$$\begin{bmatrix} A^T A + a_1 a_1^T & a_1 \\ a_1^T & 0 \end{bmatrix} \begin{bmatrix} x \\ z \end{bmatrix} = \begin{bmatrix} A^T b + a_1 b_1 \\ b_1 \end{bmatrix}$$

$$\left(A^T A + a_1 a_1^T \right) x + a_1 z = A^T b + a_1 b_1$$

$$a_1^T x = b_1$$

$$z =$$

$$\underline{x = \left(A^T A + a_1 a_1^T \right)^{-1} \left(A^T b + a_1 b_1 - a_1 z \right)}$$

$$A^T A x + a_1 \cancel{b_1} + a_1 z = A^T b + a_1 \cancel{b_1}$$

$$a_1 z = A^T b - A^T A x = (A^T A)(\hat{x} - x)$$

$$\hat{x} = (A^T A)^{-1} A^T b$$

$$A^T A x = A^T b - a_1 z$$

$$x = (A^T A)^{-1} (A^T b - a_1 z)$$

$$x = \hat{x} - (A^T A)^{-1} a_1 z$$

$$\boxed{= \hat{x} - z (A^T A)^{-1} a_1}$$

$$= \quad \text{almost!!} \ \overset{..}{\frown}$$