



**WEST BENGAL STATE UNIVERSITY**  
B.Sc. Honours 3rd Semester Examination, 2020, held in 2021

**MTMACOR07T-MATHEMATICS (CC7)**

Time Allotted: 2 Hours

Full Marks: 40

*The figures in the margin indicate full marks.  
Candidates are required to give their answers in their own words as far as practicable.  
All symbols are of usual significance.*

**Answer Question No. 1 and any *four* questions from the rest**

1. Answer any *four* questions from the following: 2×4 = 8

- (a) What is the midpoint rule for numerical integration?
- (b) Prove that  $\Delta \cdot \nabla = \Delta - \nabla$ .
- (c) Prove that  $\left(\frac{\Delta^2}{E}\right)x^3 = 6x$ , where interval of differencing is taken as 1.
- (d) Evaluate first two divided difference of  $f(x) = x^3$  for arguments  $x_0, x_1, x_2$ .
- (e) Find iterative formula for finding real root of the equation  $f(x) - g(x) = 0$  by using Newton-Raphson method.
- (f) Prove that  $f(x_0, x_1, x_2) = \frac{\Delta^2 f(x_0)}{2! h^2}$   
where  $x_1 = x_0 + h, x_2 = x_1 + h$
- (g) Solve the system of equations by Gauss-Jordan method:  $x + y = 2, 2x + 3y = 5$ .
- (h) Find the solution of the differential equation

$$\frac{dy}{dx} = 1 - y, \quad y(0) = 0$$

for  $x = 0.2$  by using Euler's method (take step-length  $h = 0.1$ ).

2. (a) To find a real root of an algebraic equation  $f(x) = 0$  by using Newton-Raphson method, iterative formula is obtained as (assuming convergence) 5

$$x_{n+1} = \frac{2x_n^3 - x_n^2 + 2}{3x_n^2 - 2x_n + 2}$$

Find the polynomial  $f(x)$  and also find the only real root of the equation.

- (b) Calculate the total number of multiplications and divisions are required in back substitution process of Gauss Elimination method to obtain the solution of the following equation. 3

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \cdots + a_{19}x_9 &= b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{29}x_9 &= b_2 \\ &\vdots \\ a_{91}x_1 + a_{92}x_2 + \cdots + a_{99}x_9 &= b_9 \end{aligned}$$

3. (a) In LU-decomposition method, it is required to compute an upper triangular matrix and lower triangular matrix. If the given matrix is singular, what will be problem for LU-decomposition? Explain it analytically. 3
- (b) Suppose you have given  $(2n + 1)$  interpolating points  $(x_i, y_i)$ ,  $i = 0, 1, \dots, 2n$  and you have determined interpolating polynomial using these interpolating points by Lagrange interpolation formula and Newton divided difference interpolation formula. Now, one extra interpolating point  $(x_{2n+1}, y_{2n+1})$  is given to you. At this moment what formula is better (compare with two said formulas) to compute updated interpolating polynomial. Explain it analytically. 5
4. (a) Find  $y'(x)$  from the following table: 3+2

$x$	0	1	2	3	4
$y(x)$	1	1	15	40	85

Hence find  $y'(x)$  at  $x = 0.5$ .

- (b) Suppose you are given four tabulated interpolating points  $(0.01, 1.2)$ ,  $(0.03, 1.5)$ ,  $(0.07, 2.5)$ ,  $(0.08, 3)$  and you have to find  $f(0.015)$  by using Lagrange interpolation formula. If you directly apply the Lagrange formula, what error may be occurred? The error can be avoided by using a suitable linear transformation. Mention what linear transformation can be suitable for this problem. Write down the new Lagrange interpolation formula after transformation. 3
5. (a) What are the advantages and disadvantages of Newton-Raphson method over a general fixed point iteration method in computing a real root of an equation  $f(x)$ , in case both the methods are applicable. 4
- (b) Define divided difference of order  $n$ . Show that it is symmetric function of its arguments. 4
6. (a) Find absolute, relative and percentage error in  $f(x) = 3 \sin x - 2x^2 - 9$  for  $x = 0$  when the error in  $x$  is 0.003. 3
- (b) Design an algorithm to find the real roots of the equation  $ax^2 + bx + c = 0$ , where  $a, b, c$  are real numbers. 5

7. Compute  $y(0.2)$  for the initial value problem  $\frac{dy}{dx} = x + y$ ,  $y(0) = 1$  by Runge-Kutta method. Use step length  $h = 0.1$ . 8

8. (a) Derive a general quadrature formula for equidistant ordinates and hence derive Simpson's  $3/8^{\text{th}}$  rule. 4+1

- (b) The velocity  $v$  of a particle at a distance  $s$  from a fixed point on its path is given by the table below 3

$s$ (in m)	0	10	20	30	40	50	60
$v$ (m/sec)	47	58	64	65	61	52	38

Estimate the time taken to travel 60 meters by using Trapezoidal rule.

9. Describe the power method for finding the largest (in magnitude) eigen value of a real square matrix  $A$ . How least eigen value (in magnitude) can be obtained by using the Power method? Explain it mathematically. 8

10. Find the inverse of the matrix 8

$$A = \begin{pmatrix} 3 & 2 & 1 \\ 2 & 3 & 2 \\ 1 & 2 & 2 \end{pmatrix}$$

using LU-decomposition method and hence solve the system of equations

$$3x + 2y + z = 6$$

$$2x + 3y + 2z = 7$$

$$x + 2y + 2z = 5$$

11. Define 'degree of precision' of a numerical integration formula. Mention the degree of precision for Trapezoidal rule and Weddle's rule. Deduce mathematically the degree of precision of Simpson's one-third rule. 8

What will be the error for computing the integral

$$\int_2^5 (6x^2 + 5x + 2) dx$$

by using Simpson's one-third rule and taking 10 sub intervals. Justify your answer.

**N.B. :** Students have to complete submission of their Answer Scripts through E-mail / Whatsapp to their own respective colleges on the same day / date of examination within 1 hour after end of exam. University / College authorities will not be held responsible for wrong submission (at in proper address). Students are strongly advised not to submit multiple copies of the same answer script.

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