ECE 141

Lecture 12

Root Locus Extensions

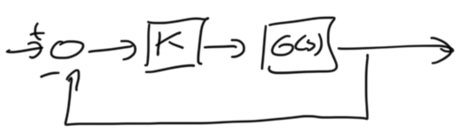
$$\boxed{1 + K L(s) = 0}$$

$$H(s) = \frac{n(s)}{\underbrace{d(s) + K_\beta \beta(s)}_{a(s)} + K_\gamma \underbrace{\gamma(s)}_{b(s)}} = \frac{\dfrac{n(s)}{d(s) + K_\beta \beta(s)}}{1 + K_\gamma \underbrace{\dfrac{\gamma(s)}{d(s) + K_\beta \beta(s)}}_{L(s)}}$$

$$= \frac{n(s)}{\underbrace{d(s) + K_\gamma \gamma(s)}_{a(s)} + K_\beta \underbrace{\beta(s)}_{b(s)}} = \frac{\dfrac{n(s)}{d(s) + K_\gamma \gamma(s)}}{1 + K_\beta \underbrace{\dfrac{\beta(s)}{d(s) + K_\gamma \gamma(s)}}_{L(s)}}$$

Negative gain

$K \leq 0$

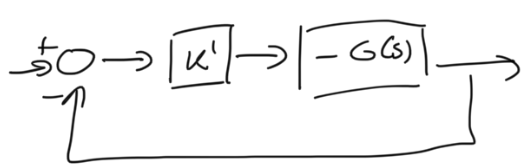


$$\frac{KG}{1 + KG}$$

$1 + K L(s) = 0$

$K = -K'$

$K' > 0$



$$\frac{-K'G}{1 - K'G}$$

$1 - K L(s) = 0$

$K L(s) = -1$

$\angle L(s) = 180^\circ$

$K L(s) = 1$

$\angle L(s) = 0^\circ$

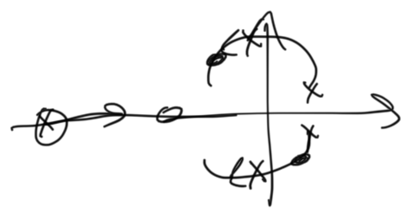
Value of K?

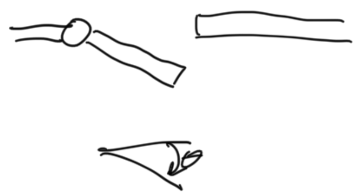
$1 + K L(s) = 0$

Given desired pole location s^*, compute k by solving $1+kL(s^*)=0$

$$K = -\frac{1}{L(s^*)} = -\frac{d(s)}{n(s)} \qquad L(s) = \frac{n(s)}{d(s)}$$

$$K = \left| \frac{d(s)}{n(s)} \right| \qquad 1 + \underset{\uparrow}{k} L(s) = 0$$





$$I\ddot{\theta} = -\alpha\dot{\theta} + u$$

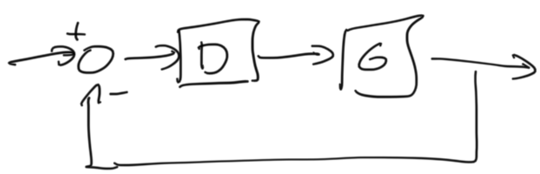
$$I s^2 \Theta(s) = -\alpha s \Theta(s) + U$$

$$\Theta(s)\left[s^2 + s\frac{\alpha}{I}\right] = \frac{1}{I} U$$

$$\frac{\Theta(s)}{U(s)} = \frac{1}{I\left(s^2 + \alpha s / I\right)}$$

$\alpha = 0.1$

$I = 1 \Rightarrow G(s) = \dfrac{1}{s(s+0.1)}$

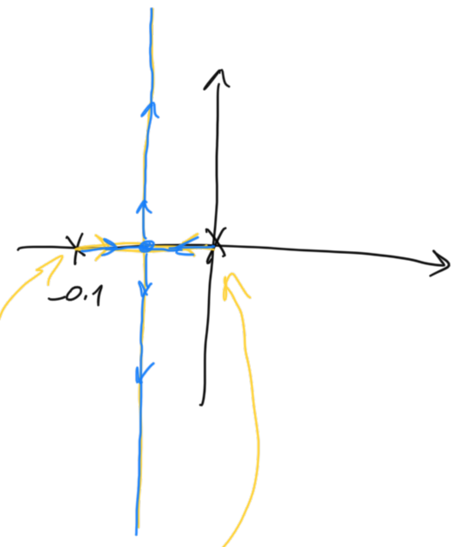


$$\frac{DG}{1+DG} \qquad D = K_p$$

$$\frac{K_p G}{1 + K_p G}$$

$$1 + K_p \underbrace{\frac{1}{s(s+0.1)}}_{L(s)}$$

$$t_s = \frac{4.6}{\sigma}$$



$$\alpha = \frac{0 - 0.1}{2} = -0.05$$

$$\phi_l = \frac{180 + 360(l-1)}{n-m} \qquad l = 1,2$$

$$\phi_1 = 90^\circ$$

$$\phi_2 = 90^\circ + 180^\circ$$

$$\phi_{1,dep} = -0 - 180^\circ = -180^\circ$$

$$\phi_{2,dep} = -(180^\circ) - 180^\circ = -360^\circ = 0$$

$$s^2 + 0.1s + K_p$$

$$\frac{K_p G}{1 + K_p G} = \frac{K_p \frac{1}{s^2+0.1s}}{1 + K_p \frac{1}{s^2+0.1s}}$$

$$= \frac{K_p}{\underbrace{s^2 + 0.1s + K_p}}$$

2	1	K_p
1	0.1	
0	K_p	

$$b\frac{da}{ds} - a\frac{db}{ds} = 0 \qquad b = 1, \quad a = s^2 + 0.1s$$

$$2s + 0.1 = 0 \iff s = -0.05$$

$q = 2$

$s^* = -0.05 \Rightarrow 1 + K_p L(s^*) = 0$

$$\downarrow$$

$$\frac{1}{s^2 + 0.1s} \qquad 1 + K_p \frac{1}{(0.05)^2 - 0.005} = 0$$

$$K_p = (0.05)^2 - 0.005$$

$$t_s = \frac{4.6}{\sigma} = \frac{4.6}{0.05} \cong 92 \text{ sec.}$$

$$\frac{(K_p + K_D s)\dfrac{1}{s^2 + 0.1s}}{1 + (K_p + K_D s)\dfrac{1}{s^2 + 0.1s}} = \frac{K_p + K_D s}{s^2 + 0.1s + K_p + K_D s}$$

Root locus for K_p

$a(s) = s^2 + 0.1s + K_D s$

$b(s) = 1$

Root locus for K_D

$a(s) = s^2 + 0.1s + K_p$

$b(s) = s$

$$\underbrace{s^2 + 0.1s}_{a(s)} + K_p \cdot \underbrace{1}_{b(s)} + K_D s$$

$a(s) + K_p\, b(s)$

$s^2 + 0.1s + K_p + K_D s$

$a(s) + K_D\, b(s)$