Desarrollo y simulación del movimiento Browniano

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Ecuación de Langevin

$$\frac{d\vec{p}}{dt} = \sum \vec{F}$$

$$\frac{d}{dt}(m\vec{v}) = -\zeta\vec{v} + \vec{f}(t)$$

$$m\frac{d\vec{v}}{dt} = -\zeta\vec{v} + \vec{f}(t)$$

$$\dot{\vec{v}}(t) = -\frac{\zeta}{m}\vec{v} + \frac{1}{m}\vec{f}(t)$$

$$\zeta = 6\pi\eta r$$

Fuerza Estocástica

$$\langle \vec{f}(t) \rangle = 0$$

$$\langle \vec{f}(t)\vec{f}(t') \rangle = 2\zeta k_B T \delta(t - t')$$

Proceso Wiener

$$\langle \Delta \vec{W}_i \rangle = 0$$

$$\langle \Delta \vec{W}_i \Delta \vec{W}_j \rangle = 2\zeta k_B T \delta_{ij} \Delta t$$

Metodo de Euler

$$\frac{d\vec{x}}{dt} = v$$

$$\boxed{\therefore \quad x_{i+1} = x_i + v_i \Delta t}$$

$$\frac{d\vec{v}}{dt} = -\frac{\zeta}{m}\vec{v} + \frac{1}{m}\vec{f}(t)$$

$$\Delta\vec{v} = -\frac{\zeta}{m}\int_0^t dt'\vec{v}_i + \frac{1}{m}\int_0^t dt'\vec{f}(t)$$

$$\Delta\vec{v} \neq -\frac{\zeta}{m}\vec{v}_i\Delta t + \frac{1}{m}\vec{f}(t)\Delta t$$

$$\because \quad \int_0^t dt'\vec{f}(t) \neq \vec{f}(t)\Delta t$$

$$\int_0^t dt'\vec{f}(t) = \Delta\vec{W}_i$$

$$\therefore \quad \Delta\vec{v} = -\frac{\zeta}{m}\vec{v}_i\Delta t + \frac{1}{m}\Delta\vec{W}_i$$

$$\vec{v}_{i+1} = \vec{v}_i - \frac{\zeta}{m}\vec{v}_i\Delta t + \frac{1}{m}\Delta\vec{W}_i$$

$$\vec{v}_{i+1} = \left(1 - \frac{\zeta}{m}\Delta t\right)\vec{v}_i + \frac{1}{m}\Delta\vec{W}_i$$

Proceso Wiener

$$\langle \Delta \vec{W}_i \rangle = 0 \quad \rightarrow \quad \mu = 0$$

$$\langle \Delta \vec{W}_i \Delta \vec{W}_j \rangle = 2\zeta k_B T \delta_{ij} \Delta t \quad \rightarrow \quad \sigma^2 = 2\zeta k_B T \Delta t$$

$$\therefore \quad \vec{v}_{i+1} = \left(1 - \frac{\zeta}{m}\Delta t\right) \vec{v}_i + \frac{1}{m} N(\mu, \sigma^2)$$

$$\boxed{\vec{v}_{i+1} = \left(1 - \frac{\zeta}{m}\Delta t\right) \vec{v}_i + \frac{1}{m} N(0, 2\zeta k_B T \Delta t)}$$