EE 113 HW 5

1. $x(n) = \underbrace{u(n+3)}_{\text{right hand exp}} - \underbrace{\left(\tfrac{1}{2}\right)^n u(n-3)} = u(n+3) - \left(\tfrac{1}{2}\right)^3 \cdot \left(\tfrac{1}{2}\right)^{n-3} u(n-3)$

$= z^3 \cdot \frac{z}{z-1} - \left(\tfrac{1}{2}\right)^3 \cdot z^{-3} \frac{z}{z-1/2}$

$|z|>1 \qquad |z|> 1/2$

$|z|>1, \quad z \neq \pm\infty$

2. $y(n) = \underbrace{\left(\tfrac{1}{2}\right)^{n-1} u(n-2)} * \underbrace{\left[1 + \left(\tfrac{1}{3}\right)^{n+1}\right] u(n)}$

$\left(\tfrac{1}{2}\right) \cdot \left(\tfrac{1}{2}\right)^{n-2} u(n-2) \Rightarrow \frac{1}{2} z^{-2} \frac{z}{z-1/2} \quad |z|>1/2$

$u(n) + \left(\tfrac{1}{3}\right)^{n+1} u(n) \Rightarrow \frac{z}{z-1} + \frac{1}{3} \cdot \cancel{\left(\tfrac{1}{3}\right)^n u(n)}$

$z/z-1/3 \quad |z|>1$

$\frac{\cancel{z}}{2z^{\cancel{2}}(z-1/2)} \left(\frac{z}{z-1} + \frac{z}{3z-1} \right)$

$\lim_{z\to\infty} Y(z) = y(0) = 0\left(1+\tfrac{1}{3}\right) = \boxed{0}$

3. a. $\sum_{n=2}^{\infty} n^2 \left(\tfrac{1}{3}\right)^n \rightarrow$ z-transform of $n^2 u(n-2)$ @ $z=3$

$xn^2 =$ linear mod.

$u(n-2) \rightarrow z^{-2} \frac{z}{z-1} \quad |z|>1$

$nu(n-2) =$ derivative $\cdot -z$ $= \frac{d}{dz}\left(\frac{1}{z^2-z}\right) = -1(z^2-z)^{-2} (2z-1) \cdot -z = \frac{2z^2-z}{(z^2-z)^2}$

$n^2 u(n-2) = \frac{2z^2-z}{z^2(z-1)^2} = \frac{2z-1}{z(z-1)^2} = \frac{2}{(z-1)^2} - \frac{1}{z(z-1)^2}$

$= -4(z-1)^{-3} + \left(z(z-1)^2\right)^{-2} \cdot \left((z-1)^2 + z(2z-2)\right)$

@ $z=3 \rightarrow -4 \cdot \frac{1}{8} + (12)^{-2} \cdot (4 + 3 \cdot 4)$

$= -\frac{1}{2} + \frac{1}{144} \cdot 16 = \frac{1}{9} - \frac{1}{2} = \left(\frac{-7}{18}\right) \cdot -z = \frac{-7}{18} \cdot -3 = \frac{7}{6} = \boxed{1.167}$

b. $\sum_{n=0}^{\infty} n(\frac{1}{3})^n - \sum_{n=100}^{\infty} n(\frac{1}{3})^n$

$= \sum n(\frac{1}{3})^n u(n) - \sum n(\frac{1}{3})^n u(n-100)$

z transform @ $z=3$

$u(n) = \frac{z}{z-1}$ $|z|>1$ L1 mod.

$-z \cdot \frac{z-1-z}{(z-1)^2} = \frac{+z}{(z-1)^2} \rightarrow \frac{3}{4} = \boxed{0.75}$

$u(n-100) = z^{-101} \frac{z}{z-1} = \frac{1}{z^{100}(z-1)}$

$-z \frac{-(101 z^{100} - 100 z^{99})}{(z^{101} - z^{100})^2} = 0$ @ $z=3$.

$\boxed{0.75}$

c. $\sum_{n=999}^{\infty} n(\frac{1}{2})^n = \sum n(\frac{1}{2})^n u(n-999)$

$u(n-999) \rightarrow z^{-999} \frac{z}{z-1}$ $|z|>1$

linear mod: $-z \frac{-999 z^{998} - 998 z^{997}}{(z^{999} - z^{998})^2}$

$= \boxed{0}$ 1.1

4. a. z.i $x=0 \rightarrow \lambda^2 - \lambda + 1/4 = 0 \rightarrow \lambda = 1/2$

$y(n) = c_1 (1/2)^n + c_2 n \cdot (1/2)^n$

$c_1 \cdot 4 + c_2 \cdot n \cdot 4 = -7/2$
$c_1 2 + c_2 n = 0$ $\} \; c_1 = c_2 = 7/8$

$\boxed{y(n) = \left(\frac{7}{8} \cdot (\frac{1}{2})^n + \frac{7}{8} n (\frac{1}{2})^n\right) u(n)}$

b. $h(n) * x(n)$

if $x(n) = \delta(n) \rightarrow h(n) - h(n-1) + \frac{1}{4} h(n-2) = \delta(n-1)$

$H(z) - H(z) z^{-1} + H(z) \cdot \frac{1}{4} z^{-2} = z^{-1}$

$H(z) = \frac{z}{z^2 - z + 1/4}$

if $x(n) = (\frac{1}{2})^{n-1} u(n) \rightarrow \frac{1}{2} \cdot \frac{z}{z - \frac{1}{2}}$ $|z| > \frac{1}{2}$

$y_{zs}(n) = \frac{1}{2} \cdot \frac{z^2}{(z - \frac{1}{2})^3}$ $|z| > \frac{1}{2}$

5. $X(z) = \dfrac{12z^3 - 4z^2 + 12z - 3}{(z-\frac{1}{2})^2\, 12\, (z-\frac{1}{3})(z-\frac{1}{4})}$ → partial fraction decomposition

$$\frac{A}{z-1/3} + \frac{B}{z-1/4} + \frac{C}{z-1/2} + \frac{D}{(z-1/2)^2}$$

if $z = \frac{1}{3}$ $\quad \frac{1}{12} = A \cdot \frac{1}{12} \cdot \frac{1}{36} \rightarrow A = 36$

$z = \frac{1}{2}$ $\quad \frac{1}{8} - \frac{1}{12} + \frac{1}{2} - \frac{1}{4} = D \cdot \frac{1}{6} \cdot \frac{1}{4} \rightarrow \frac{7}{24} = D \cdot \frac{1}{24} \quad D = 7$

$z = \frac{1}{4}$ $\quad \frac{1}{64} - \frac{1}{48} = B\left(-\frac{1}{12}\right)\left(\frac{1}{4}\right)^2 \quad \frac{-1}{12} = -B\frac{1}{12},\ B = 1$

$$X(z) = \frac{36}{z-1/3} + \frac{1}{z-1/4} - \frac{37}{z-1/2} + \frac{7}{(z-1/2)^2}$$

1: $|z| > \frac{1}{2}$ $\quad x(n) = 36\cdot\left(\frac{1}{3}\right)^{n-1}u(n-1) + \left(\frac{1}{4}\right)^{n-1}u(n-1) - 36\cdot\left(\frac{1}{2}\right)^{n-1}u(n-1) + 7(n-1)\left(\frac{1}{2}\right)^{n-2}u(n-1)$

$|z| < \frac{1}{4}$ $\quad x(n) = -36\cdot\left(\frac{1}{3}\right)^{n-1}u(-n) - \left(\frac{1}{4}\right)^{n-1}u(-n) + 36\cdot\left(\frac{1}{2}\right)^{n-1}u(-n) - 7(n-1)\left(\frac{1}{2}\right)^{n-1}u(-n)$

$\frac{1}{3} < |z| < \frac{1}{2}$ $\quad x(n) = 36\left(\frac{1}{3}\right)^{n-1}u(n-1) + \left(\frac{1}{4}\right)^{n-1}u(n-1) + 36\left(\frac{1}{2}\right)^{n-1}u(-n) - 7(n-1)\left(\frac{1}{2}\right)^{n-1}u(-n)$

$\frac{1}{4} < |z| < \frac{1}{3}$ $\quad x(n) = 36\left(\frac{1}{3}\right)^{n-1}u(-n) + \left(\frac{1}{4}\right)^{n-1}u(n-1) + 36\cdot\left(\frac{1}{2}\right)^{n-1}u(-n) - 7(n-1)\left(\frac{1}{2}\right)^{n}u(-n)$