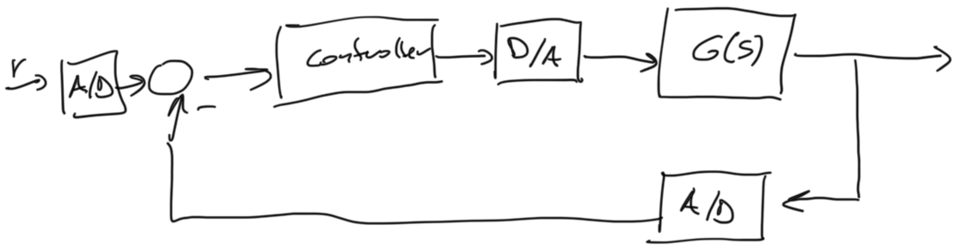
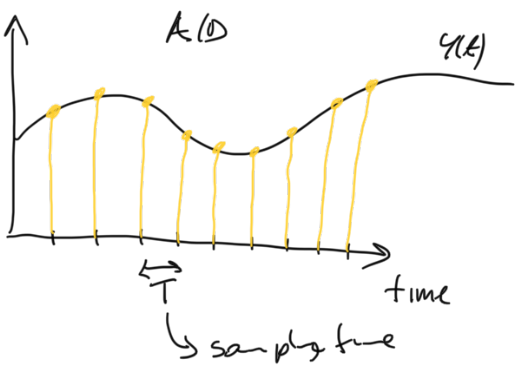
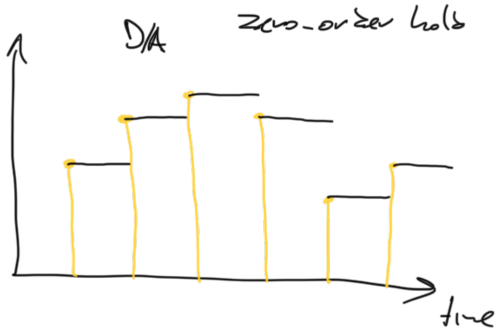
ECE 141

Lecture 17



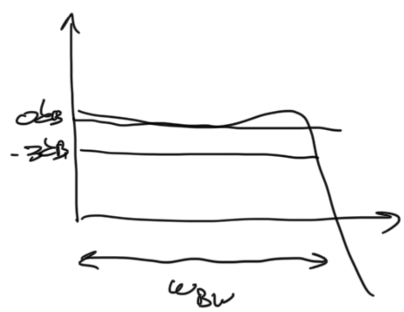


Sample-and-hold
Zero-order hold



choice of T

T



$\frac{1}{T}$ could be 20 times the desired closed-loop bandwidth

Design controller in s-domain	Model in the z-domain
Convert to z-domain	Design controller in the z-domain

$$\mathcal{L}\{f(t)\} = \int_0^{+\infty} f(t)\, e^{-st}\, dt$$

$$\mathcal{Z}\{f(k)\} = F(z) = \sum_{k=0}^{+\infty} f(k)\, z^{-k}$$

$$f(k) = f(kT)$$

$$\mathcal{L}\left\{\frac{df}{dt}\right\} = s\,F(s)$$

$$\mathcal{Z}\{f(k-1)\} = z^{-1} F(z)$$

$$y(k) = -5y(k-1) - 3y(k-2) + 4u(k) - 2u(k-1) + 2u(k-2)$$

$$Y(z) = -5z^{-1}Y(z) - 3z^{-2}Y(z) + 4U(- 2z^{-1}U(z) + 2z^{-2}U(z)$$

$$Y(z)\left[1 + 5z^{-1} + 3z^{-2}\right] = U(z)\left[4 - 2z^{-1} + 2z\right.$$

$$\frac{Y(z)}{U(z)} = \frac{4 - 2z^{-1} + 2z^{-2}}{1 + 5z^{-1} + 3z^{-2}} = G(z)$$

$f(t) = e^{-at}$ $\quad \mathcal{L}\{f\} = \dfrac{1}{s+a}$ $\quad$ pole at $-a$

$f(kT) = e^{-akT}$ $\quad \mathcal{Z}\{f\} = \sum_{k=0}^{+\infty} e^{-akT} z^{-k}$

$$= \sum_{k=0}^{+\infty} \left(e^{-aT} z^{-1}\right)^{n}$$

$$= \frac{1}{1-e^{-aT}z^{-1}} = \frac{z}{z-e^{-aT}}$$

pole at e^{-aT}

pole at $s=-a$ $\iff$ pole at $z=e^{-aT}$

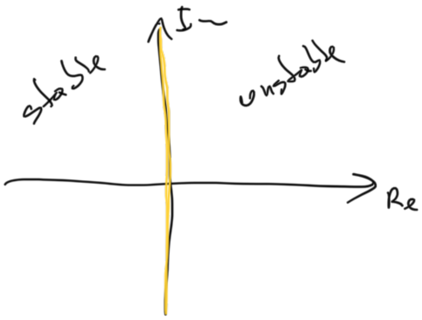
Imaginary axis $\iff$

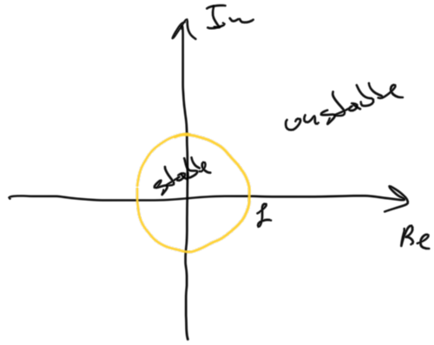
$s=j\omega$ $\qquad$ $z=e^{-j\omega T}$

$$\left|e^{-j\omega T}\right| = \left|\cos(\omega T) - j\sin(\omega T)\right|$$

$$= \sqrt{\cos^2(\omega T)+\sin^2(\omega T)}$$

$$= 1$$





$s=-a+jb$

$a>0$

$$z = e^{(-a+jb)T}$$

$$\left|e^{(-a+jb)T}\right| = \left|e^{-aT}e^{jbT}\right|$$

$$= |e^{-aT}|\underbrace{|e^{jbT}|}_{1}$$

$$= |e^{-aT}| < 1$$

Final Value Theorem

$\lim_{t \to \infty} x(t) = \lim_{s \to 0} sX(s)$ provided that $sX(s)$ is stable

$\lim_{k \to \infty} x(k) = \lim_{z \to 1} (1 - z^{-1}) X(z)$ provided that all the poles of $(1-z^{-1})X(z)$ are inside the unit disc.

Emulation — Tustin's method

$$\frac{U}{E} = K_p + K_I \frac{1}{s} + K_D s$$

Proportional

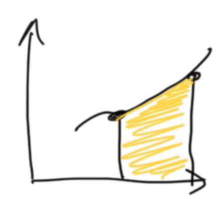
$$u(t) = K_p e(t)$$

$$u(kT) = K_p e(kT)$$

$$u(k) = K_p e(k)$$

integral

$$u(kT+T) = K_I \int_0^{kT+T} e(\tau)\, d\tau$$



$$= \underbrace{K_I \int_0^{kT} e(\tau)\, d\tau}_{} + K_I \int_{kT}^{kT+T} e(\tau)\, d\tau$$

$$\approx u(kT) + K_I T \frac{1}{2}\left(e(kT+T) + e(kT)\right)$$

$$u(k) = u(k-1) + k_I \frac{T}{2}\left(e(k) + e(k-1)\right)$$

$$U(z) = z^{-1} U(z) + k_I \frac{T}{2}\left(E(z) + z^{-1} E(z)\right)$$

$$\frac{U(z)}{E(z)} = k_I \frac{1}{\frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}}} \qquad k_I \frac{1}{s}$$

derivative

$$u(t) = k_D \frac{de}{dt} \Longrightarrow \int_0^t u(\tau)\, d\tau = k_D\, e(t)$$

$$\boxed{\frac{1}{k_D}\int_0^t u(\tau)\, d\tau = e(t)} \qquad \text{Der.}$$

$$\boxed{k_I \int_0^t e(\tau)\, d\tau = u(t)} \qquad \text{Int.}$$

$$e(k) = e(k-1) + \frac{1}{k_D}\frac{T}{2}\left(u(k) + u(k-1)\right)$$

$$E(z) = z^{-1} E(z) + \frac{1}{k_D}\frac{T}{2}\left(U(z) + z^{-1} U(z)\right)$$

$$\frac{U(z)}{E(z)} = k_D \frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}}$$

PID

$$k_P + k_I \frac{1}{s} + k_D s \longrightarrow k_P + k_I \frac{1}{\frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}}} + k_D \frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}}$$

1) pick T

$$" \quad 2 \quad 1-z^{-1}$$

2) Replace s with $\frac{\;}{T}\,\frac{\;}{1+z^{-1}}$

Matched pole zero Method

$$\underbrace{D(s) = K_c \frac{s+2}{s+3}}_{} \qquad D(z) = K_D \frac{z-e^{-2T}}{z-e^{-3T}}$$

Same DC gain

$$\lim_{s\to 0} s\,\frac{1}{s}\,D(s) = \lim_{z\to 1} (1-z^{-1})\left(\bigcirc \; D(z) \right)$$

$$\mathcal{Z}\{1(k)\} = \sum_{k=0}^{\infty} z^{-k} = \sum_{k=0}^{\infty} (z^{-1})^{k} = \frac{1}{1-z^{-1}}$$

$$\lim_{s\to 0} D(s) = \lim_{z\to 1} D(z)$$

$$\lim_{s\to 0} K_c \frac{s+2}{s+3} = \lim_{z\to 1} K_D \frac{z-e^{-2T}}{z-e^{-3T}}$$

$$K_c \frac{2}{3} = K_D \frac{1-e^{-2T}}{1-e^{-3T}}$$

$$K_D = \frac{2}{3} K_c \frac{1-e^{-3T}}{1-e^{-2T}}$$

1) Given $G(s)$, compute $G(z)$ by matching poles and zeros

2) Compute K_D by $\lim_{s\to 0} G(s) = \lim_{z\to 1} G(z)$