ECE 141

Lecture 13

$\omega \longmapsto \frac{A'}{A}$

$\omega \longmapsto \phi$

Bode plot $|G|$ magnitude / amplitude

$\angle G$ phase

dB - deciBells

$$|G|_{dB} = 20\log_{10}|G|$$

$G(s)$ $\downarrow$ Laplace transform

$s = \sigma + j\omega$

$\sigma = 0 \Rightarrow G(j\omega) = G(\omega)$ $\downarrow$ Fourier transform

$$G = K\frac{(s-z_1)(s-z_2)\cdots}{(s-p_1)(s-p_2)\cdots} \overset{\sigma=0}{=} K\frac{(j\omega-z_1)(j\omega-z_2)\cdots}{(j\omega-p_1)(j\omega-p_2)\cdots}$$

$$= K_0\frac{(j\omega\tau_1+1)(j\omega\tau_2+1)\cdots}{(j\omega\tau_a+1)(j\omega\tau_b+1)\cdots}$$

$$G = 5\frac{s-2}{(s-1)(s+3)} \overset{\sigma=0}{=} 5\frac{j\omega-2}{(j\omega-1)(j\omega+3)} = 5\frac{-2\left(j\omega\left(-\frac{1}{2}\right)+1\right)}{-3\left(j\omega(-1)+1\right)\left(j\omega\left(\frac{1}{3}\right)\right.}$$

$$|G|_{dB} = 20\log_{10}\left[\left|\frac{10}{3}\,\frac{j\omega(-\frac{1}{2})+1}{(j\omega(-1)+1)(j\omega(\frac{1}{3})+1)}\right|\right]$$

$$= 20\log_{10}\left|\frac{10}{3}\right| + 20\log_{10}\left|j\omega(-\tfrac{1}{2})+1\right| - 20\log_{10}\left|j\omega(-1)+1\right|$$
$$-20\log_{10}\left|j\omega(\tfrac{1}{3})+1\right|$$

3 types of terms:

1) $K_0(j\omega)^n \quad n \in \mathbb{Z}$

2) $(j\omega\tau+1)^{\pm 1}$

3) $\left[\left(\frac{j\omega}{\omega_n}\right)^2 + 2\zeta\frac{j\omega}{\omega_n} + 1\right]^{\pm 1}$

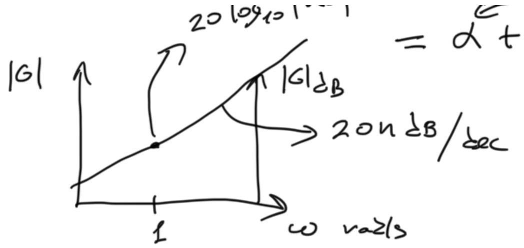
$\boxed{K_0(j\omega)^n}$

$$\left|K_0(j\omega)^n\right|_{dB} = 20\log_{10}|K_0| + \underbrace{20\log_{10}\left|(j\omega)^n\right|}$$

$$20\log_{10}\left|j^n\omega^n\right|$$
$$= 20\log_{10}\left(|j^n||\omega^n|\right)$$
$$= 20\log_{10}\left(|\omega^n|\right)$$
$$= 20\,n\log_{10}|\omega|$$
$$= 20\,n\log_{10}\omega$$

$$= 20\log_{10}|K_0| + 20\,n\log_{10}\omega$$

$|K_0|$



$= \alpha + \beta x \leftarrow x$

$\angle G$

$$\angle K_0 (j\omega)^n = \angle (j\omega)^n = \angle j^n \omega^n = \angle j^n$$

Assume $K_0 > 0$

$K_0 < 0 \quad K_0 = -K_0', \; K_0' > 0$

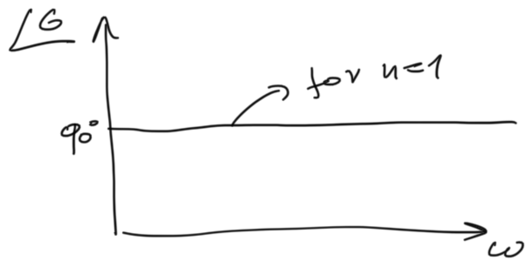
$n=2, \quad \phi = 180°$

$n=1, \quad \phi = 90°$

$n=0, \quad \phi = 0°$

$n=-1, \quad \phi = -90°$

$n=-2, \quad \phi = -180°$



$\boxed{j\omega\tau + 1}$

$\omega\tau \ll 1 \qquad j\omega\tau + 1 \cong 1$

$\omega\tau \gg 1 \qquad j\omega\tau + 1 \cong j\omega\tau$

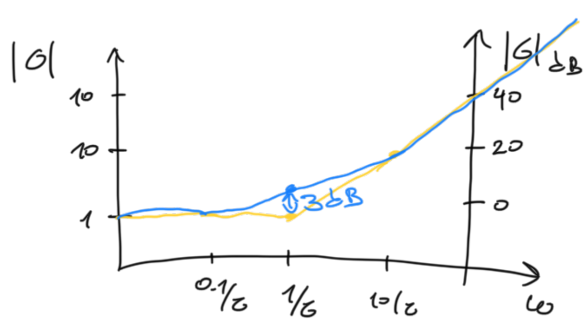
for $\omega = \frac{1}{\tau} \quad j\omega\tau + 1 = j + 1$

$|j+1| = \sqrt{1^2 + 1^2} = \sqrt{2}$

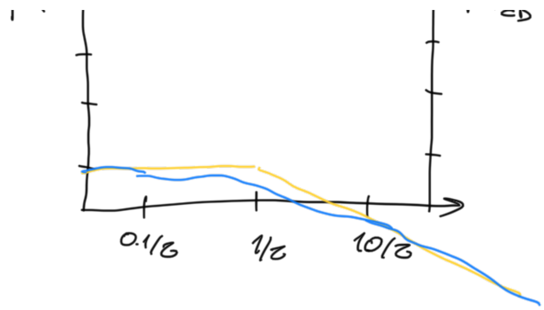
$\cong 1.4$

$20\log_{10}\sqrt{2} \cong 3\,dB$

$\omega = \frac{1}{\tau}$ break point



← plot for a zero



← plot for a pole (mirror image)

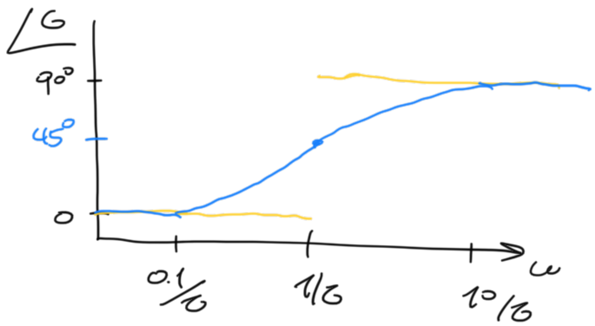
Phase

$\omega\tau \ll 1$ $\qquad \angle j\omega\tau+1 \cong \angle 1 = 0°$

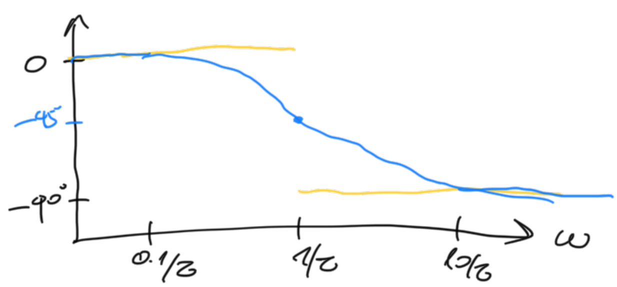
$\omega\tau \gg 1$ $\qquad \angle j\omega\tau+1 \cong \angle j\omega\tau = 90°$

$\omega = \frac{1}{\tau}$ $\quad \angle j\omega\tau+1$

$= \angle j+1 = 45°$



← plot for a zero

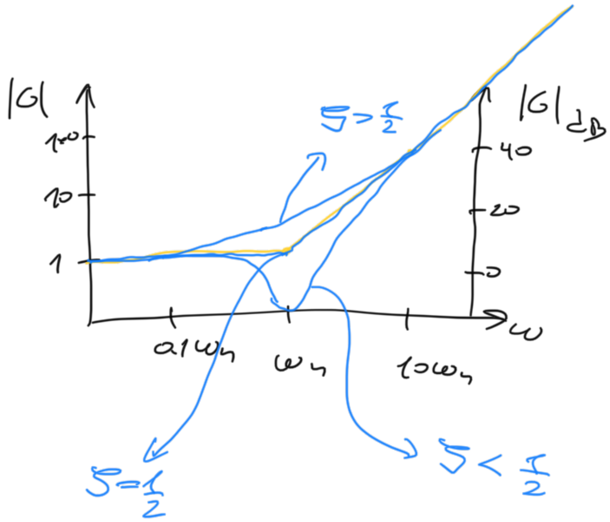


← plot for a pole

$$\boxed{\left(\frac{j\omega}{\omega_n}\right)^2 + 2\zeta\frac{j\omega}{\omega_n} + 1}$$

$\frac{\omega}{\omega_n} \ll 1$ $\qquad \left(\frac{j\omega}{\omega_n}\right)^2 + 2\zeta\frac{j\omega}{\omega_n} + 1 \cong 1$

$\frac{\omega}{\omega_n} \gg 1$ $\qquad \left(\frac{j\omega}{\omega_n}\right)^2 + 2\zeta\frac{j\omega}{\omega_n} + 1 \cong \left(\frac{j\omega}{\omega_n}\right)^2$

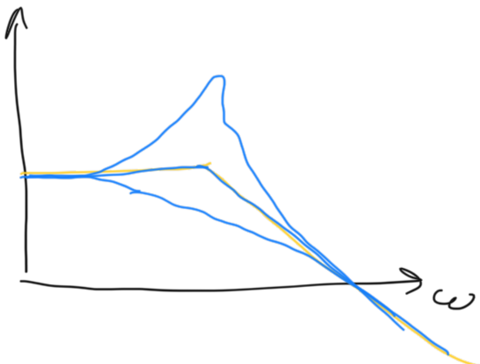


$\omega = \omega_n$

$$\left(j\frac{\omega}{\omega_n}\right)^2 + 2\zeta\,\frac{j\omega}{\omega_n} + 1$$

$$= j^2 + 2\zeta j + 1 = 2\zeta j$$

$$|2\zeta j| = 2\zeta$$



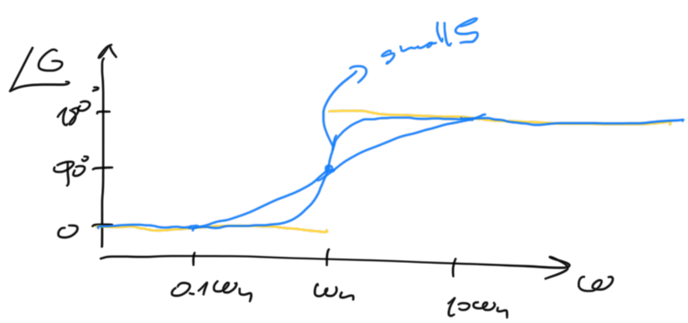
← plot for complex conjugate poles

phase

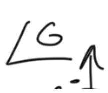
$\frac{\omega}{\omega_n} \ll 1 \qquad \angle 1 = 0^\circ$

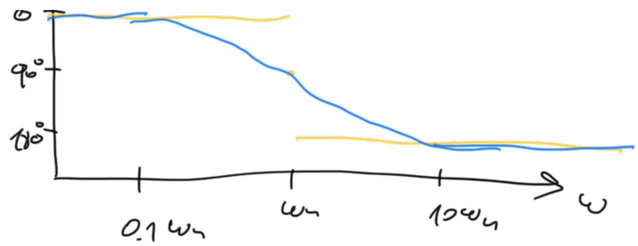
$\frac{\omega}{\omega_n} \gg 1 \qquad \angle \left(\frac{j\omega}{\omega_n}\right)^2 = 180^\circ$

$\omega = \omega_n \qquad \angle 2\zeta j = 90^\circ$



← plot for zeros

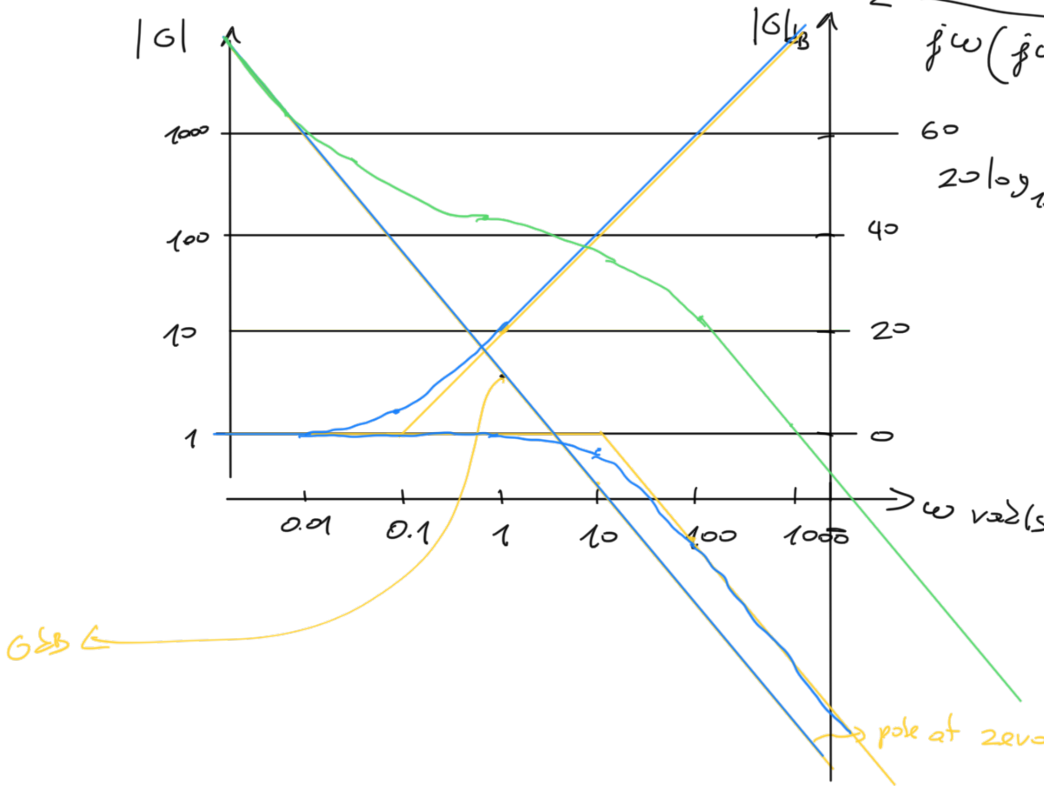




← plot for poles

$$G(s) = 200 \frac{s+0.1}{s(s+10)} \overset{s=j\omega}{=} 200 \frac{j\omega + 0.1}{j\omega(j\omega+10)} = 200 \frac{0.1\left(j\omega \frac{1}{0.1} + 1\right)}{j\omega\, 10 \left(j\omega \frac{1}{10} + 1\right)}$$

$$= 2 \frac{j\omega \frac{1}{0.1} + 1}{j\omega\left(j\omega \frac{1}{10} + 1\right)}$$



$$20 \log_{10} 2 \simeq 6$$