Problem 1:

1). $\dot{x}_1 = u - kx_1$ $\quad$ $\dot{x}_2 = kx_1 - kx_2$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} u - kx_1 \\ kx_1 - kx_2 \end{bmatrix}$$

2) $sX_1(s) = \frac{u}{s} - kX_1(s)$

$$(s+k)X_1(s) = \frac{u}{s} \Rightarrow \frac{X_1(s)}{u} = \frac{1}{s(s+k)}$$

$$s(X_2(s)) = kX_1(s) - kX_2(s)$$

$$(s+k)X_2(s) = kX_1(s) \rightarrow \frac{X_2(s)}{u} = \frac{k}{s+k} \cdot \frac{1}{s(s+k)}$$

$$= \frac{k}{s(s+k)^2}$$

$$T_1 = \frac{X_1(s)}{u} = \frac{1}{s(s+k)}$$

$sT_1 = \frac{1}{s+k}$ which is $\boxed{\text{not oscillating}}$ $\frac{1}{s+k} \leftrightarrow e^{-k \cdot t}$

$$T_2 = \frac{X_2(s)}{u} = \frac{k}{s(s+k)^2}$$

$sT_2 = \frac{k}{(s+k)^2} = \frac{k}{s^2+2sk+k^2}$ $\quad$ $\omega_n = k$ & $\zeta = 1 \Rightarrow M_p = 0$

so $\boxed{\text{no oscillation}}$

$\dot{x}_1 = u - k_1 x_1 \qquad \dot{x}_2 = k_1 x_1 - k_2 x_2$

$s X_1(s) = \frac{u}{s} - k_1 X_1(s) \Rightarrow (s+k_1) X_1(s) = \frac{u}{s} \Rightarrow X_1(s) = \frac{u}{s(s+k_1)}$

$s X_2(s) = k_1 X_1(s) - k_2 X_2(s) \Rightarrow X_2(s) = \frac{k_1 X_1(s)}{s+k_2}$

$= \frac{k_1}{s+k_2} \cdot \frac{u}{s(s+k_1)}$

X_1 will still ~~no~~ be oscillatin

$\frac{X_2}{u} = \frac{k_1}{s(s+k_1)(s+k_2)} \Rightarrow s T_2 = \frac{k_1}{s^2 + (k_1+k_2)s + k_1 k_2}$

$\omega_n = \sqrt{k_1 k_2} \quad \zeta = \frac{k_1 + k_2}{2 \cdot \sqrt{k_1 k_2}}$

is $k_1 \neq k_2$, $\zeta \neq 1$ so $\cancel{X_2}$ will oscillate.

3) $t_r < 1.8 \Rightarrow \frac{1.8}{\omega_n} < 1.8 \Rightarrow \omega_n > 1 \qquad \zeta = \frac{\sigma}{\omega_n} = \frac{1}{2}$

$t_s < 9.2 \quad \frac{4.6}{\sigma} < 9.2 \Rightarrow \sigma > 0.5$

$2k = 2\zeta\omega_n \quad k^2 = \omega_n^2 \qquad \omega_n > 1$

$k = \omega_n \qquad \boxed{k > 1}$

$\zeta = 1$

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4.

$$\dot{y} = -\lambda^2 y + x_2$$

$$sY(s) = -\lambda^2 Y(s) + \cancel{\ } X_2(s)$$

$$Y(s) = \frac{1}{s+\lambda^2} X_2(s) = \frac{1}{s+\lambda^2} \cdot \frac{ku}{s(s+k)^2}$$

$$\cancel{T_y = \frac{Y(s)}{u} = \frac{k}{s(s+\lambda^2)(s+k)^2}}$$

$$\cancel{u = \lambda k} \Rightarrow \cancel{Y(s)} =$$

$$T_Y = \frac{Y(s)}{u} = \frac{k}{s(s+\lambda^2)(s+k)^2}$$

poles stable? yes b/c roots all $Re(s) < 0$
$s = -\lambda^2, -k$

poles are stable $\Rightarrow$ proceed with FVT

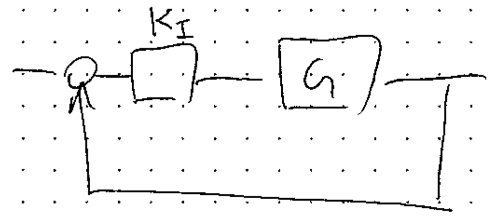
$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} Y(s) = \frac{1}{s+\lambda^2} \cdot \frac{k \cdot \lambda k}{s(s+k)^2}$$

No. will not converge
$\lim \to \infty$

5. we want to regulate SSE: so integral controller.

OS transfer func Y. $T_y = \dfrac{k}{s(s+\lambda^2)(s+k)^2}$

$$T_y = \frac{k}{s(s+\lambda^2)(s+k)^2}$$



$$\frac{K_I G}{1+K_I G} \Rightarrow \frac{\frac{k_I}{s}\cdot\frac{k}{s(s+\lambda^2)(s+k)^2}}{1+\frac{k_I k}{s\cdot s(s+\lambda^2)(s+k)^2}}$$

$$= \frac{k_I\cdot k}{s^2(s+\lambda^2)(s+k)^2+k_I k}$$

$$= \frac{k_I}{s^2(s+1)(s+1)^2+k_I}$$

$$= \frac{k_I}{s^5+3s^4+3s^3+s^2+k_I}$$

Stable poles?

$$\begin{array}{c|ccc} s^5 & 1 & 3 & 0 \\ s^4 & 3 & 1 & k_I \\ s^3 & 8/3 & -k_I/3 & \\ s^2 & \frac{3k_I+8}{8} & k_I & \\ s^1 & \frac{k_I^2+24k_I}{3k_I+8} & & \\ s^0 & k_I & & \end{array}$$

If $k_I > 0$

$\rightarrow$ no change in sign ✓

stable

$\rightarrow$ proceed w/ FVT

$$\lim_{t\to\infty} e(t) = \lim_{s\to 0} s E(s)$$

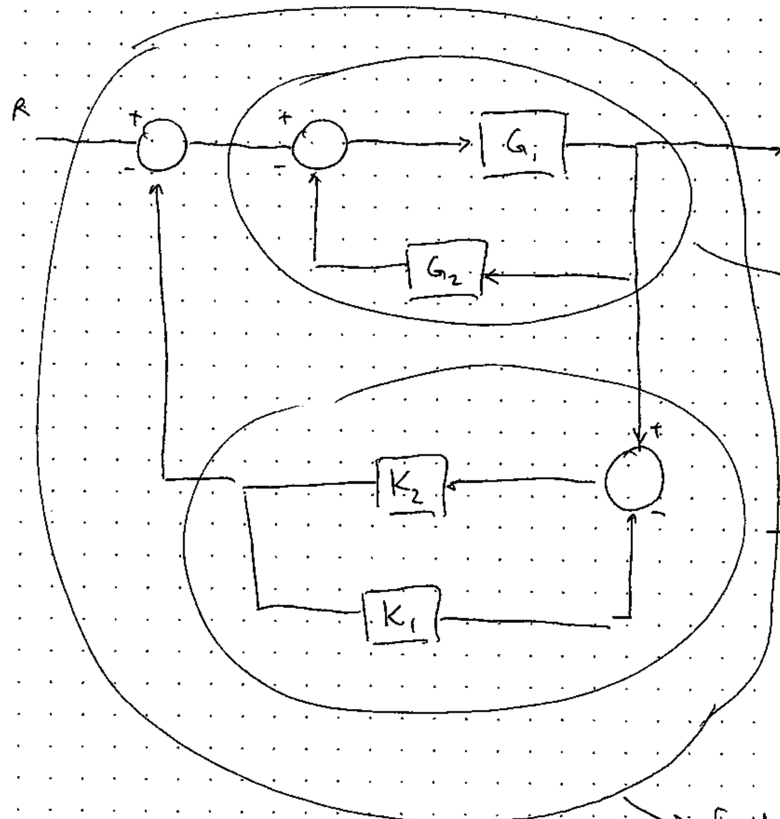
$$= \lim_{s\to 0} s(R)\,\frac{s^5+3s^4+3s^3+s^2}{s^5+3s^4+3s^3+s^2+k_I}$$

$R = \frac{1}{s} \Rightarrow$

$$\lim_{s \to 0} \frac{s^5+3s^4+3s^3+s^2}{s^5+3s^4+3s^3+s^2+k_I} = 0 \checkmark$$

YAY!

Problem 2



feedback loop

$$G = \frac{G_1}{1+G_1G_2}$$

feedback loop :

$$K = \frac{K_2}{1+K_1K_2}$$

feedback loop

$$X = \frac{G}{1+GK}$$ where G, K

$$\frac{Y}{R} = \frac{G}{1+GK} \cdot \frac{27G_2}{7} = \frac{\frac{G_1}{1+G_1G_2}}{1+\frac{G_1}{1+G_1G_2}\cdot\frac{K_2}{1+K_1K_2}} \cdot \frac{27G_2}{7} = \frac{G_1(1+K_1K_2)\cdot 27G_2}{7G_1K_2+7(1+G_1G_2)(1+K_1K_2)}$$

$$= \frac{27(1+K_1K_2)}{\underbrace{(K_1K_2+1)}_{K_1K_2+1}s^2 + \underbrace{(6K_1K_2+K_2+6)}s + \underbrace{(12K_1K_2+5K_2+12)}}$$

$$= \frac{27}{s^2 + \frac{6+6K_1K_2+K_2}{K_1K_2+1}s + \frac{12K_1K_2+5K_2+12}{K_1K_2+1}}$$

Coefficients = 1 , $\frac{6k_1k_2+k_2+6}{k_1k_2+1}$, $\frac{12k_1k_2+5k_2+12}{k_1k_2+1}$

$$\begin{array}{c|cc} 2 & 1 & \frac{12k_1k_2+5k_2+12}{k_1k_2+1} \\ 1 & \frac{6k_1k_2+k_1+6}{k_1k_2+1} & 0 \\ 0 & \frac{12k_1k_2+5k_2+12}{k_1k_2+1} & \end{array}$$

No sign change:

$6k_1k_2+k_2+6>0$

$12k_1k_2+5k_2+12>0$

$\cancel{12k_1k_2+2k_2+12>0}$

$\cancel{3k_2>0}$

$k_2\cancel{(12k_1+5)}>-12$

$\cancel{k_2(12k_1+2)>-12}$

$E = R-Y$

$$= R\left(1-\frac{Y}{R}\right) = \frac{1}{s}\left(1-\frac{27(k_1k_2+1)}{(k_1k_2+1)s^2+(6k_1k_2+k_2+6)s+12k_1k_2+5k_2+12}\right)$$

$$= \frac{1}{s}\left(\frac{(k_1k_2+1)s^2+(6k_1k_2+k_2+6)s-15k_1k_2+5k_2-15}{(k_1k_2+1)s^2+(6k_1k_2+k_2+6)s+12k_1k_2+5k_2+12}\right)$$

cancel

$$\lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s) = \frac{..s^2+..s-15k_1k_2+5k_2-15}{..s^2+..s+12k_1k_2+5k_2+12}$$

$-15k_1k_2+5k_2-15=0$

$3k_1k_2-k_2+3=0$

$k_2(3k_1-1)\cancel{+3} = \cancel{0}-3$

$k_1 = 0.01$ $k_2 =$ works ? ?

3.093

$t_s = \frac{4.6}{\sigma} < 1.3$ $\qquad M_p = e^{\frac{-\pi \zeta}{\sqrt{1-\zeta^2}}} < 0.03$

calculator has CAS.

$\sigma > 3.538$ $\qquad 0.744804 < \zeta < 1$

$\zeta > 0.744804$

$\omega_n > 3.538$ b/c ζ max $= 1$ $\quad \sigma$ min $= 3.538$

$2\zeta\omega_n = 6k_1k_2 + k_2 + 6 = 9.279 \rightarrow$ $\zeta\omega_n =$ ~~[illegible]~~ 4.639

$\omega_n^2 = 12k_1k_2 + 5k_2 + 12 = 27.84 \rightarrow$ $\omega_n =$ ~~[illegible]~~ 5.276

$\zeta =$ ~~[illegible]~~ 0.8793

$\omega_n =$ ~~[illegible]~~ 5.276

Lets check to see if it all works: $\rightarrow$ $K_1 =$ ~~[illegible]~~ 0.01 $\quad k_2 =$ ~~[illegible]~~ 3.093 $\rightarrow$ SSE $= 0$ ✓

look at end of prev page.

$M_p = e^{\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}} =$ ~~[illegible]~~ $0.303\% < 3\%$ ✓

$t_s = \frac{4.6}{\sigma} = \frac{4.6}{\zeta\omega_n} = \frac{4.6}{0.8793 \cdot 5.276} =$ ~~[illegible]~~ $0.99155 < 1.3s$ ✓

all conditions satisfied:

$$\boxed{\begin{aligned} K_1 &= 0.01 \\ K_2 &= 3.093 \end{aligned}}$$