Homework 1.   CH 1-4   T1.17, 2.5, 3.25, 4.3   A 1.10

T1.17)

$$\begin{array}{cccccc} 1 & 2 & 3 & \cdots & & T \\ & & & \uparrow & & \\ & & & t & & \end{array}$$

$$l_t = SPL(t) = \underbrace{\begin{bmatrix} \\ \\ \end{bmatrix}}_{T}$$

$$l_1 = \begin{bmatrix} 1 \\ -(1+r) \\ 0 \\ 0 \\ 0 \\ \vdots \end{bmatrix} \quad l_2 = \begin{bmatrix} 0 \\ 1 \\ -(1+r) \\ 0 \\ \vdots \end{bmatrix} \quad \cdots$$

$$l_{T-1} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \\ -(1+r) \end{bmatrix}$$

$$C \left\{ \begin{array}{l} C_1 = \begin{bmatrix} 1 \\ -(1+r) \\ 0 \\ \vdots \end{bmatrix} \quad C_2 = \begin{bmatrix} 0 \\ (1+r) \\ -(1+r)^2 \\ 0 \\ \vdots \end{bmatrix} \\ \cdots \\ C_{T-1} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ (1+r)^{T-2} \\ -(1+r)^{T-1} \end{bmatrix} \quad C_T = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ (1+r)^{T-1} \end{bmatrix} \end{array} \right.$$

$$C = C_1 + C_2 + C_3 \dots + C_{T-1} + C_T = l_1 + l_2 \cdot (1+r) + l_3 \cdot (1+r)^2 \dots + l_{T-1} \cdot (1+r)^{T-2} + \cancel{\ }$$

$=$

2.5) $\psi : R^2 \to R$

$\psi(1,0) = 1 \qquad \psi(1,-2) = 2$

$$\psi(x) = a^T x + b$$

$$= [a_1 \quad a_2] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + b$$

$$a_1 \cdot 1 + a_2 \cdot 0 + b = 1$$

$$a_1 \cdot 1 - a_2 \cdot 2 + b = 2$$

$$2a_2 = -1 \qquad a_2 = -1/2$$

$$a_1 = 1 - b$$

(a) $\psi(1,-1) =$ ~~[illegible]~~

$$[a_1 \quad a_2] \begin{bmatrix} 1 \\ -1 \end{bmatrix} + b$$

$$= a_1 - a_2 + b = 1 - a_2$$

$$= 1 - (-1/2) = \boxed{3/2}$$

(b) $\psi(2,-2) = 2a_1 - 2a_2 + b$

$$= 1 + b + 2(1-b)$$

$$= 3 - b \quad \text{but idk } b$$

so ~~UNDET~~I
INDETERMINATE

3.25

a) $\boxed{\mu_p = \theta \cdot \mu + (1-\theta) \cdot \mu^{rf}}$

$\boxed{\sigma_p = \sigma_1 + \sigma_2 = |\theta|\sigma}$

where $\sigma_1 = |\theta|\sigma$ and $\sigma_2 = 0$

when $\theta = 0$: $\mu_p = \mu^{rf}$ $\sigma_p = 0$ ✓

$\theta = 1$ $\mu_p = \mu$ $\sigma_p = |\theta|\sigma = \sigma$ ✓

b) $\sigma_p = \sigma^{tar} = |\theta|\sigma \rightarrow |\theta| = \dfrac{\sigma^{tar}}{\sigma}$

$\theta = \pm \dfrac{\sigma^{tar}}{\sigma}$ as $\theta \uparrow$, $\sigma_p \uparrow$ but $\mu_p \uparrow$

as $\theta \downarrow$, $\sigma_p \downarrow$ but $\mu_p \downarrow$

$\boxed{\theta = \pm \sigma^{tar} / \sigma}$ if $\mu > \mu_{rf}$ then σ^{tar}/σ

$\mu < \mu_{rf}$ then $\theta = -\sigma^{tar}/\sigma$ optimal.

c) we use leverage aka $\theta > 1$ when $\sigma^{tar} > \sigma$, or when target risk is greater than asset risk.

we use shorting when $\theta < 0$. ~~can also choose to short stock~~

— in the case of (b) where

$\theta = \cancel{\mu}\sigma^{tar}/\sigma$, ~~we never short the stock~~.

we hedge when $0 < \theta < 1$, or when $\sigma^{tar} < \sigma$ aka target risk is less than asset risk

4.3

Let us take a look at the set of vectors that lie on the ~~bisecting~~ perpendicular bisector of z_1 and z_2.

Aka the set of all points equidistant from z_1 and z_2.

$$\text{if} \quad \|x-z_1\|^2 = \|x-z_2\|^2, \quad \|x-z_1\|^2 - \|x-z_2\|^2 = 0.$$

Now, for the equation:

$$d = \|x-z_1\|^2 - \|x-z_2\|^2$$

as we saw before, if $d=0$, x is equidistant from both z_1 and z_2

if $d<0$ x is closer to z_1.

$d>0$ x is close to z_2.

if x is closer to z_1, it will fall into G_1 after K-means.

closer to z_2 " " G_2 after K-means.

$$\|x-z_1\|^2 = (x-z_1)^T(x-z_1) = x^Tx - x^Tz_1 - z_1^Tx + z_1^Tz_1 = \cancel{\|x\|^2} - x^Tz_1 - z_1^Tx + \|z_1\|^2$$

$$-\ \|x-z_2\|^2 = \quad \text{"} \quad \text{"} \quad = \cancel{\|x\|^2} - x^Tz_2 - z_2^Tx + \|z_2\|^2$$

$$\|x-z_1\|^2 - \|x-z_2\|^2 = x^Tz_2 - x^Tz_1 + z_2^Tx - z_1^Tx + \|z_1\|^2 - \|z_2\|^2$$

$$= x^T(z_2-z_1) + (z_2-z_1)^T x + \|z_1\|^2 - \|z_2\|^2$$

$$= 2(z_2-z_1)^T x + \|z_1\|^2 - \|z_2\|^2$$

OOPS my function $d = \|x-z_1\|^2 - \|x-z_2\|^2$ actually does <0 for G_1 and >0 for G_2. It is supposed to do the opposite I think.

REVERSING SIGNS $\Rightarrow$ $(2z_1-2z_2)^T x + \|z_2\|^2 - \|z_1\|^2 = \|x-z_2\|^2 - \|x-z_1\|^2$

$$w = 2(z_1-z_2) \quad \text{and} \quad v = \|z_2\|^2 - \|z_1\|^2$$

Problem A 1.10:

```
digits = digits(:, 1:10000);
group = randi(20, 1, 10000);
Z = zeros(784, 20);
Jprev = -1;

while 1
    % compute reps (Z)
    for i = 1:20
        I = find(group == i);
        Z(:, i) = mean(digits(:, I), 2)';
    end
    J = 0;
    for i = 1:size(digits, 2)
        mingroup = -1;
        minval = 100000;
        for j = 1:20
            tv = (norm(Z(:, j) - digits(:, i)))^2;
            if (tv < minval)
                minval = tv;
                mingroup = j;
            end
        end
        J = J + minval;
        group(i) = mingroup;
    end
    if (abs(J - Jprev) <= J / 100000)
        break;
    end
    Jprev = J;
end

for k=1:20
    subplot(4,5,k)
    imshow(reshape(Z(:,k), 28, 28));
end
```