

EE 131A  
Probability and Statistics  
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Homework 3  
Wednesday, January 20, 2021  
Due: Monday, February 1, 2021  
before class begins  
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**Please upload your homework to Gradescope by February 1, 3:59 pm.**

**Please submit a single PDF directly on Gradescope**

**You may type your homework or scan your handwritten version. Make sure all the work is discernible.**

Reading: 2.6, 3 & 4 of *Probability, Statistics, and Random Processes* by A. Leon-Garcia

1. A salesman has scheduled two appointments to sell encyclopedias. His first appointment will lead to a sale with probability .3, and his second will lead independently to a sale with probability .6. Any sale made is equally likely to be either for the deluxe model, which costs \$1000, or the standard model, which costs \$500. Determine the probability mass function of  $X$ , the total dollar value of all sales.
2. Suppose that a die is rolled twice. Let  $X$  and  $Y$  denote the maximum and minimum value to appear in the two rolls respectively. What are the possible values that  $X$  and  $Y$  can take? Compute the pmf of  $X$  and  $Y$ . What is the expectation of  $X + Y$ ?
3. Two coins are simultaneously tossed until one of them comes up a head and the other a tail. The first coin comes up a head with probability  $p$  and the second with probability  $q$ . All tosses are assumed independent.
  - (a) Find the PMF, the expected value, and the variance of the number of tosses.
  - (b) What is the probability that the last toss of the first coin is a head?

**Hint:** Model the tossing of the two coins as a Bernoulli trial.

4. Suppose  $X$  is a Binomial random variable with parameters  $n = 4$ , and  $p$ .
  - (a) Express  $E[\sin(\pi X/2)]$  in terms of  $p$ .
  - (b) Express  $E[\cos(\pi X/2)]$  in terms of  $p$ .
5. A modem transmits over a binary error-prone channel. To improve reliability, it transmits each bit, i.e "0" or "1", five times. We call each such group of five bits a "code-word." The channel changes an input bit to its complement with probability  $p = 1/10$  and it does so independently of its treatment of other input bits. The modem receiver takes a majority vote of the five received bits to estimate the input bit. Find the probability that the receiver makes the wrong decision.

6. Consider a random variable  $X$  with pdf given by

$$f_X(x) = \begin{cases} cx(1-x^2) & 0 \leq x \leq 1 \\ 0 & \text{elsewhere.} \end{cases}$$

- (a) What is  $c$ ? Plot the pdf of  $X$  using MATLAB.
- (b) Plot the cdf of  $X$  using MATLAB.
- (c) Find  $P(0.25 < X < 0.3)$ .