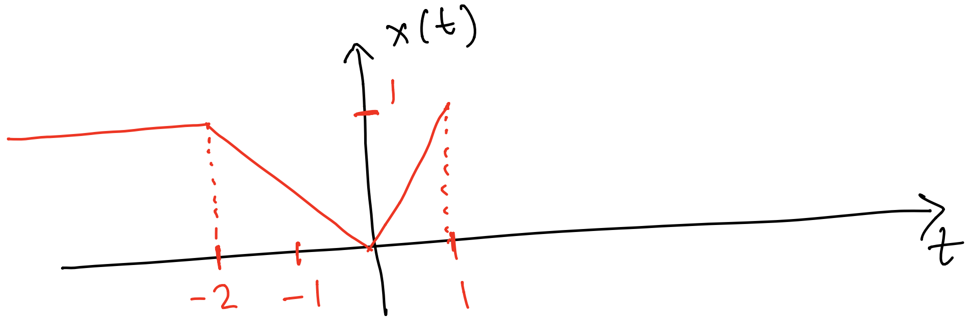
Problem 1: Basic Signal operations

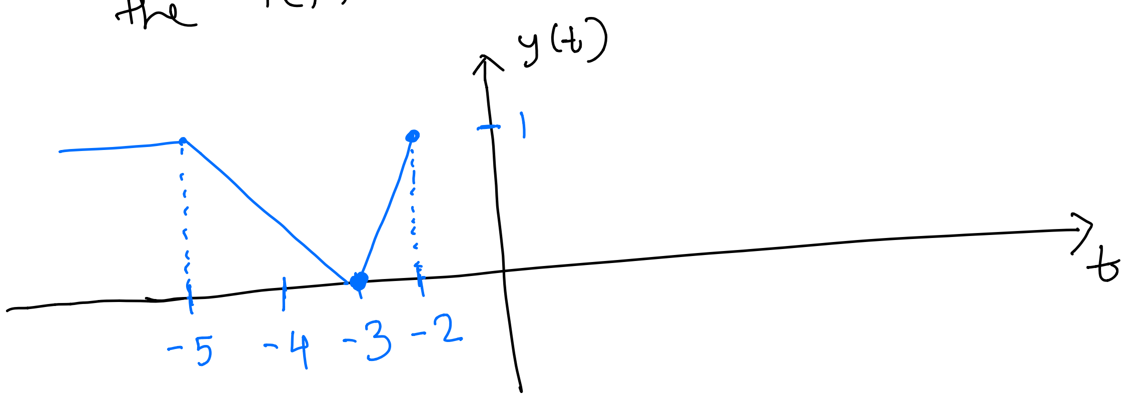
a)



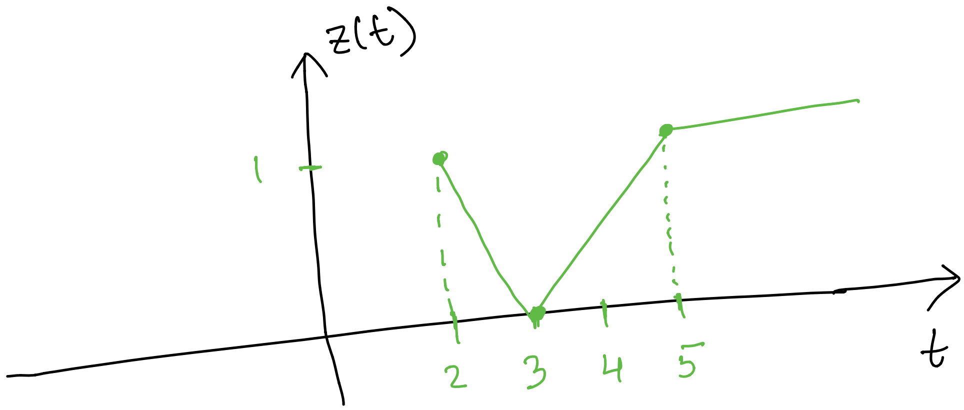
A. Let $y(t) = x(t+3)$

$z(t) = y(-t) = x(-t+3)$

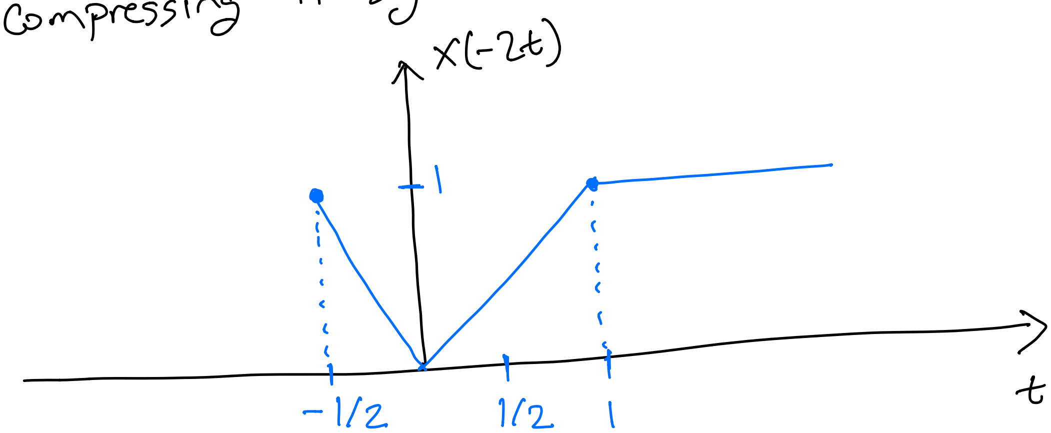
Using time-shifting operation, we can obtain $y(t)$ by shifting $x(t)$ by 3 units to the left



Using time-reversal operation, we can obtain $z(t)$ by reflecting $y(t)$ on the vertical axis



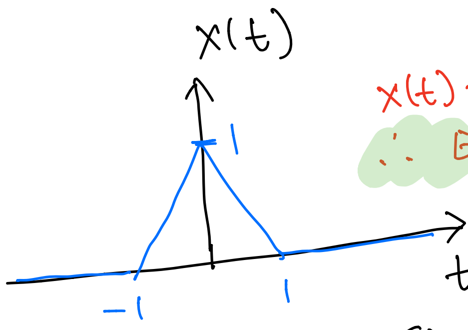
B. Using time-reversal and time-scaling operation, we can obtain $x(-2t)$ by reflecting $x(t)$ on the vertical axis and then compressing it by a factor of 2



b) By inspecting (a) and (b), we can observe that the signal in figure (b) is an advanced version of the signal in figure (a). Since the ramp down in (a) starts @ $t=1$ and the ramp down in (b) starts @ $t=0$, so signal in figure (b) is $x(t+1)$

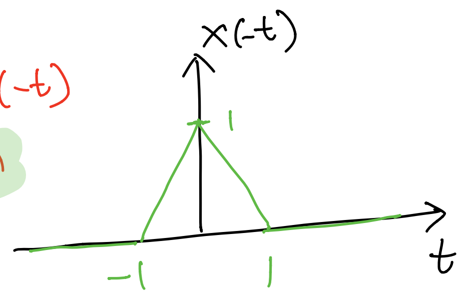
Problem 2: Even- Odd decomposition of Signals

a)

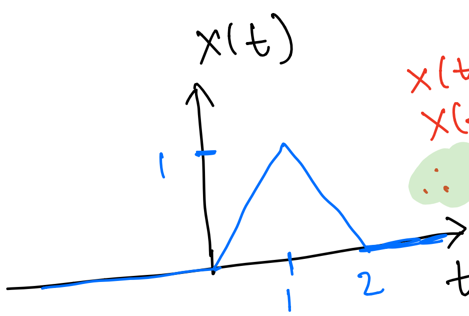


$x(t) = x(-t)$

$\therefore$ Even



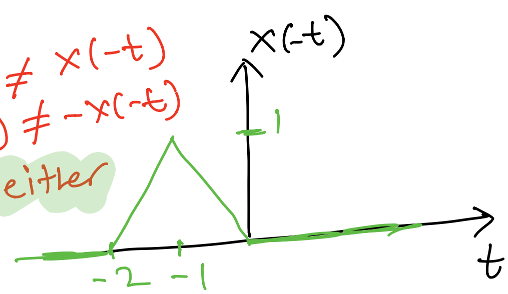
Signal 1



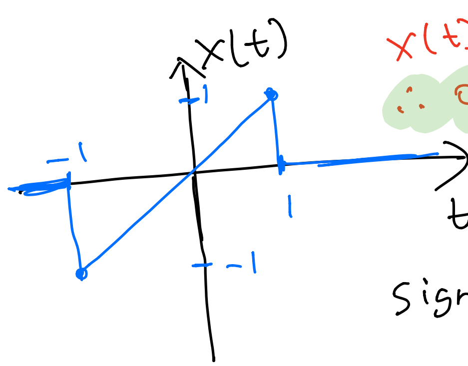
$x(t) \neq x(-t)$

$x(t) \neq -x(-t)$

$\therefore$ neither

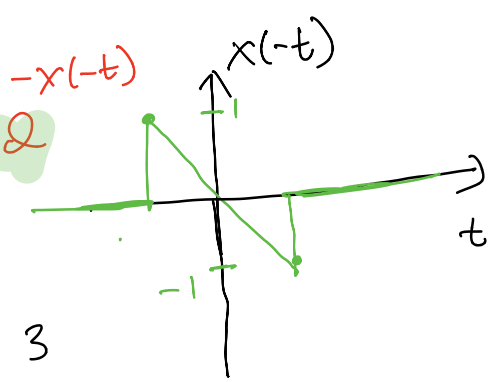


Signal 2



$x(t) = -x(-t)$

$\therefore$ odd



Signal 3

5) $x(t) = \cos(t) + \sin(t) + \sin(t)\cos(t)$

from lecture we know,

$$x_e(t) = \frac{1}{2}x(t) + \frac{1}{2}x(-t)$$

$$= \frac{1}{2}\cos(t) + \frac{1}{2}\sin(t) + \frac{1}{2}\sin(t)\cos(t)$$
$$+ \frac{1}{2}\cos(-t) + \frac{1}{2}\sin(-t) + \frac{1}{2}\sin(-t)\cos(-t)$$

Since $\cos(-t) = \cos(t)$, $\sin(-t) = -\sin(t)$

$$x_e(t) = \cos(t)$$

From lecture, we know

$$x_o(t) = \frac{1}{2}x(t) - \frac{1}{2}x(-t)$$

$$= \frac{1}{2}\cos(t) + \frac{1}{2}\sin(t) + \frac{1}{2}\sin(t)\cos(t)$$
$$-\frac{1}{2}\cos(-t) - \frac{1}{2}\sin(-t) - \frac{1}{2}\sin(-t)\cos(-t)$$

$$x_o(t) = \sin(t) + \sin(t)\cos(t)$$

Problem 3: Periodic Signals

Let $x(t)$ be a continuous time signal. We define $y_1(t)$ and $y_2(t)$ as follows

$$y_1(t) = x(2t)$$
$$y_2(t) = x\left(\frac{t}{2}\right)$$

1. From the problem statement, we know that $x(t)$ is periodic. Hence, there exists $T_0 > 0$ such that

$$x(t+T_0) = x(t) \quad \forall t$$

where T_0 is the fundamental period of $x(t)$

Now, for any t,

$$\begin{aligned} y_1(t+T_0/2) &= x(2(t+T_0/2)) \\ &= x(2t+T_0) \\ &= x(2t) \\ &= y_1(t) \end{aligned}$$

$\therefore$ $y_1(t)$ is periodic with fundamental period $T_0/2$

2. From problem statement, we know that $y_1(t)$ is periodic. Hence, there exists $T_0 > 0$ such that

$$y_1(t+T_0) = y_1(t) \quad \forall t$$

where T_0 is the fundamental period of $y_1(t)$

Now, for any t,

$$x(t+2T_0) = x\left(2\left(\frac{t}{2}+T_0\right)\right)$$

$$= y_1\left(\frac{t}{2}+T_0\right)$$

$$= y_1\left(\frac{t}{2}\right)$$

$$= x(t)$$

$\therefore$ $x(t)$ is periodic with fundamental period $2T_0$

3. From the problem statement, we know that $x(t)$ is periodic. Hence, there exists $T_0 > 0$ such that

$$x(t+T_0) = x(t) \quad \forall t$$

where T_0 is the fundamental period of $x(t)$

Now, for any t,

$$\begin{aligned} y_2(t+2T_0) &= x\left(\tfrac{1}{2}(t+2T_0)\right) \\ &= x(t/2 + T_0) \\ &= x(t/2) \\ &= y_2(t) \end{aligned}$$

$\therefore$ $y_2(t)$ is periodic with fundamental period $2T_0$

4. From problem statement, we know that $y_2(t)$ is periodic. Hence, there exists $T_0 > 0$ such that

$$y_2(t+T_0) = y_2(t) \quad \forall t$$

where T_0 is the fundamental period of $y_2(t)$

Now, for any t,

$$\begin{aligned} x(t+T_0/2) &= x\left(\tfrac{1}{2}(2t+T_0)\right) \\ &= y_2(2t+T_0) \\ &= y_2(2t) \\ &= x(t) \end{aligned}$$

$\therefore$ $x(t)$ is periodic with fundamental period $T_0/2$.