ECE 131A
Probability
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Discussion Set 9
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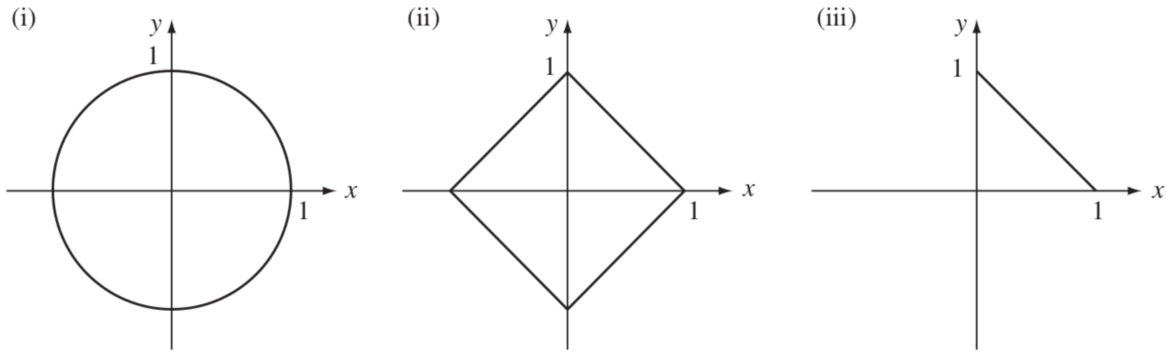
Chapters 5.1-5.10 and 6.4 of *Probability, Statistics, and Random Processes* by A. Leon-Garcia

1. *(Problem 5.28 and Problem 5.81 of ALG)* The random vector (X, Y) is uniformly distributed (i.e., $f(x, y) = k$) in the regions shown in the following figures and zero elsewhere.



   (a) Find the value of k for each case.
   (b) Find the marginal pdf for X and the marginal pdf for Y in each case.
   (c) Find $P(X > 0, Y > 0)$ for each case.
   (d) Find $f_{Y|X}(y|x)$ for each case.
   (e) Find $\mathbb{E}[Y|X = x]$ and $\mathbb{E}[Y]$ for each case.

2. *(Problem 5.23 and Problem 5.84 of ALG)*Let the number of uses logged onto a system be the RV N and the time until the next user logs off is the RV T. A joint probability is provided in the following:

$$P(N = n, T \le t) = (1-p)p^{n-1}(1 - e^{-n\lambda t}) \text{ for } n = 1, 2, 3, 4, \ldots \quad t > 0$$

   where $0 \le p \le 1$ and $\lambda > 0$ are parameters.

   (a) Find the marginal pmf of N.
   (b) Find the marginal cdf of T.
   (c) Find the conditional pdf $f_{T|N}(t|N = n)$.
   (d) Find $\mathbb{E}[T|N = n]$.

3. Let X be a zero-mean, unit variance Gaussian RV and A be a Bernoulli random variable with parameter 0.5. Define a new random variable Y such that Y is X when $A = 0$ and $-X$ when $A = 1$. Is Y a Gaussian RV? Are X and Y uncorrelated? Are X and Y independent? Are X and Y jointly gaussian RVs?

4. *(Problem 5.111 of ALG)*
   Let X and Y be jointly Gaussian random variables with PDF

$$f_{X,Y}(x,y) = \frac{\exp\left\{-\frac{1}{2}[x^2+4y^2-3xy+3y-2x+1]\right\}}{2\pi c} \quad \text{for all } x,\, y.$$

   (a) Find $E[X]$, $E[Y]$,$VAR[X]$,$VAR[Y]$, and $COV[X,Y]$ by pattern matching the above expression with the expression for jointly Gaussian random variables. Additionally, determine c.

   (b) Confirm the value of $\mathbb{E}[Y]$ by determining the marginal pdf of Y.

   (c) Find $\mathbb{E}[X|Y]$.