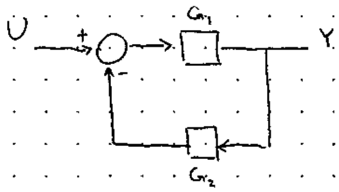
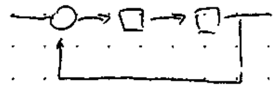
Lecture #3



$$\frac{Y}{U} = \frac{G_1}{1+G_1 G_2}$$



← unity feedback loop.

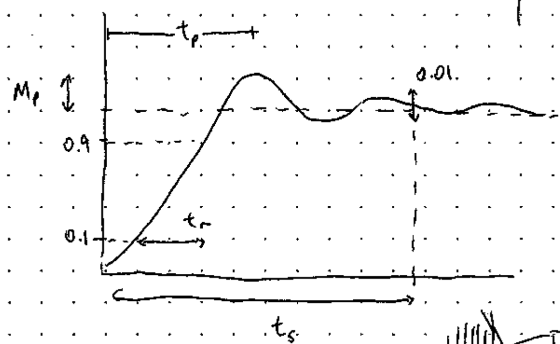
Lecture #4

$$\mathcal{L}^{-1}\left\{\frac{1}{s-p}\right\} = e^{pt} \cdot u(t)$$

2nd order system w/ no zeros:

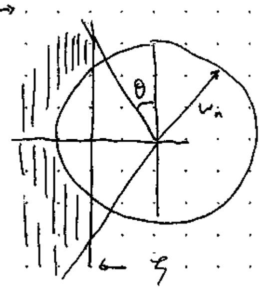
$$H(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} = \frac{\omega_n^2}{(s+r-j\omega_d)(s+r+j\omega_d)}$$

damping ratio · natural freq · damped freq.



$$t_r = \frac{1.8}{\omega_n} \quad t_p = \frac{\pi}{\omega_d} \quad t_s = \frac{4.6}{\zeta\omega_n} = \frac{4.6}{r}$$

$$\ln(M_p) = \frac{-\pi\zeta}{\sqrt{1+\zeta^2}} \Rightarrow \sin\theta = \zeta$$



region of close-looped poles in s-plane

Lecture #5

Neglect poles > 10x larger than other poles.

Stability = all poles left side of complex plane → coefficients of denom > 0

necessary NOT sufficient

sufficient: Routh Criterion

n	1	a_2	a_4	a_6	...
$n-1$	a_1	a_3	a_5	a_7	...
$n-2$	b_1	b_2			
⋮					

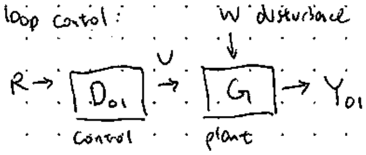
$$b_1 = \frac{-\det\begin{bmatrix} 1 & a_2 \\ a_1 & a_3 \end{bmatrix}}{a_1} \qquad b_2 = \frac{-\det\begin{bmatrix} 1 & a_4 \\ a_1 & a_5 \end{bmatrix}}{a_1}$$

changes w/.

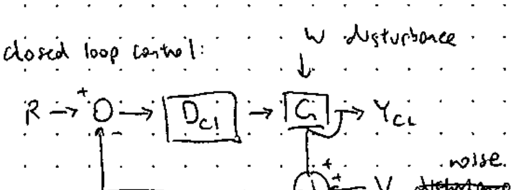
sign changes = # poles on RHS

Lecture #6

open loop control:



closed loop control:



open-loop:

$E_{OL} = [1 - G D_{OL}] R - GW$

$\downarrow$

$R - Y_{OL}$

closed loop:

$E_{CL} = \dfrac{R - GW + G D_{CL} W}{1 + G D_{CL}}$

$\downarrow$

$R - Y_{CL}$

Disturbance rejection = $(R = 0, V = 0)$

sensor ~~rejection~~ noise = $(R = 0, W = 0)$

sensitivity to parameter changes

$\dfrac{dT}{dG}$ where T = transfer func $\dfrac{Y}{R}$

PID controller on unit feedback loop:

proportional: $D = K_p$ $\downarrow$ system response

proportional & integral: $D = K_p + \underbrace{\dfrac{K_I}{s}}$ ↳ eliminates steady state error.

PID: prop. integ. deriv.: $K_p + \dfrac{K_I}{s} + K_D s$

$E = R - Y$: if only P:

$\rightarrow E = R - Y = \dfrac{R}{1 + DG} = \dfrac{R}{1 + K_p G} \rightarrow \dfrac{E}{R} = \dfrac{1}{1 + K_p G} = T(s)$

$\lim_{t \to \infty} e(t) = \lim_{s \to 0} sE(s) = \lim_{s \to 0} s \cdot \underbrace{R \cdot T(s)}_{E}$ ← REMEMBER TO TEST FOR STABILITY FIRST!!

LECTURE 7-8 skipped

LECTURE #9

RLOCUS: $\{s\} \mid 1 + KL(s) = 0$ as $K \to [0 \to +\infty]$

assume denominator $D(s)$ numerator $N(s)$ transfer func. (n poles m zeros)

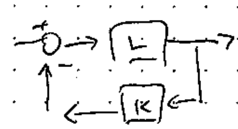
RULE #1: n branches start @ poles. m end at zeros

RULE #2: loci on $\mathbb{R}$ axis left of odd # of poles, zeros.

RULE #3: asymptotes @ $\angle$ angles $\phi_\ell = \dfrac{180° + 360(\ell - 1)}{n - m}$ $\ell = 1, 2, 3 \dots n-m$

radiating from α where $\alpha = \dfrac{\sum p_i - \sum z_i}{n - m}$

$L(s)$ is plant



$\phi = \angle$ from poles $\psi = \angle$ from zeros.

RULE #4: Pole Departure angle for pole w/ multiplicity q =

$$q\phi_{\ell,dep} = \sum_i \psi_i - \sum_{j\neq\ell} \phi_j - 180 - 360(\ell-1) \quad \ell = 1,2 \ldots q$$

for zeros:

$$q\psi_{\ell,arr} = \sum_j \phi_j - \sum_{i\neq\ell} \psi_i + 180 + 360(\ell-1) \quad \ell = 1,2 \ldots q$$

RULE #5: Locus crosses $j\omega$ axis where Routh shows roots from left → right half of plane.

transition.

aka a 0 in 1^{st} column of Routh table.

$K=0$ ⇒ START @ poles.

if pole @ origin, also on Im axis

for other $K \neq 0$, solve $1+K \cdot L(s) = 0$

assuming that $s = j\omega$.

RULE #6:

Locus has $\underbrace{\text{multiple roots}}_{\text{multiple branches meet}}$ @ $b\frac{da}{ds} - a\frac{db}{ds} = 0$

$$\frac{180+360(\ell-1)}{q} \quad \ell = 1 \ldots q$$

b is numerator

a is denominator

branches $\boxed{\text{approach}}$ a point of q roots @ angles separated by

depart @ angles with the same separation

?,? where to find q though?

LECTURE #12:

Assume multiple K gains:

$$H(s) = \frac{n(s)}{\underbrace{\alpha(s) + K_\beta \beta(s)}_{a(s)} + K_\gamma \underbrace{\gamma(s)}_{b(s)}} = \frac{n(s)/a(s)}{1 + K_\gamma \underbrace{\boxed{\frac{\gamma(s)}{a(s)}}}_{L(s)}}$$

Compute K using a single pole (any). All poles move together as K ↑

$s^* \Rightarrow 1 + K(L(s^*)) = 0$

DOOR EXAMPLE

$$K = -\frac{d(s^*)}{n(s^*)}$$

CONFUSING

AB

ANALYSIS

Lec 13.

$A\sin(\omega t) \rightarrow \boxed{} \rightarrow A'\sin(\omega t+\Phi)$

ω influences A'/A and ω influences Φ } Bode plot.

Understand how 0's and poles change the $\underbrace{\text{Bode plot}}$.

$\downarrow$

$|G|$ magnitude / amplitude (dB) $= 20\log_{10}|G| = G_{DB}$

$\angle G$ phase

$G(s)$: $s=\sigma+j\omega \rightarrow \sigma=0 \rightarrow G(\omega)$

write $G(\omega)$ as: $K_0 \cdot \dfrac{(j\omega\tau_1+1)(j\omega\tau_2+1)\ldots}{(j\omega\tau_a+1)(j\omega\tau_b+1)\ldots} \rightarrow |G|_{dB} = 20\log_{10}K_0 + 20\log_{10}(j\omega\tau_1+1) + 20\log_{10}(j\omega\tau_2+1) - 20\log_{10}(j\omega\tau_a+1) - 20\log_{10}(j\omega\tau_b+1)\ldots$

3 types of terms:

1) $K_0(j\omega)^n \quad n\in\mathbb{Z}$

2) $(j\omega\tau+1)^{\pm1}$ ← 1 = zero, −1 = pole.

3) $\left[\left(\frac{j\omega}{\omega_n}\right)^2 + 2\zeta\frac{j\omega}{\omega_n}+1\right]^{\pm1}$ ← complex conj poles / zeros.

1) $\log_{10}|1| = \underbrace{20\log_{10}|K_0|}_{b} + \underbrace{20n}_{m}\underbrace{\log_{10}\omega}_{x}$ ← mag

phase → $\angle j^n = 0$.

n	Φ
$n=2$	$\Phi=180$
$n=1$	90
$n=0$	0
$n=-1$	−90
$n=-2$	−180

2) $(j\omega\tau+1)^1$ ← zero.

2 cases:

$\omega\tau \ll 1 \rightarrow 1$

$\omega\tau \gg 1 \rightarrow j\omega\tau$

$\angle j\omega\tau+1 \approx 0$

$\angle j\omega\tau+1 \approx 90°$

$\omega = \frac{1}{\tau}$ = break point

↓

plug in

$j\omega\tau+1 = j+1$.

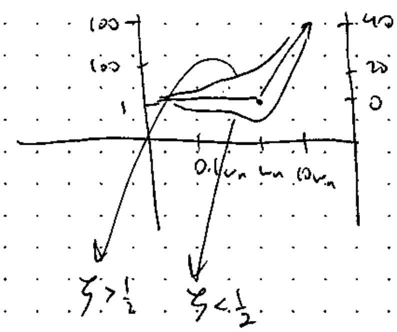
$|j+1| = 1.4 \rightarrow 3\text{dB}$

$\angle j+1 = \pi/4$

$\angle \frac{j\omega+1}{j+1} = \nearrow$

3) $\frac{\omega}{\omega_n} \ll 1$ #3 → 1 phase = 0

$\frac{\omega}{\omega_n} \gg 1$ #3 → $\left(\frac{j\omega}{\omega_n}\right)^2$ phase = 180



$\omega = \omega_n$ phase = 90°

Lecture 14:

when $\zeta = 1$, critical damp, two poles at exact same point on real axis

Non minimal phase zeros:

like if $s+2$ instead of $s-2$

$$H_1(s) = \frac{s+2}{s+1} \qquad H_2(s) = \frac{s-2}{s+1}$$

No changes for mag plot

phase = $-90°$

$$\frac{1+\frac{j\omega}{2}}{j\omega+1} \qquad \frac{-\left(\frac{j\omega}{2}\right)+1}{j\omega+1}$$

$\tau > 0$ $\qquad$ $\tau < 0$

Lecture 15:

routh → test stability
rootlocus → test stability

$$\left\{\begin{array}{l} |KL(j\omega)| = 1 \\ \angle KL(j\omega) = 180° \end{array}\right] \rightarrow \text{stability MARGINAL}$$

K does not affect phase, K raises the graph of bode magnitude

$\begin{cases} \text{stable} & \text{gain margin} > 0 \\ \text{unstable} & \text{gain margin} < 0 \end{cases}$

gain margin = amt. you raise the dB level

phase margin (if top holds) bottom does not hold.

stable : phase margin > 0
unstable phase margin < 0.

40-60 $\boxed{\text{models stuff}}$

$\underbrace{}_{50}$ increasing gain