WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 5th Semester Examination, 2020, held in 2021

MTMACOR11T-MATHEMATICS (CC11)

Time Allotted: 2 Hours   Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* questions from the rest

1. Answer any ***five*** questions from the following: 2×5 = 10

(a) Obtain the order and degree of the following partial differential equations:

(i) $k\left(\frac{\partial^2\phi}{\partial^2 x}+\frac{\partial^2\phi}{\partial^2 y}+\frac{\partial^2\phi}{\partial^2 z}\right)=\frac{\partial\phi}{\partial t}$ (ii) $\left(\frac{\partial z}{\partial x}\right)^3+\frac{\partial z}{\partial y}=0$

(b) Obtain the partial differential equation by eliminating arbitrary function f from the following equation $z=xy+f(x^2+y^2)$.

(c) If u is a function of x, y and z which satisfies the partial differential equation

$$(y-z)\frac{\partial u}{\partial x}+(z-x)\frac{\partial u}{\partial y}+(x-y)\frac{\partial u}{\partial z}=0$$

Show that u contains x, y and z only in combinations of $(x+y+z)$ and $(x^2+y^2+z^2)$.

(d) Explain briefly the relationship between the surfaces represented by $Pp+Qq=R$ and $Pdx+Qdy+Rdz=0$.

(e) Define quasi linear partial differential equation of first order. Give an example.

(f) A particle of mass m describes a circle of radius a under a central attractive force $m\mu(2a^2u^5-u^3)$. Find the velocity of the particle at any point in the orbit.

(g) A point moves in a curve so that its tangential and normal acceleration are equal and the angular velocity of the tangent is constant. Find the curve.

(h) Using Kepler's second law prove that $T^2\propto a^3$, where T is the time period and a represents the length of semi-major axis of the orbit.

(i) Determine the type (parabolic, hyperbolic or elliptic) of the following equation:

$$x^2\frac{\partial^2 u}{\partial^2 x}+(x^2+y^2)\frac{\partial^2 u}{\partial x\partial y}+4y^2\frac{\partial^2 u}{\partial^2 y}=x^2+y^2$$

(j) State Newtonian law of gravitation.

2. Solve the following partial differential equation 8

$$\frac{\partial u}{\partial x}=4\frac{\partial u}{\partial y}$$

by using the method of separation of variables.

3. Classify the wave equation $u_{tt}=c^2u_{xx}$, where c is a constant. Find the characteristics and reduce it to canonical form. Draw the characteristics of the wave equation. 1+2+3+2

4. Find the temperature $u(x, t)$ in a bar of length 20 cms that is perfectly insulated laterally, if the ends are kept at 0ºC and initially the temperature is 10ºC at the centre of the bar and falls uniformly to zero at its ends. 8

5. Solve the Boundary Value Problem 8

$$\frac{\partial u}{\partial t}=\alpha^2\frac{\partial^2 u}{\partial^2 x}$$

along with the conditions $u(0, t)=u(l, t)=0$ and $u(x, 0)=lx-x^2$ for $0<x<l$.

6. Find the general integral of the equation $yzp+zxq=xy$ and hence find the integral surface which passes through $z^2-y^2=1$, $x^2-y^2=4$. 8

7. (a) Reduce the following partial differential equation into canonical form and then solve it $yu_x+u_y=x$. 4

   (b) Solve by method of separation of variables: $u_x=2u_y+u$. 4

8. Find the values of $u(1/2, 1)$ and $u(3/4, 1/2)$ where $u(x, t)$ is the solution of the equation $\dfrac{\partial^2 u}{\partial^2 t}=\dfrac{\partial^2 u}{\partial^2 x}$, $0<x<1$, $t>0$ 8

   which satisfies the following boundary conditions:

   (i) $u(x, 0)=x^2(1-x)$, $0<x<1$

   (ii) $u_t(x, 0)=0$, $0<x<1$

   (iii) $u_x(0, t)=u_x(1, t)=0$, $t\geq 0$

9. (a) Determine the type of equation $u_{xx}+4u_{xy}+4u_{yy}=0$ by reducing it to a canonical form. 4

   (b) Find the solution of the Cauchy problem $(y+u)u_x+yu_y=x-y$ with $u=1+x$ on $y=1$. 4

10. A particle describes the curve $r^n = A\cos n\theta + B\sin n\theta$ under a central force F to the pole. First find out pedal equation of the curve. Then find the law of force. 8

11. A rocket whose mass at time t is $m_0(1-\alpha t)$, where m_0 and α are constants, travels vertically upwards from rest at $t=0$. The matter emitted has constant backward speed $4g/\alpha$ relative to the rocket. Assuming that the gravitational field g is constant and that the resistance of the atmosphere is $2m_0 v\alpha$, where v is the speed of the rocket, show that half of the original mass is left when the rocket reaches a height $g/3\alpha^2$. 8

12.(a) A particle falls down a cycloid $s = 4a\sin\psi$ under its own weight starting from the cusp. Show that when it arrives at the vertex the pressure on the curve is twice the weight of the particle. 4

(b) The path of a projectile is a parabola. Prove it. 4

13. A curve is described by a particle having a constant acceleration in a direction inclined at a constant angle to the tangent. Show that the curve is an equiangular spiral. 8

N.B. : *Students have to complete submission of their Answer Scripts through E-mail / Whatsapp to their own respective colleges on the same day / date of examination within 1 hour after end of exam. University / College authorities will not be held responsible for wrong submission (at in proper address). Students are strongly advised not to submit multiple copies of the same answer script.*

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