EE 131A
Probability
Instructor: Lara Dolecek

Discussion Set 1
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TAs: Lev Tauz, Debarnab Mitra

Chapter 2.1-2.3 of *Probability, Statistics, and Random Processes* by A. Leon-Garcia

1. *Venn diagram.* Let A and B be events. Find an expression for the event "exactly one of the events A and B occurs." Draw a Venn diagram for this event.

2. Find $P(A \cup (B^c \cup C^c)^c)$ in each of the following cases:

   (a) A, B, C are mutually exclusive events and $P(A) = 4/7$.

   (b) $P(A) = 1/2, P(B \cap C) = 1/3, P(A \cap C) = 0$.

   (c) $P(A^c \cap (B^c \cup C^c)) = 0.65$.

3. Suppose A and B are two events. Prove the following:

   (a) $P(A)P(B) = P(A \cap B) \iff P(A^c)P(B) = P(A^c \cap B)$

   (b) $P(A \cap B) = 0 \implies P(A) \leq P(B^c)$

   (c) $P(A) = P(B) = P(A \cap B) \implies P((A \cap B^c) \cup (B \cap A^c)) = 0$

4. An urn contains 40 red balls and 60 green balls. What is the probability of getting exactly k red balls in a sample of size 30 if the sampling is done without replacement? Assume $0 \leq k \leq 30$. How does the answer change if we have a sample size of 50?

5. *Inclusion-exclusion principle.* Let $A_1, A_2, \ldots, A_n$ be a set of n events. Then prove that

$$P(A_1 \cup A_2 \cup \cdots \cup A_n) = \sum_{i=1}^{n} P(A_i) - \sum_{1 \leq i_1 < i_2 \leq n} P(A_{i_1} \cap A_{i_2}) + \sum_{1 \leq i_1 < i_2 < i_3 \leq n} P(A_{i_1} \cap A_{i_2} \cap A_{i_3})$$
$$- \cdots + (-1)^{n+1} P(A_1 \cap A_2 \cap \cdots \cap A_n).$$