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Homework 1 Problems.

2.2)

$$m_1 \ddot{x}_1 = -b_2(\dot{x}_1 - \dot{x}_2) - K_2(x_1 - x_2) - K_1(\overset{x_1}{\cancel{x_1 - x_2}})$$

$$m_1 \ddot{x}_2 = -b_2(\dot{x}_2 - \dot{x}_1) - K_2(x_2 - x_1) - K_1(x_2)$$

if initial movement is m_1 & m_2 moving in sync, left and right, no energy ~~damper uses up energy, so total E of system will just decrease monotonically~~ loss and it will oscillate

$\boxed{\text{not decay}}$

2.12) if ideal, $V_+ = V_- \Rightarrow V_{in} = V_{out}$.

non-ideal:

$$V_{out} = \frac{1e7}{s+1}\left[V_+ - V_-\right] = \frac{1e7}{s+1}\left[V_{in} - V_{out}\right] \Rightarrow \boxed{V_{in} = V_{out}\left(1 + \frac{s+1}{1e7}\right)}$$

3.3) a) $f(t) = 4\cos 6t = 4 \cdot \frac{s}{s^2+36}$

b) $f(t) = \sin 3t + 2\cos 3t + e^{-t}\sin 3t$

$$= \frac{3}{s^2+9} + \frac{2s}{s^2+9} + \frac{3}{(s+1)^2+9}$$

$$= \frac{2s+3}{s^2+9} + \frac{3}{s^2+2s+10}$$

3.7) f) $F(s) = \frac{2(s+3)}{(s+1)(s^2+16)} = \frac{A}{s+1} + \frac{C + Bs}{s^2+16} = \cancel{\frac{As^2+16A+Bs^2+Bs}{(s+1)(s^2+16)}} \Rightarrow$ ~~$A+B=0$~~ ~~$16A=6$~~ ~~$B=2$~~

$$= As^2 + 16A + Bs^2 + C + Bs + Cs = 2s+6$$

$A+B=0 \quad 16A+C=6 \quad B+C=2$

$16A - B = 4$

$A = 4/17$

$B = -4/17$

$C = 38/17$

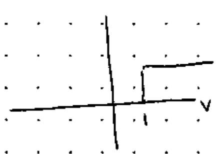
$$F(s) = \frac{\frac{4}{17}}{s+1} + \frac{\frac{38}{17}}{s^2+16} + \frac{-\frac{4}{17}s}{s^2+16}$$

$$= \left(\frac{4}{17}e^{-t} - \frac{4}{17}\cos(4t) + \frac{19}{34}\sin(4t)\right)u(t)$$

(j) $F(s) = \frac{e^{-s}}{s^2}$

$$\mathcal{L}\left(u(t-1)\right) = \frac{e^{-s}}{s}$$

$$\int_0^t u(v-1)\,dv =$$



$$f(t) = \begin{cases} t-1 & t \geq 1 \\ 0 & t < 1 \end{cases}$$

3.8) b) $F(s) = \frac{s^2+s+1}{s^3-1} = \frac{1}{s-1}$

$f(t) = \mathcal{L}^{-1}\left(\frac{1}{s-1}\right) = \boxed{e^t} \cdot u(t)$

3.9) b) $\mathcal{L}^{-1}(\ddot{y}(t)) \underset{-2}{\cancel{m}} \mathcal{L}^{-1}(\dot{y}(t)) + 4 \cdot \mathcal{L}^{-1}(y(t)) = 0$

$s^2 \cdot Y(s) - s \cdot \cancel{M} \; y(0) - y'(0) \; - 2(s \cdot Y(s) - y(0)) + 4 \cdot Y(s) = 0$

$(s^2 - 2s + 4)\, Y(s) - s \cdot 1 - \cancel{2} + \cancel{2 \cdot 1} = 0$

$Y(s) = \frac{s}{s^2-2s+4} = \frac{s}{(s-1)^2+3} = \frac{s-1}{(s-1)^2+3} + \frac{1}{(s-1)^2+3}$

$y(t) = e^t \cos(t\sqrt{3}) + \frac{1}{\sqrt{3}} \cdot e^t \sin(t\sqrt{3})$

(c) $s^2 Y(s) - s \cdot \cancel{y(0)}^{2} - \cancel{y'(0)} + s \cdot Y(s) - \cancel{y(0)}^{1} = \mathcal{L}(\sin t)$

$s^2 Y(s) - s - 2 + s \cdot Y(s) - 1 = \frac{1}{s^2+1}$

$Y(s) = \frac{\frac{1}{s^2+1} + s + 3}{s^2+s} = \frac{(s+3)(s^2+1)+1}{(s^2+s)(s^2+1)} = \frac{s^3+3s^2+s+4}{s(s+1)(s^2+1)}$

$\frac{A}{s} + \frac{B}{s+1} + \frac{C+Ds}{(s^2+1)} \quad \frac{(A(s+1)+Bs)(s^2+1)+(C+Ds)s(s+1)}{s(s+1)(s^2+1)}$

$= \frac{1}{s(s^2+1)(s+1)} + \frac{s+3}{s(s+1)}$

$= \frac{1}{s} - \frac{1/2}{s+1} + \frac{(-s-1)/2}{s^2+1} + \frac{3}{s} - \frac{2}{s+1}$

$= \frac{4}{s} - \frac{2.5}{s+1} - \frac{1/2\,(s+1)}{s^2+1}$

$y(t) = 4 - 2.5 \cdot e^{-t} - \frac{1}{2}(\cos(t) + \cancel{4}\sin(t))$ $\boxed{\left(4 - \frac{5}{2}e^{-t} - \frac{1}{2}(\sin t + \cos t)\right) u(t)}$