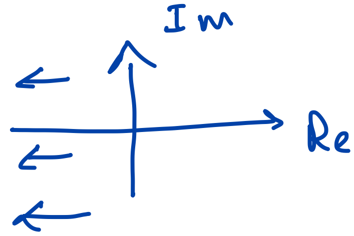
Discussion Week 4

$$H(s) = \frac{N(s)}{D(s)}$$

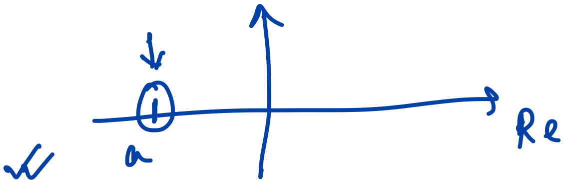


Eg $\quad H(s) = \frac{1}{s+a} \Rightarrow$ Pole at $s = -a$

$$\mathcal{L}^{-1}(H(s)) = \underline{\underline{e^{-at}}}\, u(t)$$

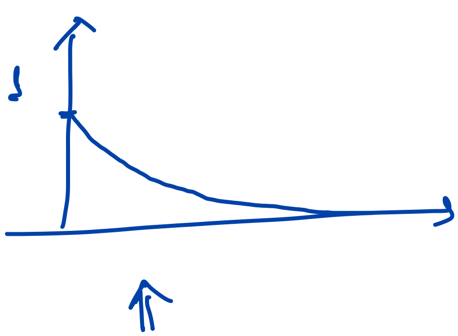
$\underline{\underline{a > 0}}$

Pole at $s = -a$



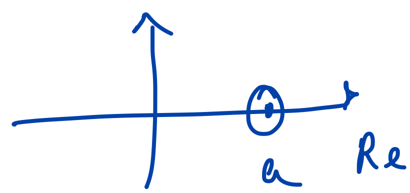
$\propto$

$e^{-at}\, u(t)$



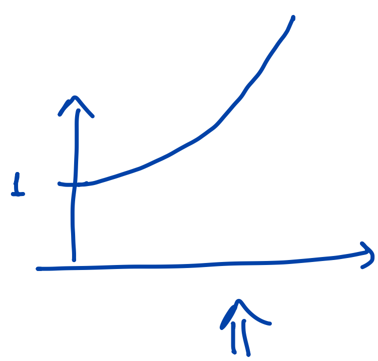
$a < 0$

Pole at $s = a$



$\propto$

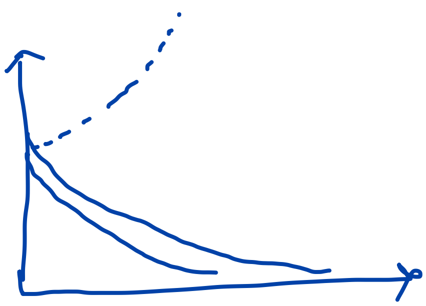
$e^{-at}\, u(t)$



$$H(s) = \frac{1}{(s+1)(s+2)(s-3)}$$

$$\frac{A}{s+1} + \frac{B}{s+2} + \frac{C}{s-3}$$

$$\mathcal{L}^{-1}(H(s)) = \underset{= \propto}{A}e^{-t} + \underset{= \propto}{B}e^{-2t} + \underline{\underline{Ce^{3t}}}$$



$$D(s) = s^2 + 2s + 1$$

$$D(s) = s^5 + 4s^4 + 10s^2 + 9s^3 + 2s + 1$$

$$H(s) = \frac{N(s)}{s^4 + 3s^3 - 5s^2 + s + 2} \Rightarrow D(s)$$

$\propto$

$$G(s) = \frac{1}{s^2 - s + 4} \times \frac{1}{(s+2)} \times \frac{1}{(s+1)}$$

$s^2 - s + 4 \uparrow (0.5 \pm j\,0.5\sqrt{15}) \Downarrow$ $\underline{\underline{\text{RHP}}}$

$(s+2) \uparrow (-2)$ $\propto$ LHP

$(s+1) \uparrow (-1)$ LHP

$$= \frac{1}{s^4 + 2s^3 + 3s^2 + 10s + 8}$$

Routh-Hurwitz Criteria :

$$s^4 + 2s^3 + 3s^2 + 10s + 8$$

s^4	1	3	8
s^3	2	10	0
s^2	-2	8	0
s^1	18	0	0
s^0	8		

2 sign changes

number of sign changes ⇒ number of poles in RHP.

s^4	A	B	C	D
s^3	E	F	G	0
s^2	□	○	△	
s^1				
s^0				

$$\square = \frac{-\det\begin{bmatrix} A & B \\ E & F \end{bmatrix}}{E} = \frac{EB - AF}{E}$$

3.53) (e) $KG(s) = \dfrac{4(s^3 + 2s^2 + s + 1)}{s^2(s^3 + 2s^2 - s - 1)}$

$$KG(s) = \frac{4s^3 + 8s^2 + 4s + 4}{s^5 + 2s^4 - s^3 - s^2}$$

$$H(s) = \frac{KG(s)}{1 + KG(s)} = \frac{N(s)}{D(s)}$$

$$D(s) = 1 + KG(s)$$

$$= s^5 + 2s^4 + \underline{\underline{3}}s^3 + 7s^2 + 4s + 4$$

s^5	1	3	4	0
s^4	2	7	4	0
s^3	$-1/2$	2	0	0
s^2	15	4	0	0
s^1	$32/15$	0	0	0
s^0	4			

$\therefore$ 2 poles in RHP

$\therefore$ Closed loop system will not be stable.

3.54 b) $s^5 + 10s^4 + 30s^3 + 80s^2 + 344s + 480 = 0$

s^5	1	30	344
s^4	10	80	480
s^3	22	296	0
s^2	$-\frac{1200}{22}$ ✓	480	0
s^1	+ ve.	0	0
s^0	+ ve ✓	0	0

$\left(-\frac{1200}{22} \times 296 - 22 \times 480\right)$

$\frac{-ve}{-ve} \Rightarrow +ve$

2 sign changes $\Rightarrow$ 2 poles in RHP.

3.56) $$KG(s) = \frac{K(s+4)}{s\left[(s+0.5)(s+1)(s^2+0.4s+4)\right]}$$ ✓

$1 + KG(s) = 0$

$= s^5 + 1.9s^4 + 5.1s^3 + 6.2s^2 + (2+K)s + 4K = 0$

$$s^5 + 1.9s^4 + 5.1s^3 + 6.2s^2 + (2+k)s + 4k = 0$$

s^5	1	5.1	$(2+k)$
s^4	1.9	6.2	$4k$
s^3	a_1	a_2	0
s^2	b_1	$4k$	0
s^1	c_1	0	0
s^0	$4k$		

$4k > 0$

$k > 0$

$$a_1 = \frac{(1.9)(5.1) - (6.2)}{1.9} = 1.837 \quad \text{+ve}$$

$$a_2 = \frac{(1.9)(2+k) - (4k)}{1.9} = 2 - 1.1k$$

$$b_1 = \frac{(a_1)(6.2) - (a_2)(1.9)}{a_1} = 1.138(k+3.63)$$

$$c_1 = \frac{(b_1)(a_2) - (a_1)(4k)}{b_1}$$

$$= \frac{-(1.25k^2 + 9.61k - 8.26)}{1.138(k+3.63)}$$

$$= \frac{-(K+8.47)(K-0.78)}{0.91\,(K+3.63)}$$

$$b_1 = K + 3.63 > 0$$

$$\Rightarrow \boxed{K > -3.63} \qquad b_1 > 0$$

$$c_1 > 0$$

$$\boxed{[-8.47 < K < 0.78]}$$

$$\boxed{K > 0}$$

$$\boxed{0 < K < 0.78}$$

Some Special Cases :

Case I : If I have a 0 in a row with at least one non zero entry appearing later in that row :

$(s^4 + 2s^3 + 0.s^2 + 3s + 4)$

1	0	4
2	3	

1	2	5
2	4	0
0	5	0

ε

$\varepsilon \to 0$

$\lim \varepsilon \to 0^+$

1	2	5
2	4	0
ε	5	0
$\frac{4\varepsilon - 10}{\varepsilon}$	0	0
5	0	0

$\Rightarrow \lim_{\varepsilon \to 0}$

1	2	5
2	4	0
0^+	5	0
$\boxed{\begin{matrix}-ve \\ \to -\infty\end{matrix}}$	0	0
5	0	0

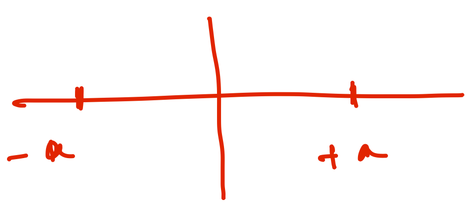
Case II : If entire row is zero

$$s^5 + 2s^4 + 6s^3 + 10s^2 + 8s + 12 = Q(s)$$

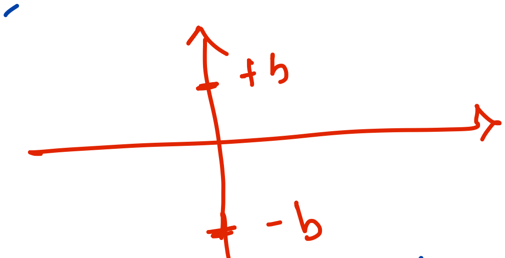
s^5	1	6	8
s^4	2	10	12
s^3	1	2	0
s^2	6	12	0
s^1	0	0	0
s^0			

Auxillary Polynomial

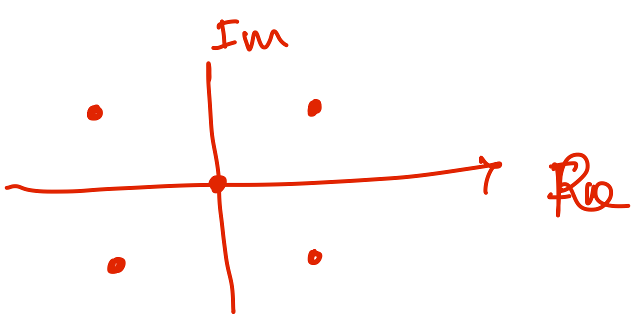
situation 1 :



situation 2 :



situation 3 :



$$6s^2 + 0.s^1 + 12 = 0$$

$$\Rightarrow \quad s^2 + 2 = 0 \quad \Rightarrow \quad P(s) : \text{Auxillary Polynomial}$$

Differentiate this term :

$$\checkmark \Rightarrow \quad 2s$$

$$\begin{array}{llll}
s^5 & 1 & 6 & 8 \\
s^4 & 2 & 10 & 12 \\
s^3 & 1 & 2 & 0 \\
s^2 & 6 & 12 & 0 \\
s^1 & 2 & 0 & 0 \\
s^0 & 12 & 0 & 0
\end{array}$$

$$\Uparrow$$

$$Q(s) = P(s) \,.\, R(s)$$

$$\underline{\underline{R(s)}} = \frac{Q(s)}{P(s)}$$