ECE M146 Spring 2021 Diagnostic Quiz
Machine Learning
Instructor: Lara Dolecek Maximum score: 45 points

Instructions

- The purpose of the diagnostic quiz is to assess whether you have the necessary background knowledge to succeed in this class. The exam is mandatory but will not count towards your grade. Do the best you can.
- Please time and grade yourself for this diagnostic quiz. Solution will be uploaded on Saturday.
- You can use any one hour window until Friday midnight to do this. You should submit it within one hour of opening this quiz. The time limit is enforced by Gradescope.
- You are advised to drop if you find it difficult to do this quiz.
- We reserve the right to view and grade your submitted quiz.
- You may find the following useful.
  - $\int \exp(ax)dx = \frac{\exp(ax)}{a} + C$ where C is an arbitrary constant
  - $||\boldsymbol{x}||_1 = \sum_{i=1}^{n} |x_i|$, $||\boldsymbol{x}||_2 = \sqrt{\sum_{i=1}^{n} x_i^2}$
  - $\frac{d(\ln(x))}{dx} = \frac{1}{x}$

1. Calculus **(10 pts)**

   (a) Let $f(\boldsymbol{x}) = \ln(ax_1x_2 + bx_1 + cx_1^2)$. What is the partial derivative of f with respect to x_1?

   Solution:

$$\begin{aligned}\frac{\partial f(\boldsymbol{x})}{\partial x_1} &= \frac{1}{ax_1x_2 + bx_1 + cx_1^2}\frac{\partial(ax_1x_2 + bx_1 + cx_1^2)}{\partial x_1} \\ &= \frac{ax_2 + b + 2cx_1}{ax_1x_2 + bx_1 + cx_1^2}\end{aligned}$$

   (b) Evaluate $\int_b^\infty ae^{-a(x-b)}dx$ where $a > 0$.

   Solution:

   Substitute $x - b = t$, $dx = dt$. We have

$$\begin{aligned}\int_b^\infty a\exp(-a(x-b))dx &= \int_o^\infty a\exp(-at)dt \\ &= a\Big[\frac{\exp(-at)}{-a}\Big]_0^\infty \\ &= 1\end{aligned}$$

2. Probability **(15 pts)**
Suppose A and B are two events. Which of these statements is **true/false**? Explain.

(a) A and B are mutually exclusive events then $P(A \cup B) = P(A) + P(B) - P(A)P(B)$
Solution:
False. $P(A \cup B) = P(A) + P(B) - P(A \cap B) = P(A) + P(B)$.

(b) $P(A \cap B \cap C) = P(A)P(B|A)P(C|B)$
Solution:
False. $P(A \cap B \cap C) = P(A)P(B|A)P(C|A \cap B)$.

(c) A is a Gaussian random variable with $\mu = 0$ and $\sigma^2 = 1$ then $P(A = 0) = 0.5$
Solution:
False. For continuous random variable $P(A = c) = 0$.

3. Linear Algebra **(10 pts)**

   Consider the vector $\boldsymbol{x} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

   (a) Compute $\frac{||\boldsymbol{x}||_2}{||\boldsymbol{x}||_1}$. **Solution:**

$$||\boldsymbol{x}||_2 = \sqrt{x_1^2 + x_2^2} = \sqrt{5}$$
$$||\boldsymbol{x}||_1 = |x_1| + |x_2| = 3$$
$$\frac{||\boldsymbol{x}||_2}{||\boldsymbol{x}||_1} = \frac{\sqrt{5}}{3}$$

   (b) Compute $\boldsymbol{x}\boldsymbol{x}^T$.
   Solution:

$$\boldsymbol{x}\boldsymbol{x}^\top = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$$

4. MATLAB Question (You may also do this in Python) **(10 pts)**
Read the following MATLAB code and answer the questions.

```
A = [1, 2; 3, 4];
B = [1, 0; 0, 2];
C = A * B;
D = A .* B;
```

(a) Write A and B in matrix form.
Solution:
$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$

(b) Write C and D in matrix form.
Solution:
$C = \begin{bmatrix} 1 & 4 \\ 3 & 8 \end{bmatrix}$ $D = \begin{bmatrix} 1 & 0 \\ 0 & 8 \end{bmatrix}$