

2-3 Tree

①

insertion rule: A 2-3 tree maintains balance by being built from the bottom-up, not the top down. Instead of hanging a new node onto leaf, we insert the new node into a leaf, we search the insertion node using the normal process of 2-3 tree.

* There are some special cases which defined in the book, while constructing tree, these are will be really helpful.

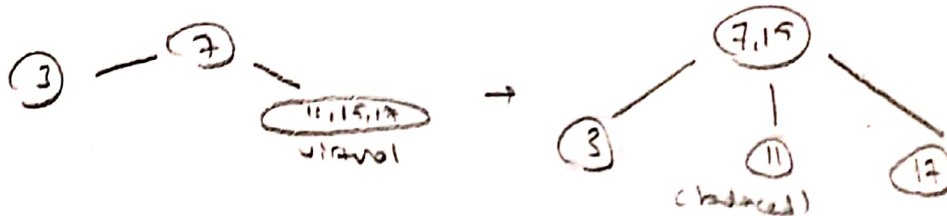
Case 1 (inserting into 2-Node Leaf)

* We can insert given node directly as a data field of a appropriate right or left child.



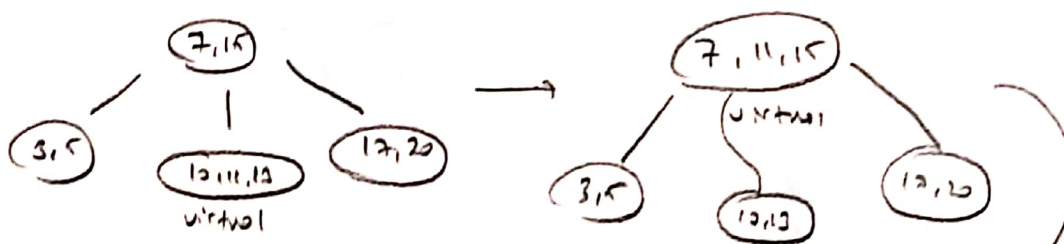
Case 2 (inserting into 3-Node Leaf with a 2-Node parent)

* Firstly we insert given element to appropriate position. But node cannot keep 3 values in it, middle value propagate up to 2-Node parent. Then we need to split executed node of 2-node.

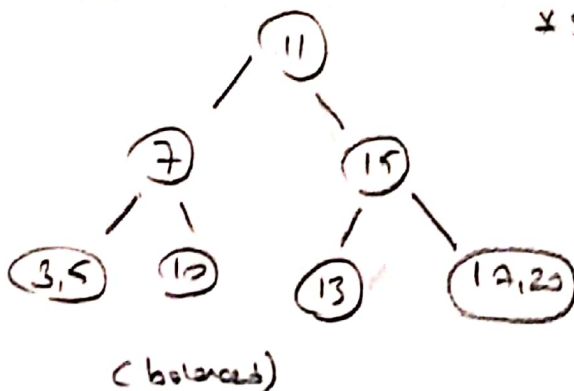


Case 3 (inserting into 3-Node leaf with 3-Node parent)

* All leaf nodes are full, so we insert any other number, one of the leaf nodes will need to be virtually split, and its middle value will propagate to the parent. Because the parent is already a tree node, it will also need to be split.



* Splitting required for same nodes.



input = { 20, 30, 8, 47, 39, 18, 40, 24 }

(2)

↳ last four digit of my number.

add(20):

* tree is empty, initialize 3 node tree and put 20 as a data field.

add(30)

(20, 30) * root was 3-node and 30 was added as a second data field

add(8)



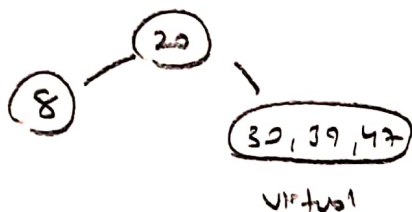
* 8 is less than 20 and we need to split up the tree as 2-node tree.

add(47)

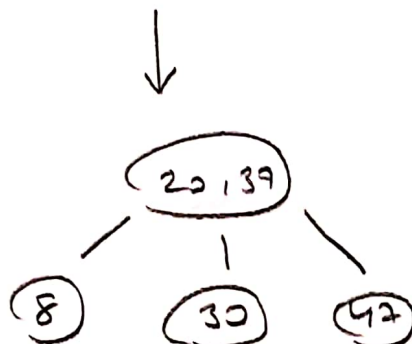


* 47 is greater than 30, and it can be represented as right data field of 3-node.

add(39)



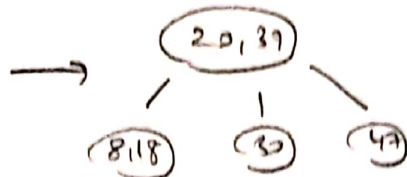
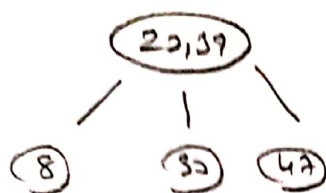
* According to case 2 we need to know width value to one degree top of current node. And split the current node as a 2-node.



* Now root node became 3-node and it has to be 3 different child so right most of tree splitted.

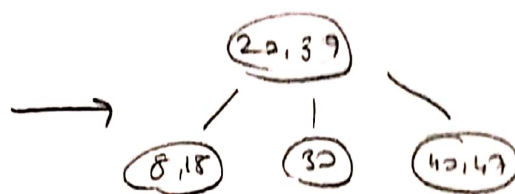
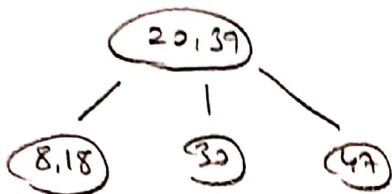
add(18)

3



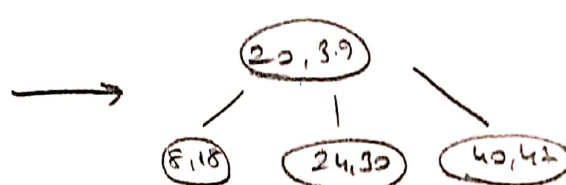
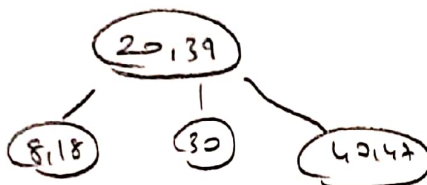
* According to de facto binary-search-tree rule 18 can be pulled or a data field of left most appropriate node.

add(40)



same rule is valid for "40".

add(24)



Final Tree

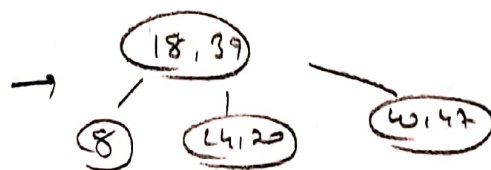
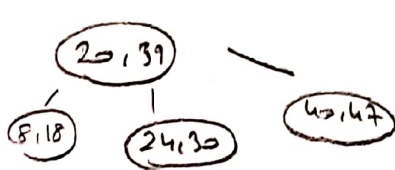
same rule is valid for middle value 18 can be assigned or value left data field of middle node.

* Removing from B+ tree is somewhat the reverse of the insertion process. To remove an item we must search for it.

→ if the item to be removed is in a leaf, we simply delete it.

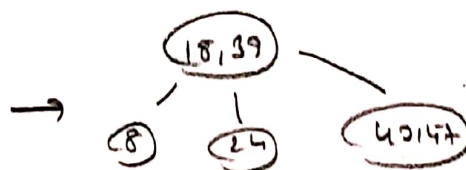
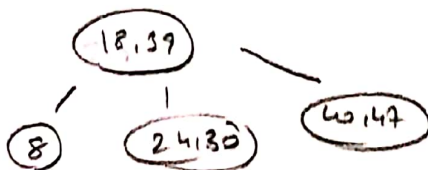
→ if the item to be removed is not a leaf, we remove it by swapping with its in-order predecessor.

remove(20)



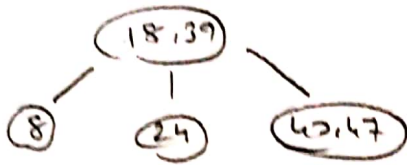
* 18 is the in-order predecessor of 20. It moves to the top.

remove(30)

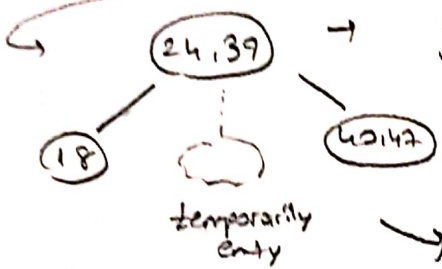


* 30 is a leaf node data field and it can be removed directly.

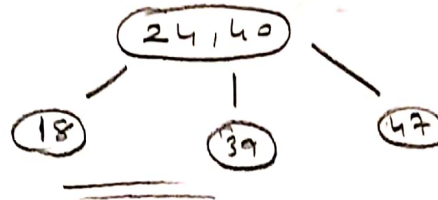
remove (8)



* Removing 8 will cause some swap operations.
 * 18 is not lower bound of root any more and it is needed to be change of lower of middle node. After that new lower bound is 18. It can substitute of removed node data field.



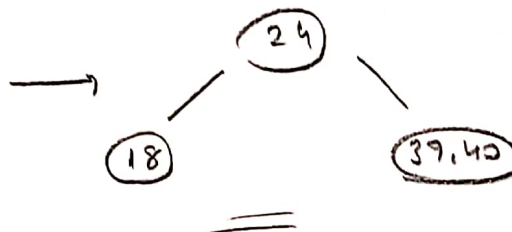
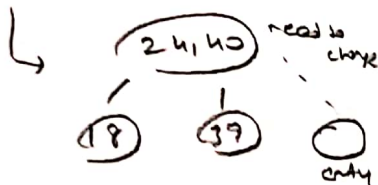
* After this some operation can be applied for lower value of right most child which is 42. It can be change as upper bound of the root node. 39 can be moved to middle node, which is empty.



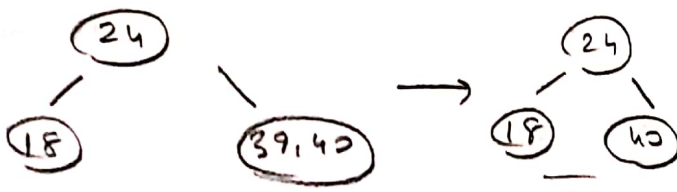
remove (47)



* After that removing right most node will be empty and root can not be 3-Node any more because 4 data will be remain for constructing, we can make root node 2-Node then middle node can be used to keep removed element from root node. Left most node also could be chose but, I choose middle one.



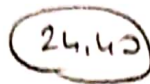
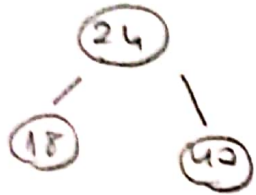
remove (39)



* It can be directly removed because, it was located in a leaf node.

remove (18)

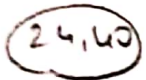
9



* After removing "18", there will not be enough data to construct a 2-node tree.
* we can also remove right most tree value and add it to root for constructing 3-node.

* Our purpose is keeping the subtree height same. By doing that our search will be $O(\log n)$ obvious.

remove (40)



* Just remove data in the last node.

remove (24)



* All elements are removed one by one first-in first-out rule.

* Advantage of 2-3 tree:

- 1) It has fewer complicated operation according to AVL and Red-Black tree.
- 2) There were no rotation.
- 3) we say that the time complexity of 2-3 tree is $O(\log n)$ with no doubt.