Discussion 8

Lecture 12 Recap

- Joint RVs $(X, Y)$
  - Discrete Joint pmfs $P(X=x, Y=y)$

$$P(X=x) = \sum_{y \in S_y} P(X=x, Y=y)$$

$P(Y=y)$

Marginal pmfs

- Independence

$$P(X=x, Y=y) = P(X=x)\,P(Y=y)$$

for all $(x,y)$ samples

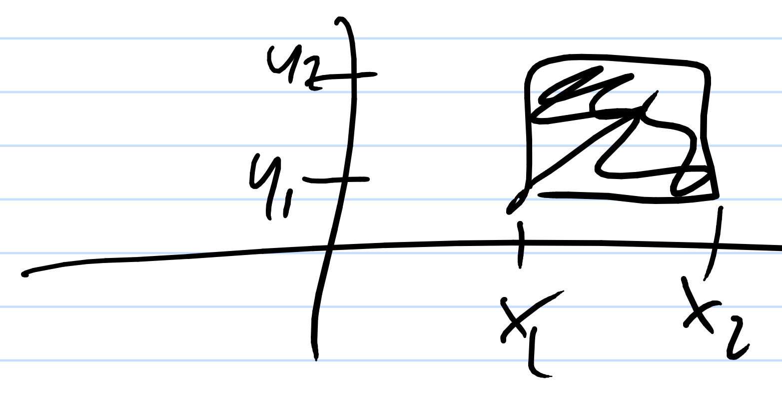
$X \perp Y \Rightarrow X, Y$ independent

- Joint cdf

$$P(X \le x, Y \le y) = F(x,y)$$

$$F_X(x) = \lim_{y \to \infty} F(x,y)$$

$$P(x_1 \le X \le x_2, y_1 \le Y \le y_2) \quad (A)$$




$$(A) = F(x_2, y_2) - F(x_1, y_2) - F(x_2, y_1) + F(x_1, y_1)$$

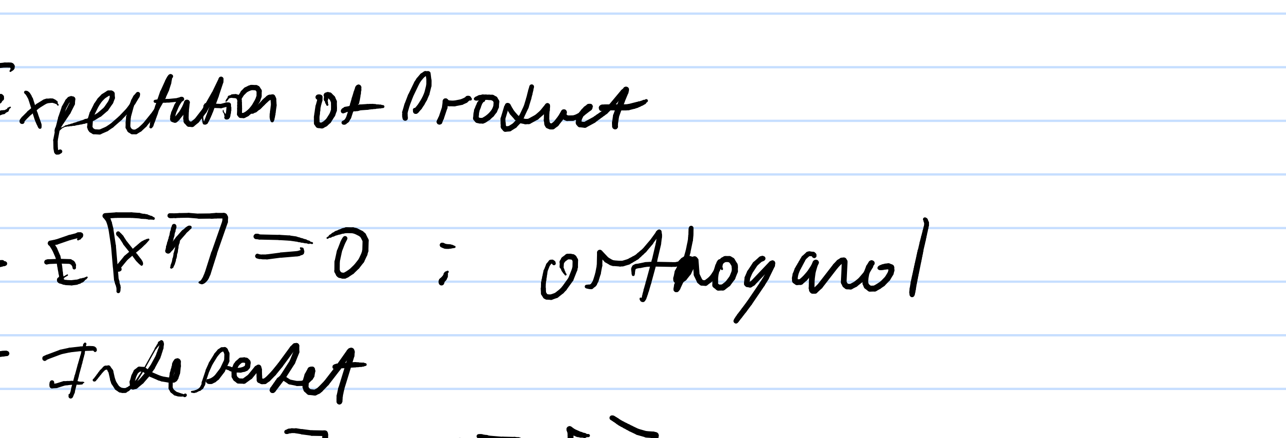
- Joint pdf

$$f(x,y) = \frac{\partial^2}{\partial x \partial y} F(x,y)$$

- $\iint f(x,y)\,dx\,dy = 1$

2D $P((X,Y) \in B) = \iint_{(x,y) \in B} f(x,y)\,dx\,dy$

1D $P(X \in I) = \int_{x \in I} f(x)\,dx$




- Independence in Joint pdf

$$f_{X,Y}(x,y) = f_X(x) \cdot f_Y(y)$$

for all $(x,y)$ samples

- Expectation of Product
  - $E[XY] = 0$ : orthogonal
  - Independent

$E[XY] = E[X]E[Y]$

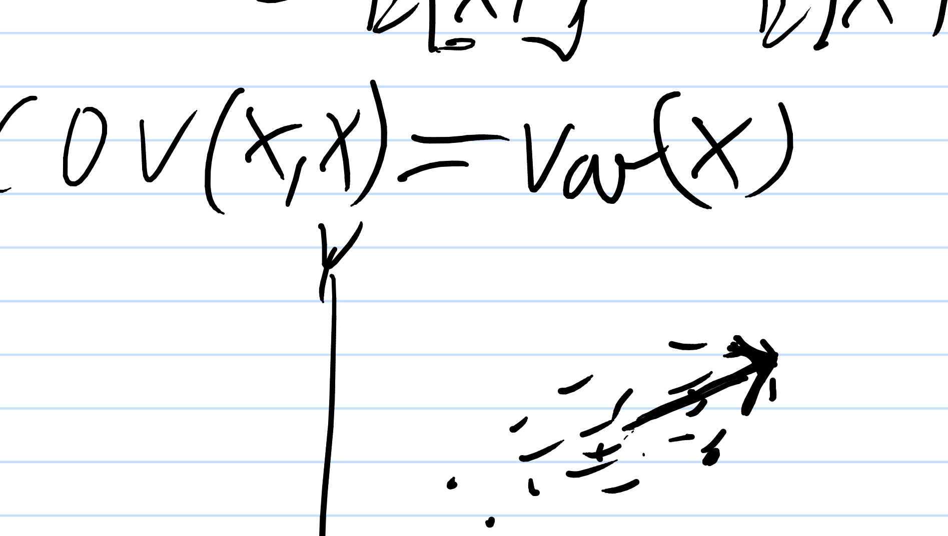
$E[g(X)f(Y)] = E[g(X)]E[f(Y)]$

  - Covariance

$$\mathrm{Cov}(X,Y) = E\big((X - E[X])(Y - E[Y])\big)$$

$$= E[XY] - E[X]E[Y]$$

$$\mathrm{COV}(X,X) = \mathrm{Var}(X)$$




- Correlation Coefficient

$$\rho_{X,Y} = \frac{\mathrm{cov}(X,Y)}{\sqrt{\mathrm{Var}(X)}\sqrt{\mathrm{Var}(Y)}}$$

- $\mathrm{COV}(X,Y) = 0$ : uncorrelated
- Uncorrelated and orthogonal

$\mathrm{Cov}(X,Y) = 0 = E[XY] - E[X]E[Y]$

$E[XY] = 0$

- Either $E[X] = 0$ or $E[Y] = 0$