Nevin Liang HOMEWORK 1

ECE 131 A

1\. (a)

1 1	2 1	3 1	4 1	5 1	6 1
1 2	2 2	3 2	4 2	5 2	6 2
1 3	2 3	3 3	4 3	5 3	6 3
1 4	2 4	3 4	4 4	5 4	6 4
1 5	2 5	3 5	4 5	5 5	6 5
1 6	2 6	3 6	4 6	5 6	6 6

} 36 outcomes.

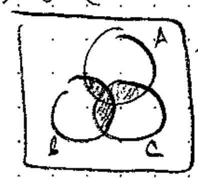
(b)

1 4
2 5
3 6
4 1
5 2
6 3

} 6 outcomes

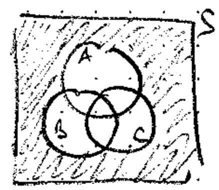
$P(|T_1 - T_2| = 3) = \frac{6}{36} = \boxed{\frac{1}{6}}$

2\. (a) $(A \cap B \cap C^c) \cup (A \cap C \cap B^c) \cup (A^c \cap B \cap C) \cup (A \cap B \cap C)$



OR

$(A \cap B) \cup (B \cap C) \cup (A \cap C)$



(b) $A^c \cap B^c \cap C^c$

(c) $(A \cap B^c \cap C^c) \cup (A^c \cap B \cap C^c) \cup (A^c \cap B^c \cap C)$

3\. (a) $P(A) = P(A \cap B^c) + P(A \cap B)$
$P(B) = P(B \cap A^c) + P(B \cap A)$
} from Venn Diagram AND definition of Complement.

$P(A) + P(B) - P(A \cap B) = P(A \cup B)$

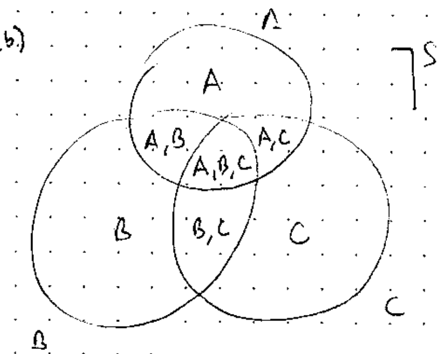
Now, $(A \cup B) \subset S \rightarrow P(S) = P(A \cup B) + P((A \cup B)^c \cap S)$
$\geq P(A \cup B)$

$P(S) = 1$ by Axiom II. Thus, $P(A \cup B) \leq 1$.

Thus, $P(A) + P(B) - P(A \cap B) \leq 1$

So, $P(A \cap B) \geq P(A) + P(B) - 1$

(b)



Let E_1 be event $A \cap B$

E_2 be event $B \cap C$

$$P(E_1) + P(E_2) - P(E_1 \cap E_2) = P(E_1 \cup E_2)$$

$$P(E_1 \cup E_2) = P(\underbrace{(A \cap B) \cup (B \cap C)}_{\cancel{\hookrightarrow \in S}}) \geq 0 \text{ by Axiom I.}$$

~~(To make writing easier, let $(A \cap B) \cup (B \cap C)$ be F_1~~)

$\cancel{F_1 \subset S}$

$\cancel{P(S) = P(F_1) + P(F_1^c \cap S) \geq P(F_1)}$

$\cancel{P(S) = 1 \text{ by Axiom II, so } P(F_1)}$

Thus, $P(E_1) + P(E_2) - P(E_1 \cap E_2) = P(A \cap B) + P(B \cap C) - P(A \cap B \cap A \cap C)$

$$= P(A \cap B) + P(B \cap C) - P(A \cap B \cap C) \geq 0$$

$P(A \cap C) \geq 0$ by Axiom I, so

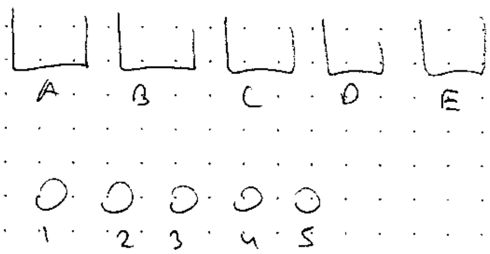
$$P(A \cap B) + P(B \cap C) + P(A \cap C) - P(A \cap B \cap C) \geq 0.$$

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$$

but second half of equation is ≥ 0. Thus,

$$P(A \cup B \cup C) \leq P(A) + P(B) + P(C) \quad \square$$

4.



Each ball can pick a bucket to go into.

Total possibilities $= 5^5$

What if each bucket has 1 ball?

either $1 \mid 2 \mid 3 \mid 4 \mid 5$

or some permutation. $5! = 120$ possibilities.

$$\frac{120}{5^5} = \boxed{\frac{24}{625}} \qquad \frac{\text{success}}{\text{total}} = \frac{P(\text{one in each bucket})}{P(\text{total}) = 1}$$

5. (a) How many ways to choose 5 cards? $\binom{4+3+5}{5} = \binom{12}{5}$

$$= \frac{\cancel{12}^{3} \cdot 11 \cdot 10 \cdot \cancel{9}^{3} \cdot 8}{\cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot 1} = 792.$$

How many ways to choose 2 white and 3 other colors?

$$\binom{4}{2} \cdot \binom{3+5}{3} = 6 \cdot \frac{8 \cdot 7 \cdot \cancel{6}}{\cancel{3 \cdot 2} \cdot 1}$$

$$= 336$$

$$\frac{\text{success}}{\text{total}} = \frac{336}{792} = \boxed{\frac{14}{33}}$$

(b) at least 2 are white: 2, 3, or 4. 3 cases.

if 2, 336 possibilities. if 3, $\binom{4}{3} \cdot \binom{3+5}{2} = 4 \cdot \frac{8 \cdot 7}{2 \cdot 1} = 112$

if 4, $\binom{4}{4} \binom{3+5}{1} = 1 \cdot 8 = 8$

$$\frac{\text{success}}{\text{total}} = \frac{336 + 112 + 8}{792} = \frac{456}{792} = \boxed{\frac{19}{33}}$$

6. (a)

	(a)	(b)
HHHH		
HHHT		
HHTH		
HTHH		
THHH		
HHTT	✓	
HTHT		
THHT		
HTTH	✓	✓
THTH		✓
TTHH	✓	✓
TTTH	✓	
TTHT	✓	✓
THTT	✓	✓
HTTT	✓	✓
TTTT	✓	

$\frac{8}{16} = \boxed{\frac{1}{2}}$ $\frac{6}{16} = \boxed{\frac{3}{8}}$

bashy method

better method to check

$P(TT + TTT + TTTT)$

$= 1 - P(\text{no successive tails})$

if 0 T's: 1
1 T's: 4
2 T's: 3
3 T's: 0
4 T's: 0

8 ✓

if 5 tosses, $P(\geq 2 \text{ successive tails}) = 1 - P(\text{no successive tails})$

=
0 T's: 1
1 T's: 5
2 T's: _ T H T _ 6
3 T's: 1
4 T's: 0
5 T's: 0

$\frac{13}{32} \neq \frac{1}{2}$ so

different for (a).

(b) should be same b/c sample size only is first 3 tosses.
as long as total tosses >3, probability should not change.