EE 131A
Probability and Statistics
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Homework 4
Monday, February 1, 2021
Due: Monday, February 8, 2021
before class begins

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Please upload your homework to Gradescope by February 8, 3:59 pm.
Please submit a single PDF directly on Gradescope
You may type your homework or scan your handwritten version. Make sure all the work is discernible.

Reading: 4 of *Probability, Statistics, and Random Processes* by A. Leon-Garcia

1. The PDF of a continuous random variable X is given by

$$f_X(x) = \begin{cases} c & 0 < x \leq 2 \\ 2c & 4 < x \leq 6 \\ c & 7 < x \leq 9 \\ 0 & \text{otherwise} \end{cases}$$

where c is a constrant.

   (a) Find the numerical value of c.
   (b) Compute $P(1 < X \leq 8)$.

2. Let M be a geometric random variable with parameter p and let X be an exponential random variable with parameter λ.

   (a) Compute the tail probabilities $P(M > k)$ and $P(X > t)$ for the geometric and the exponential random variable where k is a positive integer and t is a non-negative real number.
   (b) Plot $P(M > k)$ as a function of k and $P(X > t)$ as a function of t. Use $p = 0.6$ and $\lambda = 1$. Compare the two plots.
   (c) A continuous random variable X is said to satisfy the memoryless property if for all $t, h \geq 0$, $P(X > t+h|X > t) = P(X > h)$. Prove that the exponential random variable satisfies the memoryless property.
   Recall that in Discussion 3, you proved that the geometric random variable satisfies the memoryless property.

3. Consider a random variable X with PDF given by $f_X(x) = be^{-a|x|}$.

   (a) If $P(|X| \leq 1) = 1 - e^{-2}$, find a and b.
   (b) Find the CDF of X.

4. Let $X = U^n$ where n is a positive integer. Find the CDF and PDF of X when

(a) U is a uniform random variable in $[0, 1]$

(b) U is a uniform random variable in $[-1, 1]$

(c) U is an exponential random variable with parameter 1.

5. Find and plot the PDF of $X = -\ln(4-4U)$, where U is a continuous random variable, uniformly distributed on the $[0, 1]$ interval.

6. A point is selected at random inside a square defined by $\{(x, y) : 0 \leq x \leq b, 0 \leq y \leq b\}$. Assume the point is equally likely to fall anywhere in the square. Let the random variable Z be given by the minimum of the two coordinates of the point.

   (a) Find the region in the square corresponding to the event $\{Z \leq z\}$.

   (b) Find and plot the CDF of Z.

   (c) Use the CDF to find $P[Z > 0], P[Z > b], P[Z \leq b/2], P[Z > b/4]$.

   (d) Find and plot the PDF of Z.