EE3: Introduction to Electrical Engineering

Lecture 2: Circuits II

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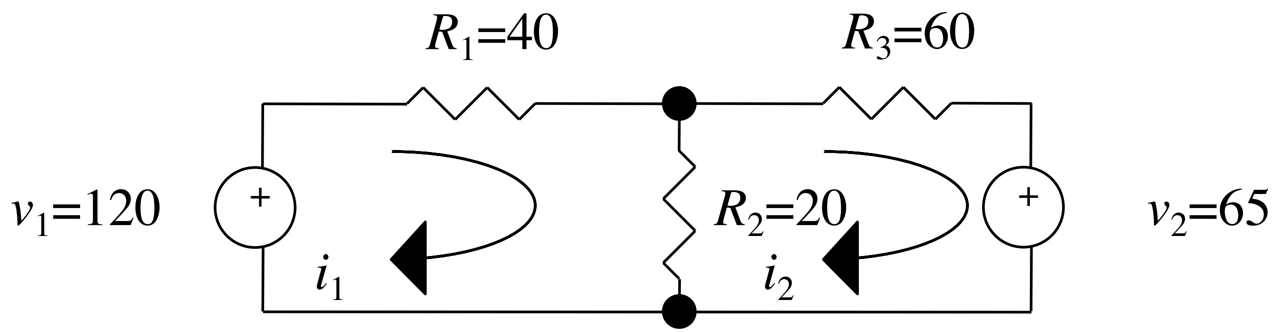
Outline

- Mesh and Node Equations
- Black boxes in circuits:
  - Example: Operational Amplifiers (Op-amps)
- Background to Communication Circuits: Fourier Theory
  - Meaning of frequency domain

Mesh Currents=Loop Currents

Example: 2-mesh network



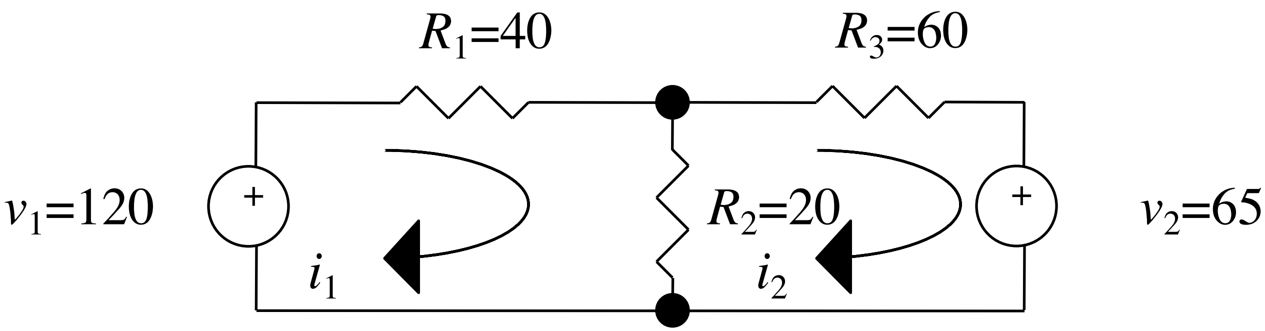
We suppose that there are independent currents flowing in each loop, as indicated. We apply KVL to each:

$$0 = i_1R_1 + (i_1 - i_2)R_2 - v_1 = i_1(R_1 + R_2) - i_2R_2 - v_1$$

$$0 = (i_2 - i_1)R_2 + i_2R_3 + v_2 = -i_1R_2 + i_2(R_2 + R_3) + v_2$$

Note the negative sign on v_1; there is a voltage gain in the indicated current direction (consistent with KVL convention).

Mesh Currents II



We now have two equations in two unknowns, i.e., $\boldsymbol{Ri}=\boldsymbol{v}$:

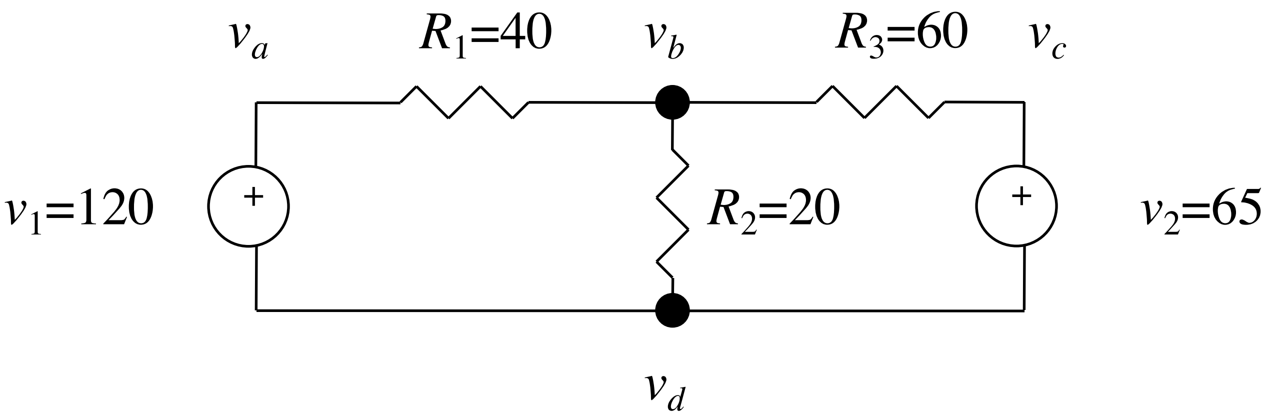
$$\begin{bmatrix} R_1+R_2 & -R_2 \\ -R_2 & R_2+R_3 \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} v_1 \\ -v_2 \end{bmatrix} \quad \text{or} \quad \begin{bmatrix} 60 & -20 \\ -20 & 80 \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 120 \\ -65 \end{bmatrix}$$

We could use MATLAB $\boldsymbol{v}=\boldsymbol{i}/\boldsymbol{R}$; or Gauss elimination:

$$\left[\begin{array}{cc|c} 20 & -20/3 & 40 \\ 0 & 80-20/3 & -25 \end{array}\right] \qquad \begin{array}{l} \Rightarrow i_2 = -0.341;\ \text{back substitute in r} \\ 20 i_1 + 20/3 \bullet 0.341 = 40;\ i_1 = 1.886 \end{array}$$

The negative sign on i_2 indicates flow is in the opposite direction to that drawn.

Node Voltages

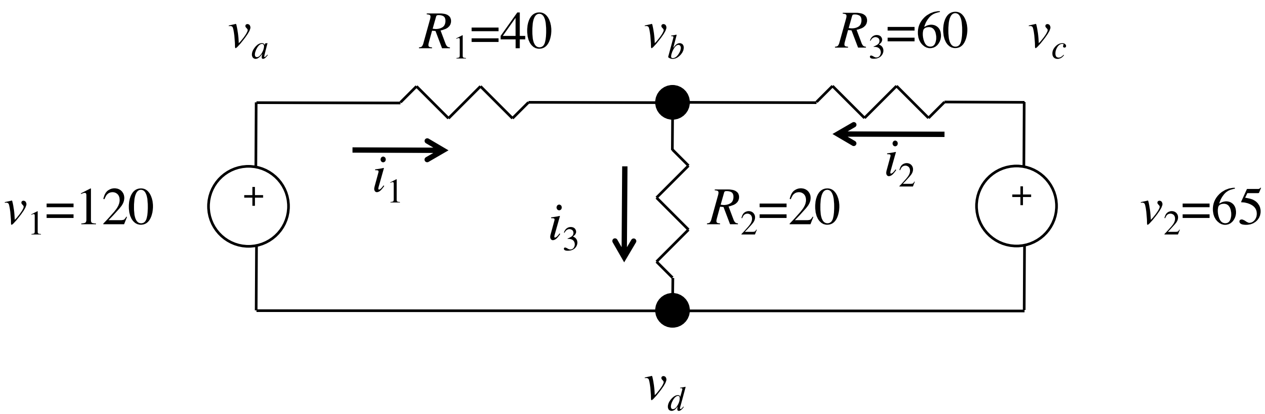


Another way to solve the circuit that sometimes reduces the number of equations is to solve for voltages at the nodes.

We begin by assigning one node to have a voltage=0 or be designated as "ground." Suppose we pick v_d=0.

Then due to the ideal voltage sources, v_a=120, v_c=65, and the only unknown is v_b. How do we proceed?

Node Voltages II



Using KCL, $0 = -i_1 - i_2 + i_3 \quad (1)$

Using OL, $i_1 = \dfrac{v_a - v_b}{40},\ i_2 = \dfrac{v_c - v_b}{60},\ i_3 = \dfrac{v_b}{20}$

Subst for i_3, $i_1 = 3 - \dfrac{v_b}{40},\ i_2 = 1.083 - \dfrac{v_b}{60}$

Subst in (1), $-3 + \dfrac{v_b}{40} - 1.083 + \dfrac{v_b}{60} + \dfrac{v_b}{20} = 0$

Mult by -120, $490 - 11 v_b = 0;\ v_b = 445$

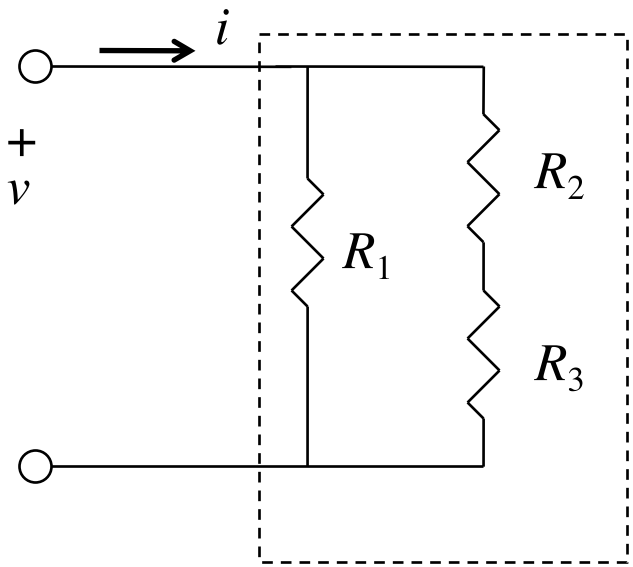
$\therefore i_1 = 1.886; i_2 = 0.341; i_3 = 2.23$

Considering directions of the currents, these results agree with the mesh current method

Black Boxes

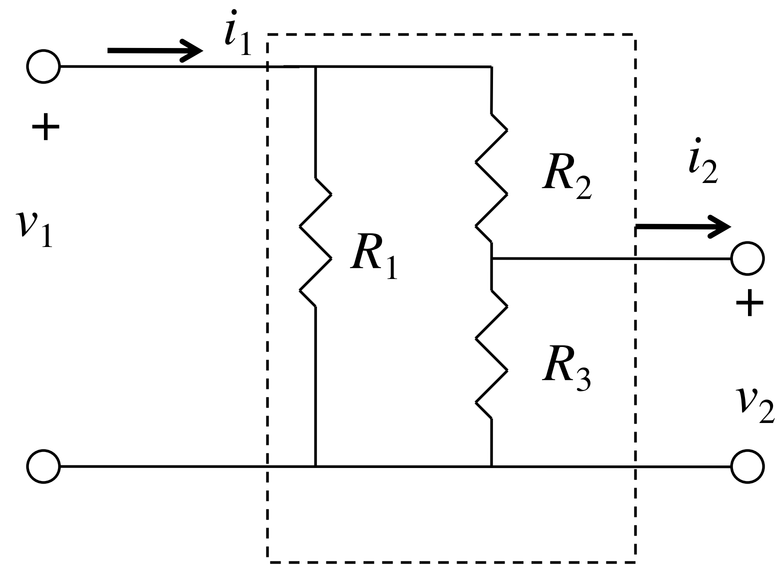
Recall that two sets of circuits are considered equivalent if they present the same *v*-*i* relations to circuits that are attached to them (e.g., parallel/series resistors).

One-Port Network



1 *v*-*i* relation needed

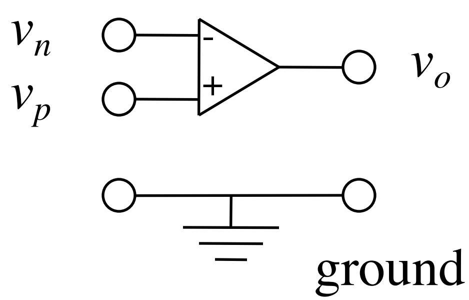
Two-Port Network



2 *v*-*i* relations needed

Operational Amplifiers

This is one of the most useful "black boxes". Later on we will outline their internal structure.

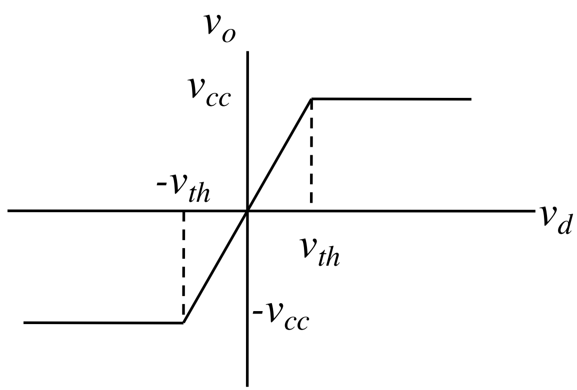


The op amp has supply voltages at $+v_{cc}$ and $-v_{cc}$ with respect to the ground (e.g., v_{cc}=15 V), and thus the max and min values of the output voltage v_o.

For any v_o within this range, $v_o = A(v_p - v_n) = Av_d$

A is typically very large (e.g., 100,000); to achieve linear amplification for practical difference voltages, bias circuits are required.

Op Amps II



The slope yields A. Difference voltages larger than v_{th} produce saturation at the supply voltages.

Certain characteristics lead to useful approximations:

1) Resistance presented to a source at either input is very high (approximated as infinite). Thus can assume no current enters.

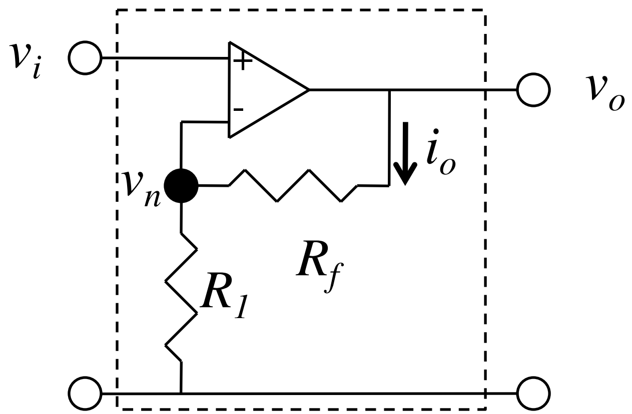
2) Output resistance is close to zero (actually 100 Ohms)

3) A is treated as being infinite.

Bias Circuits

Resistive circuits are used to bias the op amp to produce a number of useful configurations.

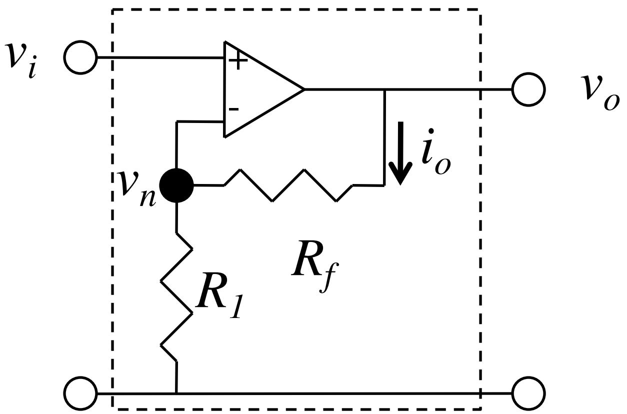
1) Non-inverting mode=voltage controlled current source



This is viewed externally as a two-port network

Because of the infinite input resistance, no current flows into the negative pin, and so the resistive feedback path acts as a voltage divider.

Non-Inverting Mode



$$v_n = i_o R_1 = \frac{R_1}{R_f + R_1} v_o$$

$$\text{Since } v_o = v_d A = \left(v_i - \frac{R_1}{R_f + R_1} v_o \right) A$$

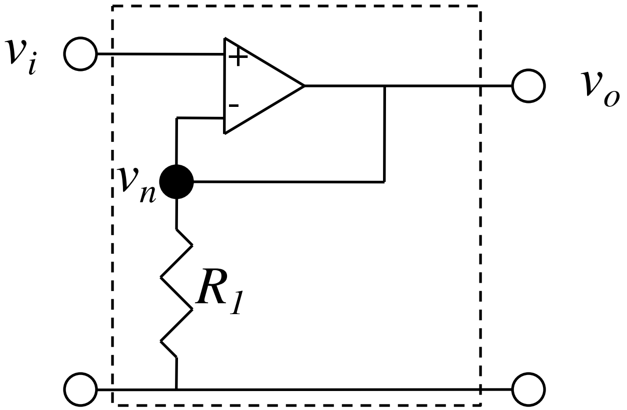
$$\text{then } \frac{v_o}{A} = v_i - \frac{R_1}{R_f + R_1} v_o$$

$$\text{But } \frac{v_o}{A} \approx 0 \text{ since } A \to \infty \qquad \text{Thus: } v_o \approx \left(1 + \frac{R_f}{R_1} \right) v_i$$

This yields linear amplification that does not (to first order) depend on the load. Of course we are limited by the supply voltage.

$$\textit{Example: } R_f = 19k\Omega,\ R_1 = 1k\Omega;\ v_o \approx 20 v_i$$

Voltage Follower



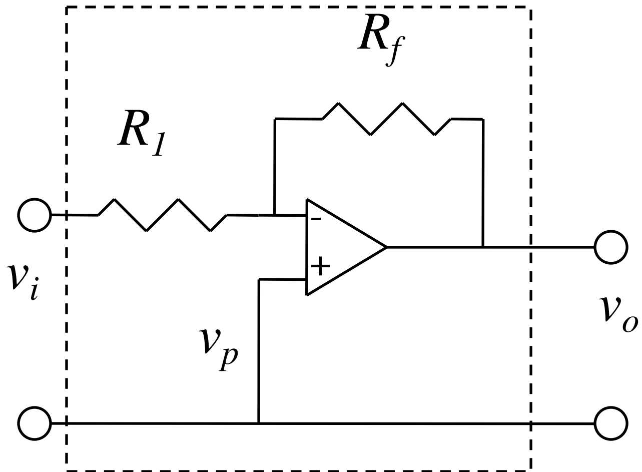
It is still true that $v_o \approx \left(1 + \frac{R_f}{R_1}\right) v_i$

But since $R_f = 0$, $v_o = v_i$

Why do this?

The answer is that the load appearing across the output is now *buffered*; no current directly flows from the input. We can thus handle large loads without worrying about affecting the input voltage level.

Inverting Mode



$$v_p = 0$$

$$\text{Since } v_o = A(v_p - v_n), \quad v_o = -Av_n$$

However, this does not really lead anyw

since A is large and thus $v_n \approx 0$

A more practical expression results from observing that no input current flows into the op amp.

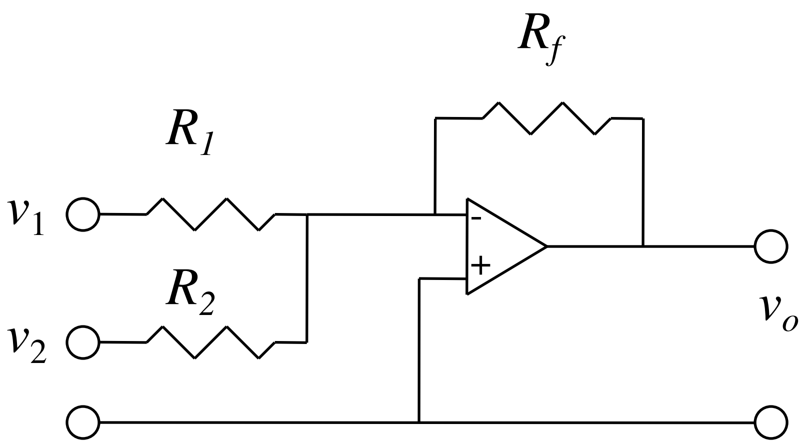
$$\text{Then by KCL, } \frac{v_i}{R_1} + \frac{v_o}{R_f} = 0; \quad v_o = -\frac{R_f}{R_1} v_i$$

This is similar to the first case, but the input resistance is now R_1 and so if the source resistance is non-negligible,

$$v_o = -\frac{R_f}{R_1 + R_s} v_i$$

EFTS

Show that the following circuit forms a weighted sum of v_1 and v_2.



Solution: Begin by observing that $v_n = 0$ as befor

By KCL at the negative terminal, $\frac{v_1}{R_1} + \frac{v_2}{R_2} + \frac{v_o}{R_f} = 0$

$$v_o = -\left(\frac{R_f}{R_1} v_1 + \frac{R_f}{R_2} v_2\right)$$

Example: $R_f = R_1 = R_2 = 10k\Omega;\ v_o = -(v_1 + v_2)$

Amplifier Basic Facts

We may regard the amplifier input (voltage or current) as a control signal that affects how large the output is (voltage or current)

The output of an amplifier has more power (P=VI) than at the input. How is this possible?

Amplification is effected by non-linear devices such as transistors. The power increase is possible only by connecting the amplifier to a power supply

Figures of merit are the power efficiency (<1), linearity (match of input and output waveforms), and amplification. It is difficult to achieve excellence in all of these at once.

Frequency Domain

Filters, transmission lines, and other circuits commonly used in communication systems are most easily analyzed in the frequency domain.

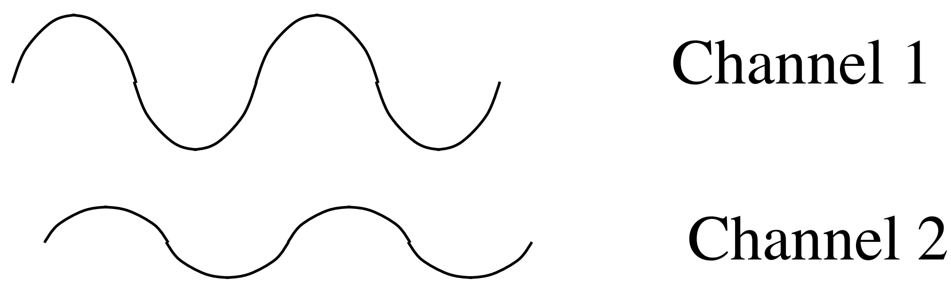
We exploit the fact that circuits containing inductors and capacitors do not respond instantly to voltage/current changes, and thus provide non-uniform frequency response.

This enables design of tuned RLC circuits, essential for communications applications.

Methods derived from the frequency domain representation allow solution of these circuits (lecture 3)

Measurement of Frequency Response

Take a sinusoidal output from a function generator at some frequency f, and use it to drive a circuit, and to be input channel 1 to an oscilloscope. Take the output of the circuit and feed it to input channel 2 of the scope.



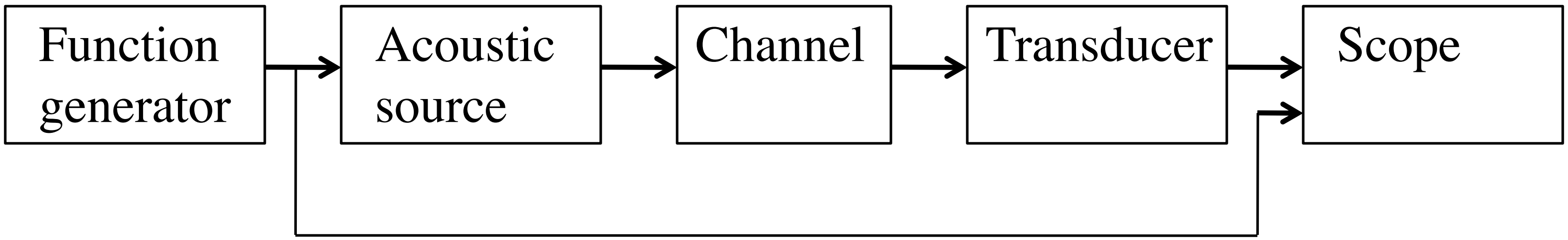
The output will in general have a different amplitude and phase, but will be at the same frequency except in certain non-linear circuits.

The ratio of the peak-peak voltages together with this phase offset are together called the frequency response of the circuit at frequency f.

Frequency Response

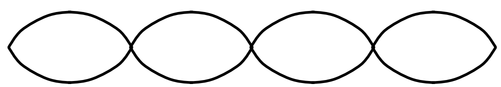
We could in principle sweep the frequency over the full range from DC to where the circuit has a small response to get its entire frequency response. This is accomplished nearly automatically in spectrum analyzers.

Example: Lab 3

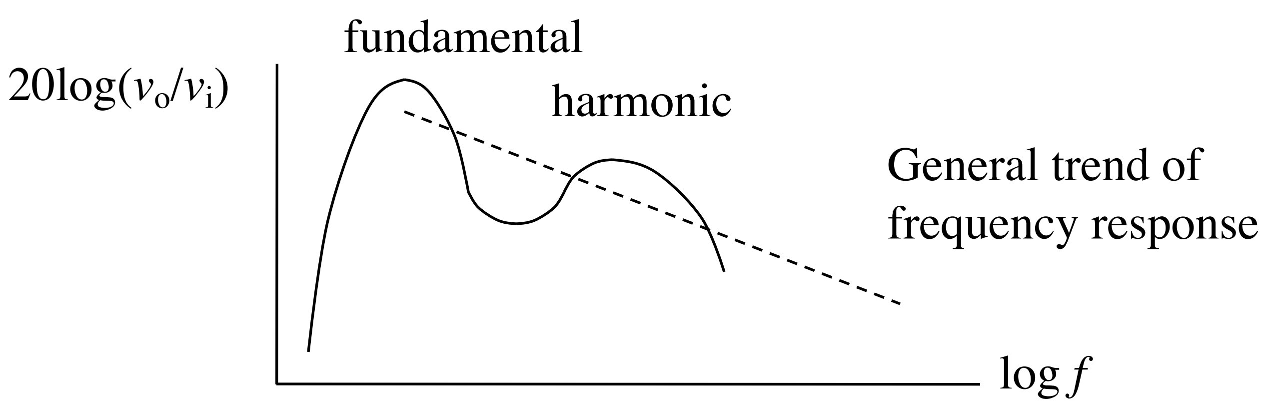


There are two frequency effects observed, in addition to simple attenuation by the transducer pair/channel combo:

(1) Standing waves (fundamental, harmonics)



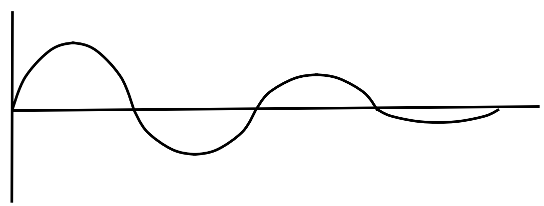
(2) Frequency response of transducers



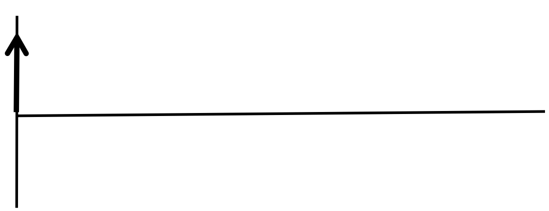
Impulse Response

We can fully describe the behavior of circuits in either the time domain or the frequency domain.

The impulse response of a circuit is the time domain waveform $h(t)$ observed when a very short pulse of unit area (e.g. a Dirac delta function) is applied at t=0. For example:



(circuit with R,L,C)



(circuit with R only)

Fourier Transform

Formally, the two domains are related using the Fourier Transform

$$X(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi ft} dt$$

The inverse transform is given by

$$x(t) = \int_{-\infty}^{\infty} X(f) e^{j2\pi ft} df$$

How does this work?

Complex Exponentials

Complex exponentials appear in both the forward and inverse transforms

$$\cos x = \frac{e^{jx} + e^{-jx}}{2}; \sin x = \frac{e^{jx} - e^{-jx}}{2j}; e^{jx} = \cos x + j \sin x$$

The complex exponential enables the representation of the phase of a sinusoid in a compact notation, with the cosine component being denoted "in phase" and the sine (imaginary) component being denoted "in quadrature", i.e., orthogonal to the in-phase component

This magnitude and phase of a complex exponential at a particular frequency yield a complete description of the response at that frequency (as for the oscilloscope).

Transform interpretation

The Fourier transform is

$$X(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi ft} dt$$

For a given frequency f_c, the Fourier transform is roughly speaking the circuit response when excited by a cosine wave of frequency f_c and unit amplitude. The phase is the offset from this reference sinusoid.

Transforms and Circuit Analysis

ANY waveform $x(t)$ of finite energy can be thought of as being composed as a set of sinusoids, as described by $X(f)$.
THUS: if we can solve the circuit response for a sinusoidal input, we can use the inverse transform to get the time domain response to any waveform.

As we will see in Lecture 3, linear circuits with R, L, C elements excited by sinusoids can be solved using the same methods as for resistors, using frequency dependent expressions called "impedance". This fact plus Fourier theory allows us to extend the methods from constant voltage sources to any excitation.

Filters

A filter is any circuit or system for which some frequencies are emphasized more than others.

Convolution arises when a signal passes through a linear system such as a filter:

$$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t)$$

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$$

It is often much easier to deal with this in the frequency domain, since we need only multiply the transforms together:

$$Y(f) = X(f)H(f)$$

The frequency domain is also a more natural setting for thinking about filters.

EFTS

In strong earthquakes, near the epicenter there are very sharp ground motions while far away the earthquake is perceived as a rolling motion. What kind of filter is the earth with respect to seismic waves?

Solution: it is lowpass, since the fast ground motions have been damped out. Most physical media are low pass filters with respect to mechanical waves.

Summary

- Equivalent circuits can reduce the complexity of analysis
- Op amps can be configured to buffer circuits, provide linear amplification, or perform mathematical operations (more later)
- Fourier theory presents an alternative representation to the time domain, that is often conceptually and computationally convenient
- Next will apply this theory to explain the first global telecommunications systems

Appendix: Computation of Fourier Transforms

- Integration is tedious. Fortunately, many Fourier transforms have already been computed and put in tables (next slide). Together with a set of properties (following slide) a large number of useful transforms can be computed without integrating. You will get practice in EE 102.
- A highly efficient algorithm that approximates the Fourier transform numerically from sampled data has been developed, known as the FFT (fast Fourier transform). This is what is used in digital spectrum analyzers, including in the MyDAQ. You will see how it is done in EE 113.
- The Fourier transform is not the only transform method used in circuits and systems theory. The Laplace transform (the s-domain) is also useful.

Fourier Transform Pairs

We use tables of Fourier transform pairs to avoid actually integrating, whenever possible.

$$A\,rect_T(t) \leftrightarrow AT\,\mathrm{sinc}(fT) \qquad rect_T(t) = 1 \text{ on } (-T/2, T/2)$$

$$A\,\mathrm{sinc}(2Wt) \leftrightarrow \frac{A}{2W} rect(f/2W) \text{ [ideal filte} \qquad \mathrm{sinc}(x) = \frac{\sin(\pi x)}{\pi x}$$

$$\delta(t - t_0) \leftrightarrow \exp(-j2\pi f t_0)$$

$$\exp(j2\pi f_c t) \leftrightarrow \delta(f - f_c)$$

$$\cos(2\pi f_c t) \leftrightarrow \frac{1}{2}\left[\delta(f - f_c) + \delta(f + f_c)\right]$$

$$\sum_{n=-\infty}^{\infty} \delta(t - nT) \leftrightarrow \frac{1}{T} \sum_{m=-\infty}^{\infty} \delta(f - m/T)$$

The last one says that a train of samples spaced at T transform to a set of samples spaced at 1/T in the frequency domain.

Fourier Transform Relations

Various properties are also used, in conjunction with pairs.

1. Linearity:
if $x_1(t) \leftrightarrow X_1(f)$ and $x_2(t) \leftrightarrow X_2(f)$ then
$c_1 x_1(t) + c_2 x_2(t) \leftrightarrow c_1 X_1(f) + c_2 X_2(f)$

2. Time scaling: $x(at) \leftrightarrow \frac{1}{|a|} X\left(\frac{f}{a}\right)$

3. Duality: $X(t) \leftrightarrow x(-f)$

4. Time Shifting: $x(t - t_0) \leftrightarrow X(f) e^{-j2\pi f t_0}$

5. Frequency Shifting: $e^{j2\pi f_c t} x(t) \leftrightarrow X(f - f_c)$

6. Differentiation in time domain: $\frac{d^n}{dt^n} x(t) \leftrightarrow (j2\pi f)^n X(f)$

7. Integration in time domain: $\int_{-\infty}^{t} x(t) dt \leftrightarrow \frac{1}{j2\pi f} X(f)$

8. Conjugate functions: $x^*(t) \leftrightarrow X^*(-f)$

9. Multiplication in time domain (convolution in frequency domain):

$$x_1(t) x_2(t) \leftrightarrow \int_{-\infty}^{\infty} X_1(\lambda) X_2(f - \lambda) d\lambda$$

10. Convolution in time domain (multiplication in frequency domain):

$$\int_{-\infty}^{\infty} x_1(t) x_2(t - \tau) d\tau \leftrightarrow X_1(f) X_2(f)$$