

1. In class, you learned that the direction that maximize the variance of the projection onto a one-dimensional space is the eigenvector that corresponds to the largest eigenvalue of the data covariance matrix  $S = \frac{1}{N}X^TX$ . Formally, the solution to the following maximization problem

$$\max_{u_1} u_1^T S u_1 \quad \text{subject to } \|u_1\|^2 = 1,$$

is the eigenvector that corresponds to the largest eigenvalue of  $S$ .

Suppose  $u_2$  is orthogonal to  $u_1$  and have unit norm. We want to maximizes the variance of the data projected on  $u_2$ . Show that the optimal  $u_2$  is defined by the second eigenvectors of the data covariance matrix  $S$  that corresponds to the second largest eigenvalues.

## 2. Minimum Error Formulation of PCA