

Chapter 5.6-5.10 of *Probability, Statistics, and Random Processes* by A. Leon-Garcia

1. *The Joint pdf of Two Continuous Random Variables.* Problem 5.26, page 291 of ALG.
Let X and Y have joint pdf:

$$f_{X,Y}(x, y) = k(x + y) \quad \text{for } 0 \leq x \leq 1, 0 \leq y \leq 1.$$

- (a) Find k .
 - (b) Find the joint cdf of (X, Y) .
 - (c) Find the marginal pdf of X and of Y .
2. *Conditional Probability and Conditional Expectation.* Problem 5.80, page 296 of ALG.
- (a) Find $f_Y(y|x)$ in Problem 1.
 - (b) Find $P[Y > X|x]$.
 - (c) Find $P[Y > X]$ using part b.
 - (d) Find $E[Y|X = x]$.
3. *(Problem 5.58 of ALG)*
Find $E[X^2 e^Y]$ where X and Y are independent random variables, X is a zero-mean, unit-variance Gaussian random variable, and Y is a uniform random variable in the interval $[0, 3]$.
4. For two random variables X and Y ,
- (a) Express $E[(X + Y)^2]$ in terms of means, variances, and covariances for X, Y .
 - (b) Find the variance of $X + Y$ in terms of means, variances, and covariances of X, Y .
 - (c) When is the variance of the sum of RVs the same as the sum of the individual variances of each RV?
5. *Sum of uniform RVs.* Suppose X and Y are independent uniform random variables in the interval $[0, 1]$, and $Z = X + Y$. Find $f_Z(z)$.
6. Suppose X and Y are independent exponential random variables with common parameter λ , and $Z = \frac{X}{X+Y}$. Find the PDF $f_Z(z)$.