# WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 5th Semester Examination, 2022-23

## PHSACOR11T-PHYSICS (CC11)

## QUANTUM MECHANICS AND APPLICATIONS

Time Allotted: 2 Hours | Full Marks: 40

*The figures in the margin indicate full marks.*
*Candidates should answer in their own words and adhere to the word limit as practicable.*
*All symbols are of usual significance.*

**Question No. 1 is compulsory and answer any *two* from the rest**

1. Answer any *ten* questions from the following: $2\times 10=20$

(a) Explain the physical significance of energy time uncertainty relation.

(b) Give the physical interpretation of the wave function $\psi(x,t)$.

(c) What are stationary states in quantum mechanics?

(d) If the commutation relation between $x$ and $p$ is $[x,p]=i\hbar$. Find the commutation value of $[x^2, p]$.

(e) What is the implication of the result: $[\hat{H}, \hat{L}]=0$?

(f) Consider the operator $\hat{Q}=i\dfrac{d}{d\phi}$ where $\phi$ is usual polar coordinates in two dimensions. Write down its eigenvalue equation and find its eigenvalues.

(g) What is normal Zeeman effect? Under what conditions it may be observed?

(h) What is Larmor precession of electron in an atom?

(i) Explain bound and unbound states in quantum mechanics.

(j) A wavefunction $\psi$ is constructed as a linear combination of a set of orthonormal eigenfunctions $\psi_n$ :

$$\psi=\sum_{n=1}^{\infty}c_n\psi_n$$

where $c_n$ are constants. Show that if $\psi$ is normalized then $\sum_{n=1}^{\infty}|c_n|^2=1$

(k) If the wavefunction of a particle trapped in space between $x=0$ and $x=L$ is given by $\psi(x)=A\sin\dfrac{2\pi x}{L}$, where $A$ is a constant, for which value(s) of $x$ will the probability of finding the particle be maximum?

(l) Electron configuration of Sodium is given by $1s^2\ 2s^2\ 2p^6\ 3s^1$. Find the ground state term symbol of Sodium.

(m) What is Stark effect?

(n) A beam of spin $\frac{1}{2}$ particle is prepared in the state $|\psi\rangle = \frac{3}{\sqrt{34}}|+\rangle + i\frac{5}{\sqrt{34}}|-\rangle$; where $|+\rangle$ and $|-\rangle$ are eigen states of $\hat{S}_z$ with eigenvalues $+\frac{\hbar}{2}$ and $-\frac{\hbar}{2}$ respectively. Find the average value in $S_z$ measurement.

2. (a) If $\psi_1$ and $\psi_2$ are two eigen states with energy $E_1$ and $E_2$ respectively, check whether the state $(\psi_1 + \psi_2)$ is stationary or not. 2

(b) (i) Prove that the time rate of change of the expectation value of a dynamical variable satisfies the following relation 3+2

$$\frac{d}{dt}\langle \hat{A} \rangle = -\frac{i}{\hbar}\langle [\hat{A}, \hat{H}] \rangle + \left\langle \frac{\partial A}{\partial t} \right\rangle$$ where the symbols have their usual meanings.

(ii) Using the above relation show that the time rate of change of expectation value of momentum is equal to the average value of force.

(c) Prove that the parity of spherical harmonics $Y_{l,m}(\theta, \phi)$ is $(-1)^l$. 2

(d) What do you mean by degenerate wavefunction? 1

3. (a) The potential in a region is given as:

$$V(x) = 0 \text{ for } x < 0$$
$$= V_0 \text{ for } 0 \le x \le a$$
$$= 0 \text{ for } x > a$$

A particle of mass $m$ and energy $E < V_0$ travelling from left to the right is incident on the potential barrier.

(i) Write down Schrodinger equations in three regions of the potential. 2

(ii) Write down appropriate boundary conditions. 2

(b) The wavefunction of a hydrogen atom is given by the following superposition of energy eigenfunctions $\psi_{nlm}(\vec{r})$ ($n$, $l$, $m$ are the usual quantum numbers):

$$\psi(\vec{r}) = \sqrt{\frac{2}{7}}\psi_{100}(\vec{r}) - \frac{3}{\sqrt{7}}\psi_{210}(\vec{r}) + \frac{1}{\sqrt{14}}\psi_{322}(\vec{r})$$

(i) Determine the ratio of expectation value of the energy to the ground state energy. 2

(ii) What are the expectation value of $\vec{L}^2$ and $\hat{L}_z$ operators? 2

(iii) What is the probability that the atom is found in a state of even parity? 2

4. (a) Hamiltonian for the linear harmonic oscillator is given by $\hat{H} = \frac{1}{2}\hat{p}^2 + \frac{1}{2}m\omega^2\hat{x}^2$, where the symbols have usual meanings. Using the basic commutation relation between $\hat{x}$ and $\hat{p}$ show that,

(i) $[\hat{a}, \hat{a}^{\dagger}] = 1$ and 2+2

(ii) the Hamiltonian is given by 3

$$H = (\hat{a}^{+}\hat{a} + 1/2)\hbar\omega$$

given that $\hat{a} = \left(\frac{m\omega}{2\hbar}\right)^{1/2}\left(\hat{x} + \frac{i}{m\omega}\hat{p}\right)$ and $\hat{a}^{+} = \left(\frac{m\omega}{2\hbar}\right)^{1/2}\left(\hat{x} - \frac{i}{m\omega}\hat{p}\right)$

Then find the normalized ground state wavefunction of linear harmonic oscillator.

(b) A particle constrained to move along $x$-axis in the domain $0 \le x \le L$ has a wavefunction $\psi(x) = \sqrt{\frac{2}{L}}\sin\left(\frac{n\pi x}{L}\right)$, where $n$ is an integer. What is the expectation value of its momentum? 3

5. (a) State Moseley's Law. Derive this law from Bohr's theory. 1+4

(b) Considering the L-S coupling scheme for helium atom, find the spectroscopics terms for (i) $1s^1\ 2s^1$ and (ii) $1s^1\ 2p^1$ configurations. 3

(c) In a Stern-Gerlach experiment on turning on the magnetic field, the beam splits into seven components. What is the angular momentum of the atoms in the beam? 2

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