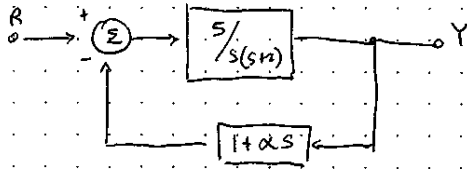


S.9)



$$G(s) = \frac{s}{s(s^2+2s+5)} \quad H(s) = 1 + \alpha s$$

$$1 + G(s) \cdot H(s) = 0$$

$$1 + \frac{s(1+\alpha s)}{s(s^2+2s+5)} = 0$$

$$s(s^2+2s+5) + s(1+\alpha s) = 0$$

$$s^3 + 2s^2 + 5s + s + \alpha s^2 = 0$$

$$1 + \frac{s \alpha s}{s^3 + 2s^2 + 5s} = 0 \rightarrow K \cdot \frac{b(s)}{a(s)}$$

$$\begin{aligned} b(s) &= s\alpha s \\ a(s) &= s^3 + 2s^2 + 5s \end{aligned} \quad \text{assuming } K = \alpha$$

$$L(s) = \frac{b(s)}{a(s)} = \frac{s\alpha s}{s^3 + 2s^2 + 5s}$$

$$T(s) = \frac{s}{s(s^2+2s+5)} = \frac{s}{s^3 + (2+5\alpha)s^2 + 5s}$$

closed loop pole locations:

$\alpha = 0$	$s^2 + 2s + 5 = (-1 \pm 2i)$
$\alpha = 0.5$	$s^2 + 4.5s + 5 = (-2, -2.5)$
$\alpha = 2$	$s^3 + 12s + 5 = (-0.432, -11.5567)$

SKETCHES ON
NEXT PAGE

$$L(s) = \frac{s\alpha s}{s^3 + 2s^2 + 5s}$$

$$\text{zeros} = s = 0$$

$$\text{poles} = \frac{-2 \pm \sqrt{4-20}}{2} = -1 \pm 2i$$

$$\alpha = \frac{-2-0}{2-1} = -2 \text{ asymptotes}$$

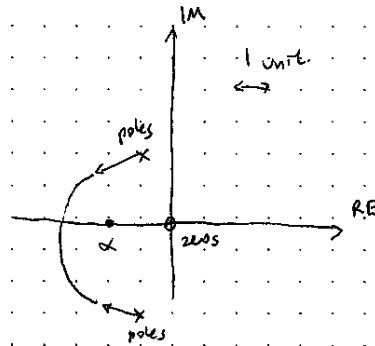
$$\phi_1 = 180 \left(\frac{1+2(1-1)}{1} \right) = 180^\circ$$

$$\theta_1 = \tan^{-1} \left(\frac{-2}{-1} \right) = -63.43^\circ$$

$$\theta_2 = \tan^{-1} \left(\frac{2}{-1} \right) = 90^\circ \quad \angle = 116.57^\circ$$

$$\theta_d = 180 - 90 + 116.57 = 206.57^\circ @$$

$$s = -1 \pm 2j$$



$$Y(s) = R(s) \cdot \frac{s}{s^3 + (2+5\alpha)s^2 + 5s}$$

$$\alpha = 0: \quad \frac{1}{s} - \frac{5s+1}{(s+1)^2+4} - \frac{1}{(s+1)^2+4}$$

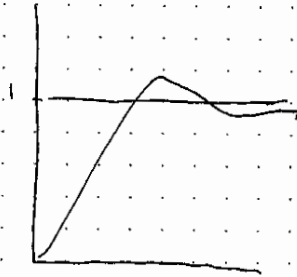
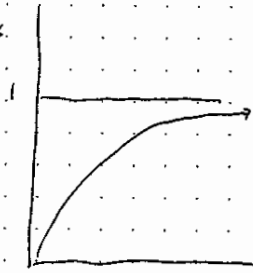
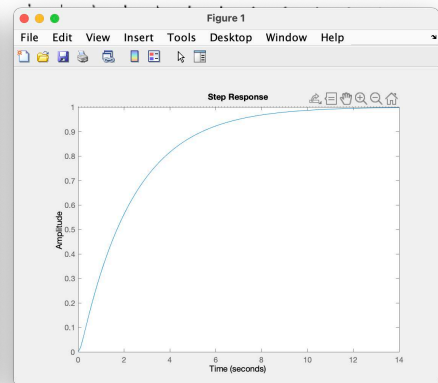
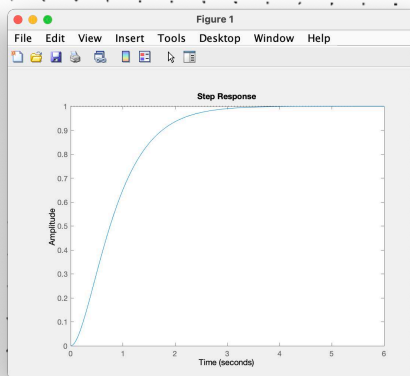
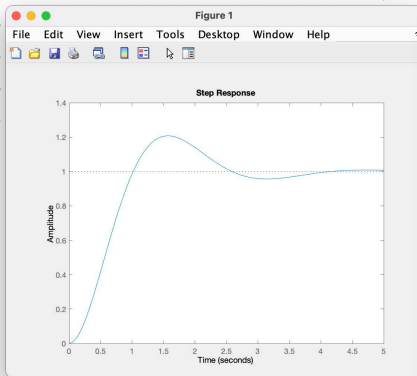
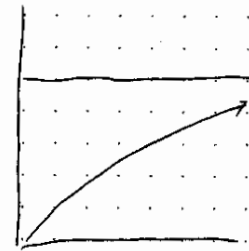
$$y(t) = u(t) - e^{-t} \cos(2t) u(t) - \frac{1}{2} e^{-t} \sin(2t) u(t)$$

$$\alpha = 0.5: \quad \frac{1}{s} - \frac{5}{s+2} + \frac{4}{s+2.5}$$

$$y(t) = u(t) - 5e^{-2t} u(t) + 4e^{-2.5t} u(t)$$

$$\alpha = 2: \quad \frac{1.0015}{s} - \frac{1.0404}{s+0.432} + \frac{0.039}{s+11.5567}$$

$$y(t) = (1.0015 - 1.0404 + 0.039) e^{-0.432t} e^{-11.5567t} u(t)$$

$\alpha = 0$  $\alpha = 0.5$  $\alpha = 2$ 

$$5.13) \quad a) \quad G(s) = K \frac{s+1}{s+13} \cdot \frac{s^2+81}{s^4(s^2+100)} \rightarrow T(s) = \frac{G(s)}{1+G(s)} \times$$

$$\text{characteristic: } 1 + K \frac{s+1}{s+13} \cdot \frac{s^2+81}{s^4(s^2+100)} = 0 \Rightarrow L = \frac{(s+1)(s^2+81)}{(s+13)s^4(s^2+100)}$$

$$\alpha = \frac{-13 - (-1)}{5-3} = -6$$

$$\phi_1 = 180 \left(\frac{1+2(8-1)}{n-m} \right) = 180 \cdot \frac{1}{2} = 90^\circ, \quad \phi_2 = 180 \cdot \frac{(1+2 \cdot 1)}{2} = 270^\circ = -90^\circ$$

zeros: $-1, \pm 9i$ poles: $-13, 0, \pm 10i$ let's use $-10i$:

$$\begin{array}{l|l} \theta_1 = \tan^{-1}\left(\frac{10}{13}\right) = 37.57^\circ & (-13) \\ \theta_2 = 90^\circ & (0) \\ \theta_3 = 90^\circ & (-10i) \end{array} \quad \left| \quad \begin{array}{l} \theta_4 = \tan^{-1}(10) = 84.29^\circ \quad -1 \\ \theta_5 = \tan^{-1}(90) = 90^\circ \quad 9i \\ \theta_6 = 90^\circ \quad -9i \end{array} \right.$$

$$\theta_d = 180 - (180 + 37.57) + 180 + 84.29 = 180 - 46.72 = 226.72^\circ = -133.28^\circ$$

$$K = \frac{-s^2(s+13)(s^2+10s)}{(s^2+8s+13)(s+1)}$$

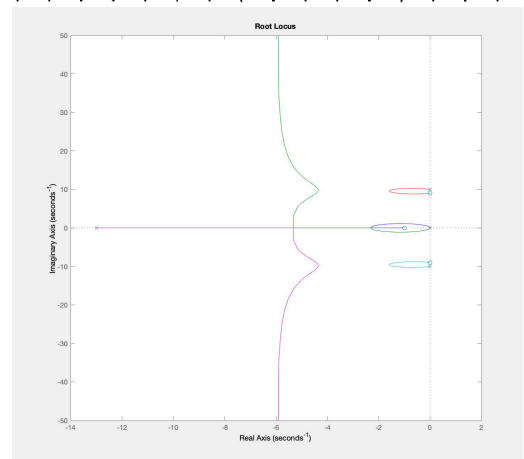
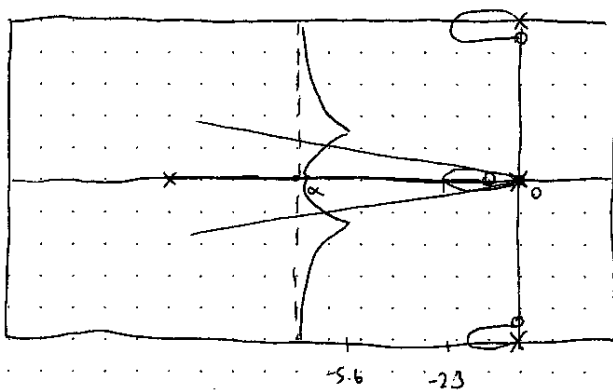
$$= \frac{-s^5 + 13s^4 + 100s^3 + 130s^2}{s^3 + 8s^2 + 13s + 13}$$

$$\frac{\partial K}{\partial s} = 0 \Rightarrow \text{MATLAB} \Rightarrow$$

$$s = 0,$$

$$s = -5.642 \pm j2.311$$

real mode



(b) $\zeta = 0.5$ the asympt. cuts the locus @ $\zeta = \zeta_{crit}$ where $\zeta \in [0, 1]$
this, system is underdamped

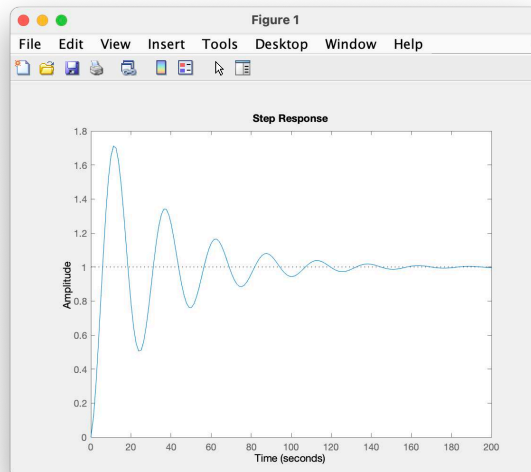
THERE EXISTS A VALUE OF K.

$$(c) \zeta = 0.707 \rightarrow \theta = 45^\circ$$

Intersection at $\theta = 45^\circ$.

$$s = -5.6 \pm j6.5 \text{ plus into K equation. } \Rightarrow K = 103.685$$

(d)



$$K = \frac{-s^2(s+13)(s^2+100)}{(s^2+181)(s+1)}$$

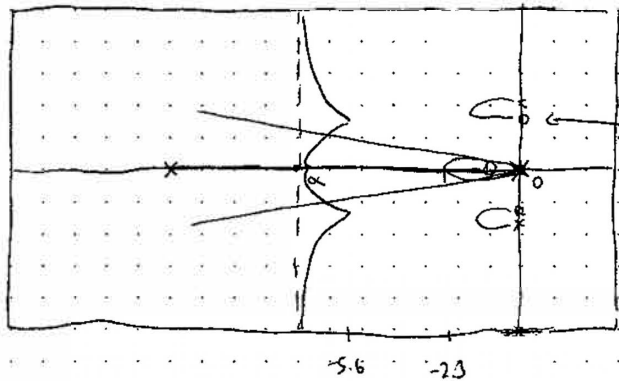
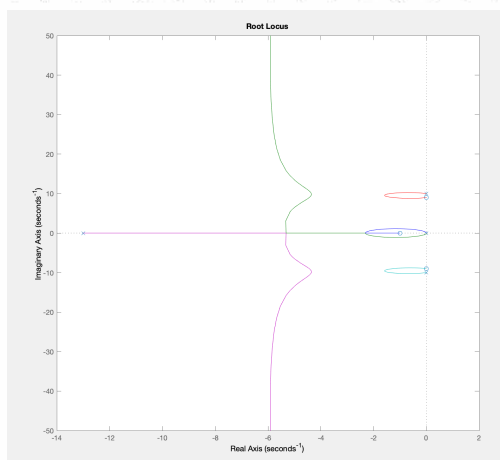
$$= \frac{-s^5 + 13s^4 + 100s^3 + 1300s^2}{s^3 + s^2 + 181s + 181}$$

$$\frac{\partial K}{\partial s} = 0 \Rightarrow \text{MATLAB} \Rightarrow$$

$$s = 0$$

$$s = -5.642 \pm j2.311$$

real roots



poles are so over 0.5

these branches are not in range

(b) $\zeta = 0.5$ the asympt. cuts the locus @ $\zeta = 0.5$ but $\zeta \in [0, 1]$
~~this system is underdamped~~ THERE DOES NOT EXIST A VALUE OF K.

$$(c) \zeta = 0.707 \rightarrow \theta = 45^\circ$$

intersect at $\theta = 45^\circ$

plus into K equation. $\Rightarrow K = 82.56$
 $s = -5.18 \pm j5.181$ or 88.3

(d)

