ECE 141

Lecture 5

$$H(s) = \frac{3s+2}{s^2+5s+6} = \frac{3s+2}{(s+2)(s+3)} = \frac{-4}{s+2} + \frac{7}{s+3}$$

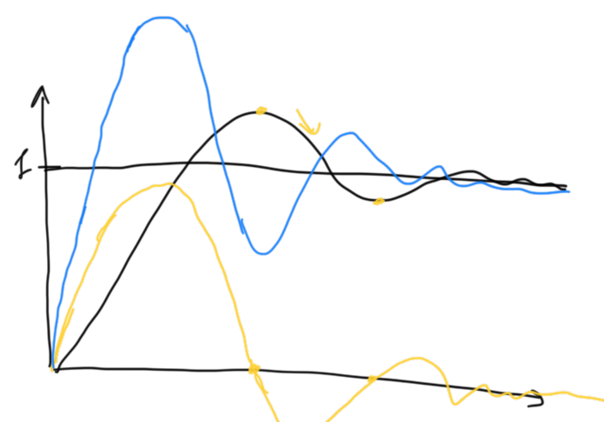
$$G(s) = \frac{s+2.1}{(s+2)(s+3)} = \frac{0.1}{s+2} + \frac{0.9}{s+3}$$

$$h(t) = \left(-4e^{-2t} + 7e^{-3t}\right)u(t)$$

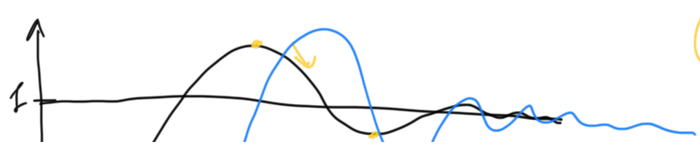
$$g(t) = \left(0.1e^{-2t} + 0.9e^{-3t}\right)u(t)$$

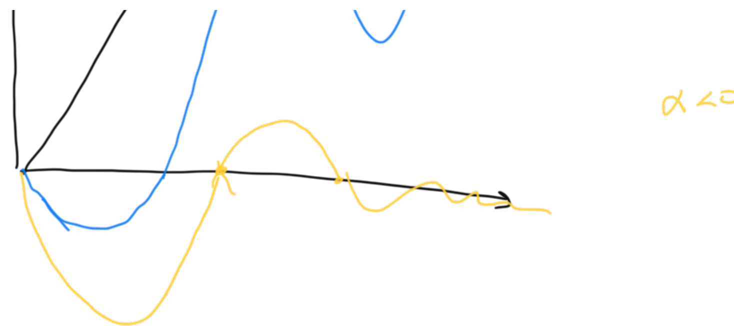
$$H(s) = \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}\left(\frac{s}{\alpha}+1\right)$$

$$= \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2} + \frac{1}{\alpha}\, s\, \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$$



$\alpha > 0$





$$\frac{\omega_n^2}{(s^2+2\zeta\omega_n s+\omega_n^2)(s+\beta)} = \underbrace{\frac{c_1}{s^2+2\zeta\omega_n+\omega_n^2}} + \underbrace{\frac{c_2}{s+\beta}}$$

$$c_2 e^{-\beta t}\, 1(t)$$

Can neglect fast poles

A pole is fast if it is at least 10 larger than the other poles

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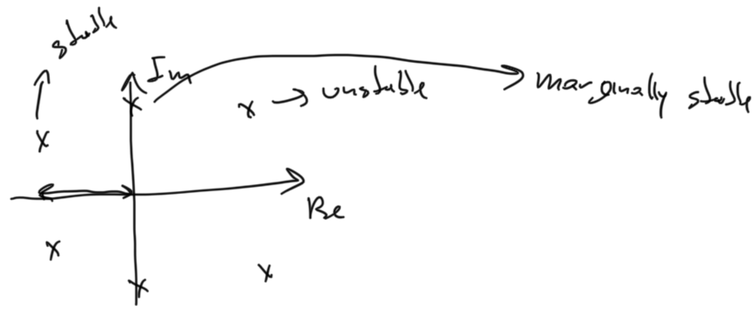
Stability: A linear time-invariant system is stable if all the poles of its transfer function are on the left part of the s-plane.



$$d(s) = s^n + a_1 s^{n-1} + a_2 s^{n-2} + \dots + a_{n-1} s + a_n$$

A necessary but not sufficient condition for stability is that all the a_i are positive

$d(s) = s + \sigma \qquad s = -\sigma \quad$ stability $\Rightarrow \sigma > 0$

$d(s) = (s+p_1)(s+p_2) = s^2 + (p_1+p_2) \dots = 0$

$$d(s) = (s+\sigma_1)(s+\sigma_2) = s^2 + \underbrace{(\sigma_1+\sigma_2)}_{>0}s + \underbrace{\sigma_1\sigma_2}_{>0}$$

stability $\Rightarrow$ $\sigma_1 > 0$
$\sigma_2 > 0$

$$d(s) = (s+\sigma_1)(s+\sigma_2)(s+\sigma_3) = s^3 + \underbrace{(\sigma_1+\sigma_2+\sigma_3)}_{>0}s^2 + \underbrace{(\sigma_1\sigma_2+\sigma_3\sigma_1+\sigma_3\sigma_2)}_{>0}s + \underbrace{\sigma_1\sigma_2\sigma_3}_{>0}$$

$s = -\sigma$ $\qquad$ $s - \sigma \Rightarrow s = \sigma$ $\qquad$ $\sigma = -5$

$s - (-5) = s + 5$

$\Rightarrow -\sigma < 0$
$\sigma > 0$

Routh Criterion

$$H(s) = \frac{n(s)}{d(s)} \qquad d(s) = s^n + a_1 s^{n-1} + a_2 s^{n-2} + \dots + a_{n-1}s + a_n$$

$$\begin{array}{c|ccccc}
n & 1 & a_2 & a_4 & a_6 & \dots \\
n-1 & a_1 & a_3 & a_5 & a_7 & \dots \\
n-2 & b_1 & b_2 & b_3 & b_4 & \dots \\
n-3 & c_1 & c_2 & c_3 & c_4 & \dots \\
\vdots & \vdots & & & & \\
2 & * & * & & & \\
1 & * & & & & \\
0 & * & & & &
\end{array}$$

$$b_1 = -\frac{\det\begin{bmatrix} 1 & a_2 \\ a_1 & a_3 \end{bmatrix}}{a_1}$$

$$b_2 = -\frac{\det\begin{bmatrix} 1 & a_4 \\ a_1 & a_5 \end{bmatrix}}{a_1}$$

$$b_3 = -\frac{\det\begin{bmatrix} 1 & a_6 \\ a_1 & a_7 \end{bmatrix}}{a_1}$$

$$c_1 = -\frac{\det\begin{bmatrix} a_1 & a_3 \\ b_1 & b_2 \end{bmatrix}}{b_1}$$

$$c_2 = -\frac{\det\begin{bmatrix} a_1 & a_5 \\ b_1 & b_3 \end{bmatrix}}{b_1}$$

$$c_3 = -\det\begin{bmatrix} a_1 & a_7 \\ b_1 & b_4 \end{bmatrix}$$

b_1 —

Example

$$d(s) = s^5 + 3s^4 + 2s^3 - s^2 - 2s + 1$$

5	1	2	-2
4	3	-1	1
3	7/3	-7/3	
2	2	1	
1	-7/2		
0	1		

$$b_1 = -\frac{\det\begin{bmatrix}1 & 2\\ 3 & -1\end{bmatrix}}{3} = -\frac{-1-6}{3} = \frac{7}{3}$$

$$b_2 = -\frac{\det\begin{bmatrix}1 & -2\\ 3 & 1\end{bmatrix}}{3} = -\frac{1+6}{3} = -\frac{7}{3}$$

$$b_3 = -\frac{\det\begin{bmatrix}1 & 0\\ 3 & 0\end{bmatrix}}{3} = 0$$

$$c_1 = -\frac{\det\begin{bmatrix}3 & -1\\ 7/3 & -7/3\end{bmatrix}}{7/3} = -\frac{-7+7/3}{7/3} = -\frac{-3\cdot 7+7}{7} = -\frac{-3+1}{7} = 2$$

$$c_2 = -\frac{\det\begin{bmatrix}3 & 1\\ 7/3 & 0\end{bmatrix}}{7/3} = -\frac{-7/3}{7/3} = 1$$

$$d_1 = -\frac{\det\begin{bmatrix}7/3 & -7/3\\ 2 & 1\end{bmatrix}}{2} = -\frac{\frac{7}{3}+2\frac{7}{3}}{2} = -\frac{3\frac{7}{3}}{2} = -\frac{7}{2}$$

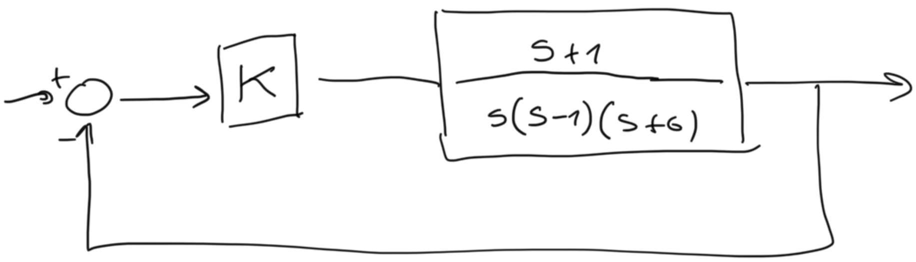
$$d_2 = 0 = -\frac{\det\begin{bmatrix}7/3 & 0\\ 2 & 0\end{bmatrix}}{}$$

$$e_1 = -\frac{\det\begin{bmatrix}2 & 1\\ -7/2 & 0\end{bmatrix}}{-7/2} = -\frac{7/2}{-7/2} = 1$$

5	1	>0	
4	3	>0	
3	7/3	>0	
2	2	>0	
1	-7/2	<0	1 sign change
0	1	>0	2 sign changes $\Longrightarrow$ 2 poles on the [illegible]

right half of
the complex plane.

Roots: $-2.084, -0.957 \pm 0.899i, 0.499 \pm 0.170i$



which values of k make this system stable?

$$G(s) = \frac{K \frac{s+1}{s(s-1)(s+6)}}{1 + K \frac{s+1}{s(s-1)(s+6)}} = \frac{K(s+1)}{s(s-1)(s+6) + K(s+1)}$$

$$s(s^2 + 6s - s - 6) + ks + k = s^3 + 5s^2 + (k-6)s + k$$

3	1	k−6
2	5	K
1	b_1	
0	K	

$$b_1 = -\frac{\det\begin{bmatrix} 1 & k-6 \\ 5 & k \end{bmatrix}}{5} = -\frac{k-5(k-6)}{5}$$

$$c_1 = -\frac{\det\begin{bmatrix} 5 & k \\ b_1 & 0 \end{bmatrix}}{b_1} = -\frac{-k\cancel{b_1}}{\cancel{b_1}} = k$$

$$-\frac{k-5(k-6)}{5} > 0 \Rightarrow k - 5(k-6) < 0 \Rightarrow -4k + 6\cdot 5 < 0$$

$k > 0$

$$\boxed{k > \frac{15}{2}}$$

$$\Rightarrow -4k < -6 \cdot 5$$

$$k > \frac{6 \cdot 5}{4} = \frac{3 \cdot 5}{2}$$