EE 131A
Probability
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Discussion Set 3
January 22, 2021
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Chapter 2.1-2.3 of *Probability, Statistics, and Random Processes* by A. Leon-Garcia

1. *Memoryless property of Geometric RV.*
   A discrete random variable M is said to satisfy the memoryless property if $P[M \geq k+j|M \geq j+1] = P[M \geq k]$ for all $j, k \geq 1$. Show that the geometric random variable satisfies the memoryless property.

2. You take an exam that contains 20 multiple-choice questions. Each question has 4 possible options and only one correct answer. You have no idea about any of the questions, so you choose answers randomly. Your score X on the exam is the total number of correct answers.

   (a) Find the PMF of X.

   (b) What is your expected score on the test?

   (c) What is $P(X > 16)$?
   Hint: *You may approximate* 32551 *as* 2^{15}.

3. *A Game with Marbles.* A box contains 5 red and 5 blue marbles. Two marbles are withdrawn randomly, one at a time without replacement. If they are the same color, then you win \$1.10; if they are different colors, then you lose \$1.00. Calculate the expected value of the amount you win and the variance of the amount you win.

4. *(Problem 3.9 and 3.26 of ALG)* A coin is tossed n times. Let the random variable Y be the difference between the number of heads and the number of tails in the n tosses of a coin. Assume $P[\text{heads}] = p$.

   (a) Describe the sample space of S.

   (b) Find the probability of the event $\{Y = 0\}$.

   (c) Find the probabilities for the other values of Y.

   (d) Find $E[Y]$ and $VAR[Y]$. In a large number of repetitions of this random experiment, what is the meaning of $E[Y]$?

5. *Bonus:* n people arrive at a restaurant and give their hats to a hat-check person. The hat-check person losses the receipts of who the hats belong to and returns the hats randomly. What is the expected number of people who get their own hat?