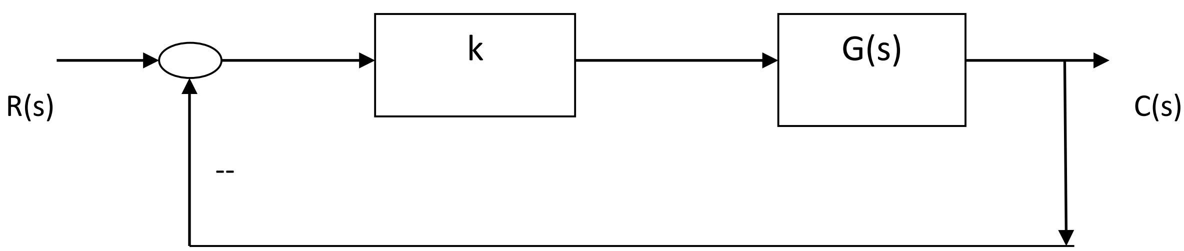
Bangladesh University of Business & Technology
Control System Lab
EEE 402

Experiment No: 06
Experiment Name: Introduction to root locus technique.

The poles and zeros of an open-loop transfer function do not depend on the value of the gain.

But in case of closed-loop transfer function, the location of the poles and zeros changes with the value of k (gain).



- For the above system-

  Open loop transfer function is = K G(s)

  Closed loop transfer function is = $\frac{KG(s)}{1+KG(s)}$

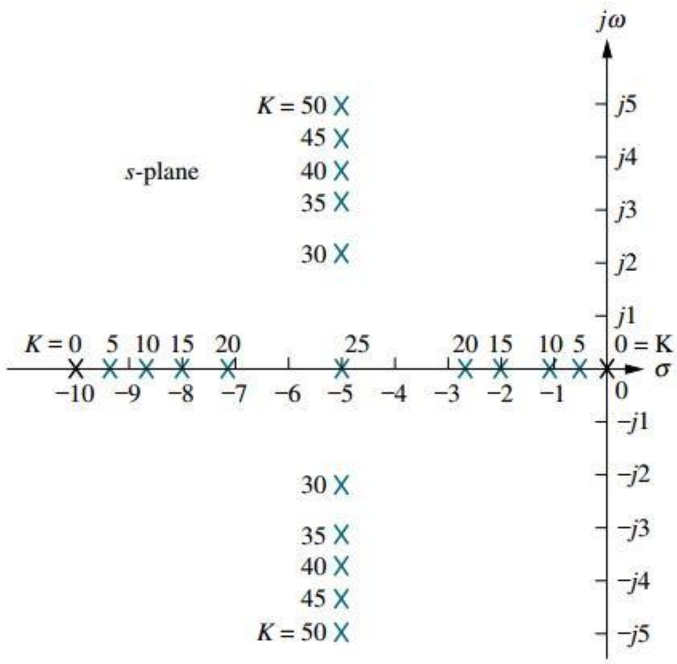
- Root locus is the representation of the paths of the closed loop poles as the gain is varied. So, as the value of k is changed, the poles change and so the response of the system.
- Let's look into an example –

$$\text{c.l. tf} = \frac{k}{s^2+10s+k}$$

Now, plot the pole-zeros of the system using 'pzmap' function for different k values e.g. k= 0, 10, 20, 25, 30, 40, 50.
You'll see—
For K<25, the poles are real. So you've overdamped response.
For K=25, the poles are real and multiple. So you've critically damped response.

For K>25, the poles are complex. So you've under-damped response.

- Regardless of the value of gain, the real parts of the complex poles are always the same. Since the settling time is inversely proportional to the real part of the complex poles for this second-order system, the conclusion is that regardless of the value of gain, the settling time for the system remains the same under all conditions of underdamped responses.
- Also, as we increase the gain, the damping ratio diminishes, and the percent overshoot increases. The damped frequency of oscillation, which is equal to the imaginary part of the pole, also increases with an increase in gain, resulting in a reduction of the peak time.
- Finally, since the root locus never crosses over into the right half-plane, the system is always stable, regardless of the value of gain, and can never break into a sinusoidal oscillation.



$$Ts = \frac{4}{\zeta\omega_n}$$

$$Tp = \frac{\pi}{\omega_d}$$

$$\zeta = cos\theta$$

$$\%OS \ \alpha \ \frac{1}{\zeta}$$

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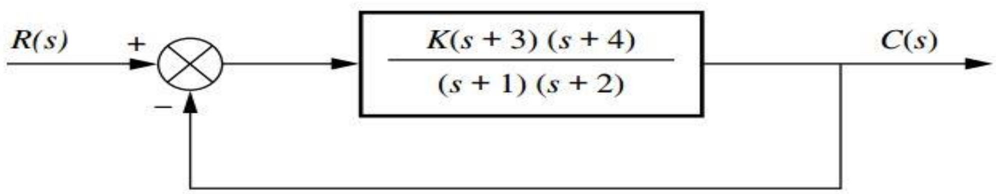
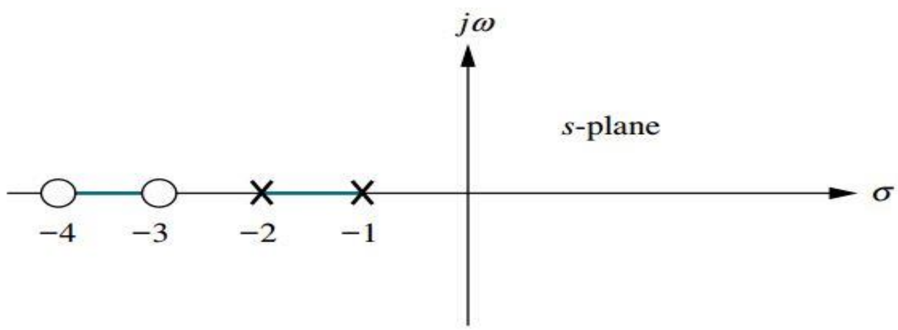
Type equation here.

Sketching the Root Locus:

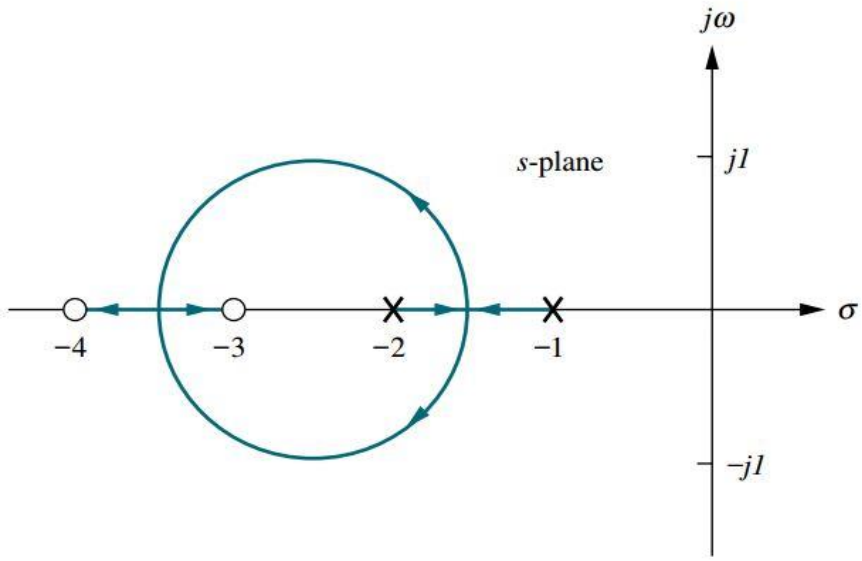
- The number of branches of the root locus equals the number of closed-loop poles.
- The root locus is symmetrical about the real axis.
- Root locus exists to the left of an odd number of real axis, finite open-loop poles and/or finite open-loop zeros.
- The root locus begins at the finite and infinite poles of G(s)H(s) and ends at the finite and infinite zeros of G(s)H(s).
- The root locus approaches straight lines as asymptotes as the locus approaches infinity.



(a)



(b)

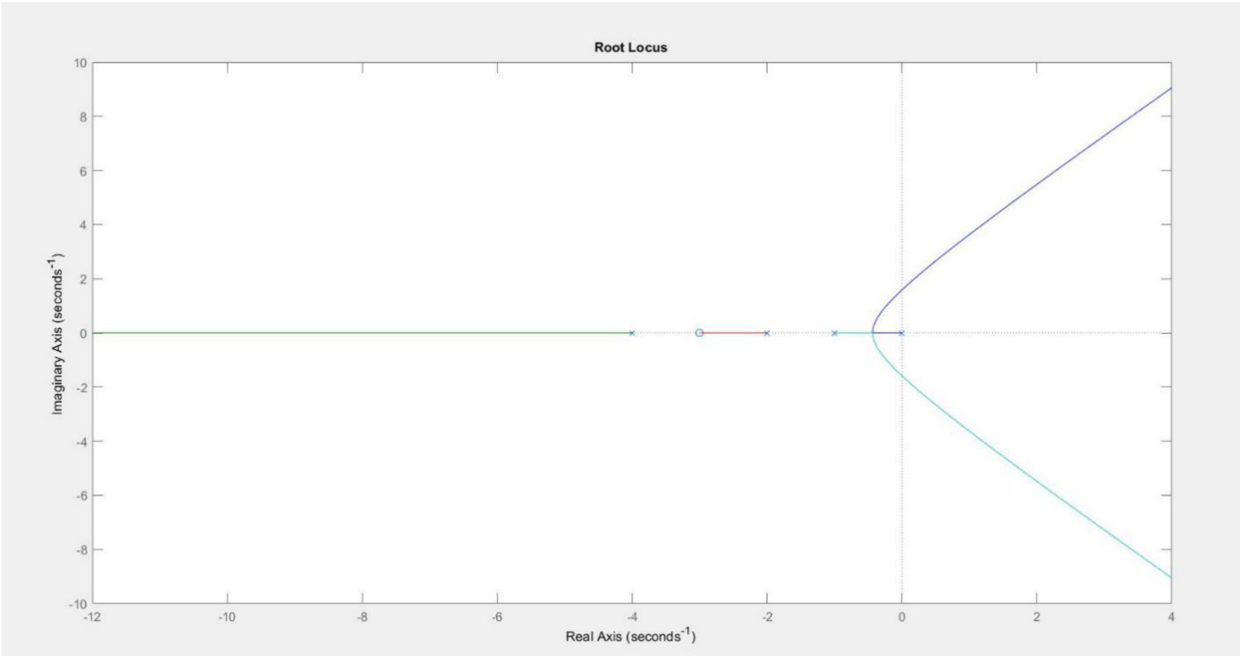


Plotting the root locus using Matlab:

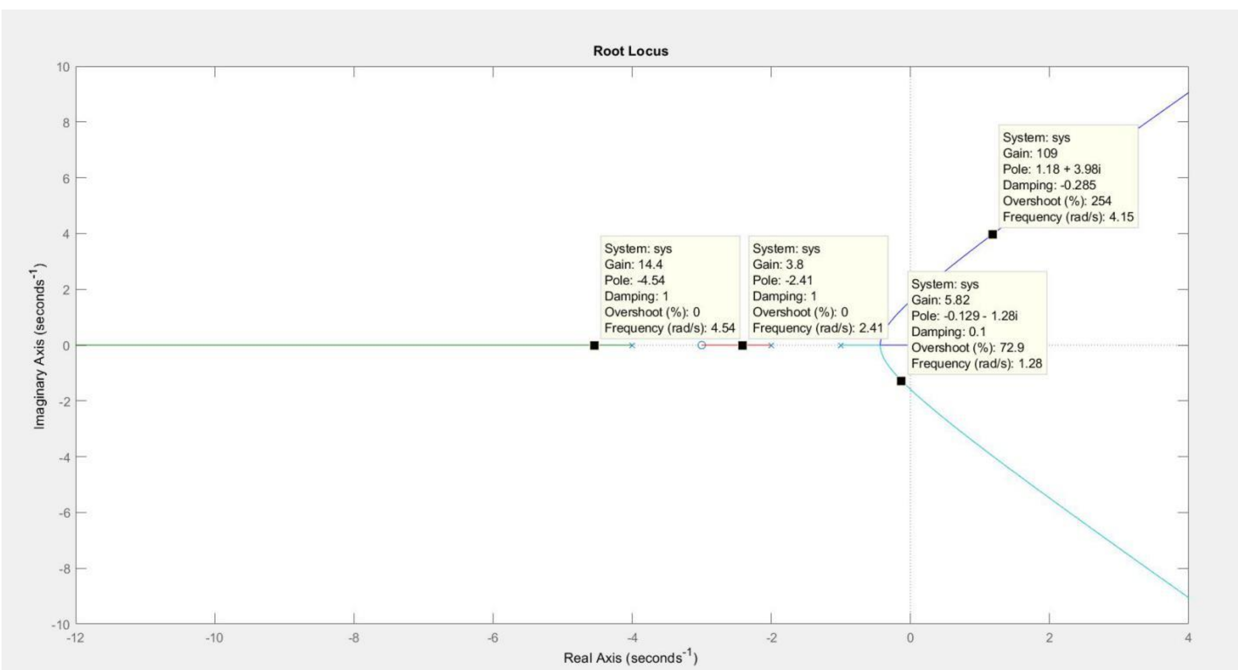
Let's assume the open loop transfer function of a system is –

$$G(s) = \frac{s+3}{s(s+1)(s+2)(s+4)}$$

```
num= [1,3];
den= conv(conv([1,0],[1,1]),conv([1,2],[1,4]));
g=tf(num,den);
rlocus(num,den)
```



➢ By clicking on any point on the root locus, you can find out the value of the gain, %OS and other information from the figure. For example --



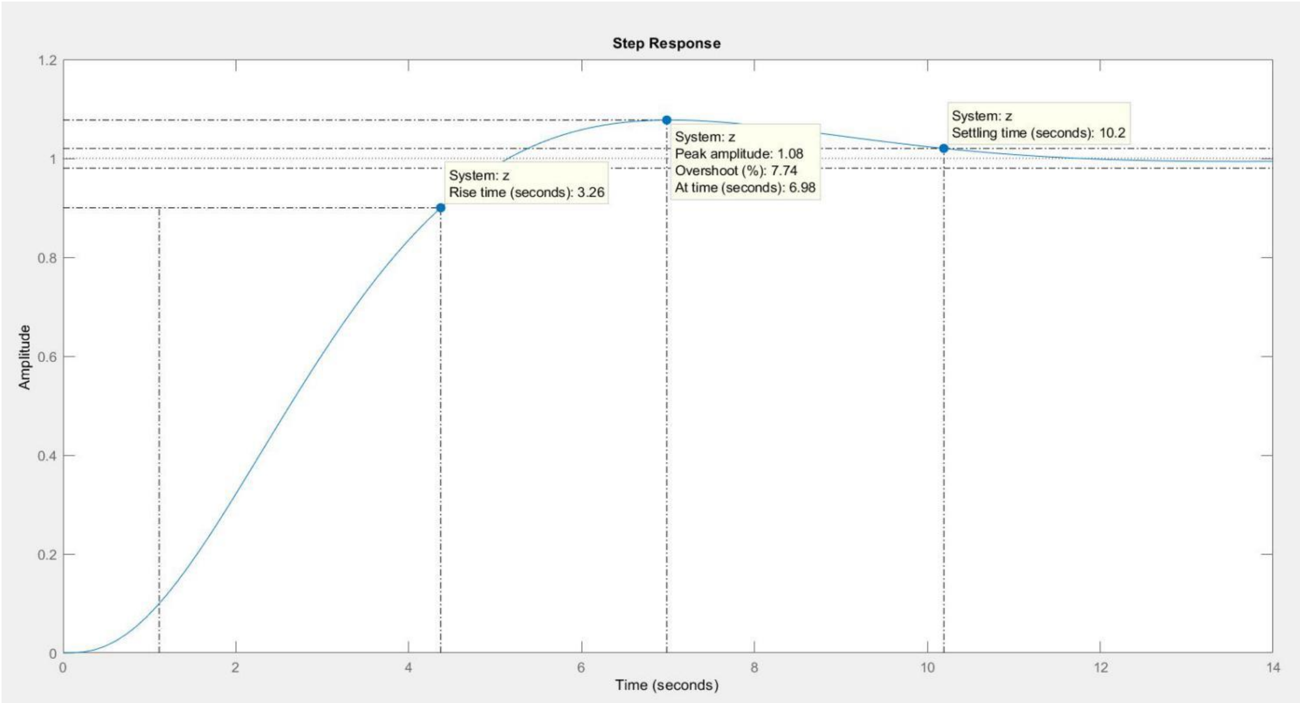
➢ You can calculate $j\omega$ crossing value and the gain associated with it.
➢ Using the Matlab command 'rlocfind', you can select a point on the root locus and find the gain and closed loop poles associated with that gain.

```
[k,poles]= rlocfind(num,den)
[nc,dc]= cloop(k*num,den)
z= tf(nc,dc)
step(z)
```

- For any value of the gain, you'll get 4 closed loop poles since the order of the characteristic equation is 4.
- 'z' is the closed loop transfer function.
- From the time response, you can find out different parameters and also check the stability of the system.

For instance –

```
selected_point =

  -0.3720 + 0.4743i


k =

    1.1020


poles =

  -4.0457 + 0.0000i
  -2.1894 + 0.0000i
  -0.3824 + 0.4764i
  -0.3824 - 0.4764i
```



Exercise:

$$G(s)= \frac{K(s+2)}{(s+3)(s^2+2s+2)}$$

$$G(s) = \frac{k(s^2 - 4s + 20)}{(s+2)(s+4)}$$

$$G(s) = \frac{k}{(s+2)(s+4)(s+6)}$$