HOMEWORK 4 SOLUTIONS

4.11)

(a)

$$Y = \frac{1}{s^2}(W + D(R - Y) - KY)$$

$$Y(\frac{s^2 + D + K}{s^2}) = \frac{W + DR}{s^2}$$

$$Y = \frac{D}{s^2 + D + K}R + \frac{1}{s^2 + D + K}W$$

$$E(s) = R(s) - Y(s) = \frac{-D + s^2 + D + K}{s^2 + D + K}R(s)$$

$$= \frac{s^2 + K}{s^2 + D + K}R(s)$$

for constant steady-state error to a ramp,

$$\lim_{s \to 0} s(\frac{s^2 + K}{s^2 + D + K})\frac{1}{s^2} = \text{constant}$$

$$\lim_{s \to 0} s(s^2 + D + K) = \text{constant}$$

$$\lim_{s \to 0} sD(s) = \text{constant}$$

$D(s)$ must have a pole at the origin.

(b)

$$Y(s) = \frac{1}{s^2 + D(s) + K}W(s)$$

$$\lim_{s \to 0} s(\frac{1}{s^2 + D(s) + K})\frac{1}{s^\ell} = 0$$

iff

$$\lim_{s \to 0} s^{\ell - 1}D(s) = \infty$$

iff $\ell = 1$ since $D(s)$ has a pole at the origin. Therefore system will reject step disturbances.

4.26)

(a)

$$\begin{aligned} m\ddot{x} &= \sum F = K_a u - D\dot{x} \\ \mathcal{L}\{m\dot{v} &= K_a u - Dv\} \\ \frac{V}{U} &= \frac{K_a}{ms+D} = \frac{0.01}{s+0.01} \end{aligned}$$

(b) Error:

$$\begin{aligned} E(s) &= V_d - V = V_d - \frac{\frac{k_p}{s+0.02}}{1+\frac{k_p}{s+0.02}} V_d + \frac{0.05\frac{1}{s+0.02}}{1+\frac{k_p}{s+0.02}} G(s) \\ &= \frac{(s+0.02)V_d - 0.05G}{s+0.02+k_p} \end{aligned}$$

If we want error < 1 m/sec in presence of grade, we in fact need $|e_{ss}(\text{step})| < 1$. Assume no input : $(V_d = 0)$

$$e_{ss}(\text{step}) = \lim_{s\to 0} s\left(\frac{-0.05}{s+0.02+k_p}\right)\frac{2}{s} = \frac{-0.1}{0.02+k_p}$$

$$\left|\frac{-0.1}{0.02+k_p}\right| < 1$$

While solving the inequality apply (or check) restriction that poles are in LHP.

$$\Longrightarrow k_p > 0.08$$

(c) The obvious advantage of integral control would be zero s.s. error for step input (Type 1 system would result).

(d) Pure integral control: $k_p \rightarrow \frac{k_I}{s}$

$$E(s) = \frac{s(s+0.02)V_d - 0.05sG(s)}{s^2 + 0.02s + k_I}$$

$$\zeta = 1 \Longrightarrow \omega_n = 0.01 \Longrightarrow k_I = 0.0001$$

4.29)

(a) TF for disturbance:

$$\frac{Y}{W} = \frac{\frac{1}{\zeta s + b}}{1 + \frac{1}{\zeta s + b} \cdot \frac{10k_p}{0.5s + 1}} \qquad b = 1 \ , \ \zeta = 0.1$$

$$e_{ss}(\text{step in W}) = \lim_{s \to 0} s \frac{1}{s} \frac{Y}{W} = \frac{1}{1 + 10k_p}$$

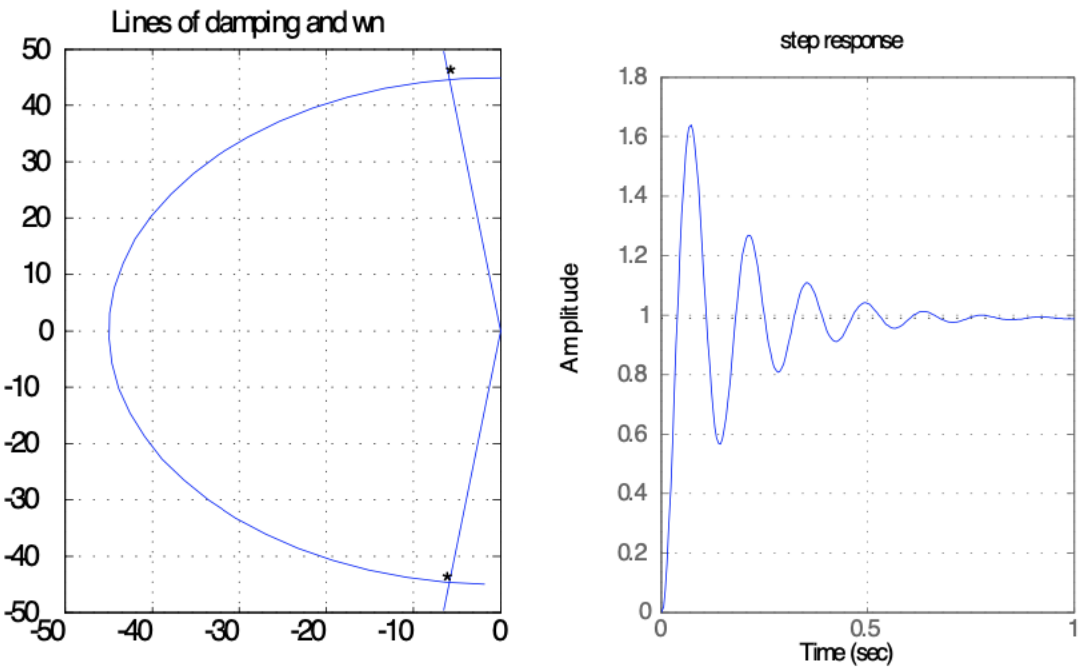
$e_{ss} \leq 0.01 \ , \ k_p \geq 9.9$ pick $k_p = 10.$

(b)

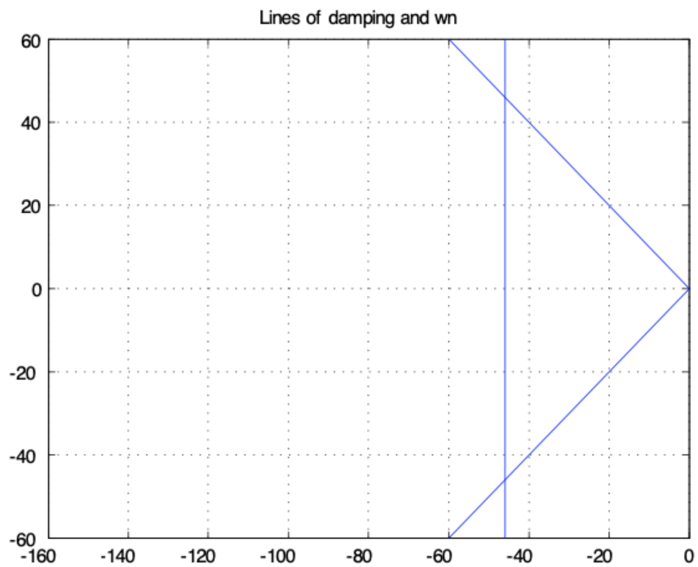
$$\frac{Y(s)}{\Omega_r(s)} = \frac{\frac{10k_p}{0.5s+1} \cdot \frac{1}{\zeta s + b}}{1 + \frac{1}{\zeta s + b} \cdot \frac{10k_p}{0.5s+1}} = \frac{2000}{s^2 + 12s + 2020}$$

$$\omega_n = \sqrt{2020} \simeq 45 \ \ , \ \zeta = \frac{12}{2\sqrt{2020}} \cong 0.13$$

The roots are undesirable (damping too low, high overshoot).



(c) For $t_s \leq 0.1 \Longrightarrow \sigma \geq 46$ For $M_p \leq 0.05 \Longrightarrow \zeta \geq 0.7$



s-plane for part(c)

d)

We know that larger ω_n and ζ are needed. This can be achieved by increasing k_p and adding derivative feedback:

$$\frac{Y(s)}{\Omega_r(s)} = \frac{\frac{10k_p}{0.5s+1} \cdot \frac{1}{\zeta s + b}}{1 + \frac{10k_p(k_D s + 1)}{(0.5s+1)(\zeta s + b)}} = \frac{200k_p}{s^2 + (12 + 200k_p . k_D)s + 20(1 + 10k_p)}$$

By choosing k_p and k_D any ζ and ω_n may be achieved.

e)

The TF to disturbance with new control:

$$\frac{Y}{W}=\frac{\frac{1}{\zeta s+b}}{1+\frac{1}{\zeta s+b}\cdot\frac{10k_p(k_Ds+1)}{(0.5s+1)}}=\frac{20(0.5s+1)}{s^2+(12+200k_pk_D)s+20(1+10k_p)}$$

$$e_{ss}(\text{step in W})=\frac{1}{1+10k_p}$$

As before derivative feedback affects transient response only. To eliminate steady-state error we can add an integrator to the loop. This can be represented by replacing k_p with $k_p+\frac{k_I}{s}$ and

$$\frac{Y}{W}=\frac{20(0.5s+1)s}{s^3+(12+200k_pk_D)s^2+(20+10k_p+200k_Ik_D)s+200k_I}$$

$$e_{ss}(\text{step in W})=0.$$