WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 2nd Semester Examination, 2022

MTMACOR04T-MATHEMATICS (CC4)

DIFFERENTIAL EQUATION AND VECTOR CALCULUS

Time Allotted: 2 Hours — Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any ***five*** questions from the following: $2\times 5=10$

(a) Show that the equation $\dfrac{dy}{dx}=\dfrac{1}{y}$, $y(0)=0$ has more than one solution and indicate the possible reasons.

(b) Find all ordinary and singular points of the differential equation

$$2x^2\frac{d^2y}{dx^2}+7x(x+1)\frac{dy}{dx}-3y=0$$

(c) Solve: $\dfrac{dx}{dt}-7x+y=0$; $\dfrac{dy}{dt}-2x-5y=0$

(d) Reduce the equation $2x^2\dfrac{d^2y}{dx^2}+4y^2=x^2\left(\dfrac{dy}{dx}\right)^2+2xy\dfrac{dy}{dx}$ to Euler's homogeneous equation by the substitution $y=z^2$.

(e) Show that if $y=y_1$ is a solution of $\dfrac{d^2y}{dx^2}+P\dfrac{dy}{dx}+Qy=0$, then another solution is $y=y_2$, where

$$y_2=y_1\int\frac{W(y_1,\,y_2)}{y_1^2}\,dx$$

P and Q being functions of x and the Wronskian $W(y_1,\,y_2)$ satisfies the equation $\dfrac{dW}{dx}+PW=0$.

(f) If $\boldsymbol{u}=t\boldsymbol{i}-t^2\boldsymbol{j}+(t-1)\boldsymbol{k}$ and $\boldsymbol{v}=2t^2\,\boldsymbol{i}+6t\boldsymbol{k}$, evaluate $\displaystyle\int_0^2(\boldsymbol{u}\times\boldsymbol{v})\,dt$.

(g) Find the unit vector in the direction of the tangent at any point on the curve given by

$$\vec{r}=(a\ \cos t)\hat{i}+(a\sin t)\hat{j}+bt\,\hat{k}$$

(h) Find the volume of the parallelepiped whose edges are represented by

$$\boldsymbol{a} = 2\boldsymbol{i} - 3\boldsymbol{j} + 4\boldsymbol{k} \quad \text{and} \quad \boldsymbol{b} = \boldsymbol{i} + 2\boldsymbol{j} - \boldsymbol{k} \quad \text{and} \quad \boldsymbol{c} = 3\boldsymbol{i} - \boldsymbol{j} + 2\boldsymbol{k}$$

(i) Find $\boldsymbol{r}$ from the equation $\dfrac{d^2r}{dt^2} = \boldsymbol{a}t + \boldsymbol{b}$, given that both $\boldsymbol{r}$ and $\dfrac{dr}{dt}$ vanish when $t = 0$.

2. (a) Find the necessary and sufficient condition that the three non-zero non-collinear vectors $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$ to be coplanar. 4

(b) If $\boldsymbol{a}$ and $\boldsymbol{b}$ be two non-collinear vectors such that $\boldsymbol{a} = \boldsymbol{c} + \boldsymbol{d}$, where $\boldsymbol{c}$ is a vector parallel to $\boldsymbol{b}$ and $\boldsymbol{d}$ is a vector perpendicular to $\boldsymbol{b}$, then obtain expressions for $\boldsymbol{c}$ and $\boldsymbol{d}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$. 4

3. (a) Prove that the necessary and sufficient condition that a vector function $\boldsymbol{f}(\boldsymbol{t})$ has a constant direction is $\boldsymbol{f} \times \dfrac{d\boldsymbol{f}}{dt} = \boldsymbol{0}$. 3

(b) (i) If $\boldsymbol{r} = (\cos nt)\boldsymbol{a} + (\sin nt)\boldsymbol{b}$, where n is a constant, show that 3

$$\boldsymbol{r} \times \frac{dr}{dt} = n(\boldsymbol{a} \times \boldsymbol{b}) \quad \text{and} \quad \frac{d^2\boldsymbol{r}}{dt^2} + n^2\boldsymbol{r} = 0$$

(ii) If $\boldsymbol{r}(\boldsymbol{t}) = 5t^2\boldsymbol{i} + t\boldsymbol{j} - t^3\boldsymbol{k}$, then find the values of $\displaystyle\int_1^2 \left(\boldsymbol{r} \times \frac{d^2\boldsymbol{r}}{dt^2}\right) dt$. 2

4. (a) Prove that: $[\boldsymbol{a}+\boldsymbol{b} \;\; \boldsymbol{b}+\boldsymbol{c} \;\; \boldsymbol{c}+\boldsymbol{a}] = 2[\boldsymbol{a} \;\; \boldsymbol{b} \;\; \boldsymbol{c}]$ 4

(b) Show that the four points $4\boldsymbol{i} + 5\boldsymbol{j} + \boldsymbol{k}$, $-\boldsymbol{j} - \boldsymbol{k}$, $3\boldsymbol{i} + 9\boldsymbol{j} + 4\boldsymbol{k}$ and $4(-\boldsymbol{i} + \boldsymbol{j} + \boldsymbol{k})$ are coplanar. 4

5. (a) Reduce the equation 4

$$2x^2 y \frac{d^2y}{dx^2} + ky^2 = x^2\left(\frac{dy}{dx}\right)^2 + 2xy\frac{dy}{dx}$$

to homogeneous form and hence solve it.

(b) Find the necessary and sufficient condition that the two solutions y_1 and y_2 of the equation 4

$$\frac{d^2y}{dx^2} + P(x)\frac{dy}{dx} + Q(x)y = 0$$

are linearly dependent.

6. (a) Solve the differential equation 4

$$\frac{d^2y}{dx^2} - 9y = x + e^{2x} - \sin 2x$$

by using the method by undetermined coefficients.

(b) Show that the equation 4

$$x^3\frac{d^3y}{dx^3}-6x\frac{dy}{dx}+12y=0$$

has three independent solutions of the form $y=x^r$, given that $y=x^2$ is a solution.

7. Solve:

(a) $(D^2+2D+1)y=e^{-x}\log x$, (by the method of variation of parameters). 4

(b) $(D^2-1)y=x^2\sin x$ 4

8. (a) Solve: $(D^2+4)x+y=te^{3t}$; $(D^2+1)y-2x=\cos^2 t$; by operator method. 4

(b) Solve: $(D^4-n^4)y=0$ completely. Prove that if $Dy=y=0$ when $x=0$ and $x=l$, then 4

$$y=c_1(\cos nx-\cosh nx)+c_2(\sin nx-\sinh nx) \quad \text{and} \quad (\cos nl\ \cosh nl)=1$$

9. (a) Obtain the power series solution of the differential equation 5

$$(1-x^2)y''+2xy'-y=0 \quad \text{about} \quad x=0$$

(b) The equation of motion of a particle is given by 3

$$\frac{dx}{dt}+\omega y=0 \quad ; \quad \frac{dy}{dt}-\omega x=0$$

Find the path of the particle.

N.B. : *Students have to complete submission of their Answer Scripts through E-mail / Whatsapp to their own respective colleges on the same day / date of examination within 1 hour after end of exam. University / College authorities will not be held responsible for wrong submission (at in proper address). Students are strongly advised not to submit multiple copies of the same answer script.*

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