1. a) $1 = \int_{-\infty}^{\infty} f_x(x)\,dx = 2c + 4c + 2c$

$$8c = 1 \Rightarrow c = 1/8$$

b)
$$P(1 \le X \le 8) = P(1 \le X \le 2) + P(4 \le X \le 6) + P(7 \le X \le 8)$$
$$= c/2 + 2c + c/2 = 3c = 3/4$$
$\rfloor\ \frac{1}{2}$ b/c linear

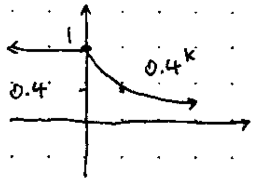
2. a) $P(M=k) = (1-p)^{k-1} \cdot p$, $P(M>k) = (1-p)^k \cdot p + (1-p)^{k+1} \cdot p + \dots$

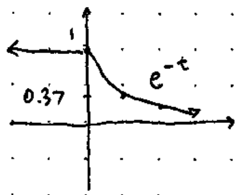
$$= \frac{p(1-p)^k}{1-(1-p)} = \boxed{(1-p)^k}$$

$P(X=t) = \lambda e^{-\lambda t}$, $P(X>t) = \int_t^{\infty} \lambda e^{-\lambda t}\,dt = e^{-\lambda t}\Big|_t^{\infty}$

$= 0 + e^{-\lambda t} = e^{-\lambda t}$ if $t > 0$. If $t < 0$, $P(X>t) = 1$.

b) $p = 0.6$ $\lambda = 1$: 0.4^k e^{-t}





e^{-t} is steeper than 0.4^k and then flattens out

c) $P(x > t+h \mid x > t) = \dfrac{P(x > t+h \ \cap x > t)}{P(x>t)} = \dfrac{P(x > h+t)}{P(x>t)}$

since $P(X>t) = e^{-\lambda t} \Rightarrow P = \dfrac{e^{-\lambda(t+h)}}{e^{-\lambda t}} = e^{-\lambda h} = P(x>h)$ ▫

3. a) $P(|x| \le 1) = 1 - e^{-2}$

$$\int_{-1}^{1} b \cdot e^{-a|x|}\,dx = \int_{-1}^{0} b \cdot e^{-a|x|}\,dx + \int_0^1 b \cdot e^{-a|x|}\,dx = \int_{-1}^{0} b \cdot e^{ax}\,dx + \int_0^1 b \cdot e^{-ax}\,dx$$

$$= \frac{-b}{a} \cdot e^{-ax}\Big|_0^1 + \frac{b}{a} \cdot e^{ax}\Big|_{-1}^{0} = \frac{b}{a}(2 - 2e^{-a}) = 1 - e^{-2} \Rightarrow a = 2, b = 1.$$

b) $F_x(x) = \int_{-\infty}^{x} e^{-2|x|}\,dx$ $\nearrow x \ge 0$, $\searrow x < 0$

$x \ge 0$:
$$\int_{-\infty}^{0} e^{2x}\,dx + \int_0^x e^{-2x}\,dx = \tfrac{1}{2}(1-0) - \tfrac{1}{2}(e^{-2x} - 1) = 1 - \tfrac{1}{2}e^{-2x}$$

$x < 0$:
$$\int_{-\infty}^{x} e^{2x}\,dx = \tfrac{1}{2}(e^{2x} - 0) = \tfrac{1}{2}e^{2x}$$

CDF:
$$F_x = \begin{cases} 1 - \frac{1}{2} \cdot e^{-2x} & x > 0 \\ \frac{1}{2} \cdot e^{2x} & x \le 0 \end{cases}$$

4. $X = U^n$ CDF: $P(X \le a) = F_x(a) = P(U^n \le a) = P(U \le \sqrt[n]{a}) = \sqrt[n]{a}/1 = \sqrt[n]{a}$

$$F_x(a) = \begin{cases} 0 & a < 0 \\ \sqrt[n]{a} & 0 \le a < 1 \\ 1 & a > 1 \end{cases} \Rightarrow f_x(a) = \begin{cases} 0 & a < 0 \\ \frac{1}{n} \cdot a^{\frac{1}{n}-1} & 0 \le a \le 1 \\ 1 & a > 1 \end{cases}$$

4b) $F_x(a) = P(U^n \le a)$:

n even: $P(-a^{1/n} \le U \le a^{1/n}) = F_U(a^{\frac{1}{n}}) - F_U(-a^{\frac{1}{n}}) = \frac{1+\sqrt[n]{a} - 1 + \sqrt[n]{a}}{2} = a^{1/n}$ $\quad 0 \le a \le 1$

$0 \quad$ otherwise.

n odd: $P(\underset{\uparrow}{0 \le U \le a^{\frac{1}{n}}} \cup \underset{\uparrow}{a \le U \le \sqrt[n]{a}}) = \begin{cases} \frac{1+a^{1/n}}{2} & |a| \le 1 \\ 0 & \text{otherwise} \end{cases}$

for positive a $\qquad$ for negative a

FINAL ANS:

$f_x(a) = \begin{cases} \frac{1}{n} \cdot a^{\frac{1-n}{n}} & 0 \le a \le 1, \ 0 \text{ otherwise} \end{cases}$ when n is even

$\begin{cases} \frac{1}{2n} \cdot a^{\frac{1-n}{n}} & |a| \le 1, \ 0 \text{ otherwise} \end{cases}$ when n is odd.

c) $F_x(a) = P(U \le \sqrt[n]{a}) = \int_0^{\sqrt[n]{a}} e^{-t}\,dt = e^{-t}\Big|_0^{\sqrt[n]{a}} = \begin{cases} 1 - e^{-\sqrt[n]{a}} & \text{when } a > 0 \\ 0 & \text{when } a < 0 \end{cases}$

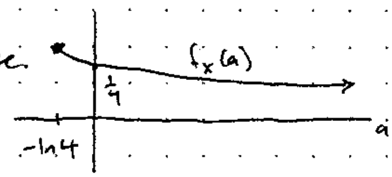
$f_x(a) = 0$ when $a < 0$ and $\frac{1}{n} a^{\frac{1}{n}-1} \cdot e^{-\sqrt[n]{a}}$ $\quad a \ge 0$

5) $X = -\ln(4-4U)$ $\quad F_x(a) = P(X \le a)$

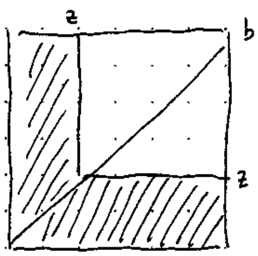
$= P(-\ln(4-4U) \le a) = P\left(U \le \frac{4-e^{-a}}{4}\right) = P\left(U \le 1 - \frac{1}{4} \cdot e^{-a}\right)$

$= \begin{cases} 1 - \frac{1}{4} e^{-a} & a \ge -\ln 4; \ 0 \text{ otherwise.} \end{cases}$

$f_x(a) = \begin{cases} \frac{1}{4} \cdot e^{-a} & a \ge -\ln 4; \ 0 \text{ otherwise.} \end{cases}$



6) a)
b)



$F_Z(z) = P(Z \le z) = \frac{b^2 - (b-z)^2}{b^2} = \frac{2bz - z^2}{b^2}$

$= \begin{cases} 0 & z < 0 \\ \frac{2bz - z^2}{b^2} & 0 \le z \le b \\ 1 & z > b \end{cases}$

c) $P(Z > 0) = 1 - \frac{0-0}{b^2} = \boxed{1}$ $\qquad P(Z > b/4) = 1 - P(Z < b/4)$

$= 1 - \frac{1}{2} \cdot 2\left(\frac{1}{4}\right)^2 = \boxed{\frac{9}{16}}$

$P(Z > b) = 1 - 1 = \boxed{0}$

$P(Z \le b/2) = \frac{b^2 - b^2/4}{b^2} = \boxed{3/4}$

d) $f_Z(z) = \frac{d}{dz}\left(\frac{2bz - z^2}{b^2}\right) = \begin{cases} \frac{2}{b} - \frac{2z}{b^2} & 0 \le z \le b \\ 0 & \text{otherwise.} \end{cases}$