Recopilacion de propiedades de funciones generalizadas

(Incompleto, falta pulir, emprolijar, completar)

(Ariel Nowik: anowik@itba.edu.ar)

- ullet f:R o R continua a trozos
- $\phi \in C_0^{\infty}$
- $oldsymbol{\cdot}$ $f',f^{(n)}$ no necesariamente existen. Pero si los funcionales notados como ellas.

$$egin{aligned} \langle f,\phi
angle &= \int_{-\infty}^{\infty} f(x)\phi(x)dx \ &\langle f',\phi
angle &= -\langle f,\phi'
angle \ &\langle f'',\phi
angle &= (-1)^2\langle f,\phi''
angle \ &\langle f^{(n)},\phi
angle &= (-1)^n\langle f,\phi^{(n)}
angle \ &\langle f,\alpha_1\phi_1+lpha_2\phi_2
angle &= lpha_1\langle f,\phi_1
angle +lpha_2\langle f,\phi_2
angle \ &\langle r',\phi
angle &= \langle u,\phi
angle \ &\langle u',\phi
angle &= \phi(0) \ &\langle \delta,\phi
angle &= \phi(0) \end{aligned}$$

Sean f1,f2 funciones generalizadas

$$\langle c_1 f_1 + c_2 f_2, \phi
angle = c_1 \langle f_1, \phi
angle + c_2 \langle f_2, \phi
angle$$

Sea $g \; \epsilon \; C^{\infty}$, f generalizada

$$\langle gf,\phi
angle = \langle f,g\phi
angle \ \langle g\delta,\phi
angle = \langle g(0)\delta,\phi
angle$$

Sea f generalizada, $g \ \epsilon \ C^{\infty}$

$$\langle (gf)', \phi
angle = \langle gf' + g'f, \phi
angle$$

Integrales delta de Dirac

Sea f continua a trozos, continua en t_0

$$\int_{-\infty}^\infty \delta(t-t_0)f(t)dt=f(t_0) \ \int_{-\infty}^\infty \delta^{(n)}(t-t_0)f(t)dt=(-1)^n\int_{-\infty}^\infty \delta(t-t_0)f^{(n)}(t)dt$$

$$\int_a^b \delta(t-t_0)f(t)dt = egin{cases} f(t_o), & a < t_0 < b \ 0 & t_0 < a ee t_0 > b \ ?? & sino \end{cases}$$

Fourier

Serie trigonometrica

Sea X(t) periodica de periodo T, frecuencia f_0

$$w_n=2\pi f_0 n=w_0 n$$

$$x(t) \sim a_0 + \sum_{n=1}^{\infty} a_n cos(w_n t) + b_n sin(w_n t)$$

Donde

$$egin{cases} a_0 = 1/T \int_{t_0}^{t_0+T} x(t) dt \ a_n = 2/T \int_{t_0}^{t_0+T} x(t) cos(w_n t) dt \ b_n = 2/T \int_{t_0}^{t_0+T} x(t) sin(w_n t) dt \end{cases}$$

Parseval serie trigonometrica

$$|2|a_0|^2 + \sum_{n=1}^{\infty} |a_n|^2 + |b_n|^2 = rac{2}{T} \int_{t_0}^{t_0+T} |x(t)|^2 dt$$

Serie exponencial

$$x(t) \sim \sum_{n=-\infty}^{\infty} X_k e^{iw_n t}$$

Donde

$$X_k = rac{1}{T} \int_{t_0}^{t_0+T} x(t) e^{-iw_n t} dt ~~orall r ~ \epsilon ~ R$$

Parseval serie exponencial

$$\sum_{k=-\infty}^{\infty}|X_k|^2=rac{1}{T}\int_{to}^{to+T}|x(t)|^2dt$$