EE3: Introduction to Electrical Engineering

Lecture 0: Physics Details

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56-147G Eng IV

Physics to EE

- Electrical and magnetic circuits can be explained at the level of physics
  - Basic Laws: Coulomb, Gauss, Faraday, Ohm, etc.
  - Maxwell's Equations
- These equations help to understand energy flow in linear circuits; this lecture will provide a review
- However, except for the simplest of systems, solution of the fundamental equations every time would be too computationally demanding to be practical
  - This course (and subsequently, the EE course sequence) will introduce a set of useful abstractions based upon these laws for analyzing circuits and designing systems

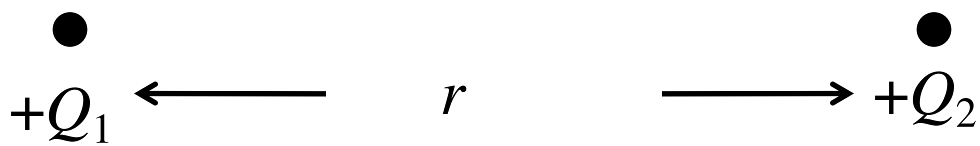
Outline

- Charge, current and fields
- Capacitors, resistors and inductors
- Energy flow in resistive and RC circuits
- EM wave propagation and antenna principles

Coulomb's Law

Consider charges Q_1 and Q_2 separated by a distance r. The force between them is

$$F = \frac{Q_1 Q_2}{4\pi\varepsilon r^2}\ N$$



where ε is the permittivity. This may also be re-written as

$$F = EQ_2 \quad N, \ \ E = \frac{Q_1}{4\pi\varepsilon r^2}\ N/C \text{ or } V/m$$

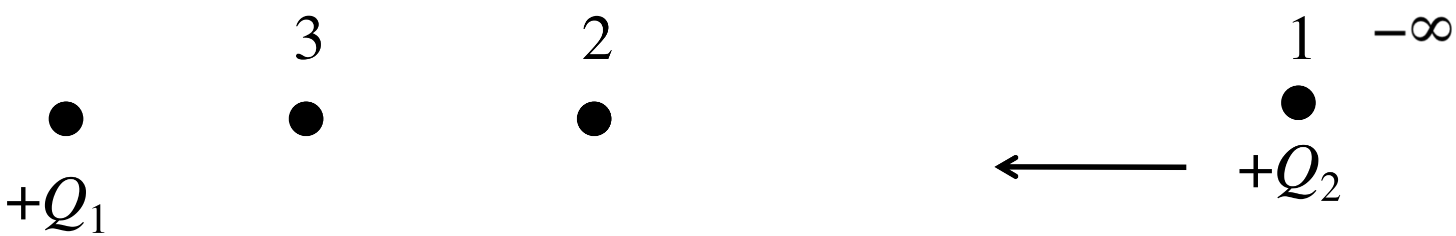
E is the electric field intensity. The value of the permittivity in free space is $\varepsilon_0 = 8.854\text{x}10^{-12}$ F/m

Coulomb's Law II

The work performed in moving a charge a distance *l* through a field is the integral of the force over the distance.

Squeezing charges together thus results in potential energy.

The work per unit charge is called the voltage, *V*.
The units are Volt=(Newton-meters)/Coulomb



Suppose a charge Q_2 is moved from position 1 (at infinity) to either position 2 or position 3; what is the voltage difference?

Coulomb’s Law III

Solution: $V_2 = \int_{-\infty}^{r_2} E_1 dr = \frac{Q_1}{4\pi\varepsilon r_2}; V_3 = \frac{Q_1}{4\pi\varepsilon r_3}$

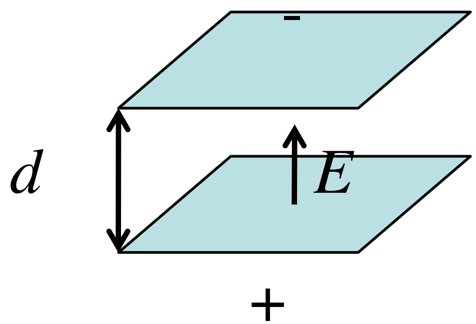
$V_3 > V_2$; this is the potential difference in the field.

Both the position and amount of charge matters for this.

The energy stored in bringing charges together is captured by the relation $Q=CV$, where C is called the capacitance; the energy (work) is then given by $W=CV^2/2$

A device that stores charge is called a capacitor. (Historically, the term “condenser” was used).

Example: Parallel Plate Capacitor



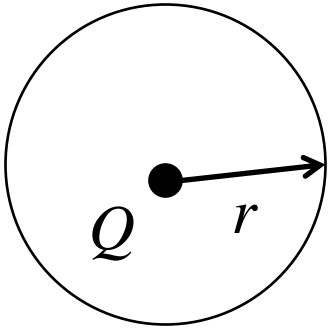
Suppose the plates have area A, electric field E

If the voltage between the plates is V, we can use the relations $V=Ed$, $C=\varepsilon A/d$ to substitute into the formula for work $W=CV^2/2$ to yield $W=\varepsilon E^2 v/2$, where v is the volume Ad.

If starting from the perspective that the charge Q is constant, we can determine the surface density of charge on the plates to work out the voltage, and come to exactly the same expression.

Note that the geometry of the plates matters for the capacitance, as does the permittivity of the dielectric in between.

Gauss's Law



Draw a sphere centered on a charge Q.

The electric field intensity at any point on the surface is then
$$E = \frac{Q}{4\pi\varepsilon r^2}$$

The electric flux is the product of the field and the area it passes through; the surface area of the sphere is $A=4\pi r^2$. Then

$$EA = \frac{Q}{4\pi\varepsilon r^2} \cdot 4\pi r^2 = \frac{Q}{\varepsilon}$$

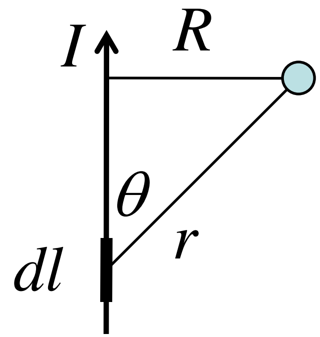
Rewrite as $Q = \varepsilon EA = DA$, where D is the electric flux de

Gauss's Law states that the total electric flux through any surface enclosing a charge is equal to the charge enclosed.

Magnetic Fields

Magnetic fields *B* arise due to motion of charges, for example, a current *I* in a wire of length *l*. For a current element *Idl*

$$dB = \frac{\mu_0}{4\pi}\frac{Idl\sin\theta}{r^2}$$



B points into page

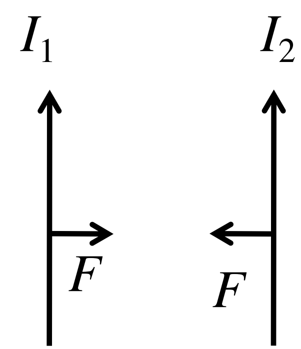
For an infinite length wire, the magnetic field circles the wire according to the right hand rule for current (thumb with the current, fingers curl with the field). Integrating the expression for the current element,

$$B = \frac{\mu_0 I}{2\pi R}$$

Magnetic Fields II

The force per unit length between two parallel infinite length wires is

$$\frac{F}{l} = I_2 \frac{\mu_0 I_1}{2\pi R}$$

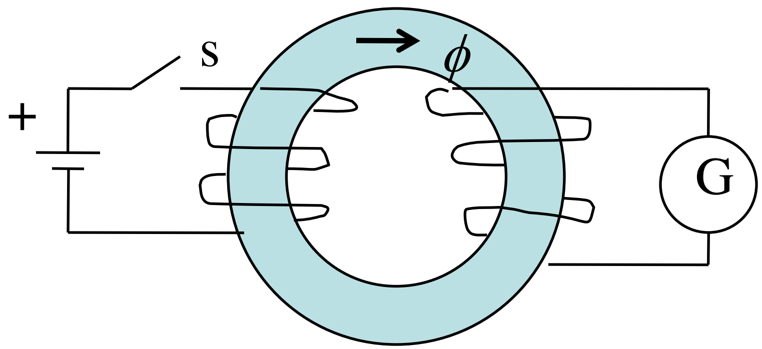


A loop in a uniform magnetic field will experience no net force. Current flowing in opposite directions in loops that are twisted close together will produce tiny magnetic fields (which is why telephone wire is twisted).

Faraday's Law I

We can store energy in magnetic fields, just as we can in electric fields. Moreover, magnetic fields can be used to induce currents and thus transfer power. Consider two experiments.

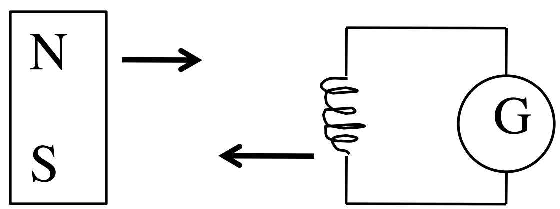
1.



In the first experiment, two coil sets are wound around an iron toroid. Closing the switch causes a current to flow through the left coil, inducing a magnetic flux in the core as shown. This produces a current in the second coil that can be measured with the galvanometer G.

Faraday's Law II

In the second experiment, a permanent magnet is moved relative to a coil of wires. This also induces a current, converting kinetic to electric energy.

2. 

The electromotive force (emf) induced in a closed circuit having a flux linkage of λ weber-turns is

$$e = -\frac{d\lambda}{dt} \quad V$$

Faraday's Law III

When the magnetic flux ϕ penetrates all N turns of a coil, the emf can be written as

$$e = -N\frac{d\phi}{dt} \text{ V}$$

The direction of the current is always to oppose the action that caused it. The emf of the self-induction for coils of wire is given by

$$e = L\frac{di}{dt} \text{ V}$$

This relation between current and voltage defines inductance.

Energy Storage in Inductors

For inductors we can write the voltage/current relations

$$v = L\frac{di}{dt} \text{ or } i(t) = \frac{1}{L}\int_0^t v\,dt + i(0)$$

Notice that a step change in current through an inductor is impossible. Assuming zero initial current, the total energy stored in the magnetic field at time *t* is

$$W = \int_0^t v\,i\,dt = \int_0^t \left(L\frac{di}{dt}\right) i\,dt = \int_0^t L\,i\,di$$

$$W = \frac{1}{2}Li^2$$

Ohm's Law

The current through a uniform material of cross sectional area A, resistivity ρ, and length l due to an applied voltage V is

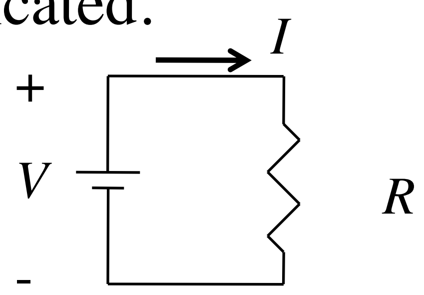
$$I = \frac{A}{\rho}\frac{V}{l} = V/R;\ i.e., V = IR$$

R is called the resistance. Energy is dissipated in resistors in the form of heat; all practical circuit elements contain some resistance, and thus there are always losses as current flows. The power dissipated is $VI=RI^2$.

Resistance generally increases with temperature. Some materials also display changes in resistance in the presence of electric or magnetic fields; this is crucial to the operation of transistors.

Energy Flow in Resistive Circuit

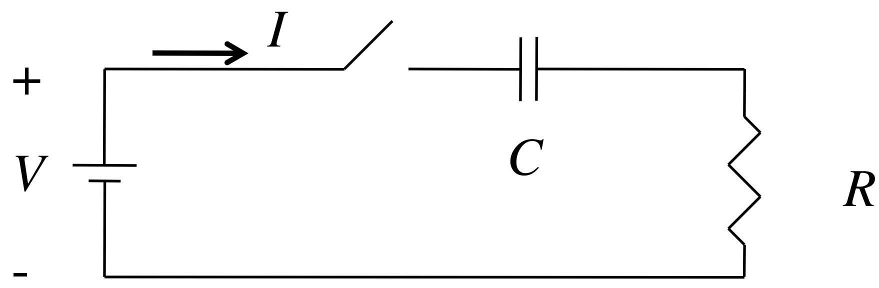
Consider a circuit with a constant voltage source (such as a battery) and a resistor. Current flows in the direction indicated.



By Ohm's law, the voltage drop across the resistor must be $V=IR$. The power dissipated by the resistor must exactly balance the power supplied by the battery. Moreover, given values for V and R we can determine the current, and thus the power as well. For example, if V=3 Volts, R=12 Ohms, then I=0.25 Amperes and power=0.75 Watts.

Energy Flow in RC Circuit I

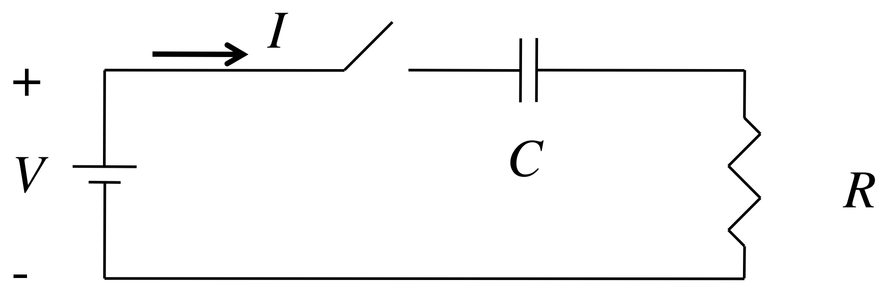
Now suppose we additionally have a capacitor as indicated



Because voltage cannot change instantaneously across a capacitor we must consider two phases: a transient, when the capacitor is charging, and a steady state, when it has reached its final voltage.

The steady state is easy to compute: no current can flow when the capacitor is fully charged, and thus the voltage drop across the capacitor is equal to the battery voltage.

Energy Flow in RC Circuit II



To find the transient current flows and voltage, we must first invoke Kirchhoff's voltage law, which states that the sum of voltages around a loop is zero. Thus $V=V_c+V_R$. Suppose the switch is open before t=0 and then closes, with the capacitor having no charge. We next invoke the voltage and current relations for capacitors and resistors to create the differential equation:

$$V = iR + \frac{1}{C}\int_0^t i\,dt$$

Energy Flow in RC Circuit III

There are many methods for solving differential equations. In this case, because current appears both within and outside the integral, an obvious guess is that the solution should involve an exponential function. This turns out to be correct, with the specific solution being:

$$i(t) = \frac{V}{R} e^{-t/RC}$$

The voltage across the resistor is given by Ohm's law:

$$v_R(t) = R\frac{V}{R} e^{-t/RC} = Ve^{-t/RC}$$

Then the voltage across the capacitor is simply

$$v_C(t) = V - Ve^{-t/RC} = V(1 - e^{-t/RC})$$

Note that this asymptotically approaches V—the steady state solution.

Energy Flow in RC Circuit IV

Since the current and voltages across the elements are variable, integration is required to work out the relative energies. At the conclusion of the process, the capacitor stores energy that could be used to perform work. For example, if the battery is replaced with a short circuit, the energy stored in the capacitor will be used up in the resistor in the form of heat (with the discharge having an exponential characteristic).

Radio Propagation and Antennas

EM Radiation from Current Element

When a time-varying current is applied to a conductor, some of the energy will radiate away in the form of electromagnetic waves, which will oscillate at the same frequency as the original current.

Similarly, a conductor exposed to EM waves will capture some of the energy, in the form of a time-varying current being induced.

Devices which are designed to radiate EM energy or capture it are called antennas; within circuits, we assign them R, L and C values.

The fraction of the energy flowing to the antenna that is radiated is called the efficiency; through careful design, this can be very high at particular resonant frequencies. But all wires can act as antennas, albeit potentially with low efficiency.

Free Space Radio Propagation

The simplest propagation model is that of a wavefront propagating from an isotropic source in free space: energy is launched from the source with equal intensity in all directions.

For any sphere centered on the source, regardless of the radius r the same total amount of energy must impinge on its surface. Since the surface area of the sphere is given by $4\pi r^2$, it then follows that if P_t is the transmitted power, the power impinging on an area A of the surface is given by

$$P_r = \frac{P_t A}{4\pi r^2}$$

Thus, the signal intensity drops as the second power of distance from the source.

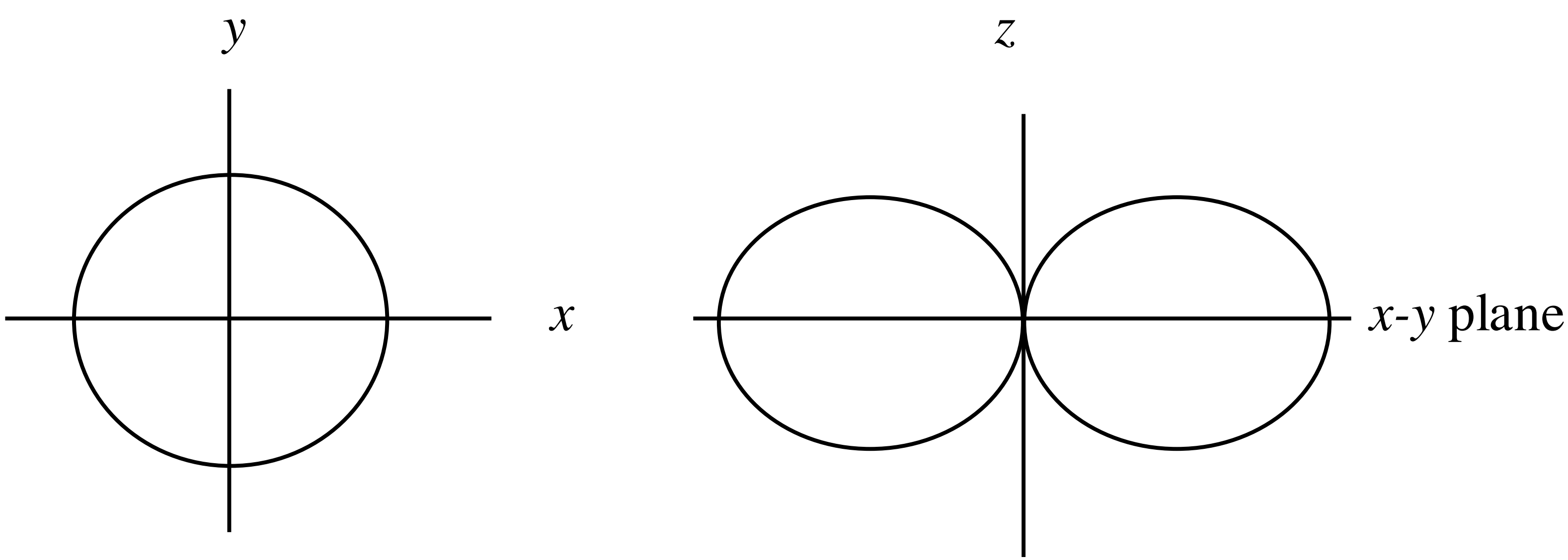
Antenna Energy Capture

A dish antenna will typically capture 80% of the energy that impinges on its area. But a whip antenna has tiny area; how does it work?

The answer lies in the fact that the inducement of currents within the antenna changes the local EM field. It is not the physical area of the antenna that matters, but how it couples into this field. For antennas, we account for this coupling by speaking of "effective area". This effective area depends upon the direction the antenna is facing.

Antenna Gain

The antenna gain is the ratio of the peak intensity of the radiated signal compared to what the intensity would have been with an isotropic source.



With antenna gains accounted for in Tx and Rx, $P_r = P_t G_t G_r \left(\frac{\lambda}{4\pi r} \right)^2$

In Reality…

Free space propagation is never actually encountered except in space. Usually there will be distortion, scattering, and attenuation caused by the propagation medium, collected together as a propagation loss L_p.

Absorption in a uniform medium results in constant fractional loss per unit distance. This amounts to a fixed loss in dB per unit distance. Multiple media may be in the signal path, each with their own loss per unit distance associated with them.

E.g. in satellite communications there is atmospheric loss, rain loss, and loss on the cable connecting an antenna to a receiver. In near-ground propagation, foliage and other obstructions can induce frequency-dependent losses, mainly due to scattering by objects of dimension on the order of a wavelength or larger; result is lower frequency radio waves penetrate structures and dense forests better than high-frequency waves.

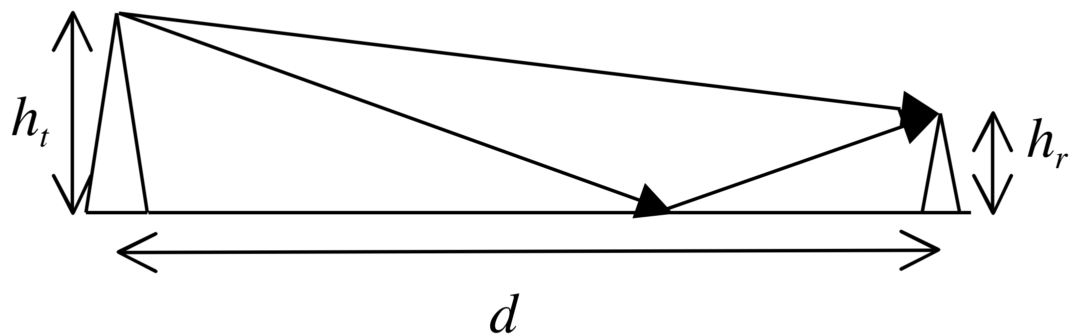
Geometric Optics Approximation

Many radio propagation phenomena can be approximated using geometric optics: rays emanating from source, affected by reflection, refraction, and diffraction

Detailed computation using finite difference solutions to Maxwell's equations is usually impractical: excessive computations, and too many unknown parameters

Example: Ground Bounce

For near-ground propagation as in broadcast radio and television, and cellular radio systems a variety of impairments must be dealt with. Of particular concern is the ground bounce ray.



When d is large compared to h the angle of incidence is very small. The ground induces a 180 degree phase shift in the reflected ray, with cancellation only partially complete because the two paths have slightly different lengths and therefore phases. The result is:

$$P_r = \frac{P_t G_t G_r h_t^2 h_r^2}{d^4}$$

Summary

- Electrical and magnetic circuits can be explained at the level of physics
  - Basic Laws: Coulomb, Gauss, Faraday, Ohm, etc.
  - Maxwell's Equations
- These equations help to understand energy flow in both lumped circuit elements (RLC) and antennas
- However, except for the simplest of systems, solution of the fundamental equations every time would be too computationally demanding to be practical
  - But we still need to go back to basics when creating new devices
- In EE, we use a sequence of abstractions of circuit theory to analyze ever more complex systems
  - This course will introduce some of them; you will see many abstractions of basic theory over the course of your career.