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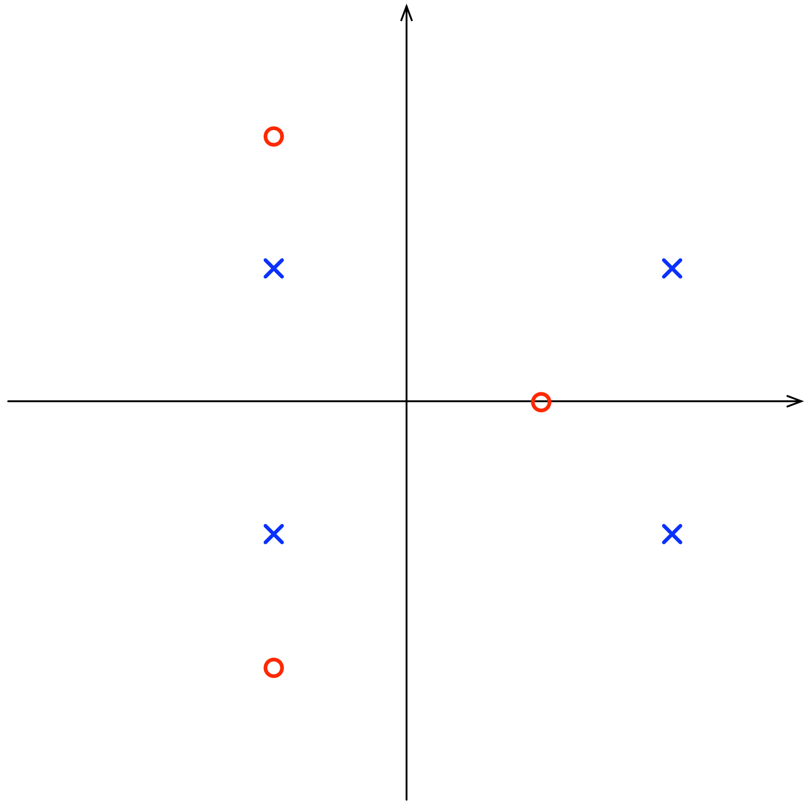

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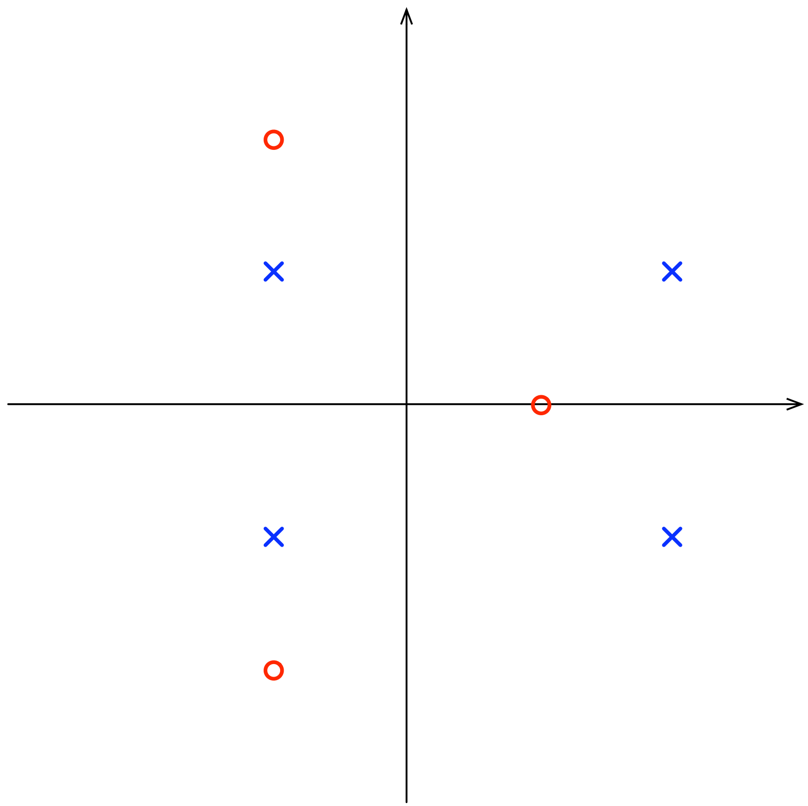

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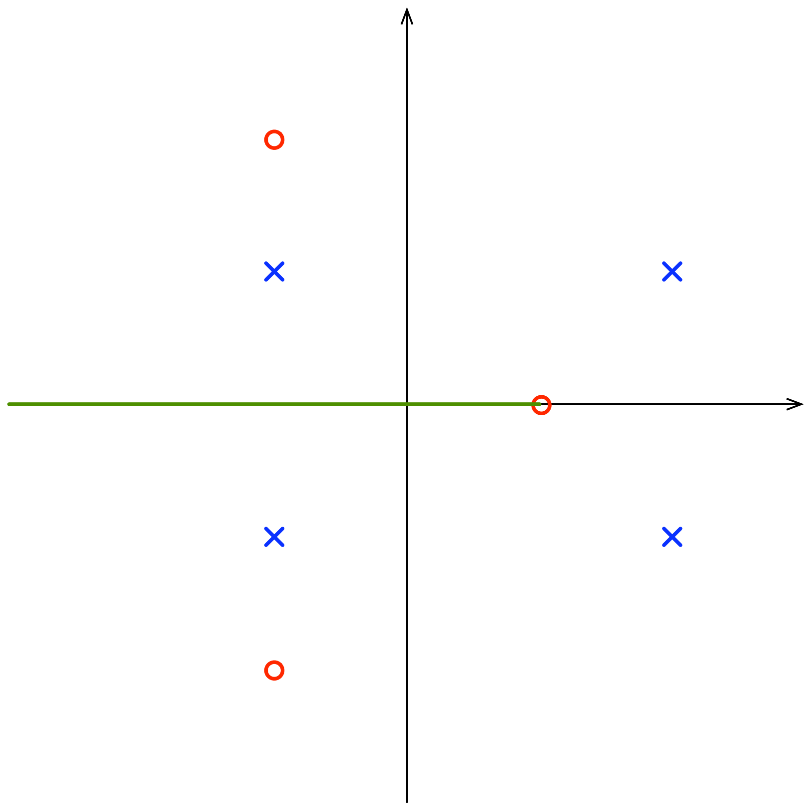

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$$\phi_l = \frac{180^o + 360^o(l-1)}{n-m}, \quad l = 1, 2, \ldots, n-m, \qquad \alpha = \frac{\sum_i p_i - \sum_j z_j}{n-m}$$

$$\phi_1 = \frac{180^o + 360^o(1-1)}{4-3} = 180^o$$

$$\alpha = \frac{(2+j+2-j-1+j-1-j)-(1-1+2j-1-2j)}{4-3} = 2-(-1) = 3$$

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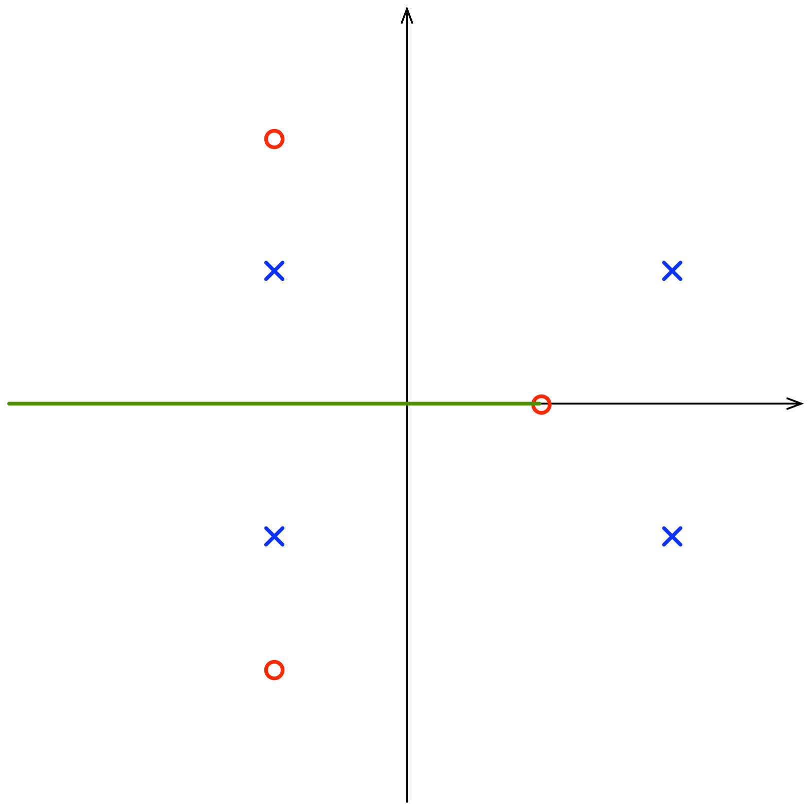

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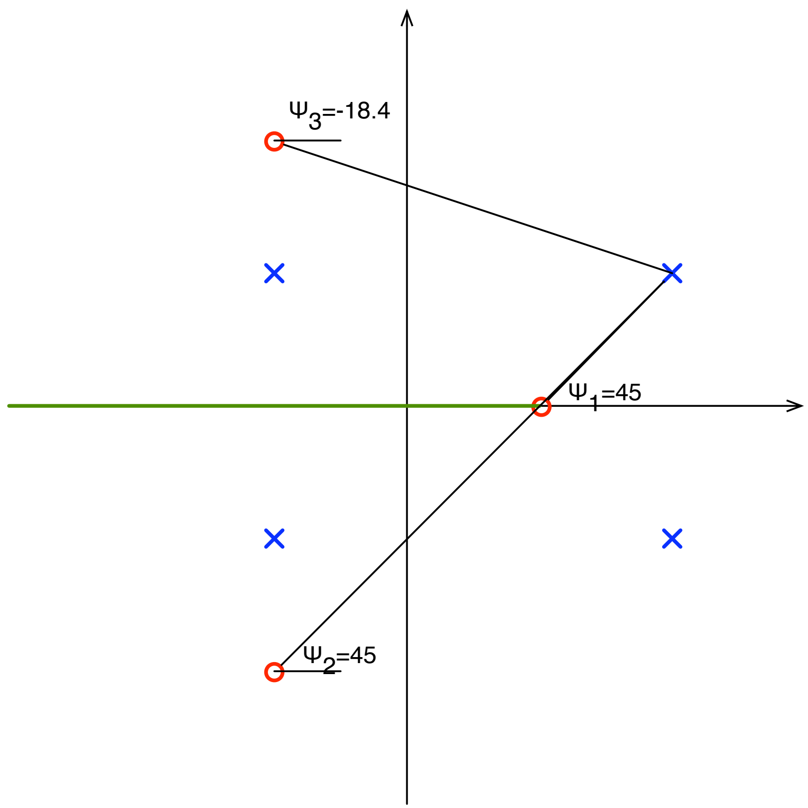

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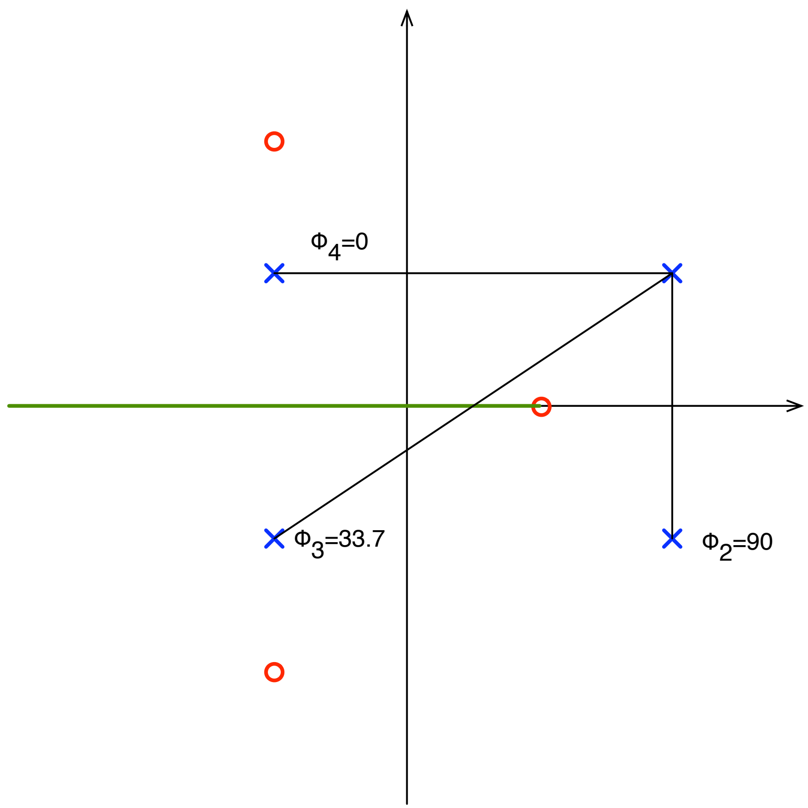

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$$\begin{aligned} \phi_{1,dep} &= (45 + 45 - 18.4) - (90 + 33.7 + 0) - 180 \\ &= 71.6 - 123.7 - 180 = -232.1^o \\ \phi_{2,dep} &= 232.1^0 \text{ (by symmetry)} \end{aligned}$$

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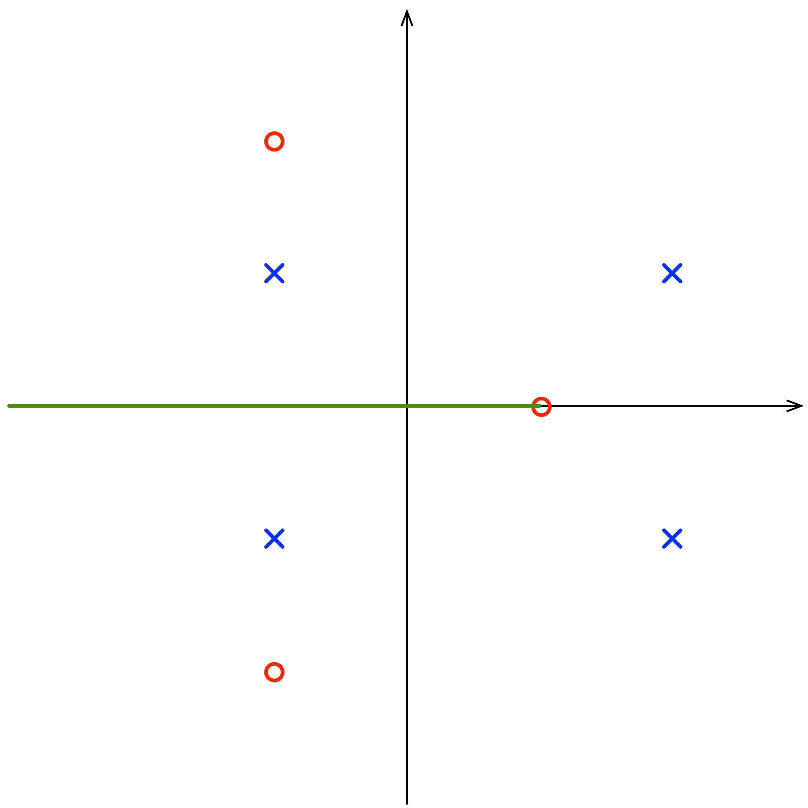

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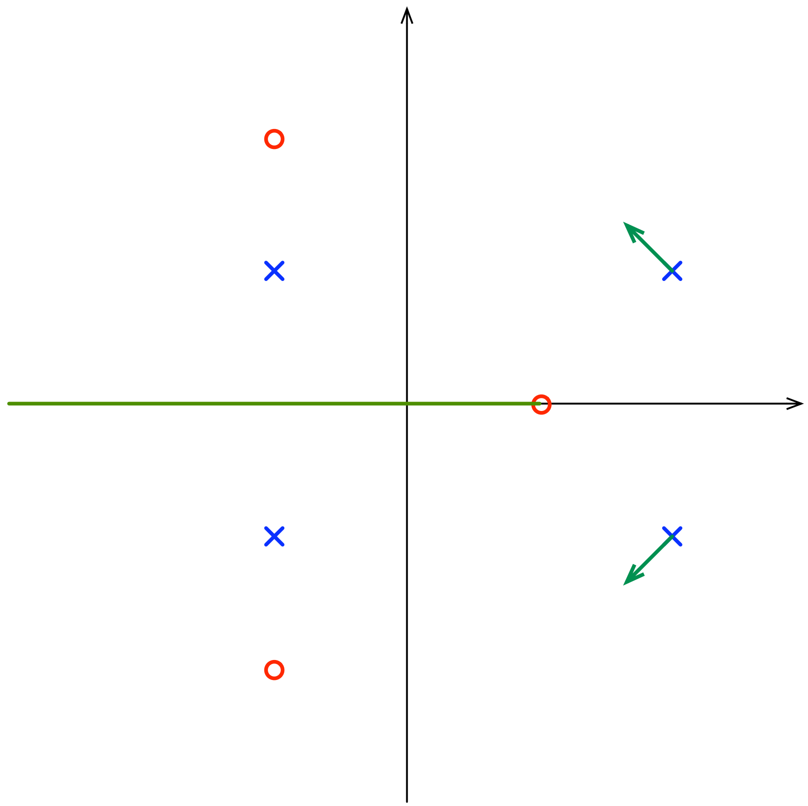

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$$\begin{aligned} \phi_{3,dep} &= -83.2^o \\ \phi_{4,dep} &= 83.2^0 \text{ (by symmetry)} \end{aligned}$$

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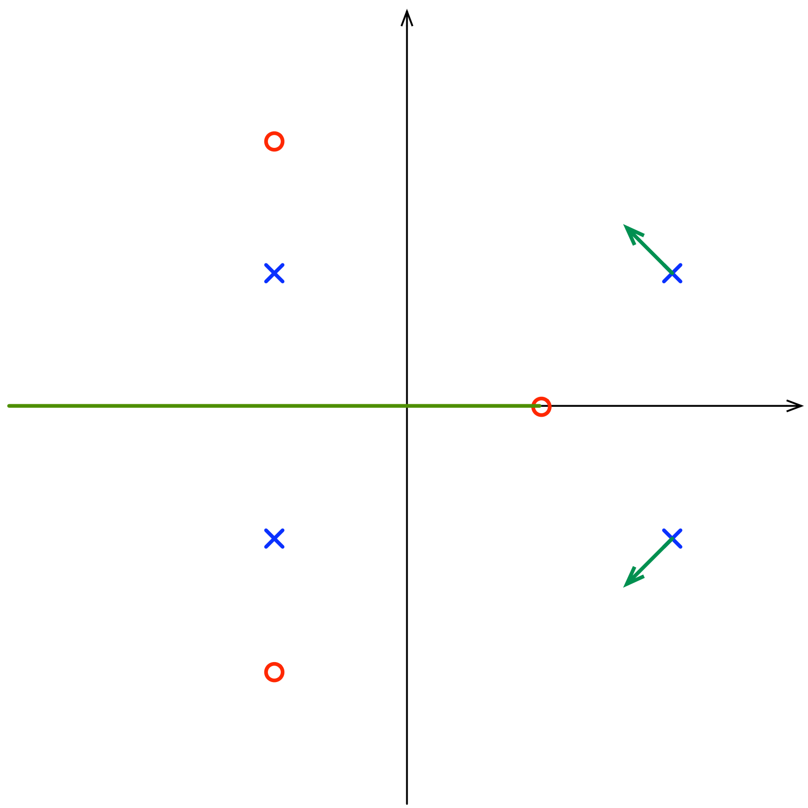

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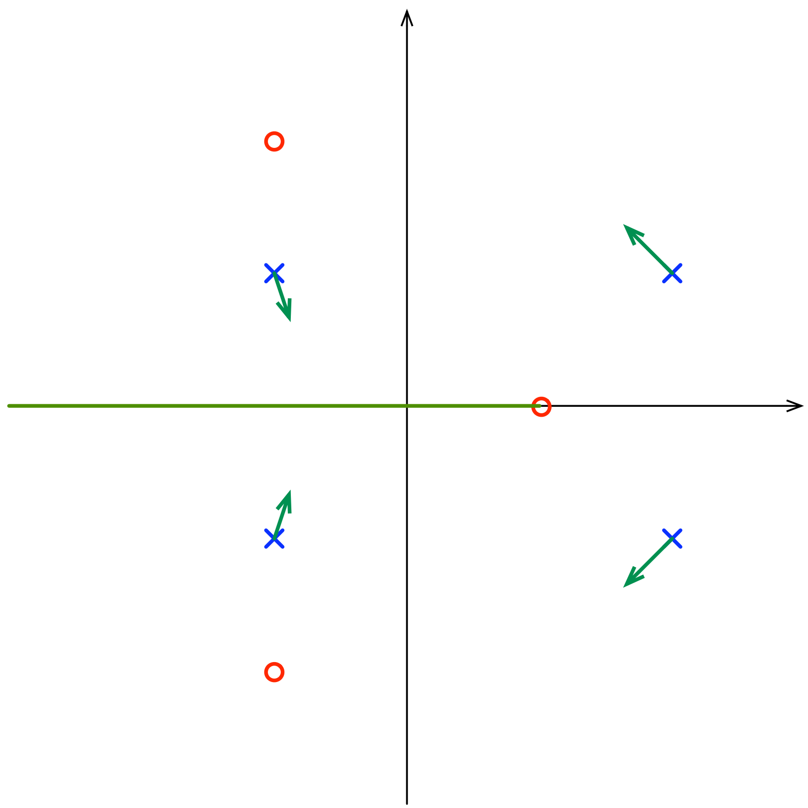

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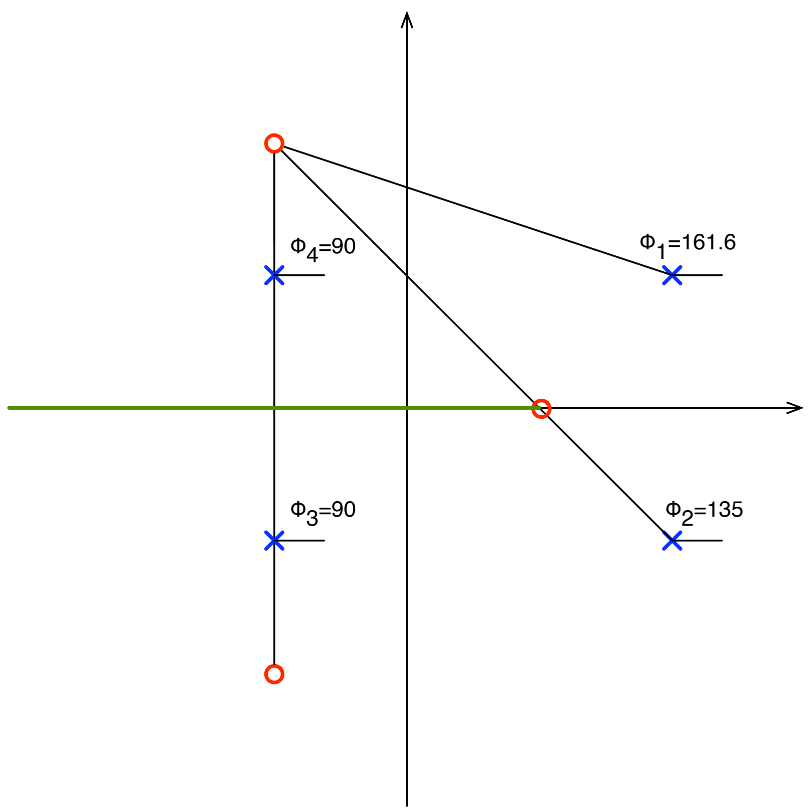

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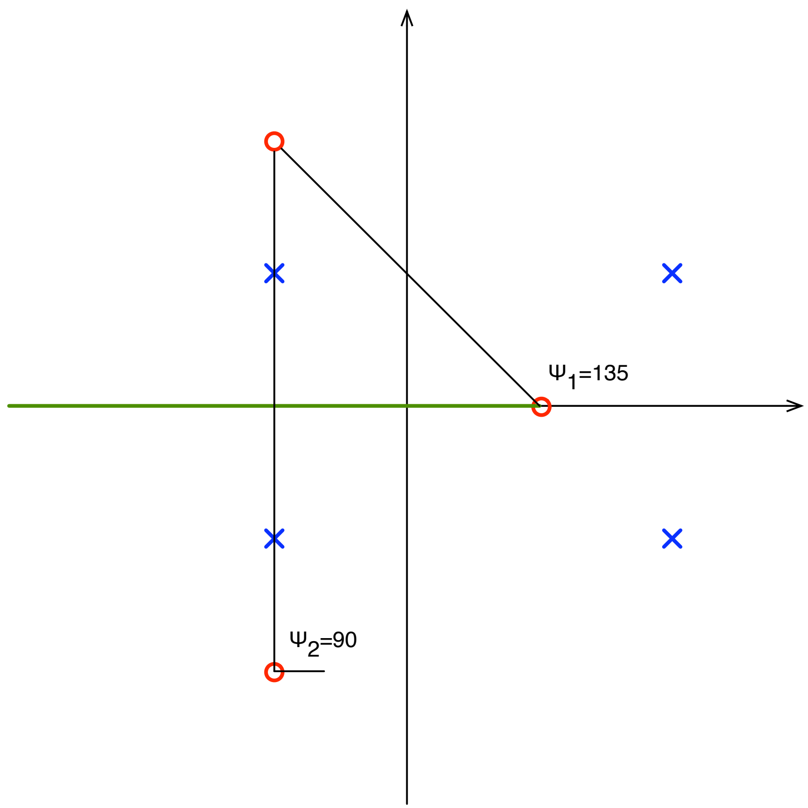

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$$q\psi_{l,arr} = \sum_j \phi_j - \sum_{i \neq l} \psi_i + 180^o + 360^o(l-1).$$

$$\begin{aligned}
\psi_{3,arr} &= (161.6 + 135 + 90 + 90) - (135 + 90) + 180 = 71.6^o \\
\psi_{2,arr} &= -71.6^o \text{ (by symmetry)} \\
\psi_{1,arr} &= 180^o
\end{aligned}$$

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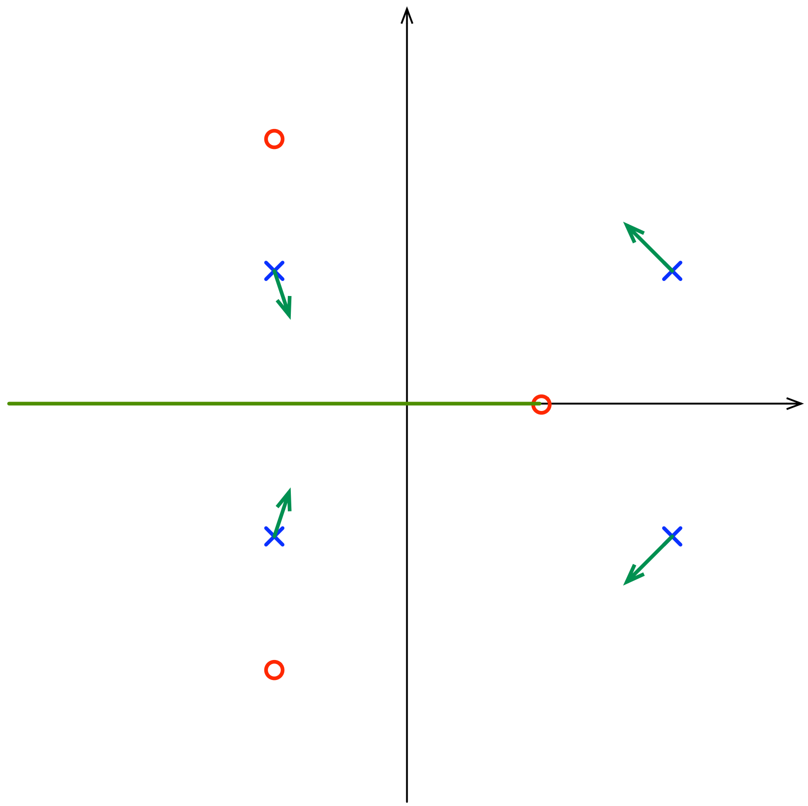

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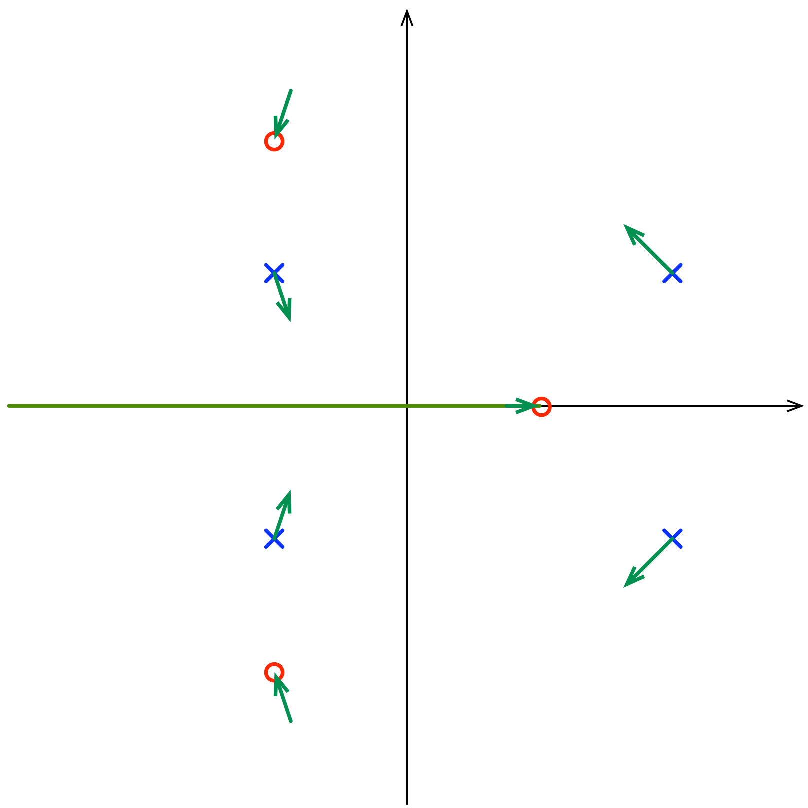

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Rule V: The locus crosses the $j\omega$ axis at points where the Routh criterion shows a transition from roots in the left half-plane to roots in the right half-plane.

Skip

$$H(s) = \frac{s^3 + s^2 + 3s - 5}{s^4 - 2s^3 - s^2 + 2s + 10} = \frac{b(s)}{a(s)}$$

Rule VI: The locus will have multiple roots at points on the locus where:

$$b\frac{da}{ds} - a\frac{db}{ds} = 0$$

and the branches will approach a point of q roots separated by:

$$\frac{180^o + 360^o(l-1)}{q}$$

and will depart at angles with the same separation.

$$\begin{aligned}(s^3 + s^2 + 3s - 5)(4s^3 - 6s^2 - 2s + 2) - (s^4 - 2s^3 - s^2 + 2s + 10)(3s^2 + 2s + 3) &= 0 \\ -40 - 10s - 5s^2 - 36s^3 + 8s^4 + 2s^5 + s^6 &= 0\end{aligned}$$

$$s = -2.1 \pm 3.5j, -0.9, 0.3 \pm j, 2.4$$

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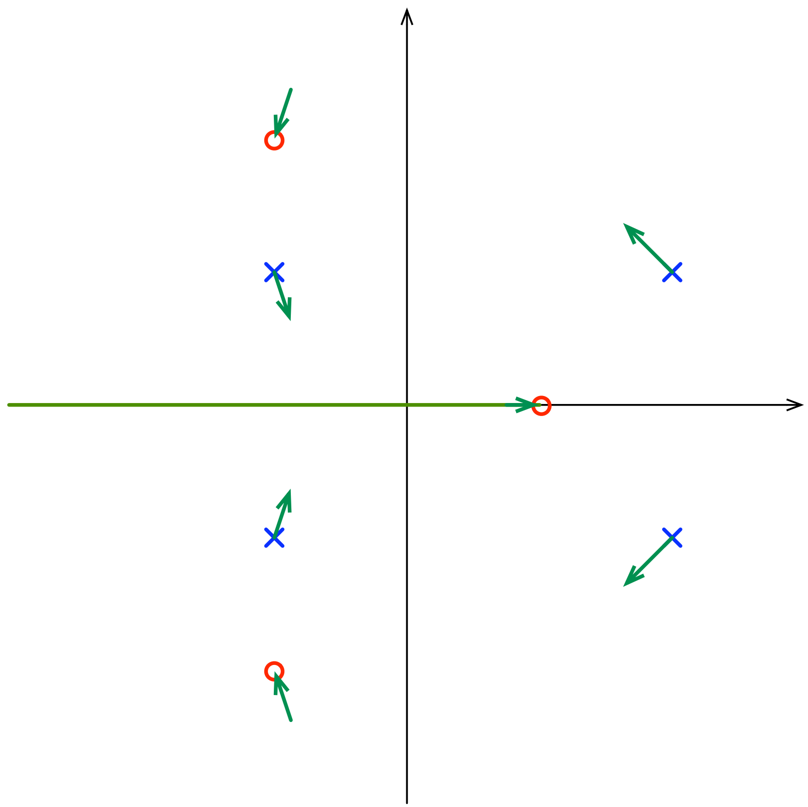

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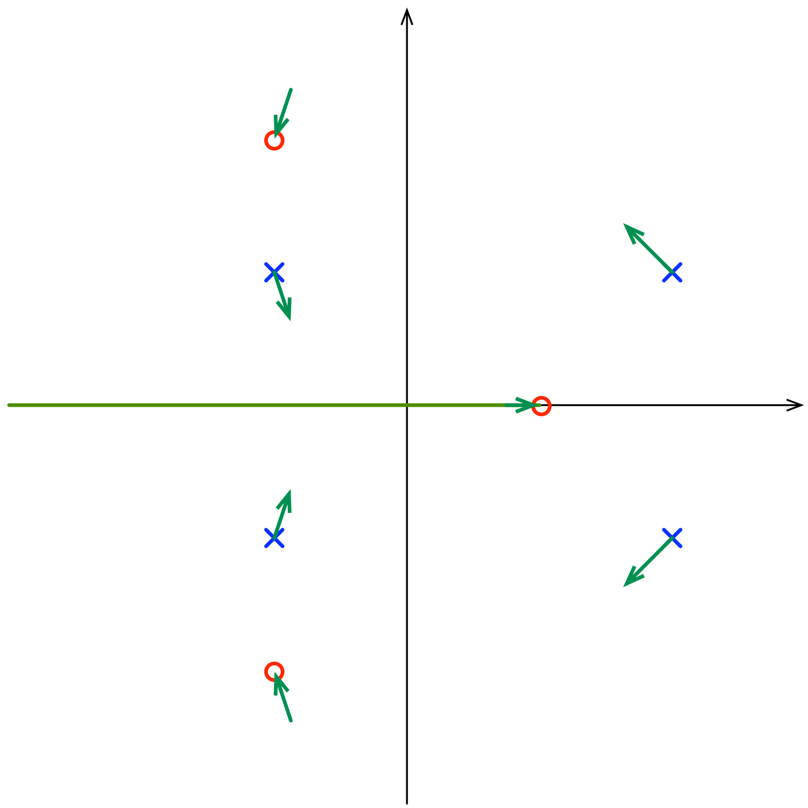

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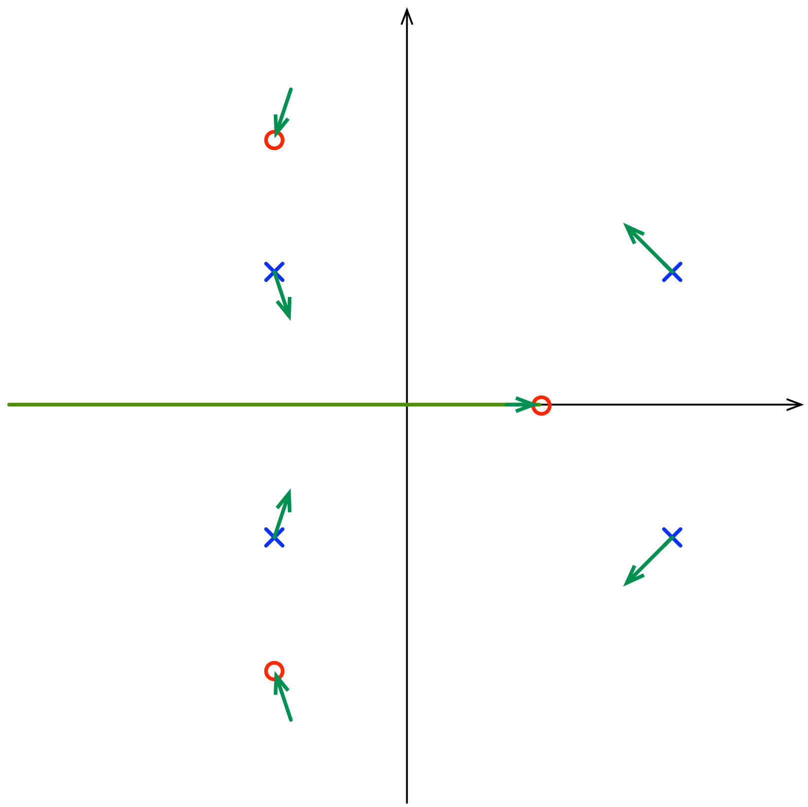

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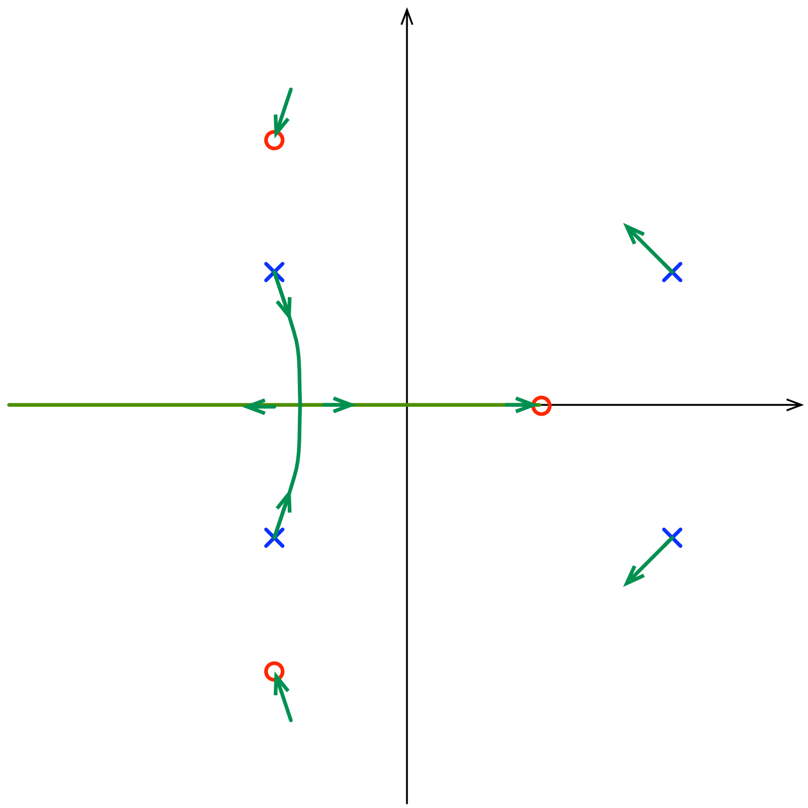

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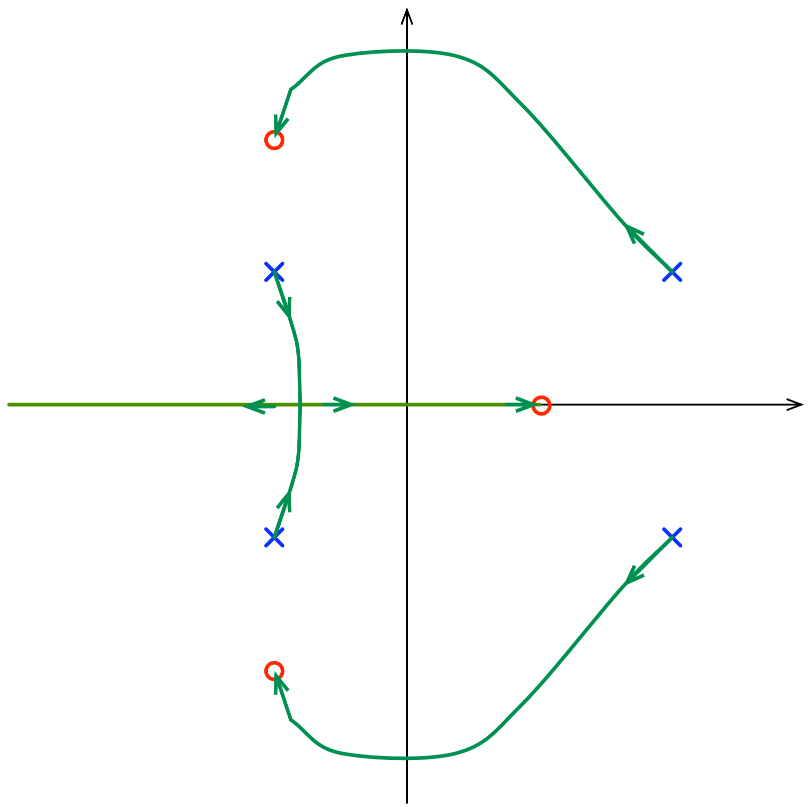
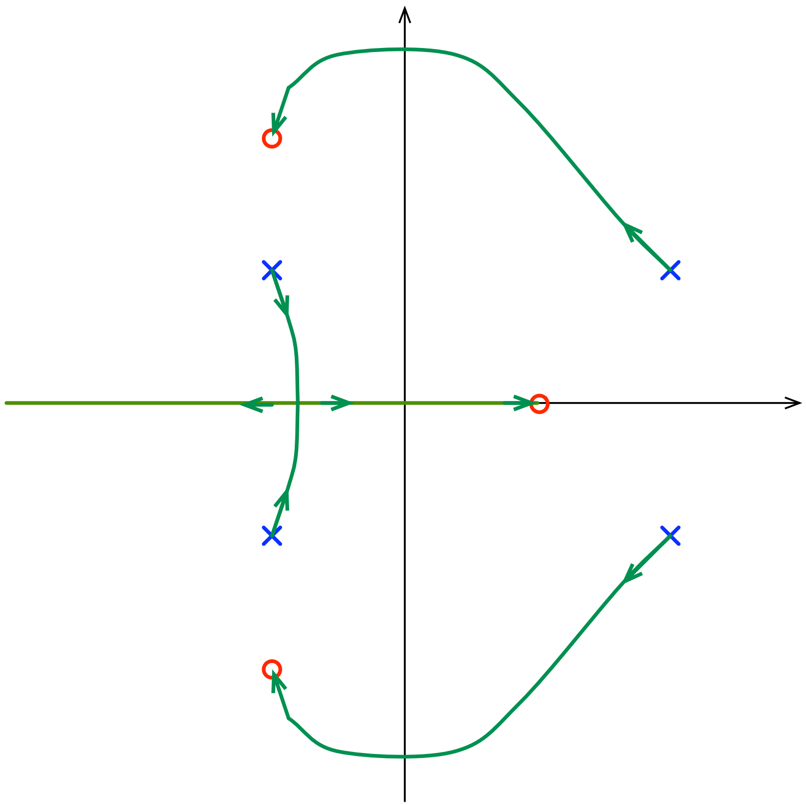
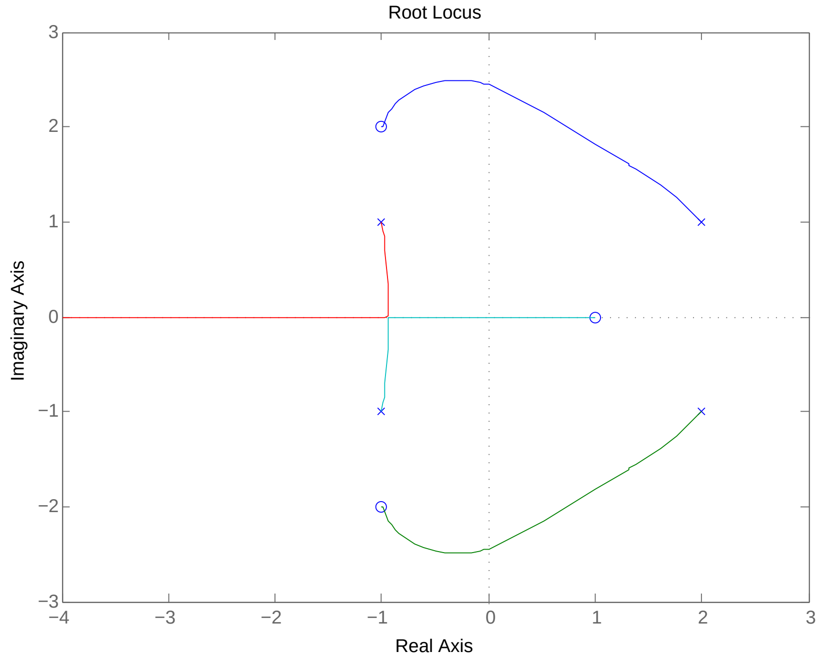


Root locus

$$H(s) = \frac{s^3 + s^2 + 3s - 5}{s^4 - 2s^3 - s^2 + 2s + 10} = \frac{b(s)}{a(s)}$$





Matlab: `rlocus(tf([1 1 3 -5],[1 -2 -1 2 10]))`

`sisotool(tf([1 1 3 -5],[1 -2 -1 2 10]))`

Recall the transfer function of the inverted pendulum with a PID controller:

$$H(s) = \frac{s^3 - 20s}{s^3 + K_D s^2 + (K_P - 20)s + K_I}$$

Nominal values $K_D = 1$, $K_I = 1$, and $K_P = 22$.

Recall the transfer function of the inverted pendulum with a PID controller:

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To plot the root locus with respect to K_D we rewrite $H(s)$ in the form:

$$\begin{aligned} H(s) &= \frac{s^3 - 20s}{s^3 + (K_P - 20)s + K_I + K_D s^2} \\ &= \frac{\frac{s^3 - 20s}{s^3 + (K_P - 20)s + K_I}}{1 + K_D \frac{s^2}{s^3 + (K_P - 20)s + K_I}} \\ &= \frac{\frac{s^3 - 20s}{s^3 + 2s + 1}}{1 + K_D \frac{s^2}{s^3 + 2s + 1}} \end{aligned}$$

Matlab: `rlocus(tf([1 0 0],[1 0 2 1]))` `sisotool(tf([1 0 0],[1 0 2 1]))`

Recall the transfer function of the inverted pendulum with a PID controller:

$$H(s) = \frac{s^3 - 20s}{s^3 + K_D s^2 + (K_P - 20)s + K_I}$$

Nominal values $K_D = 1$, $K_I = 1$, and $K_P = 22$.

To plot the root locus with respect to K_P we rewrite $H(s)$ in the form:

$$\begin{aligned} H(s) &= \frac{s^3 - 20s}{s^3 + K_D s^2 - 20s + K_I + K_P s} \\ &= \frac{\frac{s^3 - 20s}{s^3 + K_D s^2 - 20s + K_I}}{1 + K_P \frac{s}{s^3 + K_D s^2 - 20s + K_I}} \\ &= \frac{\frac{s^3 - 20s}{s^3 + s^2 - 20s + 1}}{1 + K_P \frac{s}{s^3 + s^2 - 20s + 1}} \end{aligned}$$

Matlab: `rlocus(tf([1 0],[1 1 -20 1]))` `sisotool(tf([1 0],[1 1 -20 1]))`

Recall the transfer function of the inverted pendulum with a PID controller:

$$H(s) = \frac{s^3 - 20s}{s^3 + K_D s^2 + (K_P - 20)s + K_I}$$

Nominal values $K_D = 1$, $K_I = 1$, and $K_P = 22$.

To plot the root locus with respect to K_I we rewrite $H(s)$ in the form:

$$\begin{aligned} H(s) &= \frac{s^3 - 20s}{s^3 + K_D s^2 + (K_P - 20)s + K_I} \\ &= \frac{\frac{s^3 - 20s}{s^3 + K_D s^2 + (K_P - 20)s}}{1 + K_I \frac{1}{s^3 + K_D s^2 + (K_P - 20)s}} \\ &= \frac{\frac{s^3 - 20s}{s^3 + s^2 + 2s}}{1 + K_I \frac{1}{s^3 + s^2 + 2s}} \end{aligned}$$

Matlab: `rlocus(tf([1],[1 1 2 0]))` `sisotool(tf([1],[1 1 2 0]))`