

PROBLEM 2

1. $A+A^T$ is PSD \Rightarrow for all y , $y^T(A+A^T)y \geq 0$

so, $y^T A y + y^T A^T y \geq 0$. Note that $(y^T A y)^T = y^T A^T y$. Also, $y^T A y$ is a scalar because $(1 \times m)(m \times m)(m \times 1) = (1 \times 1)$.

So, $y^T A y = y^T A^T y$.

$y^T A y + y^T A^T y = 2y^T A y \geq 0 \Rightarrow y^T A y \geq 0$ for all y .
 $x^T A x \geq 0$ for all x (accidentally used y instead of x .)

2. To prove $I+A$ is nonsingular we can prove that $\Rightarrow I+A$ is positive definite

because $Mx=0 \rightarrow \tilde{M}^T x=0 \rightarrow x=0$

$y^T(I+A)y = y^T y + y^T A y$. Now, $y^T A y \geq 0$ for all y from (1).

$y^T y = \|y\|^2 > 0$ if $y \neq 0$.

Thus, $y^T y + y^T A y > 0$ if $y \neq 0 \Rightarrow$ positive definite $(I+A) \Rightarrow$ nonsingular.

3.