ECE 141 – Midterm
Spring 2021

05/06/21

Duration: 1 hour and 40 minutes

The midterm is open book.
Please carefully justify all your answers.
You are requested to have your camera turned on so that we can see you and your work at all times and until you upload your work to Gradescope. We will be recording the exam.
You are not allowed to communicate with others, by any means, during the exam.
Once you finish the exam, scan it and upload it to Gradescope.
Note that different students will receive different exams.

Problem 1

In this problem we consider a pump subcutaneously injecting insulin to offset the glucose resulting from a meal.

1. Write down the differential equation that describes the dynamics of insulin knowing that:

   (a) its dynamics is governed by a compartmental model with two compartments described by x_1 and x_2;

   (b) the time derivative of x_1 is given by the difference between the insulin injection rate u and a term proportional to x_1 with constant of proportionality $k > 0$;

   (c) the time derivative of x_2 is given by the difference between a term proportional to x_1 and a term proportional to x_2, both with proportionality constant k.

2. If a step input is applied to the derived model (where u represents the input), will x_1 or x_2 exhibit oscillatory behavior? Would your answer change if we use a different constant k for each compartment, i.e., x_1 terms are multiplied by k_1 and x_2 terms are multiplied by k_2?

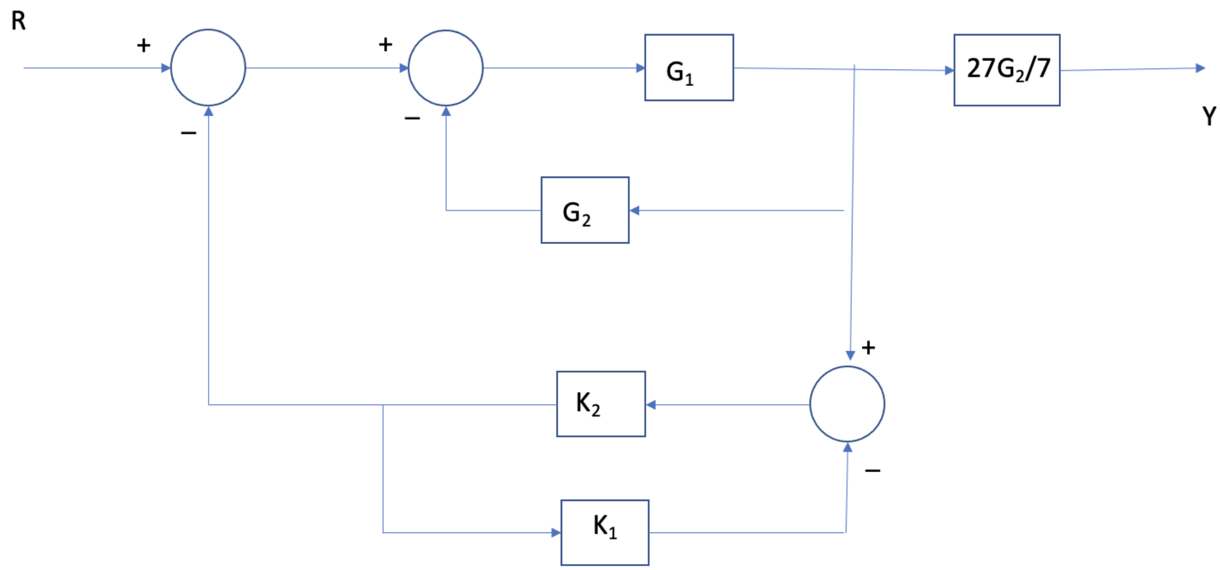
3. Considering x_2 as the output, for which values of k will the rise time be smaller than 1.8 and the settling time be smaller than 9.2 when the input is a step?

4. Assume now that we are interested in regulating the insulin concentration y related to x_2 by $\dot{y} = -\lambda^2 y + x_2$. If we use an insulin injection rate of λk, will y converge to a constant value? If so, which value?

5. Design a controller resulting in zero insulin concentration steady-state error to step inputs. Use $\lambda = k = 1$ for this question only.

Problem 2

Consider the following diagram. Let $G_1 = \frac{1}{s+1}$ and $G_2 = \frac{7}{s+5}$.



1. Compute the transfer function from R to Y.
2. For what values of K_1 and K_2 is this system stable?
3. Design the value of K_1 and K_2 and, if needed, replace one of these gains with a different type of controller, so that the closed-loop system can track a step input, i.e., the steady state error to a unit step input should be zero, the overshoot is no more than 3% and the settling time no greater than 1.3s.