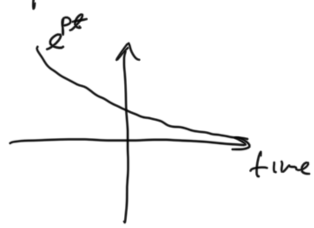
ECE 141

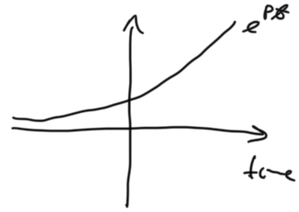
Lecture 4

$H(s) = \frac{1}{s-p}$ $\qquad h(t) = \mathcal{L}^{-1}\{H(s)\} = e^{pt}u(t)$

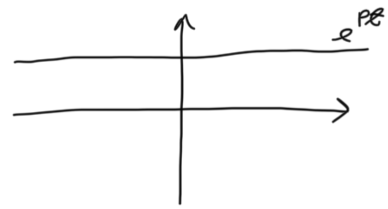
if $p<0$




if $p>0$




$p=0$




$H(s) = c\,\frac{1}{s-p}$ $\qquad h(t) = \mathcal{L}^{-1}\{H(s)\} = c\,e^{pt}u(t)$

second order systems with no zeros

$$H(s) = \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2} = \frac{\omega_n^2}{(s+\sigma-j\omega_d)(s+\sigma+j\omega_d)}$$

zeta

$\zeta$ damping ratio

$\omega_n$ natural frequency

$\omega_d$ damped frequency

$$(s+\sigma-j\omega_d)(s+\sigma+j\omega_d) = s^2 + s\sigma + \cancel{sj\omega_d} + \sigma s + \sigma^2 + \cancel{\sigma j\omega_d} - \cancel{j\omega_d s} - \cancel{j\omega_d\sigma}$$

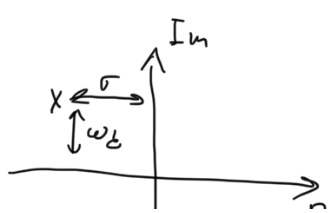
$$= s^2 + 2\sigma s + \sigma^2 + \omega_d^2$$

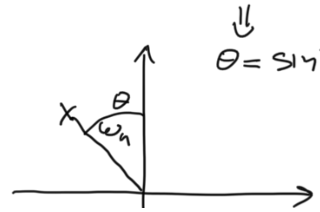
$$\begin{cases}\omega_n^2 = \sigma^2 + \omega_d^2\\ \sigma = \zeta\omega_n\end{cases}$$

$\zeta = \sin\theta$

$\Downarrow$

$\theta = \sin^{-1}\zeta$



Ke

$$\omega_n^2 = \sigma^2 + \omega_d^2 \Rightarrow \omega_d^2 = \omega_n^2 - \sigma^2 \Rightarrow \omega_d^2 = \omega_n^2 - \zeta^2\omega_n^2 = \omega_n^2(1-\zeta^2)$$
$$\Rightarrow \omega_d = \omega_n\sqrt{1-\zeta^2}$$

$\sigma = 0$ purely complex pole $\Rightarrow \theta = 0 \Rightarrow \zeta = \sin 0 = 0$

$\omega_d = 0$ purely real pole $\Rightarrow \theta = \frac{\pi}{2} \Rightarrow \zeta = \sin\frac{\pi}{2} = 1$

$$\mathcal{L}^{-1}\{H(s)\} = h(t) = \underline{e^{-\sigma t}}\ \underline{\sin(\omega_d t)}\ \underline{\frac{\omega_n}{\sqrt{1-\zeta^2}}}\ u(t)$$

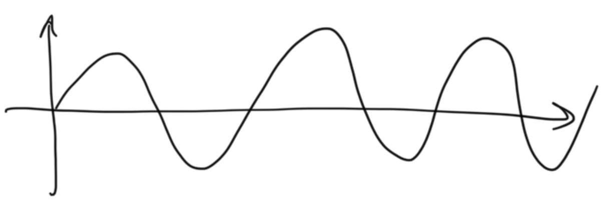
for $\sigma = 0$ $h(t) = \sin(\omega_d t)\frac{\omega_n}{\sqrt{1-\zeta^2}}u(t)$

$= \sin(\omega_n t)\, u(t)\, \omega_n$

$\sigma = 0 \Rightarrow \zeta = 0$

$\omega_d = \omega_n$

$\sqrt{1-\zeta^2} = 1$



$\underline{\underline{\zeta = 1}}$ no oscilations

fully damped

$$H(s) = \frac{c_1}{s-p_1} + \frac{c_2}{s-p_2} \xrightarrow{\mathcal{L}^{-1}} c_1 e^{p_1 t}u(t) + c_2 e^{p_2 t}u(t)$$

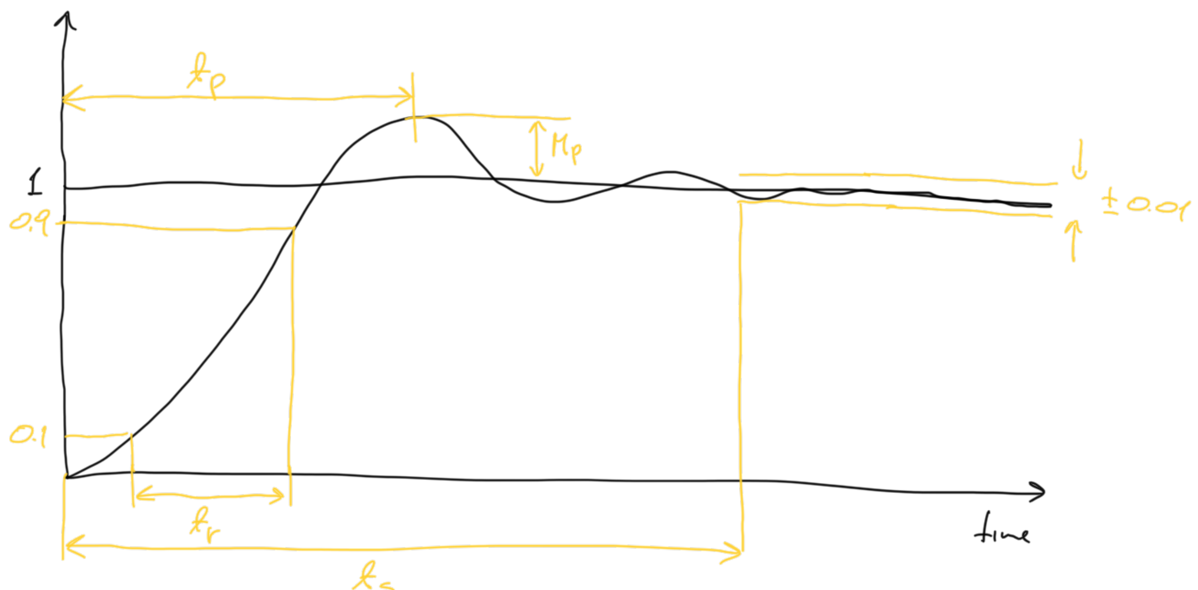
$$\frac{s-p_2}{(s-p_1)(s-p_2)} \qquad \mathcal{L}^{-1}\left\{\frac{1}{(s-p_1)(s-p_2)}\right\}$$

$$\frac{1}{s-p_1}$$

$$= \left(C_1 e^{p_1 t} + C_2 e^{p_2 t}\right) u(t)$$

$C_2 = 0$

Time domain specifications




Rise time ($t_r$): time it takes the system to reach the vicinity of its new set point.

settling time ($t_s$): time it takes the system transients to decay

overshoot ($M_p$): maximum amount the system overshoots its final value, given in percentage

peak time ($t_p$): the time it takes the system to reach its maximum overshoot point

$$t_r \cong \frac{1.8}{\omega_n} \qquad t_p = \frac{\pi}{\omega_d} \qquad t_s = \frac{4.6}{\zeta\omega_n} = \frac{4.6}{\sigma} \qquad M_p = e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}}$$

$$y(t) = 1 - e^{-\sigma t}\left(\cos(\omega_d t) + \frac{\sigma}{\omega_d}\sin(\omega_d t)\right)$$

$$\frac{dy}{dt} = 0 \qquad \frac{dy}{dt} = \sigma e^{-\sigma t}\left(\cos(\omega_d t) + \frac{\sigma}{\omega_d}\sin(\omega_d t)\right)$$

$$- e^{-\sigma t}\left(-\omega_d \sin(\omega_d t) + \sigma \cos(\omega_d t)\right)$$

$$= e^{-\sigma t}\left(\frac{\sigma^2}{\omega_d}\sin(\omega_d t) + \omega_d \sin(\omega_d t)\right) = 0$$

$$\Longrightarrow \left(\frac{\sigma^2}{\omega_d} + \omega_d\right)\sin(\omega_d t) = 0$$

$$\Longrightarrow \sin(\omega_d t) = 0 \Longrightarrow \omega_d t = \pi \Longrightarrow t_p = \frac{\pi}{\omega_d}$$

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$$y(t_p) = 1 + M_p = 1 - e^{-\sigma\pi/\omega_d}\left(\cos(\pi) + \frac{\sigma}{\omega_d}\sin(\pi)\right)$$

$$= 1 + e^{-\frac{\sigma\pi}{\omega_d}}$$

$$\Longrightarrow M_p = e^{-\frac{\sigma\pi}{\omega_d}}$$

$$\sigma = \zeta\omega_n$$

$$\omega_d = \omega_n\sqrt{1-\zeta^2}$$

$$\Longrightarrow M_p = e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}}$$

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$$e^{-\zeta\omega_n t_s} = 0.01$$

$$\zeta\omega_n t_s = 4.6$$

$$t_s = \frac{4.6}{\zeta\omega_n} = \frac{4.6}{\sigma}$$

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$$\omega_n \geq \frac{1.8}{t_r} \qquad \zeta \geq \zeta(M_p) \qquad \sigma \geq \frac{4.6}{t_s}$$

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$$t_s \leq t_s^* \qquad \frac{4.6}{\sigma} \leq t_s^* \Longrightarrow \frac{4.6}{t_s^*} \leq \sigma$$

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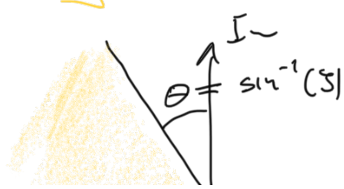


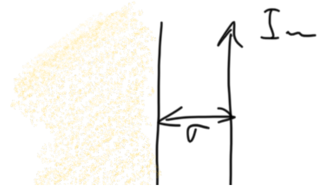

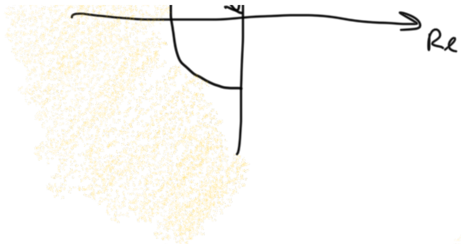
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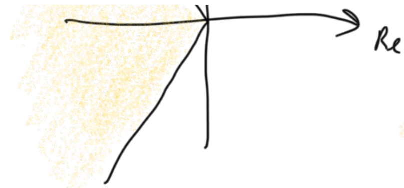
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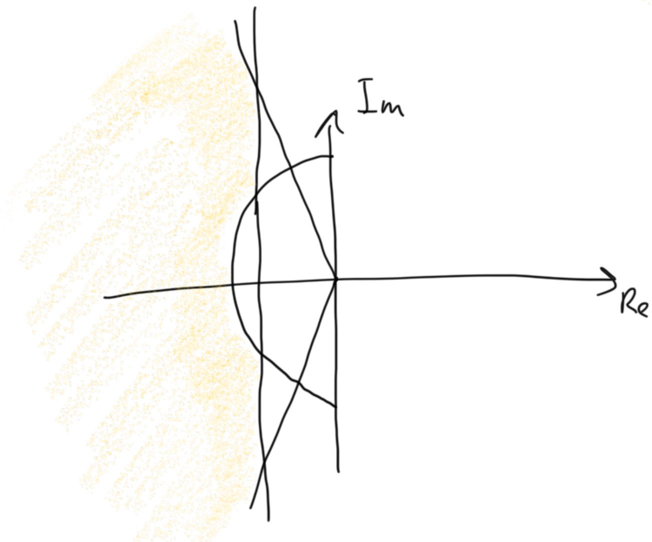
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