$$f_{X,Y}(x,y) = \frac{\partial^2 F_{X,Y}(x,y)}{\partial y\, \partial x} \Rightarrow P(x_1 \le x \le x_2,\ y_1 \le y \le y_2) = \int_{y_1}^{y_2}\int_{x_1}^{x_2} f_{X,Y}(x',y')\,dx'\,dy'$$

Marginal PDF:

$$P(X=x_i) = \sum_j P(X=x_i, Y=y_j)$$

$\rightarrow$ for continuous: $f_X(x) = \int_{-\infty}^{\infty} f_{X,Y}(x,y')\,dy'$

$$= \frac{d}{dx} F_X(x) = \frac{d}{dx} \lim_{y\to\infty} F_{X,Y}(x,y)$$

$$= \frac{d}{dx} F_{X,Y}(x, +\infty) =$$

INDEPENDENCE & PDF/CDF

$X \perp\!\!\!\perp Y$: $P(X\in A, Y\in B) = P(X\in A)\cdot P(Y\in B) \quad \forall A,B$

$$F_{X,Y}(x,y) = P(x\le X, y \le Y) = F_X(x)\cdot F_Y(y)$$
$$= P(x \le X)\cdot P(y\le Y)$$

Since $f_{X,Y}(x,y) = \dfrac{\partial^2 F_{X,Y}(x,y)}{\partial x\, \partial y}$

if $X \perp\!\!\!\perp Y$: $f_{X,Y}(x,y) = f_X(x)\cdot f_Y(y)$

Expected Value of func of 2 RVs

$Z = g(X,Y)$ continuous: discrete

$$\mathbb{E}(Z) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x,y)\, f_{X,Y}(x,y)\,dx\,dy = \sum_j \sum_k g(x_j, y_k)\, P_{X,Y}(x_j, y_k)$$

marginal: $W = h(X)$

$$\mathbb{E}(W) = \int_{-\infty}^{\infty} h(x)\, f_X(x)\,dx$$

X, Y dependent $P(X)\cdot P(Y) \neq P(X\cdot Y)$; $\mathbb{E}[X\cdot Y] \neq \mathbb{E}[X]\,\mathbb{E}[Y]$

if $X \perp\!\!\!\perp Y$, $\mathbb{E}[X\cdot Y] = \mathbb{E}[X]\cdot\mathbb{E}[Y]$ and $\mathbb{E}[g(X)\cdot h(Y)] = \mathbb{E}[g(X)]\cdot\mathbb{E}[h(Y)]$

JOINT MOMENTS

$\mathbb{E}[X^j \cdot Y^k]$: j,k - th moment of X, Y.

$j,k = 1,1 \Rightarrow \mathbb{E}[X \cdot Y]$. if $=0$, j & k orthogonal.

$\mathbb{E}[(x - m_x)^j \cdot (y - m_y)^k]$ = (j,k)-th central moment of (X, Y)

when $j=2, k=0$: variance of X

when $j=1, k=1$: covariance of X & Y.

$\mathbb{E}[XY] - m_x \cdot m_y = \mathbb{E}((x - m_x) \cdot (y - m_y))$

equal / if $X \perp\!\!\!\perp Y$, $\text{cov}(X,Y) = 0$

correlation coefficient $\rho_{X,Y} = \dfrac{\text{cov}(X,Y)}{\sqrt{\text{VAR}(X)} \cdot \sqrt{\text{VAR}(Y)}}$

and $0 \leq \rho_{X,Y} \leq 1$

if $\rho_{X,Y} = 0$, uncorrelated

CoV = 0	uncorrelated	equal when either $\mathbb{E}[x]$ or $\mathbb{E}[y] = 0$.
$\mathbb{E} = 0$	orthogonal	