EE 131A
Probability
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Discussion Set 2
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Chapters 2.4-2.5 & 3 of *Probability, Statistics, and Random Processes* by A. Leon-Garcia

1. Let G be a geometric random variable with parameter p. Recall that the pmf for G is $P(G = k) = (1-p)^{k-1}p$ for $k \geq 1$. Find the variance of G, i.e. $Var(G)$.

2. Ninety students, including Joe and Jane, are to be split into three classes of equal size, and this is to be done at random. What is the probability that Joe and Jane end up in the same class?

3. *Pairwise independence and overall independence.* Alice, Bob and Claire each throw a fair die once. Show that the events A, B and C where A: "Alice and Bob roll the same face", B: "Alice and Claire roll the same face" and C: "Bob and Claire roll the same face" are pairwise independent but not independent.

4. Each of k jars contains m white and n black balls. A ball is randomly chosen from jar 1 and transferred to jar 2, then a ball is randomly chosen from jar 2 and transferred to jar 3, etc. Finally, a ball is randomly chosen from jar k. Show that the probability that the last ball is white is the same as the probability that the first ball is white, i.e., it is $\frac{m}{m+n}$.

5. *Bonus:Monty Hall Problem.* You are a contestant on a game show, and you are given a choice of three doors. Behind one door is a car; behind the other two doors are goats. You pick a door, for example, No. 1, and the game show host, who knows what is behind the doors, opens another door, say No. 3, which has a goat. The host asks you,"Do you want to switch to door No. 2?" Should you switch your choice? Why?