WEST BENGAL STATE UNIVERSITY

B.Sc. Honours/Programme 2nd Semester Examination, 2022

STSHGEC02T/STSGCOR02T-STATISTICS (GE2/DSC2)

INTRODUCTION TO PROBABILITY

Time Allotted: 2 Hours | Full Marks: 40

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

GROUP-A

Answer any *four* questions from the following 5×4 = 20

1. (a) If $P(A)=\frac{1}{4}$, $P(B)=\frac{2}{5}$, $P(A\cup B)=\frac{1}{2}$, find $P(AB)$, $P(A\,|\,B)$ and $P(B\,|\,A)$.

   (b) If A and B are independent events, and $P(A)=\frac{2}{3}$, $P(B)=\frac{3}{5}$, find $P(A\cup B)$, $P(\overline{A}\,|B)$ and $P(\overline{A}B)$.

2. Find the mode of a geometric distribution with p.m.f given by:

$$f(x)=\begin{cases} pq^{x-1} & , \quad x=1,2,\ldots;\ 0<p<1;\ p+q=1 \\ 0 & , \quad \text{otherwise} \end{cases}$$

3. (a) Prove that two independent events, each having positive probability, can never be mutually exclusive.

   (b) State the central limit theorem in the case of i.i.d random variables.

4. For two discrete independent random variables X and Y, prove that $E(XY)=E(X)E(Y)$ and $var(X+Y)=var(X)+var(Y)$.

5. Let the distribution function of a random variable X be $F(x)=1-e^{-2x}$, $x\geq 0$. Find the mean of the random variable X.

6. (a) Define moment generating function (MGF) of a random variable.

   (b) Find out the MGF of a random variable X with pdf

$$f(x)=\begin{cases} \frac{1}{2}e^{-x/2} & , \quad x\geq 0 \\ 0 & , \quad \text{otherwise} \end{cases}$$

GROUP-B

Answer any *two* from the following questions 10×2 = 20

7. (a) Discuss with examples (i) conditional probability and (ii) independence of two random events. 4+3+3

   (b) State Bayes' theorem.

   (c) In a bolt factory, machines M_1 and M_2 manufacture respectively 40% and 60% of the total output. Of their output 5% and 10% are defective. One bolt is drawn from the product and found defective. What is the probability that it is manufactured by M_2?

8. (a) Write down the pdf of normal distribution with mean μ and variance σ^2. Show that the pdf is symmetric about μ. 2+2+3+3

   (b) Show that any odd order central moment for normal distribution is zero. If $X \sim N(0, 1)$ show that $\mu_4 = 3\sigma^4$.

9. (a) For two random variables X and Y, $E(X) = 8$, $E(Y) = 6$, $var(X) = 16$, $var(Y) = 36$ and $r_{XY} = 0.5$. Find 6+4

   (i) $E(XY)$

   (ii) $cov(X, X + Y)$

   (iii) $var(2X - 2Y)$

   (iv) Correlation coefficient between $2X + 3Y$ and $2X - 3Y$.

   (b) Prove that two uncorrelated random variables are independent, if each of the variables assumes only two distinct values.

10. (a) What is convergence in probability? 2+3+5

    (b) State Weak Law of Large Numbers. Determine whether it holds for the following sequence of independent random variables:

$$P(x_n = +1) = \frac{1}{2}(1 - 2^{-n}) = P(x_n = -1).$$

$$P(x_n = 0) = 2^{-n}$$

and $$P(x_n = x) = 0 \qquad \forall x \neq 1, -1, 0$$

N.B. : *Students have to complete submission of their Answer Scripts through E-mail / Whatsapp to their own respective colleges on the same day / date of examination within 1 hour after end of exam. University / College authorities will not be held responsible for wrong submission (at in proper address). Students are strongly advised not to submit multiple copies of the same answer script.*

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