EE3: Introduction to Electrical Engineering

Lecture 1: Circuits I

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Course Outline

- Circuit fundamentals: theory and hands-on experience to provide technical explanation of:
  - Telecommunications
  - Electrical Grid
  - Computing and Control
  - Devices: pn-junction (diodes, transistors, photonics)
  - Technology trends
- Goal is to provide a basic understanding of the great EE inventions and those to come, and provide an opportunity in the lab to become familiar with devices described in lecture, and produce a working design.
- After four weeks of set labs to illustrate equipment and device operation, will have five weeks to create own project

Course Outline III

- Grades
  - Homework 10%
  - Labs 10%
  - Project 40%
  - Quizzes 10%
  - Final 30%
- Quizzes
  - Weeks 3, 5
- Project
  - Evaluation based on performance at end, and in final oral/written reports.
  - Labs and projects illustrate concepts presented in lecture

Basic Circuit Theory

In this course, we will show how the ideas behind many of the most famous developments in electrical engineering can be explained using relatively simple circuits.

We now explore some of the basic methods for analyzing the behavior of circuits.

Basic Physics

A current consists of a flow of charges (usually electrons, but sometimes holes). While electrons are negatively charged, by convention current is positive in the direction of the arrow.

Resistors: electrons accelerated by a voltage collide with the material and dissipate energy as heat (Ohm's Law)

Capacitors: Like charges repel each other (Coulomb's Law); but a voltage can force them closer to together, storing energy.

Inductors: current produces a magnetic field (Faraday's Law); this field cannot collapse immediately, and when it does it releases energy back into the circuit

Ohm's Law

The current through a uniform material of cross sectional area A, resistivity ρ, and length l due to an applied voltage V is

$$I = \frac{A}{\rho}\frac{V}{l} = V/R;\ i.e., V = IR$$

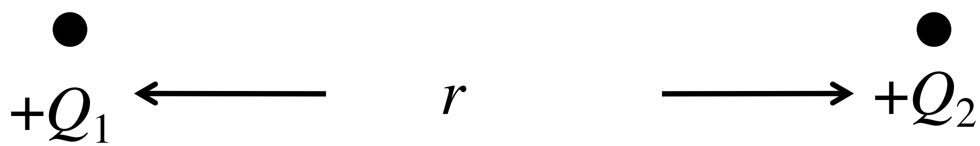
R is called the resistance. Energy is dissipated in resistors in the form of heat; all practical circuit elements contain some resistance, and thus there are always losses as current flows. The power dissipated is $VI=RI^2$.

Resistance generally increases with temperature. Some materials also display changes in resistance in the presence of electric or magnetic fields; this is crucial to the operation of transistors.

Coulomb’s Law

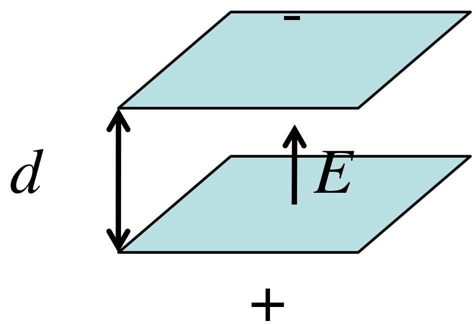
Consider charges Q_1 and Q_2 separated by a distance r.
The force between them obeys

$$F \propto \frac{Q_1 Q_2}{r^2}\ N$$



Thus, energy is required to bring the charges together.
In circuits, this is accomplished by applying a voltage.

Example: Parallel Plate Capacitor



Suppose the plates have area *A*, electric field *E*

If the voltage between the plates is *V*, then $V=Ed$. The capacitance is $C=\varepsilon A/d$. Observe that larger plates enable more charge at a given density, while smaller *d* results in more force and thus more energy stored.

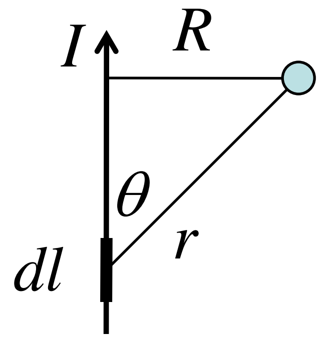
Since the accumulated charge is just the integral of the current, it may be shown that the voltage/current relations are

$$v = \frac{1}{C}\int_0^t i\,dt; \quad i = C\frac{dv}{dt}$$

Magnetic Fields

Magnetic fields *B* arise due to motion of charges, for example, a current *I* in a wire of length *l*. For a current element *Idl*

$$dB = \frac{\mu_0}{4\pi}\frac{Idl\sin\theta}{r^2}$$



B points into page

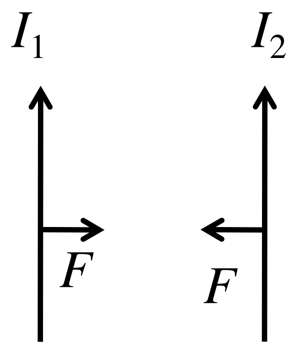
For an infinite length wire, the magnetic field circles the wire according to the right hand rule for current (thumb with the current, fingers curl with the field). Integrating the expression for the current element,

$$B = \frac{\mu_0 I}{2\pi R}$$

Magnetic Fields II

The force per unit length between two parallel infinite length wires is

$$\frac{F}{l} = I_2 \frac{\mu_0 I_1}{2\pi R}$$



Since it can produce forces, evidently the magnetic field also stores energy. This is measured by the inductance. For an inductor, the voltage/current relations are

$$v = L\frac{di}{dt};\quad i = \frac{1}{L}\int_{-\infty}^{t} v\,dt$$

Water and Electrical Current Flow Analogies

Battery; raises voltage=Pump; raises pressure
Each drive flow.

Resistance; causes voltage drop=Thinness of pipe; causes back pressure. Each resist flow.

Capacitance; stores charge in response to voltage=Storage tank; stores water in response to pressure for later release

Inductance; stores magnetic energy in response to current=Momentum of current flow; kinetic energy builds with flow volume/velocity

Amplifier; low voltage input controls high voltage output; Gate in dam; low energy control produces large water flow

Models and Abstractions

Physics models of circuits: laws by which moving charge results in EM fields, EM waves; energy conservation laws

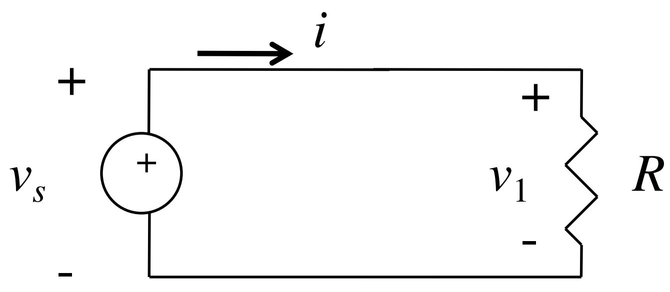
Essential for creating new devices and in understanding how circuit elements work.

But…

For building circuits, level of abstraction is too low; we need constructions that make calculations easier. This is similar to the relation of Newtonian and quantum mechanics. The latter is fundamental, but computationally intractable at large scale. In building systems, there are many levels of abstraction, suitable at different scales

Kirchhoff's Voltage Law

KVL: The sum of voltages in a closed circuit is zero

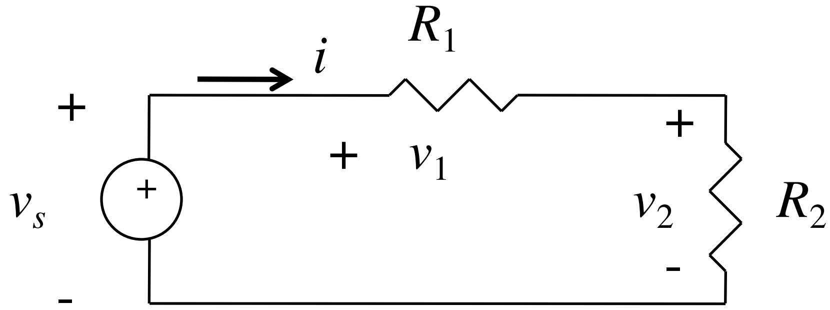


By Ohm's law, $v_1=iR$; by KVL, $v_s-v_1=0$. The signs attached to the voltage are - if it is a gain and + if a loss in the direction of the current (some books use the opposite convention).

Thus, $v_1-v_s=0$; $v_1=v_s$; $i=v_s/R$

EFTS

Determine v_1, v_2, i, in terms of v_s, R_1, R_2



Solution: By KVL and Ohm's Law,

$$v_s - v_1 - v_2 = 0; \;\; v_s = v_1 + v_2 = iR_1 + iR_2$$

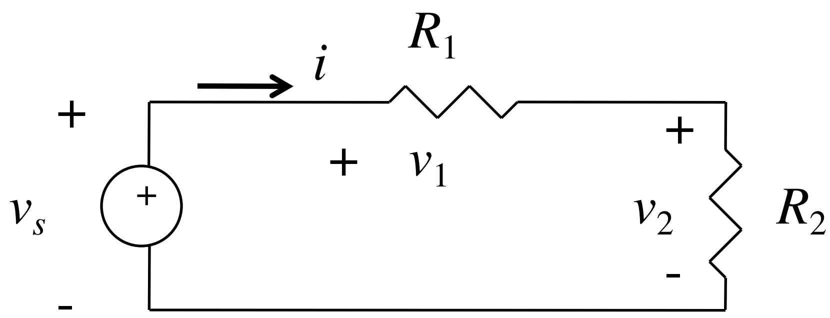
$$\therefore v_s = i(R_1 + R_2); \;\; i = \frac{v_s}{R_1 + R_2}$$

$$\therefore v_1 = iR_1 = \frac{v_s}{R_1 + R_2} R_1; \;\; v_2 = iR_2 = \frac{v_s}{R_1 + R_2} R_2$$

Example: $v_s = 12V, R_1 = 4\Omega, R_2 = 8\Omega.$

Then $i = 1A, v_1 = 4V, v_2 = 8V.$

Energy Interpretation



Q=charge moving through the circuit

W=work performed by the voltage source

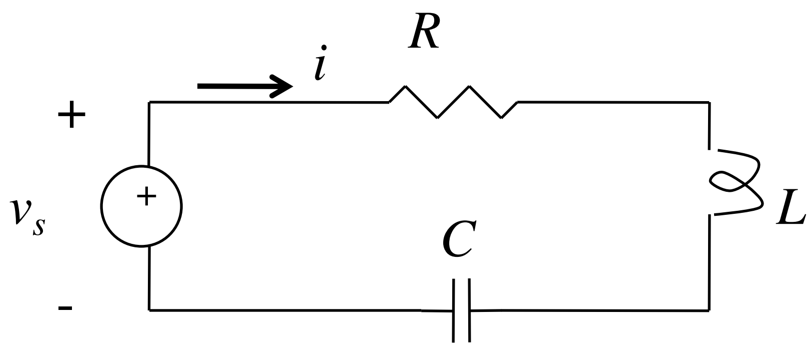
$$W = W_1 + W_2$$

$$\frac{W}{Q} = \frac{W_1}{Q} + \frac{W_2}{Q}$$

$$emf = v_s = v_1 + v_2$$

Thus, KVL is an energy conservation law.

Application to RLC Circuits

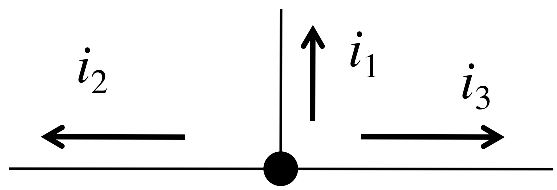


$$v_s = iR + L\frac{di}{dt} + \frac{1}{C}\int i\,dt$$

KVL applies to both static and variable sources, and to resistive and reactive loads.
We will later show how to solve such equations.

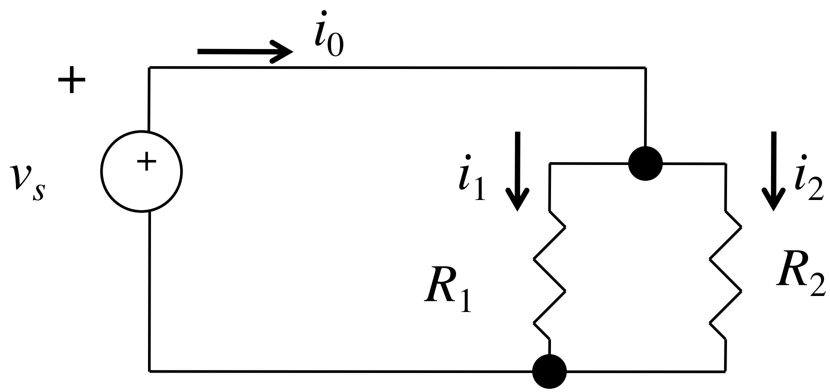
Kirchhoff's Current Law

KCL: the sum of the currents leaving a junction is zero



What does this law conserve? Obviously, charge.

Example



Apply KCL to the junction above the resistors

$$-i_0 + i_1 + i_2 = 0; i_0 = i_1 + i_2$$

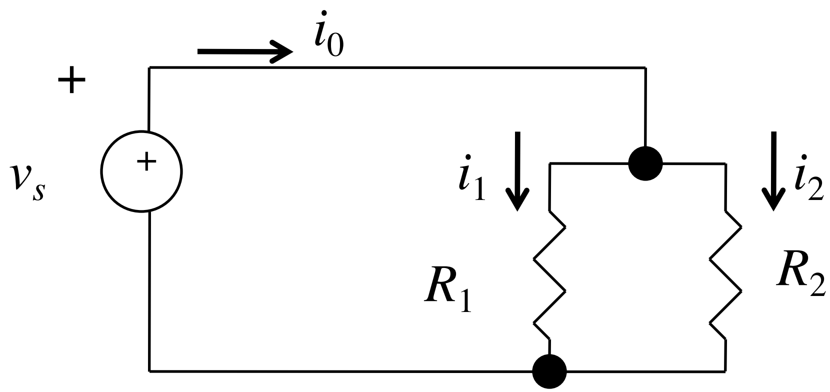
The +/- signs come from i_1 and i_2 leave while i_0 enters .

To determine the current values, use Ohm's Law and KVL

$$v_s = i_1 R_1 = i_2 R_2$$

The same conclusion is reached applying KCL to the bottom junction.

EFTS



Determine i_0, i_1, i_2 if v_s=10, R_1=10, R_2=5.

Solution:

$$v_s = i_1 R_1 = i_2 R_2$$

$$\therefore 10 i_1 = 10; i_1 = 1 \qquad 5 i_2 = 10; i_2 = 2$$

$$\therefore i_0 = 3$$

The equivalent resistive load *R'* of the parallel resistors is

$$\therefore i_0 R' = 10; R' = 10/3$$

Tricks

We can use KVL, KCL, etc. to solve for voltages and currents in complex circuits

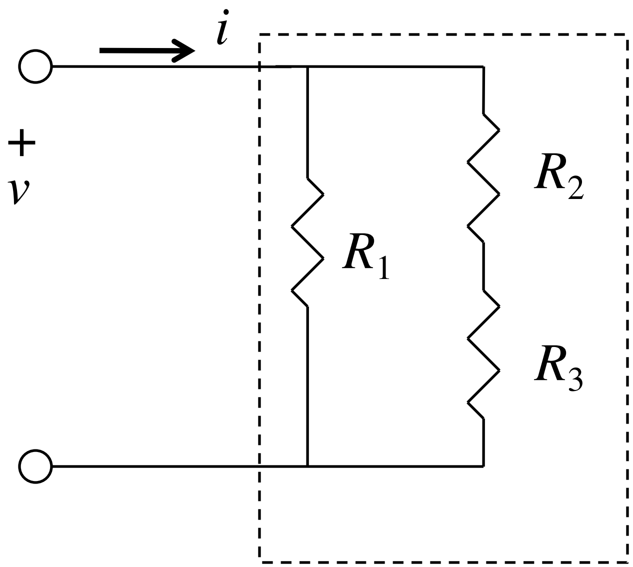
However, instead we build other methods based on them to simplify the calculations.

We will look at parallel and series circuits, mesh currents, node equations, and re-use of certain known solutions that frequently recur as circuit components.

Equivalent Circuits

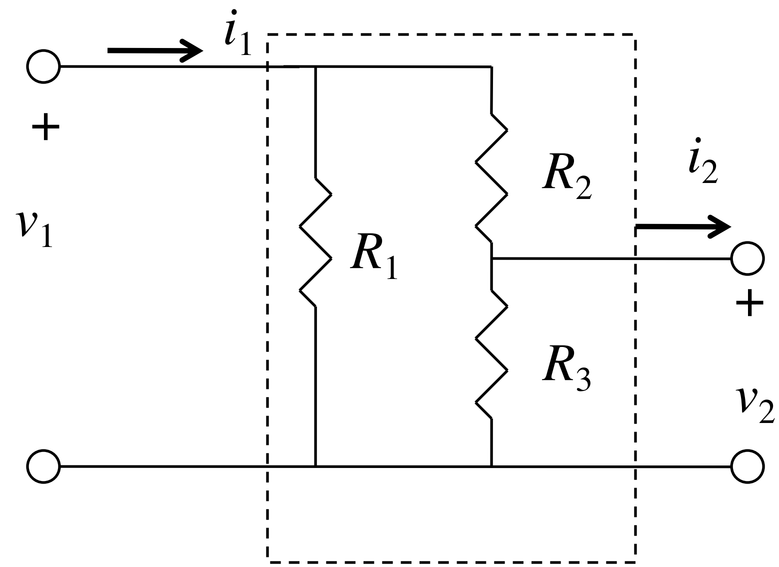
Two sets of circuits are considered equivalent if they present the same *v*-*i* relations to circuits that are attached to them

One-Port Network



1 *v*-*i* relation needed

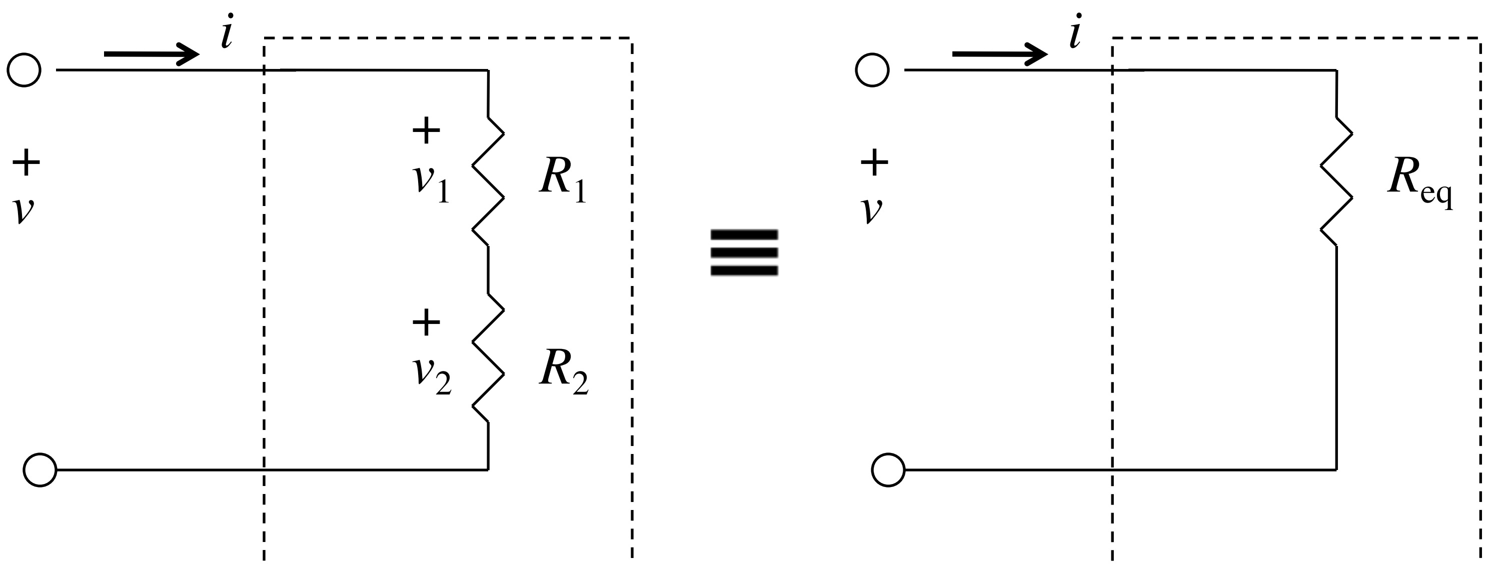
Two-Port Network



2 *v*-*i* relations needed

Series Circuits

Series resistors divide voltage among them but otherwise can be replaced with an equivalent resistor equal to their sum



Using KVL, $v = v_1 + v_2$

$$= iR_1 + iR_2$$

$$= i(R_1 + R_2) = iR_{eq}$$

Further, $v_2 = \dfrac{R_2}{R_1 + R_2} v$

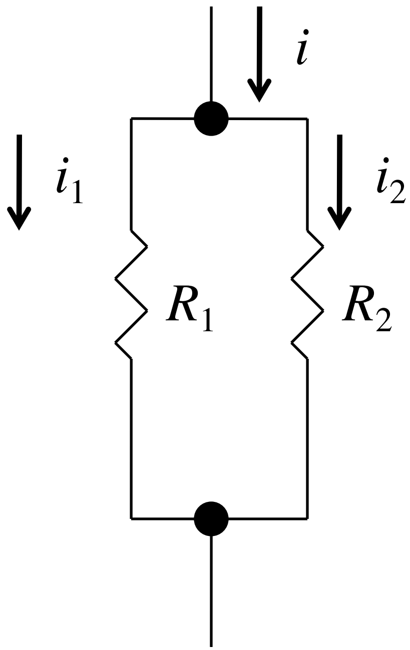
In general, $R_{eq} = R_1 + R_2 + K + R_n$

Similarly for series induct

$L_{eq} = L_1 + L_2 + K + L_n$

Parallel Circuits

Parallel resistances divide current among them



Using KCL, $i = i_1 + i_2$

$$i = \frac{v}{R_1} + \frac{v}{R_2}$$

$$i = \frac{R_1 + R_2}{R_1 R_2} v \equiv v / R_{eq}$$

Moreover, $i_2 = \dfrac{R_1}{R_1 + R_2} i$

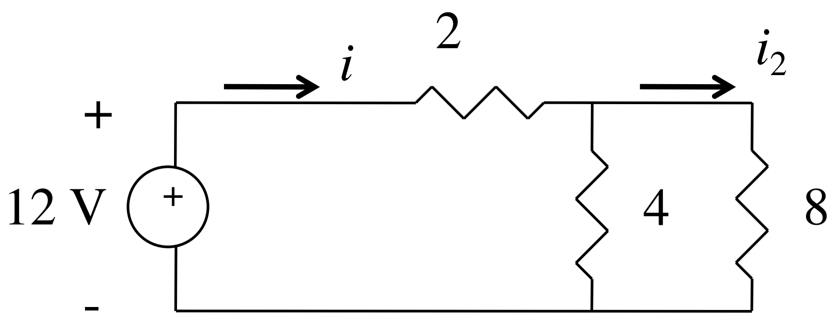
With 3 resistors, $R_{eq} = \dfrac{R_1 R_2 R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3}$

A fruitful approach is to always work with pairs, and then combine t

For parallel capacitors, $C_{eq} = C_1 + C_2 + \ldots + C_n$

Example

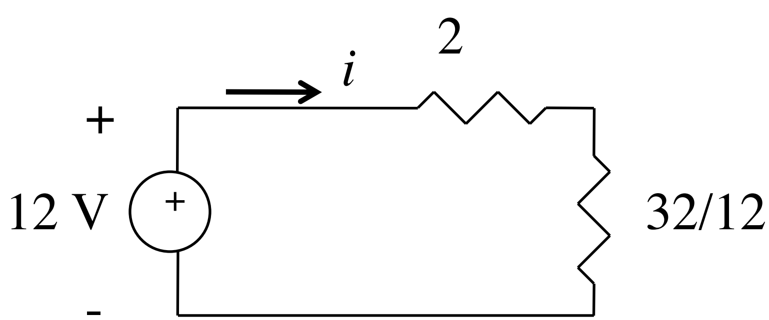
Rather than write mesh equations, we use relations for parallel and series resistances.



For the parallel resis

$$R_{par} = \frac{4(8)}{4+8} = 32/12$$

Now there are two resistors in series



For series resistors $R_{eq} = 2 + 32/12$

Since $v = iR_{eq} = 12, i = 2.57A$

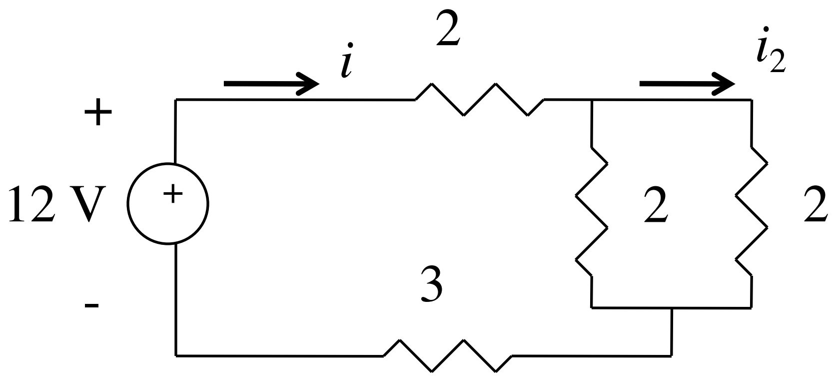
Using the voltage divider rel

$$v_{par} = \frac{32/12}{2+32/12}12 = 6.86$$

Using the current divider relation, $i_2 = \frac{4}{12} i = 0.86A$

EFTS

Determine the currents and voltages.



For the parallel resis

$$R_{par} = \frac{2(2)}{2+2} = 1$$

There are now three series resistors with values 2,1,3; R_{eq}=6

$V=iR_{eq}$; i=12/6=2 A. Thus the voltage drop across the first resistor is 4 V, 2 V across the parallel resistors, and 6 V across the last one.

Since the parallel resistances are equal, the current is equally divided, and i_2=1 A.

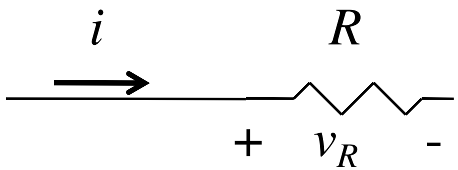
Summary

- To analyze circuits, we make use of physics and then build more abstract models
  - Lumped elements
  - KCL and KVL
  - Parallel and Series circuit
- Next we will consider "black box" approaches where combinations of elements are considered as if they were a single unit, and consider general methods for solving circuits

Appendix: Common Circuit Elements I

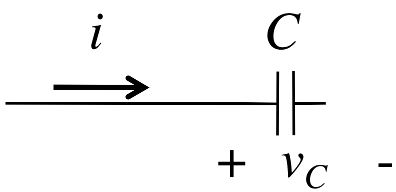
A useful abstraction is the concept of the circuit diagram with lumped circuit elements (in reality even wires have some resistance, inductance and capacitance)

Resistor



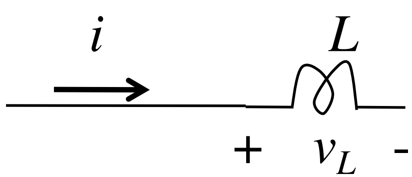
$v_R = iR$; Units = Ohms Ω

Capacitor



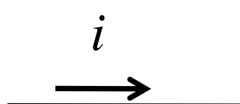
$i = C\frac{dv}{dt}$; $v = \frac{1}{C}\int_{-\infty}^{t} i\,dt$; Units = Farads

Inductor



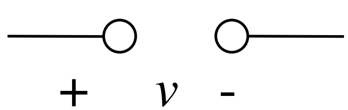
$v = L\frac{di}{dt}$; $i = \frac{1}{L}\int_{-\infty}^{t} V\,dt$; Units = Henrys

Short circuit



$v = 0$ for any i

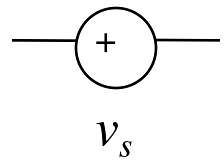
Open circuit



$i = 0$ for any v

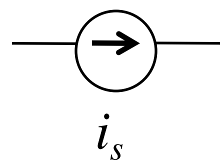
Common Circuit Elements II

Ideal voltage source



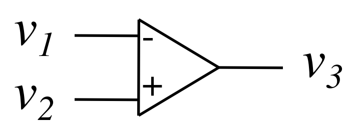
$v = v_s$ for any i

Ideal current source



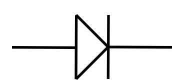
$i = i_s$ for any v

Operational amplifier



Amplification depends on bias ci

Diode



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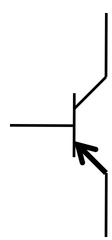
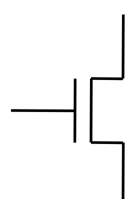
Transistor



or



Amplifies or satura