Chapter 20

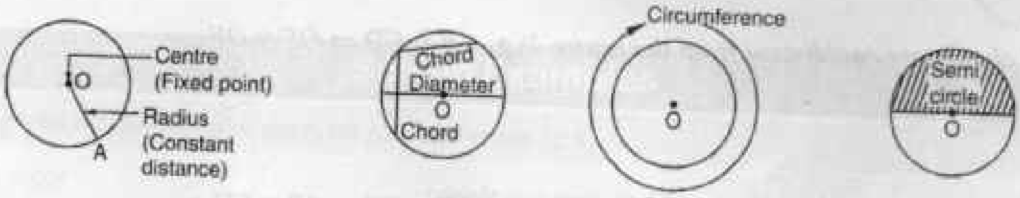
CIRCLES

KEY FACTS

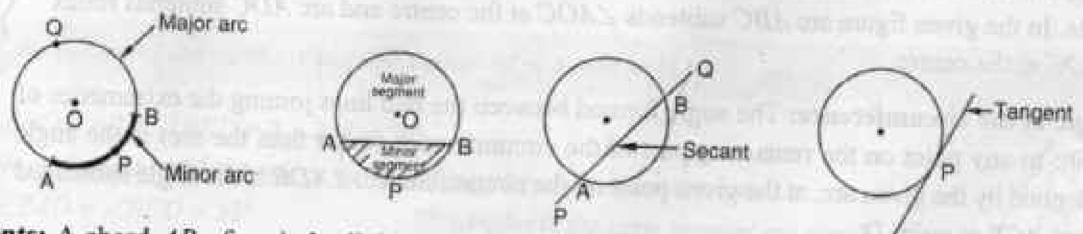
1. A **circle** is a simple closed curve all of whose points are at a constant distance from a fixed point in the same plane. The fixed point is called the **centre** of the circle.
2. **Radius:** A line segment joining the centre of the circle with any point on it is called the radius (Plural : radii) of the circle. All radii of a circle are equal.
3. **Diameter:** A line segment which passes through the centre of a circle and has the end points on the boundary of the circle is called the **diameter** of the circle.

$$\boxed{\text{Diameter} = 2 \times \text{radius}}$$

4. **Chord:** A line segment joining any two points on a circle is called a **chord** of the circle.The diameter is the longest chord.



5. **Circumference:** The distance around the circle is called its **circumference.**
6. **Semi-circle:** A diameter of a circle divides the circle into two equal parts. Each part is called a semi-circle.
7. **Arc:** An arc is a part of a circle included between two points on a circle. In the figure ***APB*** and ***AQB*** are arcs, They are denoted by $\widehat{APB}$ and $\widehat{AQB}$. $\widehat{APB}$ is less than a semi circle, and is called a **minor arc.** $\widehat{AQB}$ is greater than a semi-circle, and is called a **major arc.**



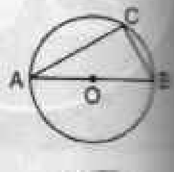
8. **Segments:** A chord *AB* of a circle divides the circular region into two parts. Each part is called a segment of the circle.The bigger part containing the centre of the circle is called the **major segment** and the smaller part which does not contain the centre is called the **minor segment** of the circle.

9. **Secant:** A line which intersects the circle at two distinct points is called a **secant.** PQ is a secant intersecting the circle at points A and B.

10. **Tangent:** A line touching a circle at a point is called a tangent.

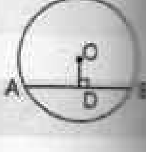
Some important properties of a circle:

(i) ***The angle in a semi circle is a right angle.*** E.g., $\angle ACB = 90^\circ$.



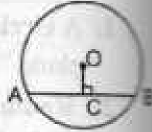
(ii) ***The line joining the centre of the circle and the mid-point of a chord (not passing through the chord) is perpendicular to the chord.***

D is the mid-point of $AB \Rightarrow OD \perp AB$.

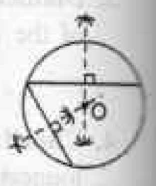


(iii) ***The perpendicular from the centre to a chord bisects the chord.***

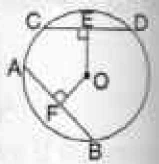
$OC \perp AB \Rightarrow AC = CB$.



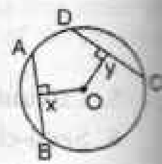
(iv) ***The perpendicular bisectors of two chords of a circle intersect at its centre.***



(v) ***Equal chords are equidistant from the centre*** E.g., $AB = CD \Rightarrow OE = OF$

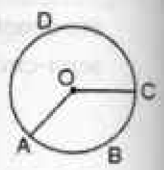


(vi) ***Chords equidistant from the centre are equal*** E.g., $OX = OY \Rightarrow AB = CD$.

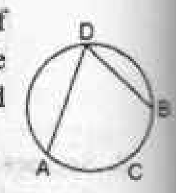


(vii) There can be one and only one circle passing through three non collinear points.

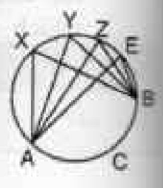
(viii) **Angle at the centre:** The angle formed between the two radii joining the extremeties of an arc to the centre of a circle is known as the angle subtended by the arc at the centre of the circle. In the given figure arc ABC subtends $\angle AOC$ at the centre and arc ADC subtends reflex $\angle AOC$ at the centre.



Angle at the circumference: The angle formed between the two lines joining the extremeties of an arc to any point on the remaining part of the circumference (other than the arc) is the angle subtended by the given arc, at the given point on the circumference. $\angle ADB$ is the angle subtended by arc ACB at point D.



The given figure shows four angles all subtended by arc ACB on the remaining part of the circumference at four different points.



Such angles are said to be angles in the same segment. Thus, $\angle AXB$, $\angle AYB$, $\angle AZB$, $\angle AEB$ are all angles subtended by arc ACB at points X, Y, Z and E.

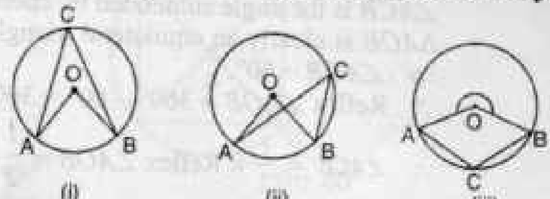
(ix) ***The angle subtended at the centre by an arc of a circle is double the angle which this arc subtends at any remaining part of the circumference.***

E.g., in Fig (i) and (ii)

$\angle AOB = 2\angle ACB$

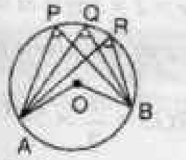
In Fig (iii)

Reflex $\angle AOB = 2\ \angle ACB$

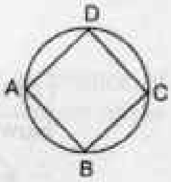


(x) ***Angles in the same segment are equal.***

E.g., $\angle APB = \angle AQB = \angle ARB$



(xi) **Cyclic quadrilateral:** If the vertices of a quadrilateral lie on a circle, it is called a cyclic quadrilateral. $ABCD$ is a cyclic quadrilateral.

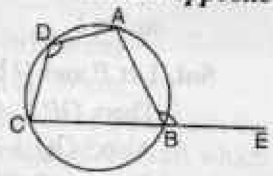


(xii) ***The opposite angles of a cyclic quadrilateral are supplementary.***

i.e., $\angle A + \angle C = 180°$, $\angle B + \angle D = 180°$.

(xiii) ***If a side of a cyclic quadrialteral is produced, the exterior angle so formed is equal to the interior opposite angle.***

E.g., $\angle ABE = \angle ADC$



Solved Examples

Ex. 1. ***In the given circle with diameter AB find the value of x.***

Sol. $\angle ADB = 90°$ *(Angle in a semi circle is a right angle)*

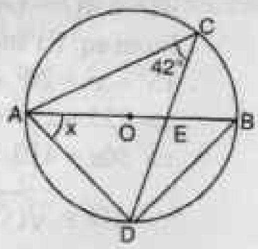
Also $\angle ABD = \angle ACD = 42°$ *(Angles in the same segment)*

$\therefore$ In $\Delta\ ADB$,

$x = \angle BAD = 180° - (\angle ADB + \angle ABD)$

$= 180° - (90° + 42°)$ *(Angle sum prop. of a Δ)*

$= 180° - 132° = \mathbf{48°}$.



Ex. 2. ***O is the centre of the circle. Find the values of p, q and r.***

Sol. $q = \angle BOD = 2 \times \angle BED = 2 \times 35° = \mathbf{70°}$.

(Angle at the centre = 2 × Angle at the remaining part of the cicumference)

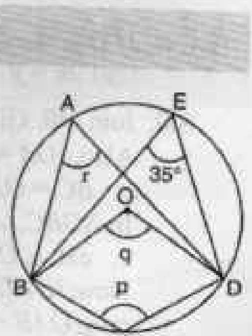
$r = \angle BAD = \angle BED = \mathbf{35°}$. *(Angles in the same segment are equal)*

In cyclic quad. $BEDC$, $\angle BCD + \angle BED = 180°$.

(opp. $\angle$s of a cyclic quad. are supplementary)

$\Rightarrow p + 35° = 180°$

$\Rightarrow p = 180° - 35° = \mathbf{145°}$.



Ex. 3. ***If the length of a chord of a circle is equal to its radius, then find the angle subtended by it at the minor arc of the circle.***

Sol. Given, a circle with centre O and chord AB. $OA = OB = AB$ = radius = r (say)

$\angle ACB$ is the angle subtended by chord AB at the minor arc.

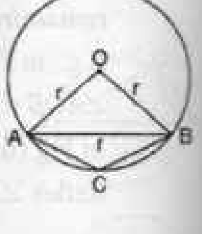
ΔAOB is clearly an equilateral triangle

$\Rightarrow \angle AOB = 60°$.

$\therefore$ Reflex $\angle AOB = 360° - 60° = 300°$

$\therefore \angle ACB = \frac{1}{2} \times$ Reflex $\angle AOB = \frac{1}{2} \times 300° = \mathbf{150°}$.

(Angle at the centre = 2 × Angle at remaining part of the circumference)



Ex. 4. ***PQ is the diameter of the given circle, whose centre is O. Given, $\angle ROS = 54°$, calculate $\angle RTS$.***

Sol. $\angle RQS = \frac{1}{2} \times \angle ROS = \frac{1}{2} \times 54° = 27° \Rightarrow \angle RQT = 27°$

($\angle$s RQS and RQT being the same angles)

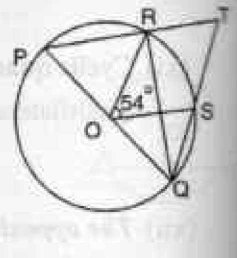
($\angle$s Angle at centre = 2 × Angle at remaining part of the circumference)

Now, in ΔPRQ, $\angle PRQ = 90°$ *(Angle in a semi circle)*

$\therefore \angle QRT = 90°$ *(Linear pair)*

So, in ΔRQT, $\angle RTQ = 180° - (\angle RQT + \angle QRT) = 180° - (27° + 90°)$

$= 180° - 117° = \mathbf{63°}$.



Ex. 5. ***What is the length of the common chord of two circles of radii 15 cm and 20 cm whose centres are 25 cm apart ?***

Sol. Let P and Q be the centres of the two circles respectively and AB be the common chord.

Then, $OP \perp AB$ and bisects AB, *i.e.*, $AO = OB$

Also, $OQ \perp AB$

Given, $AP = 15$ cm, $AQ = 20$ cm and $PQ = 25$ cm

Let $OP = x$ cm. Then $OQ = (25 - x)$ cm

In rt. $\angle d\ \Delta AOP$, $AO^2 = AP^2 - OP^2 = 15^2 - x^2$ ...(i)

In rt. $\angle d\ \Delta AOQ$,

$AO^2 = AQ^2 - OQ^2 = 20^2 - (25 - x)^2$ ...(ii)

From eq. (i) and (ii)

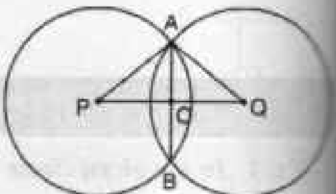
$15^2 - x^2 = 20^2 - (25 - x)^2$

$\Rightarrow 225 - x^2 = 400 - (625 - 50x + x^2) \Rightarrow 225 - x^2 = 400 - 625 + 50x - x^2$

$\Rightarrow 50x = 450 \Rightarrow x = 9$

$\therefore AO = \sqrt{15^2 - 9^2} = \sqrt{225 - 81} = \sqrt{144} = 12$ cm (From (i))

$\Rightarrow AB = 2 \times 12$ cm $= \mathbf{24}$ **cm**.



Ex. 6. ***If O is the centre of the given circle and BC = AO, then which of the following statements is true?***

(a) $2x = y$ *(b)* $x = 3y$ *(c)* $3x = y$ *(d)* $x = 2y$

Sol. Join OB. Given, $BC = OA$

Also, $OA = OB$ *(radii of same circle)*

$\therefore BC = OA = OB$

In ΔOBC, $\angle BOC = \angle BCO = y$ *($\angle$s equal sides are equal)*.

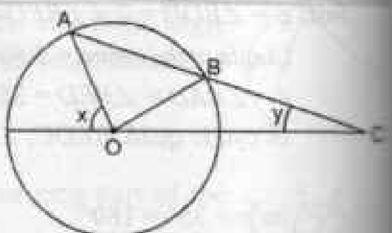
$\therefore$ ext. $\angle OBA = \angle BOC + \angle BCO = y + y = 2y$ *(Ext. $\angle$ prop. of a Δ)*

Now, in ΔOAB, $OB = OA$

$\Rightarrow \angle OAB = \angle OBA = 2y$ *(isos. Δ property)*

Again for ΔAOC, applying the exterior angle property, $x = \angle OAC + \angle OCA = 2y + y = 3y$.

Option (b) is correct.



Question Bank-20

1. If the two circles C_1 and C_2 have three points in common, then which of the following is correct?
(a) C_1 and C_2 are concentric
(b) C_1 and C_2 are the same circle
(c) C_1 and C_2 have different centres
(d) None of the above

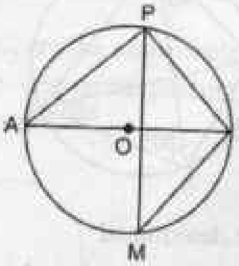
2. Which of the following pairs of lines can be parallel?
1. Two tangents to a circle.
2. Two diameters of a circle.
3. A chord of circle and a tangent to a circle.
4. Two chords of a circle.

Select the correct answer using the codes given below:
Codes:
(a) 1, 2 and 3 (b) 2, 3 and 4
(c) 1, 3 and 4 (d) 1, 2 and 4

3. Three circles with equal radii touch each other externally. The figure formed by joining the centres of these circles is
(a) an isosceles triangle
(b) an equilateral triangle
(c) a scalene triangle
(d) a right angled triangle

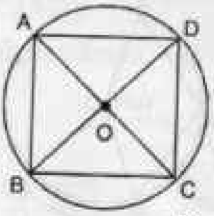
4. Two non-intersecting circles one lying inside another are of diameters a and b. The minimum distance between their circumferences is c. The distance between their centres is
(a) $a-b-c$ (b) $a+b-c$
(c) $\frac{1}{2}(a-b-c)$ (d) $\frac{1}{2}(a-b)-c$

5. In the given figure, if AB is the diameter of the circle and PM the internal bisector of $\angle APB$, then the measure of angle ABM is



(a) 15° (b) 30°
(c) 45° (d) 60°

6. A square is inscribed in a circle with centre O. What angle does each side subtend at the centre O ?



(a) 45° (b) 60°
(c) 75° (d) 90°

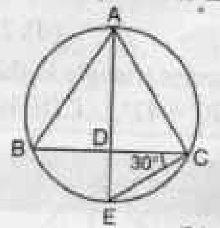
7. A regular polygon is inscribed in a circle. If a side subtends an angle of 45° at the centre. What is the number of sides of the polygon?
(a) 6 (b) 5
(c) 10 (d) 8

8. The length of a chord of a circle at a distance of 5 cm from the centre is 24 cm. The diameter of the circle is
(a) 26 cm (b) 24 cm
(c) 13 cm (d) 12 cm

9. In a circle of radius 25 cm, a chord is drawn at a distance of 7 cm from the centre. Find the length of the chord.
(a) 24 cm (b) 48 cm
(c) 50 cm (d) 36 cm

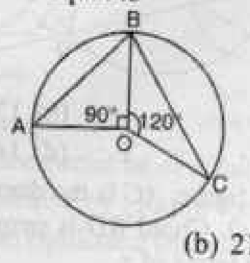
10. A chord 6 cm long is at a distance of 4 cm from the centre of a circle. Find the length of a chord which is at a distance of 3 cm from the centre of the circle.
(a) 10 cm (b) 6 cm
(c) 8 cm (d) 12 cm

11. In the given figure, ΔABC is inscribed in a circle and the bisector of $\angle A$ meets BC in D and the circle in E. If $\angle ECD = 30°$, what is $\angle A$?



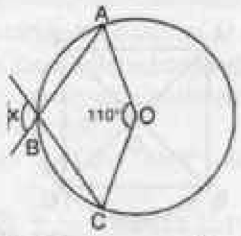
(a) 60° (b) 45°
(c) 70° (d) 150°

12. O is the centre of a circle $\angle AOB = 90°$ and $\angle BOC = 120°$. $\angle ABC$ is equal to



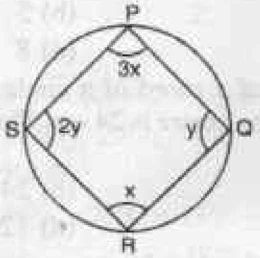
(a) 150° (b) 210°
(c) 75° (d) 105°

13. In the given figure, O is the centre of the circle. The value of x is



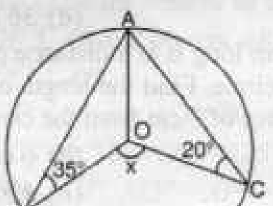
(a) 75° (b) 55°
(c) 125° (d) 110°

14. $PQRS$ is a cyclic quadrilateral. Find the measure of $\angle P$ and $\angle Q$.



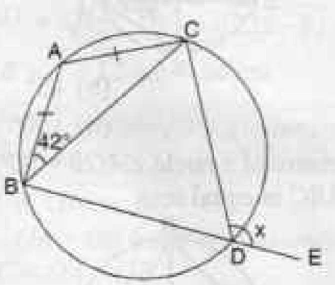
(a) 135°, 60° (b) 60°, 120°
(c) 60°, 90° (d) 100°, 120°

15. If $\angle ABO = 35°$ and $\angle ACO = 20°$, then $\angle x$ is



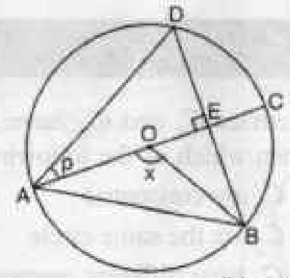
(a) 55° (b) 110°
(c) 80° (d) 70°

16. ABC is an isosceles triangle in the given circle with centre O. $\angle ABC = 42°$, $\angle CDE$ is equal to



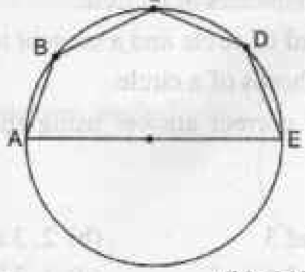
(a) 84° (b) 138°
(c) 96° (d) 148°

17. In the given figure, AC is the diameter of the circle with centre O. Chord BD is perpendicular to AC. Express p in terms of x.



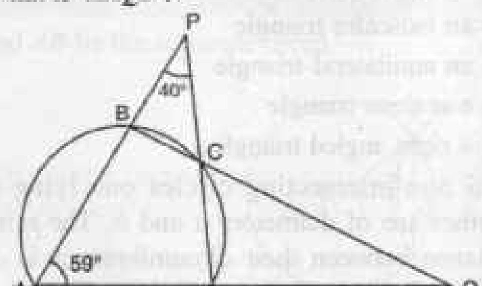
(a) $x/2$ (b) $90° + x/2$
(c) $90° - x/2$ (d) $180° - x$

18. In the given figure, AE is the diameter of the circle. Write down the numerical value of $\angle ABC + \angle CDE$.



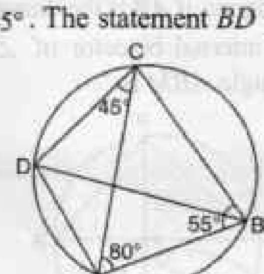
(a) 360° (b) 540°
(c) 180° (d) 270°

19. In the given figure, $\angle PAQ = 59°$, $\angle APD = 40°$, then what is $\angle AQB$?



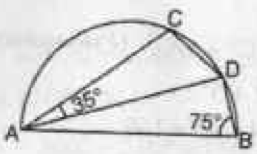
(a) 19° (b) 20°
(c) 22° (d) 27°

20. In the given figure $\angle CAB = 80°$, $\angle CBA = 55°$ and $\angle DCA = 45°$. The statement BD is the diameter is



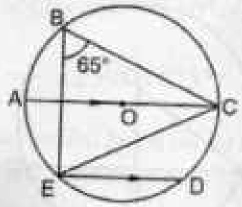
(a) False
(b) cannot be determined
(c) True
(d) Not possible

21. In the given figure, C and D are points on a semi circle described on AB as diameter. $\angle ABD = 75°$ and $\angle DAC = 35°$. What is $\angle BDC$?



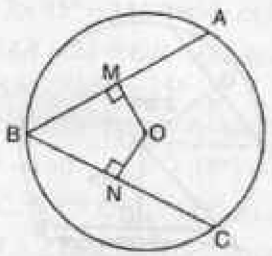
(a) 130° (b) 110°
(c) 90° (d) 100°

22. In the adjoining figure, chord ED is parallel to the diameter of the circle. If $\angle CBE = 65°$, then what is the value of $\angle DEC$?



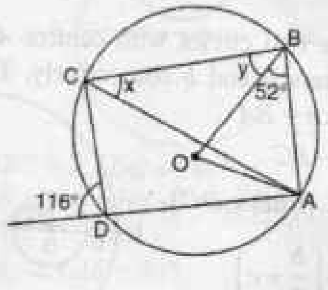
(a) 35° (b) 55°
(c) 45° (d) 25°

23. AB and BC are two equal chords of a circle with centre O. $OM \perp AB$ and $ON \perp BC$. OB is joined. State if each of the following statements is true or false. Give reasons in each case.



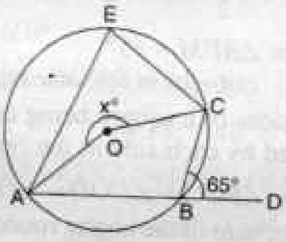
(i) $OM = ON$
(ii) $\Delta OMB \cong \Delta ONB$
(iii) BO bisects $\angle ABC$

24. In the given figure, find $x + y$.



(a) 116° (b) 102°
(c) 64° (d) 76°

25. In the given figure, O is the centre of the circle. ABD is a straight line and $\angle CBD = 65°$. Find reflex $\angle AOC$ (marked $x°$).



(a) 130° (b) 230°
(c) 190° (d) 65°

Answers

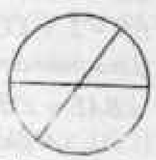
1. (b)	2. (c)	3. (b)	4. (d)	5. (c)	6. (d)	7. (d)	8. (a)	9. (b)	10. (c)
11. (a)	12. (c)	13. (c)	14. (a)	15. (b)	16. (c)	17. (c)	18. (d)	19. (c)	20. (c)
21. (a)	22. (d)	23. All are true statements.			24. (b)	25. (b)			

Hints and Solutions

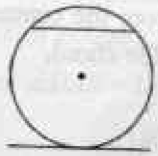
1. (b) C_1 and C_2 are the same circle as there can be one and only one circle passing through three non collinear points.

2. (c)

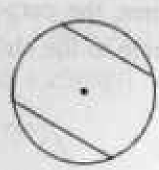
Two tangents to a circle



Two diameters of a circle

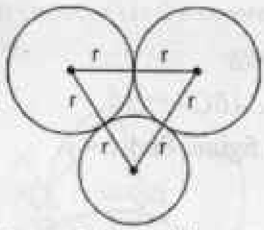


A chord and a tangent to a circle

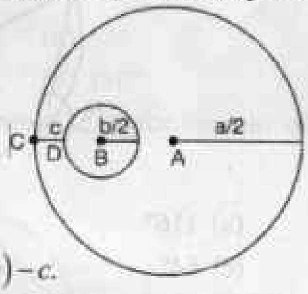


Two chords of a circle

3. (b) Let the three circles be of equal radii r. Then, the triangle formed by joining the centres of these circles is an equilateral triangle with each side $2r$.



4. (d) Let the two circles with centres A and B have diameters a and b respectively. Then required distance $= BA$



$= AC - BC$

$= AC - (BD + DC)$

$= \frac{a}{2} - \left(\frac{b}{2} + c\right)$

$= \frac{a}{2} - \frac{b}{2} - c = \frac{1}{2}(a-b) - c.$

5. (c) $\angle APB = 90°$

(Angle in a semi circle is a right angle)

$\Rightarrow \quad \angle APM = \frac{1}{2}\angle APB = 45°$ *(PM bisects $\angle APB$)*

$\therefore \ \angle ABM = \angle APM = 45°$

(Angles in the same segment are equal)

6. (d) All the sides of a square being equal, the angles subtended by each side at the centre are equal.

$\Rightarrow \ \angle AOB = \angle BOC = \angle COD = \angle DOA = x$ (say)

Since, the sum of the angles round a point = 360°, therefore,

$\angle AOB + \angle BOC + \angle COD + \angle DOA = 360°$

$\Rightarrow \ x + x + x + x = 360°$

$\Rightarrow \ 4x = 360° \Rightarrow x = \mathbf{90°}.$

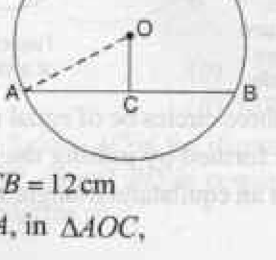
7. (d) Number of sides of the polygon

$= \dfrac{\text{Sum of the angles at the centre}}{\text{Angle subtended by one side at the centre}}$

$= \dfrac{360°}{45°} = \mathbf{8}.$

8. (a) Given $AB = 24$ cm and $OC = 5$ cm

Since, the perpendicular from the centre of the circle to the chord bisects the chord,



$AC = CB = 12\text{cm}$

Join, OA, in ΔAOC,

$AO^2 = AC^2 + OC^2$ *(Pythagoras Theorem)*

$= 12^2 + 5^2$

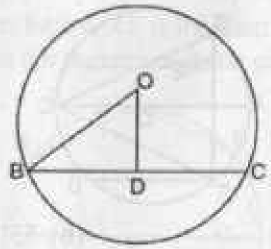
$= 144 + 25 = 169$

$\Rightarrow \ AO = \sqrt{169} = 13$ cm

$\therefore$ Diameter = 2 × radius = 2 × AO = 2 × 13 cm **= 26 cm.**

9. (b) Given, $OD = 7$ cm, $OB = 25$ cm

Since, $OD \perp BC$, it bisects BC, *i.e.*, $BD = DC$.



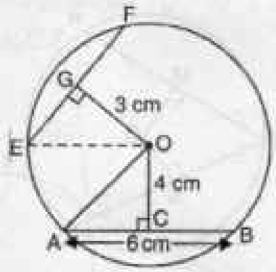
In ΔOBD,

$BD^2 = OB^2 - OD^2 = 25^2 - 7^2 = 625 - 49 = 576$

$\Rightarrow \ BD = \sqrt{576} = 24$

$\therefore$ Chord $BC = 2 \times 24$ cm $=$ **48 cm.**

10. (c) Give $OC = 4$ cm, Chord $AB = 6$ cm



$\because \ OC \perp AB \Rightarrow OC$ bisects AB

$\Rightarrow \ AC = CB = 3\text{cm}$

$\therefore$ In ΔOCA,

$OA^2 = OC^2 + AC^2$ *(Pythagoras, Theorem)*

$= 4^2 + 3^2 = 16 + 9 = 25$

$\Rightarrow \ OA = 5$ cm

Let EF be the chord at a distance of 3 cm from the centre.

Given, radius $= OE = 5$ cm

$\therefore$ In ΔOGE, $EG^2 = OE^2 - OG^2$

$= 5^2 - 3^2 = 25 - 9 = 16$

$\Rightarrow \ EG = 4$ cm

$\therefore \ EF = 2 \times EG =$ **8 cm.**

(Line from centre $\perp$ to chord bisects the chord)

11. (a) $\angle BAE = \angle BCE = 30°$

(Angles in the same segment are equal)

$\therefore \ \angle BAC = 2 \times \angle BAE = 2 \times 30° = \mathbf{60°}.$

($\because$ *AE bisects* $\angle BAC$)

12. (c) Reflex $\angle AOC = \angle AOB + \angle BOC$

$= 90° + 120° = 210°$

$\therefore\ \angle AOC = 360° - 210° = 150°$

$\therefore\ \angle ABC = \frac{1}{2} \times \angle AOC = \frac{1}{2} \times 150° = \mathbf{75°}.$

(Angle subtended by an arc at the centre is twice the angle at any remaining part of the ⊙ce of the circle).

13. (c) Reflex $\angle O = 360° - 110° = 250°$

$\therefore\ \angle ABC = \frac{1}{2} \times 250° = 125°$

(Angle subtended by an arc at the centre is twice the angle at any remaining part of the ⊙ce of the circle)

$\therefore\ x = \angle ABC = \mathbf{125°}$. *(Vertically opposite angles)*

14. (a) Opposite angles of a cyclic quadrilateral are supplementary.

$\therefore\ 3x + x = 180°$ and $2y + y = 180°$

$\Rightarrow\ 4x = 180°$ and $3y = 180°$

$\Rightarrow\ x = 45°$ and $y = 60°$.

$\therefore\ \angle P = 3 \times 45° = \mathbf{135°}$ and $\angle Q = \mathbf{60°}$.

15. (b) In ΔOAB, $OA = OB$ *(Radii of same circle)*

$\Rightarrow\ \angle OBA = \angle OAB = 35°$

(Angles opposite to equal sides are equal)

$\therefore\ \angle AOB = 180° - (35° + 35°) = 180° - 70° = 110°$

In OAC, $OA = OC$ *(Radii of same circle)*

$\Rightarrow\ \angle OCA = \angle OAC = 20°$ *(Isosceles Δ prop.)*

$\therefore\ \angle AOC = 180° - (20° + 20°) = 180° - 40° = 140°$

$\therefore$ Reflex $\angle BOC = \angle AOB + \angle AOC$

$= 110° + 140° = 250°$

$\Rightarrow\ x = 360° - 250° = \mathbf{110°}.$

16. (c) $AB = AC \Rightarrow\ \angle ACB = \angle ABC = 42°$

(Isosceles Δ prop.)

$\therefore$ In ΔABC, $\angle BAC = 180° - (42° + 42°)$

$= 180° - 84° = 96°$

$\therefore\ \angle CDE = \angle BAC = \mathbf{96°}.$

(Exterior ∠ of a cyclic quadrilateral is equal to the interior opposite angle).

17. (c) $\angle AOB = x \Rightarrow \angle ADB = x/2$

(Angle subtended by an arc at the centre is twice the angle at the remaining part of the ⊙ce of the circle).

Since, chord $BD \perp AC$, $\angle AED = 90°$

$\therefore$ In ΔAED,

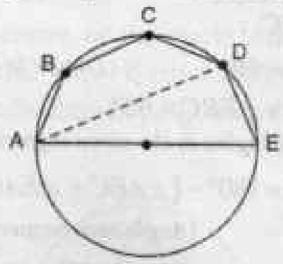
$p = \angle DAE = 180° - (\angle ADE + \angle AED)$

$= 180° - (x/2 + 90°)$

$= 90° - x/2$.

18. (d) Join AD.

In the cyclic quadrilateral $ABCD$,



$\angle ABC + \angle ADC = 180°$ ...(i)

(opp. ∠s angles of a cyclic quadrilateral are supplementary).

Also, $\angle ADE = 90°$ ...(ii)

(Angle in a semicircle is a right angle)

$\therefore$ Adding eqns. (i) and (ii), we get,

$\angle ABC + \angle ADC + \angle ADE = 180° + 90°$

$\Rightarrow\ \angle ABC + \angle CDE = \mathbf{270°}.$

19. (c) $\angle DCQ = 59°$

(Exterior angle theorem of cyclic quadrant).

In ΔADP,

Exterior $\angle CDQ$ = Int. opp. $(\angle PAD + \angle APD)$

$= 59° + 40° = 99°$

$\therefore$ In ΔDCQ, $\angle DQC = 180° - (\angle DCQ + \angle CDQ)$

$= 180° - (59° + 99°)$

$= 180° - 158° = 22°$

(Angle sum property of a Δ)

$\Rightarrow\ \angle AQB = \mathbf{22°}.$

20. (c) $\angle BAC = \angle BDC = 80°$

(Angles in the same segment are equal)

In ΔBAC, $\angle ACB = 180° - (\angle BAC + \angle CBA)$

$= 180° - (80° + 55°)$

$= 180° - 135° = 45°$

$\therefore\ \angle BCD = \angle BCA + \angle ACD = 45° + 45° = 90°$

$\Rightarrow$ BD is the diameter. *(Angle in a semi-circle = 90°)*

21. (a) In ΔADB, $\angle ADB = 90°$ *(Angle in a semi-circle)*

$\therefore\ \angle DAB = 180° - (\angle ADB + \angle ABD)$

$= 180° - (90° + 75°)$

$= 180° - 165° = 15°$

$\therefore\ \angle CAB = \angle CAD + \angle DAB = 35° + 15° = 50°$

In cyclic quadrilateral $ABDC$,

$\angle BDC + \angle CAB = 180°$

(Opp. ∠s of a cyclic quadrilateral are supplementary)

$\therefore\ \angle BDC = 180° - \angle CAB = 180° - 50° = \mathbf{130°}.$

22. (d) Join AE.

Given, $\angle EBC = 65^\circ$ and $AC \parallel ED$.

In ΔAEC,

$\angle AEC = 90^\circ$ *(Angle in a semi circle)*

$\angle EAC = \angle EBC = 65^\circ$

(Angles in the same segment are equal)

$\therefore \angle ACE = 180^\circ - (\angle AEC + \angle EAC)$

(Angle sum property of a circle)

$= 180^\circ - (90^\circ + 65^\circ) = 180^\circ - 155^\circ = 25^\circ$.

$\angle CED = \angle ACE = 25^\circ$

($AC \parallel ED$, alternate angles are equal)

23. (i) True, equal chords are equidistant from the centre.

(ii) True, In Δs OMB and ONB,

$OM = ON$ (Proved in (i))

$OB = BO$ (Common)

$\angle OMB = \angle ONB = 90^\circ$

$\therefore \Delta OMB \cong \Delta ONB$ (RHS)

(iii) $\Delta OMB \cong \Delta ONB \Rightarrow \angle OBM = \angle OBN$ (cpct)

$\Rightarrow BO$ bisects $\angle ABC$.

Hence, all the statements are true statements.

24. (b) For cyclic quadrilateral $ABCD$,

$y + 52^\circ = 116^\circ \Rightarrow y = 116^\circ - 52^\circ = 64^\circ$

(Ext. angle property of a cyclic quadrilateral)

$OB = OA$ *(radii of same circle)*

$\Rightarrow \angle OAB = \angle OBA = 52^\circ$

$\therefore \angle BOA = 180^\circ - (\angle OAB + \angle OBA)$

$= 180^\circ - (52^\circ + 52^\circ)$

$= 180^\circ - 104^\circ = 76^\circ$.

(Angle sum property of a Δ)

$\therefore x = \angle BCA = \frac{1}{2} \times \angle BOA = \frac{1}{2} \times 76^\circ = 38^\circ$

(Angle at the centre is twice the angle at any remaining part of circumference)

$\therefore x + y = 64^\circ + 38^\circ = \mathbf{102^\circ}$.

25. (b) $AECB$ being a cyclic quadrilateral,

$\Rightarrow$ interior opposite $\angle AEC$ = exterior $\angle CBD = 65^\circ$

$\therefore \angle AOC = \angle AEC \times 2 = 65^\circ \times 2 = 130^\circ$

(Angle at centre = 2 × angle at remaining part of the circumference)

$\therefore$ Reflex $\angle AOC = 360^\circ - 130^\circ = \mathbf{230^\circ}$.

Self Assessment Sheet–20

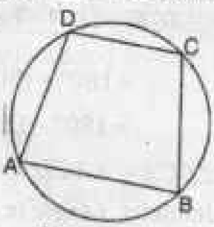
1. Which of the following statements is not TRUE ?

(a) The diameter is the greatest chord that can be drawn in a circle.

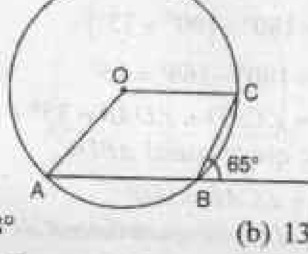
(b) A straight line cannot intersect a circle in more than two points.

(c) A diameter bisects a circle.

(d) In the figure $\angle A = \angle C$ and $\angle B = \angle D$.

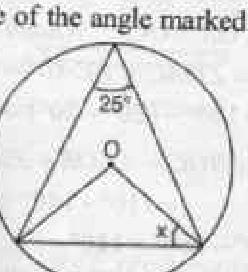


2. O is the centre of the circle, ABN is a straight line. Find $\angle AOC$.



(a) 128° (b) 132°
(c) 130° (d) 135°

3. Find the size of the angle marked x.

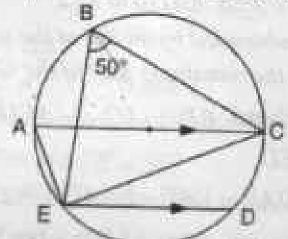


(a) 60° (b) 65°
(c) 70° (d) 55°

4. The length of a chord of a circle of radius 10 cm is 12 cm. Find the distance of the chord from the centre of the circle.

(a) 6 cm (b) 5 cm
(c) 8 cm (d) 7 cm

5. Chord $ED \parallel$ diameter AC. Determine $\angle CED$.

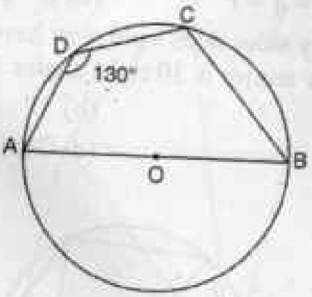


(a) 50° (b) 45°
(c) 55° (d) 40°

6. The measure of the line segment joining the centre of a circle to the mid-point of a chord is :

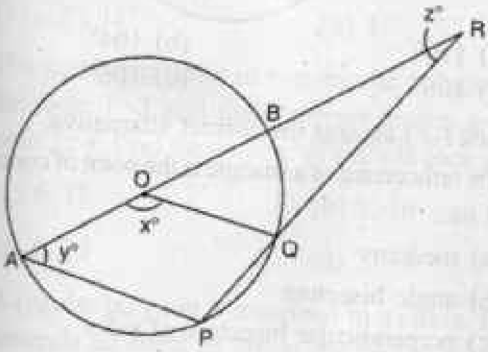
(a) twice the measure of the chord
(b) half the measure of the chord
(c) equal to the measure of the chord
(d) none of the above

7. $ABCD$ is a cyclic quadrilateral whose side AB is a diameter of the circle through A, B, C, D. If $\angle ADC = 130°$, find $\angle BAC$.



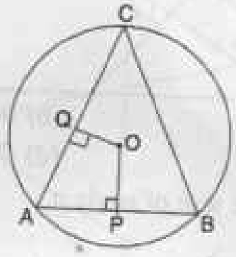
(a) 40° (b) 50°
(c) 60° (d) 30°

8. O is the centre of the circle $APQB$; $AOBR$, PQR are straight lines. Find x in terms of y and z.



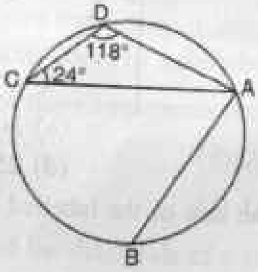
(a) $x = y + z$ (b) $x = 2y + z$
(c) $x = y + 2z$ (d) $x = 2(y + z)$

9. O is the centre of the circle ABC, radius 5 cm, $AB = 8$ cm, $AC = 6$ cm. Calculate the lengths of the perpendiculars OP, OQ from O to AB, AC.



(a) 3 cm, 4 cm (b) 4 cm, 3 cm
(c) 2 cm, 3 cm (d) 3.5 cm, 2.5 cm

10. AB, AC are equal chords of the circle $ABCD$. Calculate $\angle BAD$.



(a) 100° (b) 94°
(c) 96° (d) 80°

Answers

1. (d)	2. (c)	3. (b)	4. (c)	5. (d)	6. (d)	7. (a)	8. (d)	9. (a)	10. (b)

Unit Test–4

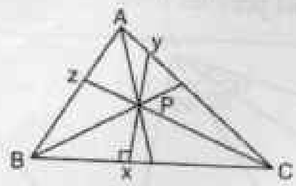
1. Which of the following is equidistant from the vertices of a triangle ?

(a) circumcentre (b) centroid
(c) orthocenter (d) incentre

2. The circumcentre in a right triangle is :

(a) inside the triangle
(b) outside the triangle
(c) on one of the perpendicular sides
(d) on the hypotenuse

3. P is the incentre of $\Delta\ ABC$. Which of the following statements is true ?



(a) $AZ = BZ$ (b) $AY = BX$
(c) $PY = PZ$ (d) $PA = PC$

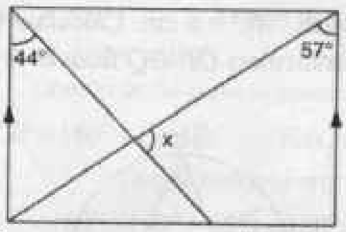
4. The incentre of a triangle coincides with the circumcentre, orthocenter and centroid in case of :

(a) an isosceles triangle
(b) an equilateral triangle

(c) a right - angled triangle

(d) a right - angled isosceles triangle

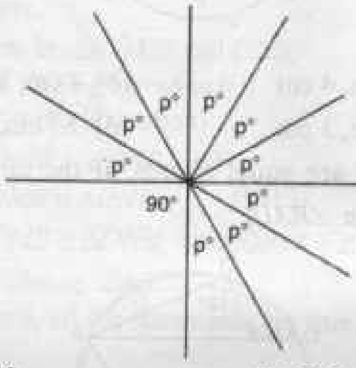
5. Find x



(a) 78° (b) 80°

(c) 75° (d) 79°

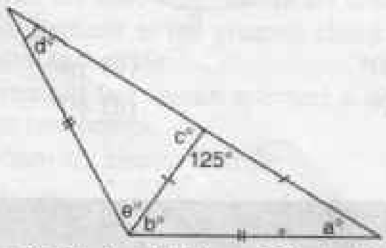
6. Calculate the size of angle p.



(a) 20° (b) 30°

(c) 40° (d) 25°

7. Calculate the size of the labelled angles



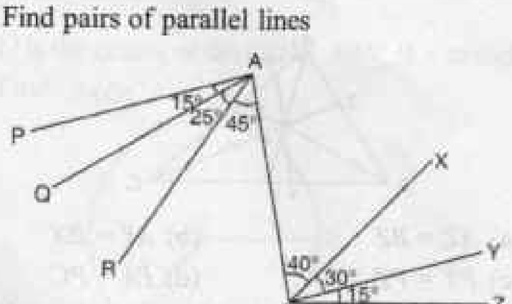
(a) $a=27.5°, b=27.5°, c=55°, d=27.5°, e=97.5°$

(b) $a=28°, b=27°, c=60°, d=28°, e=102°$

(c) $a=30°, b=25°, c=55°, d=30°, e=95°$

(d) None of these

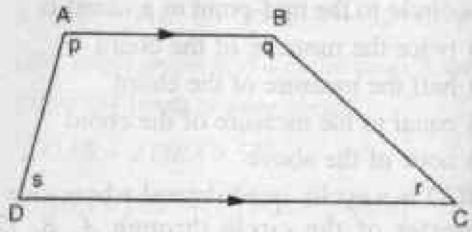
8. Find pairs of parallel lines



(a) $AR, BX; AP, BY$ (b) $AQ, BZ; AP, BX$

(c) $AQ, BY; AP, BZ$ (d) $AQ, BX; AR, BZ$

9. In the figure, $AB \parallel CD$, then



(a) $p+r=q+s$ (b) $p-r=q-s$

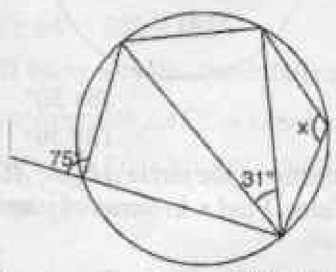
(c) $p+s=q+r$ (d) $p-q=s-r$

10. How many sides does a polygon have if the sum of its interior angles is 30 right angles ?

(a) 15 (b) 17

(c) 19 (d) 20

11. Find x



(a) 110° (b) 104°

(c) 108° (d) 106°

12. Tick (✓) against the correct alternative.

The orthocentre of a triangle is the point of concurr of its.

(a) medians

(b) angle bisectors

(c) perpendicular bisectors of sides

(d) altitudes drawn to sides from opposite vertices

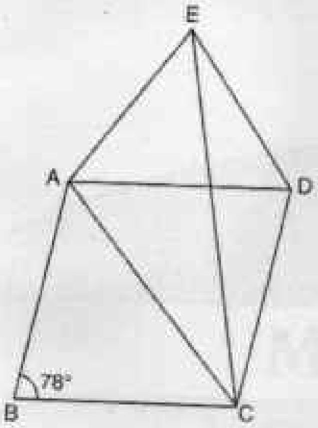
13. Match correctly

(a) centroid	(1) medians of a Δ
(b) incentre	(2) centre of the circumcircle point of intersection of the perp. bisectors of the sides of a Δ.
(c) circumcentre	(3) point of intersection of the medians of a Δ
(d) concurrent	(4) centre of the incircle point of intersection of the angle bisectors of a Δ

14. The lengths of the sides of a Δ ABC are given below. In which of these cases are angles of the triangle in the increasing order of magnitude as $\angle C, \angle B, \angle A$

(a) $BC = 5$ cm, $CA = 6.5$ cm, $AB = 7.9$ cm
(b) $BC = 10$ cm, $CA = 6.9$ cm, $AB = 5.4$ cm
(c) $BC = 3$ cm, $CA = 4$ cm, $AB = 5$ cm
(d) $BC = 3.5$ cm, $CA = 3$ cm, $AB = 4$ cm

15. $ABCD$ is a rhombus and AED is an equilateral triangle. E and C lie on opposite sides of AD. If $\angle ABC = 78°$, calculate $\angle DCE$ and $\angle ACE$.



(a) 20°, 30° (b) 21°, 31°
(c) 22°, 32° (d) 19°, 29°

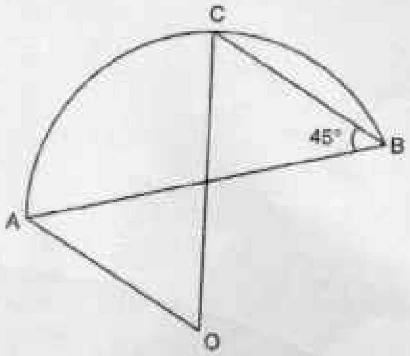
16. The number of sides of two regular polygons are in the ratio 1: 2 and their interior angles are in the ratio 3 : 4. Find the number of sides in each polygon.

(a) 6, 12 (b) 8, 16
(c) 5, 10 (d) 7, 14

17. A regular polygon is inscribed in a circle. If a side subtends an angle of 30° at the centre, what is the number of its sides ?

(a) 10 (b) 8
(c) 6 (d) 12

18. Answer True or False. ACB is an arc of a circle with centre O and $\angle ABC = 45°$, then, $AO \perp OC$.



19. Consider the following statements

(1) The bisectors of all the four angles of a parallelogram enclose a rectangle.
(2) The figure formed by joining the midpoints of the adjacent sides of a rectangle is a rhombus.
(3) The figure formed by joining the midpoints of the adjacent sides of a rhombus is a square.

Which of these statements are correct ?

(a) 1 and 2 (d) 2 and 3
(c) 3 and 1 (d) 1, 2 and 3

20. If the sum of the diagonals of a rhombus is 12 cm, and its perimeter is $8\sqrt{5}$ cm, then the lengths of the diagonals are :

(a) 6 cm and 6 cm (b) 7 cm and 5 cm
(c) 8 cm and 14 cm (d) 9 cm and 3 cm

Answers

1. (a)	**2.** (d)	**3.** (c)	**4.** (b)	**5.** (d)	**6.** (b)	**7.** (a)	**8.** (c)	**9.** (b)	**10.** (b)
11. (d)	**12.** (d)	**13.** (a) $\to 3$	(b) $\to 4$	(c) $\to 2$	(d) $\to 1$	**14.** (b)	**15.** (b)	**16.** (c)	**17.** (d)
18. True	**19.** (a)	**20.** (c)							