Probability & Statistics Nevin Liang

1. $P(S1) = .3$ Deluxe: 1000 50%
   $P(S2) = .6$ Standard: 500 50%

Total \$ vals : 2000, 1500, 1000, 500, 0

$P(2000) = 0.3 \cdot \frac{1}{2} \cdot 0.6 \cdot \frac{1}{2} = 0.045$

$P(1500) = 0.3 \cdot 1 \cdot 0.6 \cdot 1/2 = 0.09$

$P(1000) = 0.3 \cdot \frac{1}{2} \cdot 0.4 + 0.7 \cdot 0.6 \cdot \frac{1}{2} + 0.3 \cdot \frac{1}{2} \cdot 0.6 \cdot \frac{1}{2} = 0.315$

$P(500) = 0.3 \cdot \frac{1}{2} \cdot 0.4 + 0.7 \cdot \frac{1}{2} \cdot 0.6 = 0.27$

$P(0) = 0.7 \cdot 0.4 = 0.28$

$$PMF(x) \begin{cases} 0.045 & x = \$2000 \\ 0.09 & X = \$1500 \\ 0.315 & X = \$1000 \\ 0.27 & X = \$500 \\ 0.28 & X = \$0 \end{cases}$$

2. $X: \{1,2,3,4,5,6\}$ $Y: \{1,2,3,4,5,6\}$

$$PMF(X) \begin{cases} 1/36 & X=1 & 7/36 & X=4 \\ 3/36 & X=2 & 9/36 & X=5 \\ 5/36 & X=3 & 11/36 & X=6 \end{cases}$$

$$P(X=k) = \frac{k^2-(k-1)^2}{36} = \frac{2k-1}{36}$$

$$PMF(Y) = \begin{cases} 11/36 & Y=1 & 5/36 & Y=4 \\ 9/36 & Y=2 & 3/36 & Y=5 \\ 7/36 & Y=3 & 1/36 & Y=6 \end{cases}$$

$$P(Y=k) = \frac{2(7-k)-1}{36} = \frac{13-2k}{36}$$

$$E(X+Y) = E(X) + E(Y) = \frac{1+6+15+28+45+66+11+18+21+20+15+6}{36}$$

$$= \boxed{7}$$

3. $H: p$ $\quad$ $H: q$ $\quad$ $P(1H, 1T) = p(1-q) + q(1-p)$

$T: 1-p$ $\quad$ $T = 1-q$

C1 $\quad$ C2 $\quad$ $= p+q-2pq$

a) $P(N=1) = p+q-2pq$

$P(N=2) = (1-p-q+2pq)(p+q-2pq)$

$\vdots$

$PMF(N=k) = (1-p-q+2pq)^{k-1}(p+q-2pq) \qquad k \geq 1$

$$E(N) = \frac{1}{p+q-2pq}$$

$$VAR(N) = \frac{1-p-q+2pq}{(p+q-2pq)^2}$$

b) $$P(HT \mid \text{last toss}) = \frac{P(\text{1 last toss} \mid HT) \cdot P(HT)}{P(\text{1 last toss})} = \frac{p(1-q)}{p(1-q)+q(1-p)}$$

$$= \boxed{\frac{p-pq}{p+q-2pq}}$$

4. Binomial RV: $P(X=k) = \binom{n}{k} \cdot p^k (1-p)^{n-k}$

$$E(x) = \sum_{k=0}^{n} g(k) \cdot P(X=k)$$

if $\sin\left(\frac{\pi x}{2}\right) = \sum_{k=0}^{n} \sin\left(\frac{\pi k}{2}\right) \cdot \binom{n}{k} \cdot p^k (1-p)^{n-k}$

$= 0 + \binom{n}{1} \cdot p(1-p)^{n-1} + 0 - \binom{n}{3} \cdot p^3 \cdot (1-p)^1$

$= np(1-p)^3 - \binom{n}{3} \cdot p^3 (1-p) = 4p(1-p)^3 - 4p^3(1-p)$

if $\cos\left(\frac{\pi x}{2}\right) = \sum_{k=0}^{n} \cos\left(\frac{\pi k}{2}\right) \cdot \binom{n}{k} \cdot p^k (1-p)^{n-k}$

$= \binom{n}{0} \cdot (1-p)^4 - \binom{n}{2} \cdot p^2 \cdot (1-p)^2 + \binom{n}{4} \cdot p^4$

$= (1-p)^4 - \binom{n}{2} \cdot p^2 (1-p)^2 + p^4 = (1-p)^4 - 6p^2(1-p)^2 + p^4$

5. wrong decision if ≥ 3 wrong bits.

$$\binom{5}{3}\cdot\left(\tfrac{1}{10}\right)^3\cdot\left(\tfrac{9}{10}\right)^2+\binom{5}{4}\cdot\left(\tfrac{1}{10}\right)^4\cdot\tfrac{9}{10}+\binom{5}{5}\cdot\left(\tfrac{1}{10}\right)^5$$

$$= \frac{10\cdot 81+5\cdot 9+1\cdot 1}{10\cdot 10\cdot 10\cdot 10\cdot 10} = \frac{810+45+1}{10^5} = \frac{856}{10^5}$$

$$= 0.856\ \%$$

6. (a) $$\int_0^1 f_x(x)\,dx = 1$$

$$\int_0^1 cx(1-x^2)\,dx = 1 \Rightarrow c\cdot\frac{x^2}{2} - c\cdot\frac{x^4}{4}\Big|_0^1 = \frac{c}{2}-\frac{c}{4}=1$$

$$c = \boxed{4}$$

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$$\text{PDF}(x) = \begin{cases} 4x(1-x^2) & [0,1] \\ 0 & \text{otherwise.} \end{cases}$$

(b) $$F_x(x) = \frac{cx^2}{2} - \frac{cx^4}{4} = \boxed{2x^2 - x^4}$$

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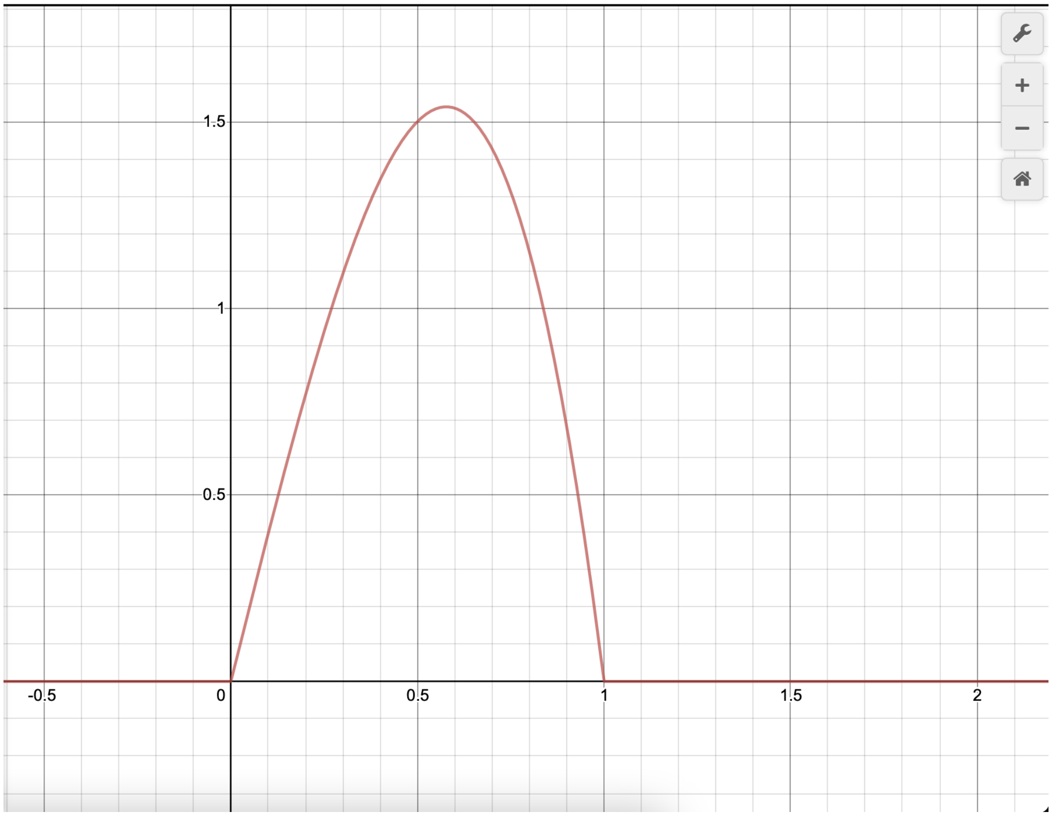
(c) $$P(0.25 < X < 0.3) = \int_{0.25}^{0.3} 4x(1-x^2) = 4\left(\frac{x^2}{2}-\frac{x^4}{4}\right)_{0.25}^{0.3}$$

$$= \boxed{0.0508}$$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0
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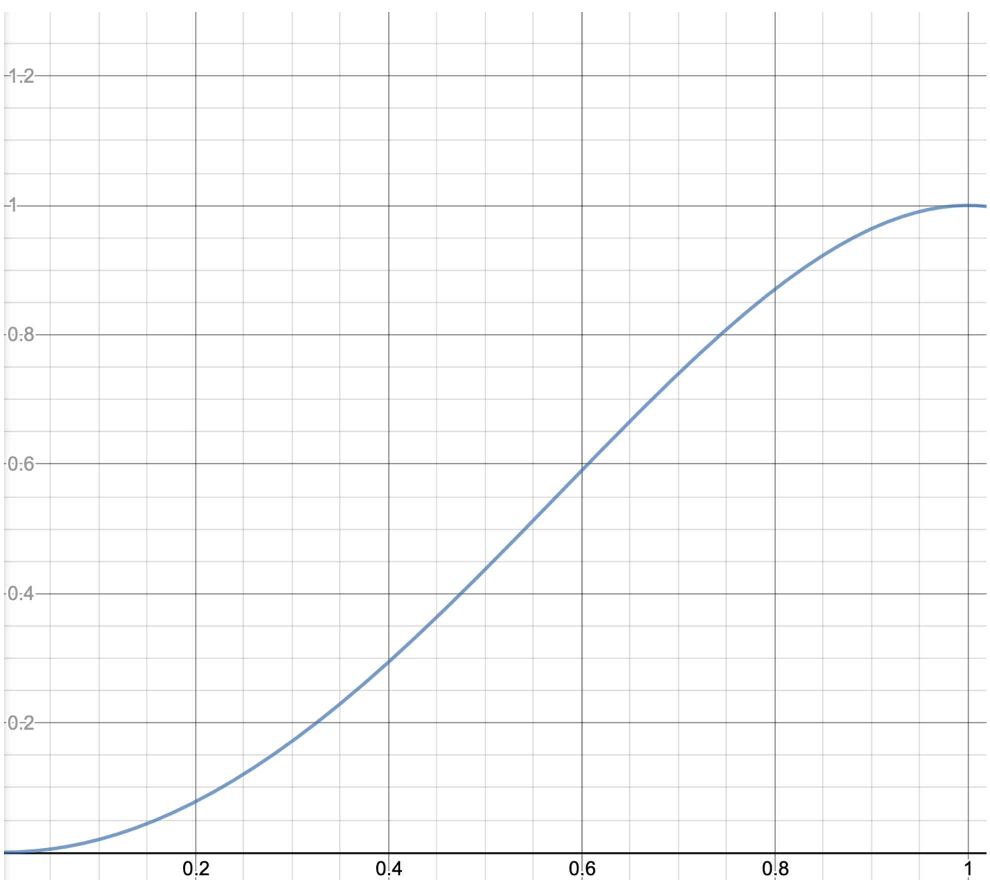
1
1.5
2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0.4
0.2
0.2
0.4
0.6
0.8
1