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Reading: 5.1-5.10 of *Probability, Statistics, and Random Processes* by A. Leon-Garcia

1. The pair (X, Y) has joint cdf given by:

$$F_{X,Y}(x, y) = \begin{cases} (1 - \frac{1}{x^2})(1 - \frac{1}{y^2}) & \text{for } x > 1, y > 1 \\ 0 & \text{elsewhere.} \end{cases}$$

- (a) Plot the joint cdf as a 3D plot using MATLAB.
 - (b) Find the marginal cdf of X and of Y .
 - (c) Find the probability of the following events: $\{X < 3, Y \leq 5\}$, $\{X > 4, Y > 3\}$.
2. Let X and Y have the joint pdf:

$$f_{X,Y}(x, y) = ye^{-y(1+x)} \quad \text{for } x > 0, y > 0.$$

Verify that $f_{X,Y}(x, y)$ is a valid pdf. Find the marginal pdf of X and of Y .

3. The joint pdf for two random variables X and Y is given below:

$$f_{XY}(x, y) = \begin{cases} \frac{x}{5} + \frac{y}{20}, & 0 \leq x \leq 1, 1 \leq y \leq 5, \\ 0, & \text{otherwise.} \end{cases}$$

- (a) Are these two random variables independent?
 - (b) Find the covariance for X and Y .
4. A point (X, Y) is selected at random inside a triangle defined by $\{(x, y) : 0 \leq y \leq x \leq 1\}$. Assume the point is equally likely to fall anywhere in the triangle.
- (a) Find the joint cdf and joint pdf of X and Y .
 - (b) Find the marginal cdf and marginal pdf of X and Y .
 - (c) Find the probabilities of the following events in terms of the joint cdf: $A = \{X \leq 1/2, Y \leq 3/4\}$; $B = \{1/4 < X \leq 3/4, 1/4 < Y \leq 3/4\}$.
 - (d) Find $P[Y < X^2]$.

5. Let X and Y be two jointly continuous random variables with joint pdf

$$f_{XY}(x, y) = \begin{cases} 6xy, & 0 \leq x \leq 1, 0 \leq y \leq \sqrt{x}, \\ 0, & \text{otherwise,} \end{cases}$$

- (a) Find $f_X(x)$.
 - (b) Find the conditional pdf of X given $Y = y$, $f_{X|Y}(x|y)$.
 - (c) Find $E[X|Y = y]$, for $0 \leq y < 1$. What is $E[X|Y]$?
 - (d) Let A be the event $\{X \geq \frac{1}{2}\}$. Find $P[A]$, $f_{X|A}(x)$, and $E[X|A]$.
6. The random variables X and Y have the joint pdf

$$f_{X,Y}(x, y) = 8xy \quad \text{for } 0 \leq y \leq x \leq 1.$$

Are X and Y independent? Find the pdf of $Z = X + Y$.

7. If you did not submit question 6 for Homework 5, you must submit it as part of this assignment. The grade will be retroactively applied to Homework 5. The grade for this question will be applied to Homework 5 when considering the lowest homework grade to drop.