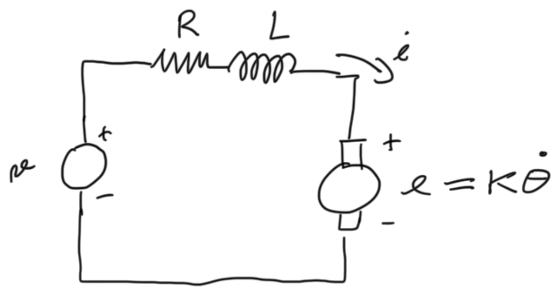
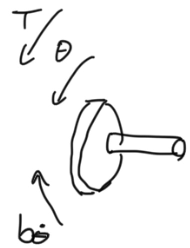
ECE 141

Lecture 7





$$J\ddot{\theta} = -b\dot{\theta} + Ki$$

$$v = L\frac{di}{dt} + Ri + K\dot{\theta}$$

L is very small $\Rightarrow L\frac{di}{dt} \cong 0$

$$\Rightarrow v = Ri + K\dot{\theta} \Rightarrow i = \frac{1}{R}\left(v - K\dot{\theta}\right)$$

$$\boxed{J\ddot{\theta} = -b\dot{\theta} + \frac{K}{R}v - \frac{K^2}{R}\dot{\theta}}$$

$$J\Theta(s)s^2 = -b\Theta(s)s + \frac{K}{R}V(s) - \frac{K^2}{R}\Theta(s)s$$

$$\Theta(s)\left[Js^2 + bs + \frac{K^2}{R}s\right] = \frac{K}{R}V(s)$$

$$\frac{\Theta(s)}{V(s)} = \frac{K/R}{s\left(Js + b + K^2/R\right)}$$

$K = 1 = R$

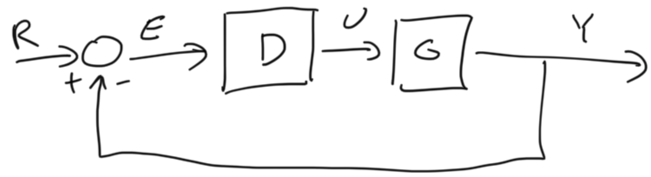
$b = 1$

$J = 1$

$$\Rightarrow \frac{\Theta(s)}{V(s)} = \frac{1}{s(s\ \ldots)} = G(s)$$

Specifications:

1) $t_s \leq 2$ sec.
2) $M_p \leq 10\%$
3) no steady state error fo step inputs



$$H(s) = \frac{Y}{R} = \frac{DG}{1+DG}$$

Proportional control

$$D(s) = K_p \qquad H(s) = \frac{\frac{K_p}{s(s+2)}}{1+\frac{K_p}{s(s+2)}} = \frac{K_p}{s^2+2s+K_p}$$

$$\begin{array}{c|cc} 2 & 1 & K_p \\ 1 & 2 & \\ 0 & K_p & \end{array}$$

$K_p > 0 \iff$ stability

steady state error

$$\lim_{t\to\infty} e(t) = \lim_{s\to 0} s\,E(s)$$

$$E = R - Y$$

$$\frac{E}{R} = 1 - \frac{Y}{R} = \frac{1+DG}{1+DG} - \frac{DG}{1+DG} = \frac{1}{1+DG}$$

$$D = K_p \Rightarrow \frac{E}{R} = \frac{1}{1 + K_p \frac{1}{s(s+2)}} = \frac{s(s+2)}{s(s+2) + K_p}$$

$$\lim_{s \to 0} s \underbrace{\left(\frac{s(s+2)}{s(s+2) + K_p} \frac{1}{s} \right)}_{E(s)}$$

Are the poles stable?
What are the poles?

$K_p > 0 \Rightarrow$ poles are stable $\Rightarrow$ final value theorem can be used.

$$\lim_{s \to 0} \frac{s^2 + 2s}{s^2 + 2s + K_p} = 0$$

$H(s)$ has no zeros and 2 poles $\Rightarrow \sigma \geq \frac{4.6}{t_s} = \frac{4.6}{2} = 2.3$

$$s^2 + 2\zeta\omega_n s + \omega_n^2$$
$$s^2 + 2s + K_p \quad \Rightarrow \zeta\omega_n = 1$$

$$\sigma = \zeta\omega_n = 1$$
$$\geq 2.3$$

Proportional and derivative

$$D(s) = K_p + K_D s$$

$$H(s) = \frac{DG}{1 + DG} = \frac{(K_p + K_D s) \frac{1}{s(s+2)}}{1 + (K_p + K_D s) \frac{1}{s(s+2)}} = \frac{K_p + K_D s}{K_p + K_D s + s^2 + 2s}$$

stability?

$$\begin{array}{c|cc} 2 & 1 & K_p \\ 1 & K_D + 2 & \end{array}$$

$K_D > -2$

$K_p > 0$

$0 \mid K_D$

$$\frac{1}{1+DG} = \frac{1}{1+(K_p+K_d s)\dfrac{1}{s(s+2)}} = \frac{s(s+2)}{s(s+2)+K_p+K_d s}$$

$$s\,\frac{s(s+2)}{s(s+2)+K_p+K_d s}\,\frac{1}{s}$$

$$\lim_{s\to 0}\ \frac{s^2+2s}{s^2+2s+K_d s+K_p} = 0$$

$$J\ddot{\theta} = T - b\dot{\theta}$$

$$\downarrow$$

$$Ki$$