CSE 287: Midterm Practice Problems

1. Below are the x, y, and z coordinates of vertices **a** and **b**.

$$\mathbf{a} = \begin{bmatrix} 4 \\ -1 \\ 6 \end{bmatrix} \quad \mathbf{b} = \begin{bmatrix} 3 \\ 2 \\ 5 \end{bmatrix}$$

a. Calculate the sum of **a** and **b**

$$a + b = \begin{bmatrix} 4 \\ -1 \\ 6 \end{bmatrix} + \begin{bmatrix} 3 \\ 2 \\ 5 \end{bmatrix} = \begin{bmatrix} 4+3 \\ -1+2 \\ 6+5 \end{bmatrix} = \begin{bmatrix} 7 \\ 1 \\ 11 \end{bmatrix}$$

b. Find a vector that runs from **a** to **b**.

$$b - a = \begin{bmatrix} 3 \\ 2 \\ 5 \end{bmatrix} - \begin{bmatrix} 4 \\ -1 \\ 6 \end{bmatrix} = \begin{bmatrix} 3-4 \\ 2--1 \\ 5-6 \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \\ -1 \end{bmatrix}$$

c. Find a unit vector (has a length of one) the points from **a** to **b**.

$$\|b-a\| = \sqrt{-1^2 + 3^2 + -1^2} = \sqrt{1+9+1} = \sqrt{11} \approx 3.32$$

$$\frac{1}{3.32}\begin{bmatrix} -1 \\ 3 \\ -1 \end{bmatrix} = \begin{bmatrix} -0.3 \\ 0.9 \\ -0.3 \end{bmatrix}$$

d. Calculate the dot product of **a** and **b**

$$a \cdot b = 4(3) + -1(2) + 6(5) = 12 - 2 + 30 = 40$$

e. Find the cosine of the angle between **a** and **b**.

$$a \cdot b = \|a\|\|b\| \cos\theta = 21 \Rightarrow \cos\theta = \frac{a \cdot b}{\|a\|\|b\|} = \frac{40}{\sqrt{53}\sqrt{38}} \approx 0.891$$

$$\|a\| = \sqrt{4^2 + -1^2 + 6^2} = \sqrt{16+1+36} = \sqrt{53} = 7.28$$

$$\|b\| = \sqrt{3^2 + 2^2 + 5^2} = \sqrt{9+4+25} = \sqrt{38} = 6.16$$

$$a\cos(0.891) = 0.471$$

f. Find the angle between **a** to **b** in degrees.

$$\angle ab = 0.471\left(\frac{180}{\pi}\right) = 26.96$$

g. Find the angle between **a** to **b** in radians.

$0.471 \leftarrow$

2. Below are the x, y, and z components of the vector **v**.

$$\mathbf{v} = \begin{bmatrix} 8 \\ -2 \\ 4 \end{bmatrix}$$

   a. Find a vector that is twice as long as **v** and points in the same direction as **v**.

$$2v = 2\begin{bmatrix} 8 \\ -2 \\ 4 \end{bmatrix} = \begin{bmatrix} 2(8) \\ 2(-2) \\ 2(4) \end{bmatrix} = \begin{bmatrix} 16 \\ -4 \\ 8 \end{bmatrix}$$

   b. Find a vector that is half as long as **v** and points opposite the direction of **v**.

$$-\frac{1}{2} v = -\frac{1}{2}\begin{bmatrix} 8 \\ -2 \\ 4 \end{bmatrix} = \begin{bmatrix} -8/2 \\ -2/-2 \\ -2/4 \end{bmatrix} = \begin{bmatrix} -4 \\ 1 \\ -2 \end{bmatrix}$$

3. Below are the x, y, and z components of the vector **v**.

$$\mathbf{v} = \begin{bmatrix} 3 \\ -2 \\ -2 \end{bmatrix}$$

   a. Find the length of **v**.

$$\|v\| = \sqrt{3^2 + -2^2 + -2^2} = \sqrt{9+4+4} = \sqrt{17} \approx 4.123$$

   b. Find a vector that points in the same direction as **v** and has a length of 3.

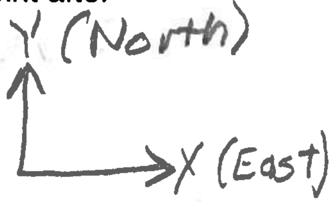
$$\frac{3}{4.123} V = \begin{bmatrix} 9/4.123 \\ -6/4.123 \\ -6/4.123 \end{bmatrix} = \begin{bmatrix} 2.183 \\ -1.123 \\ -1.123 \end{bmatrix}$$

   c. Find a points opposite the direction of **v** and has a length of 5.

$$\frac{-5}{4.123}\begin{bmatrix} 3 \\ -2 \\ -2 \end{bmatrix} = \begin{bmatrix} \frac{-5}{4.123}(3) \\ \frac{-5}{4.123}(-2) \\ \frac{-5}{4.123}(-2) \end{bmatrix} = \begin{bmatrix} \frac{-15}{4.123} \\ \frac{10}{4.123} \\ \frac{10}{4.123} \end{bmatrix} = \begin{bmatrix} -3.638 \\ 2.425 \\ 2.425 \end{bmatrix}$$

4. Use vector arithmetic to solve the following problems. Assume the positive y axis is north.

   a. Suppose someone walks three miles west, then walks two miles south, and finally walks another two miles to the east. What is their position relative to the starting point after completing the walk?



$$\begin{bmatrix}-3\\0\end{bmatrix}+\begin{bmatrix}0\\-2\end{bmatrix}+\begin{bmatrix}2\\0\end{bmatrix}=\begin{bmatrix}-3+0+2\\0+-2+0\end{bmatrix}=\begin{bmatrix}-1\\-2\end{bmatrix}$$

   b. What is their position after the walk if they start walking at a position [-5, -3].

$$\begin{bmatrix}-5\\-3\end{bmatrix}+\begin{bmatrix}-1\\-2\end{bmatrix}=\begin{bmatrix}-6\\-5\end{bmatrix}$$

5. Matching. Assume **v** and **w** are two unit length vectors.

   a. **v** and **w** are perpendicular.
   b. **v** and **w** are parallel.
   c. The angle between **v** and **w** is less than ninety degrees (π/2).
   d. The angle between **v** and **w** is greater than ninety degrees (π/2).

Dot product is equal to one, $\mathbf{v}\bullet\mathbf{w}=1$. ___b___

Dot product is positive, $\mathbf{v}\bullet\mathbf{w}>0$ ___c___

Dot product is equal to zero, $\mathbf{v}\bullet\mathbf{w}=0$ ___a___

Dot product is negative, $\mathbf{v}\bullet\mathbf{w}<0$ ___d___

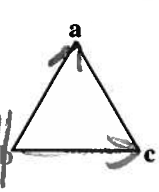
6. The vertors **a**, **b**, and **c** contain the xyz coordinates of the corners of the triangle depicted below.

   a. Write C++ code or the mathematics that you would use to find a unit normal vector that would come out of the page. If you write code, you can assume that **a**, **b**, and **c** are of type glm::vec3.

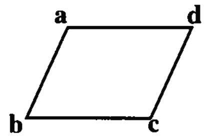
$$\frac{(c-b)\otimes(a-b)}{\|(c-b)\otimes(a-b)\|}$$

   b. Write C++ code or the mathematics that you would use to find area of the triangle.

$$\text{area}=\frac{1}{2}\|(c-b)\otimes(a-b)\|$$



7. The vertices **a**, **b**, **c**, and **d** contain the xyz coordinates of the corners of the parallelogram depicted below. Write C++ code or the mathematics that you would use to find the area of the parallelogram.



$\text{area} = \| (c-b) \otimes (a-b) \|$

8. The equation that implicitly defines a sphere is shown below. The sphere is centered on the point [3, 1, 0]. Calculate the signed distance of the point **a** = [2, 0, 1] from the surface of the sphere. Indicate whether the point is inside, on, or outside the surface of the sphere.

$$(x-3)^2 + (y-1)^2 + (z)^2 - 4 = 0$$

$(2-3)^2 + (0-1)^2 + 1^2 - 4 = -1^2 + -1^2 - 1^2 - 4 = 3 - 4 = -1$

point is inside the sphere.

~~9. The points **a** and **b** are on a LINE with an origin at the location described by a.~~

$$\cancel{\mathbf{a} = \begin{bmatrix} 4 \\ 1 \\ 6 \end{bmatrix} \quad \mathbf{b} = \begin{bmatrix} 3 \\ 2 \\ 5 \end{bmatrix} \quad \mathbf{c} = \begin{bmatrix} -1 \\ 14 \\ 1 \end{bmatrix} \quad \mathbf{d} = \begin{bmatrix} 1 \\ 8 \\ 3 \end{bmatrix}}$$

- ~~a. Create a parametric representation of the ray. The direction vector should be unit length.~~
- ~~b. Determine if the point c is on the ray.~~
- ~~c. Find the value for the parameter, *t*, that is associated with the point **d**.~~
- ~~d. Find the point that is associated with $t = 3$.~~

~~10. Identify which type of algorithm ray tracing is.~~

~~Image order~~ ~~Object order~~

~~11. Consider points of intersection with a surface that are found using the quadratic equation. How many points of intersection are there with a ray if the discriminant in the quadratic equation has the following properties?~~

- ~~a. Discriminant is greater than zero,~~ $\cancel{B^2 - 4AC > 0}$
- ~~b. Discriminant is less than zero,~~ $\cancel{B^2 - 4AC < 0}$
- ~~c. Discriminant is equal to zero,~~ $\cancel{B^2 - 4AC = 0}$

Not on exam