HOMEWORK 4

4.11 Consider the system shown in Fig. 4.29, which represents control of the angle of a pendulum that has no damping.

(a) What condition must $D_c(s)$ satisfy so that the system can track a ramp reference input with constant steady-state error?

(b) For a transfer function $D_c(s)$ that stabilizes the system and satisfies the condition in part(a), find the class of disturbances $w(t)$ that the system can reject with zero steady-state error.

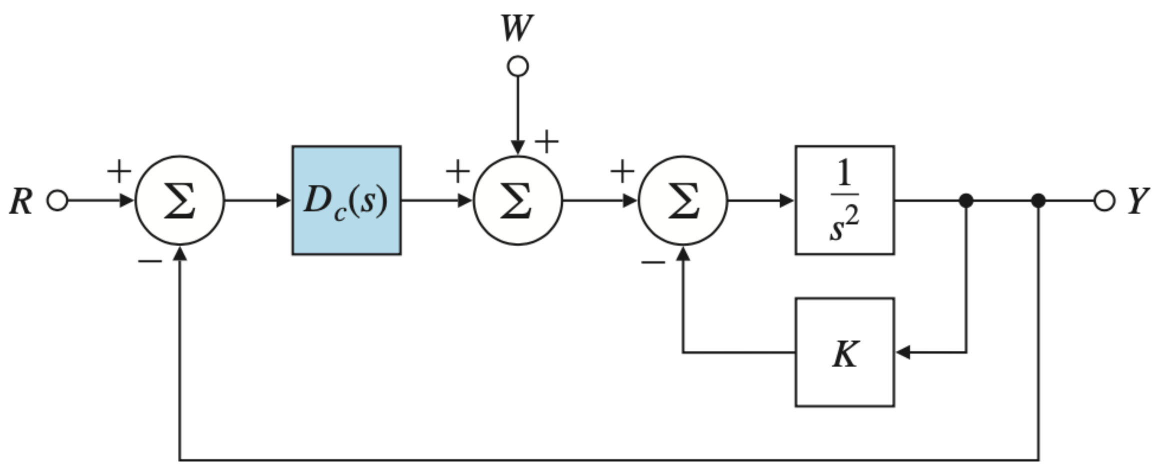

Figure 4.29
Control system for Problem 4.11

4.26 We wish to design an automatic speed control for an automobile. Assume that (1) the car has a mass m of 1000 kg, (2) the accelerator is the control U and supplies a force on the automobile of 10 N per degree of accelerator motion, and (3) air drag provides a friction force proportional to velocity of $10\ \mathrm{N \cdot sec/m}$.

(a) Obtain the transfer function from control input U to the velocity of the automobile.

(b) Assume the velocity changes are given by

$$V(s) = \frac{1}{s + 0.02} U(s) + \frac{0.05}{s + 0.02} W(s),$$

where V is given in meters per second, U is in degrees, and W is the percent grade of the road. Design a proportional control law $U = -k_P V$ that will maintain a velocity error of less than 1 m/sec in the presence of a constant 2% grade.

(c) Discuss what advantage (if any) integral control would have for this problem.

(d) Assuming that pure integral control (that is, no proportional term) is advantageous, select the feedback gain so that the roots have critical damping ($\zeta = 1$).

4.29 The transfer functions of speed control for a magnetic tape-drive system are shown in Fig. 4.42. The speed sensor is fast enough that its dynamics can be neglected and the diagram shows the equivalent unity feedback system.

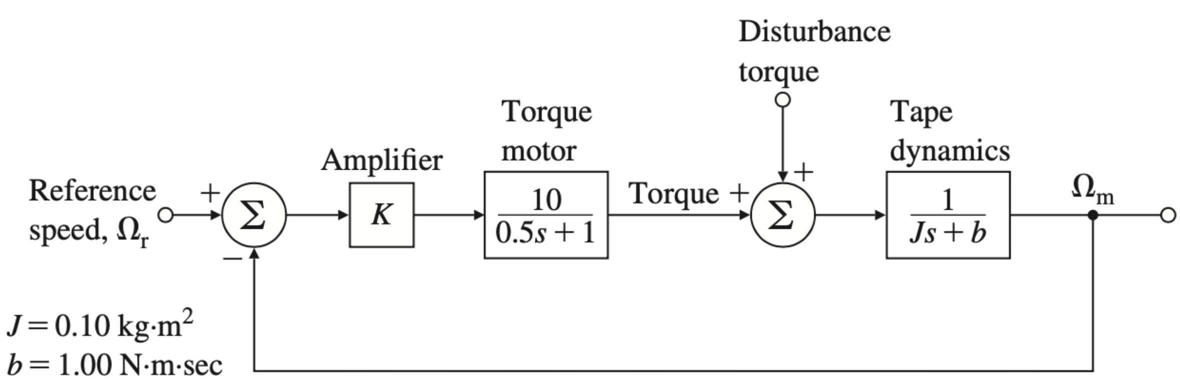

Figure 4.42
Speed-control system for a magnetic tape drive

(a) Assuming the reference is zero, what is the steady-state error due to a step disturbance torque of 1 N·m? What must the amplifier gain K be in order to make the steady-state error $e_{ss} \leq 0.01$ rad/sec?

(b) Plot the roots of the closed-loop system in the complex plane and accurately sketch the time response of the output for a step reference input using the gain K computed in part (a).

(c) Plot the region in the complex plane of acceptable closed-loop poles corresponding to the specifications of a 1% settling time of $t_s \leq 0.1$ sec and an overshoot $M_p \leq 5\%$.

(d) Give values for k_P and k_D for a PD controller, which will meet the specifications.

(e) How would the disturbance-induced steady-state error change with the new control scheme in part (d)? How could the steady-state error to a disturbance torque be eliminated entirely?