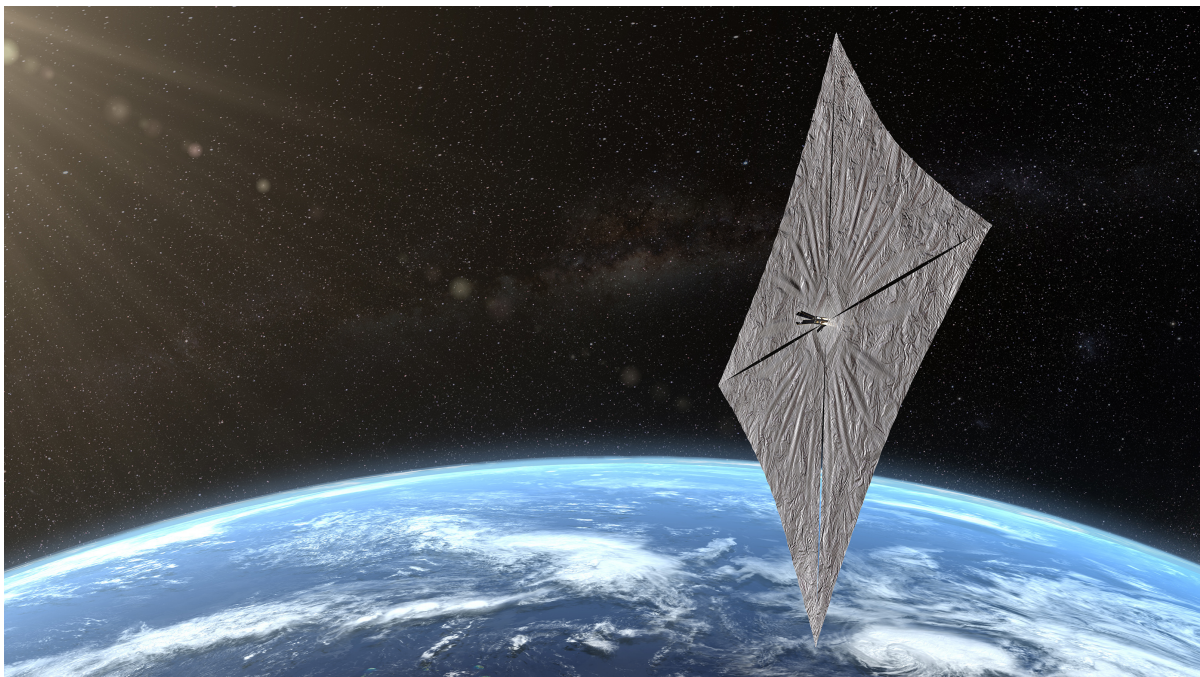


SATELLITE DYNAMICS AND ATTITUDE CONTROL

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REVISION HISTORY

VERSION	REVISION NOTES
PS1	<ul style="list-style-type: none">- Created document- Added PS1 material: Mission specifications, satellite selection. Structure and mass distribution. Inertia matrix, body axes. Orbit.

Table 1: Summary of project revisions.

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1 INTRODUCTION

We have chosen to study the LightSail 2 mission (LS2), a solar sailing proof-of-concept mission developed by The Planetary Society. Solar sails use solar radiation pressure (SRP) to propel the spacecraft, and JAXA's IKAROS mission in 2011 was the first mission to successfully do so in interplanetary space. In contrast, LS2 used solar sailing in Earth orbit to perform a controlled orbital maneuver; namely, raising spacecraft apogee.

2 PROBLEM SET 1

2.1 MISSION ADCS CHARACTERISTICS

We will model LightSail 2's mission objectives and hardware. The spacecraft was deployed into a circular, low-earth orbit at 720km altitude, 24 degrees inclination, where it then attempted to raise its apogee using solar sailing. For analysis, we will use the starting orbital parameters with arbitrary values selected for the unknown parameters, as noted in the orbital elements section.

The method by which LS2 raised its apogee is called the on/off control strategy. While the spacecraft was moving away from the sun, the sail normal vector pointed toward the sun ("on") in order to increase spacecraft velocity. While the spacecraft was moving toward the sun, the spacecraft normal pointed perpendicularly to the direction of the sun ("off"), to maintain orbit until the spacecraft was once again moving away from the sun. Thus, the spacecraft had two target attitudes: sun pointing and sun-perpendicular pointing. *The spacecraft attitude will be represented in this project using quaternions.*

In order to realize sun and sun-perpendicular pointing, the spacecraft will require at minimum sun sensors, but additional sensors for accuracy improvement and redundancy would be preferred. To move between the two target attitudes will require 90 degree slew maneuvers twice per orbit, which can be achieved with a variety of actuators. In this case, torque rods and momentum wheels were used.

The high level ADCS requirements are listed below [6]. To keep the project within a reasonable scope, we will focus on those above the line. Anything below the line will be assumed to be met.

- Provide attitude knowledge to within 5 degrees per axis during all mission phases.
- Sun sensors provide data on the angle of light incidence to the sensors to within ± 3 degrees accuracy.
- Magnetometers provide attitude knowledge of the body-fixed x-, y-, and z-axes to within ± 5 degrees relative to the Earth magnetic field.
- Utilize a momentum wheel to achieve 90-degree slew maneuvers about one axis in < 5 minutes.
- Following solar sail deployment, be capable of providing an angular acceleration of $0.0005 \text{ degrees/sec}^2$ per axis.
- Detumble from a maximum of 10 degrees per second per axis after sail deployment.
- Align +Z axis of the spacecraft with the magnetic field with maximum variation once settled of < 60 degrees.

Component	Number	Specifications
Sun sensors	5	Max error: 3 degrees
Magnetometers	4	Max error: 5 degrees
Primary Gyro	1	Max error: 3 degrees/axis
Intrepid Gyro	1	Max error: 3 degrees/axis

Table 2: LS2 Sensors

Component	Number	Specifications
Torque Rods	3	Max torque: $1 \text{ Am}^2 \times \vec{B}$
Momentum Wheel	1	Max torque: 0.06 Nm^2

Table 3: LS2 Actuators

-
- Damp attitude rates within 2 hours of P-POD deployment.
 - Accommodate a tip off rate of up to 25 degree/sec per axis from P-POD deployment.
 - Prior to sail deployment, utilize torque rods to achieve attitude control to within 10 degree per axis.
 - Prior to sail deployment, be capable of providing an angular acceleration of 0.1 degrees/s^2 per axis.
 - Prior to solar sail deployment, provide attitude control to within 10 degrees per axis.
 - Downlink telemetry for sensors, actuators and performance data.
 - Sample spacecraft angular rates using gyro sensors.
 - Be actively controllable in each of its three-axes.

To achieve these objectives, LS2 used the sensors and actuators listed in Tables 3 and 2. These components were selected for their low cost, low mass, and reliability.

2.2 SIMILAR MISSIONS

There are several satellites with similar mission objectives. LightSail 1 (LS1) was originally developed to meet the same objective, but was down-scoped to only achieve the goals listed prior to and including sail deployment. To reduce development costs, LS2 reused much of the LS1 design. Notable differences include a lower orbital altitude (400km for LS1 compared to 720km for LS2), removing the momentum wheel to simplify the spacecraft and reduce costs, and a simpler ADCS on LS1 [3].

IKAROS, the first successful solar sail mission, differed from LS2 in three major respects: first, it orbits the sun rather than Earth, and thus must survive the interplanetary, rather than near-Earth, environment. Secondly, it performs attitude control using liquid crystals in the sail that alter the diffusivity and reflectivity of the solar sail to allow for differential SRP across the sail [9]. Lastly, while LS1 and 2 used rigid booms to maintain the sail's shape, IKAROS removed these to save mass, and thus relies on centrifugal force from spacecraft spin to keep the sail extended. All of these differences have driven ADCS requirements that are different from LS2 (such as maintaining a minimum spin about the axis normal to the sail).

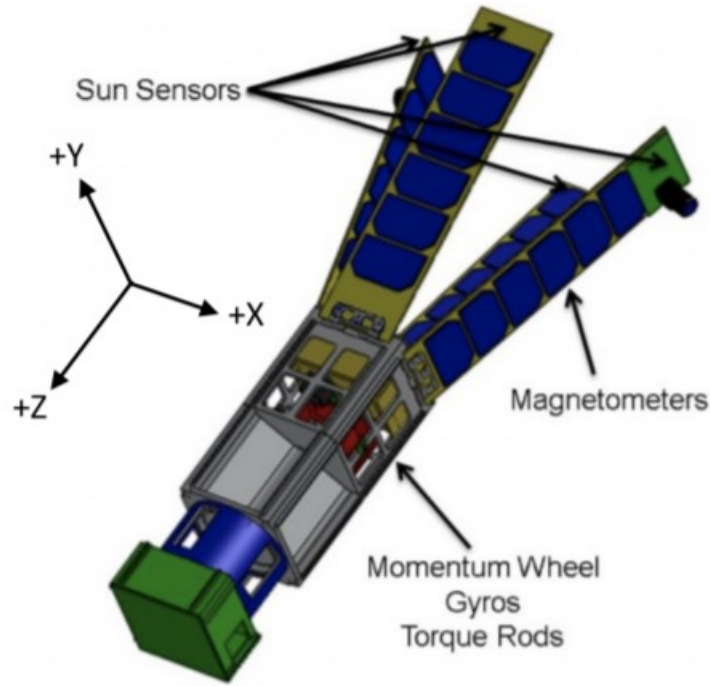


Figure 1: Annotated Full-Detail Layout of 3U Structure

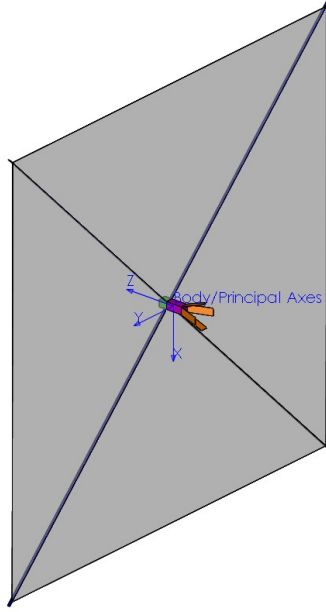
NASA has launched missions similar to LS1 and LS2, NanoSail-D, to prove usage of solar sails as passive de-orbiting mechanisms. The first NanoSail-D was lost in launch, while the second performed many of the desired functions, de-orbiting after 240 days [2].

Another relevant mission is Prox-1, LS2's launch partner. Originally developed to demonstrate automated, close-proximity maneuvers relative to another CubeSat (with LS2 as its target), the proximity maneuvers were ultimately cut and the spacecraft now serves primarily to monitor and image LS2. Prox-1's ADCS mission requirements are very different (and more strict) than LS2's, requiring greater attitude knowledge and control to perform its maneuvers and imaging [7].

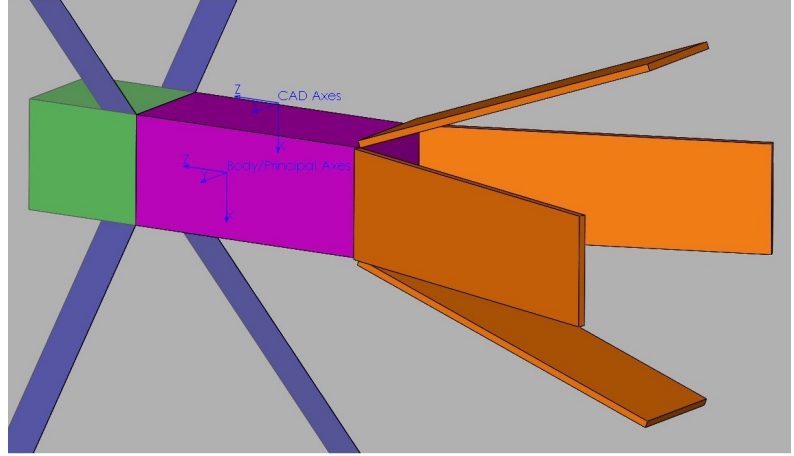
2.3 MECHANICAL LAYOUT

High-level drawings of the LS2 system in its various configurations are provided in Appendix B; note that this study will only consider the LIGHTSAIL DEPLOYED configuration. The basic layout of the 3U structure with the placement of primary ADCS components is provided in Figure 1. The body coordinates used to describe the mechanical layout are plotted in Figure 2. The body and principal axes are taken to be the same, having their origin at the system center of mass. Note that the system is symmetric over the XZ and YZ -planes, with the $+z$ axis pointing away from the deployed solar panels.

LS2 uses a motorized TRAC (Triangular Roll-able And Collapsible) boom deployment system to unfurl its 5.57m square Mylar sail. During launch, the 4 primary solar panels are folded along the length of the satellite - once deployed, the panels extend along the $-z$ axis. After de-tumbling with magnetic torquing, the booms extend and unfold the 4.5 micron thick Mylar from a central "sail housing" unit. Due to limited knowledge about the exact mass distribution of the satellite and a desire for a simplified model, the mechanical layout has been reduced



(a) Structure w/ Body Axes



(b) Detail of Simplified Cubesat Structure w/ Axes

Figure 2: Simplified Mechanical Layout of LightSail 2

Name	Color in 3D Model	Subsystems	Mass (kg)
Forward Body	Green	Comms, Boom Motors (x2), Boom Housing	1.47
Booms	Blue	TRAC Booms (x4)	0.93
Solar Sail	Grey	Mylar sail	0.20
Rear Body	Purple	Magnetorquers, Mom. Wheel, Power, Avionics	1.73
Solar Panels	Orange	Solar Cells, Magnetometers, Cameras	0.60
TOTAL	-	-	4.93

Table 4: Primary Components of Simplified Mechanical Layout

to five primary components, detailed in Table 4 below. The mass calculations are described in further detail in Appendix A; in general, each component has an associated mass and the center of mass of each component is assumed to correspond with the component's geometric center. Small symmetry-breaking structures, like the cameras at the aft end of the solar panels and corner-mounted antenna at the fore end of the 3U structure, are assumed to have negligible contribution to center of mass and inertia characteristics.

2.4 INERTIA PROPERTIES

The inertia properties provided below were computed using the 3D model shown in Figure 2. An analytical verification of these values is provided in Appendix C. Along the principal axes, the moments of the inertia for the system are

$$I_{xx} = 3.09578 \text{ kg} \cdot \text{m}^2, \quad I_{yy} = 3.09578 \text{ kg} \cdot \text{m}^2, \quad I_{zz} = 5.98190 \text{ kg} \cdot \text{m}^2$$

As our principal and body axes are identical, generalizing to the full inertia matrix w.r.t the body axes is as simple as placing these moments along the diagonal, i.e.

$$L = \begin{bmatrix} 3.09578 & 0 & 0 \\ 0 & 3.09578 & 0 \\ 0 & 0 & 5.98190 \end{bmatrix} \text{ kg} \cdot \text{m}^2$$

2.5 OUTER SURFACE DISCRETIZATION

To provide an easy way of analyzing environmental perturbative torques, we also simplify the outer geometry. The surface normals and centroids are pulled from the CAD model, where the solar panels and solar sail are taken to have a negligible thickness (effectively reducing those components to planes). Using a Solidworks VBA Macro, we export the centroid, unit normal, and area information for each outward-facing surface of interest. The MATLAB script below imports the information into MATLAB data structures for use in future simulations. Table 5 displays the centroid locations in body coordinates, the associated normal vector, and the associated area.

```

1 %% get_surfacedata()
2 % Reads surface data from CSV file.
3 % No input.
4 % Output:
5 %   - C : an (n x 3) matrix, where n is the number of surfaces, denoting
6 %       the location in body coordinates of each surface's ...
7 %       centroid in m.
8 %   - N : an (n x 3) matrix denoting the unit outward-facing normal for
9 %       each surface.
10 %   - A : an (n x 1) matrix denoting the area for each surface, in m^2.
11
12 function [C, N, A] = get_surfacedata()
13     centroid_file = 'CentroidData.csv';
14     data = readtable(centroid_file, 'ReadVariableNames', false);
15     C = [data.Var1 , data.Var2 , data.Var3];
16     N = [data.Var4 , data.Var5 , data.Var6];
17     A = data.Var7;
18 end

```

Location (m)	Normal Vec	Area (m ²)
(-0.002920162, 0.045798805, -0.049449943)	(9.05073E-16, 1, -2.41353E-15)	0.023
(0.047079838, -0.004201195, -0.049449943)	(1, 0, 2.41353E-15)	0.023
(-0.002920162, -0.054201195, -0.049449943)	(2.71522E-15, -1, 0)	0.023
(-0.052920162, -0.004201195, -0.049449943)	(-1, 1.20676E-15, -1.81015E-15)	0.023
(-0.002920162, -0.004201195, -0.164449943)	(6.93889E-16, 1.38778E-15, -1)	0.01
(-0.002920162, 0.045798805, 0.120552307)	(1.89235E-15, 1, 2.52313E-15)	0.01100045
(0.047079838, -0.004201195, 0.120554557)	(1, -2.52313E-15, 0)	0.01100045
(-0.002920162, -0.004201195, 0.175554557)	(6.93889E-16, 1.38778E-15, 1)	0.01
(-0.002920162, -0.054201195, 0.120554557)	(1.26157E-15, -1, 0)	0.01100045
(-0.052920162, -0.004201195, 0.120552307)	(-1, 1.26157E-15, 2.52313E-15)	0.01100045
(-0.002920162, 0.099837988, -0.312921377)	(-1.51514E-14, -0.939692621, -0.342020143)	0.0316
(-0.106959345, -0.004201195, -0.312921377)	(0.939692621, 3.51336E-15, -0.342020143)	0.0316
(-0.002920162, -0.108240377, -0.312921377)	(-6.80714E-15, 0.939692621, -0.342020143)	0.0316
(0.101119021, -0.004201195, -0.312921377)	(-0.939692621, -9.66175E-15, -0.342020143)	0.0316
(-0.106959345, -0.004201195, -0.312921377)	(-0.939692621, -3.51336E-15, 0.342020143)	0.0316
(-0.002920162, -0.108240377, -0.312921377)	(6.80714E-15, -0.939692621, 0.342020143)	0.0316
(0.101119021, -0.004201195, -0.312921377)	(0.939692621, 9.66175E-15, 0.342020143)	0.0316
(-0.002920162, 0.099837988, -0.312921377)	(-1.51514E-14, 0.939692621, 0.342020143)	0.0316
(-0.002920162, -0.004201195, 0.065550057)	(0, 0, 1)	31.002624
(-0.002920162, -0.004201195, 0.065550057)	(0, 0, -1)	31.002624

Table 5: Centroid, normal, and area data pulled from CAD model.

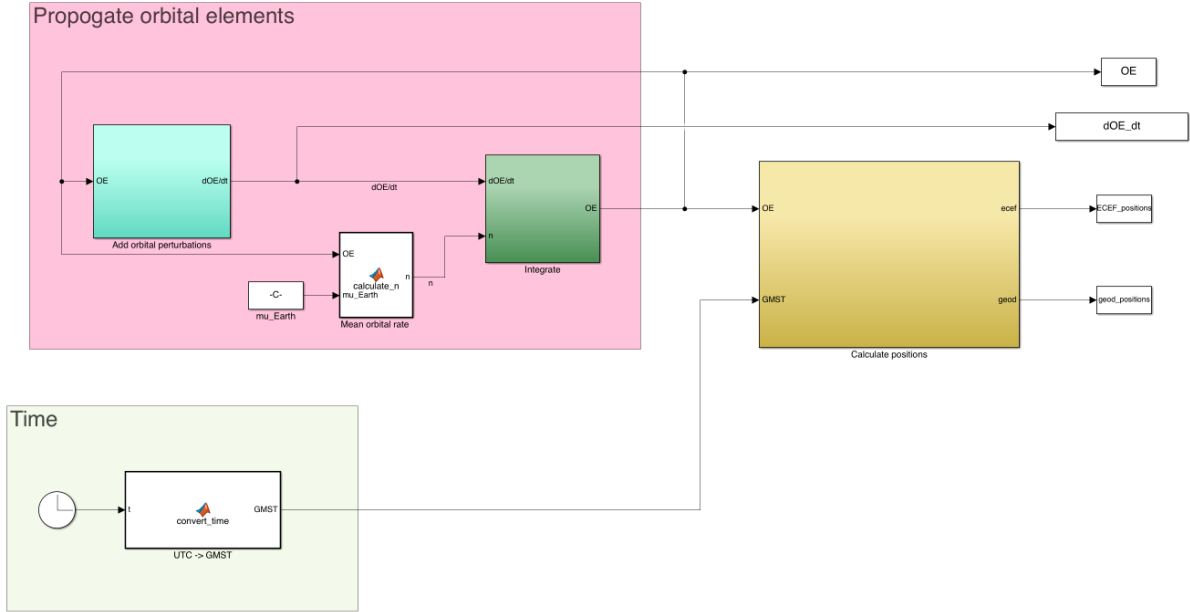


Figure 3: Top-level overview of the initial orbit propagator.

2.6 ORBITAL ELEMENTS & PROPAGATION

The initial orbit propagator was developed for AA 279A in Winter 2019, and later further supplemented to include simple orbital perturbations (namely, atmospheric drag and J2, with the infrastructure in place to add additional perturbations as relevant). SRP will be added to model the change in apogee that is one of the mission objectives. It will be incorporated into the ADCS simulation, and an overview is presented in Figure 3.

Initial values for the orbital elements are given in Table 6. These were used to produce the sample orbits given in Figure 4.

Orbital element	Symbol	Initial value	Units	Notes
Epoch	t_0	2019-07-08.20	YY-MM-DD.dd	
Semi-major axis	a	7095.553	km	$R_E + 717.4175\text{km}$
Eccentricity	e	0.0010951		
Inclination	i	24	deg	
RAAN	Ω	0	deg	Selected arbitrarily
Argument of Periapsis	ω	0	deg	Selected arbitrarily
Eccentric anomaly	E	0	rad	Selected arbitrarily

Table 6: Initial orbital elements, pulled from [8]. The arbitrarily selected values are those for which we do not have data, and have little effect on our modeling.

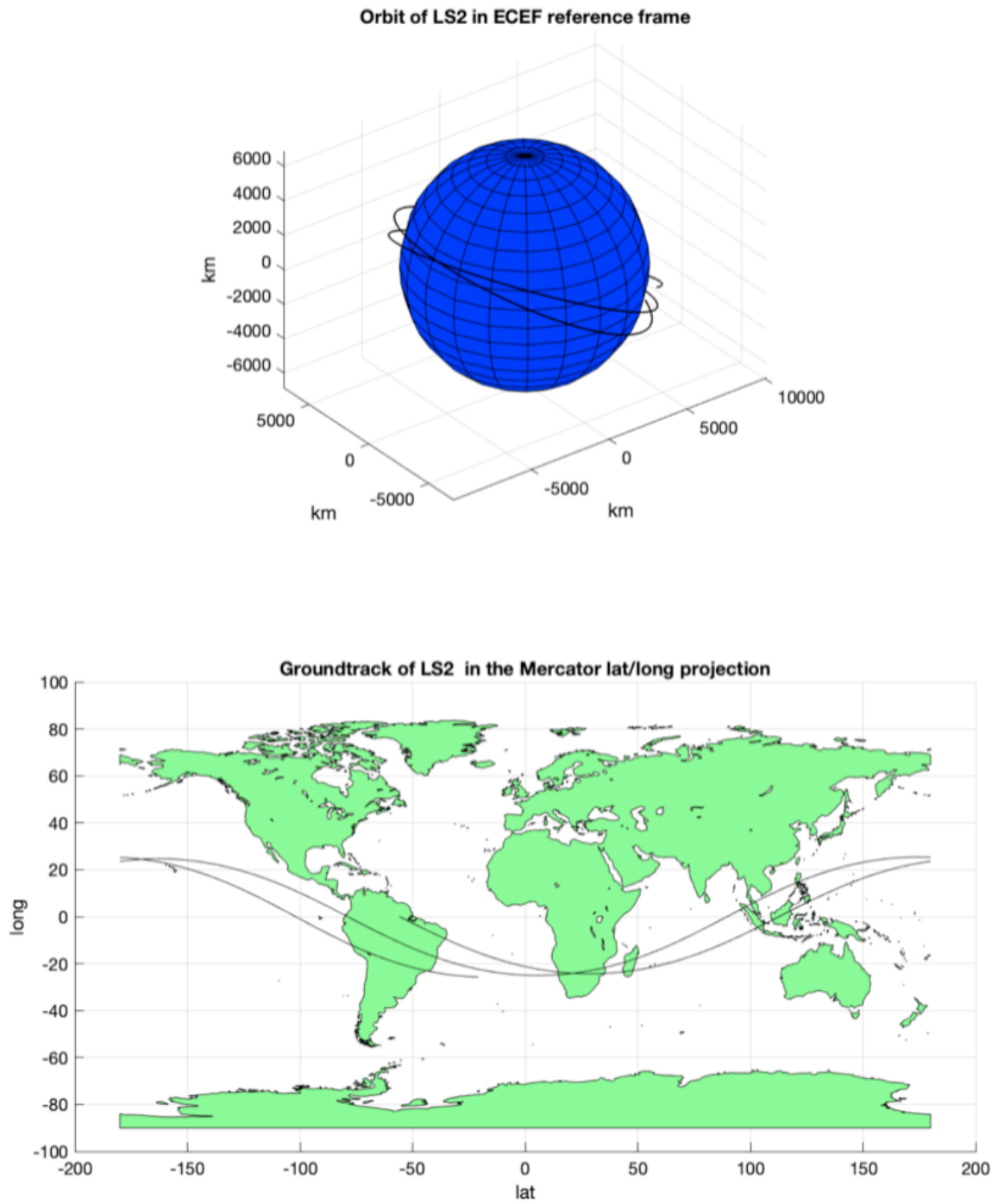


Figure 4: Initial sample orbits of LS2 given the orbital parameters in Table 6.

3 REFERENCES

- [1] Chris Biddy and Tomas Svitek. “LightSail-1 Solar Sail Design and Qualification”. In: 2012.
- [2] George C. Marshall Space Flight Center. *NASA Facts: NanoSail-D*. https://www.nasa.gov/centers/marshall/pdf/484314main_NASAFactsNanoSail-D.pdf. Accessed: 4-6-2021.
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- [4] *Endurosat 3U Solar Panel*. <https://www.endurosat.com/cubesat-store/cubesat-solar-panels/3u-solar-panel-xy/>. Accessed: 4-3-2021.
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- [8] The Planetary Society. *LIGHTSAIL 2 MISSION CONTROL*. https://secure.planetary.org/site/SPageNavigator/mission_control.html. Accessed: 4-6-2021.
- [9] Yuichi Tsuda et al. “Achievement of IKAROS – Japanese deep space solar sail demonstration mission”. In: *Acta Astronautica* 82.2 (2013), pp. 183–188. ISSN: 0094-5765. DOI: <https://doi.org/10.1016/j.actaastro.2012.03.032>. URL: <https://www.sciencedirect.com/science/article/pii/S0094576512001348>.

Appendices

A Mass Distribution Analysis

The total mass of the LS2 system is quoted as 4.93kg [7] - using various published resources from the Planetary Society and the drawings in Appendix B, we are able to make rough estimates for the masses of the five primary components: the solar sail, the sail booms, the forward body, the solar panels, and the rear body.

A.1 Solar Sail

We estimate the full mass of the solar sail system (solar sails, sail booms, boom extension mechanism, and sail housing) to be 2.9kg, based off a note from a deployment package development summary stating that the whole deployment system was contained in a package with mass “ ≈ 3 kg” [1]. The sail housing, which is located in the rear body of the system, we estimate as ≈ 500 g. The solar sails, made of mylar with density $1.38 \frac{\text{g}}{\text{cm}^3}$, have a total volume of $5.57\text{m} \times 5.57\text{m} \times 4.5\mu\text{m}$ per the LS2 drawings. The total mass of the solar sails is therefore estimated as 0.19872 kg .

A.2 Sail Booms

The sail booms, made of elgiloy with density $8.3 \frac{\text{g}}{\text{cm}^3}$ [7], have a volume of $3.5\text{cm} \times 0.02\text{cm} \times 4\text{m}$ per the LS2 drawings. The total mass of each sail boom is therefore estimated as 0.2324 kg .

A.3 Forward Body

The forward body contains the communications avionics (transceiver board and antenna) as well as the boom extension mechanism. We can estimate the mass of the boom extension mechanism by subtracting the assumed 500g mass of the sail housing, the 198.72g of solar sail mass, and the 232.4g mass of the four sail booms from the estimated solar sail system mass of 2.9kg. By adding to that an estimated ≈ 200 g for communications components, we get the estimated mass of the forward body as 1.472 kg .

A.4 Solar Panels

The mass of each 3U-long solar panel was estimated to be 0.150 kg based off the similar mass and form factor of Endurosat’s 3U solar panel system [4].

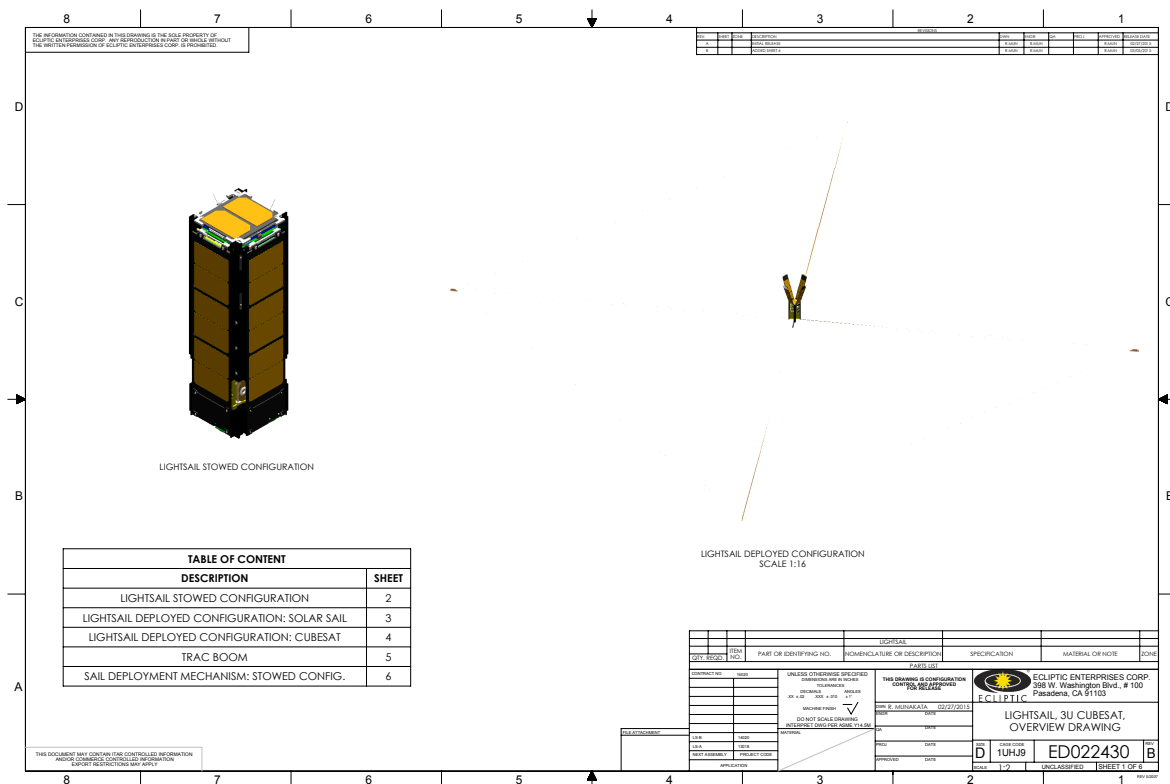
A.5 Rear Body

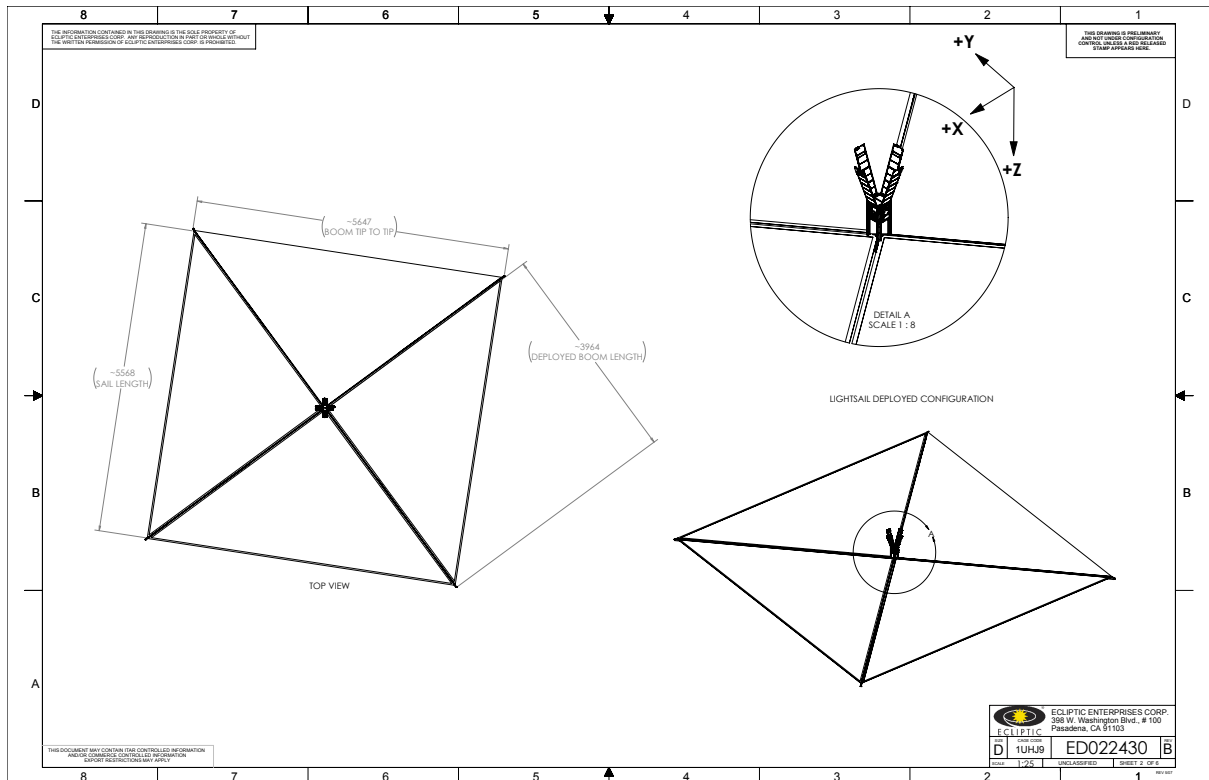
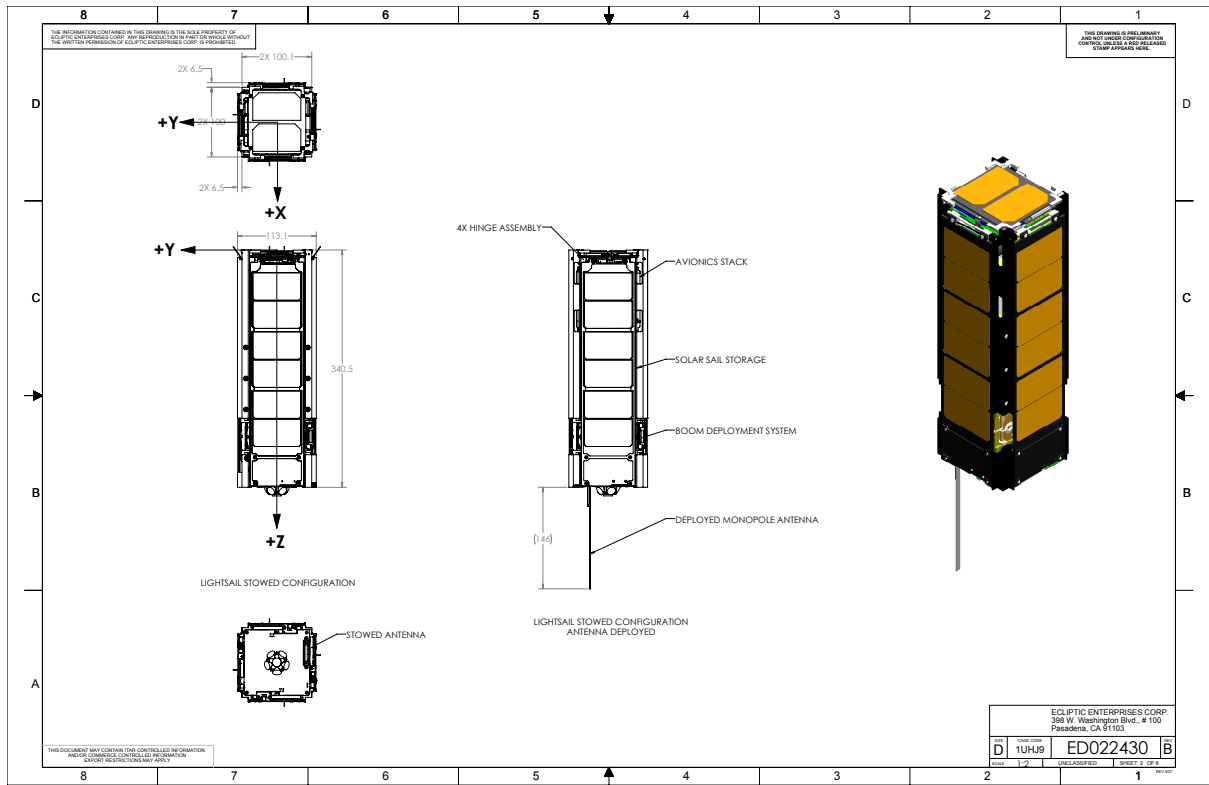
The mass of the rear body was calculated implicitly by subtracting the calculated masses of the above systems from the total satellite mass. This gives an estimated rear body mass of 1.73 kg . We give some validity to this estimate by noting that the breakdown in Table 7 provides reasonable estimates for rear body component masses that total to 1.74 kg. Note that the table lists some approximate values (indicated with \approx) and some values taken from the links on the Lightsail 2 parts list [5] where available.

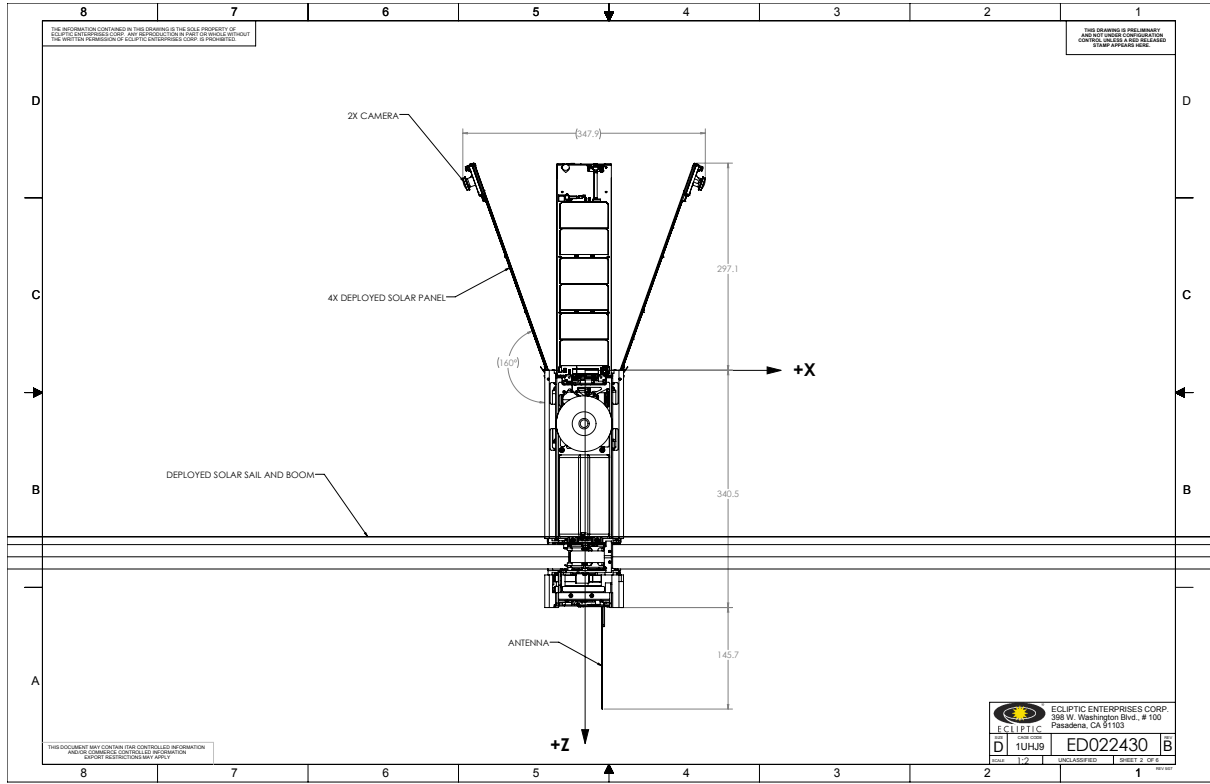
Description	Mass, kg
Momentum Wheel	0.226
Magnetorquers (x3)	0.690
Battery Cells (x8)	0.128
Avionics	≈ 0.1
Structures	≈ 0.1
Sail Housing	≈ 0.5
TOTAL	1.744

Table 7: Estimated Mass Breakdown of Rear Body

B LS2 System Drawings







C Verification of Inertia Computations

We begin with two basic equations; first, the moments of inertia of a rectangular prism with length a along its principal x axis, length b along its principal y axis, length c along its principal z axis, and mass m :

$$I_{xx} = \frac{1}{12}m(b^2 + c^2)$$

$$I_{yy} = \frac{1}{12}m(a^2 + c^2)$$

$$I_{zz} = \frac{1}{12}m(a^2 + b^2)$$

We will also need the formula for a rectangular prism tilted about an axis, as the sail booms are at a 45° angle w.r.t the z axis and the solar panels are tilted at 20° w.r.t to the x/y axes. For the sail booms, I_{zz} is the same as above, but our formula for I_{yy} will need to be modified to account for this rotation. I_{yy} for this rectangular prism (assuming the tilt angle is $\beta = 45^\circ$) is

$$I_{yy} = \frac{1}{12}m(acos\beta + bsin\beta + c)$$

For the solar panels, we will also need to use (depending on the orientation of the panel)

$$I_{zz} = \frac{1}{12}m(acos\beta + b + csin\beta)$$

if tilted w.r.t to x or

$$I_{zz} = \frac{1}{12}m(a + bcos\beta + csin\beta)$$

if tilted w.r.t to y .

Component	Mass, kg	Width a, m	Length b, m	Height c, m	// to Y-Axis, m	// to Z-Axis, m	Quantity	I _{yy} , kg m ²	I _{zz} , kg m ²
Forward Body	1.47168	0.1	0.1	0.11	0.12055	0	1	0.024097243	0.0024528
Sail Booms	0.2324	2.00E-04	4	0.035	1.324895204	1.9957	4	6.27E-01	1.24E+00
Sail	0.19872	5.57	5.57	4.50E-06	0.06555	0	1	5.15E-01	1.027544688
Rear Body	1.73	0.1	0.1	0.23	0.04945	0	1	0.013298457	0.002883333
Solar Panel X-Tilt	0.15	6.50E-03	0.1	0.316	0.312921434	0.104039183	2	1.52E-02	2.18E-03
Solar Panel Y-Tilt	0.15	0.1	6.50E-03	0.316	0.329763514	0.104039183	2	1.76E-02	2.18E-03
TOTAL	4.93	-	-	-	-	-	-	3.125909335	5.983479674

Table 8: Tabulated Inertia Calculation Data.

We also will make use of the parallel axis theorem, which states that the moment of inertia of a body about an axis B parallel to an axis, A , can be written as

$$I_B = I_A + mr^2$$

where m is the mass of the body and r is the distance between the parallel axes.

We also recognize that due to x - y symmetry, $I_{xx} = I_{yy}$ for our body, and by definition, our x and y axes are aligned with the principal axes. Furthermore, due to the alignment of our body and principal axes, we know that our full inertia matrix w.r.t the body axes is simply

$$L = \begin{bmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix} = \begin{bmatrix} I_{xx} & 0 & 0 \\ 0 & I_{xx} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix}$$

In Table 8, we tabulate the mass, a , b , c , and the distance between the principal axis and y/z body axes for each component (measured using the CAD model). Applying our equation for moment of inertia and parallel axis theorem, we are able to calculate the contribution of each item to I_{xx} and I_{zz} . Our final results are $I_{xx} = I_{yy} = 3.125 \text{ kg} \cdot \text{m}^2$ and $I_{zz} = 5.983 \text{ kg} \cdot \text{m}^2$, which have relative errors compared to the CAD-computed values of 0.97% and 0.02%, respectively.

Given these negligibly small errors (likely caused due to the fact that the distances between the parallel axes were measured assuming that the solar panels and booms were planes, rather than prisms with finite thickness), it is clear that the CAD-computed values for the inertia characteristics of our system are trustworthy.

D Additional Orbital Propagator Notes

Plots of evolution of the orbital elements used to generate Figure 4. Atmospheric drag contributes to the decrease in semimajor axis, while J2 contributes to the change in RAAN and argument of periapsis. The perturbation calculations make use of the $e \approx 0$ approximation, which is fine given the initial $e \approx 0.001$, but will need to be modified to increase fidelity as the spacecraft orbit elongates due to SRP.

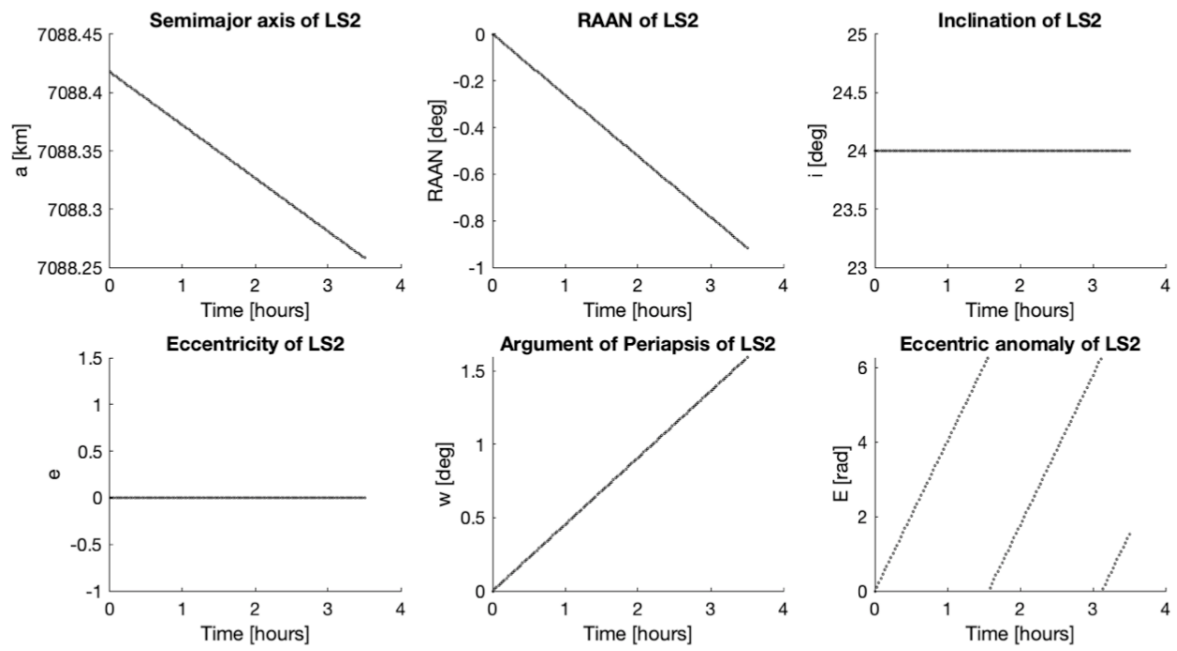


Figure 5: Keplerian orbital elements