Introduction to Electrodynamics

Fourth Edition

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Solucionario

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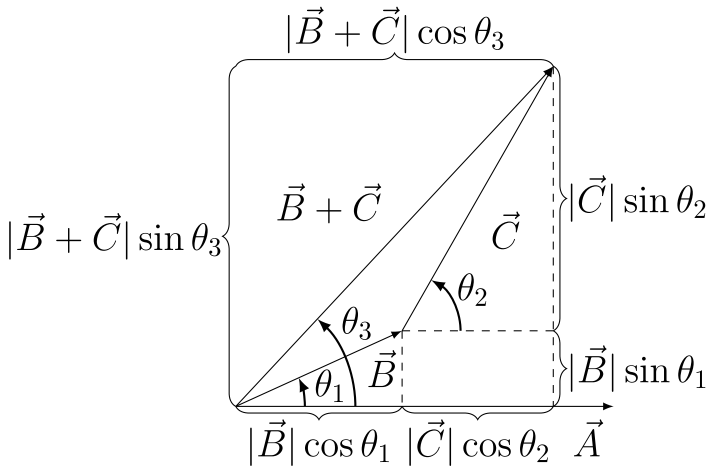
Problem 1.1 Using the definitions in Eqs. 1.1 and 1.4, and appropriate diagrams, show that the dot product and cross product are distributive,

a) when the three vectors are coplanar;

b) in the general case.

Vectores coplanares



Como podemos ver en el diagrama, cuando los vectores $\vec{A}$, $\vec{B}$ y $\vec{C}$ son coplanares se mantienen las siguientes relaciones:

$$|\vec{B}|\cos\theta_1 + |\vec{C}|\cos\theta_2 = |\vec{B}+\vec{C}|\cos\theta_3$$
$$|\vec{B}|\sin\theta_1 + |\vec{C}|\sin\theta_2 = |\vec{B}+\vec{C}|\sin\theta_3$$

Ecuación 1.1

$$\mathbf{A}\cdot\mathbf{B} \equiv AB\cos\theta$$

Ecuación 1.4

$$\mathbf{A}\times\mathbf{B} \equiv AB\sin\theta\hat{\mathbf{n}}$$

Propiedad distributiva en el producto punto

$$\vec{A}\cdot(\vec{B}+\vec{C}) = \vec{A}\cdot\vec{B}+\vec{A}\cdot\vec{C} \quad ?$$

$$\vec{A}\cdot(\vec{B}+\vec{C}) = \vec{A}\cdot\vec{B}+\vec{A}\cdot\vec{C} \quad \rightarrow \quad |\vec{A}||\vec{B}+\vec{C}|\cos\theta_3 = |\vec{A}||\vec{B}|\cos\theta_1+|\vec{A}||\vec{C}|\cos\theta_2$$

$$A = |\vec{A}|$$

$$A|\vec{B}+\vec{C}|\cos\theta_3 = A|\vec{B}|\cos\theta_1 + A|\vec{C}|\cos\theta_2 = A\left(|\vec{B}|\cos\theta_1 + |\vec{C}|\cos\theta_2\right)$$

$$A|\vec{B}+\vec{C}|\cos\theta_3 = A|\vec{B}+\vec{C}|\cos\theta_3$$

$$\boxed{\text{1.1 a)} \quad \vec{A}\cdot(\vec{B}+\vec{C}) = \vec{A}\cdot\vec{B}+\vec{A}\cdot\vec{C} \quad \blacksquare}$$

Propiedad distributiva en el producto vectorial

$$\vec{A} \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C} \quad ?$$

$$\vec{A}\times(\vec{B}+\vec{C}) = \vec{A}\times\vec{B}+\vec{A}\times\vec{C} \quad \rightarrow \quad |\vec{A}||\vec{B}+\vec{C}|\cos\theta_3\hat{n} = |\vec{A}||\vec{B}|\cos\theta_1\hat{n}+|\vec{A}||\vec{C}|\sin\theta_2\hat{n}$$

$$A = |\vec{A}|$$

$$A|\vec{B}+\vec{C}|\sin\theta_3\hat{n} = A|\vec{B}|\sin\theta_1\hat{n} + A|\vec{C}|\sin\theta_2\hat{n} = A\left(|\vec{B}|\sin\theta_1 + |\vec{C}|\sin\theta_2\right)\hat{n}$$

$$A|\vec{B}+\vec{C}|\sin\theta_3\hat{n} = A|\vec{B}+\vec{C}|\sin\theta_3\hat{n}$$

$$\boxed{\text{1.1 a)} \quad \vec{A} \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C} \quad \blacksquare}$$

Caso general

$$\vec{A} = a_x\hat{\imath} + a_y\hat{\jmath} + a_z\hat{k} \quad , \quad \vec{B} = b_x\hat{\imath} + b_y\hat{\jmath} + b_z\hat{k} \quad , \quad \vec{C} = c_x\hat{\imath} + c_y\hat{\jmath} + c_z\hat{k}$$

$$\vec{B} + \vec{C} = (b_x + c_x)\hat{\imath} + (b_y + c_y)\hat{\jmath} + (b_z + c_z)\hat{k}$$

Propiedad distributiva en el producto punto

$$\boxed{\vec{A} \cdot (\vec{B} + \vec{C}) = a_x(b_x + c_x) + a_y(b_y + c_y) + a_z(b_z + c_z)}$$

$$\vec{A} \cdot \vec{B} = a_x b_x + a_y b_y + a_z b_z \quad , \quad \vec{A} \cdot \vec{C} = a_x c_x + a_y c_y + a_z c_z$$

$$\boxed{\vec{A} \cdot \vec{B} + \vec{A} \cdot \vec{C} = a_x(b_x + c_x) + a_y(b_y + c_y) + a_z(b_z + c_z)}$$

$$\boxed{\text{1.1 b)} \quad \vec{A} \cdot (\vec{B} + \vec{C}) = \vec{A} \cdot \vec{B} + \vec{A} \cdot \vec{C} \quad \blacksquare}$$

Propiedad distributiva en el producto vectorial

$$\vec{A}\times(\vec{B}+\vec{C}) = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ a_x & a_y & a_z \\ (b_x+c_x) & (b_y+c_y) & (b_z+c_z) \end{vmatrix}$$

$$\boxed{\begin{gathered}\vec{A}\times(\vec{B}+\vec{C}) = \\ [a_y(b_z+c_z)-a_z(b_y+c_y)]\hat{\imath}+[a_z(b_x+c_x)-a_x(b_z+c_z)]\hat{\jmath}+[a_x(b_y+c_y)-a_y(b_x+c_x)]\hat{k}\end{gathered}}$$

.

$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix} = \hat{\imath}(a_yb_z-a_zb_y)-\hat{\jmath}(a_xb_z-a_zb_x)+\hat{k}(a_xb_y-a_yb_x)$$

$$\boxed{\vec{A}\times\vec{B} = (a_yb_z-a_zb_y)\hat{\imath}+(a_zb_x-a_xb_z)\hat{\jmath}+(a_xb_y-a_yb_x)\hat{k}}$$

$$\vec{A}\times\vec{C} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ a_x & a_y & a_z \\ c_x & c_y & c_z \end{vmatrix} = \hat{\imath}(a_yc_z-a_zc_y)-\hat{\jmath}(a_xc_z-a_zc_x)+\hat{k}(a_xc_y-a_yc_x)$$

$$\boxed{\vec{A}\times\vec{C} = (a_yc_z-a_zc_y)\hat{\imath}+(a_zc_x-a_xc_z)\hat{\jmath}+(a_xc_y-a_yc_x)\hat{k}}$$

$$\boxed{\begin{gathered}\vec{A}\times\vec{B}+\vec{A}\times\vec{C} = \\ [a_y(b_z+c_z)-a_z(b_y+c_y)]\hat{\imath}+[a_z(b_x+c_x)-a_x(b_z+c_z)]\hat{\jmath}+[a_x(b_y+c_y)-a_y(b_x+c_x)]\hat{k}\end{gathered}}$$

.

$$\boxed{\text{1.1 b)} \quad \vec{A}\times(\vec{B}+\vec{C}) = \vec{A}\times\vec{B}+\vec{A}\times\vec{C} \quad \blacksquare}$$

Problem 1.2 Is the cross product associative?

$$(\mathbf{A} \times \mathbf{B}) \times \mathbf{C} \stackrel{?}{=} \mathbf{A} \times (\mathbf{B} \times \mathbf{C}).$$

If so, *prove* it; if not, provide a counterexample (the simpler the better).

$$(\vec{A}\times\vec{B})\times\vec{C} = \vec{A}\times(\vec{B}\times\vec{C}) \quad ?$$

$$\vec{A} = a_x\hat{\imath} + a_y\hat{\jmath} + a_z\hat{k} \quad , \quad \vec{B} = b_x\hat{\imath} + b_y\hat{\jmath} + b_z\hat{k} \quad , \quad \vec{C} = c_x\hat{\imath} + c_y\hat{\jmath} + c_z\hat{k}$$

$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix} = \hat{\imath}(a_yb_z - a_zb_y) - \hat{\jmath}(a_xb_z - a_zb_x) + \hat{k}(a_xb_y - a_yb_x)$$

$$\boxed{\vec{A}\times\vec{B} = (a_yb_z - a_zb_y)\hat{\imath} + (a_zb_x - a_xb_z)\hat{\jmath} + (a_xb_y - a_yb_x)\hat{k}}$$

$$(\vec{A}\times\vec{B})\times\vec{C} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ (a_yb_z - a_zb_y) & (a_zb_x - a_xb_z) & (a_xb_y - a_yb_x) \\ c_x & c_y & c_z \end{vmatrix}$$

$$\begin{gathered} (\vec{A}\times\vec{B})\times\vec{C} = \\ \hat{\imath}[(a_zb_x - a_xb_z)c_z - (a_xb_y - a_yb_x)c_y] \\ -\hat{\jmath}[(a_yb_z - a_zb_y)c_z - (a_xb_y - a_yb_x)c_x] \\ +\hat{k}[(a_yb_z - a_zb_y)c_y - (a_zb_x - a_xb_z)c_x] \end{gathered}$$

$$\boxed{\begin{gathered} (\vec{A}\times\vec{B})\times\vec{C} = \\ [(a_zb_x - a_xb_z)c_z - (a_xb_y - a_yb_x)c_y]\hat{\imath} \\ +[(a_xb_y - a_yb_x)c_x - (a_yb_z - a_zb_y)c_z]\hat{\jmath} \\ +[(a_yb_z - a_zb_y)c_y - (a_zb_x - a_xb_z)c_x]\hat{k} \end{gathered}}$$

$$\vec{B} \times \vec{C} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ b_x & b_y & b_z \\ c_x & c_y & c_z \end{vmatrix} = \hat{\imath}(b_y c_z - b_z c_y) - \hat{\jmath}(b_x c_z - b_z c_x) + \hat{k}(b_x c_y - b_y c_x)$$

$$\boxed{\vec{B} \times \vec{C} = (b_y c_z - b_z c_y)\hat{\imath} + (b_z c_x - b_x c_z)\hat{\jmath} + (b_x c_y - b_y c_x)\hat{k}}$$

$$\vec{A} \times (\vec{B} \times \vec{C}) = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ a_x & a_y & a_z \\ (b_y c_z - b_z c_y) & (b_z c_x - b_x c_z) & (b_x c_y - b_y c_x) \end{vmatrix}$$

$$\begin{gathered} \vec{A} \times (\vec{B} \times \vec{C}) = \\ \hat{\imath}[(b_x c_y - b_y c_x)a_y - (b_z c_x - b_x c_z)a_z] \\ -\hat{\jmath}[(b_x c_y - b_y c_x)a_x - (b_y c_z - b_z c_y)a_z] \\ +\hat{k}[(b_z c_x - b_x c_z)a_x - (b_y c_z - b_z c_y)a_y] \end{gathered}$$

$$\boxed{\begin{gathered} \vec{A} \times (\vec{B} \times \vec{C}) = \\ [(b_x c_y - b_y c_x)a_y - (b_z c_x - b_x c_z)a_z]\hat{\imath} \\ +[(b_y c_z - b_z c_y)a_z - (b_x c_y - b_y c_x)a_x]\hat{\jmath} \\ +[(b_z c_x - b_x c_z)a_x - (b_y c_z - b_z c_y)a_y]\hat{k} \end{gathered}}$$

$$(a_zb_x - a_xb_z)c_z - (a_xb_y - a_yb_x)c_y \neq (b_xc_y - b_yc_x)a_y - (b_zc_x - b_xc_z)a_z$$
$$(a_xb_y - a_yb_x)c_x - (a_yb_z - a_zb_y)c_z \neq (b_yc_z - b_zc_y)a_z - (b_xc_y - b_yc_x)a_x$$
$$(a_yb_z - a_zb_y)c_y - (a_zb_x - a_xb_z)c_x \neq (b_zc_x - b_xc_z)a_x - (b_yc_z - b_zc_y)a_y$$

$$\therefore \quad (\vec{A} \times \vec{B}) \times \vec{C} \neq \vec{A} \times (\vec{B} \times \vec{C})$$

1.2

$$(\vec{A} \times \vec{B}) \times \vec{C} \neq \vec{A} \times (\vec{B} \times \vec{C}) \quad \blacksquare$$

El producto vectorial es no asociativo.

Problem 1.3 Find the angle between the body diagonals of a cube.

Supondremos un cubo con arista de longitud a, con vertices en $(0,0,0)$, $(a,0,0)$, $(0,a,0)$, $(0,0,a)$, $(a,a,0)$, $(0,a,a)$, $(a,0,a)$, y (a,a,a), siendo las diagonales aquellos vectores $\vec{A}$ de $(0,0,0)$ a (a,a,a), y $\vec{B}$ de $(0,0,a)$ a $(a,a,0)$.

Vectores posicionados en el origen

$$\vec{A} = a\hat{\imath} + a\hat{\jmath} + a\hat{k} \quad , \quad \vec{B} = a\hat{\imath} + a\hat{\jmath} - a\hat{k}$$

Magnitudes

$$A = |\vec{A}| = \sqrt{a^2 + a^2 + a^2} = \sqrt{3}a$$

$$B = |\vec{B}| = \sqrt{a^2 + a^2 + (-a)^2} = \sqrt{3}a$$

Producto punto

$$\vec{A} \cdot \vec{B} = a^2 + a^2 - a^2 = a^2$$

$$\vec{A} \cdot \vec{B} = AB\cos\theta \quad \rightarrow \quad \theta = \text{acos}\left(\frac{\vec{A} \cdot \vec{B}}{AB}\right)$$

$$\theta = \text{acos}\left(\frac{a^2}{(\sqrt{3}a)(\sqrt{3}a)}\right) = \text{acos}\left(\frac{1}{2}\right)$$

$$\boxed{1.3 \quad \theta \approx 1.231\text{rad}}$$

Problem 1.4 Use the cross product to find the components of the unit vector $\hat{\mathbf{n}}$ perpendicular to the shaded plane in Fig 1.11.

En el plano de la figura 1.11 están ubicados los vertices $(1,0,0)$, $(0,2,0)$, y $(0,0,3)$, por lo que tomaremos los vectores $\vec{A}$ de $(1,0,0)$ a $(0,2,0)$, y $\vec{B}$ de $(1,0,0)$ a $(0,0,3)$.

Vectores posicionados en el origen

$$\vec{A} = -1\hat{\imath} + 2\hat{\jmath} + 0\hat{k} \quad , \quad \vec{B} = -1\hat{\imath} + 0\hat{\jmath} + 3\hat{k}$$

Producto vectorial - vector normal

$$\vec{n} = \vec{A} \times \vec{B}$$

$$\vec{n} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -1 & 2 & 0 \\ -1 & 0 & 3 \end{vmatrix} = [2(3) - 0]\hat{\imath} - [-1(3) - 0]\hat{\jmath} + [0 - 2(-1)]\hat{k}$$

$$\vec{n} = 6\hat{\imath} + 3\hat{\jmath} + 2\hat{k}$$

$$n = |\vec{n}| = \sqrt{6^2 + 3^2 + 2^2} = 7$$

Vector normal unitario

$$\hat{n} = \frac{\vec{n}}{n} = \frac{1}{7}(6\hat{\imath} + 3\hat{\jmath} + 2\hat{k})$$

$$\boxed{1.4 \quad \hat{n} = \frac{6}{7}\hat{\imath} + \frac{3}{7}\hat{\jmath} + \frac{2}{7}\hat{k}}$$

Problem 1.5 Prove the **BAC-CAB** rule by writing out both sides in component form.

$$\vec{A} \times (\vec{B} \times \vec{C}) = \vec{B}(\vec{A} \cdot \vec{C}) - \vec{C}(\vec{A} \cdot \vec{B}) \quad ?$$

$$\vec{A} \times (\vec{B} \times \vec{C}) =$$
$$[(b_x c_y - b_y c_x)a_y - (b_z c_x - b_x c_z)a_z]\hat{\imath}$$
$$+[(b_y c_z - b_z c_y)a_z - (b_x c_y - b_y c_x)a_x]\hat{\jmath}$$
$$+[(b_z c_x - b_x c_z)a_x - (b_y c_z - b_z c_y)a_y]\hat{k}$$

$$\vec{B}(\vec{A} \cdot \vec{C}) = (a_x c_x + a_y c_y + a_z c_z)(b_x \hat{\imath} + b_y \hat{\jmath} + b_z \hat{k})$$

$$\vec{C}(\vec{A} \cdot \vec{B}) = (a_x b_x + a_y b_y + a_z b_z)(c_x \hat{\imath} + c_y \hat{\jmath} + c_z \hat{k})$$

$$\vec{B}(\vec{A} \cdot \vec{C}) - \vec{C}(\vec{A} \cdot \vec{B}) =$$
$$[(a_y c_y + a_z c_z)b_x - (a_y b_y + a_z b_z)c_x]\hat{\imath}$$
$$+[(a_x c_x + a_z c_z)b_y - (a_x b_x + a_z b_z)c_y]\hat{\jmath}$$
$$+[(a_x c_x + a_y c_y)b_z - (a_x b_x + a_y b_y)c_z]\hat{k} =$$

$$(a_y c_y b_x + a_z c_z b_x - a_y b_y c_x - a_z b_z c_x)\hat{\imath}$$
$$+(a_x c_x b_y + a_z c_z b_y - a_x b_x c_y - a_z b_z c_y)\hat{\jmath}$$
$$+(a_x c_x b_z + a_y c_y b_z - a_x b_x c_z - a_y b_y c_z)\hat{k}$$

$$\vec{B}(\vec{A} \cdot \vec{C}) - \vec{C}(\vec{A} \cdot \vec{B}) =$$
$$[(b_x c_y - b_y c_x)a_y - (b_z c_x - b_x c_z)a_z]\hat{\imath}$$
$$+[(b_y c_z - b_z c_y)a_z - (b_x c_y - b_y c_x)a_x]\hat{\jmath}$$
$$+[(b_z c_x - b_x c_z)a_x - (b_y c_z - b_z c_y)a_y]\hat{k}$$

$$\boxed{1.5 \quad \vec{A} \times (\vec{B} \times \vec{C}) = \vec{B}(\vec{A} \cdot \vec{C}) - \vec{C}(\vec{A} \cdot \vec{B}) \quad \blacksquare}$$

Problem 1.6 Prove that

$$[\mathbf{A} \times (\mathbf{B} \times \mathbf{C})] + [\mathbf{B} \times (\mathbf{C} \times \mathbf{A})] + [\mathbf{C} \times (\mathbf{A} \times \mathbf{B})] = \mathbf{0}$$

Under what conditions does $\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = (\mathbf{A} \times \mathbf{B}) \times \mathbf{C}$?

$$[\vec{A} \times (\vec{B} \times \vec{C})] + [\vec{B} \times (\vec{C} \times \vec{A})] + [\vec{C} \times (\vec{A} \times \vec{B})] = 0 \quad ?$$

$$\vec{A} \times (\vec{B} \times \vec{C}) = \vec{B}(\vec{A} \cdot \vec{C}) - \vec{C}(\vec{A} \cdot \vec{B})$$

$$\vec{B} \times (\vec{C} \times \vec{A}) = \vec{C}(\vec{B} \cdot \vec{A}) - \vec{A}(\vec{B} \cdot \vec{C})$$

$$\vec{C} \times (\vec{A} \times \vec{B}) = \vec{A}(\vec{C} \cdot \vec{B}) - \vec{B}(\vec{C} \cdot \vec{A})$$

$$\begin{gathered}
\vec{A} \times (\vec{B} \times \vec{C}) + \vec{B} \times (\vec{C} \times \vec{A}) + \vec{C} \times (\vec{A} \times \vec{B}) = \\
\vec{B}(\vec{A} \cdot \vec{C}) - \vec{C}(\vec{A} \cdot \vec{B}) + \vec{C}(\vec{B} \cdot \vec{A}) - \vec{A}(\vec{B} \cdot \vec{C}) + \vec{A}(\vec{C} \cdot \vec{B}) - \vec{B}(\vec{C} \cdot \vec{A}) = \\
\vec{B}(\vec{A} \cdot \vec{C}) - \vec{C}(\vec{A} \cdot \vec{B}) + \vec{C}(\vec{A} \cdot \vec{B}) - \vec{A}(\vec{B} \cdot \vec{C}) + \vec{A}(\vec{B} \cdot \vec{C}) - \vec{B}(\vec{A} \cdot \vec{C})
\end{gathered}$$

$$\boxed{1.6 \quad [\vec{A} \times (\vec{B} \times \vec{C})] + [\vec{B} \times (\vec{C} \times \vec{A})] + [\vec{C} \times (\vec{A} \times \vec{B})] = 0 \quad \blacksquare}$$

$$\vec{A} \times (\vec{B} \times \vec{C}) = (\vec{A} \times \vec{B}) \times \vec{C} \quad ?$$

$$\vec{A} \times (\vec{B} \times \vec{C}) = \vec{B}(\vec{A} \cdot \vec{C}) - \vec{C}(\vec{A} \cdot \vec{B})$$

$$(\vec{A} \times \vec{B}) \times \vec{C} = -\vec{C} \times (\vec{A} \times \vec{B}) = \vec{B}(\vec{C} \cdot \vec{A}) - \vec{A}(\vec{C} \cdot \vec{B})$$

$$\boxed{\vec{A} \cdot \vec{B} \neq \vec{C} \cdot \vec{B} \neq 0}$$

$$\vec{C}(\vec{A} \cdot \vec{B}) = \vec{A}(\vec{C} \cdot \vec{B}) \quad \rightarrow \quad \vec{A} \parallel \vec{C} \quad \rightarrow \quad \vec{A} \cdot \vec{C} = |\vec{A}||\vec{C}|$$

1.6

$$(\vec{A} \times \vec{B}) \times \vec{C} = \vec{A} \times (\vec{B} \times \vec{C}) \quad : \quad \vec{A} \cdot \vec{B} = \vec{C} \cdot \vec{B} = 0 \quad \cup \quad \vec{A} \cdot \vec{C} = \pm|\vec{A}||\vec{C}|$$

La propiedad se cumple si $\vec{B}$ es perpendicular con $\vec{A}$ y $\vec{C}$, o que $\vec{A}$ y $\vec{C}$ sean paralelos.

Problem 1.7 Find the separation vector $\boldsymbol{\mathscr{r}}$ from the source point $(2, 8, 7)$ to the field point $(4, 6, 8)$. Determine its magnitude ($\mathscr{r}$), and construct the unit vector $\hat{\boldsymbol{\mathscr{r}}}$.

Vector de separación

$$\vec{\mathscr{r}} = \vec{r} - \vec{r}'$$

$$\vec{r} = (4,6,8) \quad , \quad \vec{r}' = (2,8,7) \quad \rightarrow \quad \vec{\mathscr{r}} = (2,-2,1)$$

$$\mathscr{r} = |\ \vec{\mathscr{r}}\ | \quad \rightarrow \quad \mathscr{r} = \sqrt{2^2 + (-2)^2 + 1^2} = 3$$

$$\hat{\mathscr{r}} = \frac{\vec{\mathscr{r}}}{\mathscr{r}} = \frac{1}{3}(2,-2,1)$$

$$\boxed{1.7 \quad \hat{\mathscr{r}} = \left(\frac{2}{3}, -\frac{2}{3}, \frac{1}{3}\right)}$$

Problem 1.8

(a) Prove that the two-dimensional rotation matrix (Eq. 1.29) preserves dot products. (That is, show that $\overline{A}_y\overline{B}_y + \overline{A}_z\overline{B}_z = A_yB_y + A_zB_z$.)

(b) What constraints must the element (R_{ij}) of the three-dimensional rotation matrix (Eq. 1.30) satisfy, in order to preserve the length of $\mathbf{A}$ (for all vectors $\mathbf{A}$)?

Ecuación 1.29

$$\begin{pmatrix} \overline{A}_x \\ \overline{A}_z \end{pmatrix} = \begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} A_x \\ A_z \end{pmatrix}$$

$$P_\phi = \begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix}$$

$$\vec{A}\cdot\vec{B} = \left(P_\phi\vec{A}\right)\cdot\left(P_\phi\vec{B}\right) \quad ?$$

$$\bar{A}_x = A_x\cos\phi + A_z\sin\phi \quad , \quad \bar{A}_z = -A_x\sin\phi + A_z\cos\phi$$
$$\bar{B}_x = B_x\cos\phi + B_z\sin\phi \quad , \quad \bar{B}_z = -B_x\sin\phi + B_z\cos\phi$$

$$\left(P_\phi\vec{A}\right)\cdot\left(P_\phi\vec{B}\right) = \bar{A}_x\bar{B}_x + \bar{A}_z\bar{B}_z =$$
$$(A_x\cos\phi + A_z\sin\phi)(B_x\cos\phi + B_z\sin\phi) + (-A_x\sin\phi + A_z\cos\phi)(-B_x\sin\phi + B_z\cos\phi)$$

$$\left(P_\phi\vec{A}\right)\cdot\left(P_\phi\vec{B}\right) =$$
$$A_xB_x\cos^2\phi + A_xB_z\cos\phi\sin\phi + A_zB_x\cos\phi\sin\phi + A_zB_z\sin^2\phi$$
$$A_xB_x\sin^2\phi - A_xB_z\cos\phi\sin\phi - A_zB_x\cos\phi\sin\phi + A_zB_z\cos^2\phi$$

$$\left(P_\phi\vec{A}\right)\cdot\left(P_\phi\vec{B}\right) = (A_xB_x + A_zB_z)(\cos^2\phi + \sin^2\phi) = A_xB_x + A_zB_z$$

$$\boxed{1.8\ \text{(a)} \quad \vec{A}\cdot\vec{B} = \left(P_\phi\vec{A}\right)\cdot\left(P_\phi\vec{B}\right) \quad \blacksquare}$$

Ecuación 1.30

$$\begin{pmatrix} \overline{A}_x \\ \overline{A}_y \\ \overline{A}_z \end{pmatrix} = \begin{pmatrix} R_{xx} & R_{xy} & R_{xz} \\ R_{yx} & R_{yy} & R_{yz} \\ R_{zx} & R_{zy} & R_{zz} \end{pmatrix} \begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix}$$

$$\bar{A}_i = \sum_j R_{ij} A_j$$

$$|\vec{A}|^2 = \vec{A} \cdot \vec{A} = A_x^2 + A_y^2 + A_z^2 = \bar{A}_x^2 + \bar{A}_y^2 + \bar{A}_z^2$$

$$\sum_i A_i^2 = \left(\sum_j R_{xj} A_j \right)^2 + \left(\sum_j R_{yj} A_j \right)^2 + \left(\sum_j R_{zj} A_j \right)^2$$

$$\sum_i A_i^2 = \sum_i \left(\sum_j R_{ij} A_j \right)^2 = \sum_i \left(\sum_j R_{ij} A_j \right) \left(\sum_k R_{ik} A_k \right)$$

$$\sum_i A_i^2 = \sum_i \sum_{j,k} R_{ij} R_{ik} A_j A_k = \sum_{j,k} A_j A_k \sum_i R_{ij} R_{ik}$$

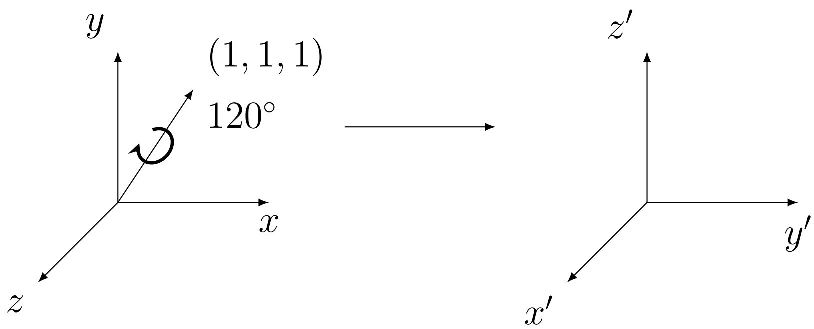
$$\boxed{\sum_i R_{ij} R_{ik} = \delta_{jk}}$$

$$\sum_i A_i^2 = \sum_{j,k} A_j A_k \delta_{jk} = \sum_i A_i A_i = \sum_i A_i^2$$

$$\boxed{\text{1.8 (b)} \quad \text{Restricción:} \quad \sum_i R_{ij} R_{ik} = \delta_{jk} = \begin{cases} 0 & : \ j \neq k \\ 1 & : \ j = k \end{cases}}$$

Problem 1.9 Find the transformation matrix R that describes a rotation by 120° about an axis from the origin through the point $(1, 1, 1)$. The rotation is clockwise as you look down the axis toward the origin.

Transformación de la matriz R



$$\boxed{\therefore \quad \bar{A}_x = A_z \quad , \quad \bar{A}_z = A_y \quad , \quad \bar{A}_y = A_x}$$

$$\bar{A}_i = \sum_j R_{ij} A_j$$

$$\bar{A}_x = R_{xx}A_x + R_{xy}A_y + R_{xz}A_z = A_z \quad \rightarrow \qquad R_{xx} = R_{xy} = 0 \quad , \quad R_{xz} = 1$$

$$\bar{A}_y = R_{yx}A_x + R_{yy}A_y + R_{yz}A_z = A_x \quad \rightarrow \qquad R_{yy} = R_{yz} = 0 \quad , \quad R_{yx} = 1$$

$$\bar{A}_z = R_{zx}A_x + R_{zy}A_y + R_{zz}A_z = A_y \quad \rightarrow \qquad R_{zx} = R_{zz} = 0 \quad , \quad R_{zy} = 1$$

$$\boxed{1.9 \quad R = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}}$$