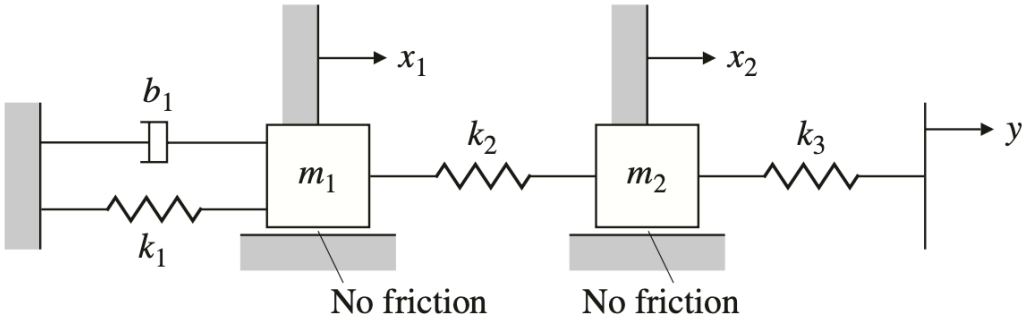
Discussion Problems

Week 2

Problems: 2.1a, 2.11, 3.3c, 3.7c, 3.8d, 3.9e

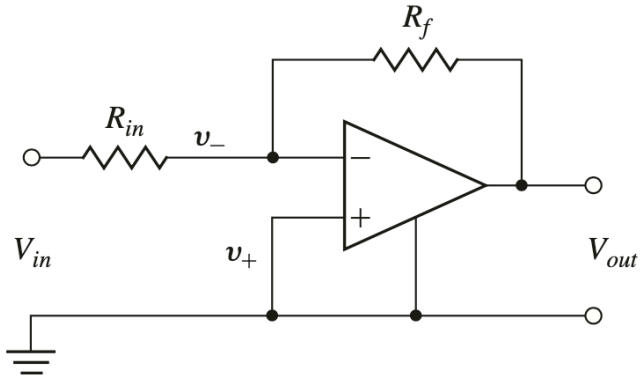
2.1 Write the differential equations for the mechanical systems shown in Fig. 2.41. For Fig. 2.41(a) and (b), state whether you think the system will eventually decay so that it has no motion at all, given that there are nonzero initial conditions for both masses and there is no input; give a reason for your answer.



(a)

2.11 A first step toward a realistic model of an op-amp is given by the following equations and is shown in Fig. 2.47:

$$V_{out} = \frac{10^7}{s+1}[v_+ - v_-],$$
$$i_+ = i_- = 0.$$



Find the transfer function of the simple amplification circuit shown using this model.

3.3 Find the Laplace transform of the following time functions:

(a) $f(t) = 4\cos 6t$

(b) $f(t) = \sin 3t + 2\cos 3t + e^{-t}\sin 3t$

(c) $f(t) = t^2 + e^{-2t}\sin 3t$

3.7 Find the time function corresponding to each of the following Laplace transforms using partial-fraction expansions:

(a) $F(s) = \frac{1}{s(s+1)}$

(b) $F(s) = \frac{5}{s(s+1)(s+5)}$

(c) $F(s) = \frac{3s+2}{s^2+2s+10}$

3.8 Find the time function corresponding to each of the following Laplace transforms:

(a) $F(s) = \frac{1}{s(s+1)^2}$

(b) $F(s) = \frac{s^2+s+1}{s^3-1}$

(c) $F(s) = \frac{2(s^2+s+1)}{s(s+1)^2}$

(d) $F(s) = \frac{s^3+s+2}{s^4-4}$

3.9 Solve the following ODEs using Laplace transforms:

(a) $\ddot{y}(t) + \dot{y}(t) + 3y(t) = 0;\ y(0) = 1,\ \dot{y}(0) = 2$

(b) $\ddot{y}(t) - 2\dot{y}(t) + 4y(t) = 0;\ y(0) = 1,\ \dot{y}(0) = 2$

(c) $\ddot{y}(t) + \dot{y}(t) = \sin t;\ y(0) = 1,\ \dot{y}(0) = 2$

(d) $\ddot{y}(t) + 3y(t) = \sin t;\ y(0) = 1,\ \dot{y}(0) = 2$

(e) $\ddot{y}(t) + 2\dot{y}(t) = e^t;\ y(0) = 1,\ \dot{y}(0) = 2$

(f) $\ddot{y}(t) + y(t) = t;\ y(0) = 1,\ \dot{y}(0) = -1$