

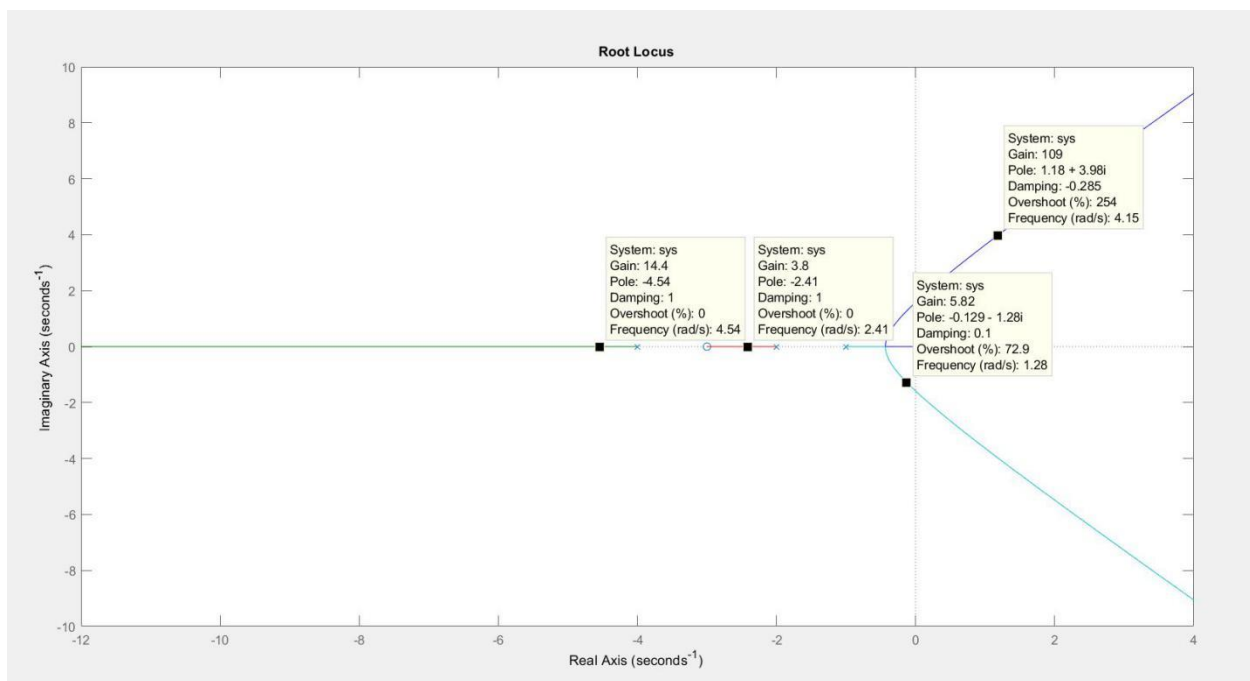
**Bangladesh University of Business & Technology**  
**Control System Lab**  
**EEE 402**

**Experiment No: 07**

**Experiment Name:** Design via root locus.

In the last lab, you were introduced with the root-locus technique. Now we'll design a control system exploiting that technique.

$$G(s) = \frac{s + 3}{s(s + 1)(s + 2)(s + 4)}$$



Now let's say we want to design a system with overshoot less than 10%

Can we design such a system?

✚ The relation between %OS and damping ratio is –

$$\zeta = \frac{\ln(OS)^2}{\sqrt{\pi^2 + \ln(OS)^2}}$$

Then the design parameter can be calculated as -

$$\zeta = 0.59$$

Now let's say we want to design a system with rise time  $T_r$  less than 1s.

✚ The relation between rise time and  $\omega_n$  is -

$$\omega_n \geq \frac{1.8}{T_r}$$

Then design parameter can be calculated as --

$$\omega_n \geq 1.8$$

Now, we can include this in Matlab this way -

```
num= [1,3];  
den= conv(conv([1,0],[1,1]),conv([1,2],[1,4]));  
g=tf(num,den);  
rlocus(num,den)  
  
zeta=0.59;  
wn= 1.8;  
sgrid(zeta,wn)  
[k,poles]=rlocfind(num,den)  
[nc,dc]=cloop(k*num,den)  
z=tf(nc,dc)  
step(z)
```

- ✚ You'll find two straight lines corresponding to the value of  $\zeta$ . Any poles located in the region bounded by these two lines will result in  $\zeta$  value of greater than 0.59. Which will make the %OS less than 10%.
- ✚ You'll find a semi-circle with a radius equal to  $\omega_n$ . So any point outside the circle will satisfy our condition of rise time  $< 1s$ .
- ✚ So, in order to design our system, we must satisfy the above mentioned criteria.

✚ For only one design parameter, use 0 in the sgrid function i.e.  
`sgrid(zeta,0)`

Using the 'rlocfind' function, you can select any point in the root locus and see the response.  
Try some points and see whether you can satisfy the design criteria.

### Exercise:

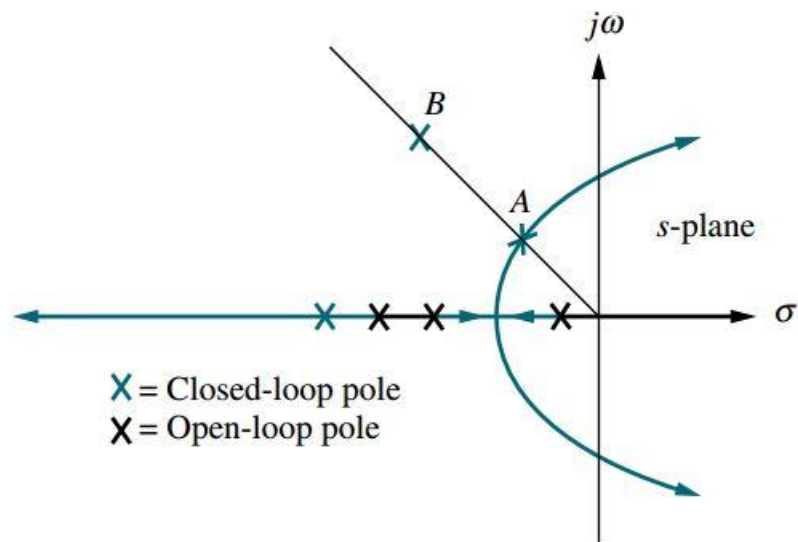
$$G(s) = \frac{K(s + 1.5)}{s(s + 1)(s + 10)}$$

Design the system for OS < 10% and rise time < 2s.

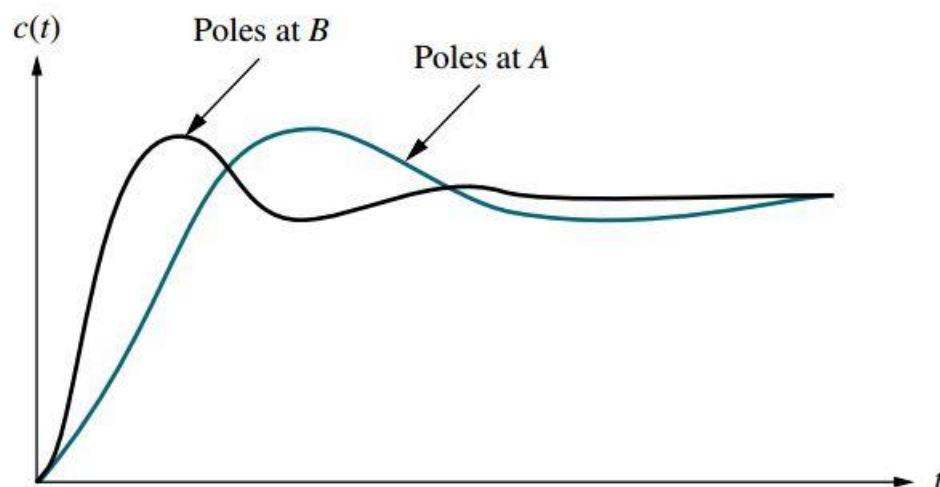
✚ Now, let's say we want faster response i.e. smaller settling time.

$$T_s = \frac{4}{\zeta \omega_n}$$

then we need larger  $\zeta \omega_n = \sigma_d$  i.e. we've to go leftwards on the plot.  
if we want this keeping the %OS constant, then we just have to follow the  $\zeta$  line.



- So, our goal is to speed up the response at A to that of B, without affecting the percent overshoot. This increase in speed cannot be accomplished by a simple gain adjustment, since point B does not lie on the root locus.
- One way to solve this problem is to replace the existing system with a system whose root locus intersects the desired design point, B. Unfortunately, this replacement is expensive and counterproductive.
- Rather than change the existing system, we augment, or compensate, the system with additional poles and zeros, so that the compensated system has a root locus that goes through the desired pole location for some value of gain.



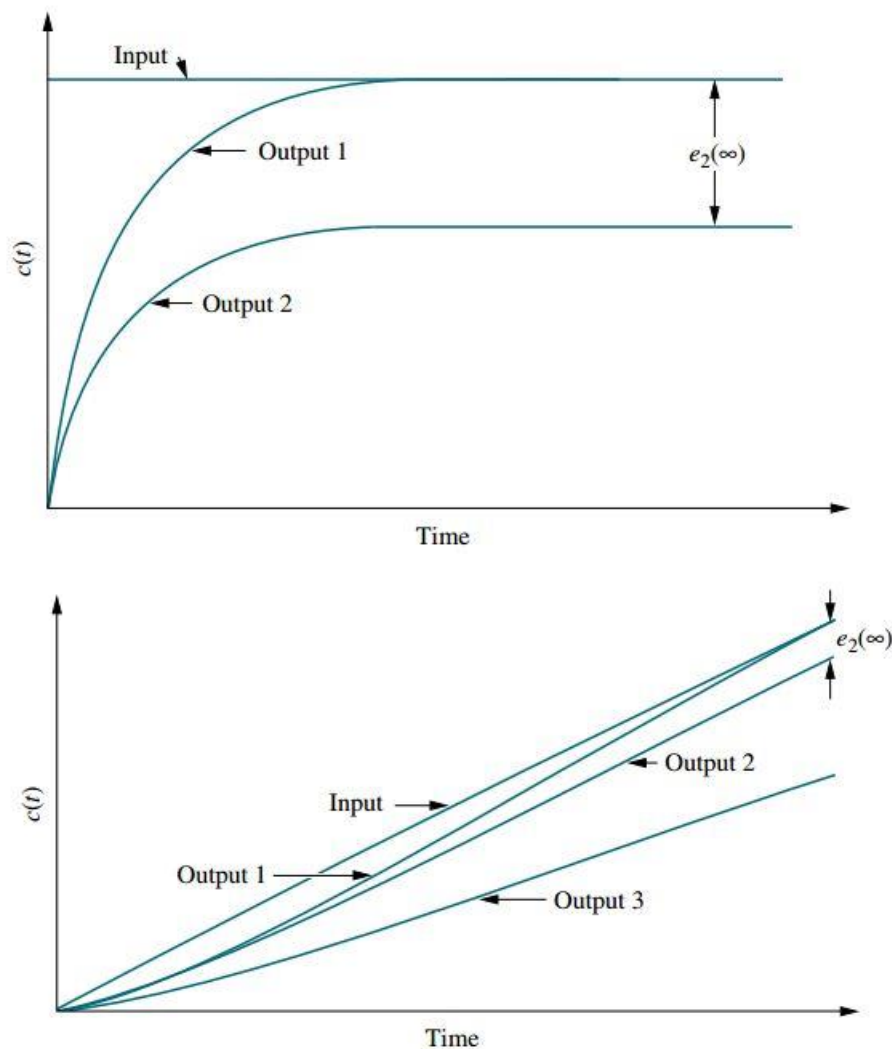
[The design procedure of these kind of compensators will be taught in the class]

## Steady-State Error:

Steady-state error is the difference between the input and the output for a prescribed test input as  $t$  tends to infinity.

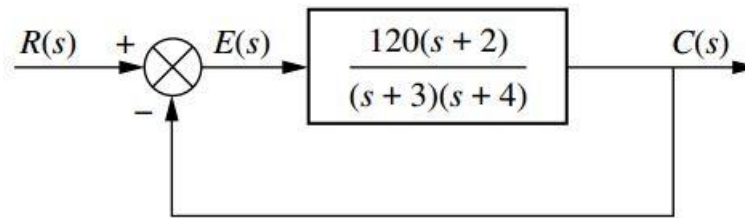
The test input can be a step function or a ramp function etc.

Error for these type of inputs looks like below --



To evaluate steady-state error from Matlab, 1<sup>st</sup> plot the step response. Then find out the steady state value. The difference from 1 gives the error.

**Exercise:** Find out the steady-state error for the system below --



Now, look at your previous design of root-locus and see how gain adjustment affects the ss error.

When the system gain was adjusted to meet the transient response specification, steady-state error performance deteriorated, since both the transient response and the static error constant were related to the gain. The higher the gain, the smaller the steady-state error, but the larger the percent overshoot. On the other hand, reducing gain to reduce overshoot increased the steady-state error.

By using dynamic compensators, compensating networks can be designed that will allow us to meet transient and steady-state error specifications simultaneously.