WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 5th Semester Examination, 2022-23

PHSADSE02T-PHYSICS (DSE1/2)

ADVANCED DYNAMICS

Time Allotted: 2 Hours | Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Question No. 1 is compulsory and answer any *two* from the rest

1. Answer any ***fifteen*** questions from the following: $2\times 15=30$

(a) Show that if the Lagrangian is independent of any 'generalised' coordinate then the corresponding 'generalised' momentum is conserved.

(b) A particle of mass m is constrained to move along a vertical circle of radius a under the field of gravity. Determine the force of constraint.

(c) The Lagrangian of a particle of mass m moving in a plane is given by $L=\frac{1}{2}m(v_x^2+v_y^2)+a(xv_y-yv_x)$ where v_x and v_y are velocity components and a is a constant.

(d) Find the canonical momenta of the particle.

(e) Show that the kinetic energy of a rigid body can be represented as $T=T=\frac{1}{2}\vec{\omega}\cdot\vec{J}$.

(f) What is meant by 'principal axes of inertia'? What is the property of a rigid body associated with them?

(g) A particle of mass m moves in one dimension with the following potential energy $V(x)=\frac{k}{2}x^2+\frac{k^2}{x}$. Find the frequency for small oscillation about position of stable equilibrium.

(h) Find the fixed points for the map $x_{n+1}=x_n^2$ and determine their stability.

(i) Show that fluid velocity $\vec{v}=\frac{-\hat{i}y+\hat{j}x}{x^2+y^2}$ is a possible motion of an incompressible ideal fluid. Is this motion irrotational?

(j) A particle of unit mass moves in a potential $V(x)=\frac{a}{x^2}+bx^2$ where a and b are positive constants. Find the angular frequency of small oscillations about the minimum of the potential.

(k) What is laminar and turbulent flow of fluid?

(l) What do you understand by bifurcation?

(m) Show that the phase trajectory for a linear harmonic oscillator is an ellipse.

(n) Derive the equation of continuity for an incompressible fluid.

(o) What is streamline motion? What is turbulent motion?

(p) Define Reynold number. How estimation of Reynold number helps to determine whether a motion is turbulent or streamline?

(q) Draw the 2D phase space diagram of a point particle of mass m falling freely under the action of earth's gravity.

(r) Classify all the fixed points of the first order differential equations.

(s) Write the dimension of the co-efficient of viscosity and the surface tension.

(t) Define Euler's angles for the orientation of a rigid body.

2. (a) A particle of mass m is constrained to move on the plane curve $xy = c (c > 0)$ under gravity (y-axis vertical). Obtain the Lagrangian of the particle. 3

(b) Show that the transformation 3

$$Q = \log(1+\sqrt{q}\cos p)$$
$$P = 2\sqrt{q}(1+\sqrt{q}\cos p)\sin p$$

is canonical.

(c) Find the moment and product of inertia of a uniform square plate of side a about X, Y and Z axes, X any axes being taken as the adjacent sides of the plate and Z axis perpendicular to its plane. 4

3. (a) What do you mean by normal modes of vibration? Explain the meaning of normal coordinates and normal frequencies. 1+1+1

(b) A massless spring of force constant k has masses m_1 and m_2 attached to its two ends. The system rests on a horizontal table. Obtain the normal frequencies of the system. 4

(c) The potential energy of a particle is given by $V = 3x^4 - 8x^3 - 6x^2 + 24x$. Find the points of stable and unstable equilibrium. 3

4. (a) Show that $Q = -p$, $P = q + Ap^2$ (where A is a constant) is a canonical transformation. The Hamiltonian for a particle moving vertically in a gravitational field g is $H = \dfrac{p^2}{2m} + mgq$. Find the new Hamiltonian for new canonical variables Q, P given above. 4

(b) Obtain the normal modes of vibration of a double pendulum, assuming equal lengths but unequal masses. Show that if the lower mass is small compared to the upper one, the two resonant frequencies are almost equal. 2+4

5. (a) Show that Poisson bracket remains invariant under canonical transformation. 5

(b) A circular disc of mass M and radius R rolls down an inclined plane. The angle of inclination is ϕ. Write the Lagrangian for the rolling disc. Write the equation of motion using Lagrange's multipliers and then find the force of constraint. 5

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