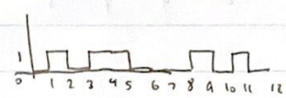
ECE 113

1.2) (a) speech signal captured by a mic is $\boxed{\text{DISCRETE}}$ because a mic samples @ 120 hz for example, so every 1/120 of a second is a discrete value that samples a continuous amplitude of speech.

(b) $\boxed{\text{digital}}$ because text files are stored in bits: either 0 or 1. the binary representation looks like 0100101 which is discrete values for each y-value (the bit value) and x-value (the index of the bits).



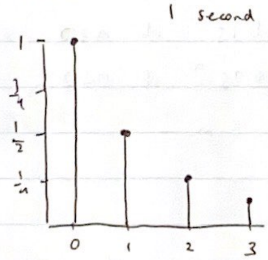
something like this

(c) $\boxed{\text{Continuous}}$. both the x-value (altitude) and the y-value (air pressure) are continuous variables (not integers, floats)

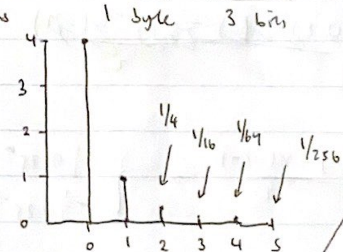
(d) the # of male votes is a discrete variable (integers). the election cycle # is also discrete (integer = 0, 1, 2, 3). So, the signal is $\boxed{\text{DIGITAL}}$. note: the # of male votes is bounded by 8 billion.

1.14) $2.1 \text{ seconds} \cdot \frac{10^6 \text{ us}}{1 \text{ second}} \cdot \frac{1 \text{ byte}}{1 \text{ us}} \cdot \frac{8 \text{ bits}}{1 \text{ byte}} \cdot \frac{1 \text{ sample}}{3 \text{ bits}} = 5.6 \cdot 10^6$ samples.

1.17) a)



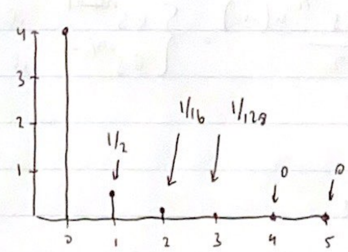
b)



it can also be $\boxed{\text{discrete}}$

if # of male votes is unbounded aka ~~not~~ no set limit

c)



1.19) $\mathcal{E}_z \triangleq \sum_{n=-\infty}^{\infty} z^2(n) = 4^2+\left(\frac{1}{2}\right)^2+\left(\frac{1}{16}\right)^2+\left(\frac{1}{128}\right)^2+0+0 = \boxed{16.25397}$ J

$$\mathcal{E}_x \triangleq \sum_{n=-\infty}^{\infty} x^2(n) = 1^2+\left(\frac{1}{2}\right)^2+\left(\frac{1}{4}\right)^2+\left(\frac{1}{8}\right)^2$$

$$= 1+\frac{1}{4}+\frac{1}{16}+\frac{1}{64} = \boxed{1.328125} \text{ J}$$

$$\mathcal{E}_y \triangleq \sum_{n=-\infty}^{\infty} y^2(n) = 4^2+1^2+\left(\frac{1}{4}\right)^2+\left(\frac{1}{16}\right)^2+\left(\frac{1}{64}\right)^2+\left(\frac{1}{256}\right)^2$$

$$= 16+1+\frac{1}{16}+\frac{1}{256}+\frac{1}{4096}+\frac{1}{65536} = \boxed{17.067} \text{ J}$$

2.7) a)

$$x(n) = \left(\frac{1}{2}\right)^n \cdot e^{j\left(\frac{\pi}{3}n+\frac{\pi}{4}\right)} \cdot e^{\frac{\pi}{3}j} = \left(\frac{1}{2}\right)^n \cdot e^{j\left(\frac{\pi}{3}n+\frac{7\pi}{12}\right)}$$

$$\boxed{\rho(n) = \left(\frac{1}{2}\right)^n} \qquad \boxed{\theta(n) = \frac{\pi}{3}n+\frac{7\pi}{12}}$$

b) $$\rho_e(n) = \frac{\rho(n)+\rho(-n)}{2} = \frac{\left(\frac{1}{2}\right)^n+\left(\frac{1}{2}\right)^{-n}}{2} = \boxed{\frac{1}{2^{n+1}}+2^{n-1}}$$

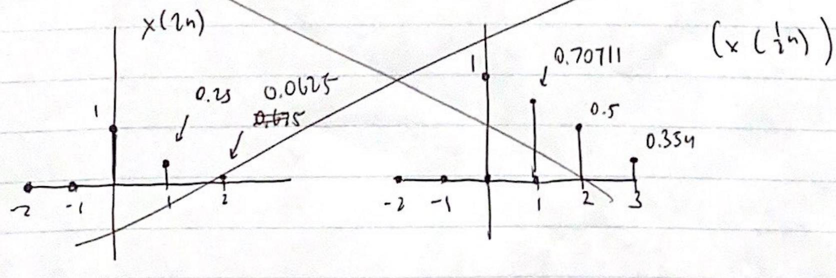
$$\rho_o(n) = \frac{\rho(n)-\rho(-n)}{2} = \frac{\left(\frac{1}{2}\right)^n-\left(\frac{1}{2}\right)^{-n}}{2} = \boxed{\frac{1}{2^{n+1}}-2^{n-1}}$$

c) $$\theta_e(n) = \frac{\frac{\pi}{3}n+\frac{7\pi}{12}+-\frac{\pi}{3}n+\frac{7\pi}{12}}{2} = \boxed{\frac{7\pi}{12}}$$

$$\theta_o(n) = \frac{\frac{\pi}{3}n+\frac{7\pi}{12}-\left(-\frac{\pi}{3}n+\frac{7\pi}{12}\right)}{2} = \boxed{\frac{\pi}{3}n}$$

2.37) $x(n) = 0.5^n u(n) \rightarrow x(2n) = 0.25^n\, u(2n) = 0.25^n\, u(n)$

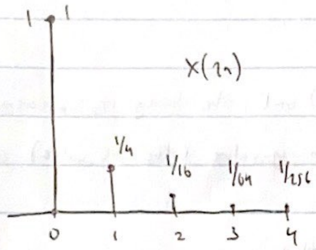
$x\left(\frac{1}{2}n\right) = 0.5^{\frac{1}{2}n}\, u\left(\frac{1}{2}n\right) = \left(\sqrt{0.5}\right)^n u(n)$

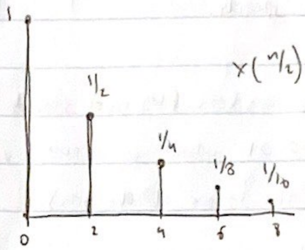


2.37) $x(n) = 0.5^n u(n)$

$x(2n) = 0.25^n u(2n) = 0.25^n u(n)$ defined @ 1, 2, 3, 4 aka $n \in \mathbb{Z}$

$x(n/2) = 0.5^{n/2} u(n/2) = 0.5^{n/2} u(n)$ defined @ $n = 0, 2, 4, 6, 8$ aka $n \in 2\mathbb{Z}$





$$E(x(2n)) = \sum_0^\infty (0.25^n)^2 = \frac{1}{1-\frac{1}{16}} = \boxed{16/15}$$

$$E(x(n/2)) = \sum_0^\infty (0.5^{2n}) = \frac{1}{1-\frac{1}{4}} = \boxed{4/3}$$

not 0.5^n because $n = 0, 2, 4, 6, 8$ NOT $n \in \mathbb{Z}$.

$$x_e(2n) = \frac{x(2n) + x(-2n)}{2} = \frac{0.25^n u(n) + 0.25^{-n} u(-n)}{2}$$

$$= \cancel{\frac{0.25^n u(n) + 0.25^{-n} u(-n)}{2}} = \begin{cases} \frac{1}{2} 0.25^n & \text{if } n \geq 0 \\ \frac{1}{2} 0.25^{-n} & \text{if } n < 0 \end{cases}$$

$$x_o(2n) = \frac{x(2n) - x(-2n)}{2} = \begin{cases} \frac{1}{2} 0.25^n & \text{if } n \geq 0 \\ -\frac{1}{2} 0.25^{-n} & \text{if } n < 0 \end{cases}$$ << edit: n > 0 strictly

Also, x_o(2n) = 0 when n = 0

$$x_e(n/2) = \frac{x(n/2) + x(-n/2)}{2} = \frac{0.5^{n/2} u(n) + 0.5^{-n/2} u(-n)}{2} = \begin{cases} 0.5^{\cancel{n/2+1}\; n/2-1} & \text{if } n = 0, 2, 4, 6, 8 \ldots \\ 0.5^{-n/2-1} & \text{if } n = -2, -4, -6 \ldots \end{cases}$$

$$x_o(n/2) = \begin{cases} 0.5^{n/2-1} & \text{if } n = 0, 2, 4, 6, 8 \ldots \\ -0.5^{-n/2-1} & \text{if } n = -2, -4, -6, -8 \ldots \end{cases}$$

It is the same but replacing n with $n/2$ or $2n$ depending on which func we are talking about.

3. a) $\sum_{n=0}^{\infty} n(0.5)^n = 0 \cdot 0.5^0 + 1 \cdot 0.5^1 + 2 \cdot 0.5^2 + 3 \cdot 0.5^3 + \dots = S$

$0 \cdot 0.5^1 + 1 \cdot 0.5^2 + 2 \cdot 0.5^3 + 3 \cdot 0.5^4 + \dots = S \cdot 0.5$

$$0.5S = 0 \cdot 0.5^0 + 1 \cdot 0.5^1 + 1 \cdot 0.5^2 + 1 \cdot 0.5^3$$

$$= 0 + \frac{0.5}{1-0.5} = 1$$

$$S = \boxed{2}$$

b) $\sum_{n=3}^{\infty} n(0.5)^{2n} = \left(\sum_{n=0}^{\infty} n(0.5)^{2n} \right) - 0.5^2 - 2 \cdot 0.5^4$

$$= \sum_{n=0}^{\infty} 2n(0.5)$$

$$0 \cdot 0.5^0 + 1 \cdot 0.5^2 + 2 \cdot 0.5^4 + \dots = S$$

$$0 \cdot 0.5^2 + 1 \cdot 0.5^4 + 2 \cdot 0.5^6 + \dots = 0.25S$$

$$0 + 1 \cdot 0.5^2 + 1 \cdot 0.5^4 + 1 \cdot 0.5^6 = 0.75S$$

$$S = \frac{4}{3}\left(\frac{0.25}{1-0.25}\right) = \frac{4}{9}$$

$$= \frac{4}{9} - \frac{1}{4} - 2 \cdot \frac{1}{16} = \frac{4}{9} - \frac{3}{8} = \frac{32-27}{72} = \boxed{\frac{5}{72}}$$