DISCUSSION PROBLEMS WEEK 3

3.22 Use block-diagram algebra to determine the transfer function between $R(s)$ and $Y(s)$ in Fig. 3.52.

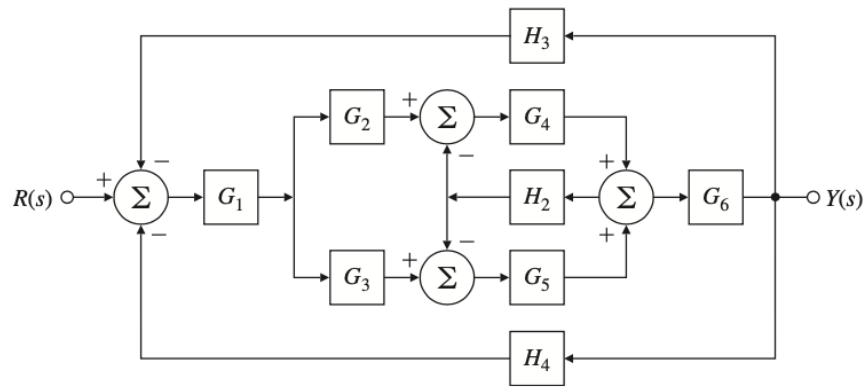


Figure 3.52
Block diagram for Problem 3.22

3.27 For the unity feedback system shown in Fig. 3.55, specify the gain and pole location of the compensator so that the overall closed-loop response to a unit-step input has an overshoot of no more than 25%, and a 1% settling time of no more than 0.1 sec. Verify your design using Matlab.

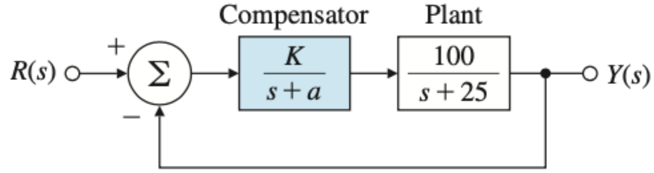


Figure 3.55
Unity feedback system for Problem 3.27

3.31 Suppose you are to design a unity feedback controller for a first-order plant depicted in Fig. 3.56. (As you will learn in Chapter 4, the configuration shown is referred to as a proportional–integral controller.) You are to design the controller so that the closed-loop poles lie within the shaded regions shown in Fig. 3.57.

(a) What values of ω_n and ζ correspond to the shaded regions in Fig. 3.57? (A simple estimate from the figure is sufficient.)

(b) Let $K_\alpha = \alpha = 2$. Find values for K and K_I so that the poles of the closed-loop system lie within the shaded regions.

(c) Prove that no matter what the values of K_α and α are, the controller provides enough flexibility to place the poles anywhere in the complex (left-half) plane.

Figure 3.56

Unity feedback system for Problem 3.31

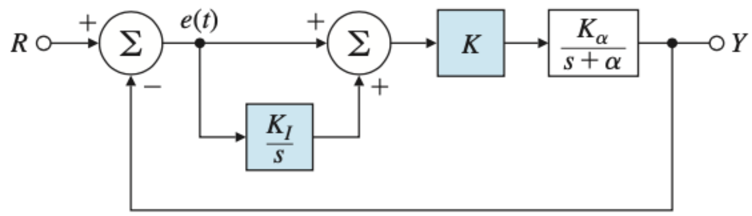
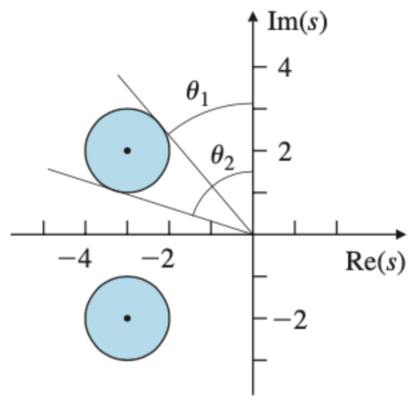


Figure 3.57

Desired closed-loop pole locations for Problem 3.31



3.35 The equations of motion for the DC motor shown in Fig. 2.32 were given in Eqs. (2.62–2.63) as

$$J_m \ddot{\theta}_m + \left(b + \frac{K_t K_e}{R_a}\right)\dot{\theta}_m = \frac{K_t}{R_a} v_a.$$

Assume that

$$\begin{aligned} J_m &= 0.01 \text{ kg}\cdot\text{m}^2, \\ b &= 0.001 \text{ N}\cdot\text{m}\cdot\text{sec}, \\ K_e &= 0.02 \text{ V}\cdot\text{sec}, \\ K_t &= 0.02 \text{ N}\cdot\text{m/A}, \\ R_a &= 10\ \Omega. \end{aligned}$$

(a) Find the transfer function between the applied voltage v_a and the motor speed $\dot{\theta}_m$.

(b) What is the steady-state speed of the motor after a voltage $v_a = 10$ V has been applied?

(c) Find the transfer function between the applied voltage v_a and the shaft angle θ_m.

(d) Suppose feedback is added to the system in part (c) so that it becomes a position servo device such that the applied voltage is given by

$$v_a = K(\theta_r - \theta_m)$$

where K is the feedback gain. Find the transfer function between θ_r and θ_m.

(e) What is the maximum value of K that can be used if an overshoot $M < 20\%$ is desired?

(f) What values of K will provide a rise time of less than 4 sec? (Ignore the M_p constraint.)

(g) Use Matlab to plot the step response of the position servo system for values of the gain $K = 0.5$, 1, and 2. Find the overshoot and rise time for each of the three step responses by examining your plots. Are the plots consistent with your calculations in parts (e) and (f)?