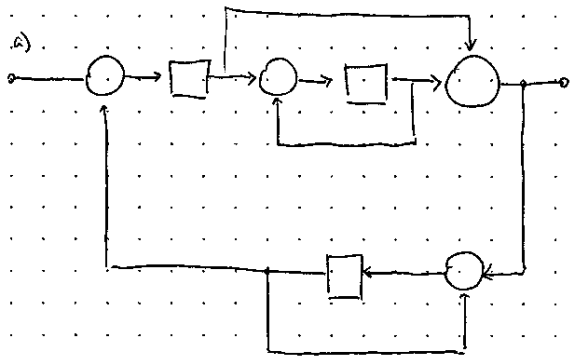
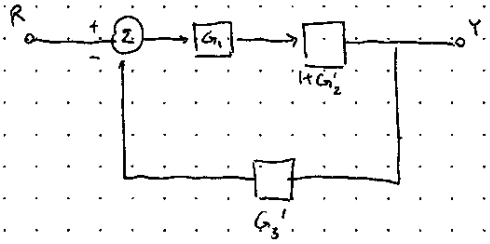
EE 141 HW 2

3.21)

a)

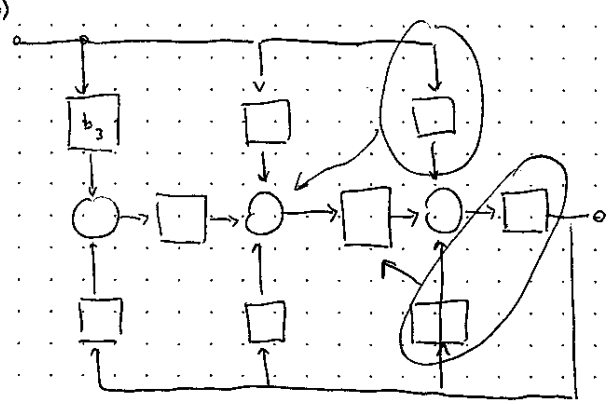


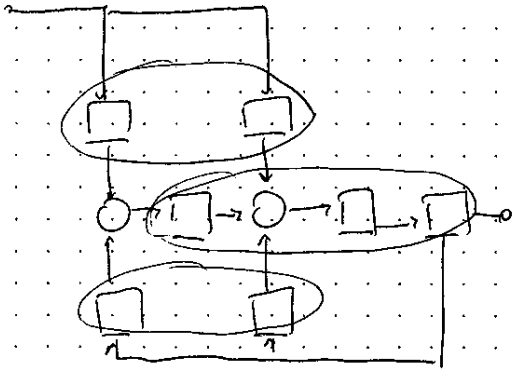


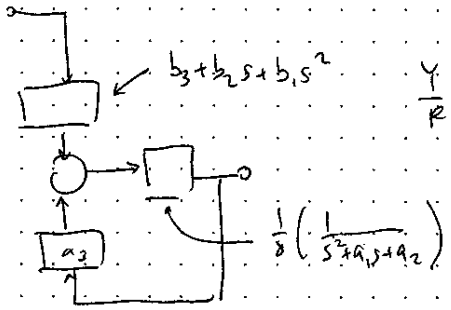
$$G_2' = \frac{G_2}{1-G_2H_2} \qquad G_3' = \frac{G_3}{1-G_3H_3}$$

$$\frac{Y}{R} = \frac{G_1\left(1+\frac{G_2}{1-G_2H_2}\right)}{1+G_1\left(1+\frac{G_2}{1-G_2H_2}\right)\left(\frac{G_3}{1-G_2H_3}\right)}$$

b)

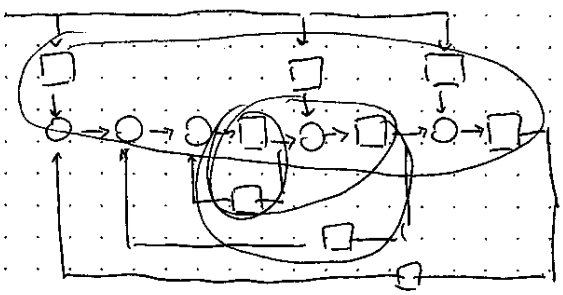


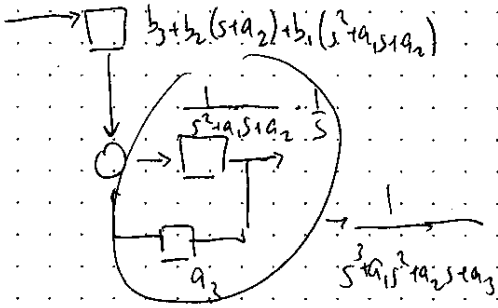




$$\frac{Y}{R} = \boxed{\frac{b_3+b_2s+b_1s^2}{s^3+a_1s^2+a_2s+a_3}}$$

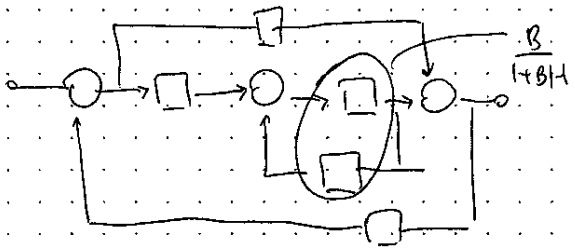
c)





$$\frac{Y}{R} = \boxed{\frac{b_3+b_2(s+a_2)+b_1(s^2+a_1s+a_2)}{s^3+a_1s^2+a_2s+a_3}}$$

(d)

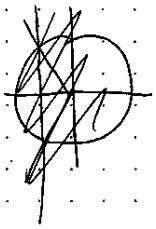


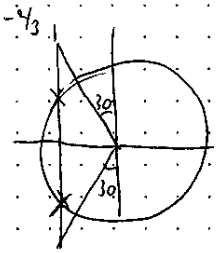
$$\frac{Y}{R} = \frac{A \cdot \frac{B}{1+BH} + D}{1+G\left(A \cdot \frac{B}{1+BH} + D\right)}$$

$$= \frac{AB + D(1+BH)}{(1+BH) + GAB + GD(1+BH)}$$

3.30) a) $\sigma = \frac{4.6}{6.9} = 0.6667 = 2/3$

$\sin^{-1}(0.5) = 30°$





b) $e^{\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}} = e^{\frac{-\pi \cdot 0.667}{\sqrt{1-4/9}}} = 0.063 \rightarrow$ **6.3%**

3.32) a) $T = \frac{Y}{R} = \frac{G}{1+G} = \frac{K}{s^2+2\zeta s+K} = \frac{\omega^2}{s^2+2\zeta\omega s+\omega^2}$ $\qquad e^{\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}} = 0.05$

$2 = 2\zeta\omega_n \quad K = \omega^2$ $\qquad (\pi\zeta) = \ln(0.05)\sqrt{1-\zeta^2}$

↳ matlab → $\zeta = 0.69$

$t_p = 1 \Rightarrow 1 = \frac{\pi}{\omega\sqrt{1-\zeta^2}} \Rightarrow \omega = \frac{\pi}{\sqrt{1-\zeta^2}} = 4.339$

$S_\omega = 29.7$ **NO**

b) $M_p = 0.05r \quad t_p = 1r$

$e^{\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}} = 0.05r \rightarrow \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = 1r$ $\qquad \frac{\pi}{\sqrt{K}\sqrt{1-\frac{1}{K}}} = 1r \rightarrow r = \frac{\pi}{\sqrt{K-1}} \rightarrow$ **K = 3.02**

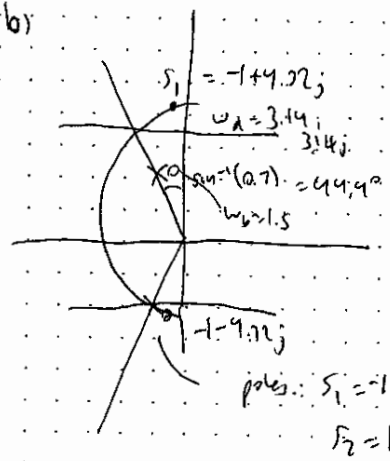
$\omega_n = \sqrt{K} \rightarrow \zeta = \frac{1}{\sqrt{K}}$ $\qquad 0.05r = e^{\frac{-\pi\cdot\zeta}{\sqrt{1-\zeta^2}}} \rightarrow$ matlab

$r = 2.21$

$M_p = 0.05r = 0.11$

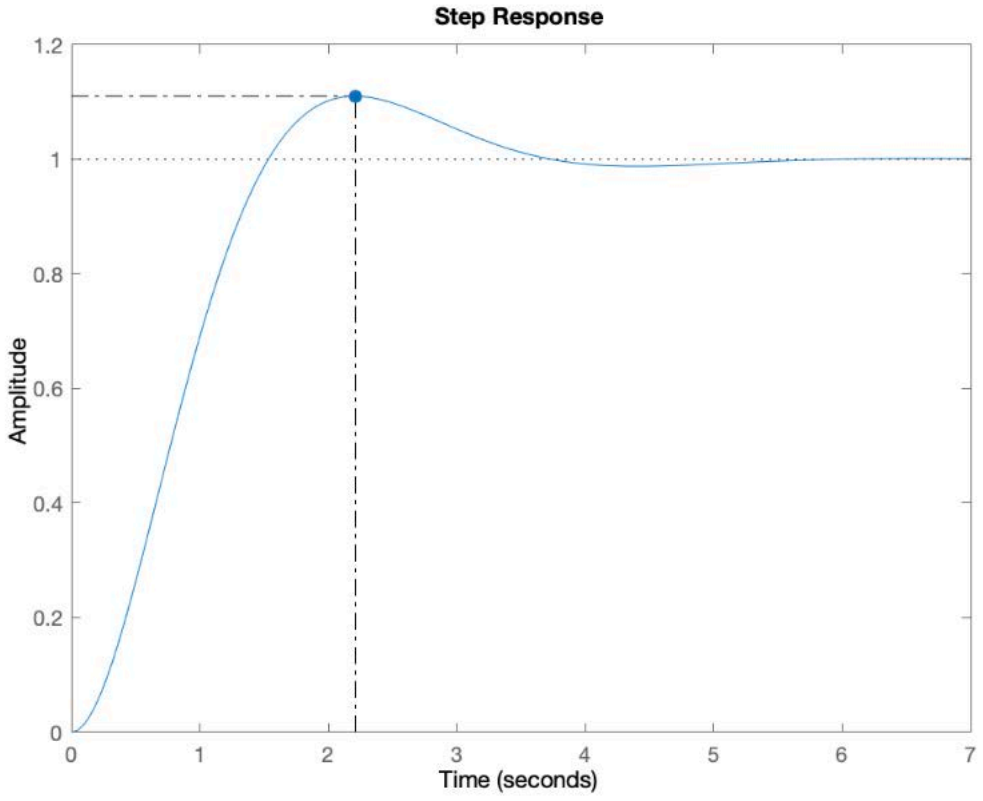
$t_p = 2.21 \cdot 1 = 2.21\,\text{sec}$

b)



c) Step Response: $K = 3.02$ Max Overshoot: 10.97%

Peak time = 2.214



3.36)

a) $J\ddot{\theta} + B\dot{\theta} = T_c$

$Js^2\theta + Bs\theta = T_c$

$\frac{\theta}{T_c} = \frac{1}{s(Js+B)} = \boxed{\frac{1.67M}{s^2 + s/30}}$

b) $T_c = k\theta_r - k\theta$

$\frac{\theta(s)}{T_c(s)} = \frac{1}{s(s+\frac{1}{30})} =$

$\theta\left(s^2 + \frac{1}{30}\right) + 1.67M \cdot k\theta = 1.67M \cdot k\theta_r$

$\frac{\theta}{\theta_r} = \frac{1.67M \cdot K}{1.67M \cdot K + s^2 + 1/30}$

a) $t_r = \frac{1.8}{\omega_n} < 80$

$\frac{1.8}{\sqrt{\frac{K}{J}}} < 80 \rightarrow \boxed{K > 303.75}$

c) $M_p = .10\%$

$e^{-\pi\zeta/\sqrt{1-\zeta^2}} = 0.1$

$\zeta = \sqrt{\frac{\ln(0.1)^2}{\pi^2 + \ln 0.1^2}} = 0.59$

$\frac{\theta}{T_c} = \frac{k}{k + Js^2 + Bs} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$

$\omega_n^2 = K/J$

$B/J = 2\zeta\omega_n$

$\zeta = \frac{\eta}{2\sqrt{KJ}} = 0.591$

$K = 476.93$

$\boxed{K < 476.93}$

c)

peak time: 2.21

for $K = 3.52$