HW 3 Problems

3.53)

(a) $KG(s) = \dfrac{4(s+2)}{s(s^3+2s^2+3s+4)}$

$1 + \dfrac{4(s+2)}{s(s^3+2s^2+3s+4)} = 0$

$s^4 + 2s^3 + 3s^2 + 8s + 8 = 0$

1	3	8
2	8	
a	b	
c		
d		

$a = \dfrac{6-8}{2} = -1$ ← sign change in 1st column.

$b = \dfrac{16-0}{2} = 8$

$c = \dfrac{8a-2b}{a} = 24$

$d = b = 8$

UNSTABLE

(b) $\dfrac{2(s+4)}{s^3+s^2} + 1 = 0$

$s^3 + s^2 + 2s + 8 = 0$

1	2	
1	8	
−6		→ sign change
8		

UNSTABLE

3.54) (a) $s^4 + 8s^3 + 32s^2 + 80s + 100 = 0$

s^4	1	32	100
s^3	8	80	
s^2	22	100	
s	43.6		
1	100		

no sign change ⇒ 0 roots w/ ⊕ real parts.

(c) $s^4 + 2s^3 + 7s^2 - 2s + 8 = 0$

s^4	1	7	8
s^3	2	−2	
s^2	9	8	
s	−4		
1	8		

2 sign changes ⇒ 2 roots w/ positive real parts

3.57) $\dfrac{Y}{R} = \dfrac{\dfrac{K(s+z)}{s+p}\cdot\dfrac{K_0}{s^2-a^2}}{1 + \dfrac{K(s+z)}{s+p}\cdot\dfrac{K_0}{s^2-a^2}} = \dfrac{K K_0 (s+z)}{(s+p)(s^2-a^2) + K K_0 (s+z)}$

$= \dfrac{K K_0 (s+z)}{s^3 + ps^2 + (K K_0 - a^2)s + K K_0 z - p a^2}$

Draw array:

s^2 p $k \cdot K_0 z - \rho a^2$

s $\frac{-kK_0 z + kK_0 \rho}{\rho}$ $\rightarrow$ $\frac{-kK_0 z \rho a^2 + kK_0 \rho - \rho a^2}{\rho}$ a $\frac{-kK_0 z + kK_0 \rho}{\rho}$

1 $kK_0 z - \rho a^2$

when is $kK_0 \rho - kK_0 z > 0$ $\rho > 0 \rightarrow k > 0 = \rho > z$

$kK_0 z - \rho a^2 > 0 \Rightarrow k > 0 = z > \frac{\rho a^2}{kK_0}$

$\boxed{\rho > z}$ $\boxed{z > \frac{\rho a^2}{kK_0}}$