1. **Properties of fourier transform**

   Suppose $x(t)$ is the input to an LTI system with transfer function $H(j\omega)$, and $y(t)$ is the output of this system, where

$$x(t) = e^{-|t|}\cos(At)$$

   and $H(j\omega) = 1 + e^{-j\omega} + e^{-3j\omega}$. Find a real number $A > 0$ such that $y(0) = 1$. Is your answer unique?

2. **Inverse fourier transform**

   Find the inverse fourier transform of the following functions

   (a) $X(j\omega) = \frac{\pi}{j}4\delta(\omega - 6) - \frac{\pi}{j}4\delta(\omega + 6)$

   (b) $X(j\omega) = \frac{12 + 7j\omega - \omega^2}{(\omega^2 - 2j\omega - 1)(-\omega^2 + j\omega - 6)}$

3. **Frequency response and differential equation description of LTI systems**

   (a) An LTI system has impulse response $h(t) = e^{-3t}u(t)$. What was the input $x(t)$, when the output is $e^{-3t}u(t) - e^{-4t}u(t)$?

   (b) Consider the following input-output pair of an LTI system:

$$x(t) = e^{-t}u(t) \to y(t) = e^{-2t}u(t)$$

   i. What is the output of the LTI system to the input $x_1(t) = e^{-t/2}u(t)$?
   ii. Find a differential equation that describes the LTI system.