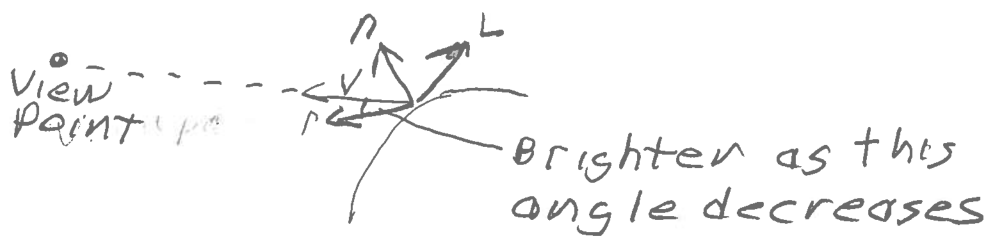
1. Top is equal to one and bottom is equal to -1. Thus, the height of the projection plane rectangle is 2. If the rendering window has an aspect ratio of 1.78. What should be width of the projection plane rectangle be set to in order to match the aspect ratio of the rendering window? (4 points)

$$\frac{W}{H} = \frac{W}{2} = 1.78 \Rightarrow W = 2(1.78) = 3.56$$

2. For a given surface and light source, describe what point on the surface you would expect specular reflection to be the brightest. Support your explanation with a diagram with labels that shows the vectors involved in the calculation. (4 points)

Specular reflection will be brightest at point were the view and reflection vectors are nearly parallel



3. What impact will decreasing the shininess exponent have on specular highlights? Why? (4 points)

Decreasing the shininess exponent will cause specular highlights to spread and become less distinct. The object will appear to be more dull.

4. Assume a viewing ray $\mathbf{e}+t\mathbf{d}$ is being traced intersects with two surfaces that are described by quadratic polynomials. Assume $\|\mathbf{d}\|=1$.The parameter for the point of intersection with surface *A* is $t=8.5$ and for surface B is $t=-4.7$.

   a. What statement can you make about the about the value of the discriminant in the quadratic formula when the parameter for the points of intersection with both surfaces was found? (2 points)

      The discriminant is greater than zero.

   b. Which surface is the closest to the view point? *A* or *B*? (2 points)

      B

   c. How far away is this closest surface? (2 points)

      4.7 units

   d. Where is surface *B* located with respect to the viewing position and direction? (3 points)

      Behind the viewpoint.

   e. What would be the parameter of the point of intersection for which local illumination be calculated and used to set the color of the pixel associated with the ray being traced? (3 points)

      $t=8.5$

   f. What is the distance to the point of intersection for which local illumination would be calculated? (3 points)

      8.5 units

   g. Write an expression that would produce XYZ coordinate of the point of intersection for which lighting calculations would be performed? (3 points)

      $e + 8.5t$

   h. Assume the local illumination for the point of intersection is held in a glm::vec4 called localLight. Write an expression that would attenuate the color held in localLight based on the distance from the view point and constant, linear, and quadratic attenuation factors. (4 points)

      $\frac{1}{K_c + 8.5K_L + 8.5^2(K_q)}$ * localLight;

5. Below are material properties for a surface, position and normal vector for a point of intersection, characteristics of a positional light source, and the direction vector of a view ray. Assume the one light source gives off ambient, diffuse, and specular light and that all vectors are unit length (they are approximately unit length). To get partial credit on calculations, show your work.

```
// Surface Material Properties
(RGB no alpha)
ambMat( 0.3, 0.4, 0.2 );
diffMat( 0.3, 0.4, 0.2 );
specMat( 0.7, 0.7, 0.7 );
emissMat( 0.0, 0.0, 0.0 );
shininess = 32.0f;
```

```
// Light properties (RGB no alpha)
ambLight(0.2, 0.2, 0.2);
diffLight( 0.6, 0.6, 0.6);
specLight( 1.0, 1.0, 1.0 );

// Points and Vectors (XYZ no homogenous
coordinate)
Intersection position ( 2.0, 2.0, 0.0 );
Intersection normal ( -0.707, 0.707, 0.0 );
View ray direction ( 0.707, 0.707, 0.0 );
Light reflection( -0.832, -0.555, 0 );
Light position ( 4.0, 5.0, 0.0 );
```

a. What is the view vector for use in lighting calculations? (4 points)

$$V = -\begin{bmatrix} 0.707 \\ 0.707 \\ 0.0 \end{bmatrix}$$

b. Calculate the light vector for use in lighting calculations. (4 points)

$$\begin{bmatrix} 4 \\ 5 \\ 0 \end{bmatrix} - \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix} \qquad \sqrt{2^2+3^2} = \sqrt{4+9} = \sqrt{13} \approx 3.61$$

$$\frac{1}{3.61}\begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.555 \\ 0.832 \\ 0 \end{bmatrix} = L$$

c. Calculate the red component of ambient reflection for the point of intersection. (3 points)

$$0.3(0.2) = 0.06$$

d. Calculate the red component of diffuse reflection for the point of intersection. (6 points)

$$n \cdot l = \begin{bmatrix} -0.707 \\ 0.707 \\ 0.0 \end{bmatrix} \cdot \begin{bmatrix} 0.555 \\ 0.832 \\ 0 \end{bmatrix} = -0.707(0.555) + 0.707(0.832) + 0(0)$$

$$= -0.277 + 0.588 = 0.865$$

$$dif = 0.865(0.3)(0.6) = 0.156$$

e. Calculate the red component of specular reflection for the point of intersection. (6 points)

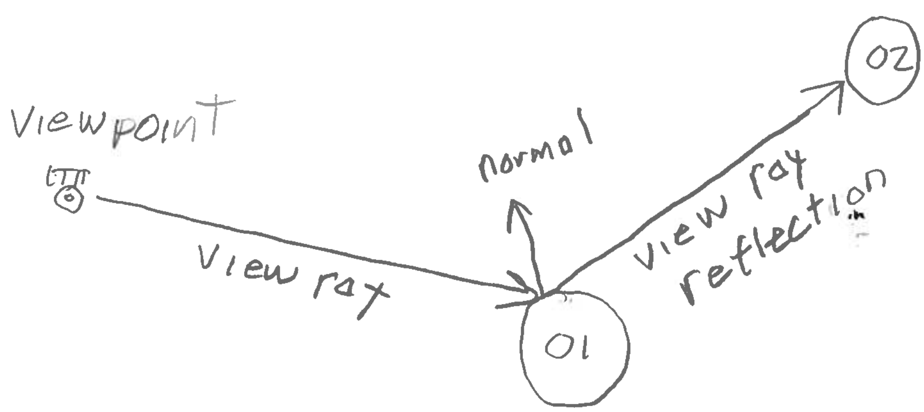
$$r \cdot v = \begin{bmatrix} -0.832 \\ -0.555 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} -0.707 \\ -0.707 \\ 0 \end{bmatrix} = 0.588 + 0.392 + 0 = 0.98$$

spec $0.98^{32}(0.7)(1.0) = 0.524(0.7) = 0.367$

f. Calculate the total red component for this light source. (4 points)

$0.0 + 0.06 + 0.156 + 0.367 = 0.583$

6. Draw a diagram that illustrates how ray tracing creates inter-object reflections. include text and labels for the vectors, surfaces, and light sources to clearly illustrate your example.(6 points)



Object one (O1) color will be a sum of its Local illumination at the point where the view ray intersects it and the local illumination at the point where the viewray reflection intersects object two (O2)

7. Correctly multiply the two matrices below. If matrices cannot be multiplied together, write "cannot be multiplied." (4 points)

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}\begin{bmatrix} 7 & 8 & 9 \\ 3 & 5 & 7 \end{bmatrix} = \begin{bmatrix} 1(7)+2(3) & 1(8)+2(5) & 1(9)+2(7) \\ 3(7)+4(3) & 3(8)+4(5) & 3(9)+4(7) \\ 5(7)+6(3) & 5(8)+6(5) & 5(9)+6(7) \end{bmatrix} = \begin{bmatrix} 13 & 18 & 23 \\ 33 & 44 & 55 \\ 53 & 70 & 87 \end{bmatrix}$$

8. Correctly multiply the matrix times the vector. If they cannot be multiplied together, write "cannot be multiplied." (4 points)

$$\begin{bmatrix} 3 & 0 & 0 & 7 \\ 0 & -4 & 0 & 4 \\ 0 & 0 & 5 & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 10 \\ 3 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 3(10)+0(3)+0(3)+7(1) \\ 0(10)+-4(3)+0(3)+4(1) \\ 0(10)+0(3)+5(3)+-2(1) \\ 0(10)+0(3)+0(3)+1(1) \end{bmatrix} = \begin{bmatrix} 37 \\ -8 \\ 13 \\ 1 \end{bmatrix}$$

9. Create the <u>transpose</u> of the matrix below." (4 points)

$$\begin{bmatrix} 4 & 9 & 7 \\ -2 & 5 & 6 \\ 3 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 4 & -2 & 3 \\ 9 & 5 & -2 \\ 7 & 6 & 1 \end{bmatrix}$$

10. In the following assume $\mathbf{A}$ and $\mathbf{B}$ are arbitrary 3 x 3 matrices, $\mathbf{I}$ is the 3 x 3 identity matrix and **a** and **b** are arbitrary vectors. (3 points each).

a. In general $\mathbf{AI} = \mathbf{A}^{-1}$ True False

b. In general $\mathbf{AA}^{-1} = \mathbf{I}$ True False

c. In general $\mathbf{AB} = \mathbf{BA}$ True False

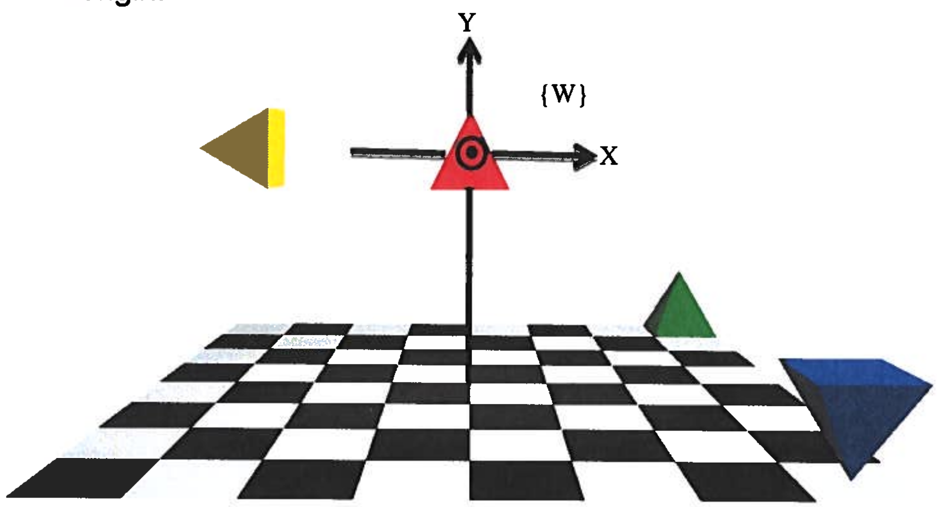
d. In general $(\mathbf{AB})^{-1} = \mathbf{A}^{-1}\mathbf{B}^{-1}$ True False

e. In general $(\mathbf{FG})^{\mathbf{T}} = \mathbf{G}^{\mathbf{T}}\mathbf{F}^{\mathbf{T}}$ True False

f. In general $\mathbf{a} \otimes \mathbf{b} = \mathbf{b} \otimes \mathbf{a}$ True False

g. In general $\mathbf{a} \bullet \mathbf{b} = \mathbf{b} \bullet \mathbf{a}$ True False

11. In the rendering below, all pyramids where rendered using the same vertices. The World coordinate origin is depicted by the bullseye in the middle of the center pyramid.. The World coordinate X axis points to the right, Y points up, and the World coordinate Z axis is coming out of the page. All of the pyramids pyramids are one unit high and one unit wide. The modeling transformation for the center pyramid is equal to the identity matrix. The checker board is eight units wide. Each square on the board is one unit square. The checker board is three units below the World coordinate origin.



a. The back right pyramid is positioned so that it exactly covers the back right square on the checker board. Write a series of C++ statements to create translation and/or rotation matrices and multiply them together to create the modeling transformation for the back right pyramid.

dmat4 trans= translate(dvec3(3.5,-2.5,-3.5));

glm::mat4 modelingTransformation = trans;

b. The tip of the front right pyramid is just touching the center of the front right square on the checker board. Write a series of C++ statements to create necessary translation and/or rotation matrices and multiply them together to create the modeling transformation for the front right pyramid.

dmat4 trans = translate(dvec3(3.5, -2.5, -3.5));
dmat4 rot = rotate(PI, dvec3(1.0, 0.0, 0.0));

could be about the x or z axis

could be positive or negative

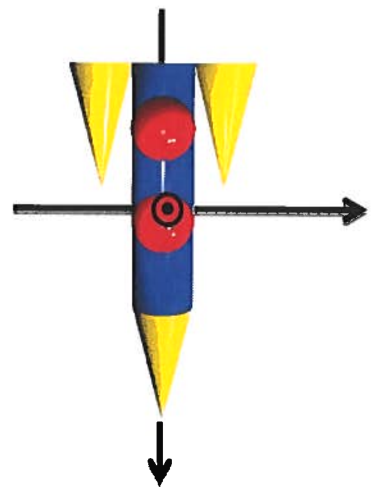
`glm::mat4 modelingTransformation =` trans * rot

c. The pyramid on the left is three units from the center pyramid in the World coordinate negative X direction. Write a series of C++ statements to create necessary translation and/or rotation matrices and multiply them together to create the modeling transformation for the pyramid on the left.

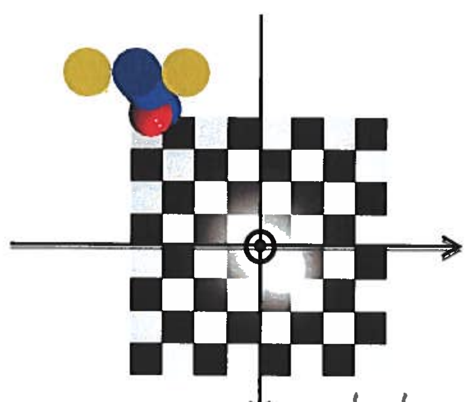
dmat4 trans = translate(dvec3(-3.0, 0.0, 0.0));
dmat4 rot = rotate(PI/2.0, dvec3(0.0, 0.0, 1.0));

`glm::mat4 modelingTransformation =` trans * rot

12. The "spaceship" model is depicted below relative to object coordinates i.e. the picture shows the position of the spaceship without any modeling transformations applied. The object coordinate Y axis is coming out of the page. The "nose" is pointing the positive Z direction. The "top" of the model is represented by the two red spheres and is oriented in the positive Y direction.



Write a series of statements to create translation and rotation matrices and multiply them together to create the composite modeling transformation that will position and orient the spaceship as depicted below. Relative to World coordinates, the nose of the spaceship is pointing in the negative Y direction.. The position of the ship is X = -2.5, Y = 3, Z = -3.5 relative to the World coordinate origin. (9 points)



dmat4 trans = translate(dvec3(-2.5, 3, -3.5));

dmat4 rot = rotate(PI/2.0, dvec3(1.0, 0.0, 0.0));

glm::mat4 modelingTransformation = trans * rot;