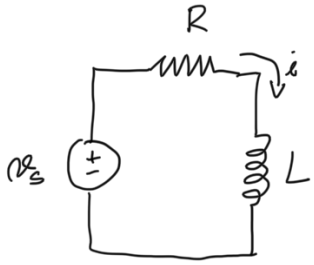
ECE 141

Lecture 3



$$v_s = Ri + L\frac{di}{dt}$$

$$\downarrow \mathcal{L}$$

$$V_s(s) = RI(s) + LsI(s) - Li(0^-)$$

assume $i(0^-) = 0 \Rightarrow V_s(s) = I(s)\left[R + Ls\right]$

$$H(s) = \frac{I(s)}{V_s(s)} = \frac{1}{R+Ls}$$

$v_s = c\,u(t)$ $\qquad \mathcal{L}\{c\,u(t)\} = c\,\mathcal{L}\{u(t)\} = c\,\frac{1}{s}$

$V_s(s) = \frac{c}{s}$

$i(t)$ $\qquad I(s) = H(s)\,V_s(s) = \frac{1}{R+Ls}\,\frac{c}{s}$

$$i(t) = \mathcal{L}^{-1}\{I(s)\} = \mathcal{L}^{-1}\left\{\frac{c}{s(R+Ls)}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{c/L}{s(s+\frac{R}{L})}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{c_1}{s} + \frac{c_2}{s+\frac{R}{L}}\right\}$$

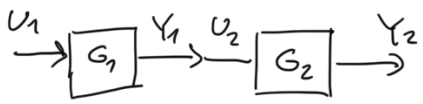
$$c_1 = H(s)\,s\Big|_{s=0} = \frac{c/L}{s+R/L}\Big|_{s=0} = \frac{c/L}{R/L} = \frac{c}{R}$$

$$c_2 = H(s)\,(s+R/L)\Big|_{s=-\frac{R}{L}} = \frac{c/L}{s}\Big|_{s=-\frac{R}{L}} = -\frac{c/L}{R/L} = -\frac{c}{R}$$

$$i(t) = \mathcal{L}^{-1}\left\{ \frac{C/R}{s} - \frac{C/R}{s+R/L} \right\} = \frac{C}{R}\mathcal{L}^{-1}\left\{\frac{1}{s}\right\} - \frac{C}{R}\mathcal{L}^{-1}\left\{\frac{1}{s+R/L}\right\}$$

$$= \frac{C}{R}u(t) - \frac{C}{R}e^{-\frac{R}{L}t}u(t)$$

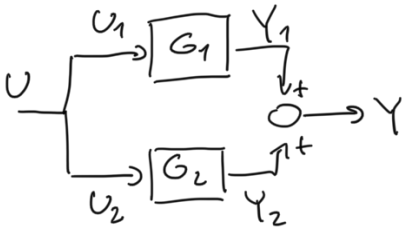
Series



$$Y_1 = U_2$$

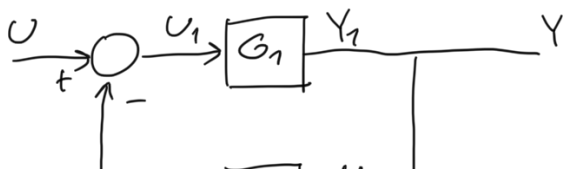
$$\begin{aligned} Y_2 &= G_2 U_2 \\ &= G_2 Y_1 \\ &= G_2 G_1 U_1 \end{aligned}$$

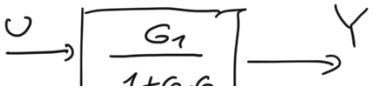
Parallel

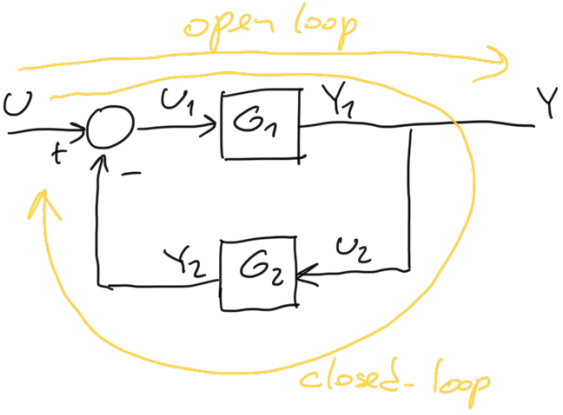


$$\begin{aligned} Y &= Y_1 + Y_2 \\ &= G_1 U_1 + G_2 U_2 \\ &= G_1 U + G_2 U \\ &= (G_1 + G_2) U \end{aligned}$$

Feedback



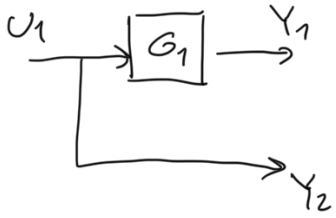


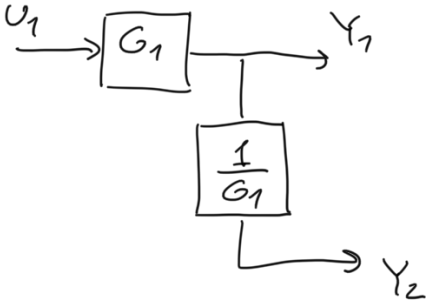


$$Y = Y_1$$
$$= G_1 U_1$$
$$= G_1 (U - Y_2)$$
$$= G_1 U - G_1 Y_2$$
$$= G_1 U - G_1 G_2 U_2$$
$$= G_1 U - G_1 G_2 Y$$

$$Y(1 + G_1 G_2) = G_1 U$$

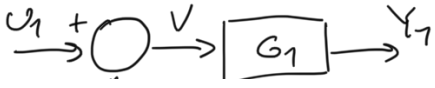
$$\frac{Y}{U} = \frac{G_1}{1 + G_1 G_2}$$

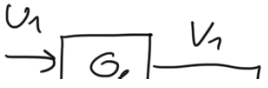


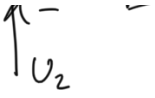


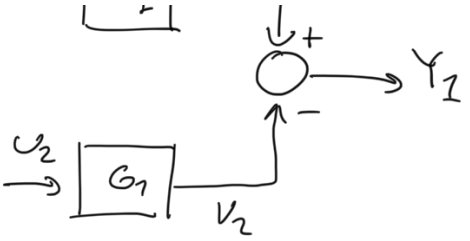
$$Y_1 = G_1 U_1$$
$$Y_2 = U_1$$

$$Y_1 = G_1 U_1$$
$$Y_2 = \frac{1}{G_1} Y_1$$
$$= \frac{1}{G_1} G_1 U_1$$
$$= U_1$$



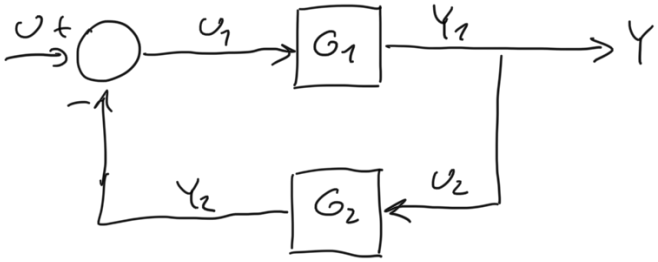




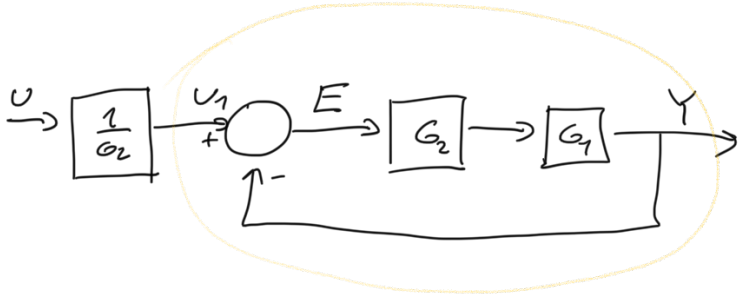


$$Y_1 = G_1 V$$
$$= G_1 (U_1 - U_2)$$
$$= G_1 U_1 - G_1 U_2$$

$$Y_1 = V_1 - V_2$$
$$= G_1 U_1 - G_1 U_2$$



$$Y = \frac{G_1}{1 + G_1 G_2} U$$



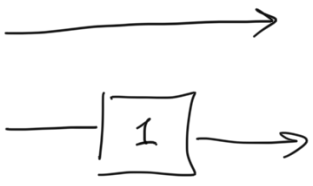
Unitary feedback loop

unit feedback loop

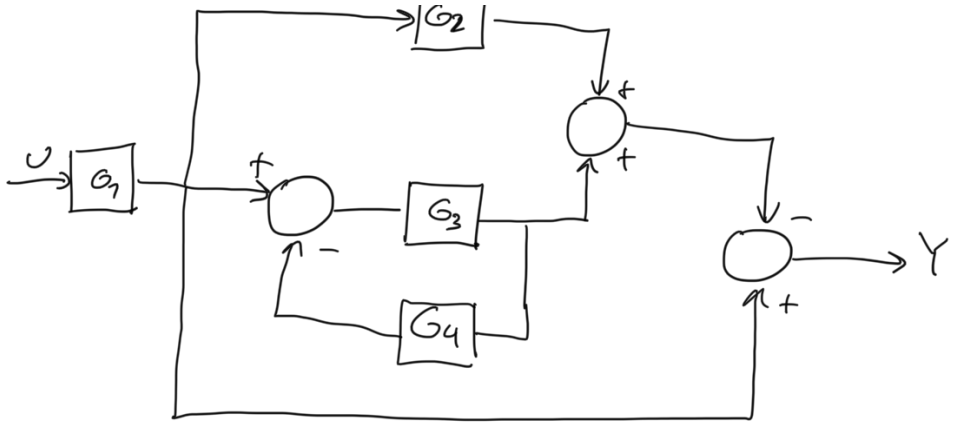
$$Y = \frac{G_1 G_2}{1 + G_1 G_2} U_1$$

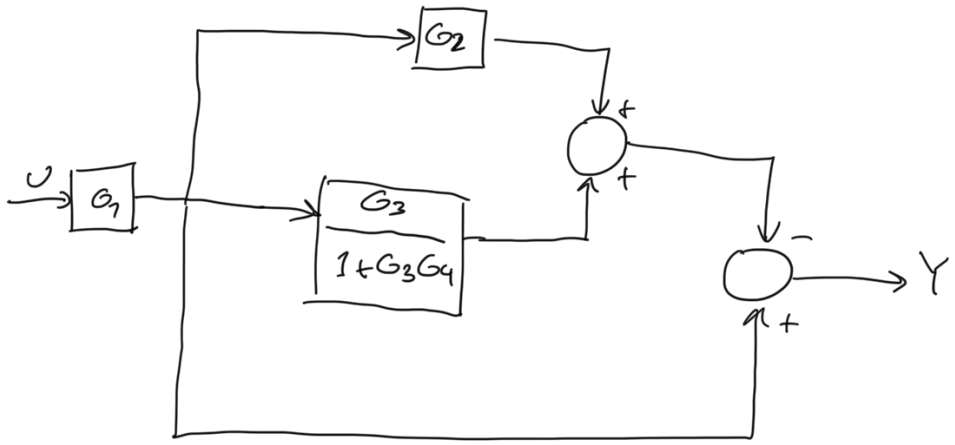
$$= \frac{G_1 G_2}{1 + G_1 G_2} \frac{1}{G_2} U$$

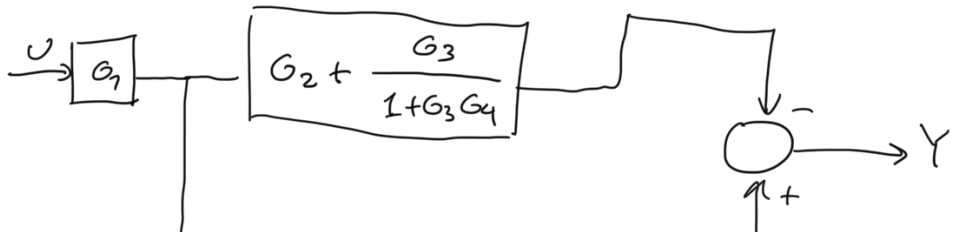
$$= \frac{G_1}{1 + G_1 G_2} U$$

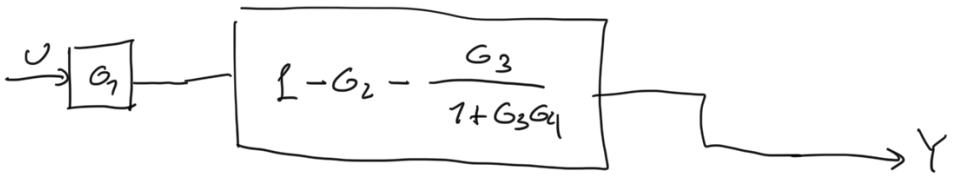


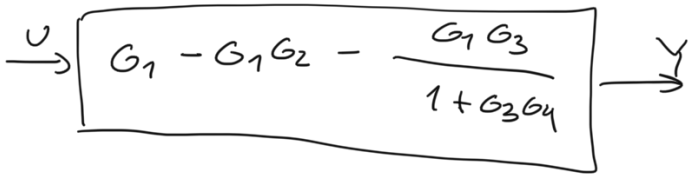
Example


U
G1
G2
G3
G4
+
+
+
-
-
+
Y


U
G1
G2
G3
1+G3G4
+
+
+
-
Y


U
G1
G2 + G3
1+G3G4
-
+
Y


U
G_1
$1 - G_2 - \frac{G_3}{1+G_3G_4}$
Y


U
$G_1 - G_1G_2 - \frac{G_1G_3}{1+G_3G_4}$
Y