EE 131A
Probability
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Discussion Set 10
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Chapter 7.1-7.3, 8 of *Probability, Statistics, and Random Processes* by A. Leon-Garcia

1. From past experience, it is known that the number of tickets purchased by a student standing in line at the ticket window for the football match of UCLA against USC follows a distribution that has mean $\mu = 2.4$ and standard deviation $\sigma = 2.0$. Suppose that few hours before the start of one of these matches there are 100 eager students standing in line to purchase tickets. If only 250 tickets remain, what is the probability that all 100 students will be able to purchase the tickets they desire?

2. A student uses pens whose lifetime is an exponential random variable with mean 1 week. Use the central limit theorem to determine the minimum number of pens he should buy at the beginning of a 15-week semester, so that with probability .99 he does not run out of pens during the semester.

3. *(CLT for a Poisson RV)* Suppose $X_1, X_2, \ldots X_n$ are n i.i.d RVs each having a Poisson distribution with parameter λ. Let $S_n = \sum_i^n X_i$. Note that the PMF of each X_i is given by

$$P(X_i = k) = \frac{\lambda^k e^{-\lambda}}{k!} \quad k = 0, 1, 2, \ldots$$

   (a) Show that S_n is another Poisson random variable.
   Hint. Use generating functions.

   (b) Find the mean and variance of S_n.

   (c) The number of messages arriving at a multiplexer is a Poisson RV with a rate of 15 messages per second. Use the central limit theorem to estimate the probability that more than 950 messages arrive in one minute.

4. *Chi-square test for testing the fit of a distribution to data*
The following histogram was obtained by counting the occurrence of the first digits in telephone numbers in one column of a telephone directory:

digit	0	1	2	3	4	5	6	7	8	9
observed	0	0	24	2	25	3	32	15	2	2

Test the goodness of fit of this data to a random variable that is uniformly distributed in the set $\{0, 1, \ldots, 9\}$ at a 1% significance level. Repeat for the case when the random variable is uniformly distributed in the set $\{2, 3, \ldots, 9\}$.

5. It is known that 90% of the cabs in a city are yellow and 10% are green. A cab hits a pedestrian at night. One witness claims that the car involved in the accident was green. Based on previous record, this witness is 80% correct, meaning that when something happens he would make the right claim 80% of the time.

   (a) Describe the corresponding hypothesis testing problem.

   (b) Based on this information, is it more likely that it was a green car or a yellow car?

6. The sum of a list of 48 real numbers is to be computed. Suppose that numbers are rounded off to the nearest integer so that each number has an error that is uniformly distributed in the interval (-0.5,0.5). Use the central limit theorem to estimate the probability that the absolute value of the total error in the sum of the 48 numbers exceeds 4.