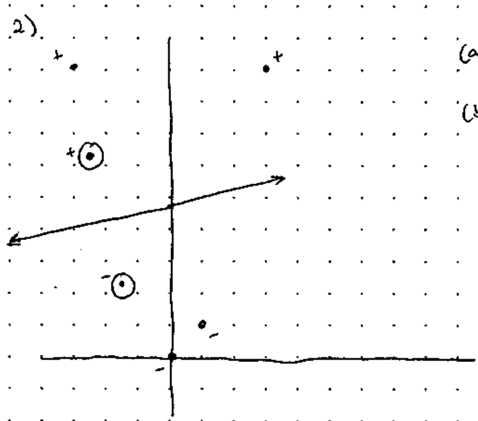
2).



(a) $(-2.5, 6.25)$, $(-1.5, 2.25)$

(b) $(-2, 4.25)$

$m = -4 \quad \perp m = \frac{1}{4}$

$y = \frac{1}{4}x + b$

$4.25 = -0.5 + b$

$b = 4.75$

$y = 0.25x + 4.75$

(c) $a_2 \cdot y^{(2)} \cdot x^T \cdot x^{(2)} + b$
$+ a_4 \cdot y^{(4)} \cdot x^T \cdot x^{(4)}$

$a_2 y^{(2)} + a_4 y^{(4)} = 0 \Rightarrow a_2 = a_4$

$y = w^T x + b$

$0 = \frac{2}{17}\begin{bmatrix} -1 & 4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} - 2.235 = 0.$

$2[-x_1 + 4x_2] - 38 = 0$

$-x_1 + 4x_2 = 19$

$x_2 = 0.25x_1 + 4.75$ ✓

Same equation.

$L = (a_4 + a_2) - \frac{1}{2}\left[a_2^2 \|x^{(2)}\|^2 + a_4^2 \|x^{(4)}\|^2 - 2a_2 a_4 x^{(2)T} x^{(4)}\right]$

$\|x^{(2)}\|^2 = 45.312 \quad \|x^{(4)}\|^2 = 7.312 \quad x^{(2)T} x^{(4)} = 17.813$

$2a_2 - \frac{1}{2} 17 \cdot a_2^2$

$\frac{\partial L}{\partial a_2} = 2 - 17a_2 = 0 \rightarrow \boxed{a_2 = a_4 = \frac{2}{17}}$

$w = \sum_{n \in S} a_n y_n x_n = \frac{2}{17} \cdot 1 \cdot \begin{bmatrix} x_2 \end{bmatrix} + \frac{2}{17} \cdot (-1) \cdot \begin{bmatrix} x_4 \end{bmatrix}$

$= \boxed{\frac{1}{17}\begin{bmatrix} -2 \\ 8 \end{bmatrix}}$

$b = \frac{1}{|S|} \sum_{n \in S} \left[t_n - w^T x_n\right] = \frac{1}{2}\left[1 - \begin{bmatrix} -2/17 & 8/17 \end{bmatrix}\begin{bmatrix} -2.5 \\ 6.25 \end{bmatrix} + (-1) - \begin{bmatrix} -2/17 & 8/17 \end{bmatrix}\begin{bmatrix} -1.5 \\ 2.25 \end{bmatrix}\right]$

$= \boxed{-2.235}$

4. $K = \begin{bmatrix} k(x_1,x_1) & k(x_1,x_2) \\ k(x_2,x_1) & k(x_2,x_2) \end{bmatrix}$ $y = \begin{bmatrix} k(x_2,x_2) \\ -k(x_1,x_2) \end{bmatrix}$

$$y^T K y = \begin{bmatrix} k(x_2,x_2) & -k(x_1,x_2) \end{bmatrix} \begin{bmatrix} k(x_1,x_1) & k(x_1,x_2) \\ k(x_2,x_1) & k(x_2,x_2) \end{bmatrix} \begin{bmatrix} k(x_2,x_2) \\ -k(x_1,x_2) \end{bmatrix}$$

$$= \begin{bmatrix} k(x_2,x_2)k(x_1,x_1) - k(x_1,x_2)k(x_1,x_2), & k(x_2,x_2)k(x_1,x_2) - k(x_1,x_2)k(x_2,x_2) \end{bmatrix} \begin{bmatrix} k(x_2,x_2) \\ -k(x_1,x_2) \end{bmatrix}$$

$$= k(x_2,x_2)k(x_1,x_1)k(x_2,x_2) - k(x_1,x_2)k(x_1,x_2)k(x_2,x_2)$$

$$\cancel{- k(x_2,x_2)k(x_1,x_2)k(x_1,x_2) + k(x_1,x_2)k(x_2,x_2)k(x_1,x_2)} \to 0$$

$$\cancel{- k(2,2)k(1,2)^2} + k$$

$$= k(x_2,x_2)\left(k(x_1,x_1)k(x_2,x_2) - k(x_1,x_2)^2\right) \geq 0$$

$\geq 0$

$k(x_1,x_2) = k(x_2,x_1)$
Symmetry.

PSD therefore,

$$k(x_1,x_1)k(x_2,x_2) - k(x_1,x_2)^2 \geq 0$$

$$k(x_1,x_2)^2 \leq k(x_1,x_1)k(x_2,x_2)$$

2

5. (a) PSD states that K is valid if $y^T K y \geq 0 \quad \forall y$

$$y^T K y = y^T \left( K_1(x,x') + K_2(x,x') \right) y$$

$$= \left( y^T K_1(x,x') + y^T K_2(x,x') \right) y$$

$$= y^T K_1(x,x') y + y^T K_2(x,x') y$$

each $\geq 0$ so total sum $\geq 0$. $\square$

(b) ~~$y^T (K) y = y^T (K_1 K_2) y$~~

~~If $K_1$ is a valid kernel, $K_1$ is PSD.~~

~~$y^T K_1 y \geq 0$~~

~~Let $y = v^T$~~

~~$v K_1 v^T \geq 0$~~

$$K_1(x,x') = a(x)^T a(x')$$

$$K_2(x,x') = b(x)^T b(x')$$

$$K = K_1 K_2 = \sum_{m=1}^{M} a_m(x) a_m(x') \cdot \sum_{m=1}^{N} b_m(x) b_m(x')$$

$$= \sum_{m=1}^{M} \sum_{n=1}^{N} a_m(x) b_n(x) \left( a_m(x') b_n(x') \right)$$

Let $c_{m,n}(x) = a_m(x) \cdot b_n x$

$$= \sum_{m=1}^{M} \sum_{n=1}^{N} c_{mn}(x) \cdot c_{mn}(x')$$

$c$ is $m \times n$ dimensional

$$= c(x)^T c(x')$$

K can be written as a inner product of feature map $c$.

so K is a kernel. $\square$

(c) $\exp(K_1(x,x')) \Rightarrow \exp(a) = 1 + a + \frac{a^2}{2} + \frac{a^3}{6} + \ldots$

$$1 + K_1(x,x') + \frac{1}{2} K_1(x,x')^2 + \frac{1}{6} K_1(x,x')^3 + \ldots$$

each of these terms $K_1(x,x')^n \cdot \frac{1}{n!}$ is a kernel by part (b).

the sum of all of these kernels is a kernel by part (a). $\square$

6.

$$t_n(w^T x_n + b) = 1 - \xi$$

$$w^T x_n + b = t_n(1 - \xi)$$

$$\left(\sum_{k \in S} a_k t_k x_k\right)^T x_n + b = t_n(1 - \xi)$$

0 / M includes all indices that $y^{(i)}(w^T x^{(i)} + b) \geq 1 - \xi_i$

$$|M| \cdot b = \sum_{n \in M} \left( t_n \boxed{(1-\xi)} - \left(\sum_{k \in S} a_k t_k x_k\right)^T x_n \right)$$

$$N_M b = \sum_{n \in M} \left( t_n - \left(\sum_{k \in S} a_k t_k x_k\right)^T x_n \right)$$

$$= \sum_{n \in M} \left( y^{(n)} - \sum_{k \in S} a_k \cdot y^{(k)} \cdot x_k^T \cdot x_n \right)$$

$$= \sum_{n \in M} \left( y^{(n)} - \sum_{m \in S} a_m \cdot y^{(m)} \langle x^{(n)}, x^{(m)} \rangle \right)$$

$$b = \frac{1}{N_M} \sum_{n \in M} \left( y^{(n)} - \sum_{m \in S} a_m \cdot y^{(m)} \langle x^{(n)}, x^{(m)} \rangle \right)$$