Islamic University of Technology (IUT)
Organization of Islamic Cooperation (OIC)
Department of Electrical and Electronic Engineering **(EEE)**

Course No	**: Math 4522**
Course Name	**: Numerical Methods Lab**
Experiment No	**: 09**
Experiment Name	**: Differentiation of Continuous Function by Numerical Techniques**

The derivative of a function at x is defined as

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

To be able to find a derivative numerically, one could make Δx finite to give,

$$f'(x) \approx \frac{f(x+\Delta x) - f(x)}{\Delta x}.$$

Knowing the value of x at which you want to find the derivative of $f(x)$, we choose a value of Δx to find the value of $f'(x)$. To estimate the value of $f'(x)$, three such approximations are suggested as follows.

Forward Difference Approximation of the First Derivative

From differential calculus, we know

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

For a finite Δx,

$$f'(x) \approx \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

The above is the forward divided difference approximation of the first derivative. It is called forward because you are taking a point ahead of x. To find the value of $f'(x)$ at $x = x_i$, we may choose another point Δx ahead as $x = x_{i+1}$. This gives

$$f'(x_i) \approx \frac{f(x_{i+1}) - f(x_i)}{\Delta x}$$

$$= \frac{f(x_{i+1}) - f(x_i)}{x_{i+1} - x_i}$$

where

$$\Delta x = x_{i+1} - x_i$$

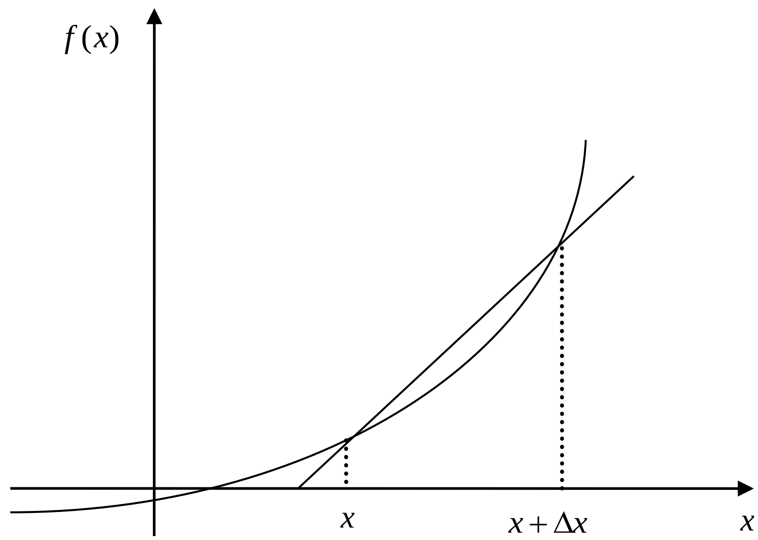

Figure 1 Graphical representation of forward difference approximation of first derivative.

Backward Difference Approximation of the First Derivative

We know

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

For a finite Δx,

$$f'(x) \approx \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

If Δx is chosen as a negative number,

$$f'(x) \approx \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \frac{f(x) - f(x-\Delta x)}{\Delta x}$$

This is a backward difference approximation as you are taking a point backward from x. To find the value of $f'(x)$ at $x = x_i$, we may choose another point Δx behind as $x = x_{i-1}$. This gives

$$f'(x_i) \approx \frac{f(x_i) - f(x_{i-1})}{\Delta x}$$

$$= \frac{f(x_i) - f(x_{i-1})}{x_i - x_{i-1}}$$

where

$$\Delta x = x_i - x_{i-1}$$

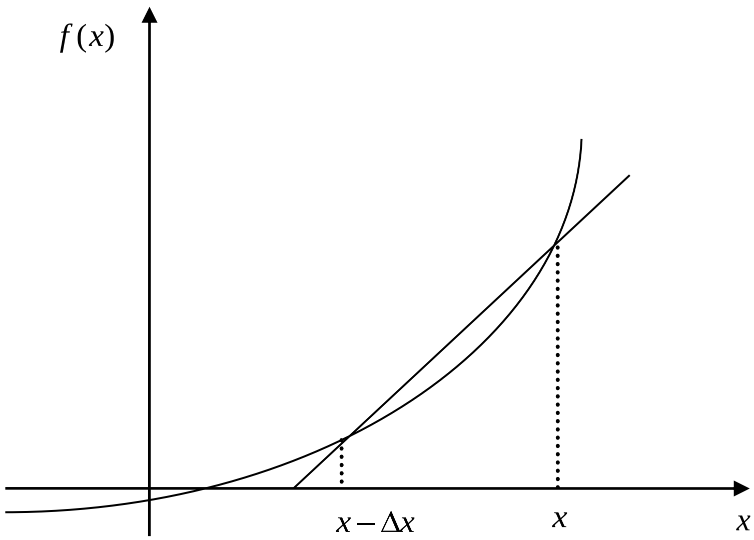

Figure 2 Graphical representation of backward difference approximation of first derivative.

Central difference approximation of the first derivative.
From the Taylor series

$$f(x_{i+1}) = f(x_i) + f'(x_i)\Delta x + \frac{f''(x_i)}{2!}(\Delta x)^2 + \frac{f'''(x_i)}{3!}(\Delta x)^3 + \dots \tag{1}$$

and

$$f(x_{i-1}) = f(x_i) - f'(x_i)\Delta x + \frac{f''(x_i)}{2!}(\Delta x)^2 - \frac{f'''(x_i)}{3!}(\Delta x)^3 + \dots \tag{2}$$

Subtracting Equation (2) from Equation (1)

$$f(x_{i+1}) - f(x_{i-1}) = f'(x_i)(2\Delta x) + \frac{2f'''(x_i)}{3!}(\Delta x)^3 + \dots$$

$$f'(x_i) = \frac{f(x_{i+1}) - f(x_{i-1})}{2\Delta x} - \frac{f'''(x_i)}{3!}(\Delta x)^2 + \dots$$

$$= \frac{f(x_{i+1}) - f(x_{i-1})}{2\Delta x} + O(\Delta x)^2$$

hence showing that we have obtained a more accurate formula as the error is of the order of $O(\Delta x)^2$.

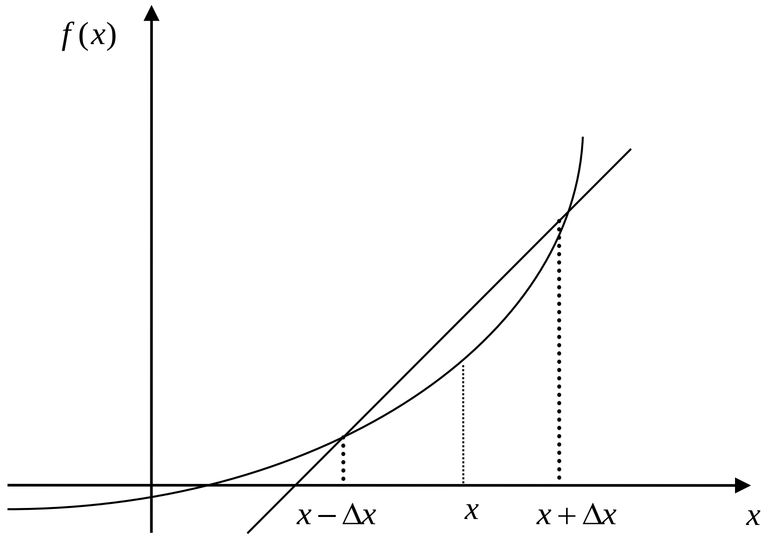

Figure 3 Graphical representation of central difference approximation of first derivative.

Code for implementing Forward, Backward and Central difference approximation of first derivative for a function of f(x)=e^-x.sin(3x). For a range of x=0 to x=4.

```
Clc
clear all
close all

Fun = @(x) exp(-x).*sin(3*x);
dFun = @(x) -exp(-x).*sin(3*x)+ 3*exp(-x).*cos(3*x);
x=linspace(0,4,101);
F=Fun(x);
h=x(2)-x(1);
xCentral=x(2:end-1);
dFCenteral=(F(3:end)-F(1:end-2))/(2*h);
xForward=x(1:end-1);
dFForward=(F(2:end)-F(1:end-1))/h;
xBackward=x(2:end);
dFBackward=(F(2:end)-F(1:end-1))/h;
plot(x,dFun(x));
hold on
plot(xCentral,dFCenteral,'r')
plot(xForward,dFForward,'k');
plot(xBackward,dFBackward,'g');
legend('Analytic','Central','Forward','Backward')
```

Task :

Write a matlab code which will show that error while using central difference method for approximating first derivative of function is $O(\Delta x)^2$ but $O(\Delta x)$ if forward or backward difference method is used.

Additional Note

You can use the 'polyder' function to find the derivative of a polynomial directly using MATLAB. The input to the function is to be a vector containing the coefficients of the polynomial.
Another approach is to use the 'syms' function. Then use the 'diff' command to find the derivatives.