Analysis of Core Radius Calculation for Mars: A Discrepancy Between Formula Application and Observational Data

Abstract

This report examines the calculation of Mars's core radius using the formula $R_c = R - d$, where R is the total planetary radius and d is the depth to the core-mantle boundary. The calculated value is compared with seismic observations from the InSight mission. The inconsistency highlights the oversimplified nature of the formula and its inability to capture the nuances of seismic data and planetary modeling.

1 Introduction

Determining the internal structure of Mars, including the core radius, is crucial for understanding its formation, evolution, and current state. The formula

$$R_c = R - d$$

is often used as a first-order approximation to estimate the core radius, where R is the total planetary radius and d is the depth to the core-mantle boundary. For Mars:

- Total planetary radius: $R = 3390$ km.
- Depth to the core-mantle boundary: $d = 560$ km.

Substituting these values yields:

$$R_c = 3390 \text{ km} - 560 \text{ km} = 2830 \text{ km}.$$

However, seismic observations indicate a core radius of approximately 1830 ± 40 km [1]. This report investigates why this formula fails to align with observational data.

2 Observational Data from Seismology

Seismic observations, particularly from the InSight mission, provide detailed information about Mars's internal structure:

- **Core Radius:** InSight's seismic data identifies the core radius as 1830 ± 40 km [1]. This value is derived using core-reflected seismic waves (ScS phases) and advanced modeling.

- **Seismic Wave Analysis:** Variations in P-wave and S-wave velocities reveal transitions between crust, mantle, and core, pinpointing the depth of the core-mantle boundary.
- **Density Profiles:** The liquid core's density is inferred to be $5.7 - 6.3$ g/cm^3, consistent with a smaller core radius.

3 Limitations of the Formula $R_c = R - d$

The formula $R_c = R - d$ fails to account for several critical aspects of planetary modeling and seismic observations. Below are the primary reasons why this formula is inadequate:

3.1 Oversimplification of Internal Structure

The formula assumes that the depth d to the core-mantle boundary is uniform and that the mantle is a single homogeneous layer. However, seismic data reveals that Mars's mantle has a complex structure with multiple transitions and variations in material properties. Seismic wave velocities and attenuation indicate significant heterogeneity, which cannot be captured by a single depth parameter d.

3.2 Neglect of Seismic Wave Behavior

Seismic wave behavior provides critical insights into internal boundaries:

- **S-Waves:** The disappearance of S-waves at the core-mantle boundary indicates the transition to a liquid core. The depth inferred from seismic wave travel times is closer to 1830 km rather than the value implied by $R - d$.
- **P-Waves:** Changes in P-wave velocities across the mantle and core refine the core's size and density, highlighting the complexity of the mantle-core boundary [1].

3.3 Misinterpretation of d

In the formula $R_c = R - d$, d is interpreted as the mantle thickness. However, d must represent the exact depth to the core-mantle boundary based on seismic wave reflections. The assumed value of $d = 560$ km is inconsistent with seismic observations, which place the boundary significantly deeper.

3.4 Role of Density and Moments of Inertia

The density of Mars's interior layers influences seismic wave speeds and planetary moments of inertia:

- A larger core radius, such as 3430 km, would imply a lower mantle density inconsistent with Mars's observed moments of inertia.
- Seismic data constrains the core density to $5.7 - 6.3$ g/cm^3, which supports a smaller core radius [1].

4 Consistency with Observational Data

The observed core radius of 1830$\pm$40 km aligns with seismic data and planetary modeling:

- **Seismic Travel Times:** Travel times of P-waves and S-waves match the predictions for a core radius of 1830 km [1].
- **Density Profiles:** The inferred density of Mars's core and mantle is consistent with a smaller core radius, supporting the seismic findings.
- **Moment of Inertia:** Mars's observed rotation dynamics further corroborate the smaller core radius, as a larger core would result in a different mass distribution.

5 Conclusion

The formula $R_c = R - d$ oversimplifies the determination of Mars's core radius by neglecting the complexity of the mantle-core boundary, seismic wave behavior, and density variations. Seismic observations from the InSight mission provide a robust estimate of the core radius at 1830 ± 40 km, highlighting the limitations of the simplistic formula. Future studies should rely on detailed seismic modeling and planetary density profiles for accurate determinations of internal structure.

References

[1] P. Lognonné et al., "Mars Seismology," *Annual Review of Earth and Planetary Sciences*, 2023.