REVIEW

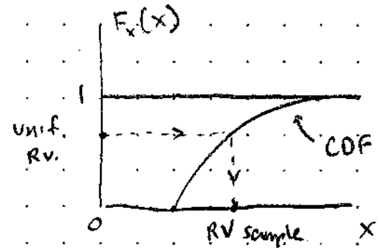
CDF & PMF are corresponding

characteristic func: $\Phi_\omega(x) = \mathbb{E}\left[e^{j\omega x}\right]$

- finds moments
- PDF/PMF of sum of RV
- Bounding methods
  - Chernoff bound useful for Gaussian
  - Chebyshev > markov inequality

GENERATING RV

- create RV X is continuous whose CDF is $F_x(x)$.
  - easiest RV to generate is uniform: $[0,1]$ ⇒ random generic var (use CDF).

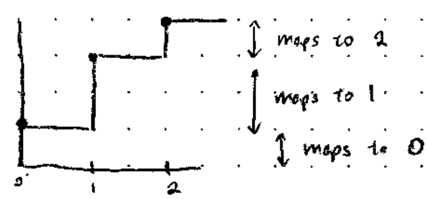


$X = F_x^{-1}(u)$ ⇒ plug in uniform RV $[0,1]$ to get random distrib of original RV, X.

$$P(X \le x) = P(F_x^{-1}(U) \le x)$$
$$= P(U \le F_x(x)) = F_x(X)$$

BOX-MULLER method for generating standard gaussian RVs.

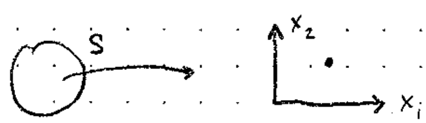
* Discrete RVs
  - use floor to map to discrete values:



Two RVs

vector RV: assigns real-valued vector to each outcome in sample space

2-dimensional RV:



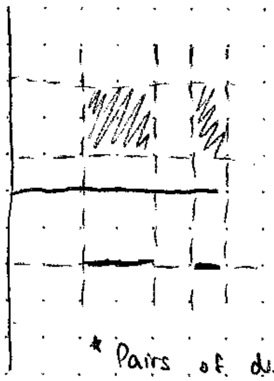
$X = (x_1, x_2)$, event A has PRODUCT FORM if it can be written as intersection of 2 1D events

$$A = \{x_1 \in A_1\} \cap \{x_2 \in A_2\}$$

2D: product looks like rectangle, line = special case.

→ next page = example.

event A: $\{ x_1 \in [2,4) \cup [5,6] \} \cap \{ x_2 \in [1,3] \cup \{-2\} \} \}$



looks like this: x axis x_1

y axis x_2

* Pairs of discrete RVs $S = \{ (x_i, y_j), i = 1,2\ldots \;\&\; j = 1,2\ldots \}$

joint PMF = P(product-form of events)

where $x = x_j \cap y = y_k$

$$p_{X,Y}(x_j, y_k) = P[\{X = x_j\} \cap \{Y = y_k\}] \triangleq P[X = x_j \cap Y = y_k]$$

P(A) = P(mutually exclusive product forms)

$$P[A] = P[\underline{X} \in A]$$

$$= \underbrace{\sum \sum}_{(x_j, y_k) \in A} p_{X,Y}(x_j, y_k)$$

sums for 1-∞

total sum = 1

MARGINAL PMFs

PMFs of individual components

↳ For P_{XY}, marginal PDF: $p_X(x_j) = \sum_{k=1}^{\infty} p_{X,Y}(x_j, y_k)$

can't find joint from marginal.. X & Y must be independent

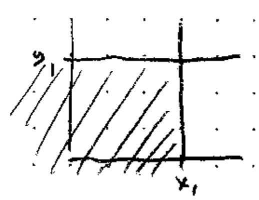
$$P(A_1 \cap A_2) = P(A_1) \cdot P(A_2)$$

if $X \perp\!\!\!\perp Y$, $P(X \in A_1, Y \in A_2) = P(X \in A_1) \cdot P(Y \in A_2)$

only if: $p_{X,Y}(x_j, y_k) = p_X(x_j) \cdot p_Y(y_k) \;\; \forall x_j, y_k$

GENERAL CASE

Joint CDF = $P(X \le x_1 \cap Y \le y_1)$. $F_{X,Y}(x_1, y_1) = P(X \le x_1, Y \le y_1)$



properties: if $x_1 \le x_2$ & $y_1 \le y_2$

$$F_{X,Y}(x_1, y_1) \le F_{X,Y}(x_2, y_2)$$

if one of x, y is $-\infty$, $F_{X,Y}$ is 0

if both ∞, $F_{X,Y} = 1$

IF 1 is ∞, use marginal.