EE 131A
Probability
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Discussion Set 5
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Chapter 4 of *Probability, Statistics, and Random Processes* by A. Leon-Garcia

1. *Gaussian RV.* If X is a normal random variable with parameters $\mu = 10$ and $\sigma^2 = 36$, compute

   (a) $P[X > 5]$

   (b) $P[4 < X < 16]$

   (c) $P[X < 8]$

   You don't have to compute the final real values. You can leave the answers in terms of the $Q(.)$ function or the standard normal CDF function $\Phi(.)$.

2. Let X be an exponential random variable with parameter $\lambda > 0$. Find the expectation and variance of X.

3. *Function of RV, $Y = g(X)$ where X is discrete and Y is continuous.*
   Assume $h << 1$ for all parts.
   Let X be a Bernoulli Random Variable with parameter p which is an input to a binary communication system. The output Y of the system is a Gaussian random variable with variance one and mean "0" when the input is "0" and mean "1" when the input is "1". In other words $Y \sim \mathcal{N}(X, 1)$.

   (a) Find $P[\text{input is } 1 | y < Y < y + h]$ and $P[\text{input is } 0 | y < Y < y + h]$.

   (b) The receiver uses the following decision rule:
   If $P[\text{input is } 1 | y < Y < y + h] > P[\text{input is } 0 | y < Y < y + h]$, decide input was 1; otherwise, decide input was 0. Show that this decision rule leads to the following threshold rule:
   If $Y > T$, decide input was 1; otherwise, decide input was 0.

   (c) What is the probability of error for the above decision rule?

4. *Max of iid. uniform.* Problem 4.174, page 231 of ALG.
   The random variable X is uniformly distributed in the interval $[0, a]$. Suppose a is unknown, so we estimate a by the maximum value observed in n independent repetitions of the experiment; that is, we estimate a by $Y = \max\{X_1, X_2, \ldots, X_n\}$.

   (a) Find $P[Y \leq y]$.

   (b) Find the mean and variance of Y, and explain why Y is a good estimate for a when n is large.

5. *Bonus:* A stick of length 1 is split at a point U that is uniformly distributed over $(0, 1)$. Determine the expected length of the piece that contains the point p, $0 \leq p \leq 1$.