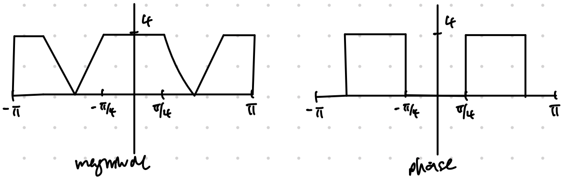
ECE 113 HW 8 Nevin Liang

1) a. $x(n) = \dfrac{\sin(\frac{\pi}{3}n)}{n} = \frac{\pi}{3}\text{sinc}(\frac{\pi}{3}n) \to X(e^{j\omega}) = \begin{cases} \pi & |\omega| < \pi/3 \\ 0 & \pi/3 \le |\omega| \le \pi \end{cases}$

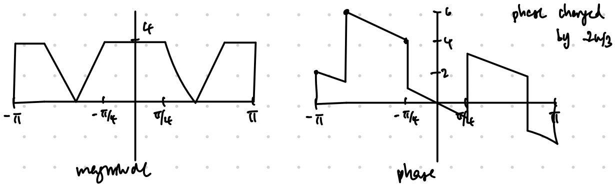
b. $y(n) = e^{-j\pi/4}\dfrac{\sin(\frac{\pi}{4}n)}{n} = e^{-j\pi/4}\cdot\frac{\pi}{4}\text{sinc}(\frac{\pi}{4}) \to Y(e^{j\omega}) = \begin{cases} \pi e^{-j\pi/4} & |\omega| < \pi/4 \\ 0 & \pi/4 \le |\omega| \le \pi \end{cases}$

c. $w(n) = e^{-j\pi n/4}\dfrac{\sin(\frac{\pi}{6}n)}{n} = \frac{\pi}{6}e^{-j\pi n/4}\text{sinc}(\frac{\pi}{6}) \to W(e^{j\omega}) = \begin{cases} \pi & -5\pi/12 < \omega < -\pi/12 \\ 0 & -\pi \le \omega \le -5\pi/12 \text{ \& } -\pi/12 \le \omega \le \pi \end{cases}$

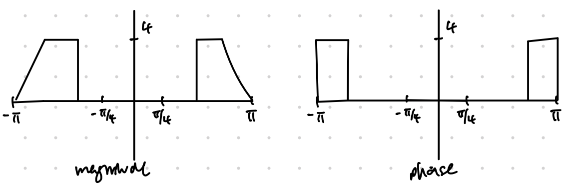
2. a. $\cos(\frac{\pi n}{2})x(n)$. $\cos(\frac{\pi}{2}n)$ = convolves w/ $\pi[\delta(\omega - \frac{\pi}{2}) + \delta(\omega + \frac{\pi}{2})]$ in F.D.



b. $\text{DTFT}[\cos(\frac{\pi}{2}n + \frac{\pi}{2})] = \pi e^{-2\pi/3\, j}[\delta(n - \frac{\pi}{2}) + \delta(n + \frac{\pi}{2})]$



c. $(-1)^n x(n)$'s DTFT is $e^{j\pi n}x(n)$ which is a $-\pi$ shift



3. $y(n) - \frac{3}{4}y(n-1) + \frac{1}{8}y(n-2) = x(n-1)$

a. $Y(z) - \frac{3}{4}z^{-1}Y(z) + \frac{1}{8}z^{-2}Y(z) = z^{-1}X(z) \to H(z) = \frac{Y}{X} = \dfrac{z^{-1}}{1 - \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2}} = \dfrac{z}{z^2 - \frac{3}{4}z + \frac{1}{8}}$

zeros @ $\boxed{z = 0}$ poles @ $(z - \frac{1}{4})(z - \frac{1}{2}) = 0 \to \boxed{z = \frac{1}{4}, \frac{1}{2}}$ $|z| > \frac{1}{2}$

b. sub $z = e^{j\omega} \to \boxed{H(e^{j\omega}) = \dfrac{e^{j\omega}}{e^{2j\omega} - \frac{3}{4}e^{j\omega} + \frac{1}{8}}}$

c. $\dfrac{z}{(z - \frac{1}{4})(z - \frac{1}{2})} = \dfrac{A}{z - \frac{1}{4}} + \dfrac{B}{z - \frac{1}{2}}$ $A = \frac{1}{4}/-\frac{1}{4} = -1$, $B = \frac{1}{2}/\frac{1}{4} = 2$

$\hookrightarrow \dfrac{-1}{z - \frac{1}{4}} + \dfrac{2}{z - \frac{1}{2}}$ ROC $|z| > \frac{1}{2}$

$h(n) = -(\frac{1}{4})^{n-1}u(n-1) + 2(\frac{1}{2})^{n-1}u(n-1) = \boxed{[-(\frac{1}{4})^{n-1} + 2(\frac{1}{2})^{n-1}]u(n-1)}$

4. $H(e^{j\omega}) = \begin{cases} -j & 0 < \omega < \pi \\ j & -\pi \le \omega < 0 \end{cases} = -j\,\text{rect}(\omega) + j\,\text{rect}(-\omega)$ $-\pi \le \omega \le \pi$

$H_1(e^{j\omega}) = \begin{cases} 0 & 0 < \omega \le \pi \\ j & -\pi < \omega \le 0 \end{cases} \to jH_1(e^{j(\omega + \frac{\pi}{2})}) = \begin{cases} 0 & \frac{\pi}{2} \le |\omega| \\ 1 & |\omega| < \frac{\pi}{2} \end{cases}$

$\hookrightarrow h_1(n) = e^{-j\frac{\pi}{2}(n-1)}\cdot\frac{1}{2}\text{sinc}(\frac{\pi}{2}n)$

$H_2(e^{j\omega}) = \begin{cases} -j & 0 \le \omega < \pi \\ 0 & -\pi \le \omega < 0 \end{cases} \to -jH_2(e^{j(\omega - \frac{\pi}{2})}) = \begin{cases} 1 & |\omega| \le \frac{\pi}{2} \\ 0 & |\omega| > \frac{\pi}{2} \end{cases}$

$\hookrightarrow h_2(n) = e^{j\frac{\pi}{2}(n-1)}\cdot\frac{1}{2}\text{sinc}(\frac{\pi}{2}n)$

$h(n) = \frac{1}{2}\text{sinc}(\frac{\pi}{2}n)[e^{j\frac{\pi}{2}(n-1)} - e^{-j\frac{\pi}{2}(n-1)}]$

$= \text{sinc}(\frac{\pi}{2}n)\cos(\frac{\pi}{2}(n-1))$

special case when n odd: $n = 1 \to h = \text{sinc}(\frac{\pi}{2})\cos(0) = 2/\pi$

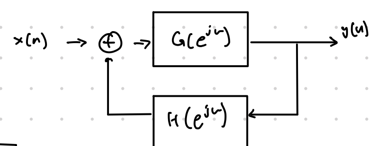
$n = 3 \to h = \text{sinc}(3\pi/2)\cos\pi = 2/3\pi$

n even: cos term is $0 \to 0$

$\boxed{h(n) = \begin{cases} 2/(\pi n) & n \text{ odd} \\ 0 & n \text{ even} \end{cases}}$

$h(n) \to \sum_{-\infty}^{\infty}|\frac{1}{n}|$ for n odd $= \infty$ so shifter not stable since not abs. summable.

5.



$y(n) = [y(n) * h(n) + x(n)] * g(n)$

$Y(e^{j\omega}) = [Y(e^{j\omega})H(e^{j\omega}) + X(e^{j\omega})]G(e^{j\omega})$

$\boxed{\dfrac{Y(e^{j\omega})}{X(e^{j\omega})} = \dfrac{G(e^{j\omega})}{1 - H(e^{j\omega})G(e^{j\omega})}}$

6. a) DTFT $\to X(e^{j\omega}) = e^{j\omega} + 2 - e^{-j\omega}$

b) 4 pt FFT @ $\omega = \frac{\pi}{2}k$ $k \in [0,1,2,3]$

$X(0) = 1 + 2 - 1 = 2$
$X(1) = j + 2 + j = 2 + 2j$
$X(2) = -1 + 2 + 1 = 2$
$X(3) = -j + 2 - j = 2 - 2j$

$\to X(k) = \{\boxed{2}, 2+2j, 2, 2-2j\}$

7. a) $X(k) = \{\boxed{1},-1,1,-1\} \rightarrow x(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(k)\, e^{2\pi kj/N}$ $k \in 0,1,2,3$

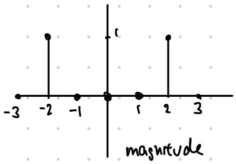
$x(n) = \frac{1}{4}\left(1 - e^{j\pi n/2} + e^{j\pi n} - e^{j3\pi n/2}\right)$

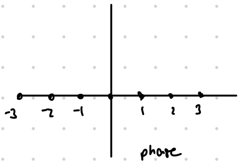
$x(0) = \frac{1}{4}(1-1+1-1) = 0$

$x(1) = \frac{1}{4}(1-j-1+j) = 0$

$x(2) = \frac{1}{4}(1+1+1+1) = 1$

$x(3) = \frac{1}{4}(1+j-1-j) = 0$





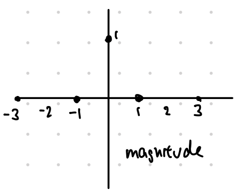
b) $Y(k) = (-1)^k X(k) = \{\boxed{1},1,1,1\} \rightarrow y(n) = \frac{1}{4}\left[1 + e^{j\pi/2\, n} + e^{j\pi n} + e^{j3\pi/2\, n}\right]$

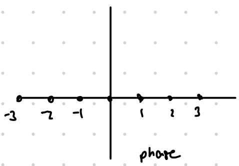
$y(0) = \frac{1}{4}(1+1+1+1) = 1$

$y(1) = \frac{1}{4}(1+j-1-j) = 0$

$y(2) = \frac{1}{4}(1-1+1-1) = 0$

$y(3) = \frac{1}{4}(1-j-1+j) = 0$





c) $Y(k) = e^{j\pi k/2} X(k)$

Time shift $x(n-n_0) \rightarrow X(k)e^{-j2\pi kn_0/N}$

$x(n+1) \rightarrow y(n) = \frac{1}{4}\left(1 - e^{j\pi(n+1)/2} + e^{j\pi(n+1)} - e^{j3\pi(n+1)/2}\right)$

$= \frac{1}{4}\left(1 - je^{j\pi n/2} - e^{j\pi n} + je^{j3\pi n/2}\right)$