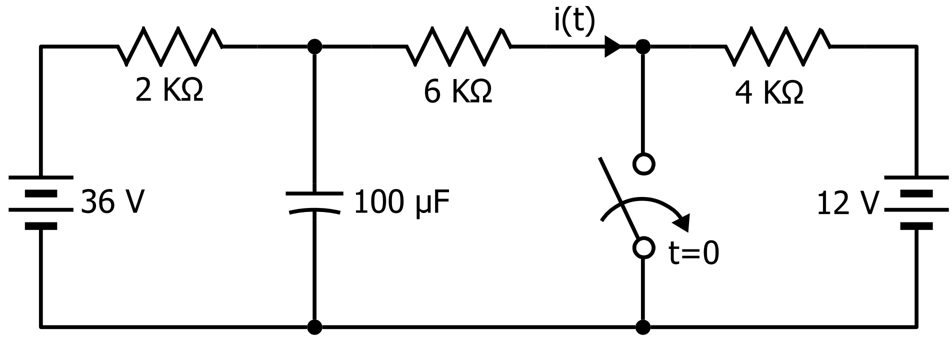
EE3 Fall 2020
Homework Problem 7

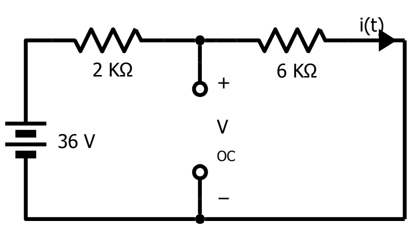
This problem is all about i(t).
a. What is i(0−)?
b. What is i(0+)?
c. What is i(∞)?
d. What is i(t), t>0?



The circuit has been in this conditon for a long time.

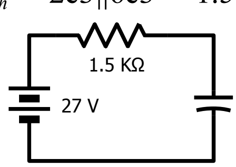
This problem can be solved without using a differential equation, though you may do so this way if you wish. Solving it without using a differential equation is done by pulling together a few ideas and bringing them to bear on the problem. That means using the answers to the first 3 parts of this problem, plus the judicious use of a Thévenin equivalent, plus the third slide of Lecture 4 in Week 4 of CCLE.

Thévenin Equivalent Solution:



$$V_{th} = V_{OC} = (36)\left(\frac{6}{6+2}\right) = 27\ \text{V}$$

$$R_{th} = 2\text{e}3 \| 6\text{e}3 = 1.5\ \text{K}\Omega$$



$$v_C(0^+) = v_C(0^-) = 12 + (36-12)\cdot\left(\frac{(6\text{e}3+4\text{e}3)}{(6\text{e}3+4\text{e}3)+2\text{e}3}\right) = 32\ \text{V}$$

By Lecture 4, third slide, $v_C(t) = V_S - [V_S - v_C(0^+)]e^{-\frac{t}{RC}}$

$$v_C(t) = 27 - [27-32]e^{-\frac{t}{(1.5\text{e}3)\cdot(100\text{e-}6)}} = 27 + 5e^{-\frac{t}{0.15}}$$

$$i(t) = \frac{v(t)}{6\text{e}3} = \left(4.5 + 0.83e^{-\frac{t}{0.15}}\right)\ \text{mA}$$

ODE Solution:

$$\frac{v-36}{2\text{e}3} + C\frac{dv_C}{dt} + \frac{v}{6\text{e}3} = 0$$

$$\frac{dv_C}{dt} + 6.67v_C = 180$$

$$P = 6.67;\ Q = 180 \rightarrow e^{\int_0^t P\,dX} = e^{6.67t}$$

$$e^{6.67t}\frac{dv_C}{dt} + 6.67e^{6.67t}\,v_C = e^{6.67t}$$

$$\frac{d}{dt}\left(e^{6.67t}\,v_C\right) = 180\ e^{6.67t}$$

$$e^{6.67t}\,v_C = 100\int e^{6.67t}\,dt = 27e^{6.67t} + C$$

$$v_C(t) = 27 + Ce^{-6.67t}$$

$$v_C(0^+) = 32 = 27 + C \rightarrow C = 5$$

$$v_C(t) = 5e^{-6.67t} + 27$$

$$i(t) = 0.83e^{-6.67t} + 4.5$$