HW #4 11/3

1. $\underbrace{y(n) + \frac{5}{6}y(n-1) - \frac{1}{6}y(n-2)} = \left(\frac{1}{2}\right)^n u(n)$ $\quad y(-2)=0 \quad y(-1)=0 \quad n\geq 0$

Homogeneous: $y(n) + \frac{5}{6}y(n-1) - \frac{1}{6}y(n-2) = 0$

$$\lambda^2 + \frac{5}{6}\lambda - \frac{1}{6} = 0$$

$$\lambda = -1, \, 1/6 \rightarrow y_h(n) = C_1(-1)^n + C_2(1/6)^n$$

Particular:

$$y_p(n) = \left(\tfrac{1}{2}\right)^n u(n) \cdot k$$

$$\left(\tfrac{1}{2}\right)^n \cdot u(n)\cdot k + \frac{5}{6}\cdot\left(\tfrac{1}{2}\right)^{n-1}\cdot u(n-1)\cdot k - \frac{1}{6}\cdot\left(\tfrac{1}{2}\right)^{n-2}\cdot u(n-2)\cdot k = \left(\tfrac{1}{2}\right)^n \cdot u(n)$$

$$\frac{1}{8}\cdot k + \frac{5}{6}\cdot\frac{1}{4}\cdot k - \frac{1}{6}\cdot\frac{1}{2}\cdot k = \frac{1}{8}$$

$$k + 5/3\, k - 2/3\, k = 1 \rightarrow k = 1/2$$

$$y(n) = C_1(-1)^n + C_2(1/6)^n + \left(\tfrac{1}{2}\right)^{n+1} u(n) \quad n\geq 2$$

$y(0) = 1 - 0 - 0 = 1 \qquad y(1) = \frac{1}{2} - \frac{5}{6}y(0) - \frac{1}{6}y(-1) = -1/3$

$$C_1 + C_2 + \tfrac{1}{2} = 1 \qquad C_1 = 4/7$$

$$-C_1 + \tfrac{1}{6}C_2 + \tfrac{1}{4} = -1/3 \qquad C_2 = -1/14$$

$$\boxed{y(n) = \left(\frac{4}{7}(-1)^n - \frac{1}{14}(1/6)^n + \left(\tfrac{1}{2}\right)^{n+1}\right)u(n) \quad \cancel{n\geq 0}}$$

2. $y(n) = \frac{3}{4}y(n-1) - \frac{1}{8}y(n-2) + x(n)$

a) $\lambda^2 - \frac{3}{4}\lambda + \frac{1}{8} = 0 \rightarrow \lambda = \frac{1}{2}, \frac{1}{4}$

b) $y_h(n) = \left(\frac{1}{2}\right)^n C_1 + \left(\frac{1}{4}\right)^n C_2$

c) i. $x(n) = u(n)$

$$y_p(n) = k u(n)$$

$$k u(n) - \tfrac{3}{4}k(u(n-1)) + \tfrac{1}{8}k u(n-2) = u(n) \quad n\geq 2$$

$$k - \tfrac{3}{4}k + \tfrac{1}{8}k = 1 \rightarrow k = 8/3$$

$$y_p(n) = 8/3 \; u(n) \quad n\geq 2$$

ii. $x(n) = (\frac{1}{3})^n u(n)$

$y_p(n) = k \cdot (\frac{1}{3})^n u(n)$

$k(\frac{1}{3})^n u(n) - \frac{3}{4}k(\frac{1}{3})^{n-1} u(n-1) + \frac{1}{8}k(\frac{1}{3})^{n-2} u(n-2) = (\frac{1}{3})^n u(n) \quad n \geq 2$

$n \geq 2$

$k - \frac{9}{4}k + \frac{9}{8}k = 1 \quad \rightarrow k = -8$

$y_p(n) = -8(\frac{1}{3})^n u(n) \qquad n \geq 2$

d. i. $y_h(n) = C_1 \cdot (1/2)^n + C_2 (1/4)^n$

$y_p(n) = 8/3 \; u(n) \qquad n \geq 2$

$y(-2) = y(-1) = 0$

$y(0) = 1 \quad y(1) = 7/4$

$y(0) \rightarrow C_1 + C_2 + 8/3 = 1$

$y(1) \rightarrow \frac{1}{2}C_1 + \frac{1}{4}C_2 + 8/3 = 7/4$

$C_1 = -2 \quad C_2 = 1/3$

$\left(-2(1/2)^n + 1/3 \cdot (\frac{1}{4})^n + 8/3\right) u(n)$

ii. same thing but initial conditions $y(0) = 1 \quad y(1) = 13/12$

$y(0) = \; C_1 + C_2 - 8 = 1$

$\frac{1}{2}C_1 + \frac{1}{4}C_2 - 8/3 = 13/12 \quad \rightarrow \quad C_1 = 6 \quad C_2 = 3$

$\left(6(\frac{1}{2})^n + 3(\frac{1}{4})^n - 8(\frac{1}{3})^n\right) u(n)$

e. $x(n) = \delta(n) \qquad y(0) = 1 \quad y(1) = 3/4$

$C_1 + C_2 = 1$

$\frac{1}{2}C_1 + \frac{1}{4}C_2 = 3/4 \quad \longrightarrow \quad C_1 = 2 \quad C_2 = -1$

$h(n) = \left(2(\frac{1}{2})^n - (\frac{1}{4})^n\right) u(n)$

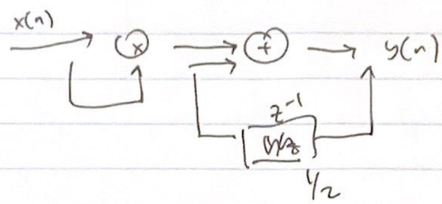
$\sum_{k=0}^{n} 2(\frac{1}{2})^k - (\frac{1}{4})^k$ is finite b/c $\lim_{n\to\infty} = 2 \cdot 1 - \frac{\frac{1}{4}}{\frac{3}{4}} = 2 - \frac{1}{3} = \boxed{\frac{5}{3}}$

which is finite $\rightarrow$ $\boxed{\text{BIBO stable}}$

3. $y(n) - \frac{1}{2} y(n-1) = x^2(n) \quad n \geq 0$

d.

a.



b. $y(-1) = 2$

$y(n) = \frac{1}{2} y(n-1)$

$\lambda - \frac{1}{2} = 0 \rightarrow \lambda = 1/2$

$y_{zi}(n) = c_1 \cdot (\frac{1}{2})^n \quad 2 = c_1 (\frac{1}{2})^{-1} \quad c_1 = 1$

$\boxed{y_{zi} = (\frac{1}{2})^n} \quad n \geq -1$ b/c looking @ responses for $n \geq 0$

c. $k(\frac{1}{2})^{n} u(n-1) - \frac{1}{2} k (\frac{1}{2})^{2n-2} u(n-2) = (\frac{1}{2})^{2n} u^2(n-1) \quad n \geq 1$

$k u(n-1) - 2k u(n-2) = u(n-1) \rightarrow k = -1$

$y_p(n) = -(\frac{1}{2})^{2n} u(n-1) = -(\frac{1}{4})^n u(n-1)$

$y_h(n) = c_1 (1/2)^n \rightarrow y_{zs}(n) = c_1 (1/2)^n - (1/2)^{2n} u(n-1) \quad n \geq 2$

$y(-1) = 0$ b/c 0-state

$y(0) = 0$ } propagation $\quad \frac{1}{4} = c_1 \cdot \frac{1}{2} - \frac{1}{4} \rightarrow c_1 = 1$

$y(1) = 1/4$

$y_{zs} = (1/2)^n - (1/2)^{2n} u(n-1)$ for $n \geq 1$

$= \boxed{\left((1/2)^n - (1/4)^n\right) u(n-1)}$