

Johnson and Shot Experiment

Newton Kwan and William Willis

October 2018

Abstract

In any electronic measurement, signal is always accompanied by noise. Although many types of noise can be reduced, there are two types that cannot be reduced. The first, Johnson noise, arises from the thermal agitations of electrons and the second, Shot noise, arises from the quantized nature of electrons. In addition to helping us understand the noise of our system, Johnson and Shot noise can be used to calculate the value of three fundamental constants – Boltzmann constant k , absolute zero T_0 , and elementary charge e . Our experiment found the Boltzmann constant $k = 1.776 \times 10^{-23} \pm 1.204 \times 10^{-24} \frac{J}{K}$, absolute zero $T_0 = -215.79 \pm 21.51^\circ C$, and fundamental charge $e = 0.8747 \times 10^{-19} \pm 6.502 \times 10^{-21} C$

Contents

1	Introduction	3
2	Theory	3
2.1	Johnson Noise	3
2.2	Shot Noise	4
2.3	Amplification and Background	5
2.4	Summary	5
3	Experimental Setup and Data	6
3.1	Attenuator Calibration	6
3.2	Gain Integral	7
3.3	Johnson Noise	10
3.3.1	Boltzmann constant k measurement	10
3.3.2	Absolute zero T_0 measurement	12
3.4	Shot Noise	14
3.4.1	Elementary charge e measurement	14
4	Summary	16
5	Conclusion	17
6	Acknowledgements	17

1 Introduction

Johnson noise is a type of irreducible noise. Discovered by John Johnson in 1926 at Bell Labs and formalized theoretically by Johnson's colleague Harry Nyquist, Johnson noise is an irreducible disturbance in electronic signal coming from the thermal agitation of electrons in electrical conductors.[2] Johnson noise is ubiquitous in any kind of electronics, regardless of the voltage applied. Johnson noise is related to the Boltzmann constant k and absolute zero T_0 .

Shot noise is another type of irreducible noise. Just as the number of photons emitted over an interval of time from a light source is not constant, the number of electrons emitted over an interval of time from a current source is not constant. This is most apparent in very small currents; if the current is reduced to a handful of electrons, the change in the number of electrons coming from the current source over a given period of time is significant. These fluctuations are what we call Shot noise. Discovered by Walter Schottky in 1918 while studying fluctuations of current in vacuum tubes, Shot noise is connected to the fundamental constant e , the charge of an electron. The magnitude of Shot noise increases with the square of the expected number of events, meaning shot noise is most frequently observed with small currents that are amplified.[3] Unlike Johnson noise, Shot noise is temperature and frequency independent.

2 Theory

2.1 Johnson Noise

The theoretical formulation of Johnson noise was done by Harry Nyquist using the second law of thermodynamics and equipartition theorem. It relates the voltage from Johnson noise to the Boltzmann constant, the resistance of the circuit, the temperature of the resistor, and the frequency bandwidth. Nyquist's formula relates these values in the following way:

$$V_j^2 = 4kTRdf \quad (1)$$

where V_j is the root mean square voltage or Johnson noise, R is resistance, T is the temperature, k is Boltzmann constant, and df is the frequency bandwidth.

At very high frequencies or at very low temperature, there are quantum effects. These effects are seen on the order of terahertz and near absolute zero. Our experimental setup is well within a classical approximation with frequencies ranging from 1k - 30kHz and temperatures ranging from -200 to 300 Kelvin, so we can ignore quantum effects.

Modifications based on our experimental setup need to be made to Nyquist's formula. The first modification we make comes from a capacitor in parallel with our resistor. To account for this, using AC circuit theory, we modify our Nyquist's formula in the following way:

$$V_k^2 = 4kT \frac{R}{1 + (2\pi fCR)^2} df \quad (2)$$

where C is capacitance.

By rearranging (2), we can determine the Boltzmann constant k and absolute zero T_0 in the following way:

$$\frac{V_j^2}{4R_m df} = kT \quad (3)$$

$$\frac{V_j^2}{4T df} = kR_m \quad (4)$$

where $R_m = \frac{R}{1+(2\pi fCR)^2}$. For the remainder of the theory, since we effectively measure R_m when we measure the resistance with an ohmmeter, we will refer to R_m simply as R . Using (3) and measuring the R_m , V_j at different values of T , the Boltzmann constant k can be determined by the slope of a linear fit of (3). Using (4) and measuring T and V_j at different values of R_m , absolute zero T_0 can be determined from the x-intercept of the linear fit of (4). These two equations serve as the basis for our experimental approach. We will further modify equations (3) and (4) and state the final equations at the end of the theory section, but we include them here to make clear how we will determine k and T_0 .

2.2 Shot Noise

Shot noise occurs when there is current. Current is not continuous, but rather the result of the motion of discrete charged particles. At microscopic levels, the current varies in unpredictable ways. This variation about the average number of electrons drifting past a point per time interval is Shot noise. [1] The magnitude of the charge of the individual carriers relates to the magnitude of the fluctuations. The Schottky formula relates these values in the following way:

$$I_s^2 = 2eI_{avg}df \quad (5)$$

where I_s is the current of the Shot noise or root mean square of the current, e is elementary charge, I_{avg} is the average current, and df is the frequency bandwidth.

Using Ohm's law and substituting $I_s^2 = \frac{V_s^2}{R^2}$ and $I_{avg}^2 = \frac{V_{avg}^2}{R^2}$ into (5), we get:

$$V_s^2 = 2eV_{avg}Rdf \quad (6)$$

Rearranging (7),

$$\frac{V_s^2}{2Rdf} = eV_{avg} \quad (7)$$

By taking measurements of V_s , equation (7) serves as the basis for calculating elementary charge e . Using (7) and measuring V_{avg} , R , V_s over a certain frequency bandwidth for different values of V_{avg} , we can perform a linear fit, which will have slope e . We will modify equation (7) as well as equations (3)

and (4) and state the final equations in the summary of the theory section. We include equation (7) before adding too many modifications to make it clear how we will determine e .

2.3 Amplification and Background

Johnson and Shot noise are both too small to observe without amplification. However, when we amplify our signal, we will need to consider a gain factor, which is essentially a measure of how much our signal gets amplified. With the addition of an amplifier, the root means square of the output voltage becomes the following:

$$V_{out}^2 = V_{in}^2 g^2(f) \quad (8)$$

where $g(f)$ is the gain factor.

Note that the equations (3), (4), and (7) are all the root mean square of the voltage, so we simplify our notation by simply letting $V_{j,rms} = V_j$. Substituting $V_{out,rms}^2$ into the Nyquist formula and Schottky formula, we get the following set of equations:

$$V_j^2 = 4kRTV_{avg} \int g^2(f)df \quad (9)$$

$$V_s^2 = 2eRV_{avg} \int g^2(f)df \quad (10)$$

In addition to amplification, background noise is present in our system, as is this case with most electronic measurements. To account for this background noise from the system, we add a term the root mean square of the background noise $V_{n,rms}$, which we notate as V_n for simplicity:

$$V_j^2 = 4kRTV_{avg} \int g^2(f)df + V_n^2 \quad (11)$$

$$V_s^2 = 2eRV_{avg} \int g^2(f)df + V_n^2 \quad (12)$$

In summary, we amplify our system because Johnson and Shot noise is too small detect without amplification. To account for amplification, we consider a gain factor that increases our signal. We also included an extra term background noise V_n term that accounted for white noise and other sources of noise outside of Johnson and Shot noise. We summarize the equations used in our experiment in the following section.

2.4 Summary

In the previous sections, we have discussed the Johnson-Nyquist equation and the Schottky equation. However, our experimental setup required a number of modifications, and we summarize the three equations that we will use to solve for the fundamental constants k , T_0 , and e .

The equation using Johnson noise to determine the Boltzmann constant k measurement is the following, where the slope of the linear fit is k

$$\frac{V_j^2 - V_n^2}{4T \int g^2(f)df} = kR \quad (13)$$

The equation using Johnson noise to determine absolute zero measurement T_0 is the following, where the x-intercept of this line will be our absolute zero measurement, T_0

$$\frac{V_j^2 - V_n^2}{4R \int g^2(f)df} = kT \quad (14)$$

The equation using Shot noise to determine the elementary charge e is the following, where the slope of the linear fit is e .

$$\frac{V_s^2 - V_n^2}{2R \int g^2(f)df} = eV_{avg} \quad (15)$$

3 Experimental Setup and Data

3.1 Attenuator Calibration

An important part of the experimental setup is the usage and calibration of an attenuator, a device that reduces the strength of an input signal. We use the attenuator to measure the integral of our gain squared, $\int g^2(f)$, and to prevent our voltmeter from overloading. We calibrated the attenuator and found the attenuating factor A , which is a scaling factor that determines how much the attenuator reduces the strength of the input signal. A diagram of our experimental setup to determine the attenuating factor is shown below:

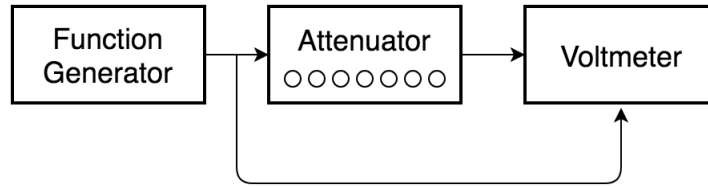


Figure 1: Experimental setup for attenuator calibration. The i th circle from the left on the attenuator represents the i th switch

In our setup, the function generator delivers a sine wave to the attenuator. Without attenuation, we measure the output voltage of the function generator V_0 . When V_0 is the input voltage into the attenuator, V_i is the output voltage of the attenuator when the i th switch from the left is on. For example, when we switch on the 1st button on the attenuator and input V_0 into the attenuator, we measure the output voltage V_1 . In order to calibrate the attenuation, we

measured V_i with the i th switch on and all others off. We measured values of V_i for switches 1, 2, 3, 4, and 7, as seen in Table 1.

$V_0(\text{mV})$	$V_1(\text{mV})$	$V_2(\text{mV})$	$V_3(\text{mV})$	$V_7(\text{mV})$
193	19.2	20.0	17.7	172

Table 1: Data of the input and output signal of the attenuator when the i th switch is on

Finding the attenuation factor A when multiple switches are on at the same time can be calculated using the following equation:

$$A = \prod \frac{V_i}{V_0} \quad (16)$$

For example, if switch 2 and 3 were switched on and the rest of the switches were off, the attenuation factor would be $A = \frac{V_2}{V_0} * \frac{V_3}{V_0}$. For our specific experiment, we turned on switches 1, 2, 3, and 7 at the same time and calculated our attenuating factor A . The attenuation factor A of our experiment is

$$A = 8.43 \times 10^{-4} \quad (17)$$

We have calculated the attenuating factor A that will help us determine the gain squared integral $\int g^2(f)$. In the next section, we will determine exactly our gain squared integral using the attenuation factor we have calculated.

3.2 Gain Integral

In this section, we will determine the integral of the the gain squared, which we refer to simply as the gain integral or G2B. Alongside every voltage measurement in this experiment, we also record the spectrum using Picoscope, a computer software used to measure the root mean square (RMS) voltage and the spectrum of the input signal. Although each spectrum in theory should be the same in all of our measurements, in practice, they are each slightly different.

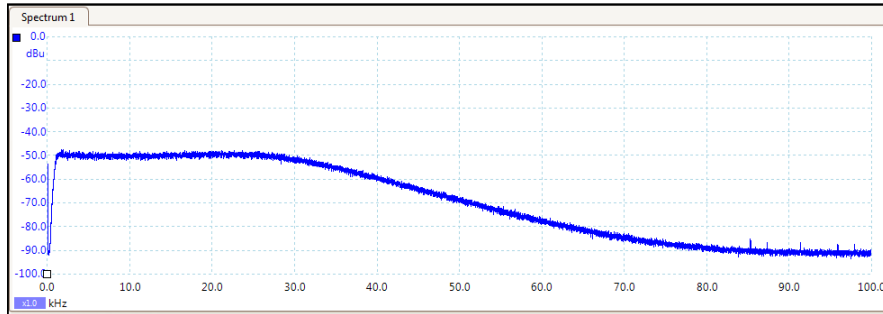


Figure 2: An example of a typical spectrum recorded by Picoscope

The spectrum we record with Picoscope is not $g^2(f)$; the shape of the spectrum is correct, but the scale is not. This is the motivation behind calculating the scaling factor g_f , a scaling factor that scales our measured spectrum to the appropriate size in order to calculate the gain integral. Using the spectrum of a measurement and g_f , $g^2(f)$ and subsequently $\int g^2(f)$ can be calculated. The equation for the gain factor g is as follows:

$$g = \frac{V_{out}^f}{AV_{in}} \quad (18)$$

where V_{out}^f is the output voltage for a certain frequency f , V_{in} is the input voltage, and A is the attenuation factor calculated earlier



Figure 3: Experimental setup for calculating the gain factor

The setup for calculating gain factor includes new equipment. The amplifier, a device that increases the strength of the input signal, is used in our experiment because the types of noise we are observing are too small to be detected without amplification. As a clarifying point, the attenuator is used to reduce the signal strength coming from the function generator and the amplifier is used to amplify the noise. The filter, a device that only allows a specific bandwidth of frequencies to pass through, is set to a bandwidth of 1 – 30kHz. For different frequency values, we measure the output voltage on Picoscope. In 1kHz increments between 1 – 30kHz, we calculate the gain factor g and plot the results.

$f(\text{kHz})$	$V_{out}^f(\text{V})$
193	19.2±0.01
0.09894	1.434±0.008
1.001	1.687±0.009
1.996	1.716±0.003
2.997	1.68±0.003
3.998	1.672±0.002
5.002	1.663±0.001
7.006	1.662±0.011
10	1.637±0.001
13.03	1.619±0.004
16	1.605±0.001

Table 2: Data of the input and output signal of the attenuator when the i th switch is on

Figure 4 shows a plot of $g(f)$ vs. f . The reason we do not simply calculate $g^2(f)$ from Figure 4 is because each measurement has a slightly different spectrum in practice. In addition, the spectrum recorded by Picoscope contains the

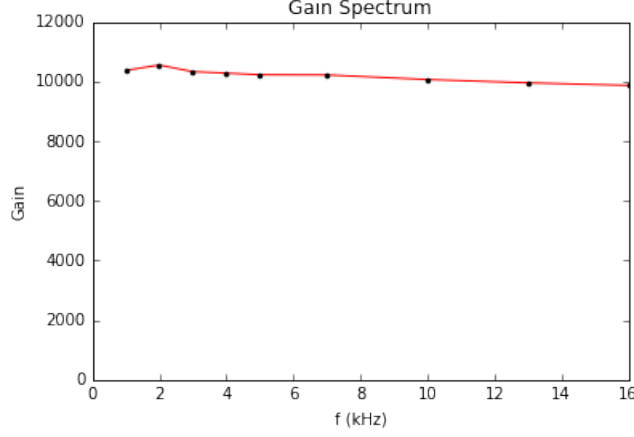


Figure 4: Spectrum of gain

whole spectrum of frequencies with thousands of frequencies, while our measurement contains less than a dozen points. By combining the scaling factor g_f and the spectrums recorded by the Picoscope, using the attenuation factor A from earlier (17) and $V_{in} = 0.193V$ we can calculate $g^2(f)$ more accurately.

frequency range (kHz)	g_f	G2B
3-4	10300	2.70385×10^{12}
5-6	10250	3.03089×10^{12}
8-9	10150	3.08871×10^{12}

Table 3: Data for determining gain factor and G2B

In order to pick a g_f to use, we pick one that is in a range of frequencies that is relatively flat so that the scaling matches the shape of the spectrum as closely as possible. We decide to use the scaling factor found in the 5-6 kHz range.

$$g_{5500} = 10250 \pm 40 \quad (19)$$

Error propagation in quadrature is encountered for the first time here. We explain briefly how this is done.

if q is any function of several variables $x...z$, then

$$\delta q = \sqrt{\left(\frac{\partial q}{\partial x} \delta x\right)^2 + \dots + \left(\frac{\partial q}{\partial z} \delta z\right)^2} \quad (20)$$

In order to estimate the error in our gain integral, we pick a spectrum that is a good representation of a majority of our data and calculate the gain integral

with three different frequency ranges. We find a conservative error using the difference of the highest and lowest gain squared integral.

$$G2B_{error} = G2B_{high} - G2B_{low} \quad (21)$$

$$G2B_{error} = G2B_{8-9kHz} - G2B_{3-4kHz} \quad (22)$$

$$G2B_{error} = 0.38486 \times 10^{12} \quad (23)$$

Ideally, we would perform this error calculation for every single spectrum in our data, but for times sake, we decided to calculate the error on one typical spectrum and assume the same error for all of the spectrums. In this section, we determined the gain factor or scaling factor that we would use to calculate the gain integral for each spectrum along with its error.

3.3 Johnson Noise

3.3.1 Boltzmann constant k measurement

The Boltzmann constant k is the first fundamental constant that we will be measuring. It is the constant that relates the average kinetic energy of a gas with its temperature most notably in the ideal gas law $PV = NkT$. It also shows up frequently in the study of blackbody radiation and thermodynamics like in the equipartition theorem we used earlier as the basis for the Johnson-Nyquist equation. In this section, we will determine the Boltzmann constant by through measurements of Johnson noise.

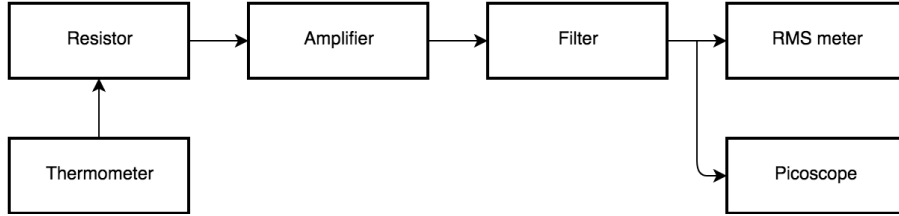


Figure 5: Experimental setup for Boltzmann constant k measurement

For this experimental setup, we add the usage of a thermometer to measure the temperature of the resistor. To gather the measurements that we will need, we measure the resistance of our different resistors with an ohmmeter, read the RMS voltage from the RMS meter, and save the spectrum of the signal using Picoscope. By measuring V_j , V_n , T , and calculating the gain integral at different resistances, we collect data that we fit with a linear function. According to equation (13), the Boltzmann constant will be the slope of a linear fit of our data. We record the noise of the system V_n by measuring voltage when we do not supply any voltage. With these measurements, we can determine the slope of the line, which gives us the Boltzmann constant k . The voltage of the noise is $V_n = 5.59 \pm 0.05$ mV

$R(\text{M}\Omega)$	V_j (V)	T (K $^\circ$)	$\int g^2(f)$
0.0820 ± 0.0001	0.06505 ± 0.00056	295.2 ± 0.1	$3.09655 \times 10^{12} \pm 0.38486 \times 10^{12}$
0.1210 ± 0.0001	0.0816 ± 0.0007	294.0 ± 0.1	$3.23520 \times 10^{12} \pm 0.38486 \times 10^{12}$
0.181 ± 0.0001	0.1015 ± 0.0007	296.3 ± 0.1	$3.32042 \times 10^{12} \pm 0.38486 \times 10^{12}$
0.22 ± 0.001	0.114 ± 0.0004	295.5 ± 0.1	$3.31051 \times 10^{12} \pm 0.38486 \times 10^{12}$
0.33 ± 0.001	0.1456 ± 0.0008	296.3 ± 0.1	$3.29273 \times 10^{12} \pm 0.38486 \times 10^{12}$
0.39 ± 0.001	0.1647 ± 0.0008	297.1 ± 0.1	$3.52032 \times 10^{12} \pm 0.38486 \times 10^{12}$
0.47 ± 0.001	0.1860 ± 0.0015	294.8 ± 0.1	$3.49177 \times 10^{12} \pm 0.38486 \times 10^{12}$
0.57 ± 0.001	0.2083 ± 0.0009	296.5 ± 0.1	$3.44091 \times 10^{12} \pm 0.38486 \times 10^{12}$

Table 4: Data for determining Boltzmann constant k

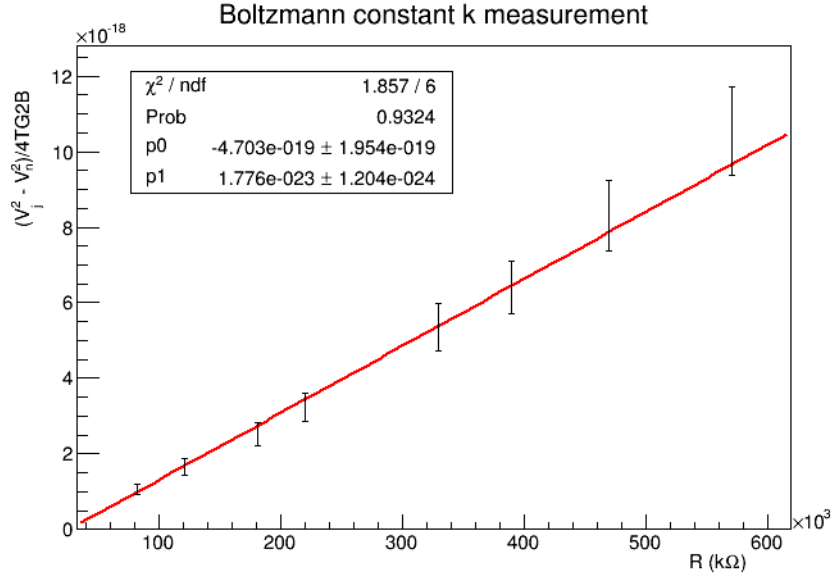


Figure 6: Line fitting result for Boltzmann constant k measurement

We calculate the gain integral by using the recorded spectrum and the scaling gain factor g_f . From the slope of our linear fit, we find the Boltzmann constant to be the following:

$$k = 1.776 \times 10^{-23} \pm 1.204 \times 10^{-24} \frac{J}{K} \quad (24)$$

In order to measure the quality of our measurement, we determine the number of sigmas we are from the internationally accepted value. The lower the number of sigmas, the more likely that our measurement is accurate, assuming the international accepted value is correct. We calculate the number of sigmas we are away from the accepted value using the following formula:

$$|k_{\text{experiment}} - k_{\text{accepted}}| < \sigma \times k_{\text{error}} \quad (25)$$

Our measurement of the Boltzmann constant is within 3.5σ of the accepted value. From previous examples of this experiment done by other groups, we can say that this is fairly good quality data.

In this section, we measured Johnson noise for a system with different resistances at a relatively stable temperature and calculated the Boltzmann constant k from the slope of linear best fit. In the next section, we will measure Johnson noise and determine the value of absolute zero.

3.3.2 Absolute zero T_0 measurement

In this section, we determine the fundamental constant absolute zero T_0 , where particles have minimal vibrational energy, a state in which the enthalpy and entropy of a cooled ideal gas reaches its minimum. The lowest theoretical temperature is internationally agreed upon value is $T_0 = -273.15^\circ$, a few degrees cooler than the average temperature of the universe. In our determination of absolute zero, we use the same experimental setup as the Boltzmann constant k measurement (Figure 5). However, instead of measuring Johnson noise with different resistors at a reasonably fixed temperature, we fix the resistor and measure Johnson noise at different temperatures.

Similar to the experimental procedure for determining the Boltzmann constant k , the signal goes through the resistor, amplifier, and filter to the Voltmeter and Picoscope. However, when determining absolute zero, we heat up the resistor and take measurements of V_j and record the spectrum at different temperatures. After heating up the resistor and taking adequate measurements, we place the resistor into liquid nitrogen and record a measurement of V_j and the spectrum. The value of the noise term is $V_0 =$ The results of our measurement are shown in Table 5.

T (C°)	R (M Ω)	V (mV)	$\int g^2(f)$ (no units)
-196 ± 0.1	0.26	93.39 ± 0.65	$3.60909 \times 10^{12} \pm 0.38486 \times 10^{12}$
28.1 ± 0.1	0.27	130.0 ± 1.9	$3.26317 \times 10^{12} \pm 0.38486 \times 10^{12}$
92 ± 0.3	0.27	140.5 ± 2.0	$3.36941 \times 10^{12} \pm 0.38486 \times 10^{12}$
129.8 ± 0.2	0.27	146.6 ± 2.0	$3.2163 \times 10^{12} \pm 0.38486 \times 10^{12}$
162.7 ± 0.2	0.27	151.2 ± 2.0	$3.99385 \times 10^{12} \pm 0.38486 \times 10^{12}$

Table 5: Data for determining absolute zero T_0

Like in the determination of the Boltzmann constant, we calculate our gain integral using our gain factor and recorded spectrum. Using equation (14) and the x intercept of the linear fit of our data, we find absolute zero to be the following:

$$T_0 = -435.9 \pm 22.7^\circ\text{C} \quad (26)$$

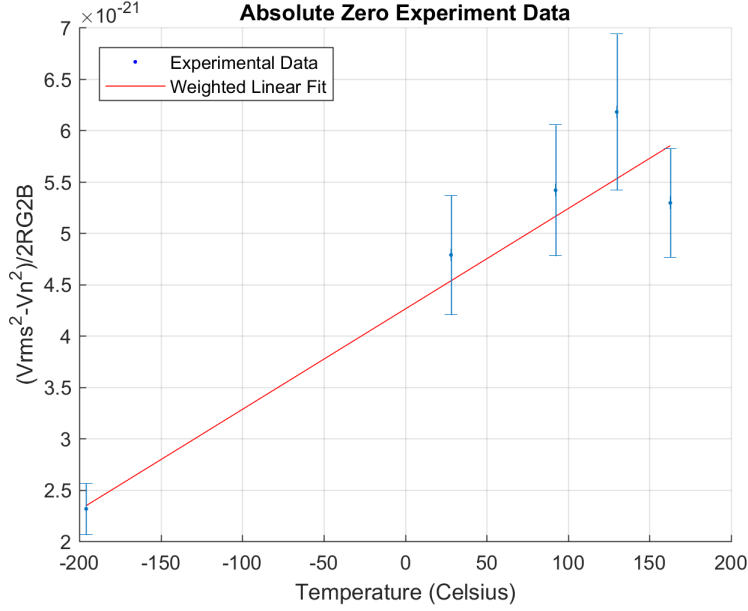


Figure 7: Line fitting result for absolute zero T_0 measurement

Our measurement of T_0 is within 10σ of the accepted value of -273.15° . Our T_0 is not within an acceptable. There are many potential sources of systematic error.

One of the systematic errors for our measurement comes from a bad spectrum, as seen in Figure 8. Notice the large bump at around 35,000 Hz. This increases G2B significantly. Comparing this figure to our typical spectrum from earlier, it is clear that there will be some systematic error in the gain integral. The source of the error likely comes a resonate frequency that arises when the temperature of the system begins to increase. Looking at all of the spectrums of the measurements we took to calculate absolute zero, the large bump at 35 kHz grows as the temperature increases. We do know why this systemic error occurs, but we can account for it in our error analysis. Further or repeated data collection would likely shed light on how our system is changed as the temperature varies.

In this section, we measured Johnson noise for a system at different temperatures T and calculated absolute zero T_0 from the x-intercept of the linear fit of our data. In the next section, we will move on to measure Shot noise and determine the value of elementary charge e .

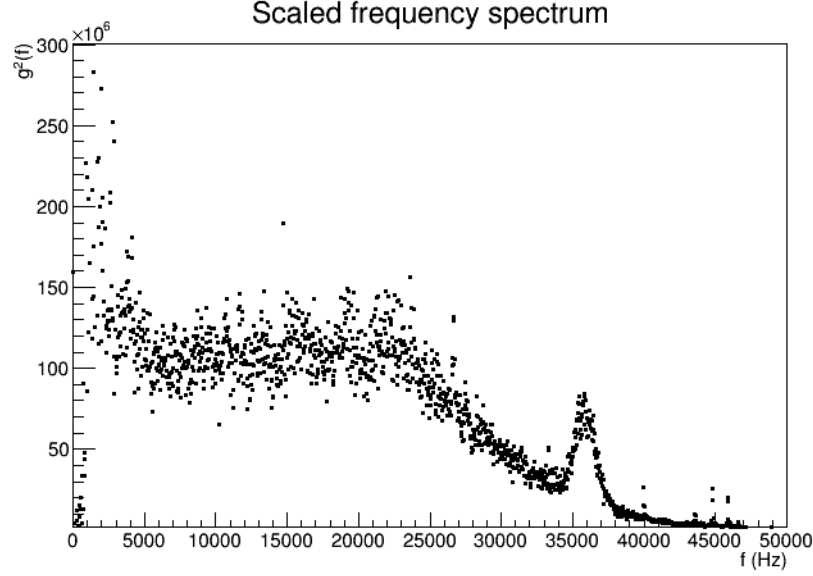


Figure 8: Example of a bad spectrum with systematic error

3.4 Shot Noise

3.4.1 Elementary charge e measurement

In this section, we explain the experimental setup and process for determining the elementary charge e . First measured by Robert A Millikan's oil drop experiment in 1909, the accepted international value is currently $e = 1.602 \times 10^{-19} \text{C}$. After the oil drop experiment, elementary charge has been determined in a number of ways including using the Josephson and von Klitzing constants, the CODATA method, and of course through Shot noise. In our experiment, we will determine the value of elementary charge e experimentally through Shot noise. Our experimental setup is shown below:

As seen in Figure 9, a power source with output V_{light} is connected to a lightbulb from a flashlight, allowing us to vary the intensity of light incident on the photodiode. Depending on the intensity of the light, the photodiode outputs V_{avg} , which we measure using a multimeter. The signal from the photodiode then goes through the amplifier and filter to our RMS meter and Picoscope, which record V_s and the spectrum, respectively. For each measurement, $R = 99.4 \pm 0.1 \text{k}\Omega$ and $V_n = 0.07887 \pm 0.00052 \text{V}$. The following table shows the results of these measurements:

Like in the Johnson noise experiment, the gain integral $\int g^2(f)$ is calculated using the scaling factor g_f and our recorded spectrum for each measurement. From the slope of the linear fit of Figure 10, we determine the value of e as the following:

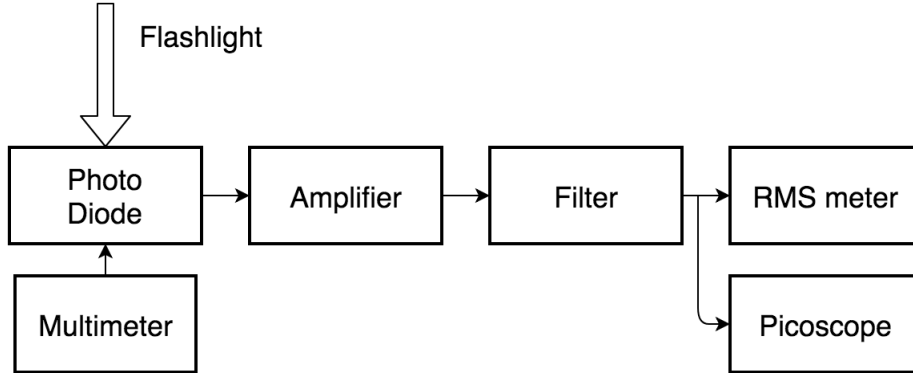


Figure 9: Experimental setup for elementary charge e through Shot noise

V_{avg} (V)	V_s (V)	V_{light} (V)	$\int g^2(f)$
0.273 ± 0.001	0.1541 ± 0.0009	0.9	$3.03089 \times 10^{12} \pm 0.38486 \times 10^{12}$
0.911 ± 0.001	0.2479 ± 0.0017	1.2	$2.96376 \times 10^{12} \pm 0.38486 \times 10^{12}$
1.994 ± 0.001	0.3480 ± 0.0024	1.5	$2.86114 \times 10^{12} \pm 0.38486 \times 10^{12}$
3.40 ± 0.01	0.4384 ± 0.0037	1.8	$2.78599 \times 10^{12} \pm 0.38486 \times 10^{12}$
5.30 ± 0.01	0.5343 ± 0.0055	2.1	$2.7013 \times 10^{12} \pm 0.38486 \times 10^{12}$
7.47 ± 0.01	0.6022 ± 0.0057	2.4	$2.54399 \times 10^{12} \pm 0.38486 \times 10^{12}$

Table 6: Data for determining absolute zero T_0

$$e = 0.8747 \times 10^{-19} \pm 6.502 \times 10^{-21} \text{C} \quad (27)$$

This data is within 10σ , meaning there is a large amount of systematic error. An in-depth exploration of systematic errors is needed. One clue that we are dealing mostly with systematic error is that our data points are a good linear fit, meaning that our data collection process is precise but not accurate. In other words, there is systematic error that is scaling or shifting our data.

One potential source of systematic error comes from an unexpected change in our amplifier before data collection. Between data collection for Johnson noise and Shot noise, a capacitor was added to the amplifier, which we hypothesize has affected our gain integral calculations. In an attempt to mitigate this, we retook parts of our gain spectrum data, but found no discernible difference in the gain factor g_f . However, one possible explanation is that the new amplifier requires modifications to our theory that we are currently unaware of.

Another source of systematic error may come from the quality of the spectrums, specifically from background noise. Upon inspecting the spectrums for our elementary charge data, we noticed a large variance in the values of the gain squared between 5000 - 20,000 kHz. Since our scaling factor g_f maps one value of the raw spectrum gain to one value of the scaled gain $g(f)$, it is possible that the high variance of points above and below gain factor led to gain integrals

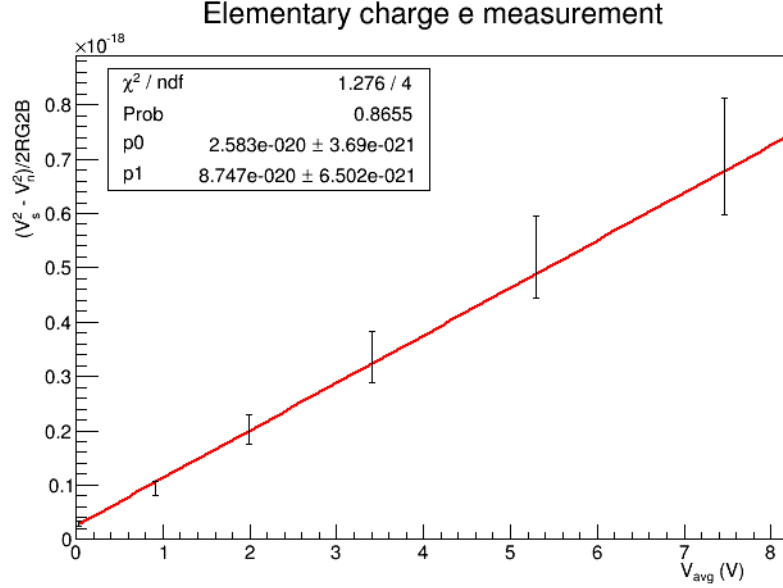


Figure 10: Linear fit for elementary charge e

that were systematically too large or too small for every measurement. In a previous iteration of our experiment, we used a different scaling factor g_f , and found better results for elementary charge e , within 3 sigmas, but worse results for the Boltzmann constant k . Possible corrections to reduce systematic errors in the future may be to determine and use different gain factors for each of the three experimental setups.

In this section, we explained our experimental process, determined the elementary charge e through Shot noise, and discussed possible sources of systematic error. In the next section, we will summarize some of the key points in our paper.

4 Summary

k (J/k)	T_0 (C°)	e (C)
$1.776 \times 10^{-23} \pm 1.204 \times 10^{-24}$	-435.9 ± 22.7	$0.8747 \times 10^{-19} \pm 6.502 \times 10^{-21}$

Table 7: Summary of Results

In summary, through the measurement of Johnson noise and Shot noise, two types of irreducible noise that arise from thermal agitations and the discrete nature of charge respectively, we determined experimentally the Boltzmann constant k , absolute zero T_0 , and elementary charge e . Though some of our

measurements have large systematic errors, we have discussed their potential sources and approaches to reducing the error. The largest source of systematic error comes from our gain integral and how it changes for each experiment. With more time in the future, one could reduce the systematic error dramatically by collecting data and understanding more thoroughly the gain affects the system. Overall, the experiment was able to determine one of the three fundamental constants k to a reasonable degree of certain (3σ), while the other two constants were left with more to be desired.

5 Conclusion

Many physicists see the connections between the macroscopic and microscopic as a cornerstone of physics. When new discoveries, like Johnson and Shot noise, can be tied to fundamental constants, the evidence and strength of physical theories strengthens. This experiment is a good example of this kind of connection. All experiments that work with electronics will need to consider Johnson and Shot noise, making this experiment especially relevant to experimentalists. Furthermore, the connection to three fundamental constants k , T_0 , and e makes it especially relevant to theorists. In conclusion, Johnson and Shot noise are two types of irreducible noise that are beneficial to understand for all physicists, both experimentalists and a theorists.

6 Acknowledgements

We want to thank Professor Oh and Dr. Bomze for their help in our experiment. I'd like to thank my lab partner William Willis for the many hours we've spent together working on this experiment.

References

- [1] Marc de Jong. Sub-poissonian shot noise, 1996.
- [2] Fan Jiang Jiyingmei Wang. Johnson and shot noise experiment, 2014.
- [3] Dennis V. Perepelitsa. Johnson noise and shot noise, 2006.