Lecture 10

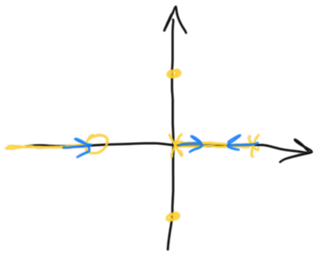
ECE 141

$$1 + K\frac{s+1}{s(s-1)}$$

2	1	K
1	$K-1$	0
0	K	

$K=0$

$K=1$



for $K=1$ $\quad s^* = j\omega_0$

$$1 + K\underbrace{\frac{s+1}{s(s-1)}}_{s \,\downarrow\, j\omega_0} = 0, \qquad K \downarrow 1$$

$$1 + \frac{j\omega_0+1}{j\omega_0(j\omega_0-1)} = 1 + \frac{j\omega_0+1}{-\omega_0^2 - j\omega_0}$$

$$= \frac{-\omega_0^2 - j\omega_0}{-\omega_0^2 - j\omega_0} + \frac{j\omega_0+1}{-\omega_0^2 - j\omega_0}$$

$$= \frac{1-\omega_0^2}{-\omega_0^2 - j\omega_0} = 0$$

$$\Rightarrow \omega_0^2 = 1 \Rightarrow \omega_0 = \pm 1$$

Rule VI: The locus will have multiple roots at points on the locus where

$$\boxed{b\frac{da}{ds} - a\frac{db}{ds} = 0}$$

and the branches will approach a point of q roots at angles separated by:

$$\frac{180^\circ + 360^\circ(l-1)}{q} \qquad l = 1, 2, \dots, q$$

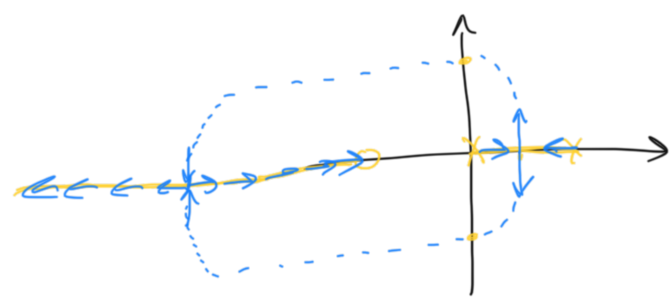
and will depart with the same separation.

$b(s) = s+1$

$a(s) = s(s-1)$

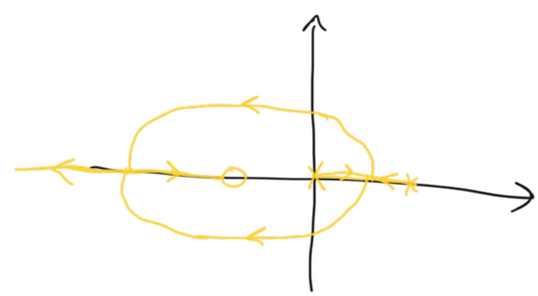
$$(s+1)(2s-1) - s(s-1)1 = 2s^2 - s + 2s - 1 - s^2 + s$$
$$= s^2 + 2s - 1 = 0$$

$$s = \frac{-2 \pm \sqrt{4-4(-1)}}{2} = -1 \pm \sqrt{2} \cong -2.41,\ 0.41$$



$q=2$

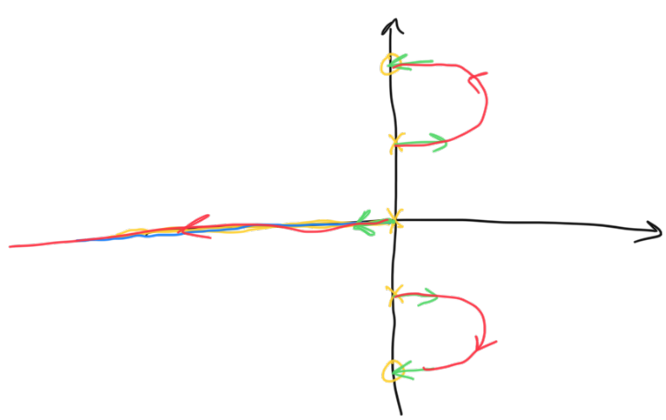
$$\frac{180^\circ + 360^\circ(0)}{2} = 90^\circ$$



$$\frac{180^\circ + 360^\circ}{2} = 90^\circ + 180^\circ$$

Example I

$$\frac{s^2+4}{s(s^2+1)} = L(s)$$



zeros: $s = \pm 2j$

poles: $s=0,\ \pm j$

$$\alpha = \frac{\Sigma p - \Sigma z}{n-m} = \frac{0+j-j-(2j-2j)}{3-2} = 0$$

$$\phi_1 = \frac{180^\circ + 360^\circ(l-1)}{n-m} = \frac{180^\circ}{1} = 180^\circ$$

$$\phi_{l,dep} = \frac{1}{q}\left[\Sigma \psi_i - \sum_{i\neq l} \phi_i - 180^\circ - 360^\circ(l-1)\right] \quad l = 1,2,\dots,q$$

pole at 0

$$\phi_{1,dep} = (90^\circ - 90^\circ) - (90^\circ - 90^\circ) - 180^\circ$$

$$= -180^\circ$$

pole at $-j$

$$\phi_{1,dep} = (90^\circ - 90^\circ) - (-90^\circ - 90^\circ) - 180^\circ$$

$$= 0^\circ$$

$$\psi_{l,arr} = \frac{1}{q}\left[\Sigma \phi_i - \sum_{j\neq l} \psi_j + 180^\circ + 360^\circ(l-1)\right] \quad l = 1,\dots,q$$

$$\psi = (-90^\circ - 90^\circ - 90^\circ) - (-90^\circ) + 180^\circ$$

$$= -90^\circ - 180^\circ + 90^\circ + 180^\circ = 0^\circ$$

$$\frac{s^2+4}{s(s^2+1)} = L(s)$$

$$\frac{G(s)}{1 + K\frac{s^2+4}{s(s^2+1)}} = \frac{s(s^2+1)\,G(s)}{s(s^2+1) + K(s^2+4)}$$

$$s^3 + s + Ks^2 + 4K$$

3	1	1	1
2	K	4K	K
1	$1 - \frac{4K-K}{K} = -3$		-3
0	4K		$4K$

$K=0$

$$b\frac{da}{ds} - a\frac{db}{ds} = (s^2+4)\left[3s^2+1\right] - s(s^2+1)\left[2s\right]$$

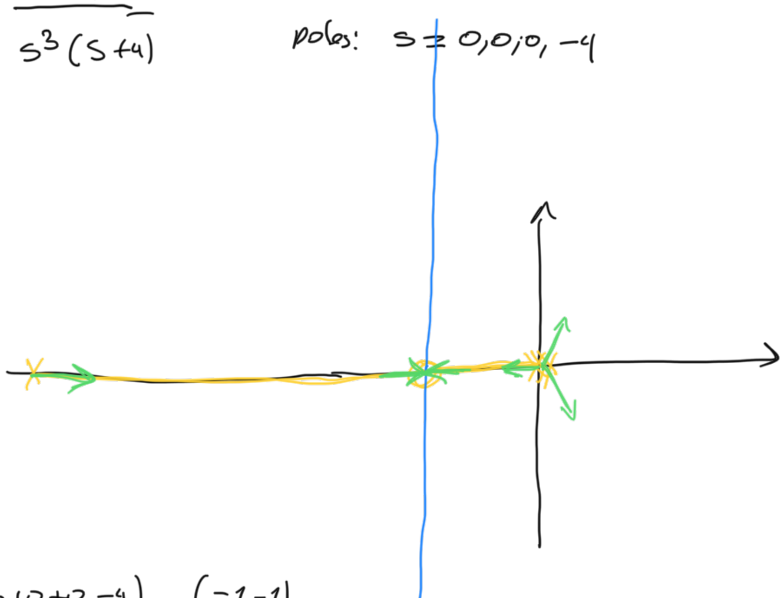
$$= 3s^4 + s^2 + 12s^2 + 4 - 2s^4 - 2s^2$$

$$= s^4 + 11s^2 + 4 = 0$$

$$s^4 + 11s^2 = -4$$

Examples $\quad \frac{(s+1)^2}{s^3(s+4)}$ $\quad$ zeros: $s = -1, -1$

poles: $s = 0, 0, 0, -4$



$$\alpha = \frac{(0+0+0-4) - (-1-1)}{4-2} = \frac{-4+2}{2} = -1$$

$$\phi_1 = \frac{180^\circ + 360^\circ \cdot 0}{2} = 90^\circ \qquad \phi_2 = \frac{180^\circ + 360^\circ(2-1)}{2} = 90^\circ + 180^\circ$$

$q = 3$

poles at 0

$$\phi_{1,dep} = \frac{1}{3}\left[(0+0) - (0) - 180^\circ - 360^\circ(1-1)\right] = \frac{-180^\circ}{3} = -60^\circ$$

$$\phi_{2,dep} = \frac{1}{3}\left[(0+0) - (0) - 180^\circ - 360^\circ(2-1)\right] = \frac{-180^\circ - 360^\circ}{3} = -180^\circ$$

$$\phi_{3,dep} = \frac{1}{3}\left[(0+0) - (0) - 180^\circ - 360^\circ \cdot 2\right] = \frac{-180^\circ - 360^\circ \cdot 2}{3} = 60^\circ$$

pole at -4

$$\phi_{1,dep} = \left[(180^\circ + 180^\circ) - (180^\circ + 180^\circ + 180^\circ) - 180^\circ\right]$$

$$= -360^\circ = 0^\circ$$

$q = 2$

$$\psi_{1,arr} = \frac{1}{2}\left[(180^\circ + 180^\circ + 180^\circ + 0) + 180^\circ + 360^\circ(1-1)\right]$$

$$= \frac{1}{2}(2 \times 360^\circ) = 360^\circ = 0^\circ$$

$$\psi_{2,arr} = \frac{1}{2}\left[(180^\circ + 180^\circ + 180^\circ + 0^\circ) + 180^\circ + 360^\circ(2-1)\right]$$

$$= \frac{1}{2}\left[3 \times 360^\circ\right] = 3 \times 180^\circ = 180^\circ + 360^\circ = 180^\circ$$