PROBLEM 1

1) looks like there is a zero @ 0.1 $+20$

a pole @ 1 -20

pole @ 100 -20

$\overline{-20}$ ✓

initial dB = 0

$$10^{\frac{0}{20}} = 1$$

so $$T = \frac{\frac{s}{0.1}+1}{\left(\frac{s}{1}+1\right)\left(\frac{s}{100}+1\right)} = \frac{1000s+100}{(s+100)(s+1)}$$

$$= \boxed{\frac{1000\left(s+\frac{1}{10}\right)}{(s+100)(s+1)}}$$

2) $\lim_{t\to\infty} e(t) = 0$

$\lim_{s\to 0} sE(s) = 0 \quad \lim_{s\to 0} s(R-Y) = 0 \quad \lim_{s\to 0} sR(1-T) = 0$

$$\lim_{s\to 0}\left(1 - \frac{K\cdot 1000\left(s+\frac{1}{10}\right)}{s^2+101s+100}\right) = 0$$

$$= \lim_{s\to 0}\left(1 - \frac{K\cdot 100}{100}\right) = 0$$

$\boxed{K=1}$ ✓

(3) minimize settling time: determined by slow pole aka

$(s+1)$ the pole at -1.

This corresponds to a time domain element of e^{-t}.

In 20 seconds, this decays to $2 \cdot 10^{-9}$ which is much

less than required value of 0.01 from target of 1.

$K=1$ is original T.F. already good enough!!!

(4) Integral controller $\frac{K_I}{s}$: $\dfrac{\frac{K_I}{s} \cdot G}{1+\frac{K_I}{s} \cdot G}$ = closed loop T.F.

$$\frac{K_I G}{s+K_I G} = \frac{K_I \cdot 1000\,(s+\frac{1}{10})}{s(s+1)(s+100) + K_I \cdot 1000\,(s+\frac{1}{10})}$$

Now $\lim\limits_{t\to\infty} e(t) = \lim\limits_{s\to 0} s \cdot E(s) = \lim\limits_{s\to 0} sR(1-T) = \lim\limits_{s\to 0} 1-T$

step input

$$= \lim_{s\to 0}\left(1 - \frac{K_I\,1000\,\cancel{(s+\frac{1}{10})}\,\frac{1}{10}}{\underbrace{\cancel{s(s+1)(s+100)}}_{0} + K_I \cdot 1000\underbrace{\cancel{(s+\frac{1}{10})}}_{\frac{1}{10}}}\right)$$

$$= \lim_{s\to 0}\left(1 - \frac{100 K_I}{100 K_I}\right) = 0$$

any value of K_I works so pick something like 10?

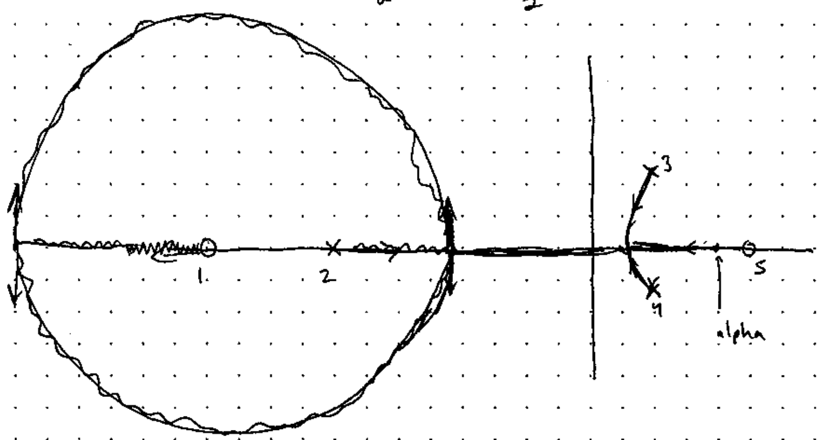
Controller $D = \boxed{\frac{10}{s}}$

Problem 2:

#1) zeros: $-12, 5$

poles: $-8,\ \frac{4 \pm \sqrt{16-40}}{2} = \frac{4 \pm 2i\sqrt{6}}{2} = 2 \pm i\sqrt{6}$

$G(s) = \frac{(s+12)(s-5)}{(s+8)(s^2-4s+10)}$



$\alpha = \frac{-4+7}{3-2} = 3$

$\phi_\alpha = \frac{180+360 \cdot 0}{1} = 180$

$\phi_1 = (180 + \cancel{9.924} + \cancel{180-9.924} + \cancel{180}) \cancel{-180} + \cancel{180}$

$= \cancel{180}\ 180$

$\tan^{-1}\left(\frac{\sqrt{6}}{14}\right) = 9.92°$

$\tan^{-1}\left(\frac{\sqrt{6}}{3}\right) = 39.23°$

$\tan^{-1}\left(\frac{\sqrt{6}}{10}\right) = 13.76$

$\psi_5 = (0 + -x + x) - 0 + 180 = 180$

$\phi_2 = (\cancel{0 + 180 + x + 180} - x) -$

$(0 + 180) - (180 + \cancel{x} + 180 - \cancel{x}) - 180 = 0$

$\phi_3 = (180 - 39.23° + 9.92°) - (90 + 13.76°) - 180 = -133.07°$

$\phi_4 = +133.07°$

$(s+8)(s^2-4s+10) + K(s+12)(s-5) = 0$

$s^3 + (k+4)s^2 + (7k-22)s - 60k + 80 = 0$

1	$7k-22$	
$k+4$	$-60k+80$	

$-\left(\frac{-7k^2 - 66k + 168}{k+4}\right)$

$+(-60k+80)$

$k+4=0 \Rightarrow k=-4$

$60k-80=0 \rightarrow k = 4/3$

$7k^2 - 66k + 168 = 0$

plug into

$1 + K \cdot G(s) = 0$

no value of pure imaginary s!!

never touches $j\omega$ axis

$$\frac{b\,da}{ds} - a\frac{db}{ds} = 0$$

$$(s^2+7s-60)\cdot(3s^2+8s-22) - (s+8)(s^2-4s+10)\cdot(2s+7) = 0$$

$$s = \boxed{-18.96,\ -4.636,\ 1.006,}$$

$$\theta = \frac{180+360(k)}{2} = \underline{270,\ 90}$$

#2) $\boxed{\text{No value}}$ one of the poles on the RHP (not sure which b/c handdrawn not sufficient info) will move to the right after reaching the $\mathbb{R}$ axis @ 1.006. This will stay on the RHP forever.

#3) No. the remove oscillatory behavior, all poles @ $\mathbb{R}$ axis.
We can do that by making the 2 poles on RHP @ 1.006 on $\mathbb{R}$.
the pole on the left will also be on $\mathbb{R}$. However it is

NOT STABLE

#4) The minimum value of overshoot occurs when two poles are @ 1.006 on the right side. In fact, these two poles on the right are not only slow but also not stable (I count them as slow).

$$1+K\cdot G(s) = 0$$

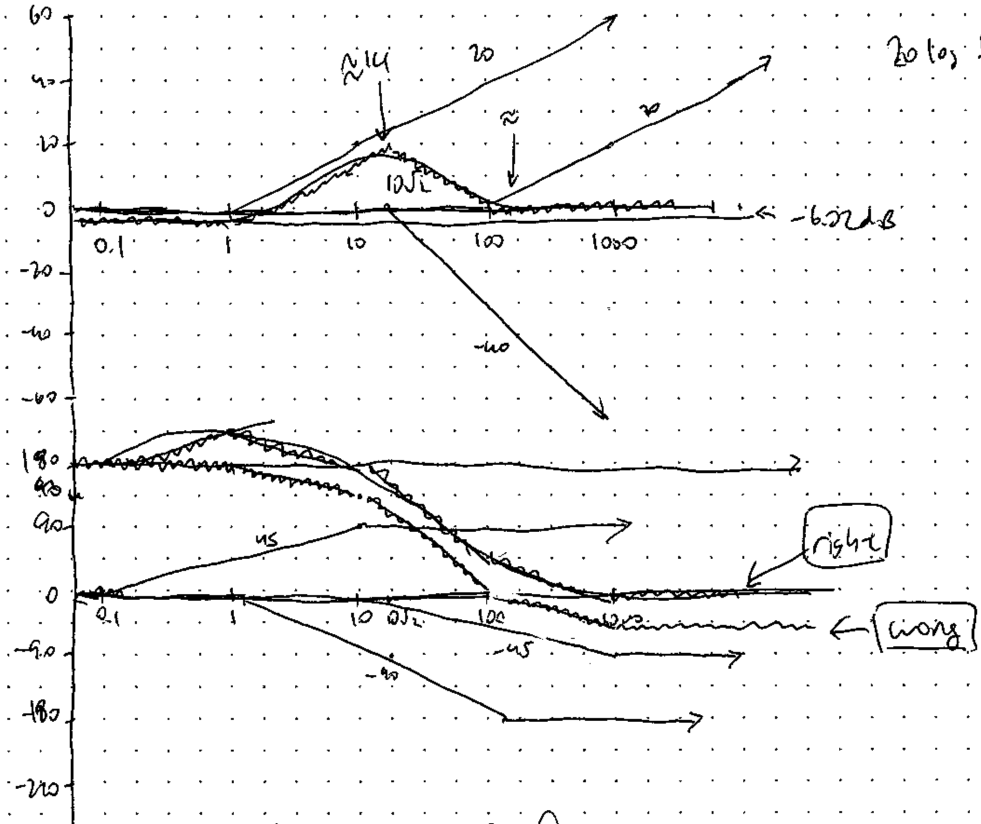
$$1+K\cdot\frac{(1.006+12)(1.006-5)}{(1.006+8)(1.006^2-4\cdot1.006+10)} = 0$$

$$K = \boxed{1.2115}$$

PROBLEM 3

1. $$\frac{(j\omega+1)(j\omega-100)}{(j\omega)^2+20(j\omega)+200} = \frac{(j\omega+1)\left(\frac{j\omega}{100}-1\right)\cdot 100}{\left(\left(\frac{j\omega}{10\sqrt{2}}\right)^2+\left(\frac{j\omega}{10\sqrt{2}}\right)\cdot\sqrt{2}+1\right)200}$$

$$= -\frac{1}{2}\,\frac{(j\omega+1)\left(-\frac{j\omega}{100}+1\right)}{\left(\frac{j\omega}{10\sqrt{2}}\right)^2+\left(\frac{j\omega}{10\sqrt{2}}\right)\sqrt{2}+1}$$



$20\log\frac{1}{2} = -6.02$

$6.02\,(j\omega)^{2 \to 1m}$

#2. phase margin ≈ @ $\omega \approx 1-2$ Let's say $\underline{2}$ for fun.

@ $\omega = 2$, ϕ really close to $180+45 = 225$

let's say $\underline{230}$

That is a phase margin of $\boxed{55^\circ}$

3) phase diagram passes 180° @ approximately $10\sqrt{2}$ or 14 rad/s.

the amplitude is close to 20, 20-3=17.

increasing this by a gain of 1.5 ~~is~~ takes $20\log(1.5)$ dB
$= 3.5$ dB

will RAISE the amplitude graph and the point is still >0 dB

so unstable

#4) Step 1: fix phase graph to get attenuation for $\omega > 100$

adding a zero @ 100 will cause the graph to flatten out at $\omega = 100$

b/c @ $\omega = 100$, the slope is $-45°$/dec.

add a $(s+100)$ zero.

~~however, we don't want it to flatten out @ ≈45°. we want it to be 0°. so, we add another pole.~~

Step 2: fix gain to get DC gain of 1

right now, it is above 0 dB (i plugged in $\omega = 100000$)
output into calc :)

input is -6.022 dB. raise by 6.022 dB.

#4) attenuate frequency > w=100

$dB < 0$ for $w > 100$

this means adding a pole @ w=100

$$\frac{K}{\left(\frac{s}{100}+1\right)}$$

this is to force the value down @ 100 since originally it was above by alittle.

Then, to make DC gain = 1, we push it back up with a zero.

@ $100\sqrt{2}\cdot\sqrt{2}$ to keep the ~~ratio~~ ratio that $100\sqrt{2}$ brought it down with.

$$\frac{1\,\cancel{K}}{\left(\frac{s}{100}+1\right)}\left(\frac{s}{100\sqrt{2}^2}+1\right) \Rightarrow \frac{1\,\cancel{K}}{\left(\frac{s}{100}+1\right)}\left(\frac{s}{200}+1\right)$$

$$\cancel{\frac{K(s+200)}{(2s+1)}} \qquad \cancel{K=1} \qquad = \cancel{\frac{0.5(s+200)}{(s+1)}}$$

$$= \frac{s+200}{2s+200} = \boxed{\frac{0.5(s+200)}{(s+100)}}$$

YAY

on second thought, the s+200 will raise the value @ 100 by alittle but cancelling out the attenuation, so not quite sure if that's fixable

Just move back 1 dec: $\boxed{\frac{0.5(s+20)}{s+10}}$