HW #2 NEVIN L

1. $x(n) = \cos\left(\frac{\pi}{3}n + \frac{\pi}{6}\right) \sin\left(\frac{\pi}{6}n + \frac{\pi}{8}\right)$

$T\left(\cos\left(\frac{\pi}{3}n + \frac{\pi}{6}\right)\right) = 2\pi \cdot \frac{3k}{\pi} = 6k$

$T\left(\sin\left(\frac{\pi}{6}n + \frac{\pi}{8}\right)\right) = 2\pi \cdot \frac{6k}{\pi} = 12k$ $\rightarrow \boxed{12}$

2. $x(n) = e^{j5\pi n/12} + e^{j\pi n/12} = \cos\left(5\pi n/12\right) + j\sin\left(5\pi n/12\right) + \cos\left(\pi n/12\right) + j\sin\left(\pi n/12\right)$

$= 2\cos\left(\frac{\pi n}{4}\right)\cos\left(\frac{\pi n}{6}\right) + 2j\sin\left(\frac{\pi n}{4}\right)\cos\left(\frac{\pi n}{6}\right) = e^{j\pi n/4} \cdot 2 \cdot \cos\left(\frac{\pi n}{6}\right)$

$\boxed{w_0 = \pi/4}$ $\boxed{w_1 = \pi/6}$ $\boxed{A = 2}$ x(n) is periodic with period $\boxed{24}$

$e^{j\pi n/4}$ has period $2\pi \cdot \frac{4k}{\pi} = 8k$

$\cos\left(\frac{\pi n}{6}\right)$ has period $2\pi \cdot \frac{6k}{\pi} = 12k$ $\rightarrow 24$

3. a. $y(n) = x(-n^2)$

$y_k(n) = x(-n^2 - k)$

$y(n-k) = x(-(n-k)^2) = x(-n^2 - k^2 + 2nk)$] not the same

$\boxed{\text{not TI}}$

b. $y(n) = x(3n-2)$

$y_k(n) = x(3n-2-k)$

$y(n-k) = x(3(n-k)-2) = x(3n-2-3k)$] not the same

$\boxed{\text{not TI}}$

c. $y(n) = x(-n/4)$ $n = 0, \pm 4, \pm 8$

$y_k(n) = x(-n/4 - k)$ where $k \equiv 0 \pmod 4$

$y(n-k) = x\left(\frac{-n+k}{4}\right) = x\left(-n/4 + k/4\right)$] not same

$\boxed{\text{not TI}}$

4. a. $\boxed{\text{No.}}$ ex: $y(n) = x(n) + k$. this is causal: y depends on present values of x.

this is not relaxed: y is nonzero for 0 values of x.

b. $\boxed{\text{Yes}}$ relaxed $\leftrightarrow$ $y(n)=0$ if $x(n)=0$ for $n<n_0$

y cannot depend on future values of x b/c otherwise

if this future value is $n>n_0$ then $x(n) \neq 0$ but $y(n)=0$

this is not possible b/c if $x(n) \neq 0$ then $y(n) \neq 0$.

c. $\boxed{\text{Yes}}$ linear $\leftrightarrow$ input signal $x(n)=0$ implies output signal $y(n)=0$. (1)

causal $\leftrightarrow$ $y(n)$ depends only on present & past values of $x(n)$. (2)

assume $x(n)=0$ for $n<n_0$ and only $n<n_0$. $y(n)$ must be 0 for $n<n_0$ by (1). ~~in addition, for $n \geq n_0$~~; this is relaxed $\square$.

5. $y(n) = x(n^2-1)$

$y_k(n) = x(n^2-1-k)$

$y(n-k) = x(n^2-2k+k^2-1) \neq x(n^2-1-k)$ $\boxed{\text{Not TI}}$

$\cancel{\mathcal{T}}(a x_1(n^2-1) + b x_2(n^2-1)) = y(n)$

$= a \cdot y_1(n) + b y_2(n)$ $\rightarrow$ $\boxed{\text{Linear}}$

$y(n) = x(n^2-1)$ so every value of output y is a different time's value of x input.

if x bounded $\rightarrow$ y bounded $\Rightarrow$ $\boxed{\text{stable}}$

$y(4) = x(15)$ $\boxed{\text{not causal}}$ b/c $4<15$