

Chapters 5.1-5.5 of *Probability, Statistics, and Random Processes* by A. Leon-Garcia

1. *Modem transmission and two RVs.* Problem 5.12, page 289 of ALG.

A modem transmits a two-dimensional signal (X, Y) given by:

$$X = r\cos(2\pi\Theta/8) \quad \text{and} \quad Y = r\sin(2\pi\Theta/8)$$

where Θ is a discrete uniform random variable in the set $\{0, 1, 2, \dots, 7\}$.

- (a) Show the mapping from S_Θ to S_{XY} , the range of the pair (X, Y) .
 - (b) Find the joint pmf of X and Y .
 - (c) Find the marginal pmf of X and of Y .
 - (d) Find the probability of the following events: $A = \{X = 0\}$, $B = \{Y \leq r/\sqrt{2}\}$, $C = \{X \geq r/\sqrt{2}, Y \geq r/\sqrt{2}\}$, $D = \{X < -r/\sqrt{2}\}$.
 - (e) Determine if X and Y are independent random variables.
 - (f) Repeat part (e) if even values of Θ are twice as likely as odd values.
2. A Bernoulli trial with $P[\text{Success}] = p$ is repeated until two successes have occurred. Let X be the trial on which the first success occurs and Y be the trial on which the 2nd success occur.
- (a) Find the joint PMF of X and Y .
 - (b) The marginal probability of X and Y .
3. Let $F_Z(z)$ and $F_Y(y)$ be valid one-dimensional CDFs. Show that $F_{Z,Y}(z, y) = F_Z(z)F_Y(y)$ satisfies the properties of a two-dimensional CDF.
4. Problem 5.18, page 290 of ALG.
- A dart is equally likely to land at any points (X_1, X_2) inside a circular target of unit radius. Let R and Θ be the radius and angle of the point (X_1, X_2) where the angle is relative to the positive-real axis.
- (a) Find the joint cdf of R and Θ .
 - (b) Find the marginal cdf of R and Θ .
 - (c) Use the joint cdf to find the probability that the dart lands in the first quadrant of the real plane and that the radius is greater than 0.5.
5. *Computing joint pdf.* Problem 5.25, page 291 of ALG.
- The amplitudes of two signals X and Y have joint pdf:

$$f_{X,Y}(x, y) = e^{-\frac{x}{2}}ye^{-y^2} \quad \text{for } x > 0, y > 0.$$

- (a) Find the joint cdf.
- (b) Find $P[X^{\frac{1}{2}} > Y]$.
- (c) Find the marginal pdfs.