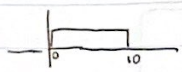
Chapter 5 Impulse Response Sequence

1. (a) $h(n) = u(n) - u(n-10)$

this is finite bc it looks like



$\boxed{\text{FIR}}$

(b) $h(n) = \left(\frac{1}{2}\right)^n \cdot u(n-3)$

this, while converging to 0, is infinite b/c $u(n-3)$ is unit step from $3 \to \infty$ $\boxed{\text{IIR}}$

$h(0) = \frac{1}{2}h(-1) + 1 = 1$ ← relaxed.

(c) $\{ y(n) = \frac{1}{2} y(n-1) + x(n)$, relaxed $\}$

$h(1) = \frac{1}{2}h(0) + 0 = \frac{1}{2}$

let $y(n) = h(n)$ and $x(n) = \delta(n)$

$h(2) = \frac{1}{2}h(1) + 0 = \frac{1}{4}$

$h(n) = \frac{1}{2}h(n-1) + \delta(n)$

$\vdots$

$h(n) = \left(\frac{1}{2}\right)^n \cdot u(n)$

we see that ~~h(n) = 2u(n)~~ since relaxed. $\boxed{\text{IIR}}$ $u(n)$ is IIR

2. ~~$y_1(n)$~~ =

$x \to a_1 x_1 + a_2 x_2$

$y(n) = \left(a_1 x_1(3n-1) + a_2 x_2(3n-1)\right) \cdot \cos\left(\frac{\pi}{3}n\right) u(n+5)$

$= a_1 x_1(3n-1)\cos\left(\frac{\pi}{3}n\right)u(n+5) + a_2 x_2(3n-1)\cos\left(\frac{\pi}{3}n\right)u(n+5)$

$= a_1 \cdot y_1(n) + a_2 \cdot y_2(n)$ ✓ $\boxed{\text{linear}}$

if $n = 5$, $y(5) = x(14) \ldots$ not causal. X

shifting x by k → $x(3n-1-k)\cos\left(\frac{\pi}{3}n\right)u(n+5)$

shift y by k → $x(3(n-k)-1)\cos\left(\frac{\pi}{3}(n-k)\right)u(n-k+5)$

} unequal X not TI

$\max(y(n)) = \max(x(3n-1))^{\max}(\cos\frac{\pi}{3}n)\max(u(n+5)) = \max(x) \cdot 1 \cdot 1 = \max(x)$ ✓ $\boxed{\text{BIBO stable}}$

3. a) $h(n) = \delta(n-1) + \frac{1}{3}\delta(n-2) - \frac{1}{4}\delta(n-3)$

input $= x(n) = e^{j\frac{\pi}{3}n}u(n)$

$y(n) = x(n-1) + \frac{1}{3}x(n-2) - \frac{1}{4}x(n-3)$

$$\boxed{e^{j\frac{\pi}{3}(n-1)}u(n-1) + \frac{1}{3}e^{j\frac{\pi}{3}(n-2)}u(n-2) - \frac{1}{4}e^{j\frac{\pi}{3}(n-3)}u(n-3)}$$

b) $2\cos\left(\frac{\pi}{6}(n-1)+\frac{\pi}{4}\right)u(n-1) + \frac{1}{3}\cdot 2\cos\left(\frac{\pi}{6}(n-2)+\frac{\pi}{4}\right)u(n-2) - \frac{1}{4}\cdot 2\cos\left(\frac{\pi}{6}(n-3)+\frac{\pi}{4}\right)u(n-3)$

4. a) $u(n) * \left(\frac{1}{2}\right)^n u(n-1) = \sum_{k=-\infty}^{\infty} u(k)\cdot\left(\frac{1}{2}\right)^{n-k}u(n-k-1)$

$= \sum_{k=0}^{\infty} \cancel{u(k)}\cdot\left(\frac{1}{2}\right)^{n-k}u(n-k-1) = \sum_{k=0}^{n-1}\left(\frac{1}{2}\right)^{n-k}\cancel{u(n-k-1)}$

$= \left(\frac{1}{2}\right)^n + \left(\frac{1}{2}\right)^{n-1} \cdots + \left(\frac{1}{2}\right)^1 = \frac{\frac{1}{2}\left(1-\frac{1}{2}^n\right)}{1-\frac{1}{2}} = \boxed{\left(1-\frac{1}{2^n}\right)u(n-1)}$

$n \geq 1$

b) $u(-n) * \left(\frac{1}{2}\right)^n u(n-1) = \sum_{k=-\infty}^{\infty} u(-k)\cdot\left(\frac{1}{2}\right)^{n-k}u(n-k-1) = \cancel{\sum_{k=-\infty}^{0}\left(\frac{1}{2}\right)^{n-k}} + \cancel{\sum_{k=0}^{n-1}\left(\frac{1}{2}\right)^{n-k}}$

if $n \leq 1$

if $n > 1$ $\quad \sum_{k=-\infty}^{0}\left(\frac{1}{2}\right)^{n-k} \rightarrow \left(\frac{1}{2}\right)^n + \left(\frac{1}{2}\right)^{n+1} + \left(\frac{1}{2}\right)^{n+2} \cdots \left(\frac{1}{2}\right)^{n+n} = \frac{\left(\frac{1}{2}\right)^n}{1-\frac{1}{2}} = \boxed{\left(\frac{1}{2}\right)^{n-1}}$

if $n \leq 1$ $\quad \sum_{k=-\infty}^{n-1}\left(\frac{1}{2}\right)^{n-k} \rightarrow \left(\frac{1}{2}\right)^1 + \left(\frac{1}{2}\right)^2 + \cdots \left(\frac{1}{2}\right)^{n+n} \cdot \frac{\frac{1}{2}}{1-\frac{1}{2}} = \boxed{1}$

$$\boxed{y(n) = \begin{cases} 1 & n \leq 1 \\ \left(\frac{1}{2}\right)^{n-1} & n > 1 \end{cases}}$$

c. $u(2n) * (\frac{1}{2})^n u(n) = \sum_{k=-\infty}^{\infty} u(2k)\cdot(\frac{1}{2})^{n-k} u(n-k)$

$= \sum_{k=0}^{\infty} (\frac{1}{2})^{n-k} u(n-k) = \sum_{k=0}^{n} (\frac{1}{2})^{n-k} = \frac{1(1-(\frac{1}{2})^{n+1})}{1-\frac{1}{2}}$

$= 2(1-(\frac{1}{2})^{n+1})$

$= \boxed{2-(\frac{1}{2})^n \quad u(n)}$

↑ b/c $n \geq 0$

otherwise 0.

5. $[\frac{1}{2}\delta(n+1) - \frac{1}{3}\delta(n)] * \underbrace{(\frac{1}{2})^{n-1}u(n) * (\frac{1}{3})^n u(n-2)}$

$\sum_{k=-\infty}^{\infty} (\frac{1}{2})^{k-1}u(k)\,(\frac{1}{3})^{n-k}u(n-k-2)$

if ~~$n \geq 2$~~ $\sum_{k=0}^{n-2} (\frac{1}{2})^{k-1} \cancel{u(k)} \cdot (\frac{1}{3})^{n-k}$

$= (\frac{1}{2})^{k-1}\cdot 3^{k-n}$

$= \frac{(3/2)^k}{\frac{1}{2}\cdot 3^n}$

$2\cdot 3^{-n}\cdot \sum_{k=0}^{n-2} (\frac{3}{2})^k = 2\cdot 3^{-n}\cdot \frac{1(1-(\frac{3}{2})^{n-1})}{1-\frac{3}{2}}$

$= -4\cdot 3^{-n}\cdot(1-(\frac{3}{2})^{n-1})$

$= -4\cdot 3^{-n} + 4\cdot 3^{-n}\cdot 3^{n-1}\cdot 2^{1-n}$

$= -4\cdot 3^{-n} + 4\cdot 3^{-1}\cdot 2^{1-n}$

$= \boxed{-4\cdot 3^{-n} + 3^{-1}\cdot 2^{3-n}}$

$[\frac{1}{2}\delta(n+1) - \frac{1}{3}\delta(n)] * [-4\cdot 3^{-n} + 3^{-1}\cdot 2^{3-n}]$

$= -2\cdot 3^{-(n+1)} + \frac{4}{3}\cdot 3^{-(n)} + \frac{1}{6}\cdot 2^{3-n-1} + \frac{1}{9}\cdot 2^{3-n}$ $\boxed{-2/9}$

$= -2\cdot 3^{-n-1} + 4\cdot 3^{-n-1} + \frac{1}{3}\cdot 2^{1-n} - \frac{1}{9}\cdot 2^{3-n} = \boxed{2\cdot 3^{-n-1} + \cancel{\frac{?}{9}}\cdot 2^{-n}}$