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Operations Research Enables Auction to Repurpose Television Spectrum for Next-Generation Wireless Technologies

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Abstract. In 2017, the U.S. Federal Communications Commission completed the world's first two-sided spectrum auction, reclaiming radio frequency spectrum from television broadcasters to meet exploding demand for mobile broadband, 5G, and other wireless services. Operations research—including a customized series of optimization models, decompositions, cuts, heuristics, large neighborhood searches, and a portfolio of propositional satisfiability solvers whose parameters were determined by machine learning—was essential to the design and implementation of the auction. The auction repurposed 84 megahertz of spectrum and generated gross revenue of nearly $20 billion, providing more than $10 billion in capital for the broadcast television industry and over $7 billion for federal deficit reduction.

Keywords: spectrum • spectrum auctions • reverse auctions • broadcast incentive auctions • optimization • satisfiability solvers

Introduction

Every day, more and more aspects of people's daily lives rely on wireless devices that use radio waves to transmit and receive information. Smartphones, over-the-air radio and television (TV), garage-door openers, remote control devices, baby monitors, and countless other devices and services require radio waves to function. The range of radio frequencies that can be used for such purposes is referred to as *radio frequency spectrum*.

The Federal Communications Commission, which we refer to as the FCC or Commission in this paper, is an independent U.S. government agency. The FCC manages radio frequency spectrum by licensing different portions of these frequencies for particular uses. Little unused spectrum suitable for the rapidly growing demand for mobile data is available. One way to relieve this shortage is to repurpose spectrum from an existing use to a higher-value purpose. Using analytics and operations research, the FCC determined that over-the-air TV stations could be rearranged (i.e., repacked) onto fewer channels, thereby using less spectrum. The recovered spectrum could then be reallocated for mobile broadband and other wireless uses. Based in part on the results of this research, a law was passed by Congress in 2012 directing the Commission to alleviate the spectrum shortage by conducting the world's first two-sided "incentive auction," using market incentives to reallocate some of the spectrum used for over-the-air broadcast television to mobile broadband uses (U.S. Congress 2012).

The FCC's Incentive Auction required more than the creation of a typical simultaneous two-sided exchange for a largely homogeneous commodity, such as those used for stock or commodity trades. First, the licenses that the television broadcasters were selling are heterogeneous products that differ in frequency (i.e., channels), power level, and location. Second, it was not clear how many TV stations would agree to receive an incentive payment in return for vacating their spectrum usage rights. Third, the licenses relinquished by broadcasters would not be usable for mobile broadband until they were reconfigured into contiguous, clear blocks of spectrum. Finally, the Commission had to ensure that thousands of television broadcasters who did not sell their spectrum in the auction could be assigned a channel in the smaller postauction TV band. Overcoming these challenges required the analysis of massive amounts of data, the creation of complex optimization and feasibility models and solution techniques, and the determination of appropriate policy objectives to serve American consumers, the wireless industry, and television broadcasters.

The innovative Incentive Auction, completed in 2017, was the first of its kind. It generated nearly $20 billion in revenue and made possible a significant increase in the capacity of the nation's wireless networks. However, the dollars raised in the auction were only a part of its lasting effects. The spectrum reallocated to wireless broadband uses will enable both the expansion of existing mobile broadband network capacity and new deployment of next-generation 5G and other wireless services. This paper documents the crucial role of operations research in the design and implementation of an auction for which statutory and practical requirements left no room for error.

FCC Spectrum Management

Radio waves travel freely through the air. The allocation of rights to particular portions of spectrum must be managed to prevent devices from interfering with each other so that, for example, a mobile telephone does not interfere with police radios or broadcast television programming, and vice versa. The FCC therefore manages the use of U.S. radio frequency spectrum by issuing licenses for users to provide specific services, on specific frequencies, in a specific area. For example, each over-the-air TV station holds a license consisting of the right to broadcast in a particular area on a specific 6-megahertz (MHz) slice of spectrum (i.e., a TV channel) within the block of the radio spectrum reserved for television use. The FCC also authorizes the use of unlicensed devices such as Wi-Fi routers and cordless phones in certain portions of the radio spectrum.

Since 1993, the FCC has conducted almost 100 spectrum auctions. These auctions have mostly been simultaneous multiple-round (SMR) auctions—an innovative auction design introduced by the FCC in 1994 for its first auction and one that has since been adopted by regulators around the world. SMR auctions assume that the FCC has an inventory of available spectrum in frequency bands needed by the participating bidders.

Different spectrum frequencies provide different benefits. Low-band spectrum of the type previously licensed to TV stations provides coverage characteristics that are also highly valued by wireless carriers, given its ability to penetrate walls and travel long distances. Prior to the Incentive Auction, there was no unused FCC inventory of low-band spectrum that could be made available for auction to meet the growing demand for mobile broadband, which Cisco forecasts will grow fivefold from 2016 to 2021, a compound annual growth rate of 35% (Cisco 2017). Therefore, a way to increase the supply of spectrum to support this growing demand by finding a novel way to reallocate it from existing uses was needed.

Why an Incentive Auction?

In March 2010, the FCC released a proposal (FCC 2010) to make additional spectrum available for wireless use by paying existing spectrum licensees market-determined incentive prices to relinquish their spectrum usage rights. The plan concluded that TV broadcasters might be ideal candidates for this innovative approach for several reasons.

1. The upper portion of the range of spectrum allocated to TV stations broadcasting on ultrahigh frequency (UHF) channels is in the 600-MHz frequency range. Those frequencies have propagation characteristics valuable for mobile broadband use.

2. These frequencies are adjacent to spectrum already allocated for mobile broadband use in the 700 MHz band.

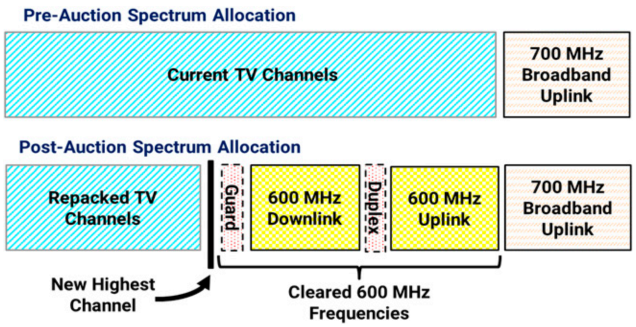
3. Over-the-air viewership for TV broadcasters was relatively small compared with their cable and satellite viewership, suggesting that a number of broadcasters might be willing to relinquish their spectrum in return for a share of the auction proceeds.

4. The transition of TV stations from analog to digital transmission, completed in 2009, made it possible for more than one TV station to share a single 6-MHz channel; that is, a station could relinquish its current channel in the auction and continue to broadcast its programming by sharing a channel with another TV station.

Although the FCC had proposed this new kind of auction as a means to repurpose spectrum, it lacked the necessary statutory authority to conduct it. The FCC's operations research studies helped demonstrate to Congress that reallocating broadcast spectrum was a viable and market-based solution. The result was bipartisan legislation, known as the Spectrum Act, in 2012 that authorized the FCC to conduct a two-sided incentive auction to reallocate a portion of the TV spectrum in the 600-MHz frequencies to wireless uses (U.S. Congress 2012). Although "600-MHz frequencies" and "600-MHz spectrum" refer to a range of frequencies that could be as broad as 566–698 MHz, the frequencies ultimately reallocated for wireless purposes were 614–698 MHz.

The FCC and Congress recognized that the broadcast spectrum relinquished by winning broadcasters—those

Figure 1. (Color online) Spectrum Allocation Before and After the Incentive Auction



broadcasters selling spectrum in the auction—would be scattered throughout the 600-MHz spectrum. Because this would not result in the contiguous block of clear spectrum needed for wireless services, Congress also empowered the Commission to relocate (i.e., repack) broadcasters that remained on the air to an equivalent channel in the portion of that band allocated to TV services following the auction. This requirement preserved the coverage area of broadcasters remaining on the air while also creating contiguous blocks of spectrum suitable for mobile broadband.

Figure 1 illustrates the spectrum allocation before and after the auction. The preauction portion of the diagram illustrates the spectrum used for over-the-air TV broadcasting before the auction and, to the right of it, the beginning of the 700-MHz band used for wireless services. This shows that prior to the auction, TV channels took up all the 600-MHz frequencies. The postauction portion of the diagram illustrates the auction's accomplishments: the spectrum located to the right, referred to as the new 600-MHz band, is now allocated for wireless uplink (i.e., mobile device sending data) and downlink (i.e., mobile device receiving data) uses. To the left at the lower frequencies is the new, smaller TV band, in which TV broadcasters that did not relinquish their spectrum rights are being repacked and will continue to broadcast. The guard band and duplex gap are narrow portions of the spectrum separating two portions of the spectrum to prevent transmissions from interfering with each other.

The Incentive Auction Design

The Incentive Auction used a series of reverse and forward auctions. The amount of spectrum repurposed and the amount paid to incumbents for relinquishing their channels were determined by the supply from TV stations in the reverse auction and the demand by wireless carriers in the forward auction, respectively. The repacking process—in which the remaining TV stations would be reassigned to channels in the new, smaller television spectrum band in a manner that protected their preauction coverage—linked and constrained the two interdependent auctions.

To bring supply and demand of spectrum into equilibrium, the incentive auction was divided into sequential stages that reduced the clearing target (i.e., the total amount of spectrum to be reallocated). The auction ended when wireless operators bid enough to pay the aggregate amount required to clear broadcasters from the spectrum and also cover certain auction-related expenses, including a reimbursement fund to cover reasonably incurred costs of repacked broadcasters assigned to new postauction channels.

Each auction stage began with a price-descending reverse auction, consisting of a series of bidding rounds in which the FCC offered participating TV station bidders progressively decreasing compensation to relinquish their spectrum usage rights. The first round of the first stage offered a predetermined initial price to broadcasters as an incentive to vacate their spectrum and resulted in more space than necessary being vacated in the initial round to meet the clearing target. In subsequent rounds, prices were reduced, bringing the total amount of vacated spectrum closer to the (final) clearing target. A broadcaster could accept reduced offers in each round either until it dropped out of the auction because it considered the offer to be too low or until it became a "provisionally winning" bidder because there was no longer any channel available for it in its preauction band. As we explain below, a provisionally winning bidder became a winning bidder when the final-stage rule was met. In each stage, the reverse auction ended when all stations participating had either been designated as provisionally winning or exited the auction.

Next, the same clearing target amount of spectrum was offered in an ascending forward auction to wireless carriers. In the forward auction, participants bid for generic blocks of spectrum in discrete geographic areas by indicating the number of blocks they would be willing to purchase at a specified price. The prices in the forward auction started low and rose as bidders indicated how many blocks they would buy at each increased price. In each stage, the forward auction ended when there was no excess demand for any of the offered blocks of spectrum.

When both the reverse and forward auctions in a stage were complete, a determination was made whether the final-stage rule had been satisfied. The final-stage rule had two parts, both of which needed to be satisfied for the auction to conclude that (1) the average bid price in the forward auction had to meet or exceed a previously established amount that guarantees that the government would receive a reasonable price for the spectrum and (2) the proceeds of the forward auction were sufficient to meet mandatory expenses set forth in the Spectrum Act; that is, the amount received would cover the amount paid to the broadcasters, the administrative cost of running the auction, and the $1.75 billion fund to reimburse the cost of repacked TV stations that would remain on air, to move to their new channels.

The auction ended once bidding satisfied the final-stage rule. If the final-stage rule was not satisfied at the end of a stage, additional stages with progressively lower spectrum clearing targets would be run. Lowering the clearing target in each stage had two effects: (1) in the reverse auction, there would be more room in the TV band in which to repack remaining stations (i.e., fewer TV stations needed to be purchased), which thereby increased price competition in the reverse auction and lowered the clearing cost (i.e., the aggregate cost of incentive payments to broadcasters offering to vacate their spectrum); and (2) in the forward auction, the supply of licenses in each market decreased, which thereby increased competition among bidders and raised the spectrum prices.

Figure 2 provides an overview of the major components of the auction design. Blocks labeled (1), (2), (3), and (4) are components of the auction where analytics, operations research, and optimization played critical roles. We describe how such tools were used below. The FCC's public web page contains additional detailed documentation regarding the procedures, with complete formulations and the rationale for the FCC's modeling choices in sufficient detail to enable the public to replicate the research (FCC 2015). To help navigate this lengthy document, readers should note that appendix B of this public notice describes interservice interference (ISIX constraints). Appendix C describes the clearing target optimization, appendix D provides the reverse auction pricing and bid processing algorithm, appendix E describes the final channel assignment optimization, appendix F details the final TV-to-TV constraints, and appendix K describes the optimization to determine stations not needed.

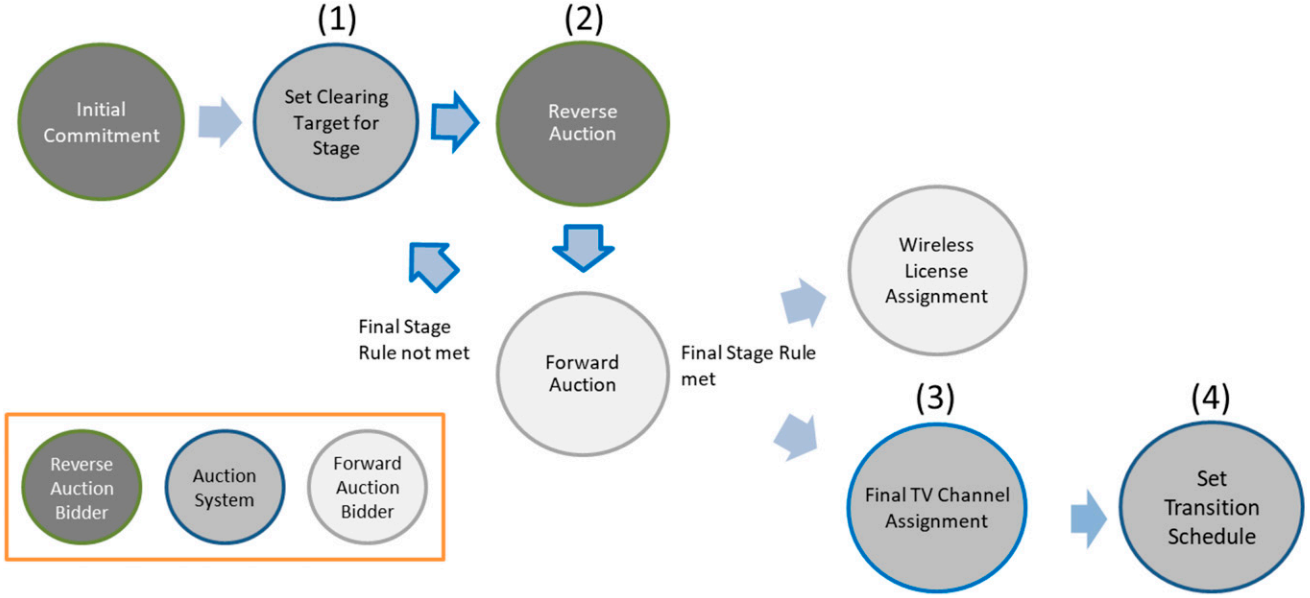
This transparency was an important part of the auction design process. As a federal government agency, all the complexities, including model formulations and solution procedures, had to be thoroughly explained to the public, including potential broadcast and wireless industry participants in the auction. At each step in the decision-making process, the FCC released public notices describing the proposed procedures in detail. All modeling assumptions and calculations were thereby published in advance, and the public was given the opportunity to comment and provide suggestions and alternatives. This allowed for external testing and validation of the FCC's research and ensured that both internal and external input informed the Commission's auction-policy decisions. Over 12,000 comments on the auction design and procedures were received in the FCC's public file.

Auction Challenges and the Role of Operations Research in Solving Them

Avoiding Interference in the Repacking Process

The overarching objective of the 2012 Spectrum Act was to use a market-based mechanism to repurpose as much of the television spectrum as possible so that it could be put to mobile broadband use while also requiring that the FCC preserve the coverage area of broadcast television licensees that would remain on the air after the auction. This requirement created a

Figure 2. (Color online) Major Components of the Incentive Auction

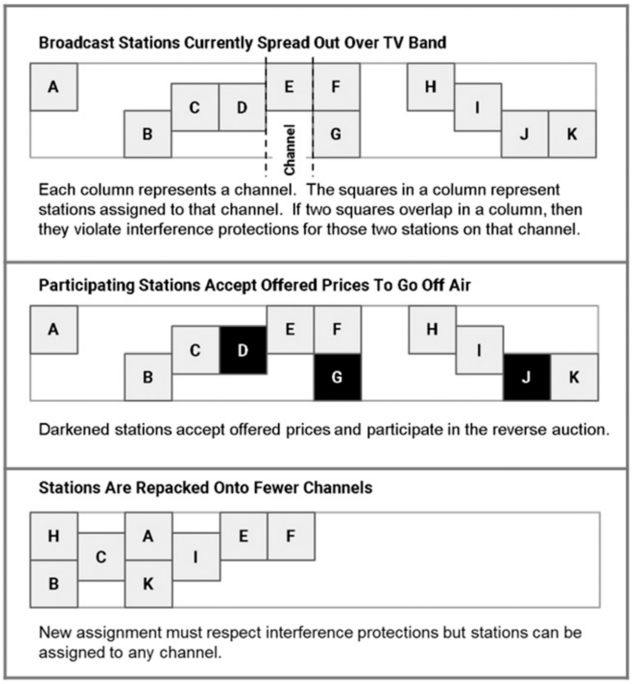


Notes. The auction components involving activity from TV broadcasters are shaded dark gray. The components involving activity from wireless providers are shaded in light gray. The other components involving the Commission's auction system are shaded in medium gray.

challenge to virtually all aspects of auction design: how to repack remaining television stations into a smaller spectrum band in a way that ensured that each station could reach its existing viewers without interfering with other stations. Figure 3 provides a simplified example of the repacking problem. Complicating this challenge was the fact that stations are heterogeneous; that is, a station's coverage area depends on its geographic location, the tower from which the station broadcasts its signals, the surrounding terrain, the channel assignment, its broadcasting transmission power, and its antenna height. With almost 2,200 stations in the United States, the FCC turned to operations research and data analytics to create a workable methodology to protect station coverage throughout the auction without unduly hindering the Spectrum Act's objective of clearing spectrum for mobile broadband use.

To accomplish this, the FCC used custom software that divided the United States into cells of 2 km × 2 km; for each cell, it determined whether a particular station has interference-free service in a cell for each available channel based on specific thresholds (FCC 2013). The FCC translated these physics-based interference calculations into constraints that could be used within both optimization and feasibility-checking routines. This translation required two tasks: (1) determining the existing coverage area for each station and (2) determining whether there would be any new interference between the stations if two stations were assigned to the same channel or to adjacent channels. Because TV stations often broadcast from the same mountaintop or

Figure 3. Example of Repacking Stations onto Fewer Channels While Protecting Stations from Interference



tower, or even the same antenna, over 2.5 million pairwise interference constraints were needed to restrict two stations from being assigned the same or adjacent channels, where such an assignment would cause more than 0.5% interference to the population in the coverage area of either station. If an assignment of TV stations to channels satisfied all pairwise constraints, then that assignment allowed all the stations to serve their existing viewership without significant interference from other stations (Figure 4).

A feasible repacking of a collection of television stations can be thought of as a generalization of the graph-coloring problem, where the vertices of the graph are TV stations and edges between TV stations indicate that they interfere with each other if assigned the same or adjacent channels. The problem of finding a feasible station-to-channel assignment is then to color the vertices of the graph such that no two neighboring vertices are colored the same (e.g., Aardal et al. 2009). The constraints describing this coloring problem are enforced in all optimization and feasibility-checking problems. The optimization problems discussed below have additional complexities, because these problems include additional constraints and objectives that must be satisfied.

International Coordination

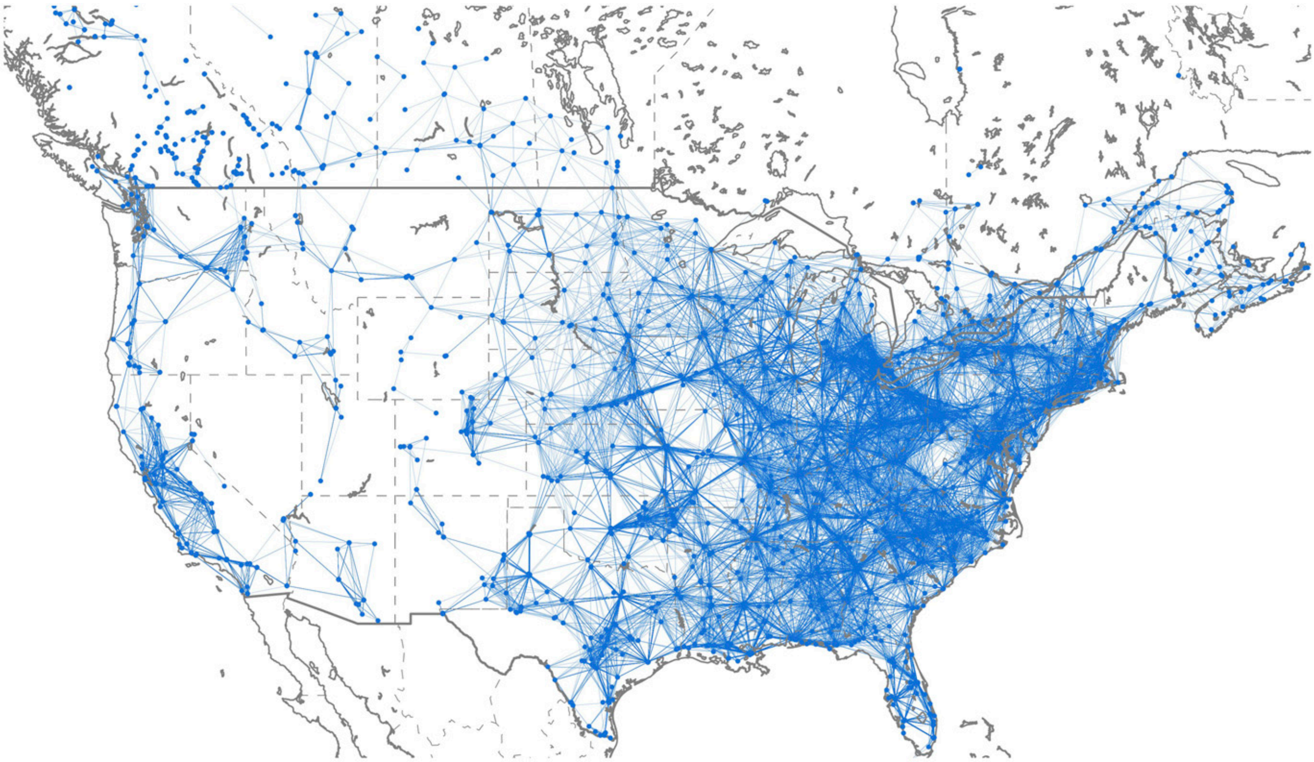
Another challenge in reallocating spectrum is that radio waves do not respect international borders. Unless Canadian and Mexican stations also moved to channels in the same reduced TV band, substantial portions of cities close to the border would have little or no 600 MHz spectrum usable for mobile broadband. The FCC therefore engaged its counterparts in Canada and Mexico to demonstrate the value of clearing spectrum in conjunction with the United States to create a uniform North American 600-MHz band. Scenario modeling enabled by operations research provided the FCC with the tools to make the necessary showing to regulators in both countries.

Negotiations with Mexico led to an agreement that all Mexican TV stations would broadcast in the spectrum below Channel 37, with all channel assignments for Mexican stations being determined ahead of the incentive auction. Negotiations with Canada resulted in a different approach where, based on numerous repacking simulations, the Canadian regulator determined that both the United States and Canada would benefit if Canadian stations were jointly repacked with U.S. stations in a unified band plan (Epstein and De La Torre 2015). Adding the 793 Canadian stations, most of which are close to the U.S. border, to the band plan resulted in an additional 150,000 pairwise interference constraints.

Establishing the Clearing Target

The auction's goal was to repurpose as much spectrum as possible on a nationwide basis. This presented the

Figure 4. (Color online) Pairwise Constraints Required Between Stations to Protect Stations from Interference



Notes. Each node represents a station, and each edge represents whether an interference protection exists between two stations on any specific channels. Note that each line on this graph represents multiple interference constraints because interference protections must be enforced for every possible channel assignment for each pair of stations.

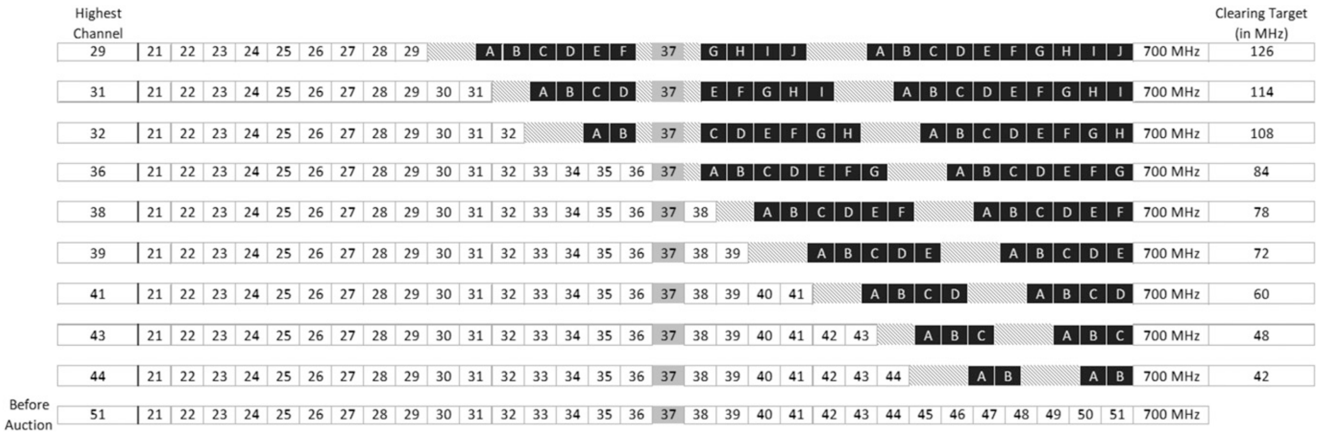
FCC with a choice: either limit the amount of spectrum repurposed in every market to the amount available in the most constrained market or allow some TV stations to be assigned channels in the new wireless band if broadcaster participation in a market was not sufficient. The FCC opted to permit some level of interference from TV stations to a few wireless licenses in the uniform nationwide band plan, because analysis showed that the amount of spectrum that could be recovered without allowing any impairments in the wireless band may have yielded too little spectrum to warrant an auction.

The Commission limited the amount of interference that could occur by determining that it would not select a clearing target (i.e., the amount of spectrum potentially being repurposed) if the target's total impairments exceeded a specified threshold. The job of the clearing target optimization process was therefore to choose, among the set of FCC-established band plans, the one that would clear the most spectrum within the specified maximum level of impairments. The forward auction would then determine the value of the impaired spectrum offered.

A band plan describes the amount of spectrum in each license, and the frequency spacing between the uplink (i.e., mobile device sending data) portion of the band and the downlink (i.e., mobile device receiving data) portion of the band. Technical constraints, together with industry feedback, had determined the band plan structure. The FCC studied the feasibility of repacking stations to create different band plans. For example, one of the initial proposals included selling both paired and unpaired licenses. The wireless industry, however, indicated a lack of interest in unpaired spectrum and a clear preference for 10-MHz licenses (5-MHz uplink and 5-MHz downlink). The final band plans were therefore designed with these characteristics. As a result of the agreement to add Canadian stations to the joint repack, the maximum amount of recoverable spectrum was 126 MHz. The two-sided auction resulted in clearing 84 MHz (i.e., frequencies 614–698 MHz), referred to as the 600-MHz band, for new wireless use.

Because licenses were to be sold in 10-MHz chunks, and because guard bands were necessary to prevent interference, recovering 126 MHz meant that 10 licenses in each market would be available for sale. The Commission created nine possible band plans, ranging from 10 licenses (126 MHz) down to 2 licenses (42 MHz). Each of the nine plans considered consisted of a downlink band, followed by a "duplex" gap to prevent interference between the downlink and uplink, and then an uplink band ending at 698 MHz (the upper limit of Channel 51). Figure 5 illustrates the band plan associated with each

Figure 5. Nine Possible Band Plans



Notes. Numbers in the white boxes represent available television channels. Dark boxes with letters represent wireless licenses available for auction. Numbers to the right of the diagram show the amount of broadcast spectrum cleared.

potential spectrum clearing target. The shaded grey areas show the guard bands that protect the downlink and uplink wireless communications from interference. In addition, Channel 37 is reserved for radio astronomy and wireless medical telemetry; therefore, it cannot be used by either TV broadcasts or wireless communications. The 6-MHz TV channels are shown in white. The new wireless licenses are shown in black, with capital letters identifying each license's downlink and uplink.

The clearing target optimization procedure was designed to choose the band plan that would repurpose the most spectrum while keeping the total impairment below a specified threshold. To make this determination, the FCC needed to quantify the potential interference between wireless services and TV transmissions. The interference data then needed to be incorporated into the optimization procedures. Finally, because there was a likelihood that the auction would require more than one stage, determining the minimum level of interference for a clearing target needed to occur quickly. Once again, the FCC relied on operations research and data analytics to accomplish these objectives.

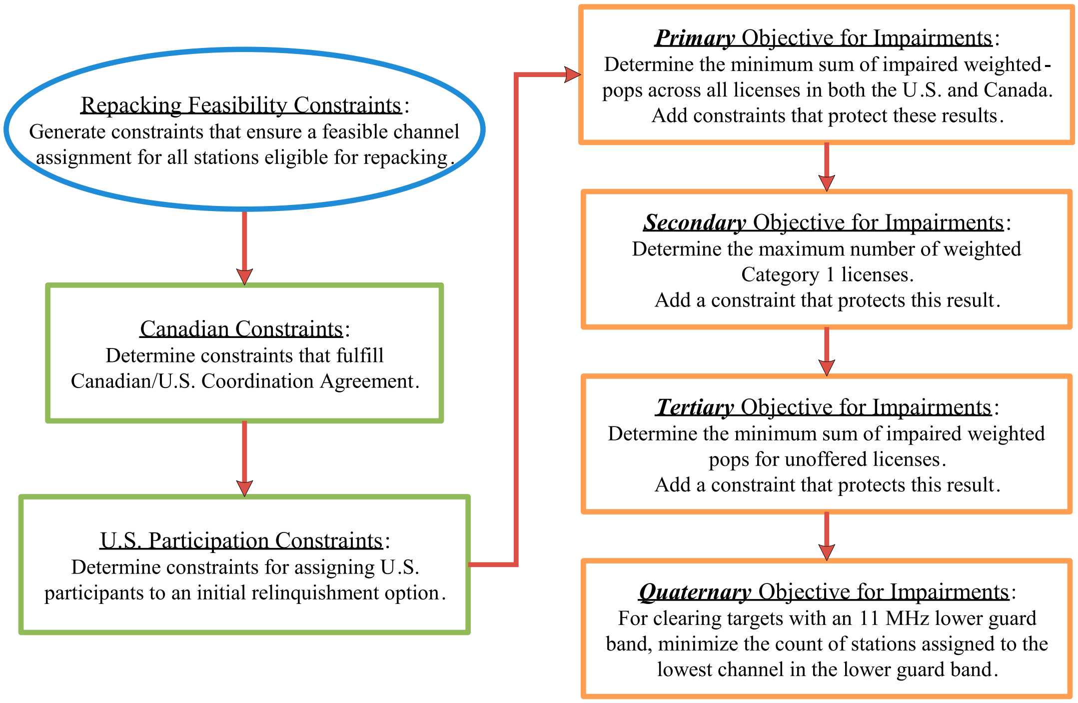
For each station on each possible channel in the wireless band, the FCC determined on a 2-km × 2-km grid level, whether any of four types of interference occurred (i.e., either uplink or downlink wireless interference to TV broadcasts and TV interference to either downlink or uplink wireless communications). The optimization-analytics challenge was to aggregate these data to a level that could be used by optimization routines and yet not miscalculate interference to the wireless uses.

There are many granularity options to quantify interservices interference, ranging from state (50 states), county (3,224 counties), census tract (74,134 tracts), 2-km × 2-km cell (3,973,715 cells in the United States only), and census block (11,166,336 blocks). The FCC chose the county level for the United States and Tier 4 market areas (i.e., wireless licensing areas as determined by Canada) for Canada, finding that 3,224 U.S. counties and 174 Tier 4 Canadian market areas would be manageable and scalable for purposes of quantifying or measuring the interference. Routines processed the data to determine the percentage of a county's population that would be inaccessible by wireless transmissions.

Upon adopting county-level aggregation, the FCC determined that any county that would receive interference of more than 10% would be considered completely impaired, finding that the county aggregation consistently overestimated the impairment, but only marginally. The FCC next generated constraints that linked television station assignments in the wireless band to county impairments, ensuring that if two station assignments impaired the same county, the percentage impairment was counted only once. To model the interservice impairment, 50,000 additional variables were needed, whereas the number of additional constraints totaled over 6.1 million.

The clearing target optimization procedure sequentially solved 14 optimization problems for each band plan. Upon solving each problem, a constraint was added to limit future assignments to maintain the previous objective. Figure 6 illustrates the process by which the clearing target steps were performed. The steps fell into three main categories of objectives: (1) where possible, assigning Canadian stations to the new reduced TV band, as opposed to the wireless band (the first box in the figure); (2) enabling maximum auction participation for U.S. stations (the second box in the figure); and (3) creating the most valuable licenses for the forward auction (the next four boxes in the figure). The first, second, and third categories of objectives involved solving five, four, and five optimization problems, respectively. For complete formulations of each

Figure 6. (Color online) Initial Clearing Target Optimization Flow



integer optimization problem, see FCC (2015). These steps were executed for each of the nine alternative band plans.

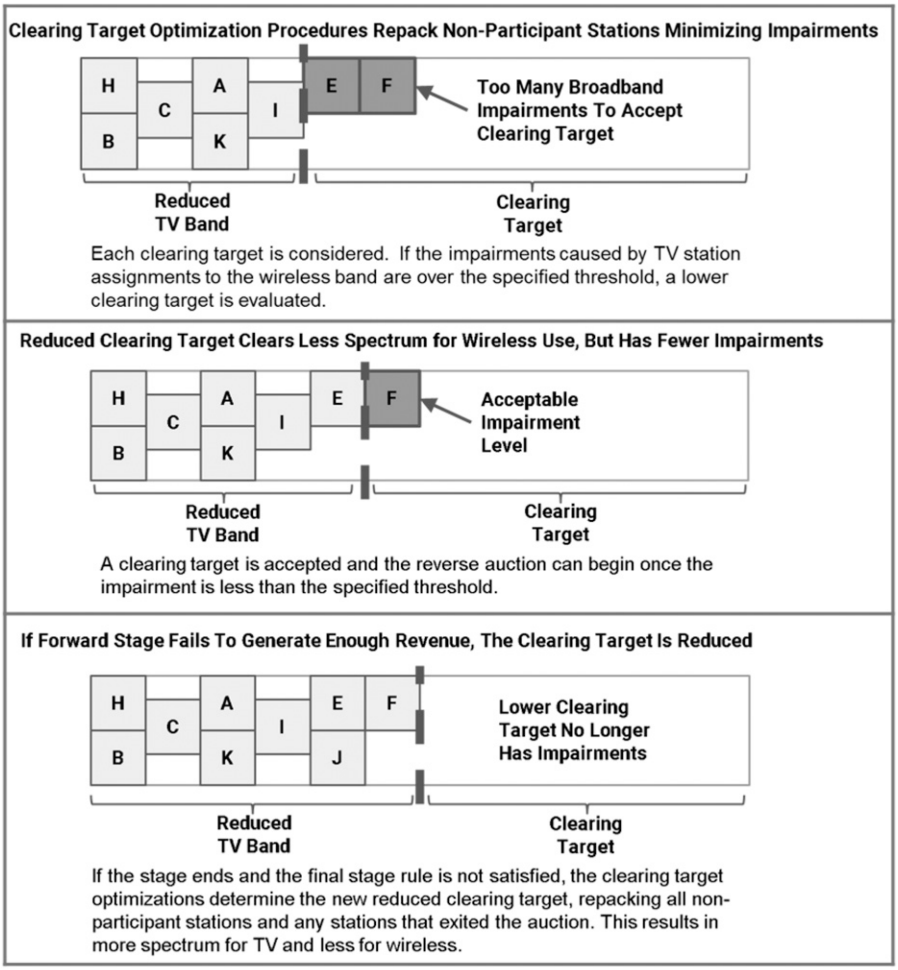
The goal of the clearing target optimization procedure was to maximize the economic value of the spectrum sold. Wireless markets were weighted based on revenues from previous spectrum auctions and the population served. Minimizing the nationwide impaired weighted population meant that the primary objective of the optimizations was to place any impairments where they would likely have the least impact on forward auction revenue. These optimizations needed to include the feasibility constraints for repacking U.S. and Canadian stations, and the interservice impairment constraints previously described.

A secondary optimization objective was to maximize the number of licenses that were designated Category 1 licenses (i.e., those having an impairment of less than 15%), as opposed to Category 2 licenses (i.e., those having an impairment between 15% and 50%). This objective sought to make the most impairment-free licenses at the cost of higher impairment in a few licenses for different assignments with the same nationwide impairment. The tertiary and quaternary objectives ensured that no station was assigned to the wireless band unless necessary. For example, the result of the secondary objective might have required two stations to be assigned to the wireless band in a market, making the new wireless licenses in that market unsellable. Because the licenses would be unsellable, the solver may have assigned a third station to the wireless band without incurring additional impairment. The optimizations for the tertiary and quaternary objectives would force this third station to be assigned to the TV band.

The system evaluated each clearing target. The auction began with the highest clearing target (i.e., the one creating the greatest number of new wireless licenses) where the impairments were below the specified threshold. In subsequent stages, the auction would move to the next-lowest clearing target that satisfied the specified threshold. Figure 7 illustrates the selection of a clearing target. The FCC's testing indicated that these complex objectives resulted in a problem that the best commercial solvers could not come close to solving, even given weeks of computing time. New formulations, decompositions, and heuristics were essential to solving these optimizations.

One of the most important solution strategies was to rewrite many of the pairwise constraints into fewer clique constraints. These specially selected cliques resulted in a reduction from over 8 million pairwise constraints to approximately 700,000 clique constraints and a significantly tighter formulation. In testing multiple approaches to generating cliques, the FCC found that its heuristics could identify more feasible solutions when using a minimal number of constraints. It therefore used a heuristic approach that resulted in a low total number of cliques while guaranteeing all edges were contained in at least one clique. However, even with this reformulation, the best commercial solvers were still incapable of finding good feasible solutions.

Figure 7. Process of Determining an Acceptable Clearing Target



The strategy for getting good feasible solutions was to use "large neighborhood search." This technique requires fixing all but a neighborhood of variables to a known solution and then having the solver provide optimal solutions to this much smaller problem. Neighborhoods were determined using characteristics such as station distance, station network designation, channel assignment, and pairwise constraints. We chose the size for each neighborhood so that the resulting subproblems could be solved within the time allotted.

Similarly, the Commission implemented a Benders decomposition approach to obtain better lower bounds. Here, the master problem determined assignments in the wireless band, and subproblems determined the regions where the master problem's assignment was infeasible. Cuts to the master problem required stations to be assigned to the wireless band only in the most congested areas.

Finally, for all of the optimizations described in this paper, the FCC created its own distributed solver system by which any results obtained from one processor could be shared with the other processors. This methodology allowed all of the heuristics and decomposition methods to share information, such as new solutions and better bounds across multiple machines. Custom profiles for each optimization approach specified the number of servers used and the parameters to use on each server. At its peak, the Commission employed over 70 extra-large cloud servers and used Gurobi 6.5 as the optimization engine. All Gurobi parameters were kept at their defaults except for specifying that the focus of the algorithm should be on finding feasible solutions (i.e., MIPfocus set to 2). These settings were confirmed through multiple simulations.

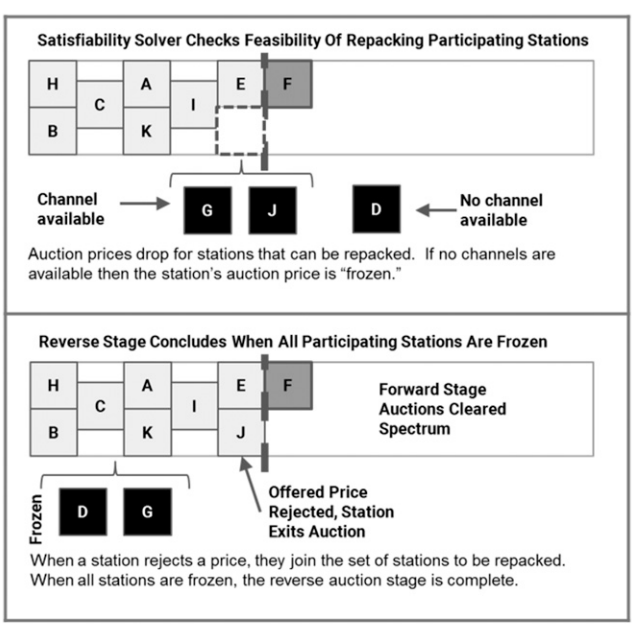
The clearing target optimization yielded impressive results. The procedures allowed the largest clearing target (126 MHz) to be chosen for stage 1 of the auction. This clearing target solution had impairments of only 8% in the United States and 5% in Canada, and 96% of the new wireless licenses had no impairment. By stage 4, the stage of the auction in which the final-stage rule was met, the clearing target optimizations determined a solution with no impairments in either the United States or Canada.

Feasibility Checking

The Incentive Auction needed to produce good economic outcomes, generate sufficient revenue to meet the closing conditions, and be tractable to run in practice. It also needed to ensure participation of broadcasters that were not experienced auction bidders and needed to understand a particularly complicated auction. To address all of these challenges, the FCC selected the descending-price clock auction format for the reverse auction. In particular, this design made it easy for a broadcaster to participate: the station could simply stay in the auction until the price dropped below the lowest price at which the broadcaster was willing to sell the station.

The reverse auction began with each participating station receiving an initial price quote and responding either that it agreed to sell its broadcast rights at the quoted price or that it had chosen to exit the auction (i.e., declined to participate at lower prices). If the latter, the station was guaranteed an assignment that preserved its service area. After each round, a feasibility checker determined whether each active bidder could be feasibly repacked along with all exited and nonparticipating stations. If so, that station received a lower price quote in the next round and again had the option of accepting the price or exiting the auction. Otherwise, repacking the station was impossible and so the station was frozen (i.e., its price stopped descending); at that level, it would sell its rights if the final-stage rule was met (Figure 8). Thus, the feasibility checks confirmed that each station not winning a bid (i.e., not frozen) in the reverse auction could continue to have a channel to broadcast and therefore could have its price decrease in the next stage.

Figure 8. Satisfiability Solver Determining Feasible Repacking for Participating Stations



These feasibility calculations differed from any of the clearing target or final channel assignment optimization problems in that the feasibility-checking problems required a yes-or-no determination that a feasible packing was possible and any feasible packing resulted in a "yes" response. It was critical that these feasibility determinations be done quickly and accurately. A false positive was unacceptable because it would mean that a television station not acquired in the auction would not have a postauction channel available to it. False negatives, although acceptable, had to be minimized because such a result would mean that the FCC might unnecessarily designate a broadcaster as a provisional winner entitled to payment, thereby increasing the clearing cost and reducing the likelihood of repurposing the highest possible amount of spectrum. Simulations indicated that, over the course of the auction, the number of checks would likely be in the tens of thousands, and a time cutoff was determined accordingly. It was therefore inevitable that some problems would remain unsolved (and thus treated as if infeasible); however, because the Commission knew the applicable interference constraints in advance, careful testing and algorithmic design minimized such instances.

The repacking constraints were encoded as a propositional satisfiability (SAT) problem, with binary variables representing the assignment of a station to a channel. To illustrate the magnitude of the analysis, SAT encoding the repacking constraints for a problem involving all stations at a clearing target of 36 channels resulted in over 73,000 variables and over 2.9 million clauses (i.e., constraints).

Given that the SAT community has developed many publicly available solvers (Järvisalo et al. 2012), the FCC could have, in principle, used all the solvers that offered even reasonable performance. However, doing so would have been too costly from the perspective of software integration and, more importantly, reliability testing. The FCC therefore conducted initial algorithm configuration experiments on 20 state-of-the-art SAT solvers, drawn mainly from SAT solver competition entries collected in AClib (Hutter et al. 2014). The Commission built the feasibility checker via an approach dubbed "deep optimization" (Newman et al. 2017); that is, it used developer insight only to identify design ideas that showed promise and relegated to an automatic search procedure the work of finding a joint parameter setting that achieved strong performance on realistic data.

The process identified two solvers that showed the strongest performance—a complete enumeration-based solver and a solver based on local search—both of which have been shown in the literature to adapt well to a wide range of SAT domains via large and flexible parameter spaces. The first solver was clasp (Gebser et al. 2007), an open-source solver based on conflict-driven nogood learning (98 parameters). The second was the open-source SATenstein framework (KhudaBukhsh et al. 2016), which allows arbitrary composition of design elements taken from a wide range of high-performance stochastic local search solvers (90 parameters).

Although adapting clasp and SATenstein to station repacking data yielded substantial performance improvements, neither reached a sufficient level of performance. It was thus necessary to leverage specific properties of the Incentive Auction problem, controlled via a wide variety of additional parameters, and falling into three key types: (1) methods leveraging the fact that feasibility problems are constructed by adding one station to a previous problem known to be feasible, (2) preprocessing techniques for simplifying repacking problems, and (3) a novel caching scheme that returns supersets of queried feasibility-checking problems, leveraging the fact that every subset of a feasible set is itself feasible.

The resulting design space had 191 parameters, nested up to four levels deep. The Commission used a tool called Sequential Model-based Algorithm Configuration (SMAC) (Hutter et al. 2011) to search this space. SMAC uses the Bayesian optimization approach of interleaving random sampling and the exploration of algorithm designs that appear promising based on a learned model.

The performance of different SAT solvers varied significantly depending on the problem. This application therefore needed to have a *collection* of algorithms that would cover the range of problems that might arise during the auction. To exploit this inherent variability and to improve performance, the Commission constructed an algorithm portfolio using a method called Hydra (Xu et al. 2010), which iteratively runs SMAC to optimize marginal gains over the portfolio produced in the previous iteration. This procedure identified algorithms that performed poorly overall but that complemented the existing portfolio. Hydra ran for eight steps, thereby producing a portfolio of novel solvers (dubbed SATFC) that could run on a standard eight-core workstation. The Incentive Auction used SATFC 2.3.1 (FCC/SATFC 2016).

SATFC exceeded expectations in the auction. The auction required over 100,000 feasibility checks, and SATFC solved nearly all the checks. Having a solver that returned definitive results in a short period allowed the FCC to conduct up to six rounds in a day. This meant the auction proceeded more quickly, creating a better bidder experience without increasing clearing costs.

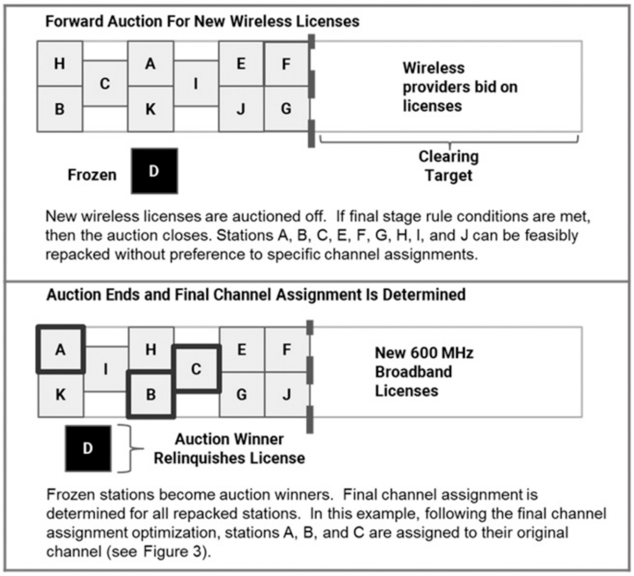
Determining the Final Channel Assignment

Once the forward auction satisfied the final-stage rule, the FCC needed to find the best channel assignment, among all feasible assignments, for the television stations that would remain on-air after the auction (Figure 9). As it did in determining the clearing target, the FCC used a hierarchical optimization procedure with three objectives. The primary objective of the final channel assignment was to maximize the number of remaining stations that could continue to broadcast on their current television channels. The FCC also needed to minimize new interference and to avoid moving stations whose move would be exceptionally difficult and (or) expensive, because this would reduce the total resources needed for transitioning to new channel assignments and minimize the time needed for the overall transition.

Modeling the primary objective of keeping the maximum number of stations on their original channels was relatively straightforward, although solving this problem was computationally complex, as we describe below; however, meeting the second and third objectives of minimizing new interference and avoiding channel changes for stations whose move would be exceptionally difficult and (or) expensive required additional study. These objectives required the FCC to determine methods (1) to capture aggregate interference for each station and (2) to measure the difficulty of moving one station to a new channel relative to another station.

For aggregate interference, stations expressed concern that by using the 0.5% pairwise interference protection, a station could be subjected to substantial new *aggregate* interference by adding together the small amounts of interference from many stations. The FCC studied this issue and determined that even though it was unlikely that a station would receive more than 2% new interference, a station might potentially receive more than 1% new interference. To decrease the chances of this happening, the model included

Figure 9. Final Channel Assignment



minimizing aggregate interference as one of the objectives. Because capturing interference on the 2-km × 2-km cell level for each station would have added billions of constraints to an already massive problem, the FCC instead used the sum of pairwise interference as an estimate for the total interference, which conservatively overestimated the actual interference because an area receiving new interference from multiple stations would have that interference counted multiple times.

Measuring the difficulty of moving one station to a new channel relative to another station required the FCC to review industry reports and consult both external and internal broadcast engineering experts to gather information about potential difficulties for a station to change channels. Using this information, the FCC created a difficulty scoring system that it could apply to each station.

As with the clearing target optimization, the FCC required reformulations, specially structured heuristics, and decomposition techniques to provide provably good solutions to these difficult optimization problems. The primary objective of determining the maximum number of stations that remained on their preauction channels was especially challenging for the solver. Using Voronoi diagrams (Fortune 2017) to determine a distance measure for the large neighborhood search techniques proved especially useful in finding feasible solutions, whereas Benders decomposition helped close the gap between lower and upper bounds. Again, using a distributed solver system enabled all components of this hybrid algorithm to share information.

The final channel optimization created a final channel plan for both U.S. and Canadian stations in just a few weeks—far fewer than the years it took the industry to develop a channel plan in 2009 when television stations nationwide converted from analog to digital television. As with the results of the clearing target optimization, the results of the final channel assignment exceeded expectations when it permitted nearly 1,700 stations to remain on their original channels and resulted in no station receiving more than 1.1% aggregate interference, with only 12 of over 2,800 stations receiving more than 1%.

Postauction Broadcast Transition Plan

A final Incentive Auction challenge remained for the FCC: how to coordinate as efficiently and expeditiously as possible the transition of the roughly 1,000 U.S. and 170 Canadian TV stations assigned to new channels in the new TV band. Without a carefully coordinated plan among all stations, interference and delay would be inevitable. The FCC again utilized operations research to develop the plan. Through extensive discussions with internal and external stakeholders, the FCC established priorities for the schedule for use in an optimization model. As with the auction, it presented the schedule-making procedure to the public for feedback and comments.

The FCC's solution was a 10-phase schedule for transitioning stations, assigning stations to a particular phase based on several objectives. These objectives included protecting stations from interference, limiting the number of stations that must coordinate with each other during a testing phase, minimizing inconvenience to TV viewers by limiting the number of TV rescans needed, clearing television stations in the new 600-MHz band first, and meeting international obligations.

Results

Benefits

The auction repurposed 84 MHz of spectrum to meet the growing demand for wireless data, raised substantial revenue for the U.S. Treasury, and provided capital to noncommercial and commercial broadcasters. The Incentive Auction was the second-largest auction in FCC history, resulting in gross revenues of $19.8 billion. Auction proceeds far exceeded the costs that needed to be covered under the Spectrum Act; these costs include (1) payments totaling $10.05 billion for winning broadcast stations, (2) funding the $1.75 billion TV Broadcaster Relocation Fund to reimburse stations for costs incurred to facilitate their moves to new channels, and (3) covering the FCC's administrative expenses. The roughly $7.3 billion remaining will be used to reduce the federal deficit.

In addition, more than 130 of the 175 winning TV stations that relinquished spectrum will continue broadcasting after the auction, most by sharing channels with other local TV stations. This means that viewer access to most TV content will remain the same and that much of the $10 billion in reverse auction winnings may remain in the industry to be reinvested by stations into new content and capital improvements. As an example, the San Bernardino Community College District called its $157 million winnings "one-time funding [that] will bolster our ability to provide the world-class programming [our area] deserves, while helping achieve our critical mission of educating, preparing and inspiring the leaders of tomorrow" (Crafton Hills College 2017). For other examples, see Payne (2017), Sostek (2017), Singleton (2017), Simpson (2017), Schmidt (2018), and University of South Florida (2017). Finally, because of improved optimization routines for the final channel assignment, 78% of the stations whose current channel was still available in the reduced TV band did not need to change channels after the auction, significantly reducing the cost of the repack and the number of times viewers would need to rescan their TVs for new channels.

But the auction was not only about raising revenues; it was primarily about making critical spectrum available to meet the demand of American consumers. Deploying that spectrum across the length and breadth of the country will create jobs. CTIA, a trade association representing the U.S. wireless communications industry, reports that in 2016, the wireless industry invested more than $26 billion in network infrastructure (CTIA 2017), and the deployment of new 600-MHz spectrum will help to provide full employment in the industry for several years. For example, T-Mobile, the largest wireless auction winner, has stated that it intends to use its new spectrum to build the first nationwide 5G network, and it has already started to deploy its new spectrum to expand and improve its existing mobile broadband network (T-Mobile 2018).

It has become abundantly clear that increasing wireless capacity spurs economic growth. The new "app economy" emerged only after spectrum became available for the 3G broadband networks needed to power smartphones. It currently accounts for 1.72 million jobs and is predicted to exceed $6.3 trillion in economic activity by 2021. Getting the repurposed 600 MHz spectrum out to the industry will continue to fuel this new segment of the economy and will also allow the United States to be at the forefront of the new 5G technology, which will power the Internet of Things and other new types of services now being developed.

Finally, the Commission's coordination with Canada and Mexico ensured that these countries and the United States will have a uniform 600-MHz band as a result of this auction. Thus, carriers and device manufacturers in these three countries will enjoy the economies of scale of a consistent spectrum band plan. Operations research was vital to achieving this outcome.

When the auction closed, a bipartisan group of senior House Energy and Commerce Committee members said that "[t]he broadcast incentive auction revolutionized the way that our nation makes spectrum allocation decisions by empowering broadcasters, businesses, networks, and consumers alike" (Committee on Energy and Commerce 2017). This result was no accident. Advancements in operations research, big data analytics, and computing power were integral to the conception, design, and implementation of the auction. As FCC Chairman Ajit Pai said, "This auction would not have been possible without the use of operations research tools to solve complicated design and implementation challenges" (FCC 2018a).

Lasting Impact

The Incentive Auction optimization work yielded several major technical innovations, such as the following:

- The FCC designed hybrid algorithms that worked far better than any previously known algorithms. Because these problems have a structure consistent with graph-coloring problems and with the set-packing problems often found in scheduling problems, the hybrid algorithms developed for this application are likely to be useful in a much broader set of applications. For example, the U.S. Marine Corps is currently looking to apply these techniques to the problem of assigning channels to radios on a battlefield, where the assignment must consider the dynamic nature of a changing battlefield. The same hybrid approaches used for the FCC are proving to be applicable for this important stochastic problem (Nicholas and Hoffman 2016).
- The feasibility checker allowed the economics community to consider breakthrough new auction-design approaches, previously considered computationally infeasible. To our knowledge, this is the first multiple-round two-sided auction that used feasibility or optimization routines within each round.
- The FCC created a distributed solver for the optimization problems, which can utilize different algorithms in real time. This allowed customization of the solving process for each step of the optimization procedure. Additionally, the plug-and-play nature of the solver allowed for continued algorithm development during the actual auction, and any new heuristics developed could be integrated immediately.
- Finally, machine learning was used not only to configure the parameters of the custom satisfiability solver but also to determine which combination of solvers, heuristics, and parameters would work best so that the greatest number of problems could be solved in the shortest period of time.

The FCC is now looking at how the operations research techniques used in the Incentive Auction and a two-sided auction structure might be applied to other spectrum bands, such as the satellite bands, to ensure that they are being used efficiently. The FCC is not alone in looking at possible ways to apply the concepts learned from the incentive auction to clear spectrum bands. Numerous government regulators around the world have sought FCC input on how they could apply market-based mechanisms such as the Incentive Auction to clear spectrum for mobile broadband use.

The FCC has recognized the value of operations research to guide other policy decisions. As Chairman Pai stated, "The success of these [operations research] tools speaks for itself, and the team's work is exemplary of the data-driven approach to policymaking that I believe should be this agency's hallmark" (FCC 2018a). On January 31, 2018, the FCC approved the creation of a new Office of Economics and Analytics (FCC 2018b) to bring together economists and data personnel from across the agency to facilitate improved integration of economics and data analysis with policy making. As part of that initiative, the FCC will establish new chief data officer and chief data scientist positions to ensure that the latest data analytics and operations research techniques are brought to bear early in FCC proceedings and at the highest levels of the Commission.

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