Islamic University of Technology (IUT)
Organization of Islamic Cooperation (OIC)
Department of Electrical and Electronic Engineering (EEE)

Course No.	Math 4522
Course Name	Numerical Methods Lab.
Experiment No.	04
Experiment Name	Introduction to Fixed-Point Iteration and Newton-Raphson Methods

Objective

- To get familiarized with the Fixed-Point Iteration and Newton-Raphson methods to find out the root of the nonlinear equation.

Theory

In the previous experiment, you were introduced to the "Bracketing" method of finding the roots. In bracketing methods, the root is located within an interval prescribed by an upper and lower bound. With each iteration, you moved closer to finding the true value of the root. This process is called "convergent" as you gradually converge towards the root.

The process is illustrated in Figure 1 (a).

In this experiment, you will learn a different technique called "Open" methods for finding the roots of non-linear equations. You start with a single starting value or two values, but they do not necessarily bracket the root. And in contrast to the bracketing method, open methods are sometimes "divergent" - that is, they diverge away from the actual value of the root. However, when they converge, they usually do so more quickly than the bracketing methods.

The process is illustrated in Figure 1 (b, c). Here, in figure 1b, the process is diverging, while in figure 1c, the process is converging.

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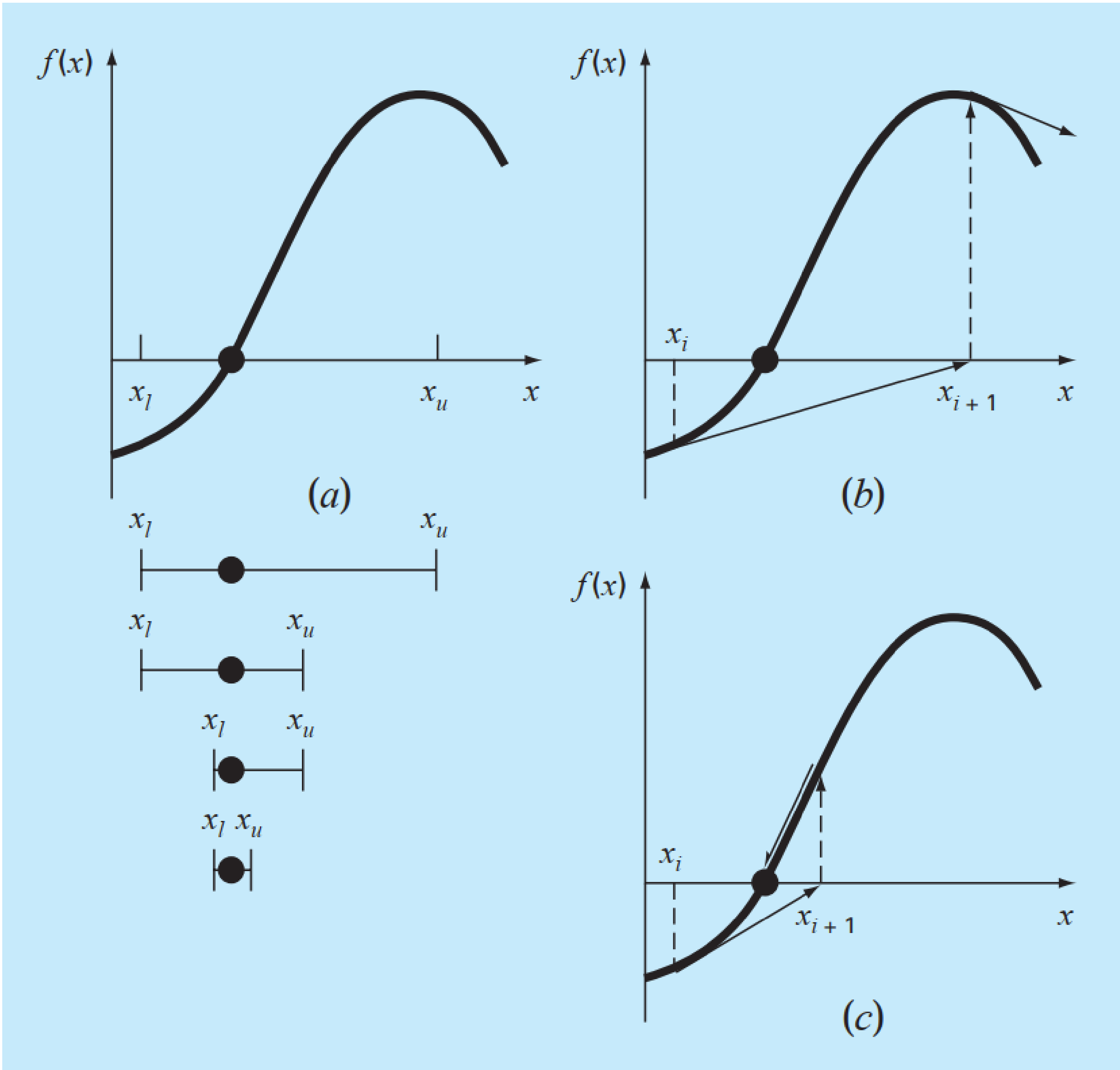

Fig 1: (a) Bracketing Method, (b, c) Open Method

Fixed-Point Iteration

In this approach, you start by rewriting the function in terms of x.

Original function, $f(x) = 0$

Rearrange it in such a way that, $x = g(x)$

For instance, if you have,

$$2x^2 - x - 1 = 0$$

It can be rewritten as,

$$x = 2x^2 - 1$$

The advantage of this is that you can obtain a new value/prediction for x from an old value of x.

- Say, for x = 0 (the initial approximation), you get the new value of x = -1.
- In the next iteration, you use -1 to find the new value of x. That is, x = 1.
- In the 3^{rd} iteration, you use 1 to find the new value of x. That is, x = 1.

This means the solution has converged and you have found the root.

MATLAB Code

```
i=1;                        % iteration number
imax=10;                    % maximum iteration number

xo=0;                       % starting value
tolerance = 0.1;

ea= zeros(1,imax);          % error array
val = zeros(1,imax);        % x-value array

f = @(x) exp(-x);           % another way of writing a function called function-handle
%f = @(x) ((2*x+5)/2)^(1/3)

while i< imax
    xnew= f(xo);            % the new value of x
    xold=xo;
    val(i)=xnew;
    ea(i)= abs((xnew-xold)/xnew)*100;   % percentage error

    if ea(i)<tolerance
        break
    end

    xo=xnew;
    i=i+1;


end
```

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This is a simple approach to finding the root. The limitation of this process is that it will not always converge. The convergence also depends on the initial choice of x. There are many possible arrangements of f(x) = 0 into x = g(x). Some arrangements will only converge given a starting value very close to the root, and some will not converge at all.

Newton-Raphson Method

This is one of the most popular methods for finding the root. It takes the tangent of the slope for a particular value of x. The value at which the tangent crosses the x-axis becomes the new estimate. This approach usually converges faster towards the root.

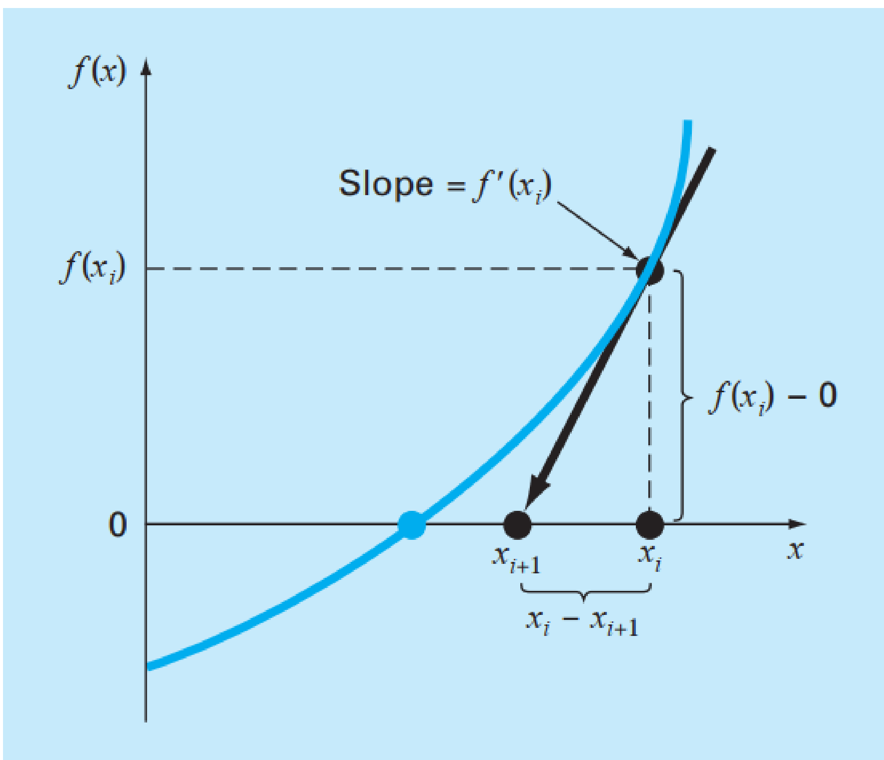

Fig 2. Newton-Raphson method

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In this algorithm, you start by finding the 1st derivative $f'(x)$ of the given equation. Then, you find the new value of x using the following equation –

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

You estimate the relative approximation error using the following equation –

$$|\in_a| = \left|\frac{x_r^{new} - x_r^{old}}{x_r^{new}}\right| * 100$$

- Compare this error with the error tolerance provided. If the obtained error value is higher than the tolerance, then repeat the process. Otherwise, stop the iteration.
- Also check if the maximum number of iterations i_{max} is reached. If the number of iterations > i_{max}, even when the desired tolerance is not achieved, stop the algorithm.

Lab Task

Write a program to find the root of a non-linear equation using the Newton-Raphson method for f(x) = $e^{-x} - x$.

Assignment

1. Explain the advantages and disadvantages of the two approaches you learned in this experiment.
2. Write the code to locate the root of $f(x)$ = $x^{10} - 1$ using the Fixed-Point Iteration and Newton-Raphson method. Take initial guess as x=0.5. Show the relative approximation error and true error at each iteration using a table created using MATLAB program. Also, provide an illustration. Comment on which method is more appropriate for this example.

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