Recopilacion de propiedades de funciones generalizadas

(Incompleto, falta pulir , emprolijar, completar)

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- $f : R \to R$ continua a trozos
- $\phi \, \epsilon \, C_0^\infty$
- $f', f^{(n)}$ no necesariamente existen. Pero si los funcionales notados como ellas.

$$\langle f, \phi \rangle = \int_{-\infty}^{\infty} f(x)\phi(x)dx$$

$$\langle f', \phi \rangle = -\langle f, \phi' \rangle$$

$$\langle f'', \phi \rangle = (-1)^2 \langle f, \phi'' \rangle$$

$$\langle f^{(n)}, \phi \rangle = (-1)^n \langle f, \phi^{(n)} \rangle$$

$$\langle f, \alpha_1\phi_1 + \alpha_2\phi_2 \rangle = \alpha_1 \langle f, \phi_1 \rangle + \alpha_2 \langle f, \phi_2 \rangle$$

$$\langle r', \phi \rangle = \langle u, \phi \rangle$$

$$\langle u', \phi \rangle = \phi(0)$$

$$\langle \delta, \phi \rangle = \phi(0)$$

Sean $f1, f2$ funciones generalizadas

$\langle c_1 f_1 + c_2 f_2, \phi \rangle = c_1 \langle f_1, \phi \rangle + c_2 \langle f_2, \phi \rangle$

Sea $g \, \epsilon \, C^\infty$, f generalizada

$$\langle gf, \phi \rangle = \langle f, g\phi \rangle$$

$$\langle g\delta, \phi \rangle = \langle g(0)\delta, \phi \rangle$$

Sea f generalizada, $g \, \epsilon \, C^\infty$

$\langle (gf)', \phi \rangle = \langle gf' + g'f, \phi \rangle$

Integrales delta de Dirac

Sea f continua a trozos, continua en t_0

$$\int_{-\infty}^{\infty} \delta(t - t_0) f(t) dt = f(t_0)$$

$$\int_{-\infty}^{\infty} \delta^{(n)}(t - t_0) f(t) dt = (-1)^n \int_{-\infty}^{\infty} \delta(t - t_0) f^{(n)}(t) dt$$

$$\int_a^b \delta(t-t_0)f(t)dt = \begin{cases} f(t_o), & a < t_0 < b \\ 0 & t_0 < a \vee t_0 > b \\ ?? & sino \end{cases}$$

Fourier

Serie trigonometrica

Sea $X(t)$ periodica de periodo T, frecuencia f_0

$$w_n = 2\pi f_0 n = w_0 n$$

$$x(t) \sim a_0 + \sum_{n=1}^{\infty} a_n cos(w_n t) + b_n sin(w_n t)$$

Donde

$$\begin{cases} a_0 = 1/T \int_{t_0}^{t_0+T} x(t)dt \\ a_n = 2/T \int_{t_0}^{t_0+T} x(t)cos(w_n t)dt \\ b_n = 2/T \int_{t_0}^{t_0+T} x(t)sin(w_n t)dt \end{cases}$$

Parseval

$$2|a_0|^2 + \sum_{n=1}^{\infty} |a_n|^2 + |b_n|^2 = \frac{2}{T} \int_{t_0}^{t_0+T} |x(t)|^2 dt$$

Serie exponencial

$$x(t) \sim \sum_{n=-\infty}^{\infty} X_k e^{iw_n t}$$

Parseval

$$\sum_{k=-\infty}^{\infty} |X_k|^2 = \frac{1}{T} \int_{to}^{to+T} |x(t)|^2 dt$$

Formula util

$$c_j = \frac{\langle v, \phi_j \rangle}{||\phi_j||^2}$$