ECE 141

Lecture 6

$$d(s) = s^5 + 3s^4 + 2s^3 + 6s^2 + 6s + 9$$

$$\begin{array}{c|ccc}
5 & 1 & 2 & 6 \\
4 & 3 & 6 & 9 \\
3 & \varepsilon & 3 & \\
2 & \frac{6\varepsilon - 9}{\varepsilon} & 9 & \\
1 & e_1 & & \\
0 & 9 & &
\end{array}$$

$$c_1 = -\frac{\det\begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}}{3} = -\frac{6-6}{3} = 0$$

$$c_2 = -\frac{\det\begin{bmatrix} 1 & 6 \\ 3 & 9 \end{bmatrix}}{3} = -\frac{9 - 6\cdot 3}{3} = -\frac{3\cdot 3 - 6\cdot 3}{3} = 3$$

$$d_1 = -\frac{\det\begin{bmatrix} 3 & 6 \\ \varepsilon & 3 \end{bmatrix}}{\varepsilon} = -\frac{3\cdot 3 - 6\cdot\varepsilon}{\varepsilon}$$

$$d_2 = -\frac{\det\begin{bmatrix} 3 & 9 \\ \varepsilon & 0 \end{bmatrix}}{\varepsilon} = \cancel{-}\frac{\cancel{-}9\cancel{\varepsilon}}{\cancel{\varepsilon}} = 9$$

$$e_1 = -\frac{\det\begin{bmatrix} \varepsilon & 3 \\ \frac{6\varepsilon-9}{\varepsilon} & 9 \end{bmatrix}}{\frac{6\varepsilon-9}{\varepsilon}} = -\frac{9\varepsilon - 3\,\frac{6\varepsilon-9}{\varepsilon}}{\frac{6\varepsilon-9}{\varepsilon}} = -\frac{9\varepsilon^2 - 3(6\varepsilon-9)}{6\varepsilon-9}$$

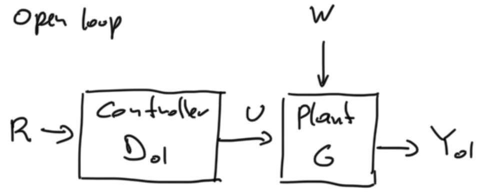
$$\begin{array}{lll}
1 & >0 & \\
 & & 0 \\
3 & >0 & \\
 & & 0 \\
\varepsilon & >0 & \\
 & & 1 \\
\frac{6\varepsilon-9}{\varepsilon} & <0 & \\
 & & 2 \\
\frac{9\varepsilon^2 - 3(6\varepsilon-9)}{9-6\varepsilon} & >0 & \\
 & & 2 \\
9 & >0 &
\end{array}$$

roots: -2.9, $-0.7 \pm 0.99i$, $0.66 \pm 1.29i$

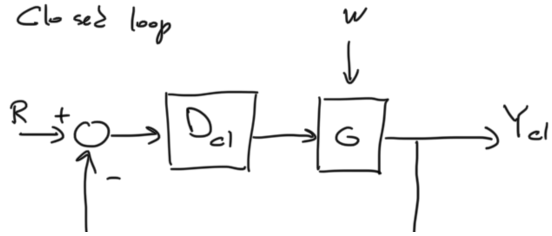
Open loop vs closed-loop control

Feed-forward vs Feed-back control

Open loop



Closed loop



$Y_{ol} = GW + G D_{ol} R$

$E_{ol} = R - Y_{ol} = R - GW - G D_{ol} R$

$= \left[1 - G D_{ol}\right] R - GW$

$Y_{cl} = GW + G D_{cl} R - G D_{cl} V - G D_{cl} Y_{cl}$

$$Y_{cl} = \frac{G D_{cl}}{1 + G D_{cl}} R + \frac{G}{1 + G D_{cl}} W - \frac{G D_{cl}}{1 + G D_{cl}} V$$

$$E_{cl} = R - Y_{cl} = \frac{1 + G D_{cl}}{1 + G D_{cl}} R - \frac{G D_{cl}}{1 + G D_{cl}} R - \frac{G}{1 + G D_{cl}} + \frac{G D_{cl}}{1 + G D}$$

$$E_{cl} = \frac{1}{1 + G D_{cl}} R - \frac{G}{1 + G D_{cl}} W + \frac{G D_{cl}}{1 + G D_{cl}} V$$

Disturbance Rejection $(R = 0, V = 0)$

$E_{ol} = -GW$

$$E_{cl} = -\frac{G}{1 + G D_{cl}} W$$

Sensor Noise $(R = 0, W = 0)$

———

$$E_{cl} = \frac{G D_{cl}}{1 + G D_{cl}} V$$

Sensitivity to parameter changes

$$T_{ol} = \frac{Y_{ol}}{R} = D_{ol} G$$

$$T_{cl} = \frac{Y_{cl}}{R} = \frac{G D_{cl}}{1 + G D_{cl}}$$

$$\frac{dT}{dG} \frac{G}{T} \equiv \frac{\frac{\delta T}{T}}{\frac{\delta G}{G}}$$

Relative change in T

Relative change in G

$$\frac{d T_{ol}}{d G} = D_{ol}$$

$$\frac{d T_{cl}}{d G} = \frac{D_{cl}(1 + G D_{cl}) - D_{cl} G D_{cl}}{(\ldots)^2}$$

$$\frac{dT_{ol}}{dG}\frac{G}{T_{ol}} = D_{ol}\frac{G}{D_{ol}G} = 1$$

$$(1 + G D_{cl})^2$$

$$\frac{dT_{cl}}{dG}\frac{G}{T_{cl}} = \frac{D_{cl} + G\cancel{D_{cl}}^2 - D_{cl}^2 G}{(1+GD_{cl})^2}\,\frac{\cancel{\phi}(1+G}{\cancel{\phi}D_{cl}}$$

$$= \frac{1}{1+GD_{cl}}$$

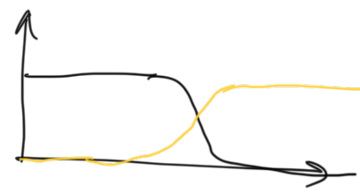
$R=0, V=0$

$$E_{cl} = \underbrace{-\frac{1}{1+GD_{cl}}}_{\to A} G W$$

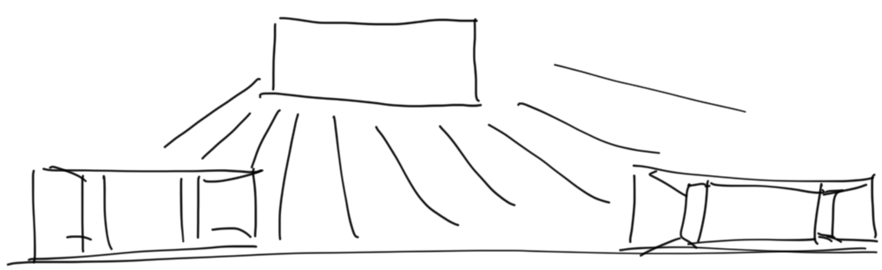
$R=0, W=0$

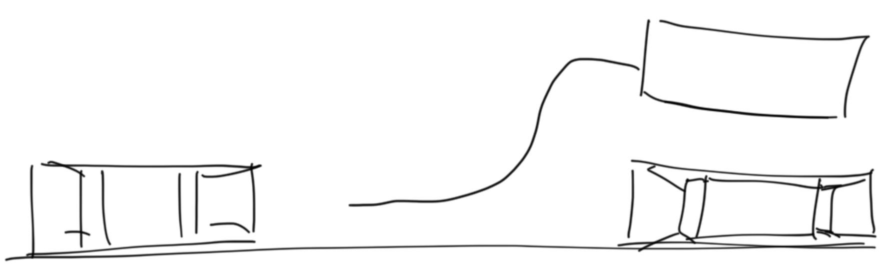
$$E_{cl} = \underbrace{\frac{GD_{cl}}{1+GD_{cl}}}_{\to B} V$$

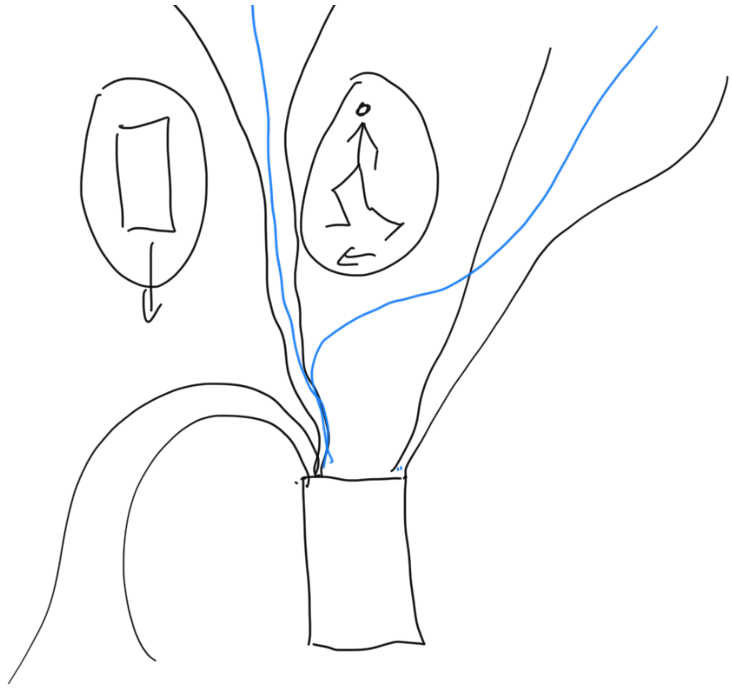
$$B - A = 1$$



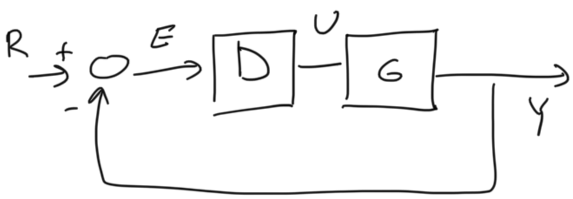
Frequency separation







PID controller



$E = R - Y$

PID → integral, derivative, proportional

Proportional control

$$D(s) = \frac{U(s)}{E(s)} = K_p \qquad u(t) = K_p\, e(t)$$

$$G(s) = \frac{b}{s^2 + a_1 s + a_2} \qquad Y = \frac{DG}{1 + DG} R$$

$$E = R - Y = \frac{1}{1 + DG} R$$

$$\frac{E}{R} = \frac{1}{1+DG} = \frac{1}{1+K_p \frac{b}{s^2+a_1 s+a_2}} = \frac{s^2+a_1 s+a_2}{s^2+a_1 s+a_2+K_p b}$$

$$d(s) = s^2 + a_1 s + a_2 + K_p b$$

$$s^2 + 2\xi\omega_n s + \omega_n^2$$

$$\begin{cases} a_1 = 2\xi\omega_n \\ \underbrace{a_2 + K_p b = \omega_n^2}_{K_p = \frac{1}{b}(\omega_n^2 - a_2)} \end{cases}$$

Proportional and integral

$$D(s) = K_p + \frac{K_I}{s}$$

$$u(t) = K_p e(t) + K_I \int_0^t e(\tau)\, d\tau$$

$$\frac{1}{1+\frac{b\left(K_p+\frac{K_I}{s}\right)}{s^2+a_1 s+a_2}} = \frac{s^2+a_1 s+a_2}{s^2+a_1 s+a_2+bK_p+b\frac{K_I}{s}}$$

$$= \frac{s^3+a_1 s^2+a_2 s}{s^3+a_1 s^2+(a_2+bK_p)s+bK_I}$$

Integral term eliminates steady state error

$$e_{ss} = \lim_{t\to\infty} e(t)$$

$$e_{ss} = \lim_{s\to 0}\left(\frac{1}{1+K_pG}\frac{1}{s}\right)s = \lim_{s\to 0}\frac{1}{1+K_pG} = \text{const}$$

$$\frac{1}{1+K_pG+K_I\frac{G}{s}} \qquad \lim_{s\to 0}\frac{\cancel{s}}{1+K_pG+K_I\frac{G}{s}}\frac{1}{\cancel{s}}$$

$$\lim_{s \to 0} \frac{s^{\to 0}}{\underbrace{s}_{\to 0} + \underbrace{k_p G s}_{\to 0} + k_I G} = 0$$

Proportional Integral and derivative

$$D(s) = K_p + \frac{K_I}{s} + K_D s$$

$$u(t) = K_p e(t) + K_I \int_0^t e(\tau)\, d\tau + K_D \frac{de}{dt}$$

$$\frac{1}{1 + DG} = \frac{s^2 + a_1 s + a_2}{s^2 + a_1 s + a_2 + b K_p + b \frac{K_I}{s} + b K_D s}$$

$$= \frac{s^3 + a_1 s^2 + a_2 s}{s^3 + (a_1 + b K_D) s^2 + (a_2 + b K_p) s + b K_I}$$