EE 113

1. $X(z) = \frac{1}{(z-1/3)^2} \cdot \frac{1}{z-1/5}$ ROC $= R_x$ $\qquad H(z) = \frac{z^{-1}}{(z-1/6)^2}$ ROC $= R_h$

a. $R_x = \{ \frac{1}{5} < |z| < 1/3 \}$ $\quad R_h = \{ |z| > \frac{1}{6} \}$

$R_x \cap R_h = \frac{1}{5} < |z| < 1/3$ since $\not\ni \infty$ $y(n)$ not causal

b. $R_x = \{ |z| > 1/3 \}$ $\quad R_h = \{ |z| > \frac{1}{6} \}$

$R_x \cap R_h = \{ |z| > 1/3 \}$

~~since~~ →

$Y(z) = \frac{1}{(z-1/3)^2(z-1/5)} \cdot \frac{1}{z(z-1/6)^2} = 0$ as $z \to \infty$

$y(0) = 0$ causal

c. $\{|z| < 1/5\} \cap \{|z| < \frac{1}{6}\} \rightarrow |z| < \frac{1}{6}$

since $\not\ni \infty$ $y(n)$ not causal

2. a. $\mathcal{Z}\left[ \left(\frac{1}{2}\right)^{n-1} u(n-3) \right] = \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^{n-3} u(n-3) = \left(\frac{1}{2}\right)^2 z^{-3} \frac{z}{z-1/2}$ $\quad |z| > \frac{1}{2}$

$H(z) = \frac{1}{4z^2(z-1/2)}$ $\quad |z| > 1/2$

b. $n\left(\frac{1}{2}\right)^{2n-1} u(n-2) = \frac{1}{8} n \left(\frac{1}{4}\right)^{n-2} u(n-2)$

$\mathcal{Z}[\;] = \frac{1}{8} \cdot -z \frac{d}{dz}\left( z^{-2} \frac{z}{z-1/4} \right] = -\frac{z}{8} \frac{-2z+1/4}{(z^2-z/4)^2} = \frac{2z^2 - z/4}{8(z^2-z/4)^2} = \frac{2z - z/4}{8z(z-1/4)^2}$ $\quad |z| > \frac{1}{4}$

c. $2\left(\frac{1}{2}\right)^n \cos\left(\frac{\pi}{3} n\right) u(n) \longleftrightarrow 2 \frac{1 - \frac{1}{2}\cos(\pi/3) z^{-1}}{1 - 2 \cdot \frac{1}{2}\cos(\pi/3) z^{-1} + \left(\frac{1}{2}\right)^2 z^{-2}} = \frac{2\left(1-\frac{1}{4}\right)z^{-1}}{1 - \frac{1}{2}z^{-1} + \frac{1}{4}z^{-2}}$

$= \frac{2z^2 - z/2}{z^2 - z/2 + 1/4}$ $\quad |z| > \frac{1}{2}$

3 a. $z^{?} \frac{1}{(z-\frac{1}{2})(z+\frac{1}{3})}$ $|z|>1/2$ $= z^{?} \frac{6}{5}\left[\frac{1}{z-\frac{1}{2}} - \frac{1}{z+1/3}\right]$

$z^{-1}[H(z)] = \boxed{\frac{6}{5}\left[\left(\frac{1}{2}\right)^{n+1}u(n+1) - \left(-\frac{1}{3}\right)^{n+1}u(n+1)\right]}$ $\frac{z+\frac{1}{3}-z+\frac{1}{2}}{(z-\frac{1}{2})(z+\frac{1}{3})}$ $\frac{5/6}{(z-\frac{1}{2})(z+\frac{1}{3})}$

right side b/c outward pointing

b. $\frac{1}{z^2+1/4} = \frac{1}{z+1/2j}a + \frac{1}{z-1/2j}b$

$a(z-1/2j)+b(z+1/2j)=1$ $a=j$ $b=j$

$h(n) = -2 \cdot |\frac{1}{2}|^{n-1}\cos(-\frac{\pi}{2}(n-1)+\frac{\pi}{2})\,u(-n)$

$= -2\,|\frac{1}{2}|^{n-1}\cos(-\frac{\pi}{2}n+\pi)u(-n) = -\frac{1}{2}^{n-2}\cos(-\frac{\pi}{2}n+\pi)u(-n)$

$= (\frac{1}{2})^{n-2}\cos(\frac{\pi}{2}n)\,u(-n)$

$\uparrow$ $\cos(a+\pi) = -\cos(a)$.

c. $H(z) = \frac{z+1/3}{(z-1/2)(z+1/4)}$ $\frac{1}{4}<|z|<\frac{1}{2}$

LSided

$\downarrow = \frac{A}{z-1/2} + \frac{B}{z+1/4} = \frac{10/9}{z-1/2} - \frac{1/9}{z+1/4} \rightarrow h(n) = -\frac{10}{9}\left(\frac{1}{2}\right)^{n-1}u(-n)$
$-\frac{1}{9}(-1/4)^{n-1}u(n-1)$

R sided

if $z=-\frac{1}{4}$ $\frac{1}{12} = B(-\frac{3}{4}) \Rightarrow B=-1/9$

$z=1/2$ $5/6 = A(3/4) \rightarrow A = 10/9$

4. LTI + causal $\rightarrow$ relaxed. $x(n) = \{1, 1, -1\}$ so Y is causal $\rightarrow y(n)=0$ $n<0$

a.) $y_e(n) = -y_o(n)$ $n<0$ since $y_e(n)+y_o(n)=0$ $n<0$.

$y(n) = 2y_o(n) \rightarrow 2n\left(\frac{1}{4}\right)^{|n-1|}u(|n|-2) \rightarrow 2n\left(\frac{1}{4}\right)^{|n-1|}u(n-2)$ since $n>0$.

$Z[y(n)] = \frac{1}{2} - z\frac{d}{dz}\left[\frac{\cancel{z^{?}}\, z^{-1}}{z-1/4}\right] = -\frac{1}{2}z\frac{-(2z-1/4)}{(z^2-1/4z)^2} = \frac{z-1/8}{z(z-1/4)^2}$ $|z|>1/4$

$H(z) = \frac{z(z-1/8)}{(z-1/4)^2(z-1)}$ $|z|>1/4$.

b. $\frac{z^2 - 7/8}{(z-1/4)^2(z-1)} = \frac{A}{z-1/4} + \frac{B}{(z-1/4)^2} + \frac{C}{z-1}$

$z = \frac{1}{4}$ $\frac{1}{?} = B \frac{-1}{4} \rightarrow B = \frac{-1}{24}$ $z=1$ $\frac{7}{8} = C \frac{9}{16}$ $C = 14/9$

$A = -5/9$

$\frac{-5/9}{z-1/4} + \frac{-1/24}{(z-1/4)^2} + \frac{14/9}{(z-1)} \leftrightarrow h(n) = \boxed{-\frac{5}{9}(1/4)^{n-1}u(n-1) - \frac{1}{24}(n-1)(1/4)^{n-1}u(n-1) + 14/9\, u(n-1)}$

c. $E = \infty$ $14/9\, u(n-1)$ goes forever to $\infty$ = $14/9$ no converging

d. $\lim_{n \to \infty} \frac{1}{2N+1} \sum_{k=-N}^{N} |h(k)|^2$ = sum of 3 terms

the ones that converge will be finite $\frac{\text{finite \#}}{2N+1}$ as

$N \to \infty$ $= 0$ , so all thats left is $\lim_{n \to \infty} \frac{1}{2n+1} \sum_{k=0}^{N} \left(\frac{14}{9}\right)^2$

$= \frac{N}{2N+1} \left(\frac{14}{9}\right)^2 = \frac{14 \cdot 14^7}{9 \cdot 9 \cdot 2} = \boxed{\frac{98}{81}}$