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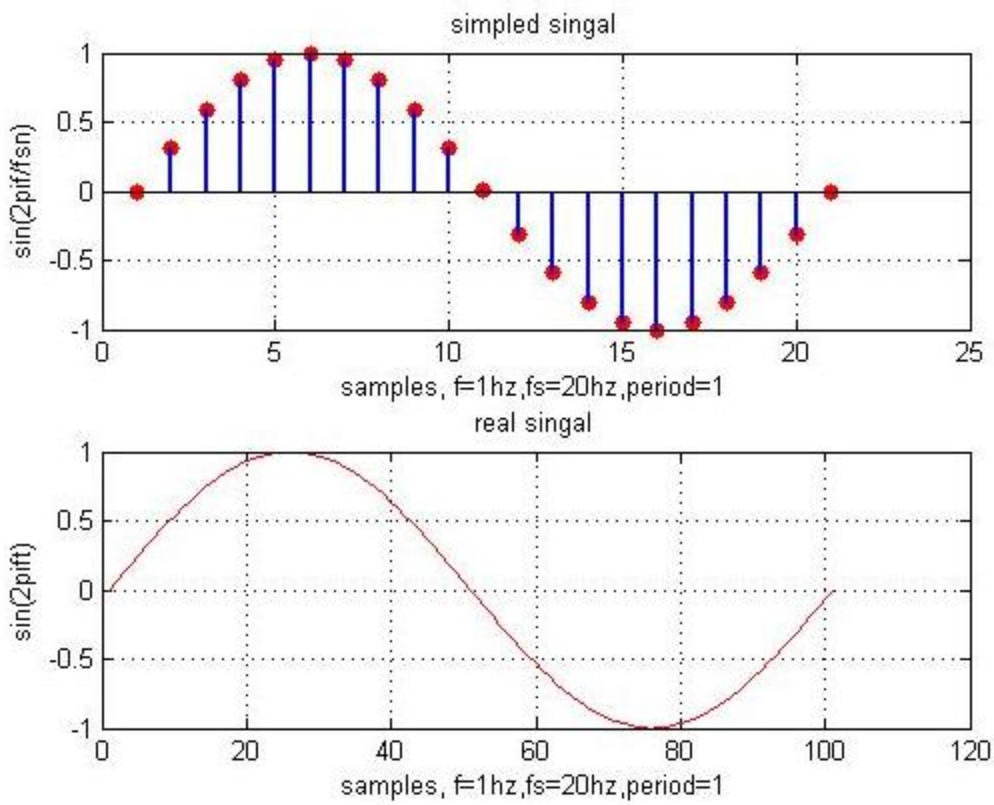
## Homework 2 DSP

- **Reminder 1** Considering a sin function $x(t) = sin(2\pi f t)$ with `f= 1Hz`, when sampled with the sampling frequency $f_s$ = 20 is equal to $x[n] = sin(2\pi \frac{f}{f_s} n)$. Plot both sin functions.

- **Exercise 1** – *Causality*

**1.1** Considering the system defined by the equation $y_k = (x_k + x_{k+1})/2$, check its causality property by examining the response to the signal $H(k-4)$ or `step(4,N)`. When plotting, include the abscissa range $[1 : N]$.

**1.2** Propose a modification to obtain a causal version and comment your observations.

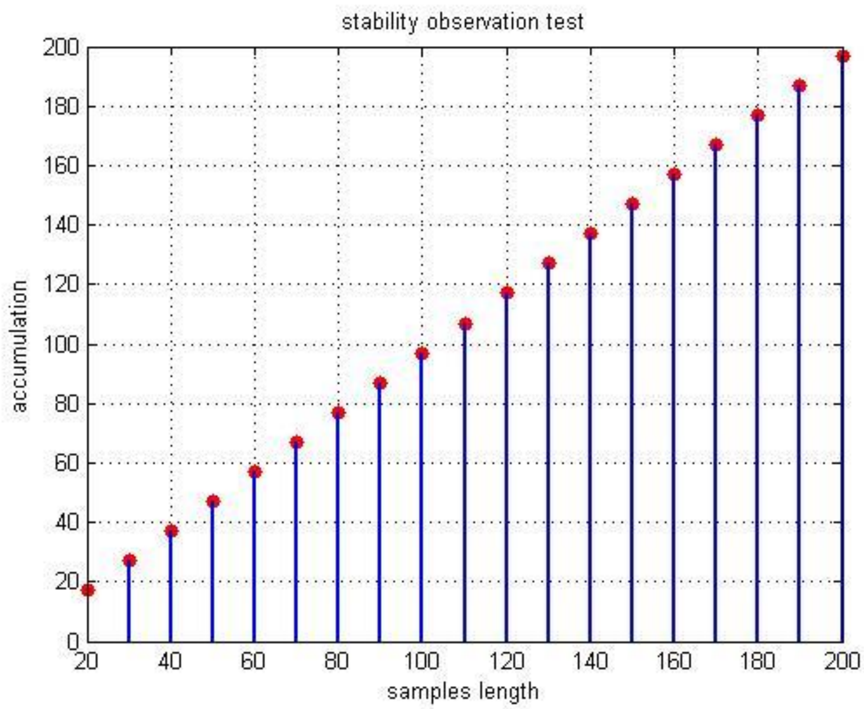



The above system is not casual because the output of the system in time k is not depends only on input in time k and before. Regardless of input shape and function, for instance

$y_k = {(x_k + x_{k+1})}/{2}, {(H(k-4) + H(k-5))}/{2} = y_{k-4}.$

Which means the output of the system at time 4 depends on time 5 which is against casualty definition. But we change the index of $x_{k-1}$then the resulting system will be casual system. Because for given example, the output of the system in instance 4 will depends on time 3 and 4 which complies with casualty definition.

**2.1** Program the primitive (accumulator) operator `prim(f)` applied on the signal `f` of length `N`. The value of the vector returned by `prim` at the index `k` will correspond to $\mathtt{F}_\mathtt{k}$ with $\mathtt{k} \leq \mathtt{N}$. Note $F_k = \sum_{q=-\infty}^{k} f_k$. Discuss on the result of the primitive operator applied to the signal $H(k-4)$. Is the primitive operator stable ?




As seen, when the number of the samples increases the output amplitude will grows linearly. For instance, in the above plot if N=40 then the primitive function will gives 40 for an input of H[k-4]. So when the number of the samples grows up to 200 this values grows 200. Which means, when the complete length of the H function is considered form –inf to +inf the output of the system will be infinite. on the other hand our input will remains bounded to 1.

The primitive operator is not stable.

**2.2** What is the impulse response of the primitive operator (in the discrete domain) ?

The inputs response of the primitive or better saying the transfer function of the system is

$$y[n] = \sum_{k=-\infty}^{n} H[k] = n + 1$$

**2.3** Test the stability of the system defined by the equation: $y_k = x_k + 2y_{k-1}$. Plot the impulse response.

Let's assume the system start from stable state or y[0]=0 and y[-1]=0. We consider a generic answer to the equation as $y = z^n$. Then we can write;

$$h[z](z^n - 2z^{n-1}) = z^{n-1} \Rightarrow h[z] = \frac{1}{z-2}$$

As seen the impulse response is bounded. Therefore, the system is stable.

**2.4** Test the stability of the system defined by the equation: $y_k = x_k + y_{k-1}/3$. Plot the impulse response.

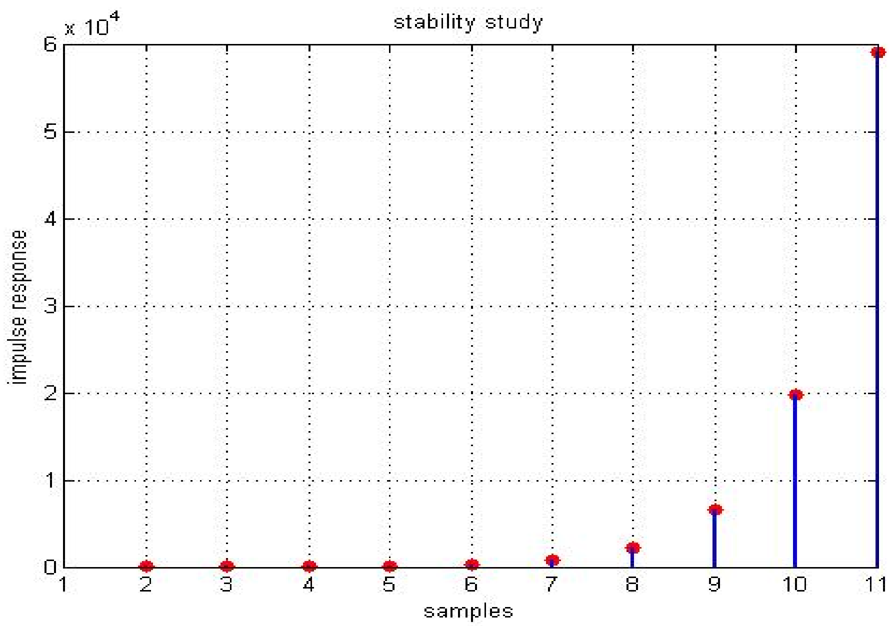
This is a first order system with following transfer function similar to those seen above.

$$h[z] = \frac{1}{z - 1/3}$$

The system has an impulse response of

$$y[n] = a * (3)^z$$

If we plot the result will be as follow;




- **Exercise 3** – *Invariance and linearity*

**3.1** Define the following signals: $\mathbf{x_a} = [0000123450000000000]$; $\mathbf{x_b} = [0000000004321000000]$;. Compute the responses $y_a$, $y_b$ according to the equation $y = 3x_{k-1} - 2x_k + x_{k+1}$

### 3.2 Prove the system defined by the previous equation is linear (and invariant).

$$\left.\begin{aligned} y_1 &= 3x_{k-1}^1 - 2x_k^1 + x_{k+1}^1 \\ y_2 &= 3x_{k-1}^2 - 2x_k^2 + x_{k+1}^2 \end{aligned}\right\} \Rightarrow 3\left(x_{k-1}^1 + x_{k-1}^2\right) - 2(x_k^1 + x_k^2) + (x_{k+1}^1 + x_{k+1}^2) = y_1 + y_2$$

Similarly can be proven that the system is scalable therefore the system is linear. If we shift the function the system will remain the response will remain the same. So the system is invariant.

### 3.3 Propose a nonlinear/noninvariant system.

$$y[n] = (x[n^2])^2$$