#4)

1. minimize $\lambda (a_1^T x - b_1)^2 + \sum_{i=2}^{m} (a_i^T x - b_i)^2 = (\lambda - 1)(a_1^T x - b_1)^2 + \sum_{i=1}^{m} (a_i^T x - b_i)^2$

equivalent to minimize

$$\boxed{\left\| \begin{bmatrix} A \\ \sqrt{\lambda-1}\ a_1^T \end{bmatrix} x - \begin{bmatrix} b \\ \sqrt{\lambda-1}\ b_1 \end{bmatrix} \right\|^2 \quad \text{so that } e_1}$$

$$\hat{x}(\lambda) = \left(\begin{bmatrix} A^T & \sqrt{\lambda-1}\ a_1 \end{bmatrix} \begin{bmatrix} A \\ \sqrt{\lambda-1}\ a_1^T \end{bmatrix} \right)^{-1} \cdot \begin{bmatrix} A^T & \sqrt{\lambda-1}\ a_1 \end{bmatrix} \begin{bmatrix} b \\ \sqrt{\lambda-1}\ b_1 \end{bmatrix}$$

$$= \left(A^T A + (\lambda - 1) a_1 a_1^T \right)^{-1} \cdot \left[A^T b + (\lambda - 1) a_1 b_1 \right]$$

$$A^T A \hat{x} = A^T b \qquad \cancel{a_1^T a_1 \hat{x} = 2 a_1^T b_1}$$

$$\left(A^T A + (\lambda - 1) a_1 a_1^T \right) \hat{x}(\lambda) = A^T b + (\lambda - 1) a_1 b_1 = \cancel{A^T A \hat{x} + (\lambda - 1)} a_1 b_1$$

$$A^T A \hat{x} + \frac{(\lambda - 1)(b_1 - a_1^T \hat{x})}{1 + (\lambda - 1) a_1^T (A^T A)^{-1} a_1} a_1 +$$