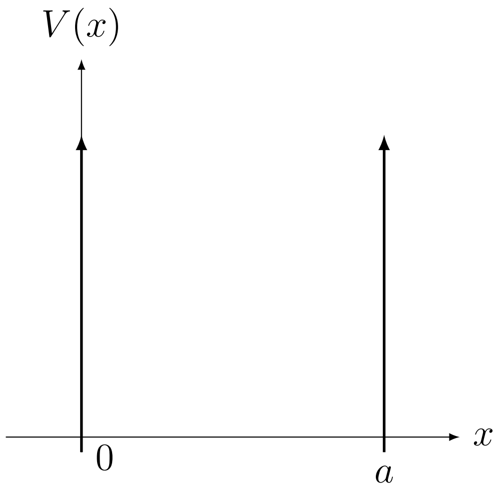
Pozo cuadrado infinito

Angel Fernando García Núñez

Potencial

$$V(x) = \begin{cases} 0 & : \ x \in (0, a) \\ \infty & : \ x \notin (0, a) \end{cases}$$



Ecuación de Schrödinger Independiente del tiempo

$$\hat{H}\psi = E\psi \quad \rightarrow \quad -\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V(x)\psi(x) = E\psi(x)$$

Regiones I $(x \leq 0)$ y III $(x \geq a)$:

$$V(x) = \infty$$

Función de onda que soluciona la ecuación de Schrödinger

$$\psi_I(x) = \psi_{III}(x) = 0$$

$$-\frac{\hbar^2}{2m}\frac{d^2\psi_I}{dx^2} + V(x)\psi_I(x) = E\psi_I(x) \quad \rightarrow \quad -\frac{\hbar^2}{2m}\frac{d^2(0)}{dx^2} + V(x)(0)(x) = E(0)(x)$$

$$\boxed{0 = 0}$$

$$-\frac{\hbar^2}{2m}\frac{d^2\psi_{III}}{dx^2} + V(x)\psi_{III}(x) = E\psi_{III}(x) \quad \rightarrow \quad -\frac{\hbar^2}{2m}\frac{d^2(0)}{dx^2} + V(x)(0)(x) = E(0)(x)$$

$$\boxed{0 = 0}$$

Región II $(x \in (0,a))$:

$$V(x) = 0$$

$$-\frac{\hbar^2}{2m}\frac{d^2\psi_{II}}{dx^2} = E\psi_{II}(x)$$

$$\frac{d^2\psi_{II}}{dx^2} + k^2\psi_{II}(x) = 0 \quad : \quad k^2 = \frac{2mE}{\hbar^2}$$

Solución:

$$\psi_{II}(x) = A\cos kx + B\sin kx$$

Condiciones de frontera por continuidad

$$\psi_I(0) = \psi_{II}(0) \quad , \quad \psi_{II}(a) = \psi_{III}(a) \quad \rightarrow \quad \psi_{II}(0) = \psi_{II}(a) = 0$$

$$\psi_{II}(0) = A\cos k(0) + B\sin k(0) = 0 \quad \rightarrow \quad A(1) + B(0) = 0 \quad \rightarrow \quad A = 0$$

$$\therefore \quad \psi_{II}(x) = B\sin kx$$

$$\psi_{II}(a) = B\sin k(a) = 0 \quad \rightarrow \quad B\sin ka = 0$$

$$B \neq 0 \quad \rightarrow \quad \sin ka = 0 \quad \rightarrow \quad ka = n\pi$$

$$\boxed{\therefore \quad k_n = \frac{n\pi}{a}}$$

Energía

$$k_n^2 = \frac{n^2\pi^2}{a^2} = \frac{2mE_n}{\hbar^2} \quad \rightarrow \quad \boxed{E_n = \frac{n^2\pi^2\hbar^2}{2ma^2}}$$

Normalización

$$\int_{-\infty}^{\infty} |\psi(x)|^2 dx = 1$$

$$\int_{-\infty}^{\infty} |\psi(x)|^2 dx = B^2 \int_0^a \sin^2 k_n x dx = 1$$

$$\cos(a+b) = \cos a \cos b - \sin a \sin b$$

$$\cos 2x = \cos^2 x - \sin^2 x = (1 - \sin^2 x) - \sin^2 x = 1 - 2\sin^2 x$$

$$\sin^2 x = \frac{1}{2} - \frac{1}{2}\cos 2x$$

$$\therefore \quad \int_{-\infty}^{\infty} |\psi(x)|^2 dx = B^2 \int_0^a \left(\frac{1}{2} - \frac{1}{2}\cos 2k_n x\right) dx = 1$$

$$B^2 \int_0^a \left(\frac{1}{2} - \frac{1}{2}\cos 2k_n x\right) dx = B^2 \left(\frac{1}{2}a - \frac{1}{2}\int_0^a \cos 2k_n x dx\right) = 1$$

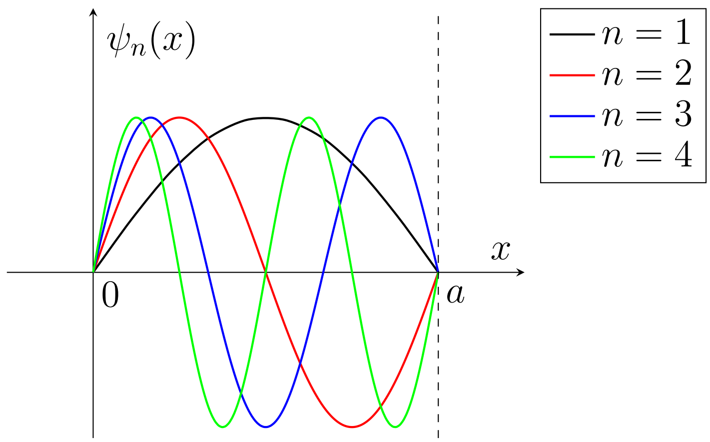
$$B^2 \left(\frac{1}{2}a + \frac{1}{4}\left|\sin 2k_n x\right|_0^a\right) = B^2 \left(\frac{1}{2}a + \frac{1}{4}\sin 2k_n a\right) = B^2 \left(\frac{1}{2}a + \frac{1}{4}\cancelto{0}{\sin 2n\pi}\right) = 1$$

$$\frac{1}{2}aB^2 = 1 \quad \rightarrow \quad \boxed{B = \sqrt{\frac{2}{a}}}$$

Función de onda y energía del sistema

$$\psi_n(x) = \begin{cases} \sqrt{\frac{2}{a}} \sin \frac{n\pi}{a} x & : \quad x \in (0,a) \\ 0 & : \quad x \notin (0,a) \end{cases} \quad , \quad E_n = \frac{n^2 \pi^2 \hbar^2}{2ma^2}$$

Función de onda



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Densidad de probabilidad