ECE 131 A
Nevin Liang

HW 5

1. $P\left[|Y-p| \geq a\right] \leq \frac{\sigma^2}{a^2}$ ← $E[Y]=E\left[\frac{X}{n}\right]=\frac{1}{n}E[X]=\frac{np}{n}=p$

$Var[Y]=VAR\left[\frac{X}{n}\right]=\frac{1}{n^2}VAR(X)=\frac{npq}{n^2}=\frac{pq}{n}$

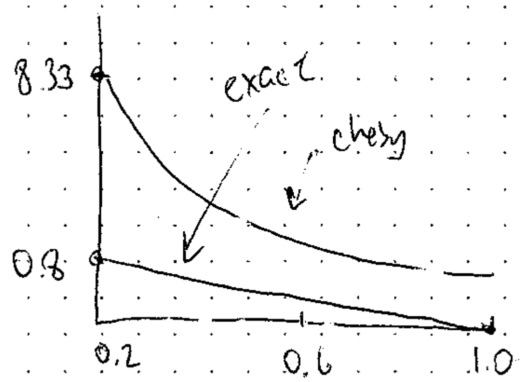
$P\left[|Y-p| \geq a\right] \leq \frac{pq}{na^2}$

$n\to\infty$, $P\left[|Y-p| \geq a\right] \leq \frac{pq}{na^2} \to \boxed{0}$ (if $a>0$)

2. $P(|X-m|>c) < \frac{\sigma^2}{c^2}$ in this case, $m=0 \Rightarrow E(X)=0$

$VAR(X)=\frac{1}{12}(2b)^2=\frac{1}{12}4b^2=\frac{b^2}{3}$ $\Rightarrow P(|X|>c)<\frac{b^2}{3c^2}$ Cheby

$P(X<-c)+P(X>c)=\frac{2|b-c|}{2b}=\boxed{\frac{b-c}{b}}$ since $|b|>|c|$



3. a) $P[X=x]=p(1-p)^x$

$E(e^{j\omega X})=\sum_{x=0}^{\infty} e^{jtx}(1-p)^x p$

$=p\sum_{x=0}^{\infty}\left((1-p)e^{jt}\right)^x$

$=\boxed{\frac{p}{1-(1-p)e^{jt}}}$

b) $E(e^{j\omega y})=\int_a^b e^{j\omega y} f(y)\,dy$

$=\int_a^b e^{j\omega y}\frac{1}{b-a}\,dy$

$=\frac{1}{(b-a)j\omega}\left[e^{j\omega b}-e^{j\omega a}\right]$

c) $\sum_{k=c}^{d} \frac{1}{d-c+1} e^{jwk}$

$$= \frac{1}{d-c+1}\left[e^{jwc} + e^{jw(c+1)} + \cdots + e^{jwd}\right]$$

$$= \frac{e^{jwc} - e^{jw(d+1)}}{(d-c+1)\,(1-e^{jw})}$$

4.
a) $\Phi_Y(w) = E(e^{jwy}) = E(e^{jw(aX+b)}) = e^{jwb}\, E(e^{jwaX})$

$y = aX+b$

$$= e^{jwb}\, \Phi_X(aw)$$

b) $X \sim Exp(\lambda)$ $\quad \Phi_X(w) = \frac{\lambda}{\lambda - wj}$ $\quad \Phi_Y(w) = e^{jw \cdot 0}\, \Phi_X(aw) = \boxed{\frac{\lambda}{\lambda - w(ja)}}$

$\boxed{\text{exp rand var}}$

c) $E[y] = E[ax+b] = a\,E[x] + b = a \cdot \frac{1}{\lambda} + b$

$\boxed{\Phi_y(w) = e^{jwb} \cdot \Phi_x(aw)}$

$\frac{d}{dw} \Rightarrow jbe^{jwb}\, \Phi_x(aw) + e^{jwb} \cdot \frac{\lambda ja}{(\lambda - jaw)^2}$

$\frac{1}{j} \cdot x$

$\Big|_{w=0} = \boxed{b + \frac{1}{\lambda} a}$

5. (a) $P(X \geq a) \leq e^{-sa} E(e^{sX}) = e^{-sa} \Phi_X(-js)$

Let be standard normal.

$$= e^{-sa}\left[e^{-j \cdot j \cdot s \cdot \mu - \sigma^2 j^2 s^2 \cdot \frac{1}{2}}\right]$$

$$= e^{-sa + s\mu + \sigma^2 s^2 \cdot \frac{1}{2}}$$

$$\frac{d}{ds}(\exp) = \mu - a + \sigma^2 s \Rightarrow s = \frac{a-\mu}{\sigma^2}$$

$$-s(a-\mu) + \frac{(a-\mu)^2}{2} = -a^2 + \frac{a^2}{2} = \frac{-a^2}{2} \Rightarrow e^{-a^2/2} \geq Q(a)$$

(b) $P(X \geq a) \leq e^{-sa} E(e^{sX})$

$$P(Y \geq (1+\delta)np) \leq \left(\frac{e^{\delta}}{(1+\delta)^{1+\delta}}\right)^{np} \quad \forall \delta > 0$$

$$\leq e^{-s(1+\delta)np} E(e^{sY})$$

$$= e^{-s(1+\delta)pn}\left(1 + p(e^s - 1)\right)^n$$

$$= \left(e^{-sp(1+\delta) + p(e^s - 1)}\right)^n$$

$$\frac{d}{ds}(\exp) = -p(1+\delta) + pe^s = 0$$

$$e^s = 1 + \delta \Rightarrow s = \ln(1+\delta)$$

$$P(Y \geq (1+\delta)np) \leq \left(\frac{e^{\delta}}{(1+\delta)^{1+\delta}}\right)^{np}$$

distribution extends to the → b/c this value goes to 0 slowly when n, δ, and p go ↑.