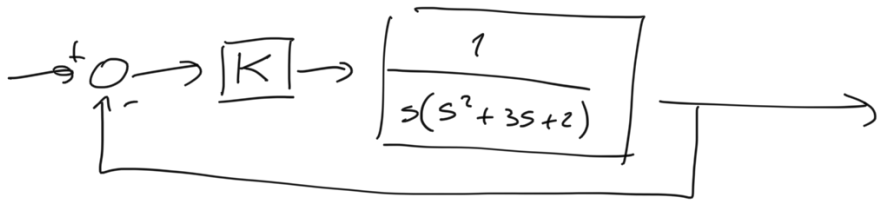
ECE 141

Lecture 15



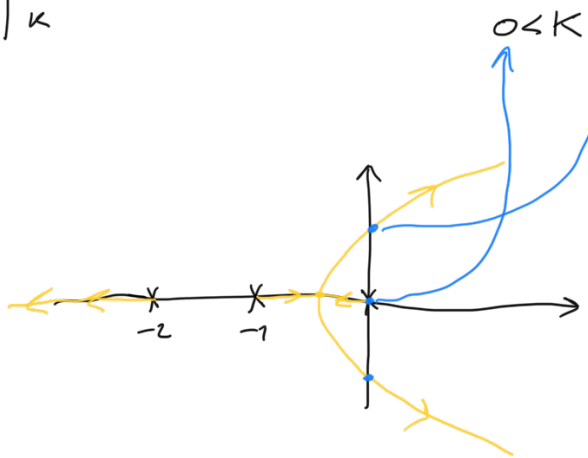
$$\frac{\dfrac{K}{s(s^2+3s+2)}}{1+\dfrac{K}{s(s^2+3s+2)}} = \frac{K}{s^3+3s^2+2s+K}$$

3	1	2
2	3	K
1	$-\frac{K-6}{3}$	
0	K	

$-(K-6)>0 \longrightarrow K-6<0 \longrightarrow K<6$

$K>0$

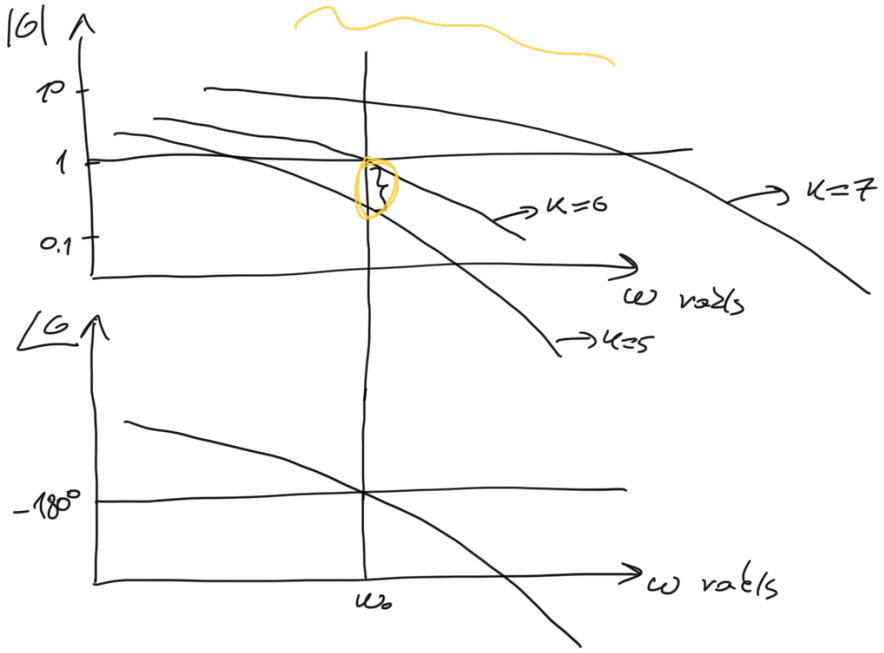
$0<K<6$



$1+KL(s)=0$

$KL(s)=-1 \Leftrightarrow \begin{cases} |KL(s)|=1 \\ \angle KL(s) = -180° \end{cases} \xrightarrow{\sigma=\omega} \begin{cases} |KL(j\omega)|=1 \\ \end{cases}$

$\angle KL(j\omega) = -180°$



Stability Margins

Gain Margin: The factor by which the gain can be raised before instability occurs.

$$G(s) = \frac{4}{s(s^2+3s+2)}$$

3.5 dB

$20\log_{10} K' = 3.5$

$K' = 10^{\frac{3.5}{20}} \cong 1.5$

stable: GM > 1 (0 dB)
unstable: GM < 1 (0 dB)

Phase Margin: The amount by which the phase of $G(\omega)$ exceeds $-180°$ when $|G(\omega)| = 1$.

PM: 12° stable: PM > 0

unstable: PM < 0

$$\frac{4s-8}{4s^4+12s^3+9s^2+25}$$

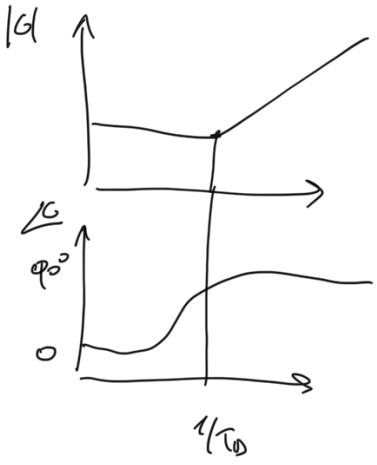
PD control (lead compensator)

$$K_p + K_D s = K_p\left(1 + \frac{K_D}{K_p}s\right) = K_p\,(1 + T_D s)$$

K_p: gain; $(1 + T_D s)$: zero

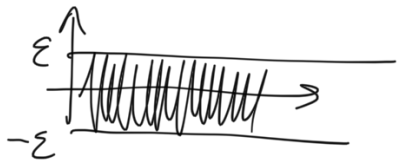
zero located at $-\frac{1}{T_D}$

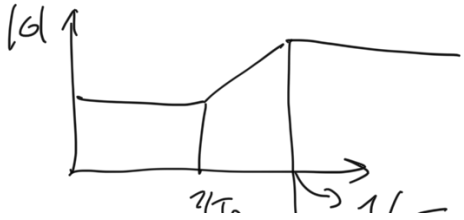
$$G(s) = \frac{15}{(s+1)^2(s+3)}$$



$|s(t)| \le \varepsilon$

$\frac{ds}{dt} >> 0$





Lead compensator

$\frac{T_D s + 1}{\quad}$ $\alpha < 1$ (lead ratio,