

EE 131A
Probability and Statistics
Instructor: Lara Dolecek

Homework 2
Monday, January 11, 2021
Due: Wednesday, January 20, 2021
before class begins
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Please upload your homework to Gradescope by January 20, 3:59 pm.
Please submit a single PDF directly on Gradescope
You may type your homework or scan your handwritten version. Make sure all the work is discernible.

Reading: Chapters 2.4-2.5 & 3 of *Probability, Statistics, and Random Processes* by A. Leon-Garcia

1. Roll two fair dice independently. In terms of the possible outcomes, define the events:

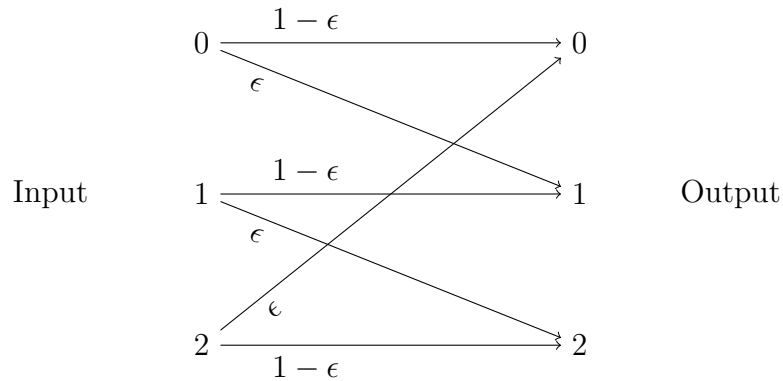
$$\begin{aligned}A &= \{\text{First die is 1, 2 or 3}\} \\B &= \{\text{First die is 2, 3 or 6}\} \\C &= \{\text{Sum of outcomes is 9}\}\end{aligned}$$

Are A , B , and C mutually independent? Hint: Three events A , B , and C are independent if all the four following constraints hold:

$$\begin{aligned}P(A \cap B) &= P(A)P(B), \\P(A \cap C) &= P(A)P(C), \\P(B \cap C) &= P(B)P(C), \\P(A \cap B \cap C) &= P(A)P(B)P(C).\end{aligned}$$

2. Assume there are 5 jars numbered 1 to 5. The i^{th} jar contains i black balls, $6 - i$ red balls, and 5 green balls. A jar is selected uniformly at random and a ball is selected uniformly at random from that jar. Let the events B , R , and G represent the events that a black, red, or green ball is chosen, respectively. Let J_k represent the event that the k^{th} jar is chosen.

- (a) What is $P(B|J_k)$?
(b) What is $P(G)$, $P(B)$, and $P(R)$?
(c) Given that the selected ball is black, what is the probability that the ball came from the k^{th} jar, i.e. $P(J_k|B)$?
3. A ternary communication channel is shown in the figure. Assume that input symbols 0, 1, and 2 are chosen for transmission with probabilities $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{1}{4}$, respectively.



- (a) Calculate the probability of each output.
 - (b) Given that the output was 1, what is the probability that the input was 0? 1? 2?
4. A family has 5 natural children and has adopted 2 girls. Each natural child has equal probability of being a girl or a boy, independent of the other children. Find the PMF of the number of girls out of the 7 children.
 5. Throw a pair of six-sided dice. Let X_1 be the number of dots on the resulting face of the first die and let X_2 be the number of dots on the resulting face of the second die. Let $Z = X_1 + X_2$ be the sum of the two dice rolls.
 - (a) What is the pmf of Z ?
 - (b) What is $\mathbb{E}[Z]$? $Var(Z)$?
 - (c) Given that $Z = 10$, what is the probability that $X_1 = k$ for $k \in \{1, 2, 3, 4, 5, 6\}$?
 6. Assume that we flip a biased coin with probability of heads being p until a 2nd head is seen. Let X be the number of flips up until and including the flip that has the 2nd head. What is the pmf and expectation of X ?