Chapter 4 of *Probability, Statistics, and Random Processes* by A. Leon-Garcia

1. *PMF of 2 RVs.* The input X to a communication channel is "-1" or "1", with respective probabilities $\frac{1}{4}$ and $\frac{3}{4}$. The output of the channel Y is given by

$$Y = \begin{cases} X & \text{wp } 1 - p - p_e \\ -X & \text{wp } p \\ 0 & \text{wp } p_e \end{cases}$$

   (a) Find S_{XY}, the range of the pair (X, Y).

   (b) Find the probabilities for all values of (X, Y).

   (c) Find $P[X \neq Y]$, $P[Y = 0]$.

2. Let X be a discrete random variable with the following PMF

$$P_X(x) = \begin{cases} 0.3 & \text{for } x = 3 \\ 0.2 & \text{for } x = 5 \\ 0.3 & \text{for } x = 8 \\ 0.2 & \text{for } x = 10 \\ 0 & \text{otherwise} \end{cases}$$

   (a) Find the CDF of X, $F_X(x)$.

   (b) Find $P(2 < X \leq 5)$ and $P(X > 4)$ using the CDF.

3. *Exponential RV.* Problem 4.61, page 221 of ALG
   Let X be an exponential random variable with parameter λ. The cdf and pdf of X are given by
$$F_X(x) = \begin{cases} 0 & \text{for } x < 0 \\ 1 - e^{-\lambda x} & \text{for } x \geq 0 \end{cases}$$

$$f_X(x) = \begin{cases} 0 & \text{for } x < 0 \\ \lambda e^{-\lambda x} & \text{for } x \geq 0. \end{cases}$$

   (a) Show that $F_X(x)$ is a valid cdf and $f_X(x)$ is a valid pdf.

   (b) For $d > 0$ and k a nonnegative integer, find $P[kd < X < (k+1)d]$.

4. *Cdf and pdf calculations.* Let ζ be a point selected at random from the unit interval. Consider the random variable $X = (1 - \zeta)^{-\frac{1}{2}}$.

(a) Sketch X as a function of ζ.

(b) Find and plot the cdf of X.

(c) Find the probability of the events $\{X > 1\}, \{5 < X < 7\}, \{X \leq 20\}$.

(d) Find and plot the pdf of X.

(e) Use the pdf to find the probabilities of the events: $\{X > a\}$ and $\{X > 2a\}$.

5. *Bonus:* Alice and Bob play a series of games with Alice winning each game with probability p. The overall winner is the first player to have won two or more games than the other. Find the probability that Alice is the overall winner.