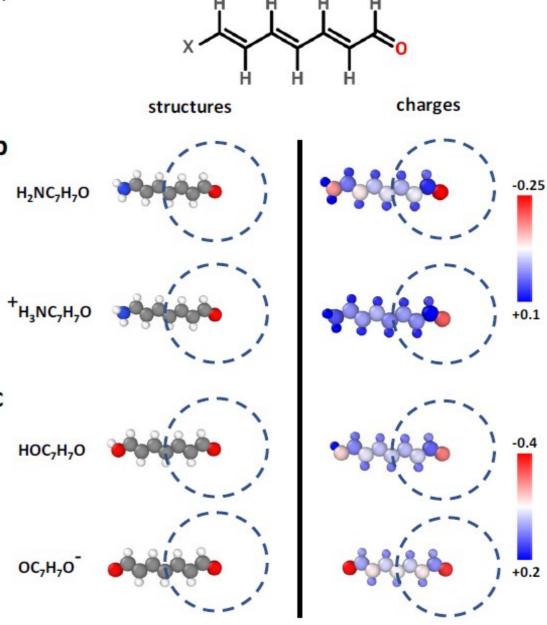
A fourth-generation high-dimensional neural network potential with accurate electrostatics including non-local charge transfer

Interatomic potentials for ionic systems with density functional accuracy based on charge densities obtained by a neural network

"In spirit of CENT"

Include description of global change in electronic structure from Long-range charge transfer or different charge states



$$E_{\text{total}}(\mathbf{R}, \mathbf{Q}) = E_{\text{elec}}(\mathbf{R}, \mathbf{Q}) + E_{\text{short}}(\mathbf{R}, \mathbf{Q}).$$

electronegativities $\{\chi_i\}$ that are adjusted to yield charges in agreement with the DFT reference charges, which here we choose to be Hirshfeld charges

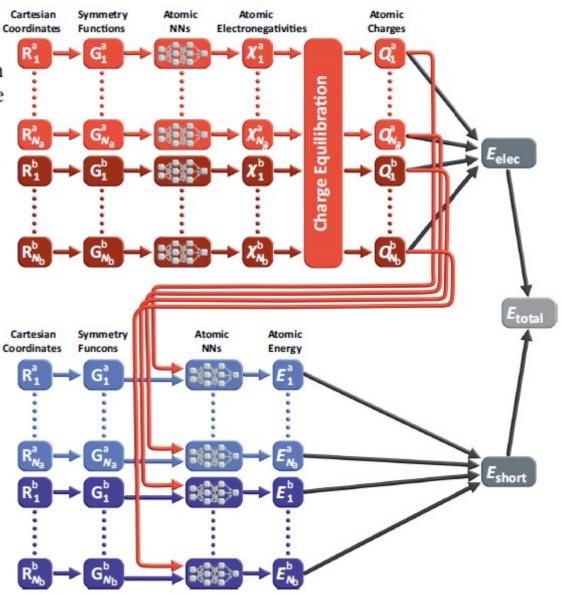
translational, rotational and permutational invariance of the electronegativities.

$$E_{\text{Qeq}} = E_{\text{elec}} + \sum_{i=1}^{N_{\text{at}}} (\chi_i Q_i + \frac{1}{2} J_i Q_i^2)$$

$$E_{\text{elec}} = \sum_{i=1}^{N_{\text{at}}} \sum_{j}^{N_{\text{at}}} \frac{\operatorname{erf}\left(\frac{r_{ij}}{\sqrt{2}\gamma_{ij}}\right)}{r_{ij}} Q_i Q_j + \sum_{i=1}^{N_{\text{at}}} \frac{Q_i^2}{2\sigma_i \sqrt{\pi}}$$

J is the hardness (here optimized during training)

$$\frac{\partial E_{\mathrm{Qeq}}}{\partial Q_i} = 0, \forall i = 1, ..., N_{\mathrm{at}} \ \Rightarrow \ \sum_{j=1}^{N_{\mathrm{at}}} A_{ij} Q_j + \chi_i = 0$$

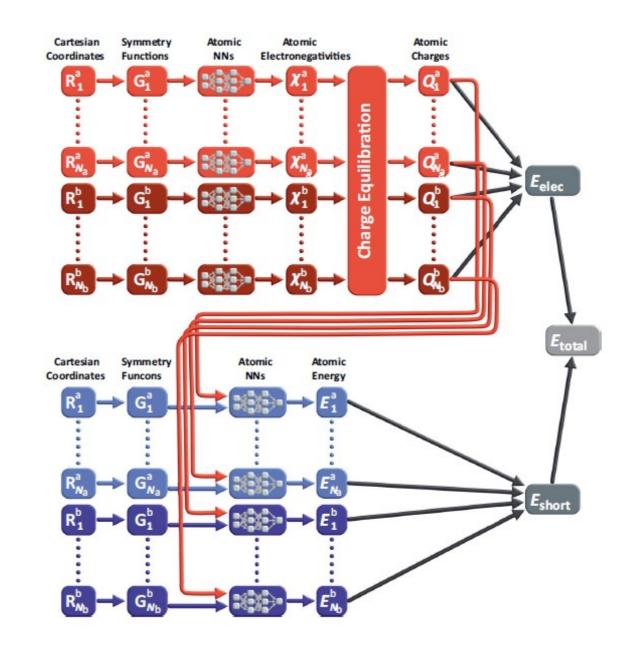


$$[\mathbf{A}]_{ij} = \begin{cases} J_i + \frac{1}{\sigma_i \sqrt{\pi}}, & \text{if } i = j \\ \frac{\text{erf}\left(\frac{r_{ij}}{\sqrt{2}\gamma_{ij}}\right)}{r_{ij}}, & \text{otherwise} \end{cases}$$

$$\begin{pmatrix} \mathbf{A} & 1 \\ \vdots \\ 1 \\ 1 & \dots & 1 \end{pmatrix} \begin{pmatrix} Q_1 \\ \vdots \\ Q_{N_{\text{at}}} \\ \lambda \end{pmatrix} = \begin{pmatrix} -\chi_1 \\ \vdots \\ -\chi_{N_{\text{at}}} \\ Q_{\text{tot}} \end{pmatrix}$$

Charges are trained vs DFT references

$$E_{\text{elec}} = \sum_{i=1}^{N_{\text{at}}} \sum_{j}^{N_{\text{at}}} \frac{\text{erf}\left(\frac{r_{ij}}{\sqrt{2}\gamma_{ij}}\right)}{r_{ij}} Q_i Q_j + \sum_{i=1}^{N_{\text{at}}} \frac{Q_i^2}{2\sigma_i \sqrt{\pi}}$$



$$E_{\text{short}} = \sum_{i=1}^{N_{\text{at}}} E_i$$

$$E_{\text{short}} = E_{\text{ref}} - E_{\text{elec}} = \sum_{i=1}^{N_{\text{at}}} E_i(\{\mathbf{G}_i\}, Q_i)$$

In Conclusion

Hardness included as parameter

Charges trained with DFT charges

E short not described through charges but own neural network

