Incentive Auction Design Alternatives: A Simulation Study

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Abstract: Over 13 months in 2016–17 the US Federal Communications Commission (FCC) conducted an "incentive auction" to repurpose radio spectrum from broadcast television to wireless internet. This paper revisits from a computational perspective the descending clock "reverse" auction used to procure broadcast rights. We investigate the quantitative significance of various aspects of the design by running extensive simulations, leveraging a reverse auction simulator and realistic models of bidder values.

1 INTRODUCTION

Over 13 months in 2016–17 the US Federal Communications Commission (FCC) conducted an "incentive auction" to repurpose radio spectrum from broadcast television to wireless internet. The result of the auction was to remove 14 UHF-TV channels from broadcast use, sell 70 MHz of wireless internet licenses for $19.8 billion, and create 14 MHz of spectrum for unlicensed uses. With fewer UHF channels remaining for TV broadcast, the TV spectrum was also reorganized. Each station was either "repacked" in the leftover channels or voluntarily sold its broadcast rights, either going off the air or switching to a different band. The volunteers received a total of $10.05 billion to yield or exchange their rights and make repacking possible.

This paper revisits from a computational perspective part of the incentive auction design: specifically the descending clock "reverse" auction used to procure broadcast rights. We investigate the quantitative significance of various aspects of the design by running extensive simulations, leveraging a reverse auction simulator and realistic models of bidder values.

What is the value in taking another look at the incentive auction? Because the design was both novel and extremely complex [17], time constraints prevented thorough consideration of every potential variation of this design. Our goal is to understand how well the auction worked in practice, which elements of the design were most important, and which variations of the design might have led to even better outcomes.

The auction mechanics are described in detail in Section 4, but roughly, the reverse auction worked by approaching stations one at a time with a series of decreasing price offers for their broadcast rights. When a station refused an offer, it exited the auction irrevocably and was guaranteed a spot in the leftover channels. As prices fell and more stations declined offers, the leftover channels "filled up". Before processing any station's bid, a "feasibility checker" ensured that the station could still fit in the leftover channels alongside the exited stations without causing undue interference; if it could not, that station "froze". Frozen stations were not asked to bid and their prices were not lowered. Stations that were frozen when the auction concluded were paid according to their most recently accepted offers. The auction included a provision to allow stations to exchange their broadcast rights for cash plus a channel in a less desirable (VHF) band and also a procedure to determine how many channels to repurpose, or "clear" (referred to as the auction's *clearing target*) in case the cost of clearing many channels proved to be too high. A "forward" ascending clock auction was used to sell licenses in the cleared spectrum to mobile carriers.

For this retrospective analysis, we rely heavily on simulations. The reverse auction design was strongly informed by theory [19]; e.g., we know that when stations' bids are binary responses to

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a series of descending offers and when stations are all independently owned, the auction design is obviously strategyproof [18] and weakly group strategy-proof. Furthermore, when the sets of repackable stations allowed by the interference constraints are substitutes, some descending clock auction can repack the efficient set of stations and another can implement the Myerson "optimal auction"; however, here the substitutes condition does not hold exactly (the interference constraints are not a matroid).

Simulations are well suited to checking whether theoretical intuitions are likely to be mirrored in actual performance and to pinpoint where further improvements would be valuable. The FCC used simulations in the design process to verify robustness of many design elements, some of which were publicly described [10]. Simulations are particularly well suited to assess designs with little or no historical data. The incentive auction was the first of its kind and so, even now we only have a single data point.

Our general simulation methodology entails a number of steps:

(1) Build an auction simulator (choosing an appropriate level of abstraction, as auction rules are often incredibly complex)
(2) Create a bidder model with parameters that control both valuations and behavior
(3) Establish a probability distribution over the bidder model parameters
(4) Draw many samples from the bidder distributions
(5) Run paired simulations holding sampled parameters fixed while varying some facet of the auction design
(6) Compare outcomes across paired samples using predetermined metrics

We built the simulator used in this paper, which can be found online at <removed for anonymity>. For the bidder modeling step, we both (1) leverage a model from the literature and (2) contribute a new model based on the released bid data; we contrast the results obtained from the two models as a robustness check. For the comparison step, we identified two key metrics to minimize when simulations all clear the same amount of spectrum: the total value of the stations removed from the airwaves (value loss) and the cost paid to stations. When assessing design elements that affect the amount of spectrum cleared, we assume that it is preferable to clear more spectrum, following the FCC's purpose in running the auction.

We consider four categories of questions—three concerning economic design and one concerning algorithmic design—for which our simulation approach is particularly well suited.

(1) *How much value was added by including the VHF option for broadcasters?* Adding this option multiplied the complexity of the auction and undermined some of its desirable theoretical properties, like obvious strategyproofness and group strategyproofness, but could have reduced the number and value of stations taken off the air and the cost of buying those stations' broadcast rights. In Section 6.1, we show that repacking the VHF band lowers costs and possibly leads to more efficient outcomes.
(2) *How much were costs reduced by offering lower auction prices to stations that reached smaller populations of viewers instead of just offering the same price to all and letting head-to-head competition set the prices?* The reduced price offers, called "pops scoring", were politically contentious and condemned by some opponents as price discrimination. In Section 6.2, we show that pops scoring was successful in reducing costs.
(3) *How was the auction's performance affected by the auctioneer's procedure for deciding the number of channels to clear, compared to what it might have been if the number of channels to be cleared were predetermined?* The actual clearing mechanism was novel and received little comment from participants. In Section 6.3 we show that the clearing procedure leads to higher

costs and less efficient outcomes than predetermining the amount to clear; in Section 6.4 we show that a simple alternative clearing procedure could have performed better.

(4) *Was auction performance significantly improved by the FCC's use of a customized feasibility checker to determine whether a station could be repacked alongside the set of stations continuing over-the-air broadcasting? How large might that effect have been?* This question is important because the design of customized feasibility checkers requires a nontrivial effort; such efforts should only be made in the future if they yield gains. We answer this question affirmatively in Section 6.5.

The rest of the paper proceeds as follows: Section 2 introduces common notation and definitions. Section 3 surveys related work. Section 4 explains the reverse auction in detail. Section 5.1 discusses how we generate values for stations using a model from the literature and Section 5.2 discusses a new value model we designed based on bid data. Section 5.3 describes stations' bidding behavior in our simulations. Lastly, Section 6 explores each of the above questions in detail using simulations.

2 NOTATION AND DEFINITIONS

In this section we introduce some notation and definitions used throughout the rest of the paper. Prior to the auction, each television station $s \in S$ was assigned to one of three bands, listed in decreasing order of desirability: UHF, high VHF ("HVHF") and low VHF ("LVHF"). We use $\text{pre}(s)$ to refer to a station's pre-auction band, sometimes called a station's home band. When an auction concludes, stations are assigned to bands according to a channel assignment γ. We let $\text{post}(\gamma, s)$ return either the band to which s is assigned under channel assignment γ or OFF if s is not assigned to a band under γ.

Throughout the auction each station is offered a series of prices. Let $P_{s;b;t}$ represent the price offered to station s in round t for selecting band b. When discussing auctions that only repack the UHF band, we omit the subscript b and use $P_{s;t}$. Let $b_{s,t} \in B_{s,t}$ represent station s's bid in round t, where $B_{s,t} \subseteq \{\text{OFF, LVHF, HVHF, UHF}\}$ denotes the options available to a station in a given round. Stations are never allowed to bid to move from higher bands to lower bands, and stations are never allowed to bid for bands higher than their home band. Stations bidding to move to a non-home band need to specify a "fallback" bid, either to drop out of the auction or maintain their currently held option. This fallback bid is used if their primary bid cannot be executed at bid processing time (because the band they want to move into is full). We denote such bids as $\text{fallback}_{s,t}$.

Stations were assigned a *score*[1] which we denote by $\text{score}(s)$ by the FCC prior to the auction via a *scoring rule*. The score was used to determine opening prices and was partially based on the population of viewers that a station reached before the auction, which we denote by $\text{Population}(s)$. We will have more to say about scoring rules in Section 6.2. We use $P_{s;b;\text{Open}}$ to refer to the auction's opening prices. The FCC publicly released the opening prices offered to 1460 UHF and 417 VHF stations [12]. Stations were allowed to participate conditionally on being initially assigned to one of a subset of bands. We let $\mathcal{P}(s)$ track a station's most recently accepted offer. After the auction, each winning station $s \in S_{\text{winners}}$ is paid $\mathcal{P}(s)$.

[1]Also referred to as a station's *volume*.

3 RELATED WORK

For the incentive auction,[2] simulations were used throughout every stage of the design process, both by the FCC and the broader research community.

Before the auction mechanism was even locked down, Kearns et al [15] used simulations to characterize the space of feasible repackings based only on the interference constraints, for example estimating the minimum number of broadcasters that would have to relinquish licenses in order for a given clearing target to be feasible. When the auction design was closer to being finalized, Cramton et al [4] used simulations to lobby for design changes. Example suggestions included changing the scoring rule and the removal of confusing components (such as Dynamic Reserve Pricing, an idea that was ultimately scrapped). After the auction concluded, Doraszelski et al [7] used simulations to estimate how profitable and how risky bidder collusion strategies might have been. They leveraged their findings to advocate for a rule change to deal with supply reduction strategies where an owner of multiple stations would face restrictions on which sets of stations they could use to participate in the auction. Ausubel et al [1] performed a post-mortem analysis of the incentive auction in a similar vein to our work, but with a primary focus on the forward auction.

The simulators used in the above projects varied greatly based on what was known about the auction at the time. Without yet knowing the auction mechanism, Kearns et al's [15] bidder modelling consisted entirely of determining which stations would participate, which they studied using independent coin flips for every station as well as more complex models in which stations belonging to network affiliates made correlated decisions. Cramton et al [4] used a (non-public) valuation model developed through "discussions with many broadcasters, taking into account revenue data, historical station sales prices, station affiliation information, total market revenue, and other factors". Notably, Doraszelski et al [7] published the value model they constructed for stations in the reverse auction. We discuss this model in Section 5.1. We leverage this model in addition to a second model we construct based on the released auction bid data.

Beyond the value model, simulation-based approaches to the reverse auction differed in their fidelity to the auction rules. In part, this was a necessity given that the design evolved over time: for example, early papers could not leverage the feasibility checker used in the auction because it did not yet exist, and so typically relied on off-the-shelf SAT solvers (e.g., [15]). The full auction rules are quite complex: while we also take some liberties outlined in Section 6, we are unaware of other work that runs simulations as complex as ours (notably, that include repacking the VHF band). Lastly, simulations are computationally expensive to run and approaches differ in their computational capabilities. For example, experiments performed by Doraszelski et al [7] were restricted to regional scale due to computational constraints. Our new simulator extends the one presented by Newman et al [21] and is closer to the actual reverse auction design than all of the previously discussed simulators. (We make no comment about the fidelity of the FCC's own simulator, since details about it are not publicly available.)

Two further papers study how prices should be set in a clock auction. First, Nguyen et al [22] considered how to set prices to minimize expected cost, using the reverse auction as a test setting. Their methods reduce prices until feasibility is violated and then use a final adjustment round to regain feasibility. Their results are therefore not directly comparable to the FCC's design which maintains feasibility throughout and never increases stations' prices.

[2]Simulations are a general tool to inform the design of markets that lack historical data. While there is relatively little literature on large-scale statistical analysis of the simulated behavior of candidate market designs in highly complex settings, of course this approach is applicable beyond the incentive auction. A prominent recent example is the New South Wales fisheries combinatorial exchange, where simulations were used to determine if imposing linear and anonymous prices would result in acceptable efficiency losses [2].

Second, Bichler et al [3] investigated the allocative efficiency of deferred acceptance auctions under different scoring rules in the problem of Steiner tree construction. It found that the choice of scoring rule tremendously impacted the efficiency of the auction and that, under some scoring rules, deferred acceptance auctions were competitive with other mechanisms based on well studied approximation algorithms.

4 REVERSE AUCTION BASICS

We now describe a simplified version of the reverse auction in which only the UHF band is repacked. The real auction also repacked two VHF bands but the inclusion of these bands complicates the auction rules significantly. The complete set of auction rules was published by the FCC [9].

First, stations respond to opening clock prices and decide whether to participate in the auction. Next, a solver finds an initial feasible channel assignment for all non-participating stations to minimize the number of channels required for those broadcasters, setting an initial clearing target. The auction then attempts to buy broadcast rights as necessary so that all stations remaining on air can fit into the available channels. It proceeds over a series of rounds, which consist of: (1) decrementing the clock and offering new prices, (2) collecting "bids" in which each station owner states whether its new price is acceptable, and (3) processing bids.

In the bid processing step, stations are considered sequentially. The feasibility checker determines whether it is possible to repack a station in each round along with the exited stations. If the feasibility checker cannot repack the station, the price for that station is not reduced and its bid is not considered further. Such a station is said to be "frozen" and its price is not subsequently reduced in the current stage. If the feasibility checker can repack the station, then the station's price is reduced and its bid is processed. If the station accepts the new clock price, it remains "active" and subject to processing again in the next round of the current stage. If the station rejects the new price, it permanently "exits" the auction. The auction stage ends when all stations are either frozen or have exited. Following each reverse auction stage is a forward auction where mobile carriers bid on licenses in the cleared spectrum. If the forward auction generates enough revenue to cover the costs of the reverse auction, the incentive auction terminates and frozen stations are paid according to their most recently accepted offer. An unsuccessful forward auction triggers another stage of the reverse auction.

The incentive auction determines the amount of spectrum to clear endogenously by iterating through several stages of reverse and forward auctions that clear progressively less spectrum until the forward auction is able to cover the costs of the reverse auction. When a new stage begins, more spectrum is added to the UHF-TV band, and stations that could not be repacked in their home band at the end of the previous stage may once again become repackable and "unfreeze". Such stations are identified at the end of each stage and said to be in "catch-up" mode. At the beginning of each stage, the base clock resets. At the start of each round of bidding, catch-up stations that face weakly lower prices than those that they froze at return to bidding status if they are repackable. Subsequent stages otherwise proceed like the initial stage.

Prices in each round $P_{s;t}$ are computed by multiplying a station's score with the base clock price c_t. c_0 was set to \$900 in the incentive auction and our simulations. The base clock price is decremented each round by d_t, the maximum of 5% of its previous value or 1% of its initial value.

We provide pseudocode in the appendix as Algorithm 1.

5 VALUE AND BIDDING MODEL

To simulate auctions, we need a model of bidding behavior, which we will describe shortly in Section 5.3; first we describe how we generate station values, upon which this model of bidding behavior depends. We present two separate value models. We ran simulations using both value models in order to assess the robustness of our conclusions. The first value model follows Doraszelski et al [7]; the second is based on our own analysis of bid data released after the auction by the FCC.

Each station s has a value $v_{s,b}$ for broadcasting in each permissible band b. A station has no value for being off air, i.e., $v_{s,\text{OFF}} = 0$. Both models only provide $v_{s,\text{UHF}}$, that is, home band values for UHF stations. For the two VHF bands in the auction, lower and higher VHF, we model a UHF station's value for switching to the HVHF band as $\frac{2}{3} \cdot v_{s,\text{UHF}} \cdot \mathcal{N}(1, 0.05)$ and similarly for the LVHF band as: $\frac{1}{3} \cdot v_{s,\text{UHF}} \cdot \mathcal{N}(1, 0.05)$—i.e., roughly two thirds and one third of the station's UHF value with some multiplicative Gaussian noise. Values for VHF stations are generated by computing a hypothetical UHF value for the VHF station and then applying the fractional reductions for VHF bands as described above.

5.1 The MCS Value Model

We have already mentioned Doraszelski et al's [7] valuation model; we now describe it in detail. The model treats a station's value as the maximum of its cash flow value as a business and its stick value[3], both of which are estimated from various sources including transaction data of station sales, advertising revenue, and station features. We refer to this value model as the MCS (max of cash flow and stick value) model for the remainder of the paper.

The MCS model does not provide values for non-mainland stations, specifically stations in Hawaii, Puerto Rico, and the Virgin Islands. We therefore excluded all non-mainland stations from our analysis completely when using both value models to preserve the same interference graph across simulations. We note that there are few such stations and they reach relatively small populations, so this exclusion is unlikely to have made a qualitative difference to our findings.

The MCS model also does not provide values for 25 mainland UHF stations. For these stations, we set $v_{s,\text{UHF}}$ to be proportional to population. We sampled the constant of proportionality from a log-normal distribution, where the mean and variance were calculated from samples of $\frac{v_{s,\text{UHF}}}{\text{Population}(s)}$ from other stations in that station's Designated Market Area (DMA)[4] or nationally if there were no other stations in the DMA.

5.2 A Novel Value Model based on Bid Data

Two years after the incentive auction concluded, partial data was revealed about the auction outcome. The data contains the selected band and price (and fallback band and price when applicable)[5] for each bid that was processed. The data also contains $\text{PermissibleStartBands}_s$, the set of bands for which each station s accepted opening prices. We use this data to construct a "realistic" model for station valuations. While the released bid data is not sufficient to reveal station values, it does make it possible to infer bounds on them. In some cases these bounds are relatively tight, bounded by a single clock interval, but most of the time they are looser or even one sided.

We inferred bounds on each UHF station's home band value, $v_{s,\text{UHF}}$. By default, $0 \le v_{s,\text{UHF}} \le \infty$. We tightened bounds by applying the following rules to the released bids:

(1) $\text{OFF} \in \text{PermissibleStartBands}_s \implies v_{s,\text{UHF}} \le P_{s;\text{OFF};\text{Open}}$
(2) $\text{OFF} \notin \text{PermissibleStartBands}_s$[6] $\implies v_{s,\text{UHF}} \ge P_{s;\text{OFF};\text{Open}}$
(3) $s \in S_{\text{winners}} \land \text{post}(s) = \text{OFF} \implies v_{s,\text{UHF}} \le \mathcal{P}(s)$ [7]

[3]The stick value represents the value of the broadcast license and tower, independent of the business; it can be more appropriate than cash flow when valuing non-commercial stations.

[4]The US is divided into 210 geographic DMAs specified by Nielsen.

[5]Prices offered for bands that were not selected by a station are not part of the dataset. The dataset also does not contain bids that were submitted but were not processed (because a station froze).

[6]This includes non-participating stations and those that participated conditional on starting in a VHF band.

[7]We exclude station 35630 that received $0 for its license.

(4) $b_{s,t} = \text{OFF} \vee \text{fallback}_{s,t} = \text{OFF} \implies v_{s,\text{UHF}} \leq P_{s;\text{OFF};t}$
(5) $\left(b_{s,t} = \text{Exit} \vee \text{fallback}_{s,t} = \text{Exit}\right) \wedge \gamma_t(s) = \text{OFF} \implies v_{s,\text{UHF}} \geq P_{s;\text{OFF};t}$

In words, we inferred that a station that included (did not include) starting off air as a permissible option had a value less than (greater than) its opening price for starting off air. A station that froze while off air and became a winner had a home band value less than its compensation. Whenever a station bid to remain off air, including as a fallback bid when attempting to move between bands, we inferred that the station's value was less than its price for remaining off air. Similarly, whenever a station bid to drop out of the auction (including as a fallback bid) while off air, we inferred that the station's value was greater than its price for remaining off air.

We then used these bounds to fit a model. We assume a model where value is proportional (in expectation) to population, $v_{s,\text{UHF}} = \text{Population}(s) \cdot n_s$. Here n_s is "noise", representing every other factor besides population that impacts the station's value, sampled from some unknown cumulative distribution function (CDF) N. Our upper and lower bounds on a given station's home band value can be translated into upper and lower bounds on a station's noise by dividing by s's population. Let x_s and y_s represent lower and upper bounds respectively on s's observed noise sample n_s, i.e., $x_s = \frac{v_{s,\text{UHF,Lower Bound}}}{\text{Population}(s)}, y_s = \frac{v_{s,\text{UHF,Upper Bound}}}{\text{Population}(s)}$. Our goal is a non-parametric maximum likelihood estimate of N. Note that by definition $N(y_s) - N(x_s) = \Pr(x_s \leq n_s \leq y_s)$. We maximize the product of these terms subject to constraints ensuring that N is a valid CDF. Let Z be a list containing all of the x_s and y_s in ascending order, such that Z_1 is the smallest element and $Z_{2|S|}$ the largest. We solve

$$\text{maximize} \prod_{s \in S} \left(N(y_s) - N(x_s)\right) \tag{1}$$

$$\text{subject to } N(x_s), N(y_s) \in [0,1]\ \forall s \in S \tag{2}$$

$$N(Z_1) \leq N(Z_2) \leq \cdots \leq N(Z_{2|S|}) \tag{3}$$

Noting that taking the log of the objective function preserves its maximum, we minimize the negative log likelihood, $-\sum_{s \in S} \log\left(N(y_s) - N(x_s)\right)$. This is a constrained optimization problem with a nonlinear convex objective function and convex constraints. The constraints ensure that the values of N must fall between 0 and 1 and that N must be non-decreasing. The solution uniquely identifies the value of N at each point x_s and y_s. We translated this problem into a second order cone program using cvxpy [5] and solved it using the ECOS [6] solver. The results are shown in Figure 1. Based on the results, N appears to be a log-linear function. We did not have any data to fit the tails of the distribution, so we extrapolated using the log-linear fit.

Finally, to generate UHF values, we multiply a station's population by a sample from N. In what follows, we refer to this model as the BD (bid data) model.

5.3 Bidding Model

We now describe the bidding behavior we used for stations.

A station participated in our simulations if its opening price for going off the air exceeded its value for continuing to broadcast in its home band, i.e. $P_{s;\text{OFF;Open}} \geq v_{s,\text{pre}(s)}$. After excluding 64 non-mainland stations, we had 1813 stations eligible to participate in our simulations, composed of 1407 UHF stations, 367 HVHF stations, and 39 LVHF stations. We note that the incentive auction had 1030 participants [13].

A key difference between the two value models is the participation rates that they elicit. In the MCS model, station values tend to be high relative to opening prices, leading to very high participation rates. For example, considering 10 000 sampled value profiles of UHF stations, 1196 of these stations participated in every sample, and only 46 had a less than 80% chance of participating.

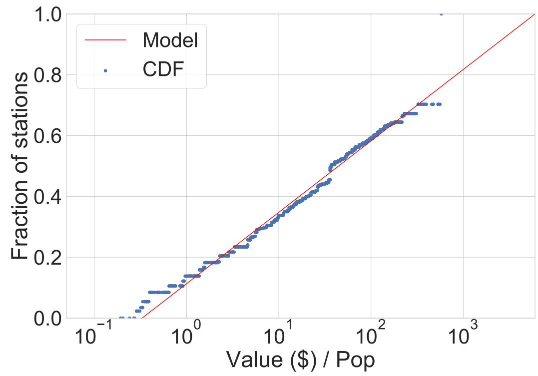


Fig. 1. CDF values for our maximum likelihood estimate of N and a log-uniform distribution fit to the results.

In contrast, when running the same experiment with the BD model, no station participated in every sample and average participation rates were 60%, closer to the 64% participation rate of UHF stations in the incentive auction.

In auctions with UHF options only, in which the only allowed bids are to remain off air or exit the auction, a single station faces a strategic situation in which its utility is "obviously" maximized by remaining off air if the price exceeds its value and by exiting otherwise—that is, by bidding myopically. The situation when VHF options are included is not obvious, but we continue to assume for simplicity that when bidding in round t, a station selects the offer that myopically maximizes its net profit, $\arg\max_{b \in B_{s,t}} P_{s;b;t} + v_{s;b}$. When fallback bids are required, we again assume that stations select the option that maximizes their net profit. We note that in the released bid data, only 52% (13%) of bids to move into LVHF (HVHF) were successful. Given that stations faced a significant risk of movements bids failing to execute, there may have been benefits to bidding on VHF bands before it was straightforwardly optimal to do so. We leave this to future work to explore.

6 EXPERIMENTS

Before describing our experiments in detail, we first formalize how we compare simulated outcomes and then discuss some simplifications our simulations make relative to the real auction process.

One efficiency-related goal for the auction is, for any given clearing target, to maximize the total value of the stations that remain on the air, or equivalently, to minimize the total value of the stations removed from the airwaves[8], i.e. $\sum_{s \in \mathcal{S}} v_{s,\mathrm{pre}(s)} - v_{s,\mathrm{post}(\gamma,s)}$. We call this value of stations removed the *value loss*. Given an efficient repacking γ^* that minimizes the value loss for a given value profile, our metric for allocative efficiency is the value loss ratio $\frac{\sum_{s \in \mathcal{S}} v_{s,\mathrm{pre}(s)} - v_{s,\mathrm{post}(\gamma,s)}}{\sum_{s \in \mathcal{S}} v_{s,\mathrm{pre}(s)} - v_{s,\mathrm{post}(\gamma^*,s)}}$ where γ is the simulation's final assignment. In general, we do not have access to γ^* because it is too difficult to compute, so instead we resort to comparing the value loss between two simulations' final assignments (i.e., the ratio of value loss ratios).

Our second metric is the cost to clear spectrum, i.e., the prices paid to all winning stations, $\sum_{s \in \mathcal{S}_{\mathrm{winners}}} \mathcal{P}(s)$.

[8]We use value lost instead of value preserved because it results in a smaller denominator and avoids exaggerating the performance of the auction. Value preserved includes the values of easy-to-repack stations, even those that do not participate in any interference constraints, and leads to efficiency estimates near 100% when few stations go off air relative to the number that remain on air, even when the number of stations going off air is large relative to the number required by an optimal solution.

We use the terms "efficiency" and "cost" below as abbreviations that refer to value loss and the total payments made to broadcasters that go off air. Outcomes with low value loss and low cost are preferred. It is straightforward to compare two outcomes if they both clear the same amount of spectrum and one is both more efficient and cheaper; otherwise, any comparison requires an opinion on how the two metrics should be traded off.

Despite our best efforts to make our simulations realistic, we could not replicate the initial optimization procedure that selects the initial clearing target because it relies on non-publicly available data—specifically the Inter-Service Interference ("ISIX") constraints that determine which geographic areas are impaired when a station is placed on the 600 MHz band (i.e., inside of the spectrum that will be resold). The FCC's optimization was permitted to "impair" some stations, placing them on channels within the 600 MHz band, even though this degrades the desirability of the mobile broadband licenses. Without the ISIX data, we also could not replicate the analogous optimizations that were repeated between stages.

In place of these optimizations, except when otherwise noted, we selected the clearing targets for simulations manually. When an initial feasible assignment cannot be found for all non-participating stations, we model impairments as follows. We add a fake "impairing" channel to each non-participating UHF stations' set of available channels. Stations placed on this special channel do not interfere with each other or with stations on the regular channels. We chose our initial assignments to minimize (given a 1 hour cutoff) the number of impairing stations, breaking ties by minimizing the aggregate population reached by impairing stations. We repeated this optimization between each stage, so that previously impairing stations could leverage the newly available spectrum to become non-impairing.

The incentive auction allowed stations to participate conditionally on being initially assigned to one of a subset of bands. The initial clearing target optimization determined how to accommodate such stations. Since we could not replicate this optimization we did not allow stations to select their preferred starting bands. Instead, in our simulations, all participating stations start off air.

We now proceed to describe our experiments in detail. As the reverse auction is simplest to reason about when only the UHF band is repacked, we ran simulations that only repack the UHF band in addition to those that also repack the VHF band in order to investigate which results generalize to the simpler setting. We ran simulations on both value models. 793 Canadian stations were involved in the repacking process (they could be moved to a new channel, but did not participate in the auction and could not be purchased). We included all of these stations in our simulations. Unless otherwise stated, in every one of our experiments we took 50 samples per treatment. Similarly, unless otherwise noted our experiments were based on a single stage of the reverse auction using the 84 MHz clearing target (the clearing target upon which that the real incentive auction concluded). We ran single stage auctions both because running multiple stages of the reverse auction is computationally expensive and because multiple-stage simulations depend on additional assumptions about the forward auction. We explore multi-stage auctions in detail in Section 6.3. We gave feasibility checks 60 seconds to complete (as in the real auction, modulo somewhat different hardware) unless otherwise stated.[9]

6.1 Repacking the VHF Band

The incentive auction reduced only the number of UHF channels, but repacked stations in three bands: UHF, HVHF, and LVHF. Repacking the two VHF bands offered potential for cost savings and

[9]We note a subtlety regarding runtime measurements. Since the feasibility checker uses a walltime cutoff, there will be some degree of unavoidable noise (i.e., problems that require time very similar to the cutoff threshold), so different auction trajectories starting from the same value profile can occasionally be caused by such measurement noise rather than a given design change being tested. This is difficult to control for, but the effect is random and averages out across samples.

efficiency gains, as UHF stations may accept a smaller payment to move to a VHF channel instead of going off the air and VHF stations may be willing to accept compensation go off the air to make space for UHF stations. An optimal repacking for a given value profile can only become weakly more efficient if the VHF bands are included as there are more configurations of stations available.

However, adding extra bands to the reverse auction complicated an otherwise elegant design. To reduce strategic options available to bidders, a rule prohibited stations from moving from lower priced options to higher priced ones. This "ladder constraint", for example, prevented a station tentatively assigned to the HVHF band from bidding to go off air. Stations no longer possessed obviously dominant strategies, as they could potentially benefit from choosing when to move up the ladder. Price calculations were more involved in the VHF setting as each option had to be priced appropriately. Fallback bids were added to the bidding language to accommodate the case where a bid to move is invalid at the time it is processed. Also, unlike in UHF-only settings where freezing is permanent, VHF stations can freeze and later unfreeze within the same stage if other stations move out of their home bands, complicating bid processing.

All of this extra complexity made the auction more difficult to explain to station owners, which mattered since some of the participants were not very sophisticated, and encouraging them to participate was a first-order concern. It is thus sensible to ask whether the additional complexity was worthwhile. More specifically, what changes to efficiency and cost arise when VHF options were included and bidders bid straightforwardly?

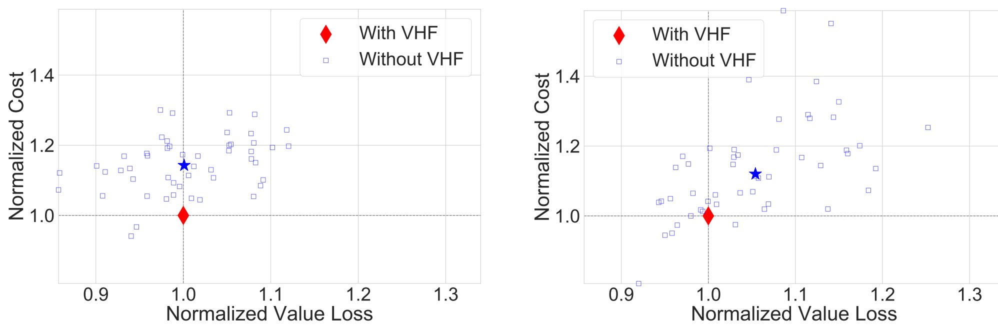
To answer this question, we ran two sets of auctions: the first repacking both the VHF and UHF bands, the other repacking only the UHF band. Our results are shown in Figure 2. In this figure, each point represents the outcome of one simulation scored according to our two metrics: efficiency on the x axis and cost on the y axis. Since it is difficult to show graphically which auctions use the same paired values for even a modest number of samples, rather than plotting raw efficiency and cost on each axis we instead plot normalized efficiency and cost. That is, we select one setting (in this case, auctions that repack VHF) as the reference treatment, and then plot the ratio of the efficiency (cost) of each simulation from additional treatments relative to the corresponding simulation using the same value profile in the reference treatment. With this choice, the reference treatment always corresponds to the point $(1, 1)$, represented in our figures by a diamond. Stars are plotted for each treatment showing the mean value according to each metric.

Altogether the experiments took a little over one CPU year to run. Using the MCS model, we observed that repacking the VHF band led to a significant reduction in payments—on average, sampled UHF-only auctions cost 1.14 times as much as their VHF-repacking counterparts. On average, efficiency did not really change, though we note that its variance was quite large. In the BD model, we observed that repacking the VHF band led to outcomes that were cheaper and more efficient: on average, sampled UHF-only auctions cost 1.12 times more and experienced 1.05 times higher value loss. On the whole, our evidence shows that if bidders continued to bid straightforwardly despite the complex design, repacking the VHF band was an important design choice that likely led to lower costs and possibly also more efficient outcomes.

6.2 Scoring Rules

Stations' starting prices in the incentive auction were not all equal. Instead, they were set proportionally to an assigned *score*, determined by a *scoring rule*. For active stations, the initial relative prices for going off-air remain constant during the auction, so those prices impact the order in which stations exit the auction and continue to broadcast.

Theoretically, scoring performs two distinct functions. First, because every descending clock auction is equivalent to a greedy algorithm for packing stations into the broadcast spectrum, it may be possible to pack a larger and more valuable set of stations if the algorithm prioritizes stations



(a) MCS

(b) BD

Fig. 2. Comparing auctions that only repack the UHF band against auctions that also repack VHF bands.

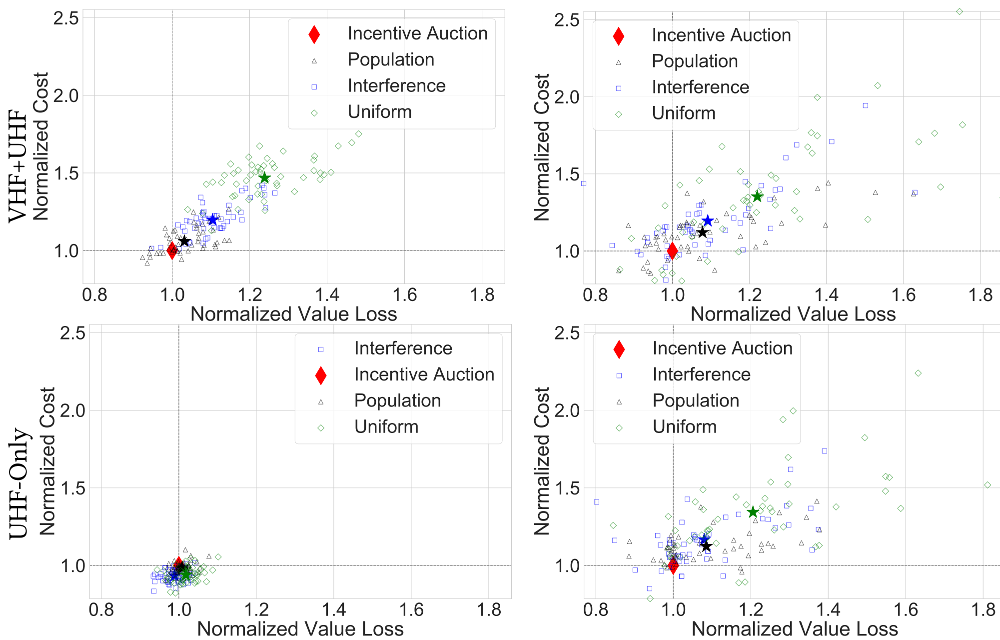
that create less interference with other stations. In the FCC's design, this idea was implemented by offering higher prices to stations with more "interference links"[10] so that those stations would be less likely to exit. The second function of scoring is to reduce the cost of the auction by offering lower prices to stations that would be likelier to accept them. In the FCC's design, this was implemented by reducing the clock prices offered to stations serving smaller populations. A formal theory of how to set prices to minimize expected total costs was developed by [20].

Putting these two together, opening prices in the incentive auction were set in proportion to the square root of the product of a station's population and interference links. The population component of scoring was extremely controversial and was vigorously opposed by a coalition of owners of lower powered stations serving smaller populations (the Equal Opportunity for Broadcasters Coalition), whose starting prices in the clock auction were reduced by this scoring rule.

In our experiments, we compared the following four scoring rules[11]:(1) "Incentive Auction" the scoring rule used in the actual auction (2) "Interference", the square root of a station's interference links (3), "Population", the square root of a station's population, and (4) "Uniform", a rule where each station has the same score. Just as in the actual auction, we normalized all scores so that the highest score UHF station had a score of 1 million.

Results for our simulations using the various scoring rules are shown in Figure 3. In general, we observed that the FCC's scoring rule, combining interference links and pops scoring, was a sensible choice across all our experiments. Scoring stations uniformly performed worst on average. Population alone was surprisingly effective as a scoring rule; we hypothesize that it may be an effective stand-in for difficulty-to-repack. In the UHF-only setting under the MCS model, the choice of scoring rule seemed to matter much less than in other settings; we have no explanation for why this was the case[12]. On average, simulations using only interference scoring under the MCS (BD) model cost 0.99 (1.17) as much as simulations that combined population and interference scoring in the UHF-only setting and 1.20 (1.21) as much in the VHF setting. On average, we conclude that population scoring succeeded in its stated aim of reducing prices and likely also increased efficiency. These experiments required 3.5 years of CPU time to run.

[10]More precisely: "an index of the number and significance of co- and adjacent channel interference constraints that station would impose on repacking."
[11]We used population and interference values directly from the FCC's opening prices document [12].
[12]Note to reviewers: we will continue trying to rule this out as experimental error.



(a) MCS (b) BD

Fig. 3. Comparing auctions using four different scoring rules.

All of the rules that we considered are static scoring rules: they assign each station a score once before the auction begins and that score never changes. In the theory of descending clock auctions, scores can be set to respond to the history of the auction and can change dynamically (provided they never increase a station's price). We leave the investigation of dynamic scoring rules for this setting as future work.

6.3 Clearing Procedure

The incentive auction alternated between the reverse and forward auctions in order to let market forces determine the appropriate amount of spectrum to clear. Each successive stage cleared a smaller amount of spectrum for a lower overall cost than the previous stage. The incentive auction went through four stages in practice.

One obvious practical concern of the multi-stage approach is the auction length: running multiple stages takes time. As noted by Ausubel et al [1] "One potential criticism of the incentive auction is that it lasted too long. If, after the initial commitment, the FCC had selected a clearing target of 84 MHz (instead of 126 MHz), it is very likely that the auction would have concluded after a single stage with significantly fewer rounds".

A less obvious concern is how the multi-stage approach impacted the final outcome. While this multi-stage design made the amount of spectrum to clear endogenous, there is reason to suspect that this property might have come at a cost, since irrevocable choices carry through the stages: stations' prices can never be raised, and exited stations can never return to bidding. A concrete example of why this might be harmful follows:

Example 6.1. Consider three identically scored stations A, B, C with values $V_A > V_B > V_C$. In the first stage, the repackable sets are $\{C, A\}$, $\{C, B\}$ and all subsets of these. The clock starts high and descends to V_A at which point station A becomes the first to exit. Since there are no repackable sets containing A and B, station B then freezes at price V_A. C is not constrained, so its price falls without bound until it eventually exits the auction. Assume that the subsequent forward auction fails to raise as much as V_A, so the clearing target is reduced and a new stage of the reverse auction begins with additional spectrum available to pack stations. Suppose that in the new stage, $\{A, B\}$ is repackable but $\{A, B, C\}$ is not. In the clock auction, since A and C have already exited, B remains frozen. The value loss of the clock auction is V_B and the cost is V_A. Now consider a single-stage auction starting from the reduced clearing target with the additional packable sets. As above, A exits first, but this does not freeze B. The price continues to fall and B exits next, freezing C. The value loss is V_C and cost is V_B, so the outcome is better according to both of our metrics.

In the example, the clock auction succeeds in optimizing the second stage target. In general, if the clock auction (nearly) optimizes for each clearing target and if the stations packed for the higher target are not a subset of those packed for the lower target, then one should expect a multi-stage auction to perform worse than an auction that starts by setting the correct target. The example suggests that the problem is that stations packed late in an early stage might be a mistake for the problem of the next stage.

Based on that intuition, we appeal to simulations to ask two kinds of questions: what was lost due to the multiple stages? Could that loss have been avoided?

To assess the loss from multiple stages, we compare single-stage simulations, in which the FCC somehow guesses the correct clearing target, against simulations that run through several stages, as happened in the real auction. We ran experiments that begin trying to clear 126, 114, 108, and 84 MHz of spectrum, each following the band plan (the ordering of clearing targets selected by the FCC, see Figure 7 in the appendix) from that point forward (leading to 4-, 3-, 2- and 1-stage auctions respectively). These experiments took 24 years of CPU time.

The results are shown in Figure 4. We observed that running the auction through multiple stages had large downsides both in terms of cost and efficiency. In simulations using the MCS model, perfectly forecasting the clearing target might have roughly halved costs and only had two thirds of the value loss of a four-stage solution. Even if the exact stage was not perfectly selected, starting the auction closer to the final stage would also have yielded significant improvements to both metrics. In simulations using the BD model, single stage auctions also outperform multi-stage auctions, though the results are slightly less dramatic in magnitude. Four-stage simulations cost 1.20 times as much their single stage counterparts and had 1.27 times as much value loss when the VHF band was repacked, and cost 1.14 times as much with 1.14 times as much value loss when it was not.

6.4 Early Stopping

The second question is: what might the FCC have done to avoid this loss if it could not guess the correct clearing target? Is there a way to keep the endogenous clearing using multiple stages while softening the cost and efficiency losses that came with it in the actual FCC design? One simple amendment we propose is to swap the ordering of the forward and reverse auctions and abort the reverse auction when its cost exceeds the previous forward auction's revenue. In the above example, once station B freezes, the provisional cost of the first stage at V_A already exceeds the first stage's forward auction revenue. With this new rule, the first clock stage would be immediately aborted and station C would not exit in that stage. Then, in the second stage, station B would exit next and the outcome would match exactly that of an auction in which the clearing target had initially been set to the final target. In general, continuing the reverse auction instead of aborting can only result in more stations exiting, reducing the possible choices for later stages. We call this amendment to the rules to avoid these additional constraints "early stopping". A suggestion along similar lines was given by Ausubel et al [1][13].

Will early stopping always outperform the original design? No—it is possible to construct examples where the commitments made in earlier stages turn out to be better than the ones that would be made in later stages. A concrete example is provided in Appendix A. Nevertheless, our intuition was that early stopping would be helpful.

To test this, we ran two sets of experiments. The first set compared early stopping auctions against single-stage stations that "knew" the correct clearing target (similar to the experiments above for multi-stage auctions). The second set of experiments compared the amount of spectrum cleared by early stopping auctions to auctions using the original design.

Both of these experiments required forward auction revenues as input: this is the part of the simulations for which data was thinnest. We adopted a convenient model that led to variations in the amount of spectrum cleared; the model is described in Appendix C. The same forward auction revenues were used within any given experiment across all sampled station value profiles. Since we had explicit forward auction values and known clearing costs, the auctions could end at any stage, meaning that the amount of spectrum being sold could differ even across paired simulations. We assume that clearing more spectrum is preferable to less, consistent with the FCC's goal in running the auction.

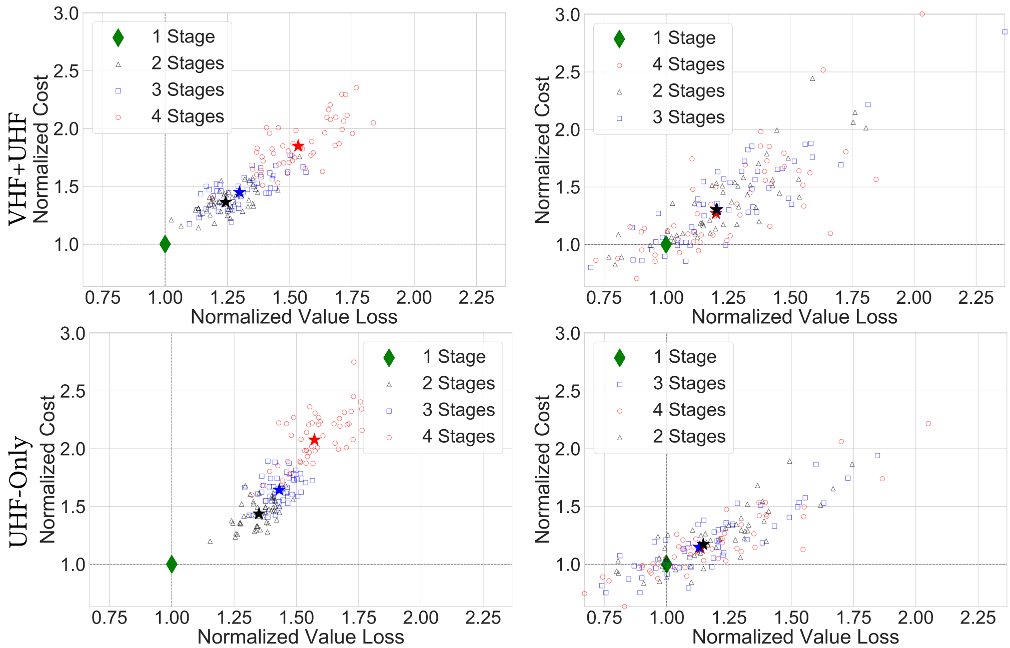
In the early stopping experiments, whenever a station became a provisional winner, we checked to see if the cost exceeded the forward auction revenue for that stage. If it had, then all remaining bidding stations immediately became provisional winners at their current prices, and the reverse auction continued on to the next stage as normal. [14]

The first set of experiments ran early stopping auctions with an initial clearing target of 126 MHz (corresponding to the first stage of the real auction) and following the band plan from then on. Given the final stage of each early stopping simulation, we ran a corresponding single-stage auction in order to check how much of a penalty early stopping had relative to perfect forecasting. The results are shown in Figure 5. For computational reasons, we only performed experiments for UHF-only simulations (the experiments took more than a decade in CPU years). We observed that in either value model, early stopping performed well relative to the single-stage auctions. In the MCS model, early stopping simulations on average only cost 1% more and had nearly identical value loss than single-stage auctions. Under the BD model, single-stage auctions on average cost 1.05 times as much and had 1.03 times the value loss of the early stopping simulations. These results are very encouraging when compared to the multi-stage experiments reported above, where the gaps between the original clearing procedure and perfect forecasting were much larger.

The second set of experiments ran simulations which compared early stopping against the original algorithm in order to determine which cleared more spectrum. Under both value models, on average more mobile licenses were created when using early stopping over the original algorithm—1.74 extra licenses under the MCS and 0.92 extra licenses under the BD model. Early stopping also led to shorter auctions under both value models: on average early stopping simulations took 70% (34%)

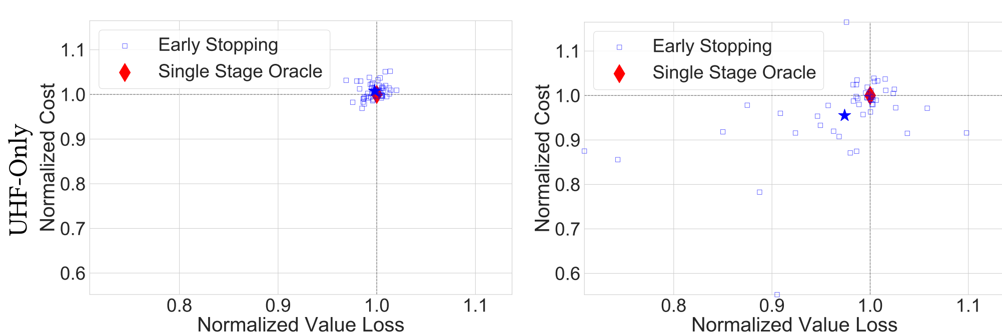
[13]Rather than swapping the order of the reverse and forward auctions, they proposed forecasting reasonable bounds on forward auction revenue (i.e., asserting that it would be unlikely for telecoms to pay more than $X for a given amount of spectrum) for each stage. The reverse auction would then terminate when the provisional cost reached $X, the following forward auction would be skipped, and the next stage of the reverse auction would be begin. They argue that first, this would have reduced the auction duration, and second, that bidders in the forward auction had a sense in early stages that the prices were too high for the auction to terminate, so they did not bid sincerely.

[14]We made one other change: we noted that the reverse auction has a step at the end of each round where it removes unconstrained stations - stations whose clocks will provably wind down to zero in a given stage and therefore might as well exit as soon as this is detected. In an early stopping auction, this no longer makes sense, as they may not be unconstrained in the next stage, so we do not perform these checks in our early stopping auctions.



(a) MCS (b) BD

Fig. 4. Comparing auctions running through 1-4 stages, ultimately ending on the same clearing target.



(a) MCS (b) BD

Fig. 5. Comparing auctions using the early stopping algorithm against single-stage auctions ending on the same clearing target as their corresponding early stopping auction.

of the total number rounds used by simulations with the original algorithm to complete under the MCS (BD) model.

In the incentive auction, at the end of the very first round of the first stage payments to frozen stations already exceeded \$50 billion; when the stage finished, the payments were \$86 billion. In the subsequent forward auction, revenues were only \$23 billion. Early stopping would have terminated the reverse auction during the very first round of bidding. The first reverse auction stage took one full month to resolve. This month, and possibly more time in future stages, could have been saved if early stopping was implemented.

6.5 Feasibility Checker

The feasibility checker determines if a station can be repacked alongside the stations that have already exited the auction. Prior to the auction, the FCC determined pairs of channel assignments that would cause harmful interference based on a complex, grid-based physical simulation ("OET-69" [8]); this pairwise constraint data is publicly available [11].

Feasibility checking was a large concern in the incentive auction because the station repacking problem is hard both theoretically (it is NP-complete) and in practice. When the feasibility checker cannot find a way to repack a station (either by proving that it cannot be done or running out of time), a station freezes—it is not asked to bid and its price stops decreasing. The feasibility checker is therefore linked to both the cost and efficiency of the auction: if the feasibility checker fails to find an assignment that repacks a station when such an assignment exists (i.e., a false negative), such a station will freeze at a higher price than it would in a future round. If the station would otherwise never freeze at all, value loss is also decreased.

Do auction outcomes always get better as the feasibility checker improves? Our intuition is that if the auction algorithm is a good one, then better feasibility checking should lead to better results, though in general this is not always true (see Appendix B for a counterexample).

SATFC 2.3.1, the feasibility checker that was used in the incentive auction, was designed in a multi-year undertaking [21]. The solver combines complete and local-search SAT-encoded feasibility checking with a wide range of domain-specific techniques, such as constraint graph decomposition. The authors used automatic algorithm configuration techniques to construct a portfolio of eight complementary algorithms from these various components. We wanted to understand whether the effort invested in making SATFC 2.3.1 was well spent, or whether a more off-the-shelf solution would have sufficed. To do so, we ran simulations swapping out SATFC 2.3.1 with different solvers.

These experiments build on prior work by Newman et al [21]. They ran 22 solvers from ACLib [14] (a managed library of solvers that can easily be plugged into algorithm configuration) on a benchmark set of sampled station repacking problems from their reverse auction simulations, showing that SATFC 2.3.1 was able to solve more problems than any other solver. They then ran reverse auction simulations comparing SATFC 2.3.1 with *PicoSAT*, another solver from this benchmark, concluding that using SATFC 2.3.1 led to better auction outcomes. We expanded on their experiments by running simulations using additional solvers and extending the experiments to our new value model.

We selected the following solvers to compare:

- SATFC 2.3.1: the feasibility checker used in the incentive auction.
- Greedy: a solver that simply checks whether the previous assignment can be augmented directly, without moving around any of the other stations. It represents the simplest reasonable feasibility checker and thus serves as a baseline.
- PicoSAT: To our knowledge, alongside MIP approaches PicoSAT is the only other solver that has been used in publications on the incentive auction, probably because it was shown to be the best among a set of alternatives in an early talk on the subject [16].
- Gurobi, CPLEX: MIP solvers considered by the FCC in initial discussions.
- Gnovelty+PCL: the best performing solver on the benchmark data.

Each problem in our simulations was always initially attempted by the greedy algorithm, so every feasibility checker can be understood as a sequential portfolio of the greedy solver and itself. It turns out that the vast majority of problems can be solved greedily.

Our experiments took just over two CPU years. Full results are shown in Figure 6. We observed that in general, stronger feasibility checkers lead to better outcomes according to both of our metrics: the relative rankings of the solvers in the benchmark study translate exactly into the

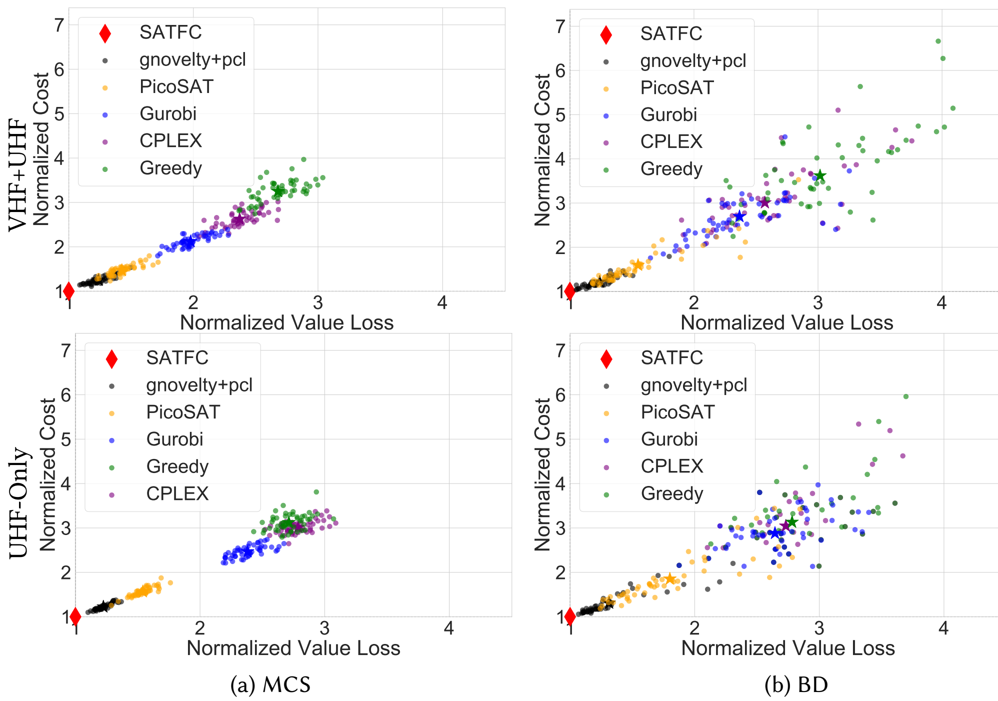


Fig. 6. Comparing auctions using different feasibility checkers.

relative rankings across both of our metrics, consistent across whether the VHF band is repacked and across both value models. In particular, we observed that SATFC 2.3.1 dominated each other solver on both metrics, not only on average but across each individual simulation and across both value models. When compared to the best alternative off-the-shelf solver, reverse auctions based on *gnovelty+pcl* cost between 1.22 and 1.31 times more on average (depending on bands being repacked and value model used) and lost between 1.22 and 1.31 times as much broadcaster value as those based on SATFC 2.3.1[15].

7 CONCLUSIONS

We used simulations to investigate previously unanswerable questions about the cost and efficiency of certain alternative designs for the incentive auction. To validate the robustness of our results, our simulations used two quite different value models: one from the empirical economics literature and another that we constructed to rationalize public bid data. At the scale of the incentive auction, even small percentage improvements in cost and welfare can translate into billions of dollars of savings.

Our main findings were that repacking VHF led to significantly lower costs and possibly more efficient outcomes, that pops scoring lowered costs as intended, that the multiple stage clearing rule increased the cost and reduced the efficiency of the auction, that a simple amendment to the clearing algorithm could both speed up the auction and nearly completely eliminates the multi-round inefficiency, and that the specialized feasibility checker developed for the auction

[15]We emphasize that these experiments compared solvers in their default, off-the-shelf parameter settings, and do not speak for what might have happened if effort were put into customizing each other solver.

significantly improved both cost and efficiency. We hope these specific insights from our simulations can help inform future auction designs. We believe our analysis demonstrates that large-scale statistical analysis of the simulated behavior of candidate market designs in highly complex settings is a practical tool to understand and evaluate alternative market designs, requiring substantial, but not unrealistic computational resources.

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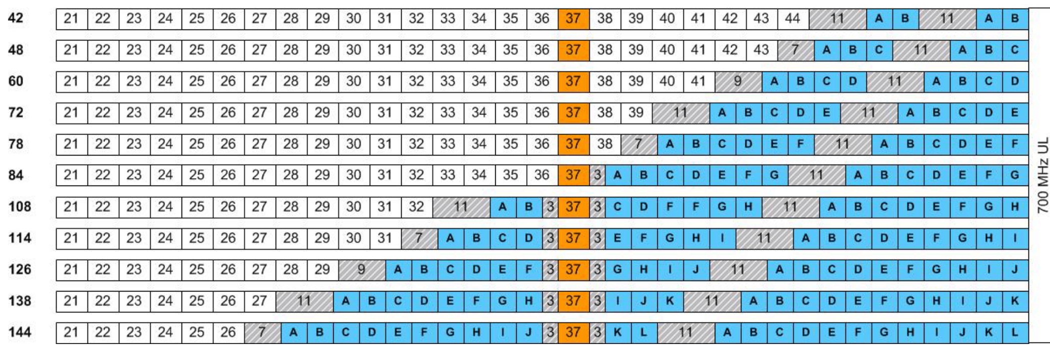

Fig. 7. The FCC's band plan for 600 MHz

A EARLY STOPPING COUNTEREXAMPLE

Example A.1. Consider four identically scored stations A, B, C, D with $V_A > V_B > V_C > V_D$, $V_B < V_C + V_D$ and $V_A < 2V_B$. Let the forward auction run first and have a purchasing price of V_A. Let the feasible sets in the first stage be $\{A, C\}$, $\{A, D\}$, $\{B\}$ and all subsets of these sets. In the second stage, the feasible sets add $\{A, B\}$, $\{A, C, D\}$ and all subsets. In both cases, the auction begins with A exiting and B freezing at price V_A. In an early stopping auction, this will trigger the end of stage one. In stage two, B unfreezes and exits. Then C and D freeze at price V_B. This leads to a value loss of $V_C + V_D$ and a cost of $2V_B$. In an auction without early stopping, C would exit, freezing D at price V_C. This would trigger the stage to end. In the next stage, B would remain frozen and D would unfreeze and exit, leading to a value loss of V_B and a cost of V_A. Using the inequalities on the values above, the auction that does not use early stopping performs better in both metrics.

B FEASIBILITY CHECKER COUNTEREXAMPLE

For a fixed cutoff, a feasibility checker can be thought of as a mapping from a set of stations to $\{Feasible, Infeasible, Unknown\}$. We can define an ordering over feasibility checkers such that a feasibility checker F_1 is strictly better than a second F_2 if and only if for all possible sets of stations $s \in 2^S$, $F_2(s) = Feasible \implies F_1(s) = Feasible$ and $\exists s$ such that $F_1(s) = Feasible$ and $F_2(s) = Unknown$. Importantly note that for the purposes of this definition we don't care about the feasibility checker's ability to prove infeasibility: while this is important for saving time and for understanding whether there is room to improve existing feasibility checkers, it does not ultimately impact the result of the auction since infeasibility and indeterminate solutions are treated identically.

Example B.1. Imagine a UHF-only setting involving three bidding stations A, B, and C. Let $V_A = V_B = V_C = V_D = V$ and let all stations have the same score. The constraints are such that the repackable sets are either $\{B, C, D\}$ or $\{A, D\}$ (and all subsets). There are two feasibility checkers: F_1 can find all feasible repackings, but $F_2(\{A, D\})$ = Unknown. Consider the first round in which each station is being offered a price p just below V and assume that the bid processing order is D, A, B, C. D exits the auction. Under F_1, A is allowed to exit the auction. This freezes B and C. The value loss for these two stations will be $2V$ and the payment will be just under $2V$. F_2, however, cannot pack A, and so A freezes. B and C then exit the auction. The value loss in this scenario is V and the payment is just under V, so this outcome is strictly better than the previous even though F_1 is strictly better than F_2.

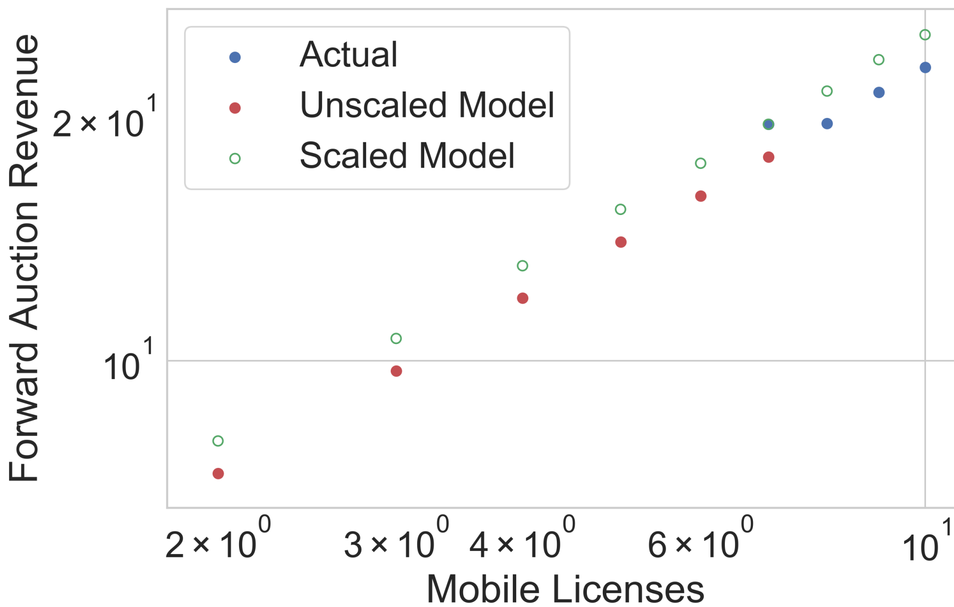

Fig. 8. Forward auction revenues observed in the incentive auction ("Actual") and our model of forward auction revenue.

C MODELING FORWARD AUCTION REVENUE

When using the early stopping algorithm, the reverse auction takes as input the output of the previous forward auction (the amount that mobile carriers will pay for the spectrum). Therefore, in order to simulate early stopping auctions, we need to model forward auction revenues.

While we have access to the real forward auction revenues, the incentive auction only went through four stages, and simulations could potentially go to stages beyond the fourth. To address this issue, we perform a log-log fit on the number of mobile licenses and forward auction proceeds in the first three stages (i.e., with 3 data points we fit $ln(\#\text{licenses}) = a \cdot ln(cost) + b$ for some constants a, b). We ignore the revenue in the fourth stage when performing this fit because the price per license rose significantly relative to the other stages, and we suspect that the price increase was likely due to an understanding among bidders that the auction would terminate in this stage. After the fit is established, we scale the entire model by a constant such that the model's prediction for the fourth stage matches the observed real revenue. Results are show below in Figure 8.

A final problem is that when using the MCS model, we observed that reverse auction simulations would rarely produce high enough procurement costs to trigger early stopping in the first stage, leading to uninteresting behavior where the auction simply ends after the first stage. To get around this, we scale our forward auction model downwards when running simulations for the MCS.

Algorithm 1 Multi-Stage UHF-Only Reverse Auction

```
function CheckCatchups()
    for $\{s | s \in S_{\text{catchup}} \wedge c_t \cdot \text{Score}(s) \leq \mathcal{P}(s)\}$ do
        $S_{\text{catchup}} \leftarrow S_{\text{catchup}} \setminus \{s\}$
        if Repackable($\{s\} \cup S_{\text{exited}}, m$) then
            $S_{\text{bidding}} \leftarrow S_{\text{bidding}} \cup \{s\}$
        else
            $S_{\text{winners}} \leftarrow S_{\text{winners}} \cup \{s\}$

function Repackable(Stations, Channels)
    Returns true if Stations can jointly broadcast in Channels

function Forward Auction(Channels)
    Returns forward auction revenue for clearing Channels

function Bid Processing()
    for $s \in S_{\text{bidding}}$ do
        if Repackable($\{s\} \cup S_{\text{exited}}, m$) then
            if $b_{s;t}$ == Exit then                                   ▷ If $s$ bid to exit
                $S_{\text{bidding}} \leftarrow S_{\text{bidding}} \setminus \{s\}, S_{\text{exited}} \leftarrow S_{\text{bidding}} \cup \{s\}$
        else
            $S_{\text{winners}} \leftarrow S_{\text{winners}} \cup \{s\}, S_{\text{bidding}} \leftarrow S_{\text{bidding}} \setminus \{s\}$      ▷ $s$ freezes
            $\mathcal{P}(s) \leftarrow P_{s;t}$

function Stage()
    $c_t \leftarrow c_0$                                                  ▷ Reset clock
    while $|S_{\text{bidding}}| > 0$ do
        $c_t \leftarrow c_t - \max(0.05 \cdot c_t, 0.01 \cdot c_0)$           ▷ Decrease clock price
        CheckCatchups()                                       ▷ Not needed in first stage
        for $s \in S_{\text{bidding}}$ do                                 ▷ Update prices
            $P_{s;t} \leftarrow c_t \cdot \text{Score}(s)$
        Collect bid $b_{s;t}$ from each station in $S_{\text{bidding}}$
        Bid Processing()
        $t \leftarrow t + 1$

$S_{\text{exited}} \leftarrow S_{\text{non-participating}}$
$S_{\text{bidding}} \leftarrow S \setminus S_{\text{exited}}$
$S_{\text{winners}} \leftarrow \{\}$                     ▷ Set of provisionally winning stations, initially empty
$\mathcal{P}(s) \leftarrow 0 \ \forall s$                                   ▷ Winning prices begin at 0
$M \leftarrow$ all channels
$m \leftarrow$ initial set of channels available for repacking
$t \leftarrow 1$
while The auction has not terminated do
    Stage()
    if Forward Auction($M \setminus m$) $\geq \sum_{s \in S_{\text{winners}}} \mathcal{P}(s)$ then   ▷ If forward auction covers costs
        Terminate the auction
    else
        $m \leftarrow m \cup$ additional channels for next stage
        $S_{\text{catchup}} \leftarrow \{s \in S_{\text{winners}} | \text{Repackable}(s \cup S_{\text{exited}}, m)\}$   ▷ Find stations that may unfreeze
        $S_{\text{winners}} \leftarrow S_{\text{winners}} \setminus S_{\text{catchup}}$
Pay $\mathcal{P}(s)$ to $s \in S_{\text{winners}}$
```