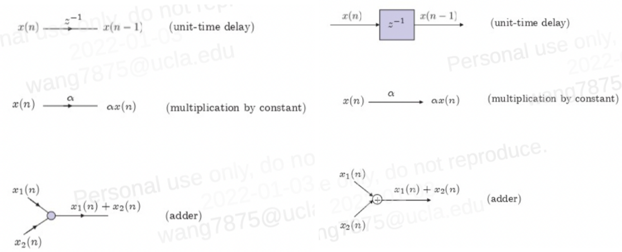
Ch. 4
Class of systems: input sequence not sufficient to evaluate response; y(n) = y(n - 1) + x(n)
- Need initial conditions for a specific system {y(n) = y(n - 1) + x(n), y(-1) = 1}
- Can also include time ranges that change the system {y(-1) = 1, n >= 0}

Relaxed system: init at rest, output sequence = 0 as long as input sequence = 0. Ex: y(n) = y(n - 1) + x(n), y(n1) = 0, n > n1
Dynamic system: a system with memory (any other value of n). Opposite = static/memoryless. Ex: y(n) = x(n - 1); y(n) = x(n^2)
TI system: y(n - K) = S[x(n - K)]. shift-invariant
- Proof: x_K(n) = x(n - K). x_K(n) is when y is shifted by K. x(n - K) is whatever inside x shifted by K

Causal system: output sample at time=n depends on present and past samples. Ex: y(n) = y(n - 1) + x(n)
Stable system: BIBO stable == bounded x(n) -> bounded y(n). Bounded = bounded by finite #. Ex: y(n) = ½ [x(n) + x(n - 1)]
Linear system: satisfies superposition & additivity/homogeneity
- Superposition: S[a1x1(n) + a2x2(n)] = a1y1(n) + a2y2(n)
  - Ex: y1(n) = 2x1(n-1), y2(n) = 2x2(n-1), y(n) = 2x(n-1). Prove y(n) = S[a1x1(n) + a2x2(n)] = a1y1(n) + a2y2(n)
- Additivity: S[x1(n) + x2(n)] = y1(n) + y2(n), Homogeneity: S[ax(n)] = ay(n)
- Satisfy excitation property: if x(n) = 0 for all n then y(n) = 0 for all n

LTI + causal means that the system is relaxed.
Block diagrams: input of x(n) output is y(n), flow diagrams



Ch. 5
Identical systems: same input -> same output, Distinct systems: same input -> different output
Convolution sum: x(n) = sum of k from -inf to inf of x(k) dirac(n - k)
- **y(n) = sum of k from -inf to inf of x(k) h(n - k) = x(n) * h(n)**
- Once we know impulse response sequence for system, y(n) = x(n) * h(n)

If S and S' are distinct LTI sys, h(n) & h'(n) are diff impulse responses
Causal sequence: x(n) = 0 for n < 0
- LTI system is causal ⇔ h(n) = 0 for n < 0 aka h is causal seq

LTI system is BIBO stable ⇔ sum of k from -inf to inf of |h(k)| is finite
Series cascading 2 LTI systems: S1 and S2 leads to h(n) = h1(n) * h2(n)
- x(n) -> S1 -> S2 -> y(n) or dirac(n) -> S1 becomes h1(n) and then -> S2 becomes h1(n) * h2(n)

Parallel cascading 2 LTI systems: S1 and S2 leads to h(n) = h1(n) + h2(n)
- S1 and S2 boxes are in parallel. Input still x(n) and output still y(n)
- If empty wire in parallel (same thing as invisible dirac(n))

FIR systems' impulse response sequence has finite duration. IIR has infinite duration
- Ex: h(n) = u(n) means IIR. h(n) = dirac(n) + dirac(n - 1) means FIR

FIR LTI system is a Moving-Average (MA) system aka tapped-delay-line
- y(n) = sum of k = 0 to N of b_k x(n - k) << MA model
- Can be implemented using finite # of delays and linear combiners (adders and multipliers)

ARMA Systems: Auto-Regressive Moving-Average
- y(n) = sum from k = 1 to M of a_k y(n - k) + sum from k = 0 to N of b_k x(n - k)
- If only dependent on past output samples and present input ^^ second term would just be b_o x(n)
- Impulse response of such representations have infinite durations and model IIR systems
  - Ex: y(n) = ½ y(n - 1) + x(n) relaxed

IIR system with the smallest number of delays = minimal
Two inverse problems: deconvolution and convolution. Find h(n) from x and y or find x from h and y
Ch. 6
Conv = commut (a * b = b * a); distrib (x * (h1 + h2) = x * h1 + x * h2); assoc (h1 * (h2 * h3) = (h1 * h2) * h3)
- h(n) * dirac(n - k0) = h(n - k0)
- Remember to include u(n - k) at the end of a conv problem if there is a bound on y caused by n

Graphical: given x and h, plot x(k) and h(k). Plot h(-k). Multiple x(k) by h(-k) sample by sample and sum to get y(0). Shift h(-k) units to the left and right. Those are the values of y(however many units shifted) << PRACTICE
Ch. 7: Homogeneous Difference Equations
M-th order homogeneous equation: y(n) + a1y(n - 1) + a2y(n - 2) + … + aMy(n - M) = 0
- Characteristic equation: p(λ) = λ^M + a1 λ^(M-1) + a2 λ^(M-2) + … + a(M-1) λ + aM = 0
- Single root at λ_l contributes a term C_l * λ_l^n
- Double root at λ_l contributes a term C_l0 * λ_l^n + C_l1 * n * λ_l^n
- Mult. m root λ_l contributes C_l0 * λ_l^n + C_l1 * n * λ_l^n + C_l2 * n^2 * λ_l^n + … + C_l(m-1) * n^(m-1) λ_l^n
- Complex roots λ_l, λ*_l contribute C_l * λ_l^n + C*_l * (λ*_l)^n

Initial conditions: y(-1), y(-2), … y(-M)
Solution 1: iterating the recursion. Use y(-1) to get y(0, 1, 2, 3, ..). As well as go backwards: -1, -2, …
Solution 2: use characteristic equation, solve for λ's. Plug in initial conditions to find C_i
LTI causal system with x(n): y(n) - ¼ y(n - 1) - ⅛ y(n - 2) = 2x(n)
- Since h(n) = 0 for n < 0, plugging in dirac(n) for x(n), then setting bounds to n >=1, we can use homogeneous
- y(n) - ¼ y(n - 1) - ⅛ y(n - 2) = x(n) - ½ x(n - 1) -> dirac(n) - ½ dirac(n - 1) for n >= 0.

LTI causal system is stable if |λ| < 1 for all λ
If the system is not relaxed, it cannot be LTI.
To find complete response, first find impulse response sequence, then convolve it with wanted input sequence
Ch. 8: Solving Difference Equations
Particular solution : zero-input solution. Homogeneous solution : zero-state solution.
Complete solution : forced solution. Transient solution : unforced solution.
Steady-state solution : natural solution.
x(n) = [Au(n) ; step-sequence] [Aλ^n u(n) : exponential sequence] [A cos(wn) u(n) ; sinusoidal sequence] [An^p u(n) ; polynomial sequence]
No initial conditions needed to find a particular solution y_p(n)
- << common particular solutions for input sequences
- If terms vanish for values of n < k, must state that n >= k or similar

	Input sequence, $x(n)$	Particular solution, $y_p(n)$
1.	$Au(n)$	$Ku(n)$
2.	$A\alpha^n u(n)$	$K\alpha^n u(n)$
3.	$A\cos(\omega_o n)u(n)$	$[K_1\cos(\omega_o n) + K_2\sin(\omega_o n)]u(n)$
4.	$A\sin(\omega_o n)u(n)$	$[K_1\cos(\omega_o n) + K_2\sin(\omega_o n)]u(n)$
5.	$A\alpha^n\cos(\omega_o n)u(n)$	$[K_1\cos(\omega_o n) + K_2\sin(\omega_o n)]\alpha^n u(n)$
6.	$A\alpha^n\sin(\omega_o n)u(n)$	$[K_1\cos(\omega_o n) + K_2\sin(\omega_o n)]\alpha^n u(n)$
7.	$Anu(n)$	$[K_1 n + K_2]u(n)$
8.	$An^2u(n)$	$[K_1 n^2 + K_2 n + K_3]u(n)$
9.	$An^p u(n)$	$[K_1 n^p + K_2 n^{p-1} + \ldots + K_{p+1}]u(n)$
10.	$An^p\alpha^n u(n)$	$[K_1 n^p + K_2 n^{p-1} + \ldots + K_{p+1}]\alpha^n u(n)$

Characterizing all solutions:
- Find particular solution (include n >= n0); find homogeneous solution; y(n) = y_p(n) + y_h(n) for n >= n0

Finding complete solutions:
- Use previous things to find sum of homogeneous and particular (DO NOT PLUG IN INITIAL CONDITIONS YET)
- Propagate initial conditions up to time prior to n0. Solve for C_i

2 special responses of a system: zero-state and zero-input
Zero-state response is a complete response of the system when it is assumed to be relaxed (zero initial conditions)
- If original initial conditions were y(-1) = 1 and y(-2) = 0, rewrite as y(-1) = 0
- Satisfies superposition principle: can be easier if x(n) is complicated like 0.5^n u(n) + 2u(n - 1)
- QUICKER WAY EXISTS

Zero-input response is homogeneous solution with constants determined from initial conditions
- Also satisfies superposition principle: zero-input response to linear combination of different sets of initial conditions = corresponding linear combination of individual zero-input responses
- ^^ REVIEW

Way #2 for finding complete solutions: y_c(n) = y_zi(n) + y_zs(n) aka sum of zero input and zero state responses
Transient response of a system = part of complete response that decays to 0 as n approaches infinity
Steady state response of system = part of complete response that persists indefinitely as n approaches infinity
Way #3 for finding complete solutions: y_zs(n) can be found quicker by conv(input sequence, impulse response seq)

Miscellaneous

Sampling theorem: signal has to be sampled at least with twice the frequency of the original signal
Continuous signal = continuous in x, y; discrete signal = discrete x, continuous y; digital signal = discrete in x , y
Energy of real valued sequence = sum of squares of samples. Sum of x^2(n) as n goes from -inf to inf
Odd part of function = (f(n) - f(-n))/2; even part of function = (f(n) + f(-n))/2
If x(n) is a function, y(n) = x(n/2) is only defined for values of n where x(_) exists
Correlation of two signals is the conv between one sig with the functional inverse version of the other sig
Modes of a system = lambdas
Series cascades of TI systems are TI. Ex: y(n) = y(n - 1) + x^2(n) is TI bc y(n) = y(n - 1) + x'(n) and x^2(n) are TI
Energy sequence means energy of sequence is finite
When flipping and dragging, the flipped motion corresponds to the values of y. If h is flipped, h moved right 1 = y(1)
If a system has nonzero initial conditions, it is not relaxed.
If a system has nonzero initial conditions, it is not linear.
CCDE needs to be relaxed to describe a LTI system. If CCDE isn't relaxed, need response for both n >= 0 and n < 0

Ch. 9: z-Transform

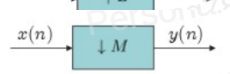
	Sequence	z-transform	ROC	Property
1.	$x(n)$	$X(z)$	ROC $= R_x \cup R$ $R_x = \{r_1 < \|z\| < r_2\}$	
2.	$y(n)$	$Y(z)$	ROC $= R_y \cup R'$ $R_y = \{r' < \|z\| < r''\}$	
3.	$ax(n) + by(n)$	$aX(z) + bY(z)$	$\{R_x \cap R_y\}$ including possibly $z = 0$ or $z = \pm\infty$	linearity
4.	$x(n - n_0)$	$z^{-n_0}X(z)$	R_x excluding possibly $z = 0$ or $z = \pm\infty$	time-shifts
5.	$a^n x(n)$	$X(z/a)$	$\{\|a\|r_1 < \|z\| < \|a\|r_2\} \cup R$	exponential modulation
6.	$(-1)^n x(n)$	$X(-z)$	$R_x \cup R$	alternating sign
7.	$x(-n)$	$X(1/z)$	$\{1/r_2 < \|z\| < 1/r_1\} \cup R$	time reversal
8.	$nx(n)$	$-z\,\frac{dX(z)}{dz}$	R_x excluding possibly $z = 0$ or $z = \pm\infty$	linear modulation
9.	$x^*(n)$	$[X(z^*)]^*$	$R_x \cup R$	conjugation
10.	$\text{Re}[x(n)]$	$\frac{1}{2}[X(z) + (X(z^*))^*]$	R_x excluding possibly $z = 0$ or $z = \pm\infty$	real part
11.	$\text{Im}[x(n)]$	$\frac{1}{2j}[X(z) - (X(z^*))^*]$	R_x excluding possibly $z = 0$ or $z = \pm\infty$	imaginary part
12.	$x(n) \star y(n)$	$X(z)Y(z)$	$\{R_x \cap R_y\}$ including possibly $z = 0$ or $z = \pm\infty$	convolution
13.	$x(n) \uparrow L$	$X(z^L)$	$\{r_1^{1/L} < \|z\| < r_2^{1/L}\} \cup R$	upsampling
14.	$x(2n)$	$\frac{1}{2}\left[X(z^{1/2}) + X(-z^{1/2})\right]$	$\{r_1^2 < \|z\| < r_2^2\} \cup R$	2-fold downsampling
15.	$x(Mn)$	$\frac{1}{M}\sum_{k=0}^{M-1} X\left(e^{-\frac{j2\pi k}{M}} z^{1/M}\right)$	$\{r_1^M < \|z\| < r_2^M\} \cup R$	M-fold downsampling

Sequence	z-Transform	ROC
$\delta(n)$	1	$\mathbb{C}$
$u(n)$	$\frac{z}{z-1}$	$\|z\| > 1$
$\alpha^n u(n)$	$\frac{z}{z-\alpha}$	$\|z\| > \|\alpha\|$
$-\alpha^n u(-n-1)$	$\frac{z}{z-\alpha}$	$\|z\| < \|\alpha\|$
$nu(n)$	$\frac{z}{(z-1)^2}$	$\|z\| > 1$
$-nu(-n-1)$	$\frac{z}{(z-1)^2}$	$\|z\| < 1$
$n\alpha^n u(n)$	$\frac{\alpha z}{(z-\alpha)^2}$	$\|z\| > \|\alpha\|$
$-n\alpha^n u(-n-1)$	$\frac{\alpha z}{(z-\alpha)^2}$	$\|z\| < \|\alpha\|$
$\cos(\omega_o n)u(n)$	$\frac{z^2 - z\cos\omega_o}{z^2 - 2z\cos\omega_o + 1}$	$\|z\| > 1$
$\sin(\omega_o n)u(n)$	$\frac{z\sin\omega_o}{z^2 - 2z\cos\omega_o + 1}$	$\|z\| > 1$
$\alpha^n\cos(\omega_o n)u(n)$	$\frac{z^2 - \alpha z\cos\omega_o}{z^2 - 2\alpha z\cos\omega_o + \alpha^2}$	$\|z\| > \|\alpha\|$
$\alpha^n\sin(\omega_o n)u(n)$	$\frac{\alpha z\sin\omega_o}{z^2 - 2\alpha z\cos\omega_o + \alpha^2}$	$\|z\| > \|\alpha\|$

Bilateral z-transform: X(z) = sum of n=-inf to inf of x(n) z^(-n) << for all z in complex plane
- When n is positive, powers of z are negative; n is negative, powers of z are positive

Region of convergence: some values of z do not work. ROC = all values of z where X(z) absolutely converges
- Finite-duration sequences = x(n) is 0 outside a finite bounded interval.
  - If n > 0, negative power of z exists, so z = 0 is excluded from ROC. If n < 0, positive power of z exists, so z = +/- inf excluded from ROC.
- Infinite-duration sequences (infinitely many nonzero samples): right sided, left sided, two sided.
  - Ex: right sided is x(n) = 0 for n < n0 aka 0.5^n u(n + 3).
  - ROC of these are discs or rings: RS: |z| > r, LS: |z| < r, TS: r1 < |z| < r2 for reals r1, r2 > 0
    - For each of these, if any of 0, +/- inf are in the range, they may or may not be included

Right sided exp seq: x(n) = a^n u(n). Increasing or decreasing based on a vs 1. ZT is X(z) = z/(z - a) w/ ROC |z| > |a|
Left sided exp seq: x(n) = -a^n u(-n-1). Opposite of RS exp seq. ZT is X(z) = z/(z - a) w/ ROC |z| < |a|
Two sided exp seq: x(n) = a^n u(n) + b^n u(-n-1). ZT is X(z) = z/(z - a) - z/(z - b) w/ ROC |a| < |z| < |b|
Boundary points of ROC need to be addressed separately R = (0, +/- inf). That is the R and R' in the first 2 lines of table
Linearity: ax(n) + by(n) ⇔ aX(z) + bY(z). ROC of sum is Rx ^ Ry ^ R
Running Sum: w(n) = sum of k = 0 to n of x(n) -> w(n) = conv(x(n), u(n))
Can use z-transform to evaluate series (PRACTICE MORE)
Initial value thm: if x(n) is causal x(n) = 0 for n < 0, X(inf) = x(0) bc X(z) = x(0) + x(1)z^-1 + x(2)z^-2… since causality
Upsampling: y(n) = x(n / L) if n/L is an integer, 0 otherwise. Time expansion: insert L-1 zeros btwn samples
Downsampling: y(n) = x(Mn). Time compression: discard other samples



Ch. 10: Inverse z-Transform

$z-$transform	Sequence	ROC
A	$A\delta(n)$	complex plane
$\frac{A}{z-\alpha}$	$A\alpha^{n-1}u(n-1)$	$\|z\| > \|\alpha\|$
$\frac{A}{(z-\alpha)^2}$	$A\cdot(n-1)\alpha^{n-2}u(n-1)$	$\|z\| > \|\alpha\|$
$\frac{A}{z-\alpha} + \frac{A^*}{z-\alpha^*}$	$2\cdot\|A\|\cdot\|\alpha\|^{n-1}\cdot\cos[\omega(n-1)+\theta]\cdot u(n-1)$	$\|z\| > \|\alpha\|$

$z-$transform	Sequence	ROC
A	$A\delta(n)$	complex plane
$\frac{A}{z-\alpha}$	$-A\alpha^{n-1}u(-n)$	$\|z\| < \|\alpha\|$
$\frac{A}{(z-\alpha)^2}$	$-A\cdot(n-1)\alpha^{n-2}u(-n)$	$\|z\| < \|\alpha\|$
$\frac{A}{z-\alpha} + \frac{A^*}{z-\alpha^*}$	$-2\cdot\|A\|\cdot\|\alpha\|^{n-1}\cdot\cos[\omega(n-1)+\theta]\cdot u(-n)$	$\|z\| < \|\alpha\|$

Inverse transforms for ^^ right-sided sequences and ^^ left-sided sequences
Partial fraction: single pole contributes A/(z-a). Double root: A/(z-a) + A'/(z-a)^2. Complex root: A/(z-a) + A*/(z-a*)
- Make sure that the ROC once inverted matches the ROC in the problem.

Ch. 11: Transfer Functions

H(z) is a transfer function, the ZT of the impulse response function h(n) and R_h is the ROC of H(z)
Eigenfunction of LTI system: exponential function x(n) = z0^n when z0 is in R_h. aka its output, y(n) = H(z0) * (z0)^n
- Eigenfunction passes through a system the same, just scaled (in this case by H(z0) the number)

Causal LTI Systems have H(z) = h(0) + h(1)z^-1 + h(2)z^-2 + ... (only negative powers) w/ R_h = |z| > r including +/- inf
Stable LTI System if sum of |h(n)| from n = -inf to +inf is < inf which is equivalent to R_h including the unit circle |z| = 1
Poles: lim as z approaches pole of H(z) = inf. Zeros if this limit is 0.
- Always remember to check +/- inf (assume this is one pole/zero). There are always equal # of poles and zeros for TF!

REVIEW MODE CANCELLATION
If a system is causal, its impulse response is right-sided. Its ROC is just |z| > whatever the biggest pole is.
3 ways to find impulse response sequence of an LTI system from a CCDE:
1) Find h(n) -> calculate H(z). useful if h(n) is known or easily findable
2) ZT both sides of CCDE -> find Y(z) / X(z) = H(z). ROC is determined by causality and pole locations
3) Use x(n) to find y(n). Then z-transform each to get Y(z) and X(z) and then do 2)

Rational TF only if IO relation can be described via CCDE
When y(n) = conv(x(n), h(n)) and Y(z) = X(z)H(z), ROC of Y(z) is R_x ^ R_h ^ {0, +/- inf}
The ROC for H(z) tells you whether the system is causal or not which tells you how to inverse ZT Y(z) and X(z)
Realizable LTI system = BIBO stable and causal -> ROC = |z| > r including +/- inf for 0 <= r < 1 (since must include r = 1)
- Which meant that poles must lie all inside |z| < 1. NOTE CONVERSE IS NOT NECESSARILY TRUE

Let G be the inverse of H aka X(z)H(z)=Y(z) and XHG = X. Thus, G(z) = 1/H(z). Zeros and poles swap.
- For G to be realizable, R_G = |z| > B including +/- inf for 0 <= B < 1 so zeros of H must be inside unit circle
- G and H must have overlapping ROC's

Ch. 12: Unilateral z-Transform

Unilateral ZT is for causal systems and sequences, but these DEs are not LTI systems. UZT STILL CAN DETERMINE ANSWERS
For bilateral ZT, find a solution to CCDE by finding sum of zero-input and zero-state response. Y_zs(z) = X(z)H(z). Remember that the ROC is exterior of disc because LTI system is causal. Unilateral does not need this 2-step process.
$X^+(z)$ = sum of x(n)z^(-n) for n=0 to inf. X(z) = $X^+(z)$ for causal sequences. $Z^+[x(n)] = Z[x^+(n)]$ = Z[x(n)u(n)] where $x^+(n)$ is x(n) for n >= 0 and 0 otherwise. ROC is similarly defined: all complex z such that sum of |x(n)z^(-n)| for n=0 to inf < inf
- UZT deals with causal right-sided sequences, ROC of $X^+(z)$ = exterior of disc and +/- inf -> |z| > r
- If z = 0 is in ROC of $X^+(z)$ ⇔ x(n) is anti-causal (nonzero samples only occur over n <= 0)
  - Which meant that $X^+(z)$ = x(0) so ROC is entire complex plane if x(n) is anti-causal

x(n) is anti causal if x(n) = 0 for n >= 1
The table below does not require x and y to be causal:

	Sequence	Unilateral z-transform	ROC	Property
1.	$x(n)$	$X^+(z)$	R_{x^+}	
2.	$y(n)$	$Y^+(z)$	R_{y^+}	
3.	$ax(n)+by(n)$	$aX^+(z)+bY^+(z)$	$\{R_{x^+}\cap R_{y^+}\}$ or $\mathbb{C}$	linearity
4.	$x(n-1)$	$z^{-1}X^+(z)+x(-1)$	R_{x^+} or $\mathbb{C}-\{0\}$	time delay
5.	$x(n+1)$	$zX^+(z)-zx(0)$	R_{x^+} or $\mathbb{C}$	time advance
6.	$a^n x(n)$	$X^+(z/a)$	$\{\ \|z\| > \|a\|r\ \}$	exponential modulation
7.	$nx(n)$	$-z\frac{dX^+(z)}{dz}$	R_{x^+}	linear modulation
8.	$x(n)$ and $y(n)$ causal: $\sum_{k=0}^{n} x(k)y(n-k)$	$X^+(z)Y^+(z)$	$\{R_{x^+}\cap R_{y^+}\}$ or $\mathbb{C}$	convolution

3 is linearity. The result's ROC is all C if it happens to be anti-causal
4 is time shift: x(n-k) = z^(-k)$X^+(z)$ + x(-1)z^(-k+1) + x(-2)z^(-k+2)+...+x(-k)
- x(n+k) = z^k$X^+(z)$ - x(0)z^k - x(1)z^(k-1) - … - x(k - 1)z
  - If this includes the point z=0, then the ROC is all C

8. The ROC is all C if the result contains z=0
Initial Value Theorem for UZT. lim of $X^+(z)$ as z->inf is x(0)
Final Value Theorem for UZT. if lim as N->inf of x(N) exists then
- Lim as N->inf of x(N) = lim z->1 of (z-1)$X^+(z)$
- Aka limit exists if ROC of (z-1)$X^+(z)$ includes unit circle

To solve CCDE with initial conditions, apply UZT to both sides, solve for $Y^+(z)$, setting $X^+(z)$ to zero is the zero input response. Setting y(*) to 0 is the zero-state response. Add Y_zi and Y_zs together, then plug in $X^+(z)$ and y(*) to get $Y^+(z)$.
Inverting Y_zi and Y_zs actually gives y_zi and y_zs!

Ch. 13: Discrete-Time Fourier Transform 341-388

DTFT: X(e^(jw)) = sum of x(n)e^(-jwn) for n=-inf to +inf
If {|z|=1} is part of ROC of X(z), the DTFT is replacing z of X(z) with e^(jw)
DTFT can be defined if |z|=1 is not part of ROC (will discuss later)
DTFT is periodic in 2pi: X(e^(jw)) = X(e^(j(w + 2pi))
x(n - n0) ⇔ e^(-jwn0) X(e^(jw))
X(e^jw) = |X(e^jw)| * e^(j(<X(e^jw))
Mag = square of real and imag parts and the phase angle is arctan(X_I / X_R)
Linearity: ax(n) + by(n) ⇔ aX(e^(jw)) + bY(e^(jw))
If DTFT is complex function, must plot both magnitude and phase both -pi to pi
- When plotting phase, if goes above pi, we can map it to -pi

If x(n) is absolutely summable → X(e^(jw)) is continuous in w and DTFT exists
Square summable sequences have DTFTs: sum of x(n)^2 for n = -inf to +inf < inf
^^ Mean-square convergence: if X_N(e^(jw)) = sum of x(n)e^(-jwn) from -N to N

If X(w) exists so that $\lim_{N\to\infty}\left(\frac{1}{2\pi}\int_{-\pi}^{\pi}\left|X_N(e^{j\omega})-X(e^{j\omega})\right|^2 d\omega\right) = 0$ then DTFT exists.

$$x(n) = \frac{1}{2\pi}\int_{2\pi} X(e^{j\omega})e^{j\omega n}d\omega$$

<< the inverse DTFT
- If inverse DTFT is not defined at n = 0, we must split it into diff case

Sinc(x) = sin(x)/x. sinc(0) = 1. (l'hopital's) -> sinc(k*pi)=0 for all nonzero k
- Sinc is not absolutely summable, but square summable

Impossible to have an ideal low pass filter, but Gibbs phenomena approximates it
x(n) = e^(jw0n) is not absolutely or square summable. Still has a DTFT though
- X(e^(jw)) = 2pi delta(w - w0) for w=[-pi, pi]

Can use PFE to find inverse DTFT's. Or, replace e^(jw) with z and inverse ZT
- Make sure we choose ROC that includes |z| = 1

Sequence $x(n)$	DTFT $X(e^{j\omega})$ over one period
$x(n)=\delta(n)$	$X(e^{j\omega})=1$
$x(n)=\begin{cases}1, & 0\le n\le L-1\\ 0, & \text{otherwise}\end{cases}$	$X(e^{j\omega})=\begin{cases}L, & \omega=0\\ e^{-j\omega\frac{(L-1)}{2}}\cdot\frac{\sin(\omega L/2)}{\sin(\omega/2)}, & \text{otherwise}\end{cases}$
$x(n)=\alpha^n u(n),\ \|\alpha\|<1$	$X(e^{j\omega})=\frac{1}{1-\alpha e^{-j\omega}}$
$x(n)=-\alpha^n u(-n-1),\ \|\alpha\|>1$	$X(e^{j\omega})=\frac{1}{1-\alpha e^{-j\omega}}$
$x(n)=\frac{\omega_c}{\pi}\text{sinc}(\omega_c n)$	$X(e^{j\omega})=\begin{cases}1, & \|w\|<w_c\\ 0, & w_c\le\|w\|\le\pi\end{cases}$
$x(n)=e^{j\omega_o n}$	$X(e^{j\omega})=2\pi\delta(\omega-\omega_o)$
$x(n)=\cos(\omega_o n),\ \omega_o\in[-\pi,\pi]$	$X(e^{j\omega})=\pi[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]$
$x(n)=\sin(\omega_o n),\ \omega_o\in[-\pi,\pi]$	$X(e^{j\omega})=-j\pi[\delta(\omega-\omega_0)-\delta(\omega+\omega_0)]$

Ch. 14: Properties of the DTFT 397-437
Shifting in the time-domain ⇔ phase change in the frequency domain. Phase change in time-domain ⇔ shifting in the frequency domain

Circular conv: $X(e^{j\omega}) \circ Y(e^{j\omega}) = \frac{1}{2\pi}\int_{2\pi} X(e^{j\lambda})Y(e^{j(\omega-\lambda)})d\lambda$

Conjugating a series is the same as replacing each term with its complex conjugate
|X(e^(jw))| and Re(X(e^(jw))) is an even func of w. <X(e^(jw)) and Im(X(e^(jw))) is an odd func of w. |X(e^(jw))| = |X(e^(-jw))| & <X(e^(jw)) = -<X(e^(-jw)).
X_R and X_I are the real and imaginary components of X(e^(jw)). x_e(n) and x_o(n) are the even and odd components of x(n). X_R is DTFT of x_e and jX_I is DTFT of x_o
x real+even -> X real+even; x imag+odd -> X real+odd; x real+odd -> X imag + odd, x imag+even -> X imag+even

$$\sum_{n=-\infty}^{\infty} x(n)y^*(n) = \frac{1}{2\pi}\int_{2\pi} X(e^{j\omega})\left[Y(e^{j\omega})\right]^* d\omega$$

Paresval's Relation on right: how to move between time domain and frequency domain quantities. When x(n) = y(n) we get: $\sum_{n=-\infty}^{\infty} |x(n)|^2 = \frac{1}{2\pi}\int_{2\pi} |X(e^{j\omega})|^2 d\omega$ whose LHS is energy. The magnitude on RHS = spectrum of x(n).
Parseval's only for square-summable seq.
Upsampling: inserting L-1 zeros between samples of x(n): Y(e^(jw)) = X(e^(jwL))
Downsampling: every term we take out goes to 0. Only select every other sample (/2)

$$Y(e^{j\omega}) = \frac{1}{M}\sum_{k=0}^{M-1} X\left(e^{\frac{j(\omega-2\pi k)}{M}}\right)$$

-

	Sequence	DTFT	Property
1.	$x(n)$	$X(e^{j\omega})$	(14.1)
2.	$y(n)$	$Y(e^{j\omega})$	(14.2)
3.	$ax(n)+by(n)$	$aX(e^{j\omega})+bY(e^{j\omega})$	linearity
4.	$x(n-n_0)$	$e^{-j\omega n_0}X(e^{j\omega})$	time-shifts
5.	$e^{j\omega_o n}x(n)$	$X(e^{j(\omega-\omega_o)})$	frequency shifts
6.	$\cos(\omega_o n)x(n)$ $\sin(\omega_o n)x(n)$	$\frac{1}{2}X\left(e^{j(\omega-\omega_o)}\right)+\frac{1}{2}X\left(e^{j(\omega+\omega_o)}\right)$ $\frac{1}{2j}X\left(e^{j(\omega-\omega_o)}\right)-\frac{1}{2j}X\left(e^{j(\omega+\omega_o)}\right)$	modulation
7.	$x(-n)$	$X(e^{-j\omega})$	time-reversal
8.	$nx(n)$	$j\,\frac{dX(e^{j\omega})}{dw}$	linear modulation
9.	$x(n)\star y(n)$	$X(e^{j\omega})Y(e^{j\omega})$	convolution
10.	$x(n)y(n)$	$X(e^{j\omega})\circ Y(e^{j\omega})$	multiplication
11.	$x^*(n)$	$\left[X(e^{-j\omega})\right]^*$	conjugation

For changing sampling rate by rational number like ⅔, upsample 2 and downsample 3

Ch. 15: Frequency Response 445-467
Frequency content of a sequence is the graph of the DTFT vs the angular freq omega
- For complex signals we need two plots: magnitude and phase
- For an LTI system, frequency responses only defined for BIBO stable systems.

H(e^(jw)) is the frequency response.
  - Exciting the system with complex exponential x(n) = e^(jw0n) gives us y(n) = e^(jw0n) H(e^(jw0)). A scaling.
  - |H(e^(jw))| = magnitude response; <H(e^(jw)) = phase response.
- Since DTFT exists for sequences h(n) that are not BIBO stable aka absolutely summable h(n), (square-summable) these sequences will not be stable. Frequency response defined as DTFT of corresponding impulse response sequences.
  - Transfer functions more general than frequency response bc they can do stable/unstable systems

Decibel plot: 20log_10|H(e^(jw))| dB vs omega (w)
Ideal low pass filter: only small w have H(e^(jw)) values aka Ae^(-jwk0) |w| <= wc, otherwise 0
Ideal high pass filter: Ae^(-jwk0) wc <= |w|, otherwise 0, ideal band pass filter: Ae^(-jwk0) w1 <= |w| <= w2, otherwise 0
Ideal band stop filter: Ae^(-jwk0) |w| <= w1 and w2 <= |w|, otherwise 0
Realizeable system = ROC |z| > r including +/- inf for some 0 <= r < 1.
Causal system = all poles of realizeable LTI system lie inside unit circle.
Transient response dies out as n increases; steady-state response stays and is called steady state solution

Ch. 16: Discrete Fourier Transform 477-507
Focusing on sequences that are absolutely summable.
DFT is a sampled version of DTFT: X(k) = X(e^(2j*k*pi/N)) for k = 0,1,...,N-1.
Assuming that x(n) is causal and duration L <= N
- N-point DFT is X(k) = sum of x(n)e^(-j*2pi*k*n/N) from n=0 to N-1 for k=0,1,...N-1

DTFT is good representation of signals in frequency domain, w is continuous and hard to process/store. x(n) and X(k) are discrete in time and frequency: w_k = 2*pi*k/N for k=0, 1,..., N-1. Can get X(k) from x(n) directly.
DFT can also recover x(n) if x(n) is causal and finite duration L <= N.
X(w) is periodic in 2pi, X(k) is periodic in N: X(k) = X(k + N) for all ints k. N is size of DFT and >= L (length of signal)
2 point DFT: X(0) = x(0) + x(1) and X(1) = x(0) - x(1 -> foundation of FFT
Zero Padding means for x(0), x(1), .. x(L - 1) keep orig seq. For x(L), x(L + 1), … x(2N - 1) put 0's.
- Interpolates additional DFT samples between orig DFT coefficients

x(n) = 1/N * (sum of X(k) e^(j*2pi*k/N * n)) for k=0 to N - 1, n = 0, 1, 2, 3, …, N - 1 << inverse DFT
- Make sure to repeat x(n) every N steps (loop it) when solving for inverse DFT
- Also remember to plot phase & magnitude

Ch. 17: Properties of the DFT 513-559
Angular frequencies close to +/- pi are high frequencies, close to 0 are low frequencies

	Causal sequences	N-point DFT	Property
1.	$x(n)$	$X(k)$	
2.	$y(n)$	$Y(k)$	
3.	$ax(n)+by(n)$	$aX(k)+bY(k)$	linearity
4.	$x[(n-n_o) \bmod N]$	$e^{-j\frac{2\pi n_o}{N}k}\cdot X(k)$	circular time shift
5.	$e^{j\frac{2\pi k_o}{N}n}\cdot x(n)$	$X[(k-k_o) \bmod N]$	circular frequency shift
6.	$\cos\left(\frac{2\pi k_o}{N}n\right)x(n)$	$\frac{1}{2}X[(k-k_o) \bmod N]+\frac{1}{2}X[(k+k_o) \bmod N]$	modulation
7.	$x(-n \bmod N)$	$X(-k \bmod N)$	time reversal
8.	$x^*(n)$	$X^*(-k \bmod N)$	conjugation in time
9.	$x^*(-n \bmod N)$	$X^*(k)$	conjugation in frequency
10.	$x(n)\circ y(n)$	$X(k)Y(k)$	circular convolution
11.	$x(n)y(n)$	$\frac{1}{N}X(k)\circ Y(k)$	product of sequences

<< properties of inverse DFT. assume x(n) is causal with L <= N
Linearity: if x and y have different lengths, pad shorter with 0. Choose N for larger one
Time shift: if signal periodic, time shift is not linear -> circular

$$\boxed{\sum_{n=0}^{N-1} x(n)y^*(n) \longleftrightarrow \frac{1}{N}\sum_{k=0}^{N-1} X(k)Y^*(k)}$$

$$\boxed{\sum_{n=0}^{N-1} |x(n)|^2 \longleftrightarrow \frac{1}{N}\sum_{k=0}^{N-1} |X(k)|^2}$$

<< Parseval's relation

When time shifting, if n0 = -1, orig values of x'(0) = x(1), x'(1) = x(2), …

Midterm Review
Just because x(2n) is a energy seq doesn't mean x(n) is one
delta(-2n+4) has to be simplified before convoluting
When proving linearity, remember to scale

cos(w) = [e^(jw) + e^(-jw)] / 2
sin(w) = [e^(jw) - e^(-jw)] / (2j)