Bangladesh University of Business & Technology
Control System Lab
EEE 402

Experiment No: 04

Experiment Name: Time Domain Analysis of an underdamped 2nd Order System.

Theory: The generalized 2nd order transfer function is –

$$\boxed{G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}}$$

Here, the term ζ is known as the 'damping ratio' and ω is known as the natural frequency of oscillation.

ζ is defined as –

$$\zeta = \frac{\text{Exponential decay frequency}}{\text{Natural frequency (rad/second)}}$$

Now let's define some other parameters associated with a 2nd order system.

1. Rise time (Tr): The time required for the waveform to go from 0.1 of the final value to 0.9 of the final value. Sometimes 0% to 100% is also used.
2. Peak time (Tp): The time required to reach the first or maximum peak.
3. Percent overshoot, %OS. The amount that the waveform overshoots the steady state, or final, value at the peak time, expressed as a percentage of the steady-state value.
4. Settling time, Ts. The time required for the transient's damped oscillations to reach and stay within 2% of the steady-state value.

The approximate equations to represent these parameters in terms of ζ and ω are as follows –

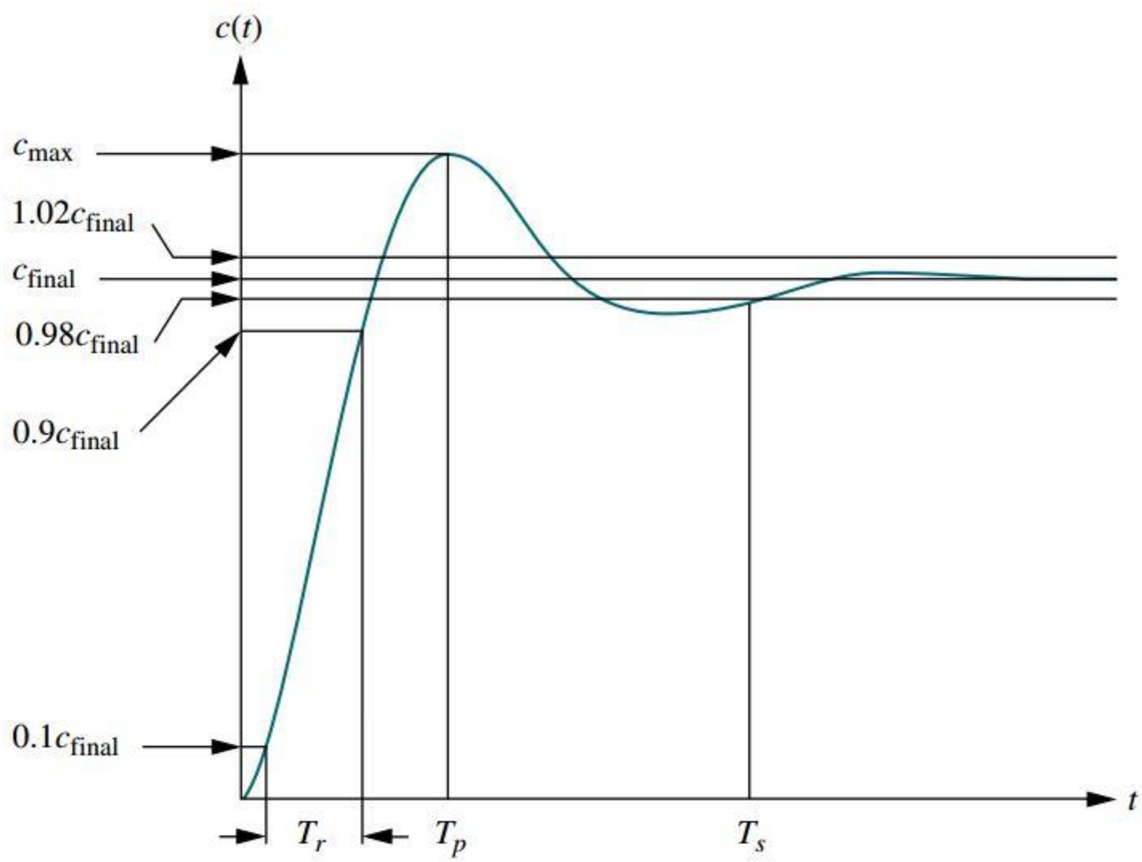
$$T_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}$$

$$\%OS = e^{-(\zeta\pi/\sqrt{1-\zeta^2})} \times 100$$

$$T_s = \frac{4}{\zeta\omega_n}$$

A precise analytical relationship between rise time and damping ratio cannot be found.

Let's look into the time response and the associated parameters of a system –



Using Matlab, we can find out those parameters from the response curve.

Example:

$$G(s) = \frac{s}{s^\wedge 2 + s + 4}$$

Plot the step response of the transfer function.

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g= tf(1,[1,1,4])
step(g)
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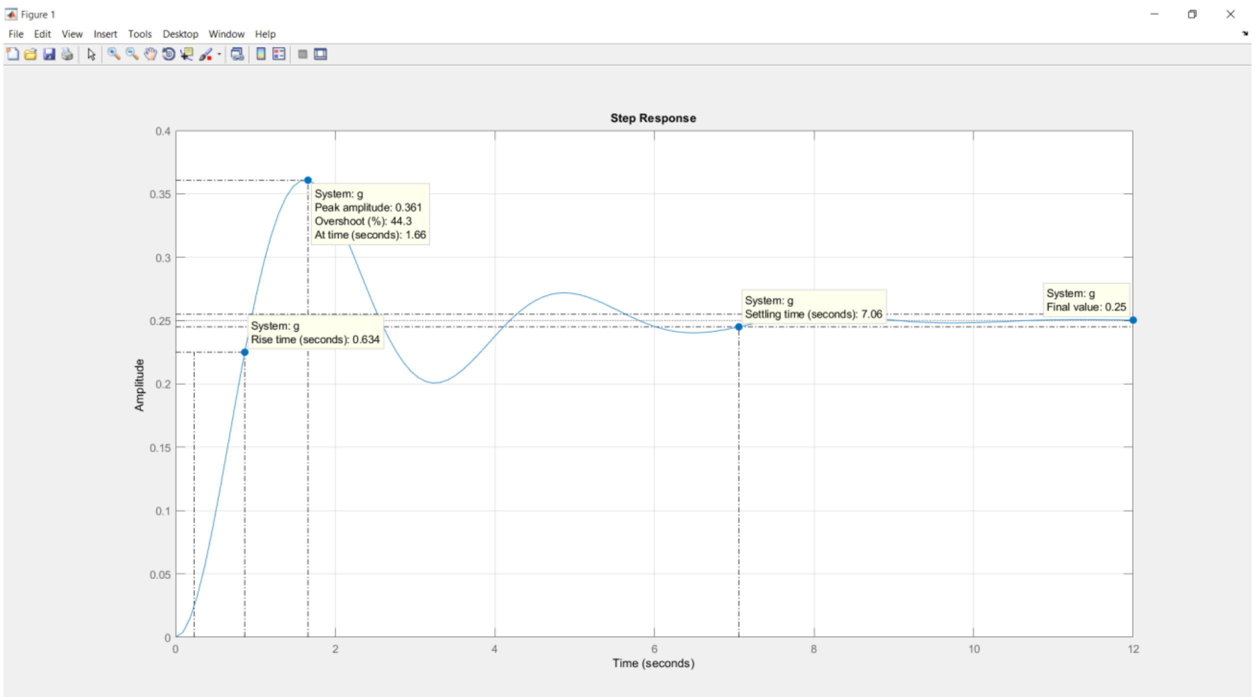

Right click on the figure and select different characteristics. Matlab will provide you with the resulting points. Click on those points to get the values.

You can check the values with the above equations. There will be some errors in the results obtained from those equations since they are approximate equations.

Exercise:

$$G(s) = \frac{361}{s^2 + 16s + 361}.$$

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