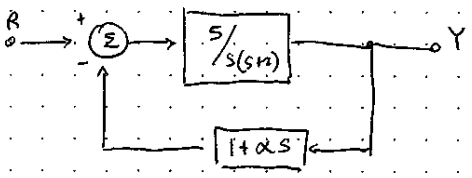
EE141 HW

5.9)




$$G(s) = \frac{5}{s(s+2)} \qquad H(s) = 1+\alpha s$$

$$1 + G(s)\cdot H(s) = 0$$

$$1 + \frac{5(1+\alpha s)}{s(s+2)} = 0$$

$$s(s+2) + s(1+\alpha s) = 0$$

$$s^2 + 2s + 5 + 5\alpha s = 0$$

$$1 + \boxed{\frac{5\alpha s}{s^2+2s+5}} = 0 \quad \rightarrow \quad K\cdot\frac{b(s)}{a(s)}$$

$\boxed{b(s) = 5s}$ assuming $K = \alpha$

$\boxed{a(s) = s^2+2s+5}$

$$L(s) = \frac{b}{a} = \boxed{\frac{5s}{s^2+2s+5}}$$

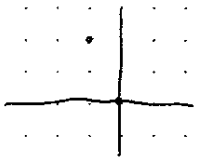
$$T(s) = \frac{\frac{5}{s(s+2)}}{1 + (1+\alpha s)\frac{5}{s(s+2)}} = \frac{5}{s^2 + (2+5\alpha)s + 5}$$

closed loop pole locations:

| | |
|---|---|
| $\alpha = 0$ | $s^2+2s+5 = (-1 \pm 2i)$ |
| $\alpha = 0.5$ | $s^2 + 4.5s + 5$, $(-2, -2.5)$ |
| $\alpha = 2$ | $s^2 + 12s + 5$ $(-0.432, -11.5567)$ |

SKETCHES ON NEXT PAGE

$$L(s) = \frac{5s}{s^2+2s+5}$$



zeros = $s = 0$

poles: $\frac{-2 \pm \sqrt{-16}}{2} = -1 \pm 2i$

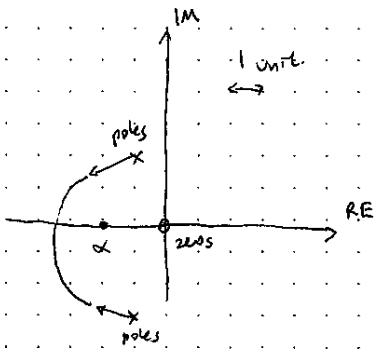
$$\alpha = \frac{-2-0}{2-1} = \frac{-2}{} \text{ asymptotes}$$

$$\phi_1 = 180\left(\frac{1+2(1-1)}{1}\right) = 180°$$

$$\theta_1 = \tan^{-1}\left(\frac{-2}{1}\right) = -63.43°$$

$$\theta_2 = \tan^{-1}\left(\frac{4}{0}\right) = 90° \quad \searrow = 116.57°$$

$$\theta_d = 180 - 90 + 116.57 = 206.57° \quad @ \; s = -1+2j$$




$$Y(s) = R(s)\cdot\frac{5}{s^2+(2+5\alpha)s+5} = \frac{5}{s(s^2+(2+5\alpha)s+5)}$$

$\alpha = 0$: $= \frac{1}{s} - \frac{s+1}{(s+1)^2+4} - \frac{1}{(s+1)^2+4}$

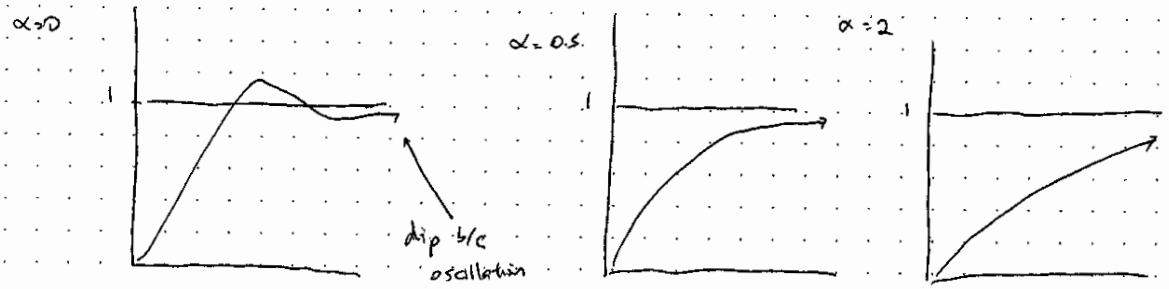
$$y(t) = u(t) - e^{-t}\cos(2t)u(t) - \tfrac{1}{2}e^{-t}\sin(2t)u(t)$$

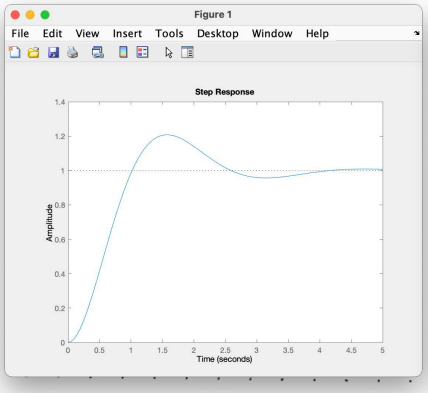
$\alpha = 0.5$: $\frac{1}{s} - \frac{5}{s+2} + \frac{4}{s+2.5}$

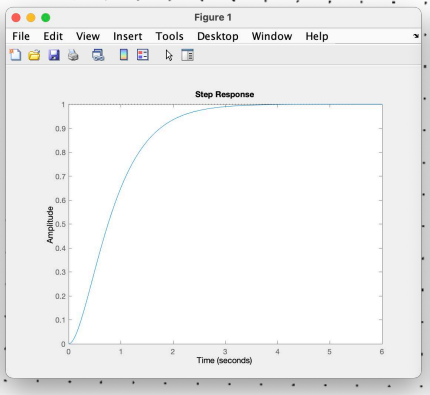
$$y(t) = u(t) - 5e^{-2t}u(t) + 4e^{-2.5t}u(t)$$

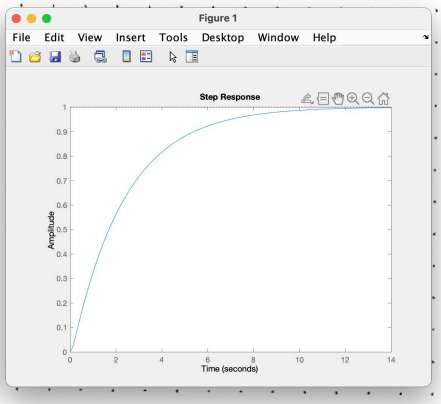
$\alpha = 2$: $\frac{1.0015}{s} - \frac{1.0404}{s+0.432} + \frac{0.039}{s+11.5567}$

$$y(t) = \left(1.0015 - 1.0404e^{-0.432t} + 0.039e^{-11.5567t}\right)u(t)$$




5.13) a) $G(s) = K \cdot \frac{s+1}{s+13} \cdot \frac{s^2+81}{s^2(s^2+100)}$ → $T(s) = \frac{G(s)}{1+G \cdot 1 \cdot 1}$ ✗

characteristic: $1 + K \cdot \frac{s+1}{s+13} \cdot \frac{s^2+81}{s^2(s^2+100)} \cdot 1 = 0 \Rightarrow L = \frac{(s+1)(s^2+81)}{(s+13)\,s^2(s^2+100)}$

$$\alpha = \frac{-13-(-1)}{5-3} = -6$$

$$\phi_1 = 180\left(\frac{1+2(1-1)}{n-m}\right) = 180 \cdot \frac{1}{2} = 90, \quad \phi_2 = 180 \cdot \frac{(1+2\cdot 1)}{2} = 270 = -90$$

zeros: $-1, \pm 9i$ poles: $-13, 0, \pm 10i$ let's use $-10i$

$\theta_1 = \tan^{-1}\left(\frac{10}{13}\right) = 37.57°$ $(-13)$

$\theta_2 = 90°$ $(0)$

$\theta_3 = 90°$ $(-10i)$

$\theta_4 = \tan^{-1}(10) = 84.29°$ $-1$

$\theta_5 = \tan^{-1}(\quad) \; 90°$ $9i$

$\theta_6 = 90°$ $-9i$

$$\theta_d = 180 - (180 + 37.57) + 180 + 84.29 = 180 + 46.72 = 226.72° = -133.28°$$

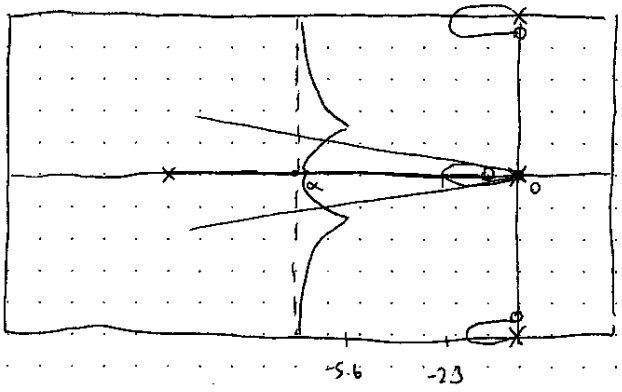
$$K = \frac{-s^2(s+13)(s^2+100)}{(s^2+81)(s+1)}$$

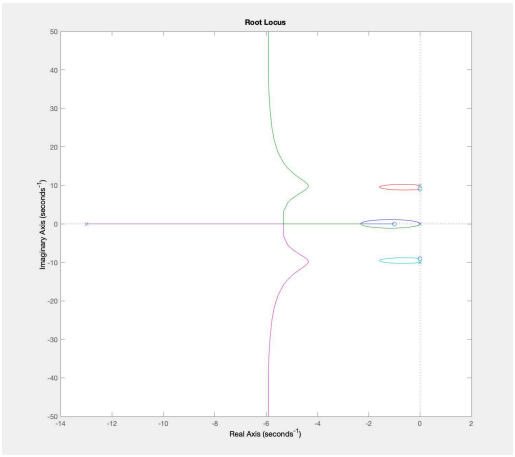
$$= \frac{-s^5+13s^4+100s^3+1300s^2}{s^3+s^2+81s+81}$$

$\frac{dK}{ds} = 0 \Rightarrow$ MATLAB $\Rightarrow$

$s = 0,$

$s = -5.642 \pm 2.311j$ } real roots




(b) $\zeta = 0.5$ the asymp. cuts the locus @ $-\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$ where $\zeta \in [0,1]$

thus, system is underdamped

THERE EXISTS A VALUE OF K.

(c) $\zeta = 0.707 \rightarrow \theta = 45°$

intersection of $\theta = 45°$.

$s = -5.6 \pm 5.5j$ plug into K equation $\Rightarrow$ $K = 103.685$

(d)