WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 2nd Semester Examination, 2021

STSACOR04T-STATISTICS (CC4)

Time Allotted: 2 Hours Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

GROUP-A

Answer any *four* from the following questions 5×4 = 20

1. Let $f:\mathbb{R}\longrightarrow\mathbb{R}$ be a surjective continuous function that takes any value at most twice. Prove that f is strictly monotone. 5

2. Cite examples of functions such that the function is (i) bounded but discontinuous at each point of the domain, and (ii) continuous in the domain but not bounded. Justify your answers. 2+3

3. Define derivative of a function. Show that, if a function is differentiable at a point then it is continuous at that point. Suppose that for a positive-valued function $f'(0)$ exists and $f(x+y)=f(x)f(y)$ for all x and y. Prove that $f'(x)$ exists for all x. 1+2+2

4. State Taylor's Theorem with Lagrange's form of remainder. Obtain Taylor's series generated from $f(x)=\ln(1+x)$ about $x_0=0$ when $|x|<1$. 2+3

5. Using the technique of double integration, show that $\Gamma\left(\frac{1}{2}\right)=\sqrt{\pi}$, where $\Gamma(p)=\int\limits_0^\infty e^{-x}x^{p-1}\,dx$. 5

6. State a necessary and sufficient condition for the existence of a solution to $\boldsymbol{A}^{n\times n}\boldsymbol{x}^{n\times 1}=\boldsymbol{b}^{n\times 1}$. Show that if a system of linear equations has two distinct solutions then there exists an infinite number of solutions. 2+3

GROUP-B

Answer any *three* from the following questions 10×3 =30

7. (a) Show that, for any differentiable function f, passing through the origin such that $1 \le f'(x) \le 2$ for all $x \in \mathbb{R}$, $x \le f(x) \le 2x$ when $x > 0$ [must state any result you need to use]. 3

   (b) Define convex and concave functions. Let $g : A \to \mathbb{R}^+$ be a differentiable function then [with full explanation] show that, (i) $f(x) = e^{g(x)}$ is convex if g is convex and (ii) $h(x) = \frac{1}{g(x)}$ is convex if g is concave. 3+2+2

8. (a) Define pointwise convergence and uniform convergence of a sequence of functions. Distinguish the two convergence with the help of example(s). 2+4

   (b) Let $\phi : [0, 1] \to \mathbb{R}$ be a continuous function. Show [with appropriate justification] that the sequence $\{x^n \phi(x)\}$ of functions is convergent uniformly on $[0, 1]$ if $\phi(1) = 0$. 4

9. (a) Define Riemann integrability of a function. Discuss Darboux approach in this connection. 2+3

   (b) Using Darboux approach, show that $f(x) = e^x$ is Riemann integrable on $[a, b]$. 5

10. (a) What do you mean by the rank of a matrix? If A' be the transpose of the matrix A, show that rank $(AA') =$ rank $(A'A) =$ rank (A). 1+4

    (b) What is the echelon form of a matrix? Prove that the rank of a matrix is equal to the number of non-zero rows in its echelon form. 2+3

11. (a) Define quadratic form of a matrix. Show that a real quadratic form $\boldsymbol{x'Ax}$ in n variables can be reduced to the form 1+4

$$\sum_{i=1}^{r} \lambda_i y_i^2 \quad , \quad \text{where } r = \text{Rank of } A^{n \times n}$$

    (b) Discuss about the definiteness of a matrix. Suppose $\boldsymbol{x'Ax}$ is a non-negative definite quadratic form with $A = ((a_{ij}))$ such that $\sum_i \sum_j a_{ij} = 0$. Show that $\sum_j a_{ij} = 0$ for every i. 1+4

N.B. : *Students have to complete submission of their Answer Scripts through E-mail / Whatsapp to their own respective colleges on the same day / date of examination within* 1 *hour after end of exam. University / College authorities will not be held responsible for wrong submission (at in proper address). Students are strongly advised not to submit multiple copies of the same answer script.*

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