HW 1
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Class Section: 802 Loop

Section 1.2

8.a : Yes. All elements in C are in A.

8.c: Yes. All elements in C are belonging to itself.

9.c: No.

9.d: Yes.

9.g: No.

Section 2.1

17: $\sim P \wedge \sim Q \equiv \sim(P \wedge Q)$?

P	Q	$\sim P$	$\sim Q$	$P \wedge Q$	$\sim P \wedge \sim Q$	$\sim(P \wedge Q)$
T	T	F	F	T	F	F
T	F	F	T	F	F	T
F	T	T	F	F	F	T
F	F	T	T	F	T	T

$\sim P \wedge \sim Q$ and $\sim(P \wedge Q)$ have different truth values in the second and third row, so they are not logically equivalent.

29: This computer program does not have logical error in the first ten lines and it is being run with complete data set.

37: $0 \leq X$ or $X < -7$.

43: $(\sim P \vee Q) \vee (P \wedge \sim Q)$

P	Q	$\sim P$	$\sim Q$	$(\sim P \vee Q)$	$(P \wedge \sim Q)$	$(\sim P \vee Q) \vee (P \wedge \sim Q)$
T	T	F	F	T	F	T
T	F	F	T	F	T	T
F	T	T	F	T	F	T
F	F	T	T	T	F	T

It's truth values are all T's, so $(\sim P \vee Q) \vee (P \wedge \sim Q)$ is a tautology.

52:
$\sim(P \vee \sim Q) \vee (\sim P \wedge \sim Q)$
$\equiv (\sim P \wedge Q) \vee (\sim P \wedge \sim Q)$ By De Morgan's law
$\equiv \sim P \wedge (Q \vee \sim Q)$ By Distributive law
$\equiv \sim P \wedge t$ By Identity law
$\equiv \sim P$
Q.E.D

Section 2.2

13.b:

P	Q	$\sim P$	$\sim Q$	$P \rightarrow Q$	$\sim(P \rightarrow Q)$	$P \wedge \sim Q$
T	T	F	F	T	F	F
T	F	F	T	F	T	T
F	T	T	F	T	F	F
F	F	T	T	T	F	F

$\sim(P \rightarrow Q)$and $P \wedge \sim Q$ are always have same truth value, so they are logically equivalent.

20.b: Today is New Year's Eve and tomorrow is not January.

38: If it not rains, Ann will go.

46.c: This statement must be true. Since "P only if Q" is logically equivalent to " if P then Q", in this case P is the statement "Compound X is boiling", Q is "its temperature is at least 150°C", so the statement is logically equivalent to given statement.

46.d: Let's assume P is the statement "Compound X is boiling" and Q is the statement "its temperature is at least 150°C". This statement could be rewrite as $\sim P \rightarrow \sim Q$, it is the inverse of the given statement so it is not necessarily true. For instance, if the actual boiling point of X were 200°C, and X's temperature is 170°C which is not less than 150°C.

50a:
$\big(p \rightarrow (q \rightarrow r)\big) \leftrightarrow \big((p \wedge q) \rightarrow r\big) \equiv$
$\equiv (p \rightarrow (\sim q \vee r)) \leftrightarrow (\sim(p \wedge q) \vee r)$
$\equiv (\sim p \vee (\sim q \vee r)) \leftrightarrow (\sim(p \wedge q) \vee r)$By Associative law
$\equiv ((\sim p \vee \sim q) \vee r) \leftrightarrow (\sim(p \wedge q) \vee r)$By De Morgan's law
$\equiv (\sim(p \wedge q) \vee r) \leftrightarrow (\sim(p \wedge q) \vee r)$
Assume
$\sim(p \wedge q) \vee r = A$
$\equiv A \leftrightarrow A$

$\equiv t$
This statement is a tautology

Section 3.1

1.d: True.

4.a:
$Q(2)$ is "$2^2 = 4 \leq 30$" which is true.
$Q(-2)$ is "$(-2)^2 = 4 \leq 30$" which is true.
$Q(7)$ is "$(7)^2 = 49 \leq 30$". This is false because $49 \nleq 30$.
$Q(-7)$ is "$(-7)^2 = 49 \leq 30$". This is false because $49 \nleq 30$.

4.c: The truth set is $\{1,2,3,4,5\}$.

8.c: The truth set is $\{-8, -6, -4, -2,0,2,4,6,8\}$.

10: Counterexample is $a = 1$.

23:
$\forall x$, if x is a computer science student then x needs to take data structures.
$\forall$ computer science students x , x needs to take data structures.