WEST BENGAL STATE UNIVERSITY

B.Sc. Honours/Programme 2nd Semester Examination, 2021

STSHGEC02T/STSGCOR02T-STATISTICS (GE2/DSC2)

INTRODUCTION TO PROBABILITY

Time Allotted: 2 Hours | Full Marks: 40

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

GROUP-A

Answer any *four* questions from the following 5×4 = 20

1. Given $P(A)=\frac{1}{2}$, $P(B)=\frac{1}{3}$, $P(A\cap B)=\frac{1}{4}$. Check whether the events A and B are

   (i) mutually exclusive
   (ii) exhaustive
   (iii) equally likely
   (iv) independent.

2. A card is drawn from a well shuffled pack of 52 cards. What is the probability of the card being black or an ace?

3. For a normal distribution with mean 3 and variance 16, find the value of y of the variate such that the probability of the variate lying in the interval $(3,\ y)$ is 0.4772. You are given $P(Z\le 2)=0.9772$.

4. If $P(X=x) = 0.1x \quad , \quad x=1,\ 2,\ 3,\ 4$
   $\qquad\qquad\quad = 0 \qquad , \quad \text{otherwise}$

   Find (i) $P(X=1 \text{ or } 2)$
   (ii) $P(\frac{1}{2}<X<\frac{5}{2}\,|\,X>1)$

5. For a Binomial $(n,\ p)$ distribution prove that $\operatorname{cov}(X,\ n-X)=-npq$, where notations have their usual meaning.

6. In a lottery n tickets are drawn at a time, out of tickets numbered $1,\ 2,\ \ldots,\ N$. Find the expectation of the sum S of the numbers on the tickets drawn.

7. Define moment generating function (mgf) of a random variable. Derive the mgf of a Poisson (λ) r.v. (random variable).

8. Let $B_1,\ B_2,\ \ldots,\ B_n$ be exhaustive and mutually exclusive events with $P(B_i)>0$, $i=1,\ 2,\ \ldots,\ n$. Show that for any event A, $P(A)=\sum_{i=1}^{n}P(B_i)P(A\mid B_i)$.

GROUP-B

Answer any *two* from the following questions 10×2 = 20

9. (a) What is convergence in probability? 2

(b) State Chebyshev's inequality. State and prove weak law of large numbers by using the Chebyshev's inequality. 3+5

10. Write down the pdf of $N(\mu,\sigma^2)$ distribution. Show that the distribution is symmetric. Calculate its median and mode. 2+2+3+3

11. The probability that a Poisson variable X takes a positive value is $(1-e^{-2})$. Calculate the (i) mean, (ii) mode, (iii) probability that X lies between –1 and 1.5. 4+3+3

12.(a) Define cumulative distribution function (cdf) of a r.v. What is the relationship of cdf with pmf and pdf for discrete and continuous random variable respectively? 5

(b) An unbiased coin is thrown three times. If the random variable X denotes the number of heads obtained, find the cdf of X. 5

N.B. : *Students have to complete submission of their Answer Scripts through E-mail / Whatsapp to their own respective colleges on the same day / date of examination within 1 hour after end of exam. University / College authorities will not be held responsible for wrong submission (at in proper address). Students are strongly advised not to submit multiple copies of the same answer script.*

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