2) (a) $P(A|C=0) = \cancel{0.8}\ \frac{0.28}{0.4} = 0.7$

$P(B|C=0) = \frac{\cancel{0.08}}{\cancel{0.32}} = \frac{\cancel{0.25}\ 0.08}{\cancel{0.320}\ 0.400} = \cancel{0.8}\ 0.2$

$P(A, B|C=0) = \frac{0.056}{0.40} = 0.14$

0.420	0.096	0.024	0.27	0.03
0.580	0.224	0.056	0.27	0.03
	0.320	0.080	0.54	0.06

$P(A=0|C=0) = 1-0.7=0.3$
$P(B=0|C=0) = 1-0.2=0.8$
$P(A=1, B=0|C=0) = 0.224/0.4 = 0.56$
$P(A=0, B=1|C=0) = 0.06$
$P(A=0, B=0|C=0) = 0.024$

(b) $P(A|C=1) = \frac{0.27+0.03}{0.6} = 0.5$

$P(B|C=1) = \frac{0.06}{0.60} = 0.1$

$P(A,B|C=1)\ \frac{0.03}{0.06} = 0.05$

$P(A=0|C=1) = 0.5$
$P(B=0|C=1) = 0.9$
$P(A=1, B=0|C=1) = 0.27/0.60 = 0.45$
$P(A=0, B=1|C=1) = 0.03/0.60 = 0.05$
$P(A=0, B=0|C=1) = 0.45$

(c) Yes conditionally independent b/c $P(A|C=0) \cdot P(B|C=0) = P(A,B|C=0)$

$0.7 * 0.2 = 0.14$ for all cases.

C=1 " " "

$0.5 \cdot 0.1 = 0.05$

is. $P(A|C) \cdot P(B|C) = P(A,B|C)$
: independent

[yes conditionally indep.]

(d) $P(A) = 0.580$ A=1 $P(A=0) = 0.42$

~~P(B) = 0.420~~ $P(B) = 0.14$ B=1 $P(B=0) = 0.86$

$P(A,B) = 0.086$ A=B=1. $P(A=0, B=0) = 0.366$
$P(A=1, B=0) = 0.494$
$P(A=0, B=1) = 0.054$

(e) $P(A) \cdot P(B) = P(A,B)$

$.58 \cdot .14 \neq .086$
NOT INDEPENDENT

does not hold for all combos of A, B

(3) (a). $P(G)$: $P(G=0) = 2/8 = 1/4$ $P(G=1) = 3/4$

$P(X|G) \rightarrow$ $P(\cancel{\text{tot}G})$ =

$P(D=0|G=0) = 0$
$P(D=1|G=0) = 1$
$P(D=0|G=1) = 0.5$
$P(D=1|G=1) = 0.5$

$P(A=0|G=0) = 1$
$P(A=0|G=1) = 1/6$
$P(A=1|G=0) = 0$
$P(A=1|G=1) = 5/6$

$P(C=0|G=0) = 1/2$
$C=0|G=1 = 1/2$
$C=1|G=0 = 1/2$
$C=1|G=1 = 1/2$

$P(B=0|G=0) = 0$
$(B=1|G=0) = 1$
$(B=0|G=1) = 2/3$
$(B=1|G=1) = 1/3$

counting in graph

(b). $P(G=0) \cdot P(D=0|G=0) \cdot P(B=1|G=0) \cdot P(C=0|G=0) \cdot P(A=1|G=0)$

$= 1/4 \cdot 0 \cdot 1 \cdot \frac{1}{2} \cdot 0 = 0$

$P(G=1) \cdot P(D=0|G=1) \cdot P(B=1|G=1) \cdot P(C=0|G=1) \cdot P(A=1|G=1)$

$= 3/4 \cdot \frac{1}{2} \cdot \frac{1}{3} \cdot \frac{1}{2} \cdot \frac{5}{6} = \frac{5}{96}$ $G=1 > G=0$

$\boxed{G=1}$ for sample 9

$P(G=0) P(D=1|G=0) P(B=1|G=0) P(C=1|G=0) P(A=1|G=0)$

$= 1 \cdot 1 \cdot \frac{1}{2} \cdot 0 \cdot \frac{1}{4} = 0$

$P(G=1) P(D=1|G=1) P(B=1|G=1) P(C=1|G=1) P(A=1|G=1)$

$= \frac{3}{4} \cdot \frac{1}{2} \cdot \frac{1}{3} \cdot \frac{1}{2} \cdot \frac{5}{6} = \frac{5}{96}$

$G=1 > G=0$

sample 10 $\boxed{G=1}$

(c). $P(D=0|G=0) = 1/4$
$P(D=1|G=0) = 3/4$
$P(D=0|G=1) = 4/8$
$P(D=1|G=1) = 4/8$

$P(A=0|G=0) = 3/4$
$P(A=1|G=0) = 1/4$
$P(A=0|G=1) = 2/8$
$P(A=1|G=1) = 6/8$

$P(B=0|G=0) = 1/4$
$P(B=1|G=0) = 3/4$
$P(B=0|G=1) = 5/8$
$P(B=1|G=1) = 3/8$

$P(C=0|G=0) = 1/2$
$P(C=1|G=0) = 1/2$
$P(C=0|G=1) = 1/2$
$P(C=1|G=1) = 1/2$

(d) $P(G=0)\, P(O=0|G=0)\, P(B=1|G=0)\, P(C=0|G=0)\, P(A=1|G=0)$

$= 2/8 \cdot 1/4 \cdot 3/4 \cdot 1/2 \cdot 1/4 = 0.00586$

$P(G=1)\, P(O=0|G=1)\, P(B=1|G=1)\, P(C=0|G=1)\, P(A=1|G=1)$

$= 1/2 \cdot 3/8 \cdot 1/2 \cdot 3/4 \cdot 6/8 = 0.0527$

For sample 9: $G_1 > G_0 \Rightarrow \boxed{G=1}$

$P(G=0)\, P(O=1|G=0)\, P(B=1|G=0)\, P(C=1|G=0)\, P(A=1|G=0)$

$= 2/8 \cdot 3/4 \cdot 3/4 \cdot 1/2 \cdot 1/4 = 0.0176$

$P(G=1)\, P(O=1|G=1)\, P(B=1|G=1)\, P(C=1|G=1)\, P(A=1|G=1)$

$= 1/2 \cdot 3/8 \cdot 1/2 \cdot 3/4 \cdot 6/8 = 0.0527$

For sample 10: $G_1 > G_0 \Rightarrow \boxed{G=1}$

(4) $\cancel{P(x^1, x^2 \ldots y^1, y^2 \ldots y^m) = \prod_{k=1}^{m} P(x^k, y^k)}$

$\cancel{= \prod_{k=1}^{m} P(y^{(k)}) \cdot P(x^{(k)} | y^{(k)}) = \prod_{k=1}^{m} P(y^{(k)}) \cdot P}$

$=$

Problem 4

(a) $$\prod_{i=1}^{m} P(x^{(i)}, y^{(i)}) = \prod_{i=1}^{m} P(y^{(i)}) \cdot P(x^{(i)} | y^{(i)})$$

$$= \prod_{i=1}^{m} \left[\theta_0^{1(y^{(i)}=0)} (1-\theta_0)^{1(y^{(i)}=1)} \prod_{j=1}^{n} \left(\prod_{k=1}^{s} \theta_{j,k|y=0}^{1[x_j^{(i)}=k, y^{(i)}=0]} \cdot \left(1 - \sum_{k=1}^{s-1} \theta_{j,k|y=0}\right)^{1[x_j^{(i)}=s, y^{(i)}=0]} \cdot \theta_{j,k|y=1}^{1[x_j^{(i)}=k, y^{(i)}=1]} \left(1 - \sum_{k=1}^{s-1} \theta_{j,k|y=1}\right)^{1[x_j^{(i)}=s, y^{(i)}=1]} \right) \right]$$

(b) $$\log((a)) = \sum_{i=1}^{m} 1[y^{(i)}=1] \log(1-\theta_0) + 1[y^{(i)}=0] \log\theta_0$$
$$+ \sum_{j=1}^{n} \sum_{k=1}^{s} 1[x_j^{(i)}=k, y^{(i)}=0] \log\theta_{j,k|y=0}$$
$$+ 1[x_j^{(i)}=s, y^{(i)}=0] \log\left(1 - \sum_{k=1}^{s-1} \theta_{j,k|y=0}\right)$$
$$+ 1[x_j^{(i)}=k, y^{(i)}=1] \log\theta_{j,k|y=1}$$
$$+ 1[x_j^{(i)}=s, y^{(i)}=1] \log\left(1 - \sum_{k=1}^{s-1} \theta_{j,k|y=1}\right)$$

$$\frac{\partial}{\partial\theta_0} = \sum_{i=1}^{m} 1[y^{(i)}=1] \log(1-\theta_0) + 1[y^{(i)}=0] \log\theta_0$$

$N_{k,0} = 1[x_j^{(i)}=k, y^{(i)}=0]$ $N_{k,1} = 1[x_j^{(i)}=k, y^{(i)}=1]$ $N_{s,0} = 1[x_j^{(i)}=s, y^{(i)}=0]$

$N_{s,1} = 1[x_j^{(i)}=s, y^{(i)}=1]$

$$\frac{-N_1}{1-\theta_0} + \frac{N_0}{\theta_0} = 0 \quad \text{where } N_1 = \sum_{i=1}^{m} 1[y^{(i)}=1] \quad N_0 = \sum_{i=1}^{m} 1[y^{(i)}=0]$$

$$\boxed{\hat{\theta}_0 = N_0/(N_0+N_1)}$$

$\sum_{j=1}^{n}$ $\sum_{k=1}^{s}$ disappears b/c w.r.t. k.

$$\frac{\partial P}{\partial\theta_{j,k|y=0}} = \sum_{i=1}^{m}\sum_{j=1}^{n} \frac{N_{k,0}}{\theta_{j,k|y=0}} + \frac{-N_{s,0}}{1 - \sum_{k=1}^{s-1} \theta_{j,k|y=0}} = 0 \implies \frac{\sum_{i=1}^{m} N_{k,0}}{\theta_{j,k|y=0}} = \frac{\sum_{i=1}^{m} N_{s,0}}{\theta_{j,s|y=0}}$$

$$\boxed{\hat{\theta}_{j,k|y=0} = \theta_{j,s|y=0} \frac{\sum_{i=1}^{m} N_{k,0}}{\sum_{i=1}^{m} N_{s,0}}} \Longrightarrow$$

Week

$$\boxed{\hat{\theta}_{j,k|y=1} = \theta_{j,s|y=1} \frac{\sum_{i=1}^{m} N_{k,1}}{\sum_{r=1}^{m} N_{s,1}}}$$ by symmetry

$\hat{\theta}_0$ the proportion of all trials whose coin flip land say $y=0$

$\hat{\theta}_{j,k|y=0}$ the proportion to the ratio of # trials where the jth dice roll is ~~not the max~~ val k over the # trials whose ~~dice~~ roll jth roll is s (the max) all with ~~dice~~ coin flip $(=0)$

$\hat{\theta}_{j,k|y=1}$ the same thing except coin flip $= \boxed{y=1}$

5.

$$f_{XY}(x,y) = \frac{\exp\left\{ \frac{-1}{2(1-\rho_{XY}^2)} \left[\left(\frac{x-m_1}{\sigma_1}\right)^2 - 2\rho_{XY}\left(\frac{x-m_1}{\sigma_1}\right)\left(\frac{y-m_2}{\sigma_2}\right) + \left(\frac{y-m_2}{\sigma_2}\right)^2 \right] \right\}}{2\pi\sigma_1\sigma_2\sqrt{1-\rho_{XY}^2}}$$

$$\Sigma = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \rightarrow |\Sigma| = ad-bc$$

$$ad-bc = \sigma_1^2\sigma_2^2 - \rho_{XY}^2\sigma_1^2\sigma_2^2$$

$ad =$ $\quad$ $bc =$

$$\boxed{\Sigma = \begin{bmatrix} \sigma_1^2 & \rho_{XY}\sigma_1\sigma_2 \\ \rho_{XY}\sigma_1\sigma_2 & \sigma_2^2 \end{bmatrix}}$$

$$\hookrightarrow \Sigma^{-1} = \boxed{\frac{1}{\sigma_1^2\sigma_2^2 - \rho_{XY}^2\sigma_1^2\sigma_2^2}\begin{bmatrix} \sigma_2^2 & -\rho_{XY}\sigma_1\sigma_2 \\ -\rho_{XY}\sigma_1\sigma_2 & \sigma_1^2 \end{bmatrix}}$$

b/c $\quad \Sigma^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

$$z = \begin{bmatrix} x \\ y \end{bmatrix} \quad \mu = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$$

$$(z-\mu)^T \Sigma^{-1} (z-\mu)$$

$$= \begin{bmatrix} x-c_1 \\ y-c_2 \end{bmatrix}^T \quad \begin{bmatrix} x-c_1 \\ y-c_2 \end{bmatrix}$$

$$= \frac{1}{(1-\rho_{XY}^2)}\left[\left(\frac{x-c_1}{\sigma_1}\right)^2 - 2\rho_{XY}\frac{(x-c_1)(y-c_2)}{\sigma_1\sigma_2} \left(\frac{y-c_2}{\sigma_2}\right)^2\right]$$

$$\boxed{\mu = \begin{bmatrix} m_1 \\ m_2 \end{bmatrix}}$$

i) $\cancel{\Sigma_{ij} = \mathrm{cov}[x_i, x_j]} =$

x_i, x_j are independent

$$E(x_i, x_j) = \underbrace{E[x_i] \cdot E[x_j]}_{} \leftarrow \iint x_i x_j \, p(x_i, x_j)$$

$$= \int x_i \, p_{x_i} \quad \int x_j \, p_{x_j}$$

$$\mathrm{cov}[x_i, x_j] = \cancel{E[x_i, x_j]} - \cancel{E[x_i] E[x_j]}$$

$$= 0$$

$$\begin{bmatrix} \sigma_1^2 & 0 & 0 & 0 & 0 \\ & \sigma_2^2 & 0 & 0 & 0 \\ 0 & & \ddots & 0 & 0 \\ & 0 & & & \\ 0 & & 0 & \sigma_N^2 & \end{bmatrix}$$