-bj)(\cos(w_0t) - j\sin(w_0t))}$

$= c^2 - c^2 = 1 \quad \cancel{= b\sin(w_0t) - b\sin(w_0t)}$

$a_1 = \pm\frac{\sqrt{2}}{2} \quad \cancel{\frac{1}{2}\int_0^2 |x(t)|^2 dt} \quad \cancel{= 2b\sin(w_0t)}$

$\cancel{= \frac{1}{2}\int_0^2 4b^2\sin^2(w_0t)\,dt = 2b^2\int_0^2 \sin^2(w_0t)\,dt = 1}$

$\boxed{\sqrt{2}\sin(\pi t)}$

$\cancel{= 2b^2\left(1 - \frac{\sin(4w_0)}{4w_0}\right) = 1}$

(c) $Y(wj) = \int_{-\infty}^{\infty} y(t)\cdot e^{-jwt} dt \quad C_k = \frac{1}{T}\int_0^T y_T(t)\cdot e^{-jkw_0t} dt$

$\frac{Y(jkw_0)}{T} = \frac{1}{T}\cdot\int_{-\infty}^{\infty} y(t)\cdot e^{-jkw_0t} dt$

$C_k = \frac{1}{T} Y(jw)$ @ $w = kw_0$

$\frac{1}{T}\int_0^T y_T(t)\cdot e^{-jkw_0t} dt$

1.2 (b) **5 / 7**

- **- 0 pts** Correct
- ✓ **- 2 pts** **Wrong answer.**
- **- 7 pts** No answer

EE102

1. (a) when $f(t)$ is pure imaginary, $R(C_k) = -R(C_{-k})$

$I(C_k) = I(C_{-k})$

$a+bi \rightarrow -a+bi$

$C_k^* = -C_{-k}$

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$x(t) = \sum_{-\infty}^{\infty} C_k e^{jw_0kt} = C_1 e^{jw_0t} + C_{-1} e^{-jw_0t}$

$\frac{1}{2}\int_0^2 x(t)^2 dt = 1 = |a_0|^2 + |a_1|^2 + |a_{-1}|^2 = \cancel{bj \cdot (\cos(w_0t) + j\sin(w_0t))}$

$2|a_1|^2 = 1 \qquad = 1 \qquad \cancel{+(-bj)(\cos(w_0t) - j\sin(w_0t))}$

$= c^2 - c^2 = 1 \qquad \cancel{= b\sin(w_0t) - b\sin(w_0t)}$

$a_1 = \pm\frac{\sqrt{2}}{2} \quad \cancel{\frac{1}{2}\int_0^2 |x(t)|^2 dt} \qquad \cancel{= 2b\sin(w_0t)}$

$\cancel{= \frac{1}{2}\int_0^2 4b^2\sin^2(w_0t)\,dt = 2b^2\int_0^2 \sin^2(w_0t)\,dt = 1}$

$\boxed{\sqrt{2}\sin(\pi t)}$

$\cancel{= b^2\left(1 - \frac{\sin(4w_0)}{4w_0}\right) = 1}$

(c) $Y(wj) = \int_{-\infty}^{\infty} y(t) \cdot e^{-jwt} dt \qquad C_k = \frac{1}{T}\int_0^T y_T(t) \cdot e^{-jkw_0t} dt$

$\frac{Y(jkw_0)}{T} = \frac{1}{T} \cdot \int_{-\infty}^{\infty} y(t) \cdot e^{-jkw_0t} dt$

$C_k = \frac{1}{T} Y(jw)$ @ $w = kw_0$

$\frac{1}{T}\int_0^T y_T(t) \cdot e^{-jkw_0t} dt$

1.3 (C) **4 / 4**

✓ **- 0 pts** Correct

- 2 pts Wrong answer

- 4 pts No answer

2. (a) i. $\boxed{a,e,d}$

$\sqrt{r^2+I^2}$ = mag

$\tan^{-1}(\frac{I}{R})$ = phase

ii. $\boxed{f}$ odd

iii. $\boxed{c,e}$

I/R

$\angle X(j\omega) = \tan^{-1}(\tan(g(\omega))) \Rightarrow -\frac{\omega}{2}$

$\frac{\sin(-\frac{\omega}{2})}{\cos(-\frac{\omega}{2})} \Rightarrow \cos(\frac{\omega}{2}) - j\sin(\frac{\omega}{2})$

so real.

$X(j\omega) \sim 1 + j$

iv $\boxed{ab}$ all others are only real / imag

v. $\boxed{e}$ X is real + not odd

" "

vi. $\boxed{f}$ odd $|X(j\omega)| \cdot e^{j\,\text{phase}}$

vii $\boxed{d}$ imag + not odd. $(-j\omega)$

viii $\boxed{b}$ $e^{-j\,\text{phase}}$

next page

2.1 (a) **16 / 16**

✓ **- 0 pts** **$$Correct\\i. a,d,e\\ii. f\\iii. c, e\\iv. a, b\\v. e\\vi. f\\vii. d\\viii. None\;or\;b$$**

- 2 pts Partially correct

- 16 pts No answer

b) i) [real + even] * [real + odd] = odd

$$\underset{f(t)}{\uparrow} \qquad \underset{g(t)}{\uparrow}$$

$f(t) \leftrightarrow F(j\omega) =$ real and even

$g(t) \leftrightarrow G(j\omega) =$ odd and imag

$$\mathcal{F}[f * g] = F \cdot G = \boxed{\text{odd}} \quad \boxed{\text{true}}$$

ii. $f(t)$ & $f(-t)$

$$\mathcal{F}(f(t) * f(-t)) = F(j\omega) \cdot F(-j\omega) = G(\omega j)$$

$$G(-j\omega) = F(-j\omega) \cdot F(--j\omega) = F(j\omega) \cdot F(-j\omega)$$

$$(+j\omega)$$

even $\boxed{\text{true}}$

c) i) $x(t) \leftrightarrow X(j\omega)$

$x^*(t) \leftrightarrow X^*(-j\omega)$

$\quad\downarrow \qquad\qquad \downarrow$

$\quad -t \qquad\quad j\omega$

$\mathcal{F}(x(t)) = x^*(-t)$

$X(j\omega) = X^*(j\omega)$

so $\boxed{\text{real}}$

ii) $X_o(j\omega) = \dfrac{X(j\omega) - X(-j\omega)}{2}$ $\quad X_e(j\omega) = \dfrac{X(j\omega) + X(-j\omega)}{2}$

real $\rightarrow X(-j\omega) = X^*(j\omega)$

$X_o(j\omega) = \dfrac{X(j\omega) - X^*(j\omega)}{2}$ $\quad X_e(j\omega) = \dfrac{X(j\omega) + X(j\omega)}{2}$

$= j \cdot k \qquad\qquad = m$

2.2 (b)(i) **4 / 4**

✓ **- 0 pts** **Correct: True**

- 2 pts Wrong Answer

- 4 pts No answer

b) i) $[\text{real} + \text{even}] * [\text{real} + \text{odd}] = \text{odd}$

$$\underset{f(t)}{\uparrow} \qquad \underset{g(t)}{\uparrow}$$

$f(t) \leftrightarrow F(j\omega) = \text{real and even}$

$g(t) \leftrightarrow G(j\omega) = \text{odd and imag}$

$$\mathcal{F}[f * g] = F \cdot G = \boxed{\text{odd}} \ \boxed{\text{true}}$$

ii. $f(t)$ & $f(-t)$

$$\mathcal{F}(f(t) * f(-t)) = F(j\omega) \cdot F(-j\omega) = G(\omega j)$$

$$G(-j\omega) = F(-j\omega) \cdot F(--j\omega) = F(j\omega) \cdot F(-j\omega)$$

$$(+j\omega)$$

even $\boxed{\text{true}}$

c) i) $x(t) \leftrightarrow X(j\omega)$

$x^*(t) \leftrightarrow X^*(-j\omega)$

$-t \qquad j\omega$

$\mathcal{F}(x(t)) = x^*(-t)$

$X(j\omega) = X^*(j\omega)$

so $\boxed{\text{real}}$

ii) $X_o(j\omega) = \dfrac{X(j\omega) - X(-j\omega)}{2}$ $\qquad X_e(j\omega) = \dfrac{X(j\omega) + X(-j\omega)}{2}$

real $\rightarrow X(-j\omega) = X^*(j\omega)$

$X_o(j\omega) = \dfrac{X(j\omega) - X^*(j\omega)}{2}$ $\qquad X_e(j\omega) = \dfrac{X(j\omega) + X(j\omega)}{2}$

$= j \cdot k \qquad\qquad = m$

2.3 (b)(ii) **4 / 4**

- ✓ **- 0 pts** **Correct: True**
- **- 2 pts** Wrong answer
- **- 4 pts** No answer

b) i) [real + even] $*$ [real + odd] = odd

$\uparrow$ $f(t)$ $\uparrow$ $g(t)$

$f(t) \leftrightarrow F(j\omega)$ = real and even

$g(t) \leftrightarrow G(j\omega)$ = odd and imag

$\mathcal{F}[f * g] = F \cdot G = \boxed{\text{odd}}$ $\boxed{\text{true}}$

ii. $f(t)$ & $f(-t)$

$\mathcal{F}(f(t) * f(-t)) = F(j\omega) \cdot F(-j\omega) = G(\omega j)$

$G(-j\omega) = F(-j\omega) \cdot F(--j\omega) = F(j\omega) \cdot F(-j\omega)$

$(+j\omega)$

even $\boxed{\text{true}}$

c) i) $x(t) \leftrightarrow X(j\omega)$

$x^*(t) \leftrightarrow X^*(-j\omega)$

$\downarrow$ $-t$ $\downarrow$ $j\omega$

$\mathcal{F}(x(t)) = x^*(-t)$

$X(j\omega) = X^*(j\omega)$

so $\boxed{\text{real}}$

ii) $X_o(j\omega) = \dfrac{X(j\omega) - X(-j\omega)}{2}$ $X_e(j\omega) = \dfrac{X(j\omega) + X(-j\omega)}{2}$

real $\rightarrow X(-j\omega) = X^*(j\omega)$

$X_o(j\omega) = \dfrac{X(j\omega) - X^*(j\omega)}{2}$ $X_e(j\omega) = \dfrac{X(j\omega) + X(j\omega)}{2}$

$= j \cdot k$ $= m$

2.4 (c)(i) **4 / 4**

- ✓ **- 0 pts** Correct
- **- 0.5 pts** No conclusion
- **- 4 pts** No answer

b) i) [real + even] $*$ [real + odd] = odd

$\uparrow$ $f(t)$ $\quad\quad$ $\uparrow$ $g(t)$

$f(t) \leftrightarrow F(j\omega)$ = real and even

$g(t) \leftrightarrow G(j\omega)$ = odd and imag

$\mathcal{F}[f * g] = F \cdot G = \boxed{\text{odd}}$ $\boxed{\text{true}}$

ii. $f(t)$ & $f(-t)$

$\mathcal{F}(f(t) * f(-t)) = F(j\omega) \cdot F(-j\omega) = G(\omega j)$

$G(-j\omega) = F(-j\omega) \cdot F(--j\omega) = F(j\omega) \cdot F(-j\omega)$

$(+j\omega)$

even $\boxed{\text{true}}$

c) i) $x(t) \leftrightarrow X(j\omega)$

$x^*(t) \leftrightarrow X^*(-j\omega)$

$\downarrow$ $-t$ $\quad\quad$ $\downarrow$ $j\omega$

$\mathcal{F}(x(t)) = x^*(-t)$

$X(j\omega) = X^*(j\omega)$

so $\boxed{\text{real}}$

ii) $X_o(j\omega) = \frac{X(j\omega) - X(-j\omega)}{2}$ $\quad\quad$ $X_e(j\omega) = \frac{X(j\omega) + X(-j\omega)}{2}$

real $\rightarrow X(-j\omega) = X^*(j\omega)$

$X_o(j\omega) = \frac{X(j\omega) - X^*(j\omega)}{2}$ $\quad\quad$ $X_e(j\omega) = \frac{X(j\omega) + X(j\omega)}{2}$

$= j \cdot k$ $\quad\quad\quad\quad$ $= m$

2.5 (c)(ii) **2 / 4**

- **- 0 pts** Correct
- **- 0.5 pts** the imaginary term should multiply with 'j'
- **- 4 pts** No answer
- **- 1 pts** No answer for$$X_o$$
- ✓ **- 2 pts** **wrong answer for $$X_e, X_o$$**

3) a)

i. $x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(jw) \cdot e^{-jwt} dw$

$\pi x(t) = \int_{0}^{\infty} X(jw) \cdot e^{jwt} dw$

$\pi(x_0) = \boxed{\pi}$

ii. $X(jw) = \int_{-\infty}^{\infty} x(t) \cdot e^{-jwt} dt = |x| + 4 \cdot 1 = \boxed{5}$

iii. $\cancel{\int_{-\infty}^{\infty} x(t) e^{-jwt} dt}$ $X(t)$ real + even.

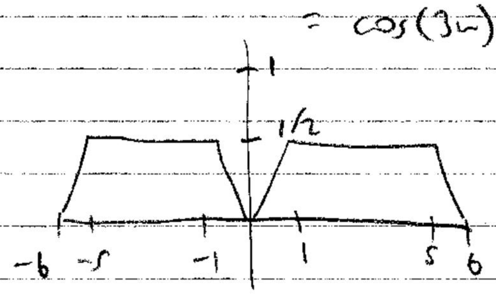
$\text{Im}(X(jw)) = 0$

so $\angle X(jw) = 0$.

iv. $2\pi \cdot x(-1) = \boxed{2\pi}$

v. $F(e^{-3jw} X(jw)) = \cos(3w) \cdot X(jw)$

$= \cos(3w) \cdot X(jw)$



b) $F(\text{rect}(\frac{1}{2}t) * \text{rect}(\frac{1}{2}t))$

$= F(\text{rect}(\frac{1}{2}t))^2$

$= \left(2\text{sinc}\left(\frac{w}{\pi}\right)\right)^2 = 4\text{sinc}^2\left(\frac{w}{\pi}\right)$

3.1 (a)(i) **2 / 2**

✓ **- 0 pts** Correct

- 1 pts wrong answer

- 0.5 pts Not finished

- 2 pts No answer

3) a)

i. $x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\cdot e^{-j\omega t}\, d\omega$

$\pi x(t) = \int_{0}^{\infty} X(j\omega)\cdot e^{j\omega t}\, d\omega$

$\pi(x_0) = \boxed{\pi}$

ii. $X(j\omega) = \int_{-\infty}^{\infty} x(t)\cdot e^{-j\omega t}\, dt = |\cdot| + 4\cdot 1 = \boxed{5}$

iii. $\cancel{\int_{-\infty}^{\infty} x(t) e^{-j\omega t}}\, dt$ $\quad x(t)$ real + even.

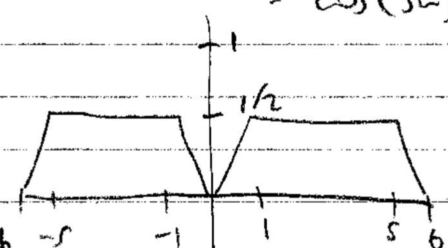
$\mathrm{Im}(X(j\omega)) = 0$

so $\angle X(j\omega) = 0.$

iv. $2\pi \cdot x(-1) = \boxed{2\pi}$

v. $F(e^{-3j\omega} X(j\omega)) = \cos(3\omega)\cdot X(j\omega)$

$= \cos(3\omega)\cdot X(j\omega)$



b) $F(\mathrm{rect}(\tfrac{1}{2}t) * \mathrm{rect}(\tfrac{1}{2}t))$

$= F(\mathrm{rect}(\tfrac{1}{2}t))^2$

$= \left(2\,\mathrm{sinc}\left(\frac{\omega}{\pi}\right)\right)^2 = 4\,\mathrm{sinc}^2\left(\frac{\omega}{\pi}\right)$

3.2 (a)(ii) **2 / 2**

✓ **- 0 pts** Correct

- 1 pts wrong answer

- 2 pts No answer

3) a)

i. $x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(jw) \cdot e^{-jwt} \, dw$

$\pi x(t) = \int_{0}^{\infty} X(jw) \cdot e^{jwt} \, dw$

$\pi(x_0) = \boxed{\pi}$

ii. $X(jw) = \int_{-\infty}^{\infty} x(t) \cdot e^{-jwt} \, dt = |\cdot| + 4 \cdot 1 = \boxed{5}$

iii. $\cancel{\int_{-\infty}^{\infty} x(t) e^{-jwt} dt}$ $x(t)$ real + even.

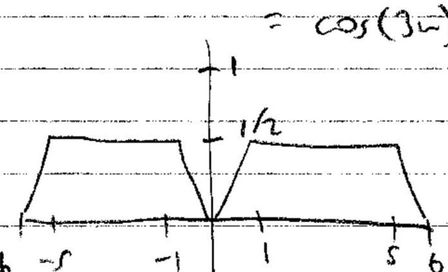
$\mathrm{Im}(X(jw)) = 0$

so $\angle X(jw) = 0$.

iv. $2\pi \cdot x(-1) = \boxed{2\pi}$

v. $F(e^{-3jw} X(jw)) = \cos(3w) \cdot X(jw)$

$= \cos(3w) \cdot X(jw)$



b) $F(\mathrm{rect}(\frac{1}{2}t) * \mathrm{rect}(\frac{1}{2}t))$

$= F(\mathrm{rect}(\frac{1}{2}t))^2$

$= \left(2\mathrm{sinc}\left(\frac{w}{\pi}\right)\right)^2 = 4\mathrm{sinc}^2\left(\frac{w}{\pi}\right)$

3.3 (a)(iii) **1.5 / 2**

- **- 0 pts** Correct
- **- 1 pts** wrong answer
- **- 2 pts** No answer
- **- 0.5 pts** Not finished
- ✓ **- 0.5 pts** **$$\pi$$ if X(jw) < 0**
- **- 0.5 pts** 0 if X(jw) > 0

3) a)

i. $x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega) \cdot e^{-j\omega t} d\omega$

$\pi x(t) = \int_{0}^{\infty} X(j\omega) \cdot e^{j\omega t} d\omega$

$\pi(x_0) = \boxed{\pi}$

ii.

$X(j\omega) = \int_{-\infty}^{\infty} x(t) \cdot e^{-j\omega t} dt = |x| + 4 \cdot 1 = \boxed{5}$

iii. $\cancel{\int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt}$ $x(t)$ real + even.

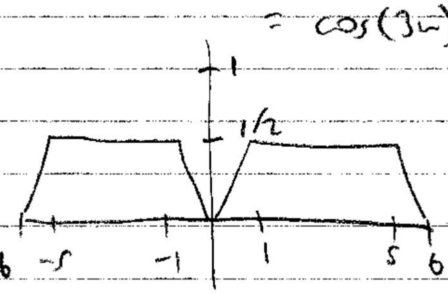
$\text{Im}(X(j\omega)) = 0$

so $\angle X(j\omega) = 0$.

iv. $2\pi \cdot x(-1) = \boxed{2\pi}$

v. $F(e^{-3j\omega} X(j\omega)) = \cos(3\omega) \cdot X(j\omega)$

$= \cos(3\omega) \cdot X(j\omega)$



b) $F(\text{rect}(\frac{1}{2}t) * \text{rect}(\frac{1}{2}t))$

$= F(\text{rect}(\frac{1}{2}t))^2$

$= \left(2\text{sinc}\left(\frac{\omega}{\pi}\right)\right)^2 = 4\text{sinc}^2\left(\frac{\omega}{\pi}\right)$

3.4 (a)(iv) **2 / 2**

- ✓ **- 0 pts** Correct
- **- 2 pts** No answer
- **- 0.5 pts** Not finished
- **- 1 pts** wrong answer

3) a)

i. $x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(jw) \cdot e^{-jwt} \, dw$

$\pi x(t) = \int_{0}^{\infty} X(jw) \cdot e^{jwt} \, dw$

$\pi(x_0) = \boxed{\pi}$

ii.

$X(jw) = \int_{-\infty}^{\infty} x(t) \cdot e^{-jwt} \, dt = |x| + 4 \cdot 1 = \boxed{5}$

iii. $\int_{-\infty}^{\infty}$ ~~x(t) e^{-jwt}~~ dt $\quad x(t)$ real + even.

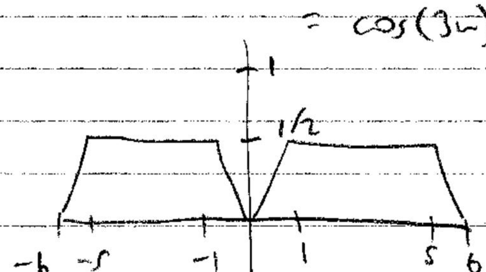
$\text{Im}(X(jw)) = 0$

so $\angle X(jw) = 0$.

iv. $2\pi \cdot x(-1) = \boxed{2\pi}$

v. $F(e^{-3jw} X(jw)) = \cos(3w) \cdot X(jw)$

$= \cos(3w) \cdot X(jw)$



b) $F(\text{rect}(\frac{1}{2}t) * \text{rect}(\frac{1}{2}t))$

$= F(\text{rect}(\frac{1}{2}t))^2$

$= \left(2\,\text{sinc}\left(\frac{w}{\pi}\right)\right)^2 = 4\,\text{sinc}^2\left(\frac{w}{\pi}\right)$

3.5 (a)(v) **2 / 2**

✓ **- 0 pts** **Correct**

- 0.5 pts not have x < 0 part / not have x > 0 part

- 2 pts No answer

- 1 pts wrong answer / no graph

3) a)

i. $x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega) \cdot e^{-j\omega t} \, d\omega$

$\pi x(t) = \int_{0}^{\infty} x(j\omega) \cdot e^{j\omega t} \, d\omega$

$\pi(x_0) = \boxed{\pi}$

ii. $X(j\omega) = \int_{-\infty}^{\infty} x(t) \cdot e^{-j\omega t} \, dt = |x| + 4 \cdot 1 = \boxed{5}$

iii. $\int_{-\infty}^{\infty} \cancel{x(t) e^{-j\omega t}} \, dt$ x(t) real + even.

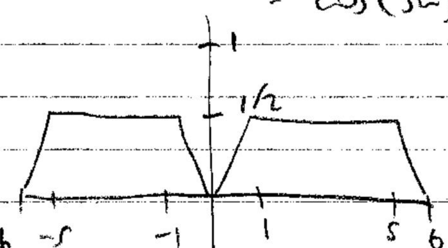
$Im(X(j\omega)) = 0$

so $\angle X(j\omega) = 0$.

iv. $2\pi \cdot x(-1) = \boxed{2\pi}$

v. $F(e^{-3j\omega} X(j\omega)) = \cos(3\omega) \cdot X(j\omega)$

$= \cos(3\omega) \cdot X(j\omega)$



b) $F(rect(\frac{1}{2}t) * rect(\frac{1}{2}t))$

$= F(rect(\frac{1}{2}t))^2$

$= \left(2 sinc\left(\frac{\omega}{\pi}\right)\right)^2 = 4 sinc^2\left(\frac{\omega}{\pi}\right)$

3.6 (b) **5 / 5**

✓ **- 0 pts** Correct

- 1 pts wrong scale

- 1 pts wrong coefficient in sinc

- 2 pts wrong answer

- 5 pts no answer

- 0.5 pts Not finished

4.1 (a)(i)(Optional) **0 / 0**

✓ **- 0 pts** **Correct**

Fourier Transforms & Inverse

4. (a) ii.

$$x_2(t) = e^{(2+3j)t} u(-t+1)$$

$$F(jw) = \int_{-\infty}^{\infty} f(t)\, e^{-jwt}\, dt$$

$$= \int_{-\infty}^{\infty} e^{(2+3j)t} \cdot u(-t+1) \cdot e^{-jwt}\, dt$$

$$= \int_{-\infty}^{1} e^{(2+3j)t} \cdot e^{-jwt}\, dt = \int_{-\infty}^{1} e^{(2+3j-jw)t}\, dt$$

$$= \frac{1}{2+3j-jw} \cdot e^{(2+3j-jw)t} \Big|_{-\infty}^{1}$$

$$= \frac{1}{2+3j-jw} \cdot \left[e^{2+3j-jw} \right]$$

iii. $x_3(t) = \begin{cases} 1+\cos(\pi t) & |t|<1 \\ 0 & \text{otherwise} \end{cases}$

$$F(jw) = \int_{-1}^{1} \left[1+\cos(\pi)t)\right] \cdot e^{-jwt}\, dt$$

$$e^{-jwt} = \cos(-jwt) + j\sin(-jwt)$$

$$F(jw) = \int_{-1}^{1} 1\, e^{-jwt} + \int_{-1}^{1} \cos \pi t \cdot e^{-jwt}\, dt$$

$$= \sin(w) \cdot \left(\frac{-1}{w-\pi} + \frac{2}{w} - \frac{1}{w+\pi} \right)$$

4.2 (a)(ii) **7 / 7**

✓ **- 0 pts** Correct

- 1 pts extra factor of -1 in the denominator

- 1 pts missing +2 in the denominator

- 2 pts missing exp(-jw) in the numerator

- 2.5 pts partially correct

- 2 pts missing a factor of exp(3j)

- 5 pts incorrect use of step function and incorrect FT of shifts

- 7 pts no answer

Fourier Transforms & Inverse

4. (a) ii.

$$x_2(t) = e^{(2+3j)t} u(-t+1)$$

$$F(jw) = \int_{-\infty}^{\infty} f(t)\, e^{-jwt}\, dt$$

$$= \int_{-\infty}^{\infty} e^{(2+3j)t} \cdot u(-t+1) \cdot e^{-jwt}\, dt$$

$$= \int_{-\infty}^{1} e^{(2+3j)t} \cdot e^{-jwt}\, dt = \int_{-\infty}^{1} e^{(2+3j-jw)t}\, dt$$

$$= \frac{1}{2+3j-jw} \cdot e^{(2+3j-jw)t} \Big|_{-\infty}^{1}$$

$$= \frac{1}{2+3j-jw} \cdot \left[e^{2+3j-jw} \right]$$

iii. $x_3(t) = \begin{cases} 1+\cos(\pi t) & |t|<1 \\ 0 & \text{otherwise} \end{cases}$

$$F(jw) = \int_{-1}^{1} \left[1+\cos(\pi)t) \right] \cdot e^{-jwt}\, dt$$

$$e^{-jwt} = \cos(-jwt) + j\sin(-jwt)$$

$$F(jw) = \int_{-1}^{1} 1\, e^{-jwt} + \int_{-1}^{1} \cos \pi t \cdot e^{-jwt}\, dt$$

$$= \sin(w) \left(\frac{-1}{w-\pi} + \frac{2}{w} - \frac{1}{w+\pi} \right)$$

4.3 (a)(iii) **5 / 7**

- **- 0 pts** Correct
- **- 1 pts** extra factor of 2pi
- **- 1 pts** added extra t
- **- 1 pts** incorrect simplification
- **- 0.5 pts** arithmetic error
- ✓ **- 2 pts incorrect integration or convolution**
- **- 2 pts** final answer not shown
- **- 1 pts** off by a factor
- **- 7 pts** no answer

iv. $x_4(t) = t\cdot e^{-2t}\cdot u(t)$

$$= \int_{-\infty}^{\infty} t\cdot e^{-2t} u(t)\cdot e^{-j\omega t}\, dt$$

$$= \int_{0}^{\infty} t\, e^{(-2-j\omega)t}\, dt$$

$$= \frac{e^{(-2-j\omega)t}\cdot t^2}{2}\Bigg|_{0}^{\infty}$$

$$F(t\cdot e^{-2t}u(t))$$

$$= j\cdot \frac{d}{d\omega} F(e^{-2t}u(t))$$

$$= j\cdot(-1)\cdot j\cdot \frac{1}{(2+j\omega)^2}$$

$$= \boxed{\frac{1}{(2+j\omega)^2}}$$

(b) $x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(j\omega)|\cdot e^{j\angle X(j\omega)}\, e^{j\omega t}\, d\omega$

$$= \frac{1}{2\pi}\left[\int_{-3}^{-2} e^{j\cdot -\omega/2}\cdot e^{j\omega t}\, d\omega + \int_{-2}^{2} \frac{1}{2}\cdot e^{j\cdot -\omega/2}\, e^{j\omega t}\, d\omega + \int_{2}^{3} 1\cdot e^{j\cdot -\omega/2}\cdot e^{j\omega t}\, d\omega\right]$$

$$= \frac{2\sin(3t-\frac{3}{2})}{2\pi t-\pi} + \left(\frac{1-2}{2\pi t-\pi}\right)\cdot \sin(2t-1)$$

$$= \frac{2\sin(3t-\frac{3}{2}) - \sin(2t-1)}{2\pi(t-\frac{1}{2})}$$

4.4 (a)(iv) **7 / 7**

✓ **- 0 pts** Correct

- 1 pts extra factor

- 1 pts copied question wrong

- 2 pts incorrect FT transform

- 1 pts not simplified

- 3 pts partially correct

- 1 pts incorrect integration

- 1 pts arithmetic error

- 3 pts Ft of *t missing

- 7 pts no answer

iv. $x_4(t) = t \cdot e^{-2t} \cdot u(t)$

$$= \int_{-\infty}^{\infty} t \cdot e^{-2t} u(t) \cdot e^{-j\omega t} \, dt$$

$$= \int_{0}^{\infty} t \, e^{(-2-j\omega)t} \, dt$$

$$= \frac{e^{(-2-j\omega)t} \cdot t^2}{2} \Big|_0^{\infty}$$

$F(t \cdot e^{-2t} u(t))$

$$= j \cdot \frac{d}{d\omega} F(e^{-2t} u(t))$$

$$= j \cdot (-1) \cdot j \cdot \frac{1}{(2+j\omega)^2}$$

$$= \boxed{\frac{1}{(2+j\omega)^2}}$$

(b) $$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} |X(j\omega)| \cdot e^{j \measuredangle X(j\omega)} \, e^{j\omega t} \, d\omega$$

$$= \frac{1}{2\pi} \left[\int_{-3}^{-2} e^{j \cdot -\omega/2} \cdot e^{j\omega t} \, d\omega + \int_{-2}^{2} \frac{1}{2} \cdot e^{j \cdot -\omega/2} \, e^{j\omega t} \, d\omega + \int_{2}^{3} 1 \cdot e^{j \cdot -\omega/2} \cdot e^{j\omega t} \, d\omega \right]$$

$$= \frac{2\sin(3t - \frac{3}{2})}{2\pi t - \pi} + \left(\frac{1-2}{2\pi t - \pi} \right) \cdot \sin(2t - 1)$$

$$= \frac{2\sin(3t - \frac{3}{2}) - \sin(2t-1)}{2\pi(t - \frac{1}{2})}$$

4.5 (b) **6 / 6**

- ✓ **- 0 pts** Correct
- **- 1 pts** incorrect compression (w/2)
- **- 1 pts** arithmetic error
- **- 2 pts** partially correct
- **- 1 pts** factor of 1/2 missing for the bound -2,2
- **- 1 pts** factor of 1/(2*pi) missing
- **- 2 pts** incorrect integration
- **- 3 pts** incorrect setup
- **- 6 pts** no answer

(c) i. $F(f_1 * f_2) = F_1(j\omega) \cdot F_2(j\omega)$

$= F_1[\text{sinc}(2t)] \rightarrow F_2\left[\begin{matrix}\text{sinc}(t) \\ \cos(3\pi t)\end{matrix}\right]$

$= \frac{1}{2} \cdot F\left(j \cdot \frac{\omega}{2}\right)$

$\text{sinc}\left(\frac{t}{2\pi} 4\pi\right) = \frac{1}{4\pi} \cdot F\left(j \cdot \frac{\omega}{4\pi}\right)$

$= \frac{1}{4\pi} \cdot 2\pi \,\text{rect}(\omega)$

cos shifts left & right so

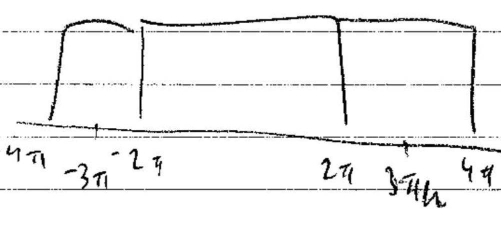
$\text{sinc}(4t) \cdot \cos(3\pi t) = k_1 \,\text{rect}\left(\frac{\omega - 3\pi}{\pi}\right)$

$k_2 \,\text{rect}\left(\frac{\omega + 3\pi}{\pi}\right)$

don't overlap, so

$F(f_1 * f_2) = 0$

ii. $f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} 0 \, d\omega = \boxed{0}$



4.6 (c)(i) **4 / 4**

- ✓ **- 0 pts** Correct
- **- 1 pts** error in F1(jw)
- **- 1 pts** error in F2(jw)
- **- 0.5 pts** F(jw)=0 (no overlap)
- **- 4 pts** no answer

(c) i. $F(f_1 * f_2) = F_1(j\omega) \cdot F_2(j\omega)$

$= F_1 [\text{sinc}(2t)] \rightarrow F_2 \left[\dfrac{\text{sinc}(t)}{\cos(3\pi t)} \right]$

$= \frac{1}{2} \cdot F\left(j \cdot \frac{\omega}{2}\right)$

$\text{sinc}\left(\frac{t}{2\pi} 4\pi\right) = \frac{1}{4\pi} \cdot F\left(j \cdot \frac{\omega}{4\pi}\right)$

$= \frac{1}{4\pi} \cdot 2\pi \,\text{rect}(\omega)$

cos shifts left & right so

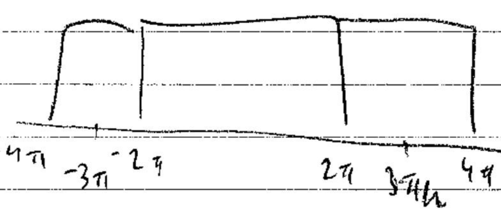
$\text{sinc}(4t) \cdot \cos(3\pi t) = k_1 \,\text{rect}\left(\frac{\omega - 3\pi}{2}\right)$

$k_2 \,\text{rect}\left(\frac{\omega + 3\pi}{2}\right)$

don't overlap, so

$F(f_1 * f_2) = 0$

ii. $f(t) = \frac{1}{2\pi} \cdot \int_{-\infty}^{\infty} 0 \, d\omega = \boxed{0}$



4.7 (c)(ii) **4 / 4**

✓ **- 0 pts** Correct

- 4 pts no or incorrect answer