21S-COMSCIM146-1 Exam 2

NEVIN LIANG

TOTAL POINTS

97 / 100

QUESTION 1

20 pts

1.1 **6 / 6**

- ✓ + **6 pts** All correct
- + **4 pts** Most line segaments are correct
- + **2 pts** Few line segaments are correct
- + **0 pts** Incorrect

1.2 **7 / 7**

- ✓ + **7 pts** Correct
- + **3.5 pts** k=1 correct
- + **2 pts** k=1 partially correct
- + **3.5 pts** k=3 correct
- + **2 pts** k=3 partially correct
- - **1 pts** Minor mistake
- - **2 pts** Major mistake
- + **0 pts** Incorrect

1.3 **7 / 7**

- ✓ + **7 pts** Correct
- + **3.5 pts** k=1 correct
- + **3.5 pts** k=3 correct
- + **2 pts** k=3 partially correct with details
- + **1 pts** k=3 partially correct wihtout details
- + **0 pts** Incorrect
- - **2 pts** Major mistake

QUESTION 2

20 pts

2.1 **3 / 3**

- ✓ + **3 pts** Correct

2.2 **5 / 5**

- ✓ + **5 pts** Correct
- + **2 pts** Primal formalation is correct
- + **2 pts** Explicit objective is correct
- + **1 pts** Explicit objective is partially correct
- + **3 pts** Explicit constrints is correct
- + **2 pts** Explicit constrains is largely correct
- + **1 pts** Explicit constrains is partially correct
- + **0 pts** Incorrect

2.3 **6 / 6**

- ✓ + **6 pts** Correct
- + **2 pts** Plot is correct
- + **1 pts** Plot is partially correct
- + **2 pts** Support vectors are correct
- + **2 pts** Normal vector is correct
- + **0 pts** Incorrect

2.4 **5 / 6**

- + **6 pts** Correct
- ✓ + **2 pts** Dual formulation is correct
- ✓ + **2 pts** alpha correct
- ✓ + **1 pts** w correct
- + **1 pts** b correct
- - **2 pts** Major calculation mistake
- - **1 pts** minor mistake
- - **1 pts** Off by a factor
- + **0 pts** Incorrect

QUESTION 3

20 pts

3.1 **3 / 3**

- ✓ + **3 pts** Correct
- + **0 pts** Incorrect

3.2 **5 / 5**

- ✓ + **5 pts** Correct

- 3 pts w not optimal
+ 0 pts Incorrect
+ 2.5 pts w correct

3.3 6 / 6

✓ + 6 pts Correct
+ 0 pts Incorrect

3.4 6 / 6

✓ + 6 pts Correct
+ 0 pts Incorrect
+ 1 pts Show some understanding of applying a kernel
+ 2 pts Classification accuracy is corret

QUESTION 4

20 pts

4.1 15 / 15

✓ + 15 pts All correct
+ 3 pts Prior correct
+ 3 pts Fatigue correct
+ 3 pts Fever correct
+ 1.5 pts Fever partially correct
+ 3 pts Cough corect
+ 3 pts Headache correct

4.2 5 / 5

✓ + 5 pts Correct
+ 2 pts Decison formula is correct
- 1 pts Correct result from undesired model

QUESTION 5

20 pts

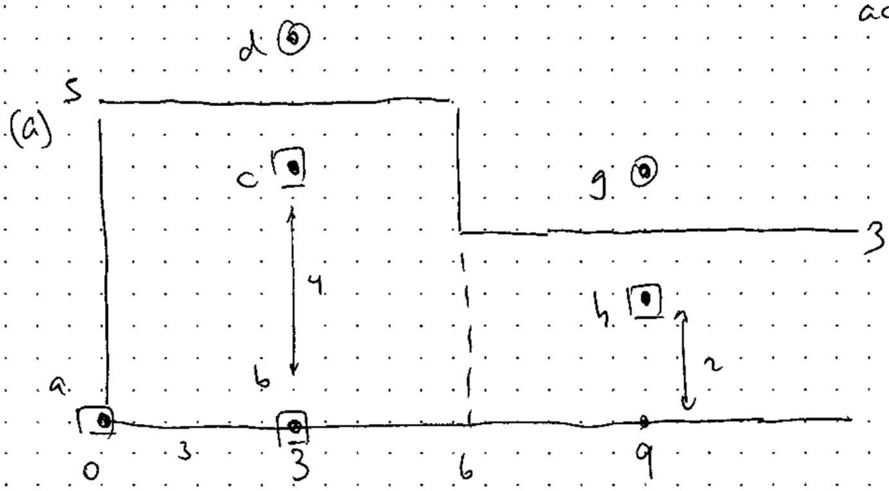
5.1 10 / 10

✓ + 10 pts All Correct
+ 2 pts mu0 correct
+ 2 pts mu1 correct
+ 2 pts prior correct
+ 4 pts Sigma correct
+ 2 pts Equation for Sigma correct
+ 0 pts Incorrect

5.2 8 / 10

+ 10 pts All correct
+ 0 pts Incorrect
✓ + 4 pts Decision boundary formulation is correct
+ 4 pts w correct
✓ + 2 pts Equation for w is correct
✓ + 2 pts b correct
+ 1 pts Equation for b is correct
- 2 pts Off by a factor
+ 1 pts Sigma inverse is correct

I Nevin Wang w/ UID 705575353 have read & understood the policy of academic integrity.



(b) for $k=1$: leave out a : ✓ b : ✓ c : ✗ d : ✗ e : ✓ f : ✓ g : ✗ h : ✗

$\boxed{50\%}$

$k=3$: a : ✓ b : ✓ c : ✓ d : ✗ e : ✓ f : ✓ g : ✓ h : ✗

$\boxed{75\%}$

(c) $k=1$: leave out a : ✓ b : ✓ c : ✗ d : ✗ e : ✓ f : ✓ g : ✗ h : ✗

$\boxed{50\%}$

$k=3$: leave out a : ✓ b : ✓ c : ✗ d : ✗ e : ✓ f : ✓ g : ✗ h : ✗

$\boxed{50\%}$

1.1 6 / 6

✓ **+ 6 pts** All correct

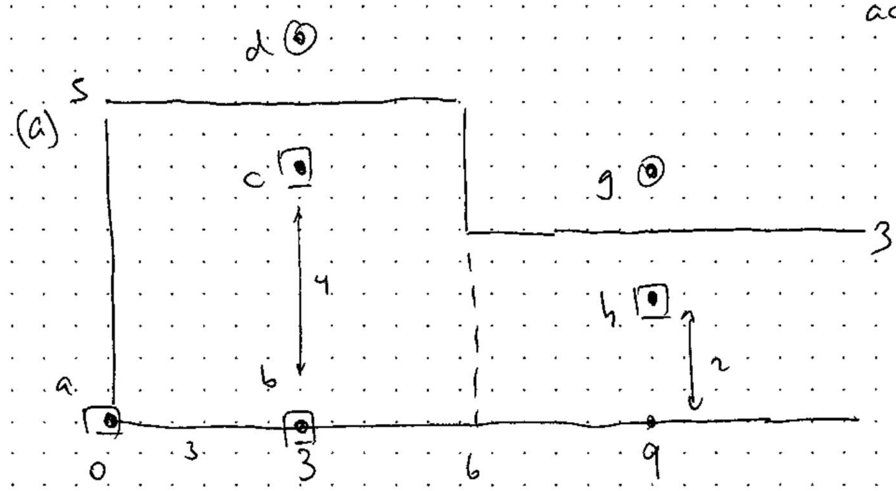
+ 4 pts Most line segaments are correct

+ 2 pts Few line segaments are correct

+ 0 pts Incorrect

e f

I Nevin Wang w/ UID 705575353 have read & understood the policy of academic integrity.



(b) for k=1 : leave out a : ✓ d: ✗ g: ✗
b : ✓ e: ✓ h: ✗
c : ✗ f: ✓

$\boxed{50\%}$

k=3 : a : ✓ c: ✓ e: ✓ g: ✓
b: ✓ d: ✗ f: ✓ h: ✗

$\boxed{75\%}$

(c) k=1 : leave out a: ✓ d: ✗ g: ✗
b: ✓ e: ✓ h: ✗
c: ✗ f: ✓

$\boxed{50\%}$

k=3: leave out a: ✓ d: ✗ g: ✗
b: ✓ e: ✓ h: ✗
c: ✗ f: ✓

$\boxed{50\%}$

1.2 **7 / 7**

✓ **+ 7 pts** Correct

+ 3.5 pts k=1 correct

+ 2 pts k=1 partially correct

+ 3.5 pts k=3 correct

+ 2 pts k=3 partially correct

- 1 pts Minor mistake

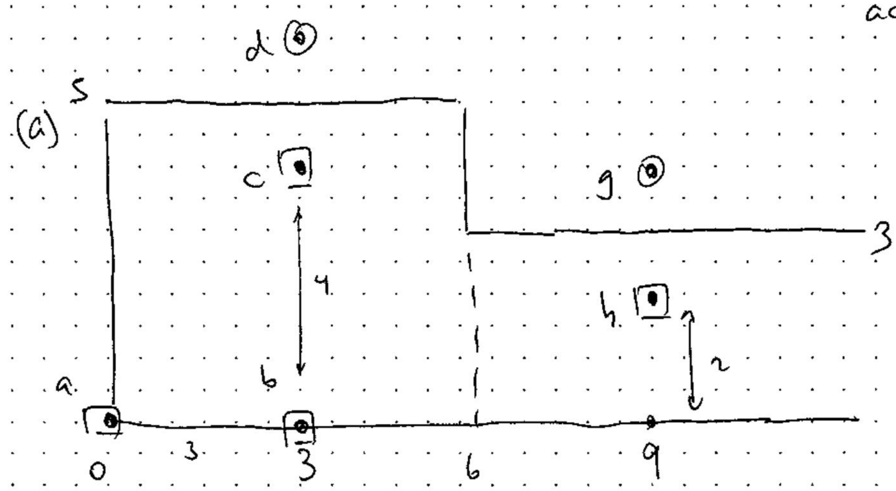
- 2 pts Major mistake

+ 0 pts Incorrect

I Nevin Wang w/ UID 705575353 have read & understood the policy of academic integrity.

11

(a)



(b) for k=1 : leave out

a : ✓ d: ✗ g: ✗

b : ✓ e: ✓ h: ✗

c : ✗ f: ✓

$\boxed{50\%}$

k=3 : a: ✓ c: ✓ e: ✓ g: ✓

b: ✓ d: ✗ f: ✓ h: ✗

$\boxed{75\%}$

(c) k=1 : leave out a: ✓ d: ✗ g: ✗

b: ✓ e: ✓ h: ✗

c: ✗ f: ✓

$\boxed{50\%}$

k=3: leave out a: ✓ d: ✗ g: ✗

b: ✓ e: ✓ h: ✗

c: ✗ f: ✓

$\boxed{50\%}$

1.3 **7 / 7**

- ✓ **+ 7 pts** Correct
- **+ 3.5 pts** k=1 correct
- **+ 3.5 pts** k=3 correct
- **+ 2 pts** k=3 partially correct with details
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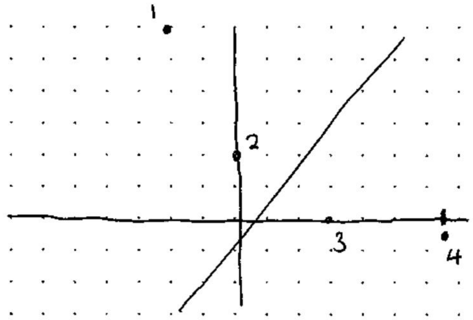
(2) (a)



2 units

yes linearly seperable

(b)

$$\min_{w,b} \frac{1}{2}\|w\|^2 \quad \text{subj. to} \quad y_n(w^T x_n + b) \geq 1 \quad (\forall n)$$

$$\min_{w_1,w_2,b} \frac{1}{2}(w_1^2+w_2^2) \quad \text{subj to} \quad \underbrace{y_n([w_1 \ \ w_2] \cdot x_n + b) \geq 1}_{\downarrow} \quad (\forall n)$$

$$\begin{bmatrix} -4w_1+12w_2+b \geq 1 \\ 4w_2+b \geq 1 \\ -6w_1+b \geq 1 \\ -13w_1+w_2+b \geq 1 \end{bmatrix}$$

(c)

$m_{23} = \frac{0-4}{6-0} = \frac{-2}{3}$

$\perp m_{23} = \frac{3}{2}$ midpoint$_{23} = (3,2)$

$y = \frac{3}{2}x + b$

$\frac{9}{2} + b = 2$ $b = -\frac{5}{2}$

$\boxed{y = \frac{3}{2}x - \frac{5}{2}}$

(d)

$$\mathcal{L} = \sum_{n=1}^{N} \alpha_n - \frac{1}{2}\sum_{n,k=1}^{N} \alpha_n \alpha_k y_n y_k x_n^T x_k \quad \boxed{\text{s.t.}} \quad \sum_{n=1}^{N} \alpha_n y_n = 0 \quad \alpha_n \geq 0$$

$\alpha_2 - \alpha_3 = 0 \rightarrow \alpha_2 = \alpha_3$

$$= (\alpha_2 + \alpha_3) - \frac{1}{2}\left(\alpha_2^2 \|x_2\|^2 - 2\alpha_2\alpha_3 x_2^T x_3 + \alpha_3^2 \|x_3\|^2\right)$$

$\|x_2\|^2 = 16$ $\|x_3\|^2 = 36$ $x_2^T x_3 = [0 \ \ 4]\begin{bmatrix}6\\0\end{bmatrix} = 0$

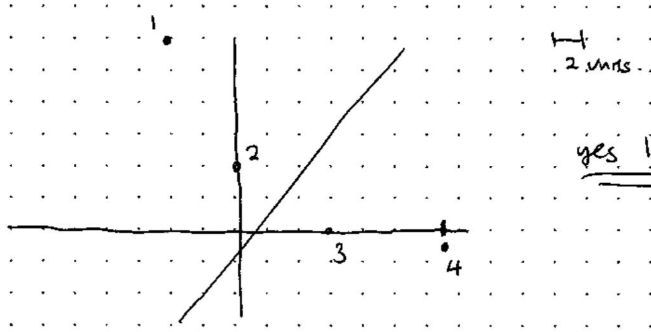
$$\mathcal{L} = \alpha_2 + \alpha_3 - 8\alpha_2^2 - 18\alpha_3^2 \rightarrow 2\alpha_2 - 26\alpha_2^2 \rightarrow \frac{\partial \mathcal{L}}{\partial \alpha_2} = 2 - 52\alpha_2 = 0$$

$$\boxed{\alpha_2 = \frac{1}{26} \rightarrow a_3 = \frac{1}{26}}$$

2.1 3 / 3

✓ **+ 3 pts** Correct

(2) (a)



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2.2 5 / 5

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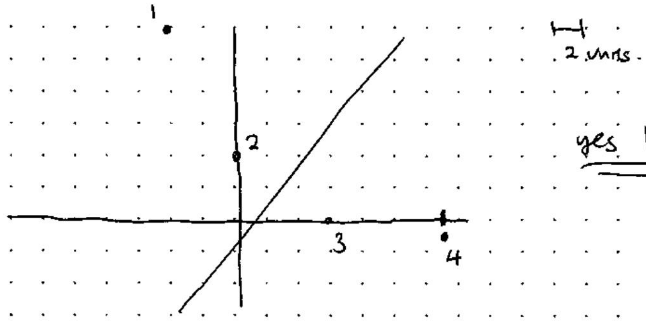

yes linearly seperable

(b)

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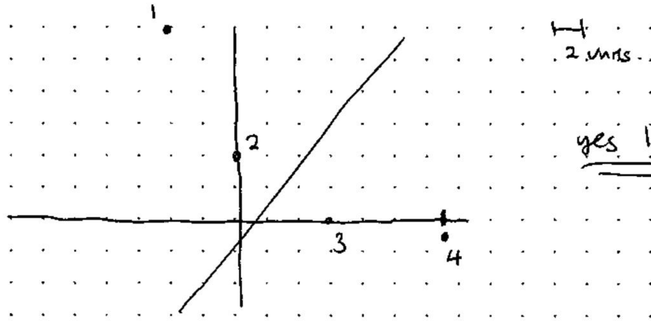
$$\boxed{\alpha_2 = \frac{1}{26} \rightarrow a_3 = \frac{1}{26}}$$

2.3 **6 / 6**

- ✓ **+ 6 pts** Correct
- **+ 2 pts** Plot is correct
- **+ 1 pts** Plot is partially correct
- **+ 2 pts** Support vectors are correct
- **+ 2 pts** Normal vector is correct
- **+ 0 pts** Incorrect

(2) (a)



yes linearly seperable

(b)

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$$\boxed{\alpha_2 = \frac{1}{26} \rightarrow \alpha_3 = \frac{1}{26}}$$

$$w = \sum_{n\in S} a_n t_n x_n = \frac{1}{26}\left(\begin{bmatrix}0\\4\end{bmatrix} - \begin{bmatrix}6\\0\end{bmatrix}\right)$$

$$= \boxed{\begin{bmatrix}-3/13\\ 2/13\end{bmatrix}}$$

$$b = \frac{1}{|S|}\sum_{n\in S}\left[t_n - w^T x_n\right] = \frac{1}{2}\left(1 - \begin{bmatrix}-3/13 & 2/13\end{bmatrix}\begin{bmatrix}0\\4\end{bmatrix} + (-1) - \begin{bmatrix}-3/13 & 2/13\end{bmatrix}\begin{bmatrix}6\\0\end{bmatrix}\right)$$

matlab

$$= \boxed{-1}$$

2.4 5 / 6

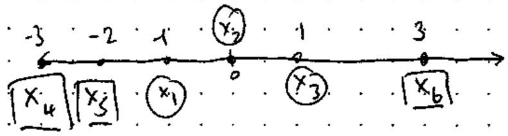
+ **6 pts** Correct
✓ + **2 pts** **Dual formulation is correct**
✓ + **2 pts** **alpha correct**
✓ + **1 pts** **w correct**
+ **1 pts** b correct
- **2 pts** Major calculation mistake
- **1 pts** minor mistake
- **1 pts** Off by a factor
+ **0 pts** Incorrect

#3)

(a)

$\bigcirc = -1$

$\square = +1$



$\boxed{\text{No.}}$ not linearly seperable

(b) ~~$-w_1+w_0=-1$~~

$\cancel{-w_1+w_0<0}$

$\cancel{w_0<0}$

$\cancel{w_1<0}$

$\cancel{w_0<w_1}$

$\cancel{-3w_1+w_0>0}$

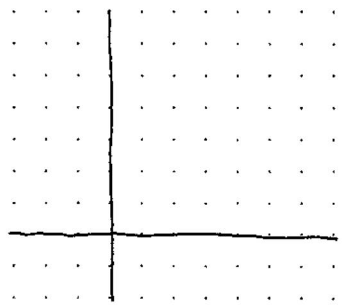
$\cancel{-2w_1+w_0>0}$

$\cancel{3w_1+w_0>0}$

$\cancel{w_0>3w_1}$

$\cancel{w_0>2w_1}$

$w_0>-3w_1$



$w_1=-1 \quad w_0=-1.5$

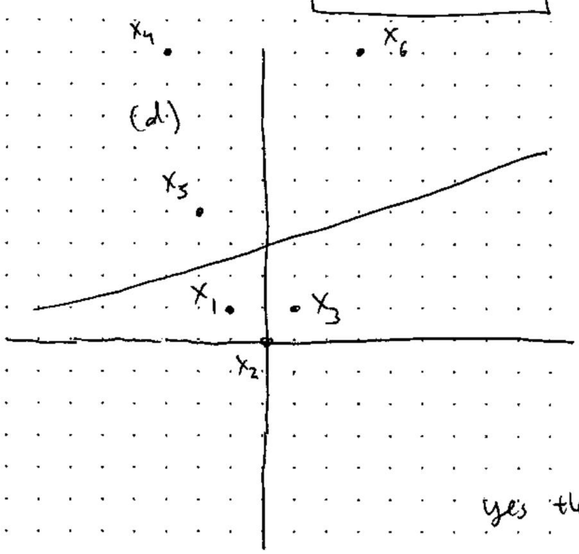
$$W=\begin{bmatrix} w_0 \\ w_1 \end{bmatrix} = \boxed{\begin{bmatrix} -1.5 \\ -1 \end{bmatrix}}$$

classification accuracy $\boxed{5/6}$

(c) if $\boxed{\phi(x)=\begin{bmatrix} x \\ x^2 \end{bmatrix}}$ b/c $\phi(u)'\cdot\phi(v)=[u \ u^2]\begin{bmatrix} v \\ v^2 \end{bmatrix} = uv+u^2v^2$

$= K(u,v)$

(d)



$$\underbrace{\begin{aligned} x_1=-1 &\rightarrow \begin{bmatrix} -1 \\ 1 \end{bmatrix} \\ x_2=0 &\rightarrow \begin{bmatrix} 0 \\ 0 \end{bmatrix} \\ x_3=1 &\rightarrow \begin{bmatrix} 1 \\ 1 \end{bmatrix} \end{aligned}}_{-1} \qquad \underbrace{\begin{aligned} x_4=-3 &\rightarrow \begin{bmatrix} -3 \\ 9 \end{bmatrix} \\ x_5=-2 &\rightarrow \begin{bmatrix} -2 \\ 4 \end{bmatrix} \\ x_6=3 &\rightarrow \begin{bmatrix} 3 \\ 9 \end{bmatrix} \end{aligned}}_{+1}$$

yes they are lin sep. $\boxed{\text{100\% accuracy}}$

3.1 3 / 3

✓ **+ 3 pts** Correct

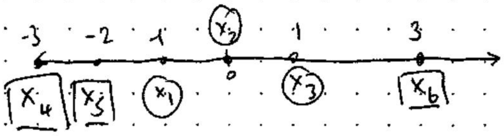
+ 0 pts Incorrect

#3)

(a)

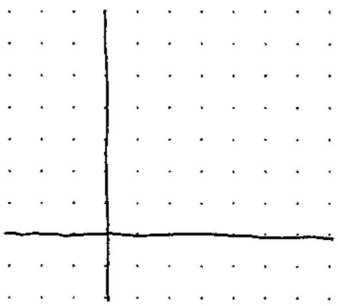
$\bigcirc = -1$

$\square = +1$



$\boxed{\text{No.}}$ not linearly seperable

(b) $\cancel{-w_1 + w_0 = -1}$



$-w_1 + w_0 < 0$

$w_0 < 0$

$w_1 < 0$

$w_0 < w_1$

$-3w_1 + w_0 > 0$

$-2w_1 + w_0 > 0$

$3w_1 + w_0 > 0$

$w_0 > 3w_1$

$\cancel{w_0 > 2w_1}$

$w_0 > -3w_1$

$w_1 = -1 \quad w_0 = -1.5$

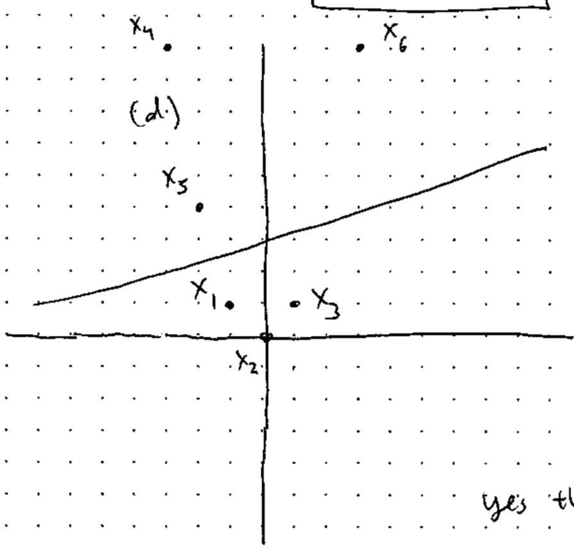
$$W = \begin{bmatrix} w_0 \\ w_1 \end{bmatrix} = \boxed{\begin{bmatrix} -1.5 \\ -1 \end{bmatrix}}$$

classification accuracy $\boxed{5/6}$

(c) if $\boxed{\phi(x) = \begin{bmatrix} x \\ x^2 \end{bmatrix}}$ b/c $\phi(u)' \cdot \phi(v) = [u \ u^2] \begin{bmatrix} v \\ v^2 \end{bmatrix} = uv + u^2v^2$

$= K(u,v)$

(d)



$$\underbrace{\begin{array}{l} x_1 = -1 \to \begin{bmatrix} -1 \\ 1 \end{bmatrix} \\ x_2 = 0 \to \begin{bmatrix} 0 \\ 0 \end{bmatrix} \\ x_3 = 1 \to \begin{bmatrix} 1 \\ 1 \end{bmatrix} \end{array}}_{-1} \qquad \underbrace{\begin{array}{l} x_4 = -3 \to \begin{bmatrix} -3 \\ 9 \end{bmatrix} \\ x_5 = -2 \to \begin{bmatrix} -2 \\ 4 \end{bmatrix} \\ x_6 = 3 \to \begin{bmatrix} 3 \\ 9 \end{bmatrix} \end{array}}_{+1}$$

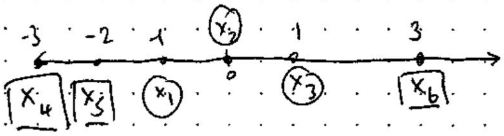
yes they are lin sep. $\boxed{\text{100\% accuracy}}$

3.2 **5 / 5**

- ✓ **+ 5 pts** Correct
- **- 3 pts** w not optimal
- **+ 0 pts** Incorrect
- **+ 2.5 pts** w correct

#3)

(a)



$\bigcirc = -1$

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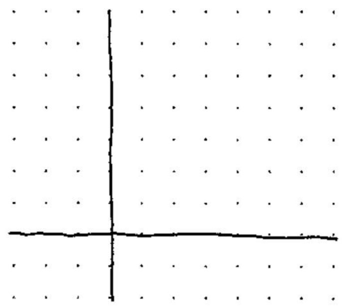
$\boxed{\text{No.}}$ not linearly seperable

(b) ~~$-w_1 + w_0 = -1$~~



$-w_1 + w_0 < 0$

$w_0 < 0$

$w_1 < 0$

$w_0 < w_1$

$-3w_1 + w_0 > 0$

$-2w_1 + w_0 > 0$

$3w_1 + w_0 > 0$

$w_0 > 3w_1$

~~$w_0 > 2w_1$~~

$w_0 > -3w_1$

$w_1 = -1 \quad w_0 = -1.5$

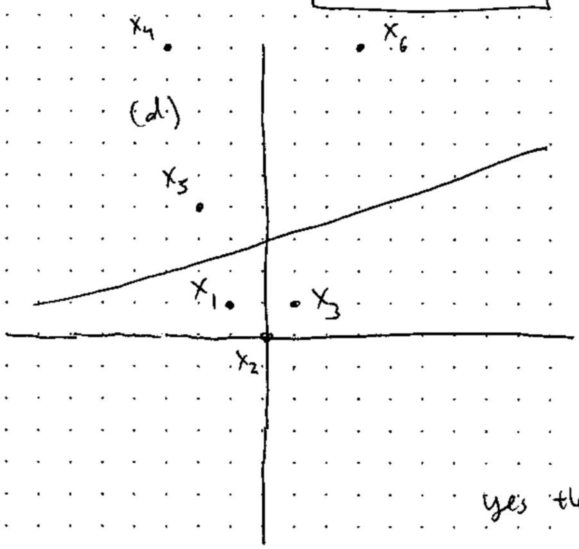
$$W = \begin{bmatrix} w_0 \\ w_1 \end{bmatrix} = \boxed{\begin{bmatrix} -1.5 \\ -1 \end{bmatrix}}$$

classification accuracy $\boxed{5/6}$

(c) if $\boxed{\phi(x) = \begin{bmatrix} x \\ x^2 \end{bmatrix}}$ b/c $\phi(u)' \cdot \phi(v) = [u \;\; u^2] \begin{bmatrix} v \\ v^2 \end{bmatrix} = uv + u^2v^2$

$= K(u, v)$

(d)



$$\underbrace{\begin{matrix} x_1 = -1 \rightarrow \begin{bmatrix} -1 \\ 1 \end{bmatrix} \\ x_2 = 0 \rightarrow \begin{bmatrix} 0 \\ 0 \end{bmatrix} \\ x_3 = 1 \rightarrow \begin{bmatrix} 1 \\ 1 \end{bmatrix} \end{matrix}}_{-1} \qquad \underbrace{\begin{matrix} x_4 = -3 \rightarrow \begin{bmatrix} -3 \\ 9 \end{bmatrix} \\ x_5 = -2 \rightarrow \begin{bmatrix} -2 \\ 4 \end{bmatrix} \\ x_6 = 3 \rightarrow \begin{bmatrix} 3 \\ 9 \end{bmatrix} \end{matrix}}_{+1}$$

yes they are lin sep. $\boxed{\text{100\% accuracy}}$

3.3 6 / 6

✓ + **6 pts** Correct

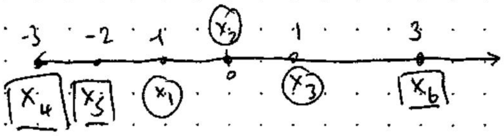
+ **0 pts** Incorrect

#3)

(a)

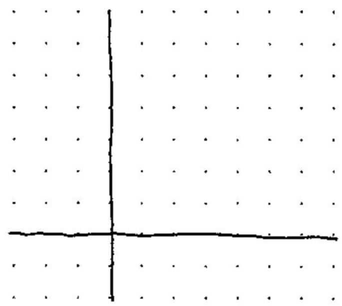
$\bigcirc = -1$

$\square = +1$



$\boxed{\text{No.}}$ not linearly seperable

(b) $\cancel{-w_1 + w_0 = -1}$



$\cancel{-w_1 + w_0 < 0}$

$\cancel{w_0 < 0}$

$\cancel{w_1 < 0}$

$\cancel{w_0 < w_1}$

$\cancel{-3w_1 + w_0 > 0}$

$\cancel{-2w_1 + w_0 > 0}$

$\cancel{3w_1 + w_0 > 0}$

$\cancel{w_0 > 3w_1}$

$\cancel{w_0 > 2w_1}$

$\cancel{w_0 > -3w_1}$

$w_1 = -1 \quad w_0 = -1.5$

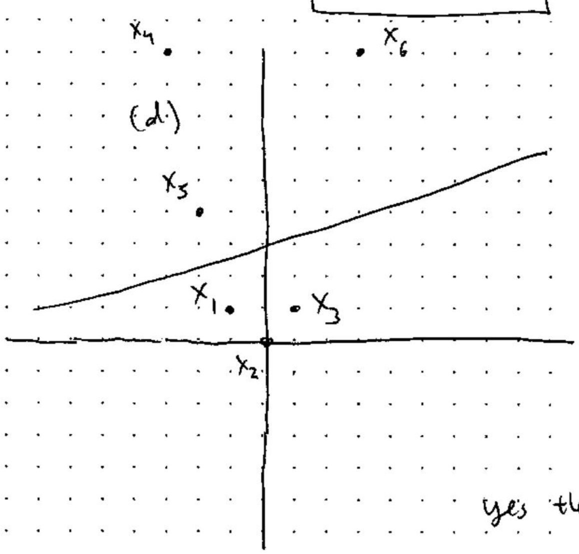
$$W = \begin{bmatrix} w_0 \\ w_1 \end{bmatrix} = \boxed{\begin{bmatrix} -1.5 \\ -1 \end{bmatrix}}$$

classification accuracy $\boxed{5/6}$

(c) if $\boxed{\phi(x) = \begin{bmatrix} x \\ x^2 \end{bmatrix}}$ b/c $\phi(u)' \cdot \phi(v) = [u \ \ u^2] \begin{bmatrix} v \\ v^2 \end{bmatrix} = uv + u^2v^2$

$= K(u,v)$

(d)



$x_1 = -1 \rightarrow \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

$x_2 = 0 \rightarrow \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$x_3 = 1 \rightarrow \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$\underbrace{}_{-1}$

$x_4 = -3 \rightarrow \begin{bmatrix} -3 \\ 9 \end{bmatrix}$

$x_5 = -2 \rightarrow \begin{bmatrix} -2 \\ 4 \end{bmatrix}$

$x_6 = 3 \rightarrow \begin{bmatrix} 3 \\ 9 \end{bmatrix}$

$\underbrace{}_{+1}$

yes they are lin sep. $\boxed{\text{100\% accuracy}}$

3.4 6 / 6

- ✓ **+ 6 pts** Correct
- **+ 0 pts** Incorrect
- **+ 1 pts** Show some understanding of applying a kernel
- **+ 2 pts** Classification accuracy is corret

#4) Fatigue : A   X Disease Y
Fever : B
Cough : C
Headache : D

(a) $P(Y=0) = 3/8$ $P(Y=1) = 5/8$

$$\begin{bmatrix} P(A=0 \mid Y=0) = 2/3 & P(C=0 \mid Y=0) = 3/3 \\ P(A=0 \mid Y=1) = 2/5 & P(C=0 \mid Y=1) = 2/5 \\ P(B=0 \mid Y=0) = 3/3 & P(D=0 \mid Y=0) = 1/3 \\ P(B=0 \mid Y=1) = 1/5 & P(D=0 \mid Y=1) = 2/5 \end{bmatrix}$$

$\{$The probabilities for $P(X=1 \mid Y=?)$ are are $1 - P(X=0 \mid Y=?)$ $\}$

(b) #9 $A=1$ $B=0$ $C=1$ $D=0$

if $Y=0$: $\frac{3}{8} \cdot \frac{1}{3} \cdot \frac{3}{3} \cdot \frac{0}{3} \cdot \frac{1}{3} = 0$

$Y=1$: $\frac{5}{8} \cdot \frac{3}{5} \cdot \frac{1}{5} \cdot 3/5 \cdot \frac{2}{3} = 0.018$

$P(Y=1) > P(Y=0)$ $\boxed{\text{PATIENT 9 HAS DISEASE}}$

4.1 15 / 15

- ✓ + **15 pts** **All correct**
- + **3 pts** Prior correct
- + **3 pts** Fatigue correct
- + **3 pts** Fever correct
- + **1.5 pts** Fever partially correct
- + **3 pts** Cough corect
- + **3 pts** Headache correct

#4) Fatigue : A   X Disease : Y
Fever : B
Cough : C
Headache : D

(a) $P(Y=0) = 3/8$ $P(Y=1) = 5/8$

$$\left[\begin{array}{ll} P(A=0 \mid Y=0) = 2/3 & P(C=0 \mid Y=0) = 3/3 \\ P(A=0 \mid Y=1) = 2/5 & P(C=0 \mid Y=1) = 2/5 \\ P(B=0 \mid Y=0) = 3/3 & P(D=0 \mid Y=0) = 1/3 \\ P(B=0 \mid Y=1) = 1/5 & P(D=0 \mid Y=1) = 2/5 \end{array}\right]$$

{The probabilities for $P(X=1 \mid Y=?)$ are $1 - P(X=0 \mid Y=?)$}

(b) #9 $A=1$ $B=0$ $C=1$ $D=0$

if $Y=0$: $\frac{3}{8} \cdot \frac{1}{3} \cdot \frac{3}{3} \cdot \frac{0}{3} \cdot \frac{1}{3} = 0$

$Y=1$: $\frac{5}{8} \cdot \frac{3}{5} \cdot \frac{1}{5} \cdot 3/5 \cdot \frac{2}{3} = 0.018$

$P(Y=1) > P(Y=0)$ $\boxed{\text{PATIENT 9 HAS DISEASE}}$

4.2 5 / 5

- ✓ **+ 5 pts** Correct
- **+ 2 pts** Decison formula is correct
- **- 1 pts** Correct result from undesired model

(5)

(a) $\phi = p(y=1) = \boxed{\frac{1}{2}}$ $(\frac{2}{4})$ b/c $y_1, y_2 = 1$

$$\mu_0 = \frac{1}{N_0} \sum_{n \in C_0} x_n = \frac{1}{2}\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix} + \begin{bmatrix} -2 \\ 1 \end{bmatrix}\right) = \boxed{\begin{bmatrix} -1 \\ 1 \end{bmatrix}}$$

$$\mu_1 = \frac{1}{2}\left(\begin{bmatrix} 1 \\ 3 \end{bmatrix} + \begin{bmatrix} 3 \\ 1 \end{bmatrix}\right) = \boxed{\begin{bmatrix} 2 \\ 2 \end{bmatrix}}$$

$$\Sigma = \frac{N_0}{N} S_0 + \frac{N_1}{N} S_1$$

$$S_0 = \frac{1}{2} \sum_{n \in C_0} (x_n - \mu_0)(x_n - \mu_0)^T = \frac{1}{2}\left[\begin{bmatrix} +1 \\ 0 \end{bmatrix}[1 \quad 0] + \begin{bmatrix} -1 \\ 0 \end{bmatrix}[-1 \quad 0]\right] = \cancel{\frac{1}{2}}$$

$$= \frac{1}{2}\left(\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$S_1 = \frac{1}{2} \sum_{n \in C_1} (x_n - \mu_1)(x_n - \mu_1)^T = \frac{1}{2}\left[\begin{bmatrix} -1 \\ 1 \end{bmatrix}[-1 \quad 1] + \begin{bmatrix} 1 \\ -1 \end{bmatrix}[1 \quad -1]\right) =$$

$$= \frac{1}{2}\left(\begin{bmatrix} +1 & -1 \\ -1 & 1 \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\right) = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$\Sigma = \frac{1}{2}\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = \boxed{\begin{bmatrix} 1 & -0.5 \\ -0.5 & 0.5 \end{bmatrix}}$$

(b) since linear decision boundary: equal cov. matrices:

$$\log\left(\frac{\theta}{1-\theta}\right) + \frac{1}{2}(x-\mu_0)^T \Sigma^{-1}(x-\mu_0) - \frac{1}{2}(x-\mu_1)^T \Sigma^{-1}(x-\mu_1) = 0$$

$$\equiv w^T x + b = 0$$

let $x = \begin{smallmatrix} 0 \\ 0 \end{smallmatrix}$: $\log(1) + \frac{1}{2}[1 \quad -1]\begin{bmatrix} 2 & 2 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} 1 \\ -1 \end{bmatrix} - \frac{1}{2}[-2 \quad -2]\begin{bmatrix} 2 & 2 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} -2 \\ -2 \end{bmatrix} = b$

$$b = -19$$

let $x = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ plug into matlab → $[w_1 \quad w_2]\begin{bmatrix} 1 \\ 0 \end{bmatrix} = -11$ $w_1 = -11$

$x = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ $[w_1 \quad w_2]\begin{bmatrix} 0 \\ 1 \end{bmatrix} = w_2 = -9$

$$\boxed{w = \begin{bmatrix} -11 \\ -9 \end{bmatrix} \quad b = -19}$$

5.1 **10 / 10**

- ✓ + **10 pts** All Correct
- + **2 pts** mu0 correct
- + **2 pts** mu1 correct
- + **2 pts** prior correct
- + **4 pts** Sigma correct
- + **2 pts** Equation for Sigma correct
- + **0 pts** Incorrect

(5)

(a) $\phi = P(y=1) = \boxed{\frac{1}{2}}$ $(\frac{2}{4})$ b/c $y_1, y_2 = 1$

$$\mu_0 = \frac{1}{N_0} \sum_{n \in C_0} x_n = \frac{1}{2}\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix} + \begin{bmatrix} -2 \\ 1 \end{bmatrix}\right) = \boxed{\begin{bmatrix} -1 \\ 1 \end{bmatrix}}$$

$$\mu_1 = \frac{1}{2}\left(\begin{bmatrix} 1 \\ 3 \end{bmatrix} + \begin{bmatrix} 3 \\ 1 \end{bmatrix}\right) = \boxed{\begin{bmatrix} 2 \\ 2 \end{bmatrix}}$$

$$\Sigma = \frac{N_0}{N} S_0 + \frac{N_1}{N} S_1$$

$$S_0 = \frac{1}{2} \sum_{n \in C_0} (x_n - \mu_0)(x_n - \mu_0)^T = \frac{1}{2}\left[\begin{bmatrix} +1 \\ 0 \end{bmatrix}[1 \quad 0] + \begin{bmatrix} -1 \\ 0 \end{bmatrix}[-1 \quad 0]\right] = \cancel{\tfrac{1}{2}}$$

$$= \frac{1}{2}\left(\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$S_1 = \frac{1}{2} \sum_{n \in C_1} (x_n - \mu_1)(x_n - \mu_1)^T = \frac{1}{2}\left[\begin{bmatrix} -1 \\ 1 \end{bmatrix}[-1 \quad 1] + \begin{bmatrix} 1 \\ -1 \end{bmatrix}[1 \quad -1]\right) =$$

$$= \frac{1}{2}\left(\begin{bmatrix} +1 & -1 \\ -1 & 1 \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\right) = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$\Sigma = \frac{1}{2}\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = \boxed{\begin{bmatrix} 1 & -0.5 \\ -0.5 & 0.5 \end{bmatrix}}$$

(b) since linear decision boundary: equal cov. matrices:

$$\log\left(\frac{\theta}{1-\theta}\right) + \frac{1}{2}(x-\mu_0)^T \Sigma^{-1} (x-\mu_0) - \frac{1}{2}(x-\mu_1)^T \Sigma^{-1} (x-\mu_1) = 0$$

$$\equiv w^T x + b = 0$$

let $x = \begin{smallmatrix} 0 \\ 0 \end{smallmatrix}$: $\log(1) + \frac{1}{2}[1 \quad -1]\begin{bmatrix} 2 & 2 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} 1 \\ -1 \end{bmatrix} - \frac{1}{2}[-2 \quad -2]\begin{bmatrix} 2 & 2 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} -2 \\ -2 \end{bmatrix} = b$

$$b = -19$$

let $x = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ plug into matlab $\rightarrow$ $[w_1 \quad w_2]\begin{bmatrix} 1 \\ 0 \end{bmatrix} = -11$ $w_1 = -11$

$x = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ $[w_1 \quad w_2]\begin{bmatrix} 0 \\ 1 \end{bmatrix} = w_2 = -9$

$$\boxed{w = \begin{bmatrix} -11 \\ -9 \end{bmatrix} \quad b = -19}$$

5.2 **8 / 10**

- \+ **10 pts** All correct
- \+ **0 pts** Incorrect
- ✓ + **4 pts** **Decision boundary formulation is correct**
- \+ **4 pts** w correct
- ✓ + **2 pts** **Equation for w is correct**
- ✓ + **2 pts** **b correct**
- \+ **1 pts** Equation for b is correct
- \- **2 pts** Off by a factor
- \+ **1 pts** Sigma inverse is correct