

HW 4)

4.11

(a) $\frac{W+D(R-Y) - KY}{s^2} = Y$

$$Y \cdot \frac{DR}{s^2} + Y = \frac{W - KY + DR}{s^2}$$

$$Y = \frac{DR+W}{s^2+D+K}$$

$$E(s) = R(s) - Y(s)$$

$$= \frac{-Ds^2+D+K}{s^2+D+K} R(s)$$

$$= \frac{s^2+K}{s^2+D+K} R(s)$$

$$\lim_{s \rightarrow 0} E(s) = \text{const} \rightarrow D(s) = \text{pole @ } (0,0)$$

(b) $Y(s) = \frac{W(s)}{s^2+D+K}$

$$\lim_{s \rightarrow 0} \frac{1}{s^2} \cdot \frac{1}{s^2+D+K} = 0$$

$$\text{if } \lim_{s \rightarrow 0} s^{l-1} D(s) = 0$$

$l=1$ $D(s)$ has pole @ 0, system rejects disturbances.

4.26

a) $m\ddot{x} = K_a u - D\dot{x}$

$$\mathcal{L}(m\ddot{x} = K_a u - D\dot{x})$$

$$\frac{V}{U} = \frac{K_a}{ms+D} = \frac{0.01}{s+0.01}$$

b)

$$E(s) = V_d - V = V_d - \frac{k_p}{s+0.02} V_d$$

$$+ \frac{0.05}{s+0.02} G(s)$$

c) 0 a s.s. error for step input

$$\lim_{s \rightarrow 0} \left(\frac{(s+0.02) V_d - 0.055}{s+0.02+k_p} \right) \cdot \frac{2}{3} = \frac{-0.05}{0.02+k_p}$$

d) $\left(\frac{P}{s} \geq 1 \rightarrow W_n = 0.01 \rightarrow K_1 = 0.0001 \right)$

$$\left| \frac{-0.1}{0.02+k_p} \right| < 1 \rightarrow k_p < 0.01$$

$$k_p > 0.08$$

9.29)

(a) TF:

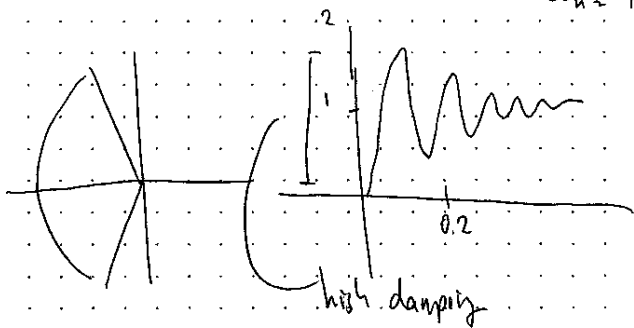
$$\frac{Y}{W} = \frac{1}{s+1} \cdot \frac{1}{s+1} \cdot \frac{10k_p}{0.5s+1} \quad b=1, \zeta=0.1$$

$$e_{ss}(\text{step}) = \lim_{s \rightarrow 0} \frac{Y}{W} = \frac{1}{1+10k_p}$$

$$e_{ss} \leq 0.01 \quad k_p \geq 9.9 \quad \boxed{k_p \approx 10}$$

$$(b) \quad \frac{Y}{X} = \frac{10k_p \cdot 1}{(0.5s+1)(s+1)} = \frac{2000}{s^2 + 1.5s + 2020} \rightarrow \omega_n^2$$

$$\omega_n = 45 \quad \zeta = \frac{1.2}{2 \cdot 45} = 0.13$$



(d) + derivative feedback

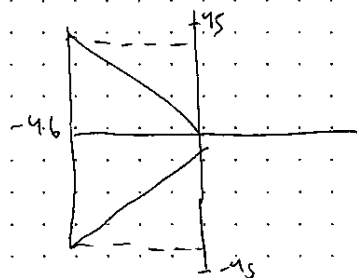
$$\frac{Y}{X} = \frac{10k_p \cdot 1}{s(s+1)(s+1)} \cdot \frac{10k_i(k_p s+1)}{(0.5s+1)(s+1)}$$

$$= \frac{200k_p}{s^2 + (12 + 200k_p k_i)s + 20 \cdot (1 + 10k_p)}$$

any values of k_p and k_i can create all values of ζ and ω_n

c) $t_s \leq 0.1 \rightarrow \sigma \geq -4.6$

$M_p \leq 0.05 \rightarrow \zeta \geq 0.7$



$$(e) \quad \frac{Y}{W} = \frac{\frac{1}{s+b}}{1 + \frac{1}{s+b} \cdot \frac{(a k_p (k_0 s + 1))}{0.5 s + 1}} = \frac{20(0.5s + 1)}{s^2 + (12 + 200k_p k_0)s + 2(-1 + 10k_p)}$$

$$e_{ss}(\text{step}) = \frac{1}{410k_p}$$

add integrator $k_p \rightarrow k_p + \frac{k_i}{s}$

$$\frac{Y}{W} = \frac{20(0.5s + 1)}{s^2 + (12 + 200k_p k_0)s^2 + (20 + 10k_p + 200k_i k_0)s + 200k_i}$$

$$e_{ss}(\text{step}) = 0$$