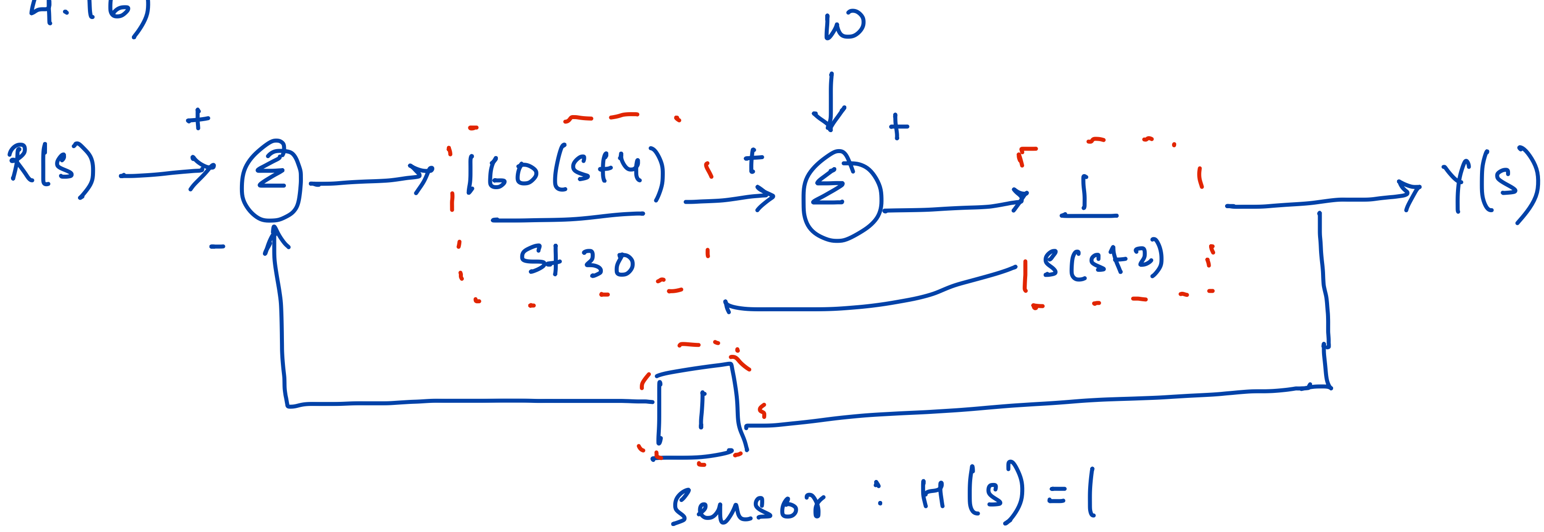
Discussion Week 5:

4.16)



(a) $\displaystyle e_{t\to\infty} = y - r$ $\qquad$ $e \to 0$ at $t \to \infty$??

if $r = u(t)$

$w = 0$

$$N(s) = \frac{160(s+4)}{s+30} \times \frac{1}{s(s+2)}$$

$$\frac{Y(s)}{R(s)} = \frac{\frac{160(s+4)}{s+30} \times \frac{1}{s(s+2)}}{1 + \frac{160(s+4)}{s+30} \times \frac{1}{s(s+2)}}$$

$$T(s) = \frac{160(s+4)}{(s+30)(s+2)s + 160(s+4)}$$

$$e = y - r$$

$$E(s) = Y(s) - R(s)$$

$$Y(s) = R(s)\,T(s)$$

$$E(s) = R(s)\,T(s) - R(s)$$

$$= R(s)\,[T(s) - 1]$$

$$= R(s)\left[\frac{160(s+4)}{(s+30)(s+2)s + 160(s+4)} - 1\right]$$

$$= R(s)\left[\frac{\cancel{160(s+4)} - (s+30)(s+2)s - \cancel{160(s+4)}}{(s+30)s(s+2) + 160(s+4)}\right]$$

$$e = -\frac{1}{s}\left[\frac{(s+30)(s+2)s}{(s+30)s(s+2) + 160(s+4)}\right]$$

At $t \to \infty$, e ?

$$\lim_{t\to\infty} e(t) = \lim_{s\to 0} s\, E(s)$$

$$\lim_{s\to 0} -\cancel{s} \cdot \frac{1}{\cancel{s}} \left[\frac{(s+30)(s+2)\,s}{(s+30)\,s\,(s+2) + 160(s+4)} \right] \to 0$$

$$= 0$$

$$\lim_{t\to\infty} e(t) = 0$$

$$r(t) = t$$

$$R(s) = 1/s^2$$

$$K_v = \left| \frac{1}{\lim_{t\to\infty} e(t)} \right|$$

$$\lim_{t\to\infty} e(t) = \lim_{s\to 0} s \cdot E(s$$

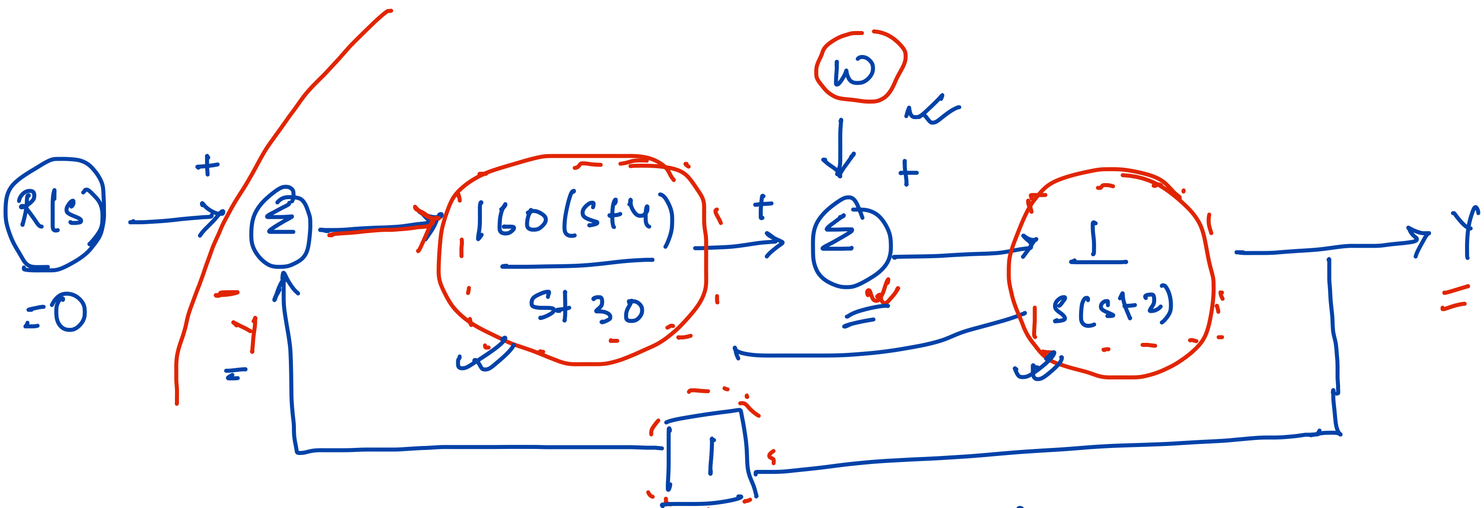
$$= \lim_{s\to 0} \cancel{s} \cdot \frac{1}{s^{\cancel{2}}} (-1) \left[\frac{(s+30)(s+2)\,s}{(s+30)\,s\,(s+2) + 160(s+4)} \right]$$

0

$$= -\frac{30 \cdot x}{160 \times 4/2} \Rightarrow -\frac{3\cancel{0}}{32\cancel{0}}$$

$$K_v = \left| \frac{1}{-3/32} \right| = 32/3$$

(b)



Sensor : $H(s) = 1$

$$\left[\underbrace{-Y \times \frac{160(s+4)}{s+30}} + W \right] \times \frac{1}{s(s+2)} = Y \rightarrow$$

$$W \times \frac{1}{s(s+2)} = Y\left[1 + \frac{160(s+4)}{s(s+2)(s+30)} \right].$$

$$W = Y\left[\frac{s(s+2)(s+30) + 160(s+4)}{(s+30)}\right]$$

$$\frac{Y}{W} = \frac{s+30}{s(s+2)(s+30) + 160(s+4)} = G(s)$$

$Y(s) \to 0$ as $t \to \infty$ when $R(s) = 0$

$$Y(s) = W(s) \cdot G(s)$$

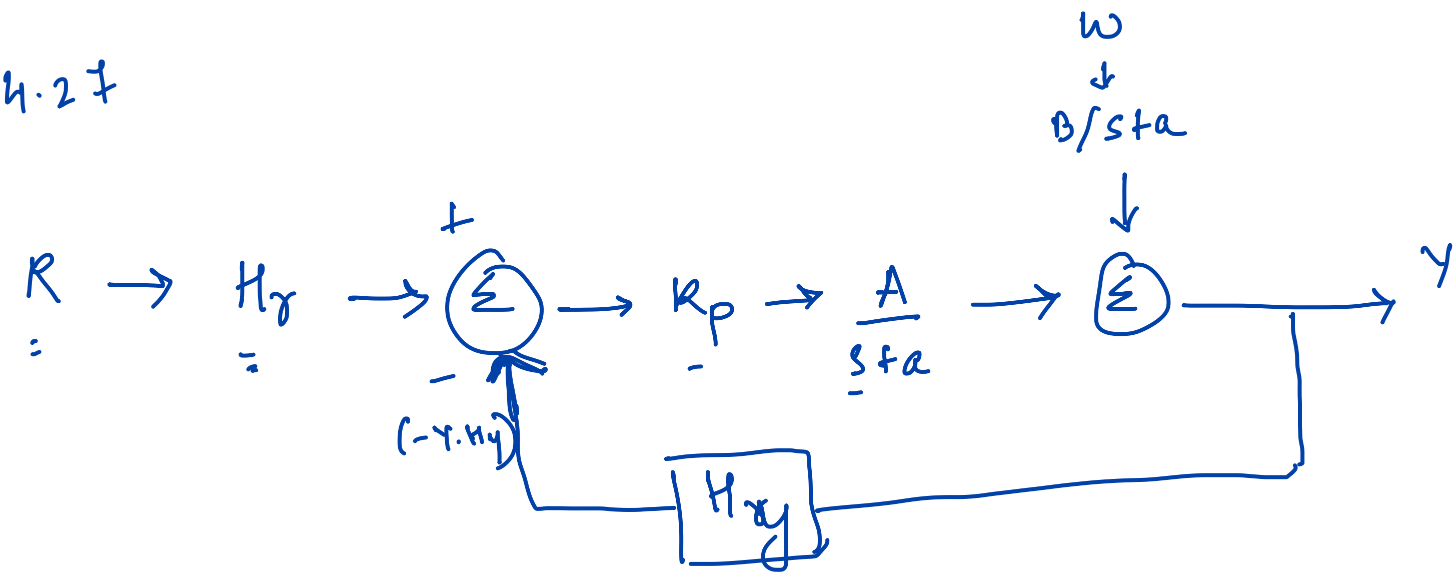
$$W(s) = 1/s$$

$$\lim_{s \to 0} sY(s) = \lim_{t \to \infty} y(t)$$

$$\lim_{s \to 0} \cancel{s} \cdot \frac{1}{\cancel{s}} \times \frac{s+30}{s(s+2)(s+30) + 160(s+4)}$$

$$= \frac{3\cancel{0}}{16\cancel{0} \times 4} = \frac{3}{64} = \lim_{t \to \infty} y(t)$$

4.27



(a) $$\left[R\cdot H_r - Y\cdot H_y\right] K_p \cdot \frac{A}{s+a} + \frac{wB}{s+a} = Y$$ ✓

$w(s) \rightarrow Y(s)$

$R(s) = 0$

$$-\frac{Y\cdot H_y K_p A}{s+a} + \frac{B\,w}{s+a} = Y$$

$$\Rightarrow \quad \frac{Bw}{s+a} = Y\left[1 + \frac{H_y K_p A}{s+a}\right]$$

$$\Rightarrow \quad B\,w = Y\left[s + a + H_y K_p A\right]$$

$$\Rightarrow \quad \frac{Y}{w} = \frac{B}{s+a+H_y K_p A}$$

$R(s) \rightarrow Y(s)$, $w(s) = 0$

$$R \cdot H_r \cdot \frac{K_p \cdot A}{s+a} - H_y \frac{Y K_p A}{s+a} = Y$$

$$R H_r K_p A = Y[s+a+H_y K_p A]$$

$$\frac{Y}{R} = \frac{H_r K_p A}{s+a+H_y K_p A} = T(s)$$

(b) $R(s) = r_0/s$

$w(t) = 0$

$$TF = \frac{Y}{R} = T(s) \Rightarrow Y(s) = T(s) \cdot R(s)$$

$$\lim_{s\to 0} sY(s) = \lim_{t\to 0} y(t)$$

$$\lim_{s\to 0} \cancel{s} \cdot \frac{r_0}{\cancel{s}} \times \boxed{\frac{H_r K_p A}{s+a+H_y K_p A}} = r_0$$

$= 1$

$$\lim_{t\to\infty} y(t) = r_0$$

$$\therefore \boxed{\frac{H_r K_p A}{a + H_y K_p A} = 1}$$ ✓

$$\Rightarrow \quad H_r K_p A = a + H_y K_p A$$ ✓

Open-loop Case:

$$H_y = 0$$

$$\frac{H_r K_p A}{a} = 1 \Rightarrow H_r K_p A = a$$

$$\Rightarrow \boxed{K_p = \frac{a}{H_r A}}$$ ✓

(c) $$[R \cdot H_r - Y \cdot H_y] K_p \cdot \frac{A}{s+a} + \frac{wB}{s+a} = Y$$ ✓

$$\Rightarrow \frac{R \cdot H_r K_p A}{s+a} + \frac{wB}{s+a} = Y \left[\frac{s+a + H_y K_p A}{s+a} \right]$$

$$\Rightarrow \quad \underline{R} H_r K_p A + W B = Y[s + a + H_y K_p A]$$

$$\lim_{s \to 0} s.Y(s) = \cancel{s}\left[\frac{r_0}{\cancel{s}} \times \frac{H_r K_p A}{(s + a + H_y K_p A)} + \frac{w_0}{\cancel{s}} \times \frac{B}{(s + a + H_y K_p A)} \right]$$

$$\lim_{t \to \infty} y(t) = \frac{r_0 H_r K_p A}{a + H_y K_p A} + \frac{w_0 B}{a + H_y K_p A}$$

Open loop Case : Feed forward case:

$$H_y = 0$$

$\propto$ variation $\Rightarrow \dfrac{w_0 B}{a}$

Feed back Case : $\dfrac{w_0 B}{a + H_y K_p A}$

$$\propto K_p \to \infty$$

4.32) $\begin{cases} \dot{y} + 60y = 600 v_a - 1500 w \\ v_a = -\left(k_p e + k_I \int_0^t e\, dt\right) \end{cases}$

$e = r - y$

(a) $sY(s) + 60Y(s) = 600 V_a(s) - 1500 W(s)$

$V_a(s) = -\left[k_p \left(\cancel{R}(s) - \cancel{Y}(s)\right) + \frac{k_I}{s}\left(\cancel{R}(s) - \cancel{Y}(s)\right)\right]$

$sY(s) + 60Y(s) = -600\left[\left(k_p + \frac{k_I}{s}\right)\left(R(s) - Y(s)\right)\right] - 1500W(s)$

$Y(s)\left[s^2 + 60s + 600k_p s + 600k_I\right]$

$= \left(600k_p s + k_I \cdot 600\right) R(s) - 1500\, s\, W(s)$

$\to 0$

$Y(s) = \left[\frac{600 K_p s + k_I \cdot 600}{s^2 + 60s + 600k_p s + 600k_I}\right] R_s - \cancel{\frac{1500\, s\, W(s)}{[\quad]}}$

$$\frac{Y(s)}{W(s)} = \frac{-1500\,s}{(s^2+60s+600\,k_p s+600\,k_I)}$$

Characteristic Eq:

$$s^2+60s+600\,k_p s+600\,k_I$$

$$= [s-(-60+60j)][s-(-60-60j)]$$

4.33) $R(s)$

$$\frac{Y(s)}{R(s)} = \frac{600\,k_p s+600\,k_I}{s^2+60s+600\,k_p s+600\,k_I}$$

(a)

$R(s) = 1/s$

(b)

$R(s) = 1/s^2$

$$E(s) = R-Y$$

$$= R(s) - R(s)\,T(s)$$

✓ $\lim_{s\to 0} sE(s) = \lim_{t\to\infty} e(t)$

(c)

$w(s) = 1/s$

(d)

$w(s) = 1/s^2$

$$\lim_{t \to \infty} y(t) = \lim_{s \to 0} s\, Y(s)$$