EE 133A HW #2: A(1.7, 1.8, 2.4, 2.8) + #5. NEVIN LIANG

1.7) $J = \frac{1}{n}\|c_1 \cdot \vec{1} + c_2 \cdot a - b\|^2 = \frac{1}{n}\|(m_b - m_a c_2)\cdot\vec{1} + c_2 \cdot a - b\|^2$

$= \cancel{\frac{1}{n}\|\frac{\rho s_b}{s_a}\vec{1}\|} + \cancel{= \frac{1}{n}\left\|\frac{\vec{1}^T b}{n}\vec{1} - \frac{\vec{1}^T a}{n}\frac{\rho s_b}{s_a}\vec{1} + \frac{\rho s_b}{s_a}a - b\right\|^2}$

$= \frac{1}{n}\|c_2(a - m_a\vec{1}) - (b - m_b\vec{1})\|^2$

$= \frac{1}{n}\left(\|c_2(a - m_a\vec{1})\|^2 - 2\|c_2(a - m_a\vec{1})\|^T\|b - m_b\vec{1}\| + \|b - m_b\vec{1}\|^2\right)$

$= \frac{1}{n}\left(c_2^2 \cdot n \cdot s_a^2 + n \cdot s_b^2 - 2\frac{\rho \cdot n \cdot s_a \cdot s_b \cdot c_2}{\cancel{c_2 \cdot s_a\sqrt{n} \cdot s_b\sqrt{n}}}\right)$

$= \frac{1}{n}\left(c_2^2 \cdot n \cdot s_a^2 + n s_b^2 - 2n\rho c_2 \cdot s_a \cdot s_b\right) = \frac{1}{n}\left(\rho^2 \cdot s_b^2 \cdot n + n \cdot s_b^2 - 2n\rho^2 s_b^2\right)$

$= s_b^2 - \rho^2 s_b^2 = \boxed{s_b^2(1-\rho^2)}$

1.8) We want to minimize $\frac{\|c_1 \cdot \vec{1} + c_2 \cdot a - b\|^2}{n} = \|c_1\vec{1} + (c_2 \cdot a - b)\|^2 / n$

$= (c_1^2 \cdot \vec{1} + 2 \cdot c_1 \|c_2 \cdot a - b\| + \|c_2 \cdot a - b\|^2)/n$

$\frac{\partial}{\partial c_1} J = (2c_1 n\vec{1} + 2\|c_2 a - b\|)/n \to 0$

$\frac{2 c_1 n}{n} + 2\|c_2 \cdot m_a - m_b\| = 0$

$\frac{c_1}{n} = \boxed{m_b - m_a \cdot c_2}$

b) $J = \frac{\|c_2(a - m_a\vec{1}) - (b - m_b\vec{1})\|^2}{n(1 + c_2^2)} = \frac{c_2^2 \cdot \|a - m_a\vec{1}\|^2 - 2 \cdot c_2(a - m_a\vec{1})^T(b - m_b\vec{1}) + \|b - m_b\vec{1}\|^2}{n(1 + c_2^2)}$

$= s_a^2 c_2^2 + s_b^2 - 2\rho s_a s_b c_2 / n(1 + c_2^2)$

Q ~~formula:~~ $\cancel{\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}} \cancel{= \left(\frac{s_b}{s_a} - \frac{s_a}{s_b}\right) \pm \sqrt{\frac{s_a^2}{s_b^2} - 2 + \frac{s_b^2}{s_a^2} + 4\rho^2}}$

If ρ and c_2 have the ~~same~~ diff sign, $s_a^2 c_2^2 + s_b^2 - 2\rho s_a s_b c_2$ will be large b/c $-2\rho s_a s_b c_2$ will be > 0. If same sign, $-2\rho s_a s_b c_2$ is < 0, and J is minimized.

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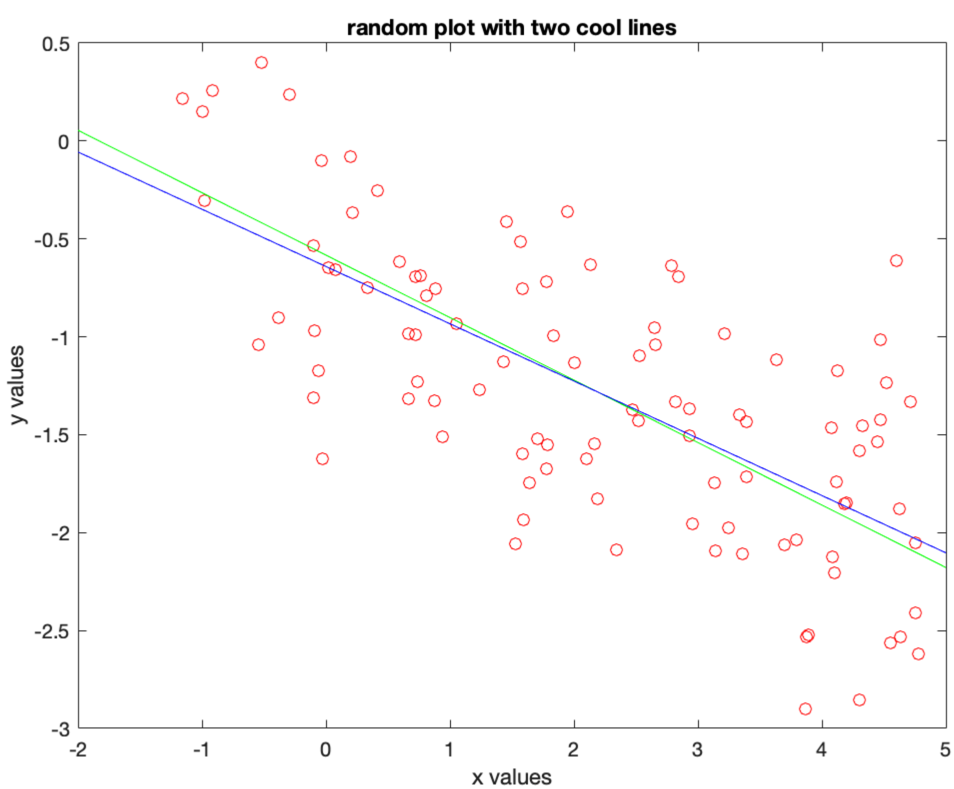
```
a;
b;
rho = corrcoef(a, b);
rho = rho(1,2);
sa = std(a); sb = std(b);
ma = mean(a); mb = mean(b);
coeff = [rho sa/sb-sb/sa -rho];
r = roots(coeff);
% rho is negative -0.6959, therefore we choose the negative coefficient aka
% the first element
c2 = r(1);
c1 = mb - ma * c2;

figure(1);
hold on;
scatter(a, b, 'red');
plot(x, x * c2 + c1, 'green');

A = [ones(length(a),1) a];
linreg = A\b;
plot(x, linreg(1) + linreg(2) * x, 'blue');

hold off;

xlabel("x values");
ylabel("y values");
title("random plot with two cool lines");
```



```
disp("green line is orthogonal and blue line is regular linreg")
```

```
green line is orthogonal and blue line is regular linreg
```

2.4) If G_A is strongly connected there is a path from all nodes i to node j. Furthermore, this path is at MOST $n-1$ steps long ∴ going through a vertex twice is a waste of steps.

The matrix $(I+A)^{n-1}$ is special. $(A)^{n-1}$ is the adjacency matrix for paths of length $n-1$. Having it be $I+A$ instead means you can have as many loops to yourself before moving. thus, instead of being ONLY paths of length $n-1$, now $(I+A)^{n-1}$ is all paths of max length $n-1$.

If the entry is 0, there is no path at all. → strongly connected.

2.8) $y = (A \otimes B)x = \begin{bmatrix} A_{11}Bx_1 + A_{12}Bx_2 + A_{13}Bx_3 + A_{14}Bx_4 + \dots \\ A_{21}Bx_1 + \dots \\ \vdots \end{bmatrix}$

$Bx_1 - Bx_n$ take n^2 operations each total n^3

$A_{xy}(Bx_i)$ takes n^2 ops. n rows, so n^3 total

$O(n^3+n^3) = O(n^3) < O(n^4)$ ✓ much more efficient

#5) (a) b/c for all k, $\underbrace{i \quad k}\underbrace{k+1 \quad j}$ C_{ik} is the min for the 1st half $C_{k+1,j}$ is the min for the second half, and $2n_{i-1}n_k n_j$ is the cost of joining either side of the C_{ij} block to itself.

(b)

$C_{12} = \text{min} \cancel{(C_{11} + C_{22}} + 2n_0n_1n_2 = 2 \cdot 5 \cdot 10^9 = 10^{10}$

$C_{23} = 2n_1n_2n_3 = 2 \cdot 5 \cdot 10^{10} = 10^{11}$ $C_{24} = \min(\cancel{C_{22}} + C_{34} + 2 \cdot n_1 \cdot n_2 \cdot n_4,$

$C_{34} = 2n_2n_3n_4 = 2 \cdot 10^8$ $\cancel{C_{23} + C_{44} + 2 \cdot n_1 \cdot n_3 \cdot n_4})$

$= \boxed{2 \cdot 10^8 + 1 \cdot 10^9}$

$C_{13} = \min(\cancel{C_{11}} + C_{23} + 2 \cdot n_0 n_1 n_3, C_{12} + \cancel{C_{33}} + 2n_0n_2n_3)$

~~$C_{14} = \min(C_{11} + C_{24} +$~~

$10^{11} + 2 \cdot 5 \cdot 10^8$ $\boxed{10^{10} + 2 \cdot 10^9}$

$= \cancel{10^{11} + 10^9}$

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$$C_{14} = \min\left(\cancel{C_{11}} + C_{24} + 2 \cdot n_0 n_1 n_4 \,,\; C_{12} + C_{34} + 2 n_0 n_2 n_4 \,,\; C_{13} + \cancel{C_{44}} + 2 n_0 n_3 n_4 \right)$$

$$2 \cdot 10^{8} + 1 \cdot 10^{9} + 2.5 \cdot 10^{6} \,,\; 10^{10} + 2 \cdot 10^{8} + \dots \,,\; 10^{10} + 2 \cdot 10^{9} + \dots$$

$$\downarrow$$

$$1 \cdot 10^{9} + 2 \cdot 10^{8} + 10^{7} = \boxed{1210000000}$$

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