ECE102, Fall 2020 **Midterm review**

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1. **Complex number, Signal and System Properties**

(a) Compute the magnitude and phase of z, where z is defined as

$$z = (\sqrt{3} + j)e^{-\frac{\pi}{3}j}$$

(b) Evaluate the expression

$$\frac{d}{dt}\left(\frac{1}{2}[1 - e^{-2t}]u(t) + \frac{1}{2}[e^{-2(t-3)} - 1]u(t-3)\right)$$

(c) For each of the following systems determine if they are: (1) Memoryless (2) Time invariant (3) Linear (4) Causal (5) Stable.

i. $y(t) = x(t-2) + x(2-t)$

ii. $y(t) = \cos(3t)x(t)$

iii. $y(t) = x(\frac{t}{3})$

iv. $y(t) = \int_{-\infty}^{2t} x(\tau)d\tau$

2. **Impulse response, LTI systems and Convolution**

Consider the LTI system characterized by the I/O relationship:

$$\text{System 1}: \quad y(t) = \int_{t-4}^{t} x(\tau)d\tau$$

(a) Determine the impulse response of the system $h_1(t)$.

(b) Is the system causal? Justify your answer using the impulse response.

(c) Is the system stable? Justify your answer using the impulse response.

(d) Determine and write a closed-form expression for the output $y(t)$ of System 1 for the input

$$x(t) = 4e^{-2t}\{u(t) - u(t-4)\}$$

(e) Determine the output $y(t)$ of System 1 for the input

$$x(t) = 3\{u(t-2) - u(t-6)\}$$

(f) Consider a second LTI system described by the following I/O relationship:

$$\text{System 2}: \quad y(t) = \int_{-\infty}^{t} e^{-\frac{1}{2}(t-\tau)} x(\tau) d\tau$$

i. Determine the impulse response for System 2, denoted by $h_2(t)$.

ii. Determine and write a closed-form expression for the output of System 2, $y(t)$, for the input

$$x(t) = 2\{u(t) - u(t-4)\} - 3\{u(t-6) - u(t-10)\}$$

(g) System 1 and System 2 are in series along with a third system (all three in series) which is a differentiator as described below. Determine a closed-form expression for the overall impulse response, denoted $h_0(t)$, for the series combination of all three systems.

$$\text{System 3}: \quad y(t) = \frac{d}{dt} x(t)$$

3. **LTI systems and Fourier series**

Consider the linear and time-invariant system described by the input-output relationship:

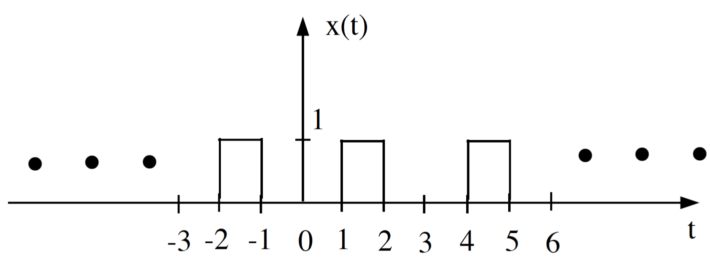
$$y(t) = \int_{t-1}^{t} x(\tau) d\tau$$

(a) Determine the impulse response $h(t)$ of this system.

(b) Is this system causal? You must use $h(t)$ to justify your answer.

(c) Is this system stable? You must use $h(t)$ to justify your answer.

(d) Consider that the input to this system is the periodic signal $x(t)$ below with period $T = 3$. Determine the fourier series coefficients, denoted a_k, $-\infty < k < \infty$, for $x(t)$.



Express your final answer for a_k as a closed-form function of k that works for all k.

(e) i. Plot several periods of the output of the system to the input signal $x(t)$ plotted above.

ii. Determine the fourier series coefficients, denoted b_k, $-\infty < k < \infty$, for $y(t)$. Express your final answer for b_k as a closed-form function of k.

iii. Is b_k real-valued for all k? Explain either why they are real-valued OR why they are not real-valued.

(f) Consider the signal

$$w(t) = x(t)\cos(\frac{2\pi}{3}t)$$

What is the period of $w(t)$? Determine the fourier series coefficients, denoted c_k, $-\infty < k < \infty$, for $w(t)$. Express your final answer for c_k as a closed-form function of k.

4. **Fourier coefficient and symmetry**

(a) Find the Fourier coefficient of the function: $f(x) = \begin{cases} 1 & \text{if } x < \frac{T}{2} \\ 0 & \text{if } x > \frac{T}{2} \end{cases}$ where T is the fundamental period of $f(x)$.

(b) Show that if $x(t)$ is a real periodic signal with fundamental period T, then

$$c_k = c_{-k}^*$$

where c_k are the fourier coefficients of $x(t)$.

(c) Show that if $x(t)$ is an even periodic signal with fundamental period T, then

$$c_k = c_{-k}$$

where c_k are the fourier coefficients of $x(t)$.