11.26)

(a) $A = \begin{bmatrix} B & -C^T \\ C & D \end{bmatrix} \overset{?}{=} \begin{bmatrix} R_{11}^T R_{11} & R_{11}^T R_{12} \\ -R_{12}^T R_{11} & -R_{12}^T R_{12} + R_{22}^T R_{22} \end{bmatrix}$

B is PD so $B = R_{11}^T R_{11}$ is possible.

$C = -R_{12}^T R_{11}$

since R_{11} has positive diagonal elements,

thus, since R_{11} is U.T. $\rightarrow$ invertible

$-R_{12}^T = C \cdot R_{11}^{-1}$

and $R_{12} = (-C \cdot R_{11}^{-1})^T$

$D = -R_{12}^T R_{12} + R_{22}^T R_{22}$ ... we want to prove that R_{22} is upper triangular

$\underbrace{R^T R}_{\text{positive definite}} + \underbrace{R_{12}^T R_{12}}_{\text{positive semidefinite from:}} = R_{22}^T R_{22}$

$a^T R_{12}^T R_{12} \cdot a = \|(R_{12} \cdot a)\|^2 \geq 0$

for all a.

PD + PSD = PD so R_{22} exists and is U.T.

(b) Step 1: $B = R_{11}^T R_{11} \rightarrow R_{11}$ takes $\boxed{\frac{1}{3}n^3}$ by cholesky.

Step 2: $C = -R_{12}^T R_{11}$ $\quad$ $R_{11} \cdot \vec{X} = \vec{y}$ takes $O(N^2)$ to solve

$\hookrightarrow$ N colums $\Rightarrow$ $\boxed{\cancel{n^3}}$ mn^2 since $\vec{X}$ has m columns.

Step 3: $\underbrace{D+}_{m^2} \; R_{12}^T R_{12} = R_{22}^T R_{22} \Rightarrow$ cholesky's $= \frac{1}{3}m^3$

$R_{12}^T R_{12} \rightarrow (n \times m)^T (n \times m) \Rightarrow 2m^2 n$

$\boxed{2mn^2 + \frac{1}{3}n^3 + \cancel{m^2} + \frac{1}{3}m^3}$ $\leftarrow$ only cubic terms.