2) $A = QR$

$$x = (A^T A)^{-1} A^T b = R^{-1} Q^T b$$

$$(A^T A)^{-1} a_1 = (R^T R)^{-1} a_1$$

$$= R^{-1} R^{-T} a_1$$

$R^{-T} \cdot a_1 = x$ $\quad R^T x = a_1$ can be computed in $\boxed{n^2}$ flops by back substitution.

$R^{-1} \cdot x = y$ $\quad Ry = x$ $\rightarrow$ n^2 flops by back sub.

$R^{-1} R^{-T} a_1 = 2n^2$ flops.

$a_1^T R^{-1} R^{-T} a_1 = \|(R^{-T} a_1)\|^2 = \|x\|^2 \approx \boxed{2n}$ flops.

3) minimize $\|Ax-b\|^2 + (a_1^T x - b_1)^2 = \left\| \begin{bmatrix} A \\ -a_1^T \end{bmatrix} x - \begin{bmatrix} b \\ -b_1 \end{bmatrix} \right\|^2$

subject to $a_1^T x = b_1$

solution =

$$\begin{bmatrix} A^T & -a_1 \\ a_1^T & 0 \end{bmatrix} \begin{bmatrix} A \\ -a_1^T \\ a_1^T \end{bmatrix} \begin{bmatrix} x \\ z \end{bmatrix} = \begin{bmatrix} A^T & -a_1 \\ a_1^T & 0 \end{bmatrix} \begin{bmatrix} b \\ -b_1 \\ b_1 \end{bmatrix}$$