WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 6th Semester Examination, 2021

STSACOR14T-STATISTICS (CC14)

Time Allotted: 2 Hours   Full Marks: 40

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

GROUP-A

Answer any *four* from the following questions $5 \times 4 = 20$

1. Write the pdf of p-variate normal distribution with mean vector $\boldsymbol{\mu}$ and dispersion matrix Σ. Let $\boldsymbol{X} = (X_1, X_2, ..., X_p)' \sim \mathcal{N}_p(\boldsymbol{\mu}, \Sigma)$, where $\Sigma = ((\sigma_{ij}))$ is p.d. and $\boldsymbol{a}$ is a fixed vector. If r_i is the correlation between X_i and $\boldsymbol{a}'\boldsymbol{X}$, show that $\boldsymbol{r} = (r_1, r_2, ..., r_p)' = c^{-1/2} D\Sigma \boldsymbol{a}$, where $c = \boldsymbol{a}'\Sigma\boldsymbol{a}$ and D is a diagonal matrix with diagonal elements $\sigma_{ii}^{-1/2}$.   1+4

2. Let X and Y have the joint pdf   5

$$f_{X,Y}(x, y) = \exp\left(c + 4x + 4y - \frac{x^2}{2} - \frac{y^2}{2} - \frac{x^2 y^2}{2} \right), \ (x, y) \varepsilon \ \mathbb{R}^2,$$

   where c is a constant. Obtain the conditional distribution of X given $Y = y$ and the conditional distribution of Y given $X = x$.

3. Let $\boldsymbol{X} = (X_1, X_2, ..., X_7)'$ be a multivariate normal with mean vector $\boldsymbol{\mu}$ and positive definite dispersion matrix Σ. Stating all necessary result(s), verify whether   5

$$P[(\boldsymbol{X} - \boldsymbol{\mu})'\Sigma^{-1}(\boldsymbol{X} - \boldsymbol{\mu}) < 15] > P[(\boldsymbol{X} - \boldsymbol{\mu})'\Sigma^{-1}(\boldsymbol{X} - \boldsymbol{\mu}) > 15]$$

   What is your conclusion if the distribution of $\boldsymbol{X}$ is not normal?

4. Define moment generating function of a random vector $\boldsymbol{X}^{p \times 1}$. The moment generating function of a bivariate normal random vector $(X, Y)'$ is   5

$$E(e^{t_1 X + t_2 Y}) = e^{t_1^2 + 5t_2^2 - t_1 t_2 - 2t_2}$$

   Find the moment generating function of $(X - 2Y, \ X + 3Y)'$.

5. Write nonparametric single sample location problem under symmetry. Justify whether your null hypothesis and alternative hypothesis are simple or composite.   3+2

6. For exporting mangoes of West Bengal to a foreign country, it is necessary to provide approximate upper and lower limits of weight of a mango. A random sample of n mangoes are available for this purpose. Without making any specific distributional assumption, estimate the limits with high confidence.   5

GROUP-B

Answer any *two* from the following questions $10 \times 2 = 20$

7. (a) Let, for a random vector $X^{p \times 1}$, the partial regression coefficient of X_i on X_j for   7+3
fixed $X_1, X_2, ..., X_{i-1}$, $X_{i+1}, ..., X_{j-1}$, $X_{j+1}, ..., X_p$ be denoted by $b_{ij.12...(i-1)(i+1)...(j-1)(j+1)...p}$.

Show that $b_{12.34...(p-1)} = \dfrac{b_{12.34...p} + b_{1p.23...(p-1)} \, b_{p2.13...(p-1)}}{1 - b_{1p.23...(p-1)} \, b_{p1.23...(p-1)}}$

(b) The random variables $Y_1 = X_1 - \mu$ and $Y_i = X_i - X_i - X_{i-1}$, $i = 2, 3, ..., p$, are independently distributed each as $\mathcal{N}(0,1)$. Find the joint distribution of $X_1, X_2, ..., X_p$.

8. (a) Suppose X_1, X_2, X_3 jointly follow a trinomial distribution with parameters n, p_1,   6+4
p_2, p_3. If ρ_{ij} is the correlation coefficient between X_i and X_j, find the expressions for ρ_{12}, ρ_{13} and ρ_{23} and hence deduce the expression for the partial correlation coefficient $\rho_{12 \cdot 3}$.

(b) Explain the concept of ellipsoid of concentration corresponding to a given p-dimensional distribution.

9. Suppose the data $\{X_1, X_2, ..., X_n\}$ relate to the diameter of pine stem in a certain   8+2
forest. From a historical study, it was found that the distribution function can be well approximated by $G(x) = \Phi((x-50)/2)$ with $\Phi(x)$: cdf of $\mathcal{N}(0,1)$ distribution. Verify on the basis of the present data, whether there is enough reason to support this believe. Formulate the problem statistically and provide a complete analysis. Also discuss the large sample situation.

Students have to complete submission of their Answer Scripts through E-mail / Whatsapp to their own respective colleges on the same day / date of examination within 1 hour after end of exam. University / College authorities will not be held responsible for wrong submission (at in proper address). Students are strongly advised not to submit multiple copies of the same answer script.

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