WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 2nd Semester Examination, 2022

STSACOR04T-STATISTICS (CC4)

Time Allotted: 2 Hours — Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

GROUP-A

Answer any *four* from the following questions

1. If a real-valued function f (not necessarily monotonic) has finite left-hand and right-hand limit at every point on an open interval then f must be continuous on it except on a countable set. — 5

2. Check the continuity of the function $f(x)=e^{\sin x}$ on the two sets $A=[2,7]$ and $B=(2,7)$. Interpret the nature of continuity based on your findings. — 4+1

3. Suppose f is a function defined on the real line with the property that $f(x+y)=f(x)f(y)$, $\forall x,\ y\in\mathbb{R}$. If f is differentiable at zero and $f'(0)=1$, show that f is everywhere differentiable and $f'(x)=f(x)$, $\forall x\in\mathbb{R}$. — 5

4. Write δ-ε definition of limit. Hence find the appropriate limits of the following functions (if exists). — 1+2+2

   (a) $f(x)=\dfrac{x^2+x-2}{\sqrt{x+2}}$ at $x_0=-2$

   (b) $f(x)=x+\dfrac{|x|}{x}$, $x\neq 0$ at $x_0=0$.

5. Define uniform convergence of a sequence of functions. Show that the sequence $\{n^2x^2e^{-nx}\}$ of functions is not convergence uniformly on $[0,\ \infty)$. — 1+4

6. (a) Define eigenvalue of a matrix. Show that the eigenvalues of an idempotent matrix is either 0 or 1. — (1+2)+2

   (b) Show that a matrix is singular if and only if at least one of its eigenvalue is zero.

GROUP-B

Answer any *three* from the following questions

7. (a) Define uniform continuity of a function. Let f and g be two uniformly continuous functions on $A(\subseteq \mathbb{R})$ and if they are both bounded on A then show that fg is also uniformly continuous on A. (1+3)+(3+3)

(b) Show that $f(x)=x$ and $g(x)=\sin x$ are both uniformly continuous on $\mathbb{R}$ but fg is not uniformly continuous on $\mathbb{R}$.

8. (a) Suppose f is continuous on a finite interval $[a, b]$ and let $\alpha = \inf\limits_{a\le x\le b} f(x)$ and $\beta = \sup\limits_{a\le x\le b} f(x)$. Show that α and β are the respective minimum and maximum of f on $[a, b]$ i.e. there exist $x_1,\ x_2 \in [a,b]$ such that $f(x_1)=\alpha$ and $f(x_2)=\beta$. 5+5

(b) Suppose that g is continuous and h is Riemann integrable and non-negative on $[a, b]$. Then show that, for some $c\in[a,b]$

$$\int_a^b g(x)\, h(x)\, dx = g(c)\int_a^b h(x)\, dx\,.$$

9. (a) Define power series and its convergence. State some properties of it. Study the convergence of the power series $\sum\limits_{n\ge 1}\frac{n^n}{n!}(x+2)^n$. 6+4

(b) Evaluate the double integral

$$\int_0^2\int_0^{2-x}(x+y)^2\exp\left(\frac{4y}{x+y}\right)dy\, dx\,.$$

10.(a) In view of vector algebra, explain the consistency of a system of linear equations $\boldsymbol{Ax}=\boldsymbol{b}$. 4+6

(b) Show that the rank of a matrix is unaltered by means of an orthogonal transformation.

11.(a) Define quadratic form of a matrix. Let M be a positive definite matrix of order n, show that there exists a lower triangular matrix L such that $M = LL^T$. Is the matrix unique? (1+4+1)+4

(b) What do you mean by echelon form of a matrix? Illustrate its use with an example.

N.B. : *Students have to complete submission of their Answer Scripts through E-mail / Whatsapp to their own respective colleges on the same day / date of examination within* 1 *hour after end of exam. University / College authorities will not be held responsible for wrong submission (at in proper address). Students are strongly advised not to submit multiple copies of the same answer script.*

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