ECE102, Fall 2020 **Homework #2**

Signals & Systems Prof. J.C. Kao

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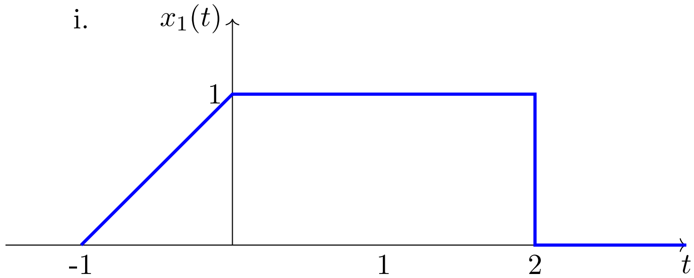
Due Friday, 23 Oct 2020, by 11:59pm to Gradescope.
Covers material up to Lecture 4.
100 points total.

1. (22 points) **Elementary signals.**

   (a) (9 points) Consider the signal $x(t)$ shown below.



Sketch the following:

i. $y(t) = x(t)\,(1 - u(t) + u(2t-1))$
ii. $y(t) = \int_{-\infty}^{t} \delta(\tau + 0.5)x(\tau)d\tau$
iii. $y(t) = x(t) - r(t+1) + r(t) + u(t)$

(b) (9 points) Evaluate these integrals:

i. $\int_{-\infty}^{\infty} f(t+1)\delta(t+1)dt$

ii. $\int_{t}^{\infty} e^{-2\tau}u(\tau-1)d\tau$

iii. $\int_{0^-}^{\infty} f(t)(\delta(t-1)+\delta(t+1)+\delta(t))dt$

(c) (4 points) Let b be a positive constant. Show the following property for the delta function:

$$\delta(bt) = \frac{1}{b}\delta(t)$$

Hint: Solve this problem by defining:

$$\delta(t) = \lim_{\Delta \to 0} \text{rect}_{\Delta}(t)$$

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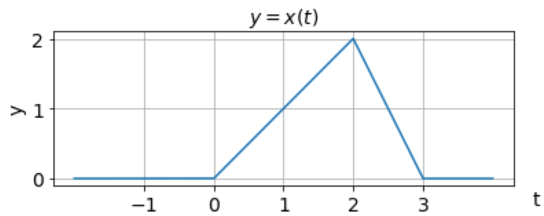
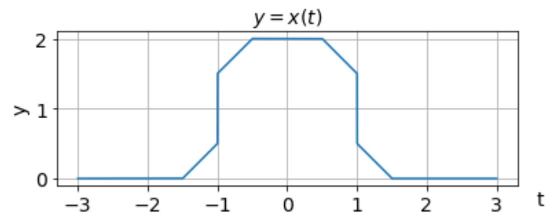
2. (23 points) **Expression for signals.**

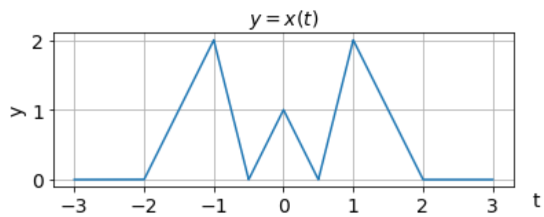
(a) (15 points) Write the following signals as a combination (sums or products) of unit triangles $\Delta(t)$ and unit rectangles $\mathrm{rect}(t)$.



a(i)

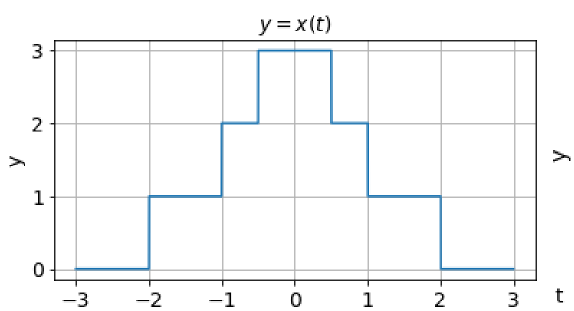


a(ii)

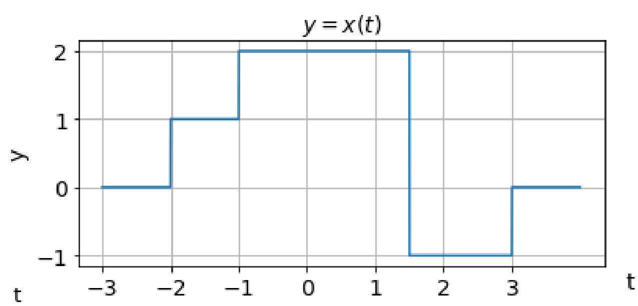


a(iii)

(b) (8 points) Express each of the signals shown below as sums of scaled and time shifted unit-step functions.



b(i)



b(ii)

3. (30 points) **System properties.**

(a) (20 points) A system with input $x(t)$ and output $y(t)$ can be time-invariant, causal or stable. Determine which of these properties hold for each of the following systems. Explain your answer.

i. $y(t) = |x(t)| + x(2t)$

ii. $y(t) = \int_{t-T}^{t+T} x(\lambda)d\lambda$, where T is positive and constant.

iii. $y(t) = (t+1)\int_{-\infty}^{t} x(\lambda)d\lambda$

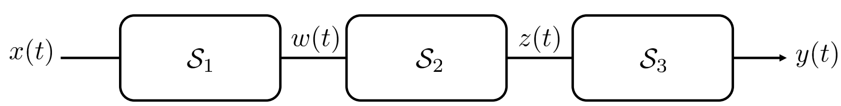
iv. $y(t) = 1 + x(t)\cos(\omega t)$

v. $y(t) = \frac{1}{1+x^2(t)}$

(b) (6 points) Consider the following three systems:

$$\mathcal{S}_1 : w(t) = x(3t)$$
$$\mathcal{S}_2 : z(t) = \int_{-\infty}^{t} w(\tau)d\tau$$
$$\mathcal{S}_3 : y(t) = \mathcal{S}_3(z(t))$$

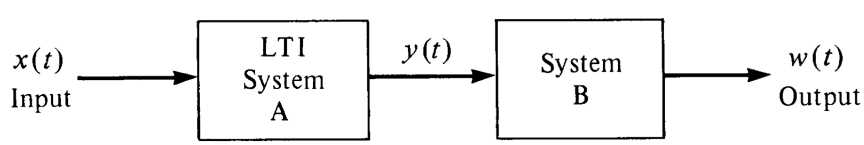
The three systems are connected in series as illustrated here:



Choose the third system $\mathcal{S}_3$, such that overall system is equivalent to the following system:

$$y(t) = \int_{-\infty}^{t-4} x(\tau)d\tau$$

(c) (4 points) Consider the cascade of two systems shown below. System B is the inverse of system A.



i. Suppose an input $x_1(t)$ produces $y_1(t)$ as System A output and an input $x_2(t)$ produces $y_2(t)$ as System A output. What is $w(t)$ if the input is such that $y(t)$, the output of System A, is $ay_1(t) + by_2(t)$ with a,b constants? *Hint: An inverse system cascaded with the original system is the identity system.*

ii. Suppose an input $x_1(t)$ produces $y_1(t)$ as System A output. What is $w(t)$ if $x(t)$ is such that $y(t) = y_1(t - \tau)$?

iii. Is System B an LTI system? Justify your answer.

4. (10 points) **Power and energy of complex signals**

   (a) (5 points) Is $x(t) = Ae^{j\omega t} + Be^{-j\omega t}$ a power or energy signal? A and B are both real numbers, not necessarily equal. If it is an energy signal, compute its energy. If it is a power signal, compute its power. (*Hint: Use the fact that the square magnitude of a complex number v is: $|v|^2 = v^* v$, where v^* is the complex conjugate of the complex number v.*)

   (b) (5 points) Is $x(t) = e^{-(1+j\omega)t}u(t-1)$ an energy signal or power signal? Again, if it is an energy signal, compute its energy. If it is a power signal, compute its power.

5. (15 points) **MATLAB**

   (a) (5 points) **Task 1**

   A complex sinusoid is denoted:
   $$y(t) = e^{(\sigma + j\omega)t}$$

   First compute a vector representing time from 0 to 10 seconds in about 500 steps (You can use `linspace`). Use this vector to compute a complex sinusoid with a period of 2 seconds, and a decay rate that reduces the signal level at 10 seconds to half its original value. What σ and ω did you choose? If your complex exponential is y, plot:

   ```
   >> plot(y);
   ```

   What is MATLAB doing here?

   (b) (5 points) **Task 2**

   Use the `real()` and `imag()` MATLAB functions to extract the real and imaginary parts of the complex exponential, and plot them as a function of time (plot them separately, you can use `subplot` for this task). This should look more reasonable. Label your axes, and check that your signal has the required period and decay rate.

   (c) (5 points) **Task 3**

   Use the `abs()` and `angle()` functions to plot the magnitude and phase angle of the complex exponential (plot them in the same figure). Scale the `angle()` plot by dividing it by `2*pi` so that it fits well on the same plot as the `abs()` plot (i.e. plot the angle in cycles, instead of radians, the function `angle(x)` returns the angle in radians).