Q1.

a) $\overset{\rightarrow f(n)}{\log_2 n^2+1} \in \overset{\rightarrow g(n)}{O(n)}$

Using limit method; $\log_2 n^2+1 = 2\log_2 n + \log_2 2 = 2\log_2 2n$

$$\lim_{n\to\infty} = \frac{2\log_2 2n}{n} = \frac{\frac{2.2}{2n\ \ln 2}}{1} = \lim_{n\to\infty} \frac{2}{n\ln 2} = \frac{2}{\ln 2} \lim_{n\to\infty} \frac{1}{n} = 0$$

$\log_2 n^2+1 \in O(n)$ = f(n) smaller order of growth then g(n)

<u>True</u>

b) $\underbrace{\sqrt{n(n+1)}}_{f(n)} \in \underbrace{\Omega(n)}_{g(n)}$

$$\lim_{n\to\infty} \frac{\sqrt{n(n+1)}}{n} = \frac{\sqrt{n^2+\cancel{1}}}{n} = \frac{n}{n} = 1 = c, \text{ so}$$

sabit

f(n) has same growth of order of g(n), true

$\sqrt{n(n+1)} \in \Omega(n)$

c)

$n^{n-1} \in \Theta(n^n)$

$$\lim_{n\to\infty} \frac{n^{n-1}}{n^n} \Rightarrow \frac{(n-1)' . n^{n-1} . \cancel{\ln(n)}}{n' . n^n . \cancel{\ln(n)}} = \frac{1 . n^{n-1}}{1 . n^n} = \frac{\cancel{n^{n-1}}}{\cancel{n^{n-1}} . n} = \frac{1}{n}$$

n^{n-1}'s growth order smaller than n^n, so <u>false</u> $n^{n-1} \in \Theta(n^n)$ statement

d) $O(\underbrace{2^n+n^3}) \subset O(4^n)$

compare these $O(2^n)$ or $O(n^3)$ faster $\rightarrow \lim_{n\to\infty} \frac{2^n}{n^3} = \frac{1.2^n \ln 2}{0.n^3 \ln n} = \infty$

2^n bigger then n^3

$O(2^n)$

$O(2^n) \overset{?}{\subset} O(4^n) \rightarrow \lim_{n\to\infty} \frac{4^n}{2^n} = \infty$ 4^n is faster than 2^n

Thus we can say that $O(2^n+n^3) \subset 2^n \rightarrow$ <u>true</u>

e) $O(2\log_3 \sqrt[3]{n}) \subset O(3\log_2 n^2)$

$$\lim_{n\to\infty} \frac{3\log_2 n^2}{2\log_3 \sqrt[3]{n}} = \frac{6\log_2 n}{2\log_3 n^{1/3}} \to \frac{6\log_2 n}{\frac{2}{3}\log_3 n} = \frac{9\,\frac{1}{n\,\ln 2}}{\frac{1}{n\,\ln 3}} = \frac{9\,\cancel{n}\,\ln 3}{\cancel{n}\,\ln 2} = c$$

it means they have same growth rate

so, $O(2\log_3 \sqrt[3]{n}) \subset O(3\log_2 n^2)$, subset can be itself so statement is true.

f) $\log_2 \sqrt{n}$ are same asymbtic order $(\log_2 n)^2$

$$\lim_{n\to\infty} \frac{\log_2 \sqrt{n}}{(\log_2 n)^2} = \frac{\log_2 n^{1/2}}{\log_2 n \, \log_2 n} = \frac{\frac{1}{2}\,\cancel{\log_2 n}}{\log_2 n \, \cancel{\log_2 n}} = \frac{\frac{1}{2}}{\log_2 n} \quad \frac{\frac{1}{2}}{1.\log_2 n \ln 2} \cong 0$$

it means $\log_2 \sqrt{n}$ has smaller growth rate then $(\log_2 n)^2$, so statement is false

2) n^2, n^3, $n^2 \log n$, $n^{1/2}$, $\log n$, 10^n, 2^n, $8^{\log n}$

$\downarrow$

$8^{\log_2 n} \to n^3$

$n^{1/2} < n^2 < n^3 = n^3$

$n^2 \log n > \log n \to$ compare $\log n$, $n^{1/2}$

$$\lim_{n\to\infty} \frac{n^{1/2}}{\log n} = \frac{\frac{1}{2} n^{-1/2}}{\frac{1}{n}\log_2 e} = \frac{\frac{1}{2}\frac{1}{\sqrt{n}}}{\frac{\log_2 e}{n}} = \infty$$

$\frac{1}{\sqrt{n}}\, n = \infty$

So $\log n < \sqrt{n}$

$2^n \cdot 5^n > 2^n$

$\log n < \sqrt{n} < n^2 < n^3 = 8^{\log_2 n}$

Compare $\lim_{n\to\infty} \frac{n^2 \log n}{n^3} = \frac{\log n}{n} = \frac{\frac{1}{n}\ln 2}{1} = \frac{1}{n} \cong 0$

$8^{\log_2 n} = n^3 > n^2 \lg n > n^2 > \sqrt{n} > \lg n$

2^n or $n^3 \Rightarrow \lim_{n\to\infty} \frac{2^n}{n^3} = \frac{2^n \ln 2}{3n^2} = \frac{2^n}{n^2} \overset{\infty}{\nearrow} = \infty$

New order

$10^n > 2^n > 8^{\log_2 n} = n^3 > n^2 \lg n > n^2 > \sqrt{n} > \lg n$

3) a)

for loop runs n times. Because of the length of the array size

In if else part doesn't affect the execution time, because they only run one times in for loop.

I did not accidentally assign values to variables first and second. But in question, they actually take the maximum value the integer can get.

In both cases, the algorithm will run in average of n time.

$\theta(n)$

b) Analysis loop

for(int i=2 ; i<=n ; i++) → input size n

if (i%2) → if i is even, then it executed one times.

else part → if i is not even, executed that parts next i values will be $\underline{i^2 - i}$.

$i_{n+1} = i_n(i_n - 1)$

→ this part dominates that parts, because exponential.

$i_{n+1} = i_n^2 - i_n$ → If we say this function run k times

i^{2^k} → k times run

→

↳ values is x^{2^k}, it increases until the value n.

$x^{2^k} = n$ find n take log

$k = \log(\log n)$

So, average time comp. $\theta(\log(\log(n))$

4) a) $\sum_{i=1}^{n} i^2 \log i$

$f(n) = i^2 \log i$ is non-decreasing function

$\int_0^n i^2 \log i \, di \leq f(n) \leq \int_1^{n+1} i^2 \log i \, di$

$u = \log i$

$v = i^2$ $v = \frac{i^3}{3}$

$\log i . \frac{i^3}{3} - \frac{\log e}{3} \leq f(n) \leq \frac{(i+1)^3 . (3\log_e(i+1) - 1) + 1}{9\log_e 2}$

$f(n) \in O(n^3 . \log n)$ upper bond

$f(n) \in \Omega(n^3 . \log n)$ lower band

so averge time is equal to $\theta(n^3 . \log n)$

4) b)

$\sum_{i=1}^{n} i^3$ is it non decreasing?

Yes, non - decreasing functions

$$\int_{0}^{n} i^3 \, di \leq f(n) \leq \int_{1}^{n+1} i^3 \, di$$

$$\frac{i^4}{4}\Big|_{0}^{n} \leq f(n) \leq \frac{i^4}{4}\Big|_{1}^{n+1}$$

$$\frac{n^4}{4} - 0 \leq f(n) \leq \frac{(n+1)^4}{4} - \frac{1}{4}$$

upper band $O(n^4)$, lower band $\Omega(n^4)$, Both upper and lower band are same so we get $f(n) \in \Theta(n^4)$

4) c)

$\sum_{i=1}^{n} \frac{1}{2\sqrt{i}}$ it is non - incrasing so;

$$\int_{1}^{n+1} \frac{1}{2\sqrt{i}} \leq f(n) \leq \int_{0}^{n} \frac{1}{2\sqrt{i}}$$

$$\sqrt{n+1} - 1 \leq f(n) \leq \sqrt{n}$$

$f(n) = \Omega(\sqrt{n})$

$f(n) = O(\sqrt{n})$

so $f(n) = \Theta(\sqrt{n})$

4) d) $\sum_{i=1}^{n} \frac{1}{i}$ non - incrasing

$$\ln\Big|_{1}^{n+1} \leq f(n) \leq \ln i\Big|_{0}^{n+1}$$

$$\ln(n+1) \leq f(n) \leq \ln(n+1) \underbrace{- \ln 0}_{\text{undefined}}$$ ⇒ change function $\sum_{i=1}^{n} \frac{1}{i} = 1 + \sum_{i=1}^{n} \frac{1}{i}$

5)

-> If we search first element of list, then our best case will be $\Omega(1)$

-> If we search first element of list and that element is repeated

$\underline{X}$ _ $\underline{X}$ $\rightarrow$ $O(1)$ -> doesn't change anything, look at first place and find it.

-> If we search last element of list, then our worst case will be $O(n)$, n refer to list size

-> If we search last element of list, and the last element repeated

_ _ _ $\underline{5}$ _ $\underline{5}$

$O(n-y)$

↳ first position of last element, this time that wouldn't be worst case.

, so worst case $O(n)$, best case $\Omega(1)$