Final Report: Seismic Analysis of Mars

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1 Executive Summary

The primary objective of the Marsquake simulation project was to investigate the core of Mars using seismic data from the InSight Mission and simulated datasets provided by leading seismologists. The project focused on understanding Mars' internal structure through seismic wave propagation, specifically studying P-waves and S-waves.

Advanced data analytics and core seismic theory were employed, utilizing various Python modules for 3D visualizations and analytics. The key outcomes included determining the radius of Mars' core, identifying the refractive indices of its layers, and proving the range of the S-wave shadow zone. Additionally, the liquid state of Mars' core was confirmed, and seismic ray refraction was observed using ray theory, allowing for an in-depth analysis of seismic wave propagation on Mars.

These findings not only validate existing hypotheses regarding Mars' surface but also offer a robust framework for future seismic wave analyses. This research has significant implications for future exploration and Marsquake analysis, enabling scientists to better understand Mars' internal structure and respond to new seismic events on the planet.

2 Introduction

The exploration of planetary interiors is crucial for understanding the formation and evolution of celestial bodies. The Marsquake Simulation Project aims to investigate the internal structure of Mars by analyzing seismic data from NASA's InSight Mission, supplemented with simulated datasets from leading seismologists. By studying the propagation of P-waves and S-waves, this research provides insights into Mars' core composition, layer boundaries, and seismic activity.

The project employs advanced data analytics, seismic theory, and Python-based 3D visualizations to interpret seismic wave behavior. Key objectives include determining the radius of Mars' core, identifying refractive indices of its layers, and analyzing the S-wave shadow zone, which provides critical evidence of Mars' liquid core. Furthermore, ray theory and seismic refraction were utilized to model wave propagation, offering a deeper understanding of the planet's subsurface structure.

The findings from this study not only confirm existing hypotheses about Mars' core but also establish a robust framework for future seismic investigations. By refining seismic wave analysis techniques, this research enhances our ability to interpret planetary interiors and contributes to the broader field of planetary seismology. The insights gained from this project have significant implications for future Mars exploration, seismic event monitoring, and comparative planetology, paving the way for new discoveries in planetary science.

3 Methodology

This section details the step-by-step approach adopted to achieve the project goals. Each subsection corresponds to a specific module:

3.1 Module 1: Differential Calculus in Wave Equations

We began by applying fundamental differential calculus to the basic wave equation and Hooke's law for stress-strain relations. Utilizing the identity of the Laplacian, we decomposed the wave equation into gradient and curl potential components. This decomposition provided a clear distinction between P-waves and S-waves, both valid solutions to the wave equation. Logically, we identified that P-waves emerge from the gradient term, while S-waves originate from the curl term. Furthermore, the differential formulation yielded the velocities of these wave types, which we then computed directly.

3.2 Module 2: Shear Waves and Shadow Zones

Understanding the shear nature of S-waves, we recognized why they are incapable of propagating through liquid media. This fundamental property explains the formation of shadow zones in seismic wave propagation. By analyzing geometric constraints, we demonstrated how regions where S-waves fail to propagate result in predictable shadow zones on a planetary scale.

3.3 Module 3: Snell's Law and Seismic Refraction

To validate the applicability of Snell's Law to seismic wave propagation, we performed a sample calculation at the Core-Mantle Boundary (CMB). By assessing the refraction of seismic waves at this boundary, we reinforced the fundamental principles governing wave transmission and bending due to changes in medium properties.

3.4 Module 4: Core Radius Determination and Shadow Zone Interpretation

Using simple arithmetic, we estimated the Earth's core radius as the difference between the total planetary radius and the mantle thickness. Additionally, we interpreted the geometric formation of seismic shadow zones, further corroborating our previous findings regarding wave propagation limits through different layers of the Earth.

3.5 Module 5: Mars InSight Rover Data Analysis

Finally, we analyzed data from the Mars InSight Rover mission. By examining seismic waveforms, we observed a significant change past a certain angular threshold: the disappearance of S-wave velocity ($v_s \to 0$) and the reduction of P-wave velocity (v_p). This observation aligned with our theoretical framework from Module 1, confirming the existence of a shadow zone and the role of a liquid core in planetary interiors. Our analysis provided empirical validation of seismic wave behavior, reinforcing the theoretical principles established in previous modules.

3.6 Module 6: Seismic Signal Processing using ML

3.6.1 Extracting the features out of given seismograph wave

The seismic waveform data used in this study was sourced from the NASA SpaceApps Challenge dataset, available on GitHub. To ensure consistency, the data was first normalized between 0 and 1. Noise reduction was performed using a Butterworth bandpass filter (0.5–10 Hz) to eliminate low-frequency and high-frequency artifacts, preserving the relevant seismic signal. A Hamming window was then applied to the filtered waveform to improve spectral analysis by reducing spectral leakage.

Feature extraction was conducted across both the time domain and frequency domain. Amplitude-based features included true peak amplitude, RMS amplitude, filtered peak amplitude, and crest factor, which describe the signal's intensity and variation. Statistical features such as zero-crossing rate (ZCR), skewness, and kurtosis were computed to characterize waveform symmetry and sharpness. In the frequency domain, Fast Fourier Transform (FFT) was applied to extract dominant frequency components, while power spectral density (PSD) analysis provided spectral centroid, bandwidth, total power, and peak frequency to quantify frequency distribution. Additionally, phase spectrum analysis identified significant phase shifts, and signal entropy was calculated from the normalized PSD, representing the complexity of the waveform.

To interpret and validate these extracted features, various graphical visualizations were employed, including waveform plots, amplitude histograms, PSD graphs, phase spectrum plots, and zero-crossing visualizations. These insights play a crucial role in seismic classification tasks, such as detecting shadow zones and analyzing planetary core properties. The extracted features provide a comprehensive understanding of seismic wave behavior, forming a foundation for further machine learning-based seismic analysis.

3.6.2 Figuring out Shadow zones and Non shadow zones

We wrote a bit of reusable code with a function to get the "slowness" (analog of refractive index at a particular point). We can enter here simulated dataset for velocities of P and S waves, real data available online, or inferred P and S wave velocity sets from papers search as the InSight Rover paper. Then, we conducted a simple mathematical simulation where we implemented Snell's law with total internal refraction to plot and display the rays. This allowed us to view the mathematically expected shadow zones (if a layer of Mars was radially homogenous and P and S waves were the only ways of energy propagation). We thus got to view the approximate angles for the shadow zone for S (82°) and P waves (105° 142°)

3.7 Predicting Core Radius using Regression

The dataset used for this study was obtained from *Updated Interior Structure Models of Mars with a Liquid Silicate Layer atop the Martian Core*, which contains 1,000 seismic models compatible with seismic body wave travel times and converted seismic waves from high-quality marsquakes. The dataset includes key planetary parameters such as mean mass, moment of inertia, pressure, temperature, and gravity, along with seismic wave velocities and core compositions derived from Ab Initio Molecular Dynamics* simulations.

To determine the radius of the core (R_c), we identified the depth at which the shear wave velocity (V_s) becomes zero, conclusively indicating the transition to a liquid core. This approach is in accordance with established geophysical principles that define the boundary of the interior solid and liquid layers of a planet.

For training of machine learning models, we applied standard regression techniques. Basic feature engineering was performed to extract relevant predictors, including:

- V_s just before it becomes zero (to capture changes that lead to the core transition).

- V_p at the core-mantle boundary (CMB) (to account for the behavior of compressional waves near the core).
- Average V_p within the mantle (to establish velocity contrasts).
- Average density (ρ) in the CMB (to incorporate density variations that affect the propagation of seismic waves).

The models were trained and evaluated using appropriate performance metrics, including ** mean absolute error (MAE), root mean square error (RMSE), and R2 score **. Cross-validation was used to ensure robustness and hyperparameter tuning was performed for optimal performance.

3.8 Anomaly Detection

The task of anomaly detection from seismic data on Mars is approached using a combination of K-Nearest Neighbors (KNN) and Autoencoders, with a particular focus on detecting rare and unusual seismic events. The methodology is inspired by several notable research papers in the field, including:

- Anomaly Detection in Seismic Data–Metadata Using Simple Machine-Learning Models
- Unsupervised Anomaly Detection for Earthquake Detection on Korea High-Speed Trains Using Autoencoder-Based Deep Learning Models
- GH-Magnitude Earthquake Identification Using an Anomaly Detection Approach on HR-GNSS Data
- RECOVAR: Representation Covariances on Deep Latent Spaces for Seismic Event Detection

After reviewing these studies, the following approach was developed and applied to the task of anomaly detection on Marsquake seismic data.

3.8.1 K-Nearest Neighbors (KNN) for Anomaly Detection

K-Nearest Neighbors (KNN) is used as a distance-based method for detecting anomalies in the seismic dataset. The algorithm works by calculating the distance between a given data point and its nearest neighbors. Normal data points are expected to be close to their neighbors, while anomalies will typically be farther away.

The steps involved in KNN anomaly detection are as follows:

- The distance between each data point and its k-nearest neighbors is computed.
- A data point is classified as an anomaly if its distance to the nearest neighbors exceeds a certain threshold.
- The parameter k (the number of nearest neighbors) is optimized to ensure a balance between sensitivity and specificity in anomaly detection.

This approach allows KNN to effectively flag rare events, such as unusual seismic activity that might indicate a Marsquake or a meteorite impact.

3.8.2 Autoencoder for Anomaly Detection

Autoencoders are employed to reconstruct seismic data and identify anomalies based on reconstruction error. The basic idea is to train the autoencoder to learn an efficient encoding of the input data, which is then used to reconstruct the original data. If the reconstruction error is large, the data point is considered an anomaly.

The key steps involved in Autoencoder-based anomaly detection are:

- The autoencoder network learns to encode and decode seismic data, minimizing the reconstruction error during training.
- The model is trained using seismic data from Mars, with the aim of reconstructing the waves up to a specified percentage of the input data.
- After training, data points with reconstruction errors above a pre-defined threshold are flagged as anomalies.

In this approach, two hyperparameters are particularly important:

- One hyperparameter controls the percentage of data up to which the autoencoder breaks down the seismic wave before reconstructing.
- The second set of hyperparameters determines the structure of the network and how sensitive it is to minor variations in the data.

This method allows the autoencoder to focus on reconstructing typical seismic signals and identifying anomalous patterns, which could represent unusual seismic events on Mars.

3.8.3 Reasons for Anomalies in Seismic Data

Anomalies in seismic data can arise due to several factors:

- **Natural Seismic Events**: Seismic activity such as Marsquakes, meteorite impacts, or volcanic events can generate signals that differ significantly from the typical seismic background.
- **Instrumental Errors**: Faulty or malfunctioning sensors, or interference from other sources, may lead to erroneous or anomalous data.
- **Environmental Factors**: Changes in the environmental conditions, such as variations in temperature, pressure, or external disturbances, can affect the seismic measurements and lead to anomalies.
- **Data Processing Issues**: Problems during data acquisition or preprocessing (e.g., missing data, noise, or incorrect calibration) can result in anomalous readings.

By utilizing both KNN and Autoencoders, this approach effectively combines distance-based anomaly detection with deep learning techniques to comprehensively identify anomalies in Marsquake seismic data. This hybrid methodology is designed to capture a wide range of potential anomalies, from subtle data inconsistencies to more significant and rare seismic events.

3.9 Methodology for Simulating Wave Propagation and Wave Potential Using PINNs

Objective: This task involves the use of Physics-Informed Neural Networks (PINNs) to predict seismic wave velocities (P-wave and S-wave) across the Martian interior, as well as to model the wave potential as a function of depth and time. The goal is to estimate refraction angles, shadow zones, and visualize the propagation of seismic waves through Mars' interior.

3.9.1 Step 1: Data Collection and Preprocessing

- **Martian Interior Properties:** Gather data on the physical properties of Mars, including the depth, P-wave velocity (Vp), S-wave velocity (Vs), and density (Rho) of each layer in Mars' crust, mantle, and core.
- **Normalization:** Normalize the data (depth, Vp, Vs, and density) to standardize the inputs and outputs for the PINN. This ensures the model receives data in a form that aids effective learning.
  - Depth, Vp, and Vs are normalized using their mean and standard deviation.

3.9.2 Step 2: Physics-Informed Neural Network (PINN) for Seismic Wave Velocities

- **PINN Architecture:** Develop a neural network that simulates seismic wave propagation in Mars' interior, predicting the P-wave and S-wave velocities (Vp and Vs). Two separate PINN models are built for Vp and Vs.
- The network has multiple layers with a tanh activation function to enable non-linear learning of the wave velocities.
- **Physics-Informed Loss Function:** The physics-informed loss function enforces physical constraints, such as ensuring non-negativity of the Lamé parameter for Vp and shear modulus for Vs.
- **Training:** Train the network to predict seismic wave velocities by minimizing a loss function that incorporates both the data-driven error and the physics-informed penalty for violating physical laws (e.g., keeping Lamé parameters and shear modulus non-negative).

3.9.3 Step 3: Wave Potential Prediction Using PINN

- **PINN for Wave Potential:** A second PINN is developed to predict the wave potential across the Martian interior as a function of depth and time.
- The network learns the behavior of seismic waves (P-waves and S-waves) as they propagate through different layers of Mars' interior.
- The PINN is trained using data that includes both the depth and time, allowing the model to capture the dynamic evolution of the seismic wave potential over time.

- The performance of the PINN is evaluated using the Mean Squared Error (MSE) metric, and the model is validated with wave potential data, achieving an MSE of 0.98, indicating good model performance.

3.9.4 Step 4: Refraction Angles and Shadow Zones

- **Refraction Behavior:** The trained PINNs are used to compute refraction angles for seismic waves as they pass through the different layers of Mars. The refraction behavior is modeled based on the velocities predicted by the PINNs and Snell's law of refraction.

- Incident and refracted angles are calculated and plotted to analyze how seismic waves bend at the boundaries between layers with different wave velocities.

- **Shadow Zones:** The model predicts the S-wave shadow zone by identifying regions where S-waves do not propagate due to the liquid core, providing insight into the core's structure.

- The S-wave shadow zone is visualized and the size and shape are analyzed.

3.9.5 Step 5: Wave Propagation Simulation and Visualization

- **Simulate Wave Propagation:** Using the PINN outputs (Vp and Vs), simulate seismic wave propagation through Mars' interior. The simulation tracks the rays' paths as they travel through the various layers.

- The wavefronts of both P-waves and S-waves are traced, and their interaction with the core-mantle boundary is visualized.

- **Ray Tracing and Visualization:** Trace the rays of seismic waves using a ray tracing algorithm, and visualize the propagation of waves within a 2D or 3D cutaway model of Mars. The visualization includes both the wave velocity profiles and the paths of the seismic waves.

- Refracted rays, shadow zones, and wavefronts are visualized to understand the seismic response of Mars' interior.

3.9.6 Step 6: Refractive Index Calculation

- **Refraction Index:** The refractive index for both P-waves and S-waves is calculated by comparing the velocities of the waves at different depths.

- The refractive index is computed using the ratio of the velocities in successive layers and is plotted against depth to visualize the wave behavior in different Martian layers.

3.9.7 Step 7: Results Analysis and Interpretation

- **Analysis of Results:** The results of the simulations, including the predicted velocities, refraction angles, and shadow zones, are analyzed. The MSE of 0.98 for wave potential prediction indicates good accuracy, and this is validated with real data or theoretical expectations.

- **Visual Outputs:** The results are visualized in plots and graphs to understand the seismic behavior of Mars, including:
  - A plot of the refracted angles as a function of incident angles.
  - A 2D cutaway of Mars with visualized ray paths.
  - A graph of refractive indices with respect to depth for both P-wave and S-wave velocities.

4 Results

This section presents the outcomes of each module.

- Module 1: The P-wave and S-wave velocities can be calculated using the bulk modulus and shear modulus, providing the speeds at which seismic waves propagate through different materials.
- Module 2: The S-wave shadow zone indicates regions where S-waves are not detected, suggesting the presence of a liquid core in the planet, as S-waves cannot propagate through liquids.
- Module 3: Using Snell's Law, the angle of refraction for P-waves at the core-mantle boundary can be calculated, which helps understand the wave propagation behavior at the CMB.
- Module 4: Core radius is calculated by the given formula which turns out to be wrong. Then by experimental research done by our team and a research paper on InSight mission proves that it is not what it is given as ouput.
- Module 5: We conclude that the core of Mars is liquid which is proved by our Module 7, our discussion in Module 2 and also by the research paper given on InSight Mission.
- Module 6: In this module we created a framework to do the seismic signal processing and then in other part we by simulating the rays made our own dataset of shadow zone and then by ML proved that S wave shadow zone occurs beyond 84 degrees.
- Module 7: Here also we made the dataset by other given 1000 datasets and then applied the standard regression techniques to obtain radius of core dependent on Vp,Vs and density. We did it under MSE of 30.
- Module 8: Here we did anomaly detection by using hyperparameter tuning on KNN and Auto-encoders and made a function which can do the same for any mseed file.
- Module 9: In this module we 2 PINN one to predict the depth vs Vp/Vs and other to predict the wavefield and wave potential. We also trained Gradient boost and compared our result to previously trained PINN.Then we used this simulation to predict the refraction angles and refractive indices across the layers and hence ran ray simulations to conclude the S wave shadow zone.

Challenges and Limitations: Martian Seismic Exploration

1. **Tiny Datasets, Big Assumptions:**

   We were working with extremely limited Martian data, mostly just raw seismic waveforms. To even start analyzing, we had to make some pretty big (and potentially shaky) assumptions:

   - *Perfect Wave Bending (Snell's Law):* Assumed seismic waves bent at boundaries exactly as predicted by simple physics.
   - *Mars as Uniform Layers:* Imagined Martian layers as perfectly uniform, like simple cake layers, ignoring real-world complexity.
   - *Just P and S Waves:* Pretended only basic seismic waves (P and S) existed, ignoring other wave types.

   What We Considered Doing:

   - *Check Assumption Impact:* See if our assumptions really messed things up.
   - *Make Up More Data (Simulations):* Generate fake Martian data to fill the gaps.
   - *Use Tougher Analysis Methods:* Find techniques that work even if assumptions are wrong.

   What We Actually Did:
   We found a dataset that itself gave the velocity of P and S waves by depth and used it to work on our models and simulations. We sidestepped the fact that any such data was heavily approximate, gave data at 200-odd intervals on 3390 km of Martian radius, and probably the result of a model itself.

2. **Simulations Gone Wild (Spirals!):**

   Earth has tons of seismic stations, Mars barely any. We had to rely on simulations to track wave paths. But our simulations were *too* perfect mathematically, leading to weird "spiral" paths for some waves – probably unrealistic.

   What We Considered Doing:

   - *Fix the Simulation:* Figure out the spiral issue and tweak the code.
   - *Cut Off Crazy Paths:* Ignore the spiral parts of the wave paths.
   - *Try Different Simulation:* Use a different simulation method altogether.

   What We Actually Did:
   [We put a simple exit conditions, that if a wave is on this radially spiralling theoretic infinite path, it stops rendering after so many (400) steps.

3. **Mystery Waveforms, No Labels, Few Stations:**

The Martian waveform data wasn't labeled with what caused each event (marsquake, impact, etc.). And with only 3 stations, we couldn't just use standard machine learning that needs labeled data.

What We Considered Doing:

- *Find Patterns Without Labels:* Use machine learning that finds hidden patterns itself.
- *Physics Hints for Learning:* Use our Martian knowledge to guide the machine learning.
- *Label a Little Data (Maybe):* Label a tiny bit ourselves to get started, if possible.

What We Actually Did:
We ran an unsupervised model to work on the waveforms to detect anomalies and classify unsupervised on the waveforms, normalized using Butterworth.

4. **Research Papers: Secrets in Abstracts:**

Many research papers only gave abstracts, no real details on code, data, or methods. They had better resources, but kept the specifics hidden – frustrating for us trying to build on their work!

What We Considered Doing:

- *Hunt for Open Research:* Look for papers with more details freely available.
- *Ask the Researchers:* Try emailing paper authors for more info.
- *Guess the Methods:* Try to figure out their methods from the little info we had.

What We Actually Did:
We... sidestepped this and focused our work on the Peter Shearer theory book and the InSight Rover report.

5. **3D Math Headache (Too Complex!):**

Seismic waves in Mars are a 3D problem, leading to really hard math equations. Simulating these in code was conceptually tough and took lots of computer power.

What We Considered Doing:

- *Simplify to 2D:* Pretend Mars was 2D to make it easier.
- *Super Computers Smart Solvers:* Use powerful computers and special math tools.
- *Focus on Parts of 3D:* Just solve the 3D problem for a small area or wave type.

What We Actually Did:
Yes, we went with the 2D solution, simulating everything on an isotropic-assumption 2D cutaway of Mars at the diameter

5 Visualizations

Important plots and simulations are attached in Github

6 References

Module 1, 2, 3: Seismology Fundamentals

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Module 4, 5: Core Radius and Core State Verification

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Module 6: Seismic Signal Processing Using ML

- Updated Interior Structure Models of Mars with a Liquid Silicate Layer atop the Martian Core. Dataset of seismic data associated with the Martian core-mantle boundary.

Module 7: Predicting Core Radius Using Regression

- Updated Interior Structure Models of Mars with a Liquid Silicate Layer atop the Martian Core. Dataset of seismic wave velocities (P-wave and S-wave) and density data to predict core radius.

Module 8: Anomaly Detection in Seismic Data

- Anomaly Detection in Seismic Data–Metadata Using Simple Machine-Learning Models.
- Unsupervised Anomaly Detection for Earthquake Detection on Korea High-Speed Trains Using Autoencoder-Based Deep Learning Models.
- GH-Magnitude Earthquake Identification Using an Anomaly Detection Approach on HR-GNSS Data.
- RECOVAR: Representation Covariances on Deep Latent Spaces for Seismic Event Detection.

Module 9: Physics-Informed Neural Networks (PINN) for Mars Seismology

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