3. 1) $\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} = $ P.D. prove A_{11}, A_{22} are P.D.

Let $V = \begin{bmatrix} X \\ 0 \end{bmatrix}$ ← N vector, ← N vector

$$V^T \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} V = \begin{bmatrix} X^T & 0 \end{bmatrix} \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} X \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} X^T A_{11} & X^T A_{12} \end{bmatrix} \begin{bmatrix} X \\ 0 \end{bmatrix}$$

$$= X^T A_{11} X > 0$$

Note: $V^T A V > 0$ for All $V \neq 0$. Thus, it is true for a subset of All V. Thus, true for a set of vectors V that can be written as $\begin{bmatrix} X \\ 0 \end{bmatrix}$ where $\vec{X}, \vec{0}$ are ~~..~~ N dimensional.

$\vec{X}$ can be ANY vector, so the proof holds.

Let $w = \begin{bmatrix} 0 \\ y \end{bmatrix}$ (same as before thingy) $\Rightarrow$ $w^T A w = y^T A_{22} y > 0$

so A_{22} P.D.

2) $B_{11} = A_{11}^{-1}$. A_{11} is P.D. For ANY vector x, let $y = A_{11}^{-1} x$

$A_{11} y = x$. $x^T A_{11}^{-1} x = y^T A_{11}^T A_{11}^{-1} A_{11} y = y^T A_{11}^T y$

$= y^T A_{11} y > 0$. So, A_{11}^{-1} is P.D. and B_{11} is P.D.

Since $y^T A_{11}^T y$ is scalar.