

lec 13.

$$A \sin(\omega t) \rightarrow \boxed{} \rightarrow A' \sin(\omega t + \phi)$$

ω influences A'/A and ω influences ϕ } Bode plot.

Understand how 0's and poles change the Bode plot.

$$|G| \text{ magnitude / amplitude (dB)} = 20 \log_{10} |G| = G_{dB}$$

$$\angle G \text{ phase}$$

$$G(s): s = \sigma + j\omega \rightarrow \sigma = 0 \rightarrow G(\omega)$$

$$\text{write } G(\omega) \text{ as: } K_0 \cdot \frac{(j\omega\tau_1 + 1)(j\omega\tau_2 + 1)}{(j\omega\tau_a + 1)(j\omega\tau_b + 1)} \dots$$

$$\rightarrow |G|_{dB} = 20 \log_{10} K_0 + 20 \log_{10} (j\omega\tau_1 + 1) + 20 \log_{10} (j\omega\tau_2 + 1) - 20 \log_{10} (j\omega\tau_a + 1) - 20 \log_{10} (j\omega\tau_b + 1) \dots$$

3 types of terms:

1) $K_0 (j\omega)^n \quad n \in \mathbb{Z}$

2) $(j\omega\tau + 1)^{\pm 1} \leftarrow 1 = \text{zero}, -1 = \text{pole}$

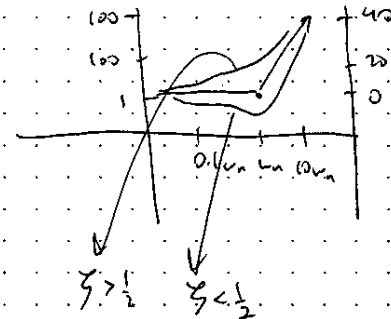
3) $\left[\left(\frac{j\omega}{\omega_n} \right)^2 + 2\zeta \frac{j\omega}{\omega_n} + 1 \right]^{\pm 1} \leftarrow \text{complex conj poles/zeros}$

$$1) \log_{10} |1| = \underbrace{20 \log_{10} |K_0|}_b + \underbrace{20n \log_{10} \omega}_{m} \underbrace{\log_{10} \omega}_{x} \leftarrow \text{mag}$$

phase $\rightarrow \angle j^n = 0$

$n=2$	$\phi=180$
$n=1$	90
$n=0$	$+0$
$n=-1$	-90
$n=-2$	-180

3) $\frac{\omega}{\omega_n} \ll 1 \quad \#3 \rightarrow 1 \quad \text{phase} = 0$
 $\frac{\omega}{\omega_n} \gg 1 \quad \#3 \rightarrow \left(\frac{j\omega}{\omega_n} \right)^2 \quad \text{phase} = 180$



$\omega = \omega_n \quad \text{phase} = 90^\circ$

2) $(j\omega\tau + 1)^{\pm 1} \leftarrow \text{zero}$

2 cases:

$\omega\tau \ll 1 \rightarrow 1$
 $\omega\tau \gg 1 \rightarrow j\omega\tau$
 $\angle j\omega\tau \approx 0$
 $\angle j\omega\tau \approx 90^\circ$

$\omega = \frac{1}{\tau} = \text{break point}$
 \downarrow
 $\text{phas } n$
 $j\omega\tau + 1 = j + 1$
 $|j + 1| = 1.4 \rightarrow 3 \text{ dB}$
 $\angle(j + 1) = \pi/4$
 $\angle(j\omega\tau + 1) = \pi/4$