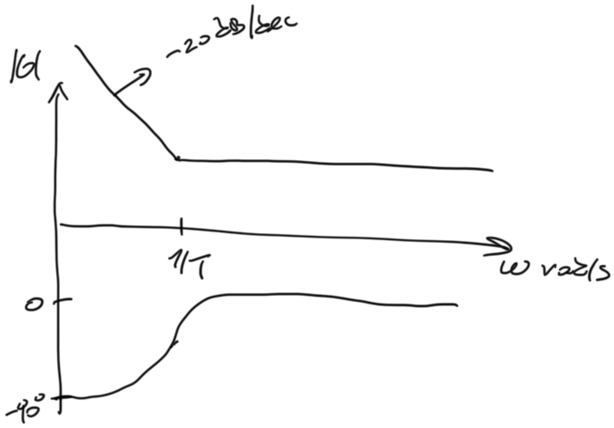
ECE 141

Lecture 16

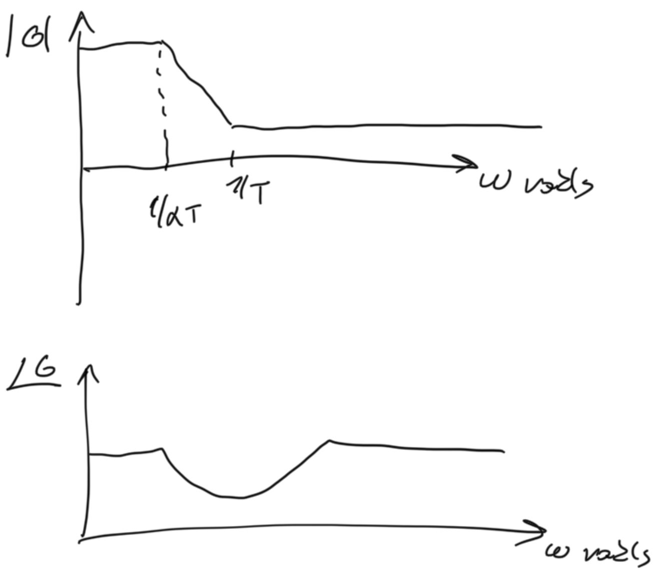
Lag compensation (PI control)

$$K_p + K_I \frac{1}{s} = \frac{K_p}{s}\left(s + \frac{K_I}{K_p}\right) = \frac{K_p}{s}\left(s + \frac{1}{T}\right)$$

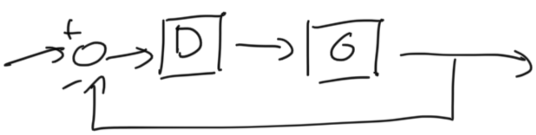


$$K_p e(t) + K_I \int_0^t e(\tau)\,d\tau$$

$$K_I \int_0^t c\,d\tau = \underline{\underline{K_I c t}}$$



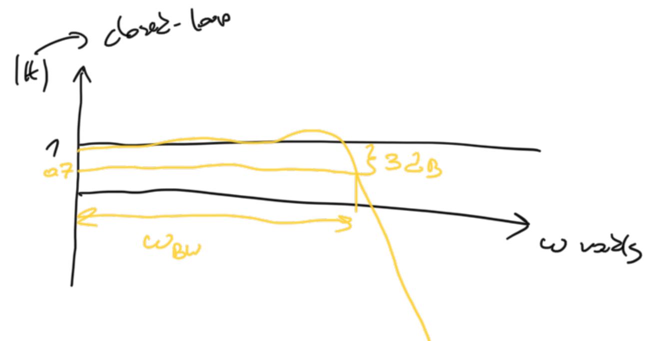
$$\frac{Ts+1}{\alpha T s+1} \qquad \alpha > 1$$



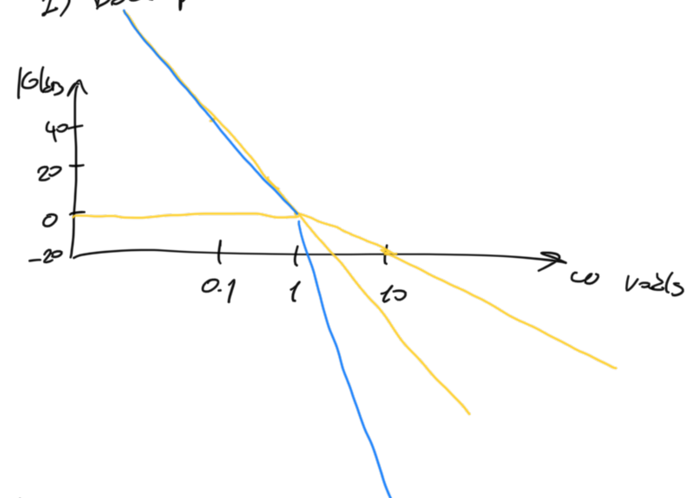
$$G = \frac{1}{s^2(s+1)}$$

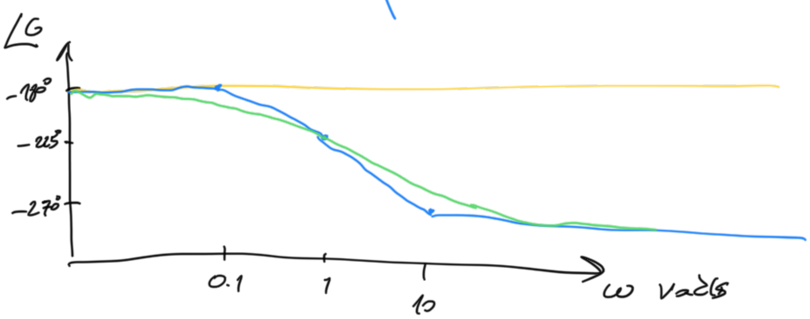
Requirements:

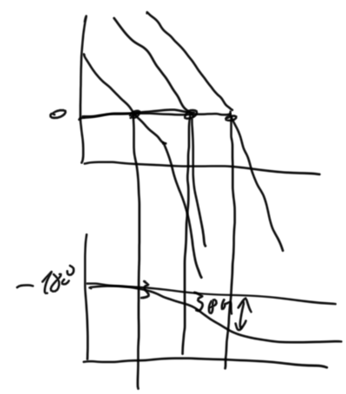
1) PM of at least 30°
2) zero steady state error to steps
3) bandwidth of no more than 1 rad/s

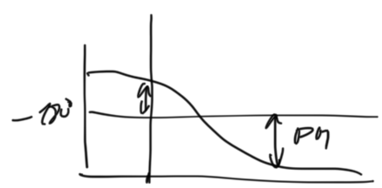


1) Bode plot



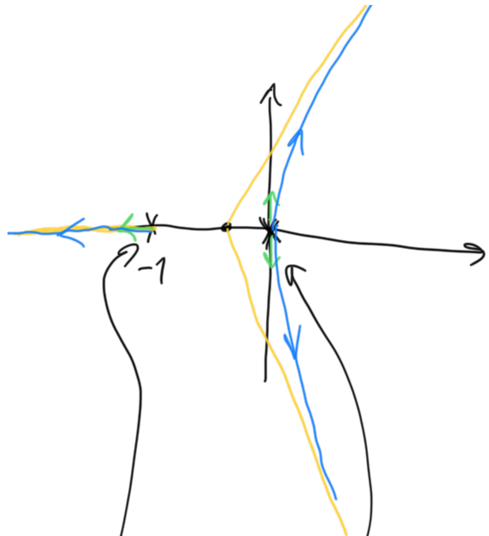






stability: no → 2 poles at the origin
s^3+s^2

Can we stabilize with a proportional controller?



$$\frac{\Sigma p - \Sigma z}{n-m} = \frac{0+0-1-(0)}{3} = -\frac{1}{3}$$

$$\phi_\ell = \frac{180^\circ + 360^\circ(\ell-1)}{n-m} \qquad \ell = 1, \dots, 3$$

$$\phi_1 = \frac{180^\circ}{3} = 60^\circ$$

$$\phi_2 = \frac{180^\circ + 360^\circ}{3} = \frac{180^\circ + 2 \cdot 180^\circ}{3}$$

$$= \frac{3}{3} 180^\circ = 180^\circ$$

$$\phi_3 = -60^\circ \text{ (by symmetry)}$$

Dept. angles

$$\begin{cases} \phi_1 = \frac{1}{2}\left(-0 - 180^\circ - 360^\circ(1-1)\right) = -90^\circ \\ \phi_2 = 90^\circ \text{ (symmetry)} \end{cases}$$

$$\phi_1 = -(180^\circ + 180^\circ) - 180^\circ - 360^\circ(1-1) = -360^\circ - 180^\circ = -180^\circ$$

$$1 + k\frac{1}{s^2(s+1)} \qquad s^3 + s^2 + k$$

3	1	0	1	$K = 0$
2	1	k	1	
1	$-\frac{k}{1}$		$-k$	
0	k		k	

Stability with PD controller?

new polynomial $s^3 + s^2 + k_D s + k_p$

$$\frac{(k_p + k_D s)\frac{1}{s^2(s+1)}}{1 + (k_p + k_D s)\frac{1}{s^2(s+1)}}$$

3	1	k_D	1

$$
\begin{array}{c|ccc}
2 & 1 & K_P & 1 \\
1 & -\frac{K_P - K_D}{1} & & K_D - K_P \\
0 & K_P & & K_P
\end{array}
\qquad \boxed{K_D > K_P \ \& \ K_P > 0}
$$

steady state error?

$$\frac{Y}{R} = \frac{DG}{1+DG} \qquad E = R - Y \Rightarrow \frac{E}{R} = 1 - \frac{Y}{R} = \frac{1}{1+DG}$$

$$\lim_{s\to 0} s\left(\frac{1}{1+DG}\,\frac{1}{s}\right) = \lim_{s\to 0} \frac{1}{1+DG}$$

$$= \lim_{s\to 0} \frac{1}{1 + (K_P + K_D s)\frac{1}{s^2(s+1)}}$$

$$= \lim_{s\to 0} \frac{s^2(s+1)}{s^2(s+1) + K_P + K_D s}$$

$$= 0$$

$$s = j\,1.5$$

$$K\frac{s+0.1}{s^2(s+1)}$$

$$\left| K \frac{j1.5 + 0.1}{(j1.5)^2 (j1.5+1)} \right| = 1$$

$$K = \frac{\left|(j\,1.5)^2 (j\,1.5+1)\right|}{\left|j\,1.5 + 0.1\right|}$$