Introduction to Machine Learning

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## 1. Linear Algebra Review

- (a) What are the four properties of a inner product i.e.  $\langle x, y \rangle$ ?
- (b) What are the three properties of a norm f(x)?
- (c) What is the L2-norm? L1-norm? L $\infty$ -norm?
- (d) What does it mean to normalize a vector?
- (e) What does it mean for two vectors to be orthogonal?
- (f) Given two vectors  $\mathbf{w}$  and  $\mathbf{v}$  that are not orthogonal, find two orthogonal vectors that span the same space as  $\mathbf{w}$  and  $\mathbf{v}$ .

2. Given a line ax + by + c = 0 and a point  $(x_0, y_0)$ , find the formula for the minimum distance between the point and the line.

3. In class, you were taught to only consider the perceptron that goes through the origin. We will now show that this formulation is sufficient to encompass the case where the perceptron does not go through the origin.

Consider the classification function of a perceptron classifier that does not go through the origin,

$$h(x) = w^T x + b$$

where w and b are the hyper plane parameters.

Now, consider the classification function of a perceptron classifier that does go through the origin

$$\tilde{h}(\tilde{x}) = \tilde{w}^T \tilde{x}.$$

Find a way to formulate  $\tilde{w}$  and  $\tilde{x}$  in terms of x, b, w.