# Johnson and Shot Noise Experiment

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## Abstract ¶

In electronic measurements, one observes both "signal" and "noise", the former usually being of interest. In most experiments, one tries to increase the signal while simultaneously decreasing noise. But, there are two types of noise that cannot be reduced: Johnson noise and Shot noise, the former coming from thermal agitations and the latter from the fact that electrons are quantized. These two irreducible noises are connected to three fundamental constants and can be used to calculate them: Boltzmann constant $k$ by measuring RMS voltage across a resistor, absolute zero $T_0$ by varying the temperature of a resistor, and fundamental charge $e$ by varying intensity of light on a photodiode. Our experiment measured the Boltzmann constant $k = 1.776 \times 10^{-23} \pm 1.204 \times 10^{-24} \frac{J}{K}$, absolute zero $T_0 = -215.79 \pm 21.51 \circ \mathrm{C}$, and fundamental charge $e = 1.301 \times 10^{-19} \pm 1.403 \times 10^{-20} \mathrm{C}$.

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# Contents



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# 1 Introduction

Johnson noise is an irreducible, unwanted disturbance in electronic signal coming from the thermal agitation of charge carriers (usually electrons) in electrical conductors in equilibrium, regardless of any applied voltage. Discovered by John Johnson in 1926 at Bell Labs and formalized theoretically by Johnson's colleague Harry Nyquist, Johnson noise is connected to two fundamental constants $k$ and $T_0$, Boltzmann constant and absolute zero.

Shot noise is another type of irreducible noise that occurs when current flows through a wire. Just as photons in a beam of light are emitted randomly, a stream of electrons (current) is not constant. This is most apparent in small currents; if the current is reduced to a handful of electrons, the relative number of electrons, will be significant. These fluctuations are what we call shot noise. Discovered by Walter Schottky in 1918 while studying fluctuations of current in vacuum tubes, Shot noise is connected to the fundamental constant $e$, the charge of an electron. The magnitude of Shot noise increases with the square of the expected number of events, meaning shot noise is most frequently observed with small currents that are amplified. Unlike Johnson noise, Shot noise is temperature and frequency independent.

# 2 Theory

## 2.1 Johnson Noise

Johnson noise was formulated theoretically by Harry Nyquist using the second law of thermodynamics and equipartition theorem.

Nyquists formula is the following:

$$V_{rms}^2 = 4kTRdf \qquad (1)$$

where $R$ is resistance, $T$ is the temperature, $k$ is Boltzmann constant, $V_{rms}$ is RMS voltage, and $df$ is the frequency range we are measuring.

Although there is an exact quantum formulation, our experimental setup is within the classical approximation: frequencies from 1k - 30kHz and temperatures ~300$K$.

In order to use the formula in practice, we will need to modify the Nyquist formula. The first modification we make comes from a capacitor in parallel to our resistor, modifying $V_{rms}$. To account for this, using AC circuit theory, we modify Nyquist theorem in the following way:

$$V_{rms}^2 = 4kT\frac{R}{1+(2\pi fCR)^2}df \qquad (2)$$

where $C$ is capacitance

The second modification and third modification to the Nyquist formula (and to the Shot noise formula) deal with amplification and background noise, respectively, which we will be discussed in the Amplification and Background section.

Using a modified version of Nyquist's formula, we will determine fundamental constants $k$ and $T_0$, Boltzmann constant and absolute zero, the former by holding $T$ constant and varying $R$ and the latter by holding $R$ constant and varying $T$:

$$\frac{V_{rms}^2}{4R_m} = kT \qquad (3)$$

$$\frac{V_{rms}^2}{4T} = kR_m \qquad (4)$$

where $R_m = \frac{R}{1+(2\pi fCR)^2}$. $k$ Boltzmann will be determined by the slope of the line of (3) and the $T_0$ will be determined the x-intercept of the linear fit of (4). In practice, we will not use the formulas (3) and (4) in their current forms, but we include them to help make clear how we will determine $k$ and $T_0$

## 2.2 Shot Noise

Shot noise is another type of irreducible noise that occurs when current flows through a wire. Just as photons in a beam of light are emitted randomly, a stream of electrons i.e. current is not constant. This is most apparent in small currents; if the current is reduced to a handful of electrons, the relative number of electrons i.e. strength of current, will be significant. These fluctuations are what we call shot noise.

Using a circuit with a photodiode:

$$dI_{rms}^2 = 2eI_{avg}df \qquad (?)$$

Using Ohm's law $V = I_{avg}R$ and substituting $Iavg^2 = \frac{V^2}{R^2}$, we get:

$$dV_{rms}^2 = 2eI_{avg}R^2df \qquad (?)$$
$$dV_{rms}^2 = 2eV_{avg}Rdf \qquad (?)$$

In practice, we will use a modified version of this that accounts for gain factor and noise. Once modified, we will be able to measure the fundamental charge, $e$.

## 2.3 Amplification and Background

Johnson and shot noise are both too small to observe without amplification. When we amplify our signal, we will need to consider a gain factor $g(f)$. With this amplification, the mean square of the voltage we observe will be:

$$dV_{out}^2 = dV_{in}^2 g^2(f) \qquad (?)$$

We substitute $V_{out}^2$ into the Nyquist formula and Shot formula:

$$V_{j,rms}^2 = 4kRTV_{avg} \int g^2(f)df \qquad (?)$$

$$V_{s,rms}^2 = 2eRV_{avg} \int g^2(f)df \qquad (?)$$

Lastly, we account for background from the system by adding a term $V_{n,rms}$, which we measure for each experiment:

$$V_{j,rms}^2 = 4kRTV_{avg} \int g^2(f)df + V_{n,rms}^2 \qquad (?)$$

$$V_{s,rms}^2 = 2eRV_{avg} \int g^2(f)df + V_{n,rms}^2 \qquad (?)$$

All of the modifications have been complete, and we will be using the formulas in their current forms for the experiment.

## 2.4 Summary

In summary, these are the modfied Nyquist formulas and Shot noise formulas we will use:

The equation we use for the Boltzmann constant measurement is

$$\frac{V_{j,rms}^2 - V_{n,rms}^2}{4T \int g^2(f)df} = kR \qquad (?)$$

The slope of the line will be fundamental constant $k$

The equation we will use for the absolute zero measurement is

$$\frac{V_{j,rms}^2 - V_{n,rms}^2}{4R \int g^2(f)df} = kT \qquad (?)$$

The x-intercept of this line will be our absolute zero measurement, $T_0$.

The equation we will use for the shot noise measurement is

$$\frac{\langle V_s^2 \rangle - \langle V_n^2 \rangle}{2R \int g^2(f)df} = eV_{av} \qquad (?)$$

The slope of the line will be fundamental constant $e$

# 3 Experimental Setup and Data

## 3.1 Attenuator Calibration

In order to measure our gain integral and not overload our $V_{rms}$ reader, we need to use an attenuator. This section will describe how we calibrated the attenuator and found $A$, the attenuating factor.

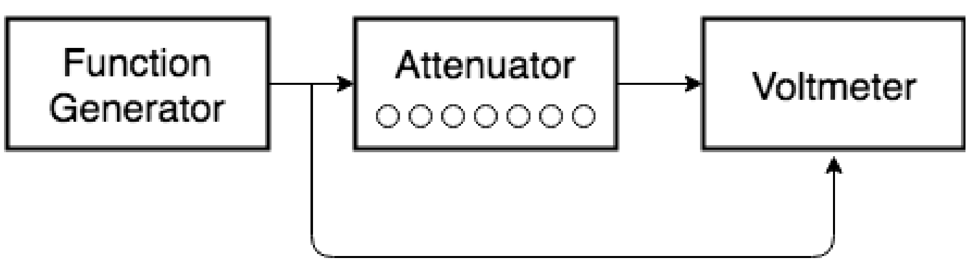



Figure ??: Experimental setup for attenuator calibration. Each circle on the attenuator represents one swit

**Measuring $A$, attenuating factor**

We have a function generator delivering a voltage with frequency that can be varied. We measure the output of the function generator first without attenuation, $V_0$. $V_0$ will be the input voltage and $V_i$ will be the output voltage when the $i$th switch from the left is on. We measure $V_i$ when the $i$th switch is on and all others are off. The formula for attenuating factor, $A$, takes the product of the $V_i$'s.

$$A = \prod \frac{V_i}{V_0} \qquad (?)$$

where $i$ is the $i$th button

These are the raw values of the voltages $V_i$ when the $i$th switch is on

| $V_0$ **(mV)** | $V_1$ **(mV)** | $V_2$ **(mV)** | $V_3$ **(mV)** | $V_4$ **(mV)** |
|---|---|---|---|---|
| 193 | 19.2 | 20.0 | 17.7 | 172 |

Table 1: Data of the input and output signal of the attenuator

The attenuator factor $A$ of our experiment is

$$A = 8.43 \times 10^{-4}$$

## 3.2 Gain Integral

The gain factor is important in Johnson and Shot noise. The method for obtaining it is described in this section.

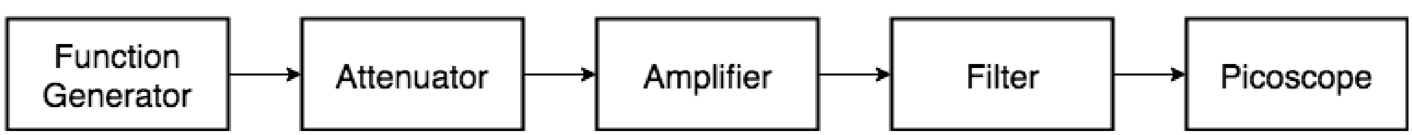



Figure ??: Experimental setup for gain factor

Because the noise we are observing is too small compared to the signal, we need to use an amplifier. The filter is set to a bandwidth of $1 - 30kHz$. When we change the frequency of the function generator, the read by the picoscope will change. The gain factor can be calculated using the input voltage $V_{in}$, the attenuation factor $A$, and the output voltage $V_{out}$, which we obtain by varying the frequency of the function generator.

$$g = \frac{V_{out}^{i}}{AV_{in}} \qquad (?)$$

We measure the gain factor using by varying the frequency of the function generator.

| $f$(kHz) | $V_{avg}$(V) |
|---|---|
| 0.09894 | 1.434±0.008 |
| 1.001 | 1.687±0.009 |
| 1.996 | 1.716±0.003 |
| 2.997 | 1.68±0.003 |
| 3.998 | 1.672±0.002 |
| 5.002 | 1.663±0.001 |
| 7.006 | 1.662±0.011 |
| 10 | 1.637±0.001 |
| 13.03 | 1.619±0.004 |
| 16 | 1.605±0.001 |

Table 3: Data of gain factor measurements

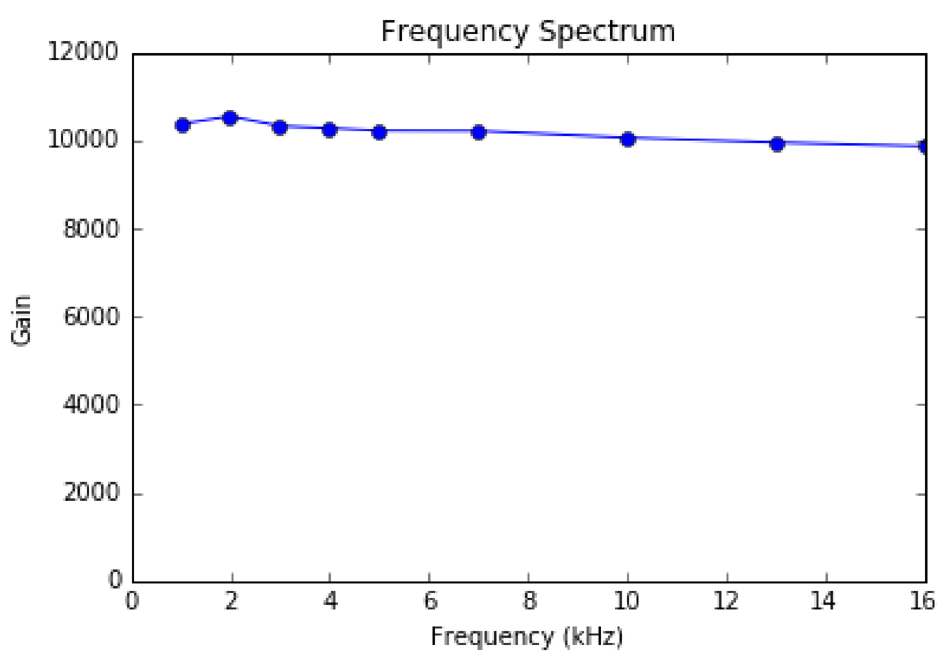



Figure ??: Experimental setup for gain factor

We decide to use the scaling factor found in the 5-6 kHz range, scaling factor = 10250

$$g_{5500} = 10250$$

Error propagation in quadrature is encountered for the first time here. We explain briefly how it this is done.

if $q$ is any function of several variables $x \ldots z$, then

$$\delta q = \sqrt{\left(\frac{\partial q}{\partial x}\delta x\right) + \ldots + \left(\frac{\partial q}{\partial z}\delta z\right)} \qquad (?)$$

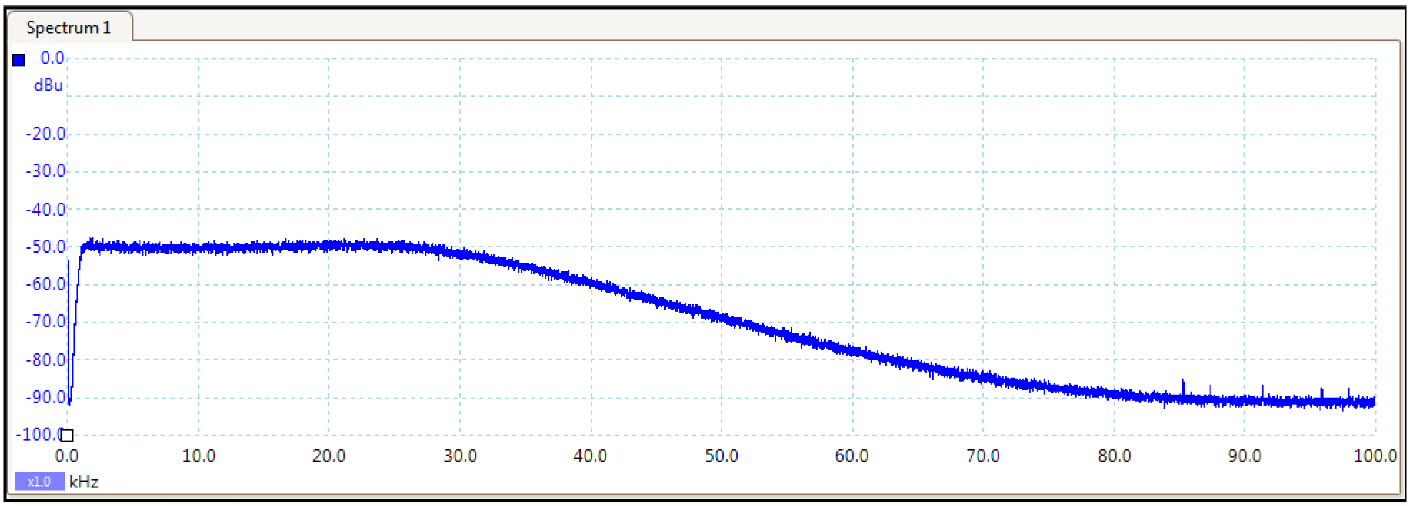



Figure ??: Typical spectrum measured from picoscope

Alongside every voltage measurement in this experiment, we recorded the spectrum using the picoscope program. This spectrum, although in theory should be similar in all the measurements we do, in practice are slightly different. But, this spectrum alone does not give us $g^2(f)$, which is $gain\ integral\ squared$; the shape of the spectrum is correct, but the scale is not. This is the motivation for calculating the scaling factor $g_n$, which is the scaling factor we will use to convert our spectrum to the correct units and scale. Through the use of a ROOT program that takes the spectrum of a measurement and the scaling factor $g_n$ we calculate, we can determine $g^2$ and take the integral to get the scaling factor $\int g^2(f)$, which we also refer to as G2B.

Our scaling $g_n$ is determined through by measuring how the voltage coming out of the amplifier is affected by frequency. In other words, we measure the output voltage of the amplifier as a function of frequency $f$. The data is shown in the table below. The data is a spectrum with the correct scaling. So, why do we not use this graph to calculate $g^2(f)$ for all of the measurements? The reason was briefly mentioned earlier: each measurement has a slightly different spectrum and contains the full spectrum of frequencies, while our crude measurement only contains less than a dozen points. By combining the scale of the graph from our $V_{out}(f)$ measurements and the spectrums recorded by the picoscope, we can calculate $g^2(f)$ more accurately.

In order to pick a scaling factor from our $V_{out}(f)$ data, we pick a range of the graph that is relatively flat, in our case between 3-4 Hz. In order to get a conservative error on our gain squared integral, we also choose a scaling factor that is between 5-6Hz and 8-9Hz. For each scaling factor, we calculate the gain squared integral of one of our spectrums that we think is a good representation of our data in general and find a conservatoive error using the difference of the highest and lowest gain squared integral.

$$G2B_{error} = G2B_{high} - G2B_{low}$$

| **frequency range (kHz)** | **scaling factor** | **G2B** |
|---|---|---|
| 3-4 | 10300 | 2.70385e12 |
| 5-6 | 10250 | 3.03089e12 |
| 8-9 | 10150 | 3.08871e12 |

$$G2B_{error} = G2B_{8-9kHz} - G2B_{3-4kHz}$$
$$G2B_{error} = 0.38486 \times 10^{12}$$

## 3.3 Johnson Noise

### 3.3.1 Boltzmann constant $k$ measurement

The Boltzmann constant relates the average kinetic energy of a gas with its temperature.

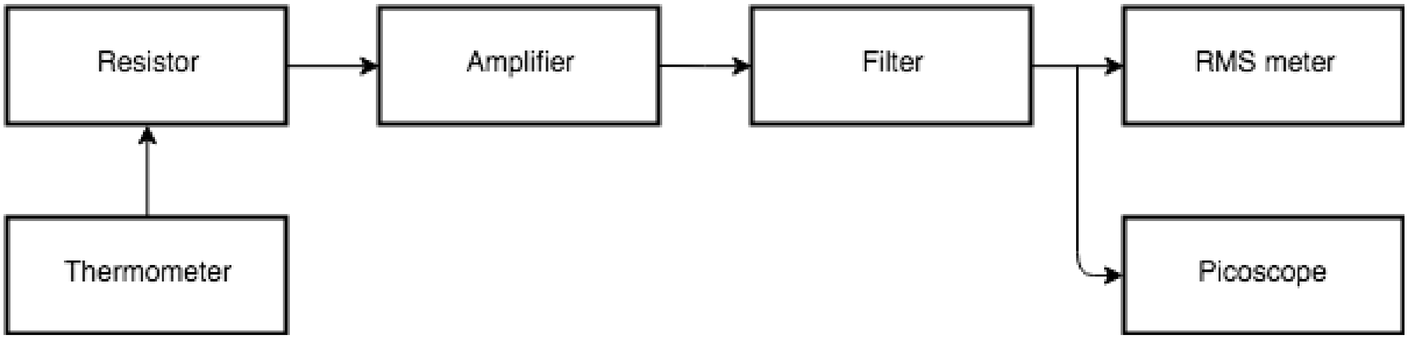



Figure ??: Experimental setup for Boltzmann constant measurement

Accepted value:

$$k = 1.381 \times 10^{-23} \text{J/K}$$

We vary the resistors, measure the temperature, and record the voltage and spectrum from the picoscope. We record the noise of the system $V_n$ by measuring voltage when we do not supply any voltage. With these measurements, we can determine the slope of the line of the following equation, which gives us the Boltzmann constant $k$.

$$\frac{\langle V_j^2 \rangle - \langle V_n^2 \rangle}{4T \int g^2(f)df} = kR \qquad (?)$$

| $R$ **(MΩ)** | $V_{rms}$ $(V)$ | $T(K\circ)$ | $g^2(f)$ |
|---|---|---|---|
| 0.0820±0.0001 | 0.06505±0.00056 | 295.2±0.1 | $3.09655\times10^{12}\pm0.38486\times10^{12}$ |
| 0.1210±0.0001 | 0.0816±0.0007 | 294.0±0.1 | $3.2352\times10^{12}\pm0.38486\times10^{12}$ |
| 0.181±0.0001 | 0.1015±0.0007 | 296.3±0.1 | $3.32042\times10^{12}\pm0.38486\times10^{12}$ |
| 0.22 | 0.114±0.0004 | 295.5±0.1 | $3.31051\times10^{12}\pm0.38486\times10^{12}$ |
| 0.33 | 0.1456±0.0008 | 296.3±0.1 | $3.29273\times10^{12}\pm0.38486\times10^{12}$ |
| 0.39 | 0.1647±0.0008 | 297.1±0.1 | $3.52032\times10^{12}\pm0.38486\times10^{12}$ |
| 0.47 | 0.1860±0.0015 | 294.8±0.1 | $3.49177\times10^{12}\pm0.38486\times10^{12}$ |
| 0.57 | 0.2083±0.0009 | 296.5±0.1 | $3.44091\times10^{12}\pm0.38486\times10^{12}$ |

$V_{noise} = 5.592mV \pm 46.47\mu V$

Table 3: Data of different resistances used for Boltzmann constant measurement

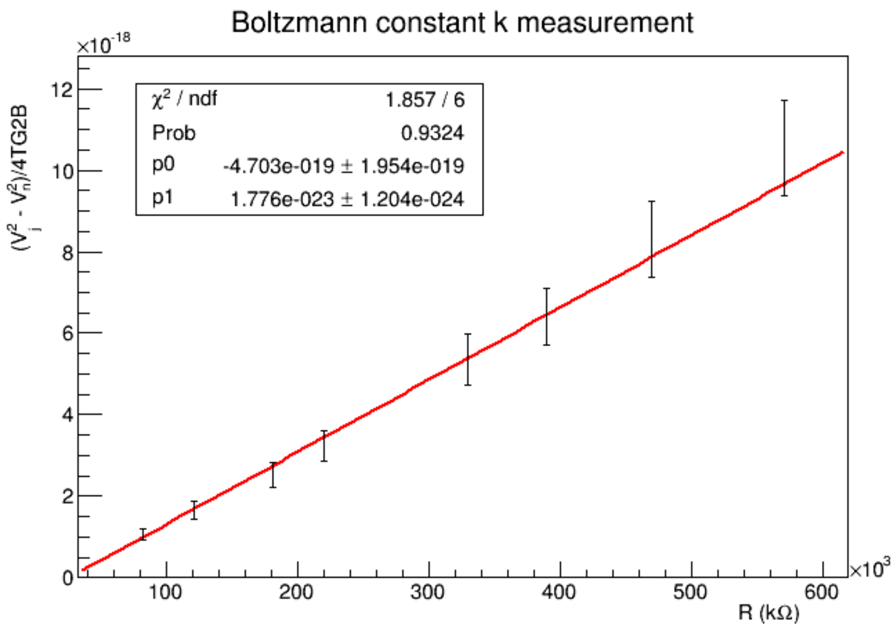



Figure ?: Line fitting result for Boltzmann constant determination

$$k = 1.776 \times 10^{-23} \pm 1.204 \times 10^{-24} \frac{J}{K}$$

We calculate the number of sigmas we are away from the accepted value using the following formula:

$$|k_{experiment} - k_{accepted}| < \sigma \times k_{error}$$

Our measurement of the Boltzmann constant is within 3.5 $\sigma$ of the accepted value.

### 3.3.2 Absolute zero $T_0$ temperature measurement

Accepted value :

$$T_0 = -273.15 \text{ C}\circ$$

Absolute zero is the lowest theoretical temperature where particles have minimal vibrational energy. We use the same set up for T_0

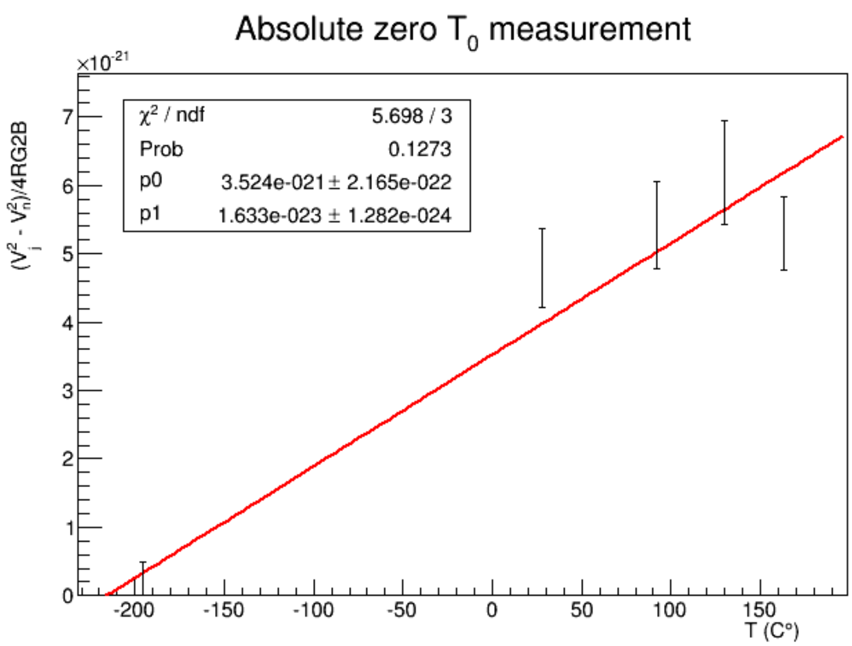
We take the same measurements as when we determined the Boltzmann constant $k$ except we heat the resistor. -add in part about liquid nitrogen

$$\frac{\langle V_j^2\rangle - \langle V_n^2\rangle}{4R\int g^2(f)df} = kT \qquad (?)$$

| $T$ (C∘) | $R$ ($M\Omega$) | $V$ (mV) | $g^2$ (no units) |
|---|---|---|---|
| $-196\pm0.1$ | 0.26 | $93.39\pm0.65$ | $3.60909\times10^{12}\pm0.38486\times10^{12}$ |
| $28.1\pm0.1$ | 0.27 | $130.0\pm1.9$ | $3.26317\times10^{12}\pm0.38486\times10^{12}$ |
| $92\pm0.3$ | 0.27 | $140.5\pm2.0$ | $3.36941\times10^{12}\pm0.38486\times10^{12}$ |
| $129.8\pm0.2$ | 0.27 | $146.6\pm2.0$ | $3.2163\times10^{12}\pm0.38486\times10^{12}$ |
| $162.7\pm0.2$ | 0.27 | $151.2\pm2.0$ | $3.99385\times10^{12}\pm0.38486\times10^{12}$ |

$V_{noise} = 5.592mV \pm 46.47\mu V$

Table ??: Different temperatures for measuring Absolute zero




$$T_0 = -215.79 \pm 21.51 \circ \mathrm{C}$$

Our measurement of $T_0$ is within 3 $\sigma$ of the accepted value.

One of the systematic errors for our measurement comes from a bad spectrum, as seen below. Notice the large bump at around 35,000 Hz. This increases G2B significantly.

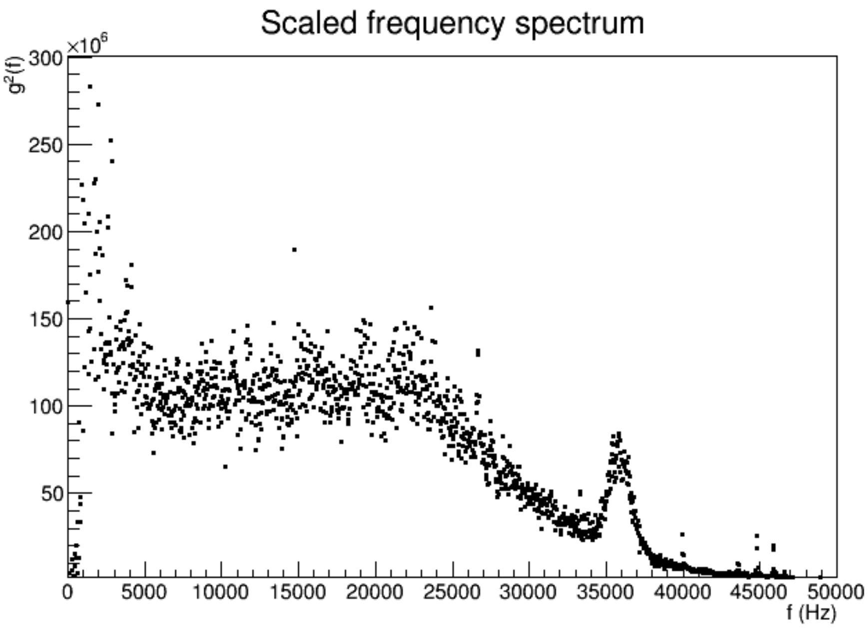



Figure ??: Example of bad spectrum

## 3.4 Shot Noise

### 3.4.1 Elementary charge $e$ measurement

The elementary charge $e$ is the charge of a single proton or the magnitude of the charge of an electron.

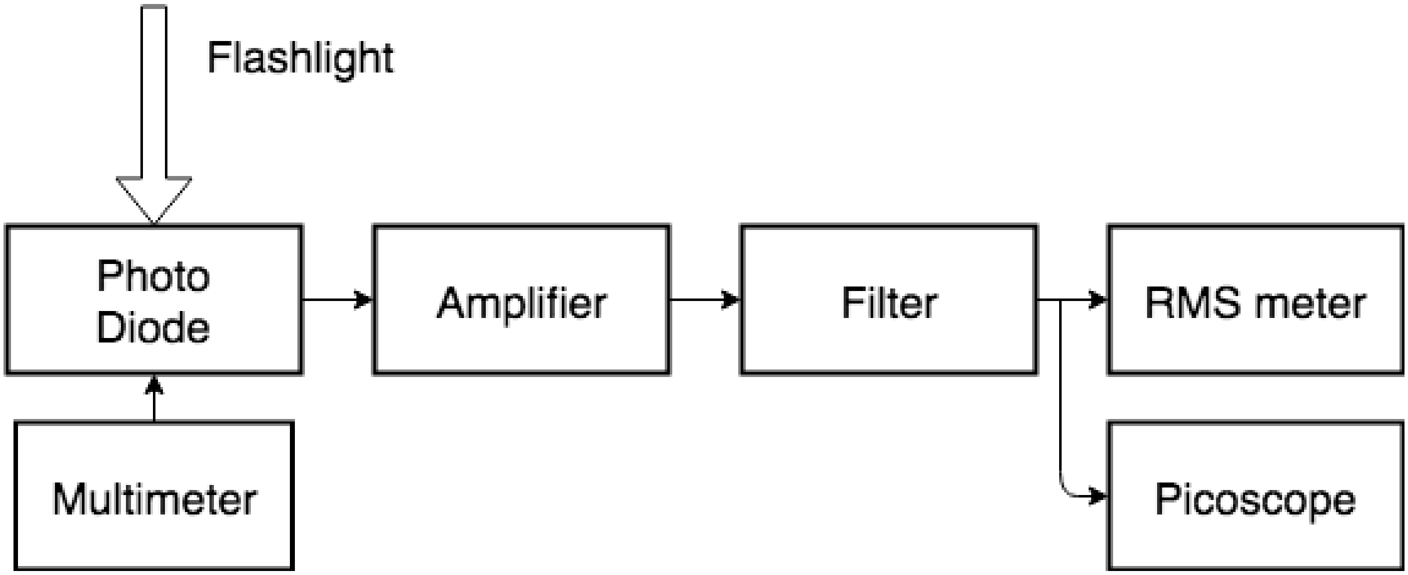



Figure ??: Experimental setup for elementary charge measurement

Accepted value:

$$e = 1.602 \times 10^{-19}$$

We connect a power source to a lightbulb which allows us to vary the light intensity. We shine the light onto the photodiode and read $V_{avg}$ using a multimeter. Then, we connect the photodiode through an amplifier and read off the $V_{rms}$ and spectrum from the computer. We measure the resistance $R$ at each measurement. This will give us the fundamental charge $e$ as the slope of a line.

$$\frac{\langle V_s^2 \rangle - \langle V_n^2 \rangle}{2R \int g^2(f) df} = eV_{av} \qquad (?)$$

This is the raw data from measuring elementary charge $e$

| $V_{av}$(V) | $R$(kΩ) | $V_{rms}$(V) | $V_n(mV)$ | $light(V)$ | $g^2$ |
|---|---|---|---|---|---|
| 0.273±0.001 | | 0.1541±0.0009 | | 0.9 | $3.03089\times10^{12}\pm0.38486\times10^{12}$ |
| 0.911±0.001 | | 0.2479±0.0017 | | 1.2 | $2.96376\times10^{12}\pm0.38486\times10^{12}$ |
| 1.994±0.001 | 99.4±0.1 | 0.3480±0.0024 | 0.07887±0.00052 | 1.5 | $2.86114\times10^{12}\pm0.38486\times10^{12}$ |
| 3.40±0.01 | | 0.4384±0.0037 | | 1.8 | $2.78599\times10^{12}\pm0.38486\times10^{12}$ |
| 5.30 ±0.01 | | 0.5343±0.0055 | | 2.1 | $2.7013\times10^{12}\pm0.38486\times10^{12}$ |
| 7.47±0.01 | | 0.6022±0.0057 | | 2.4 | $2.54399\times10^{12}\pm0.38486\times10^{12}$ |

Table ??: RMS voltage and average voltage for measuring elementary charge

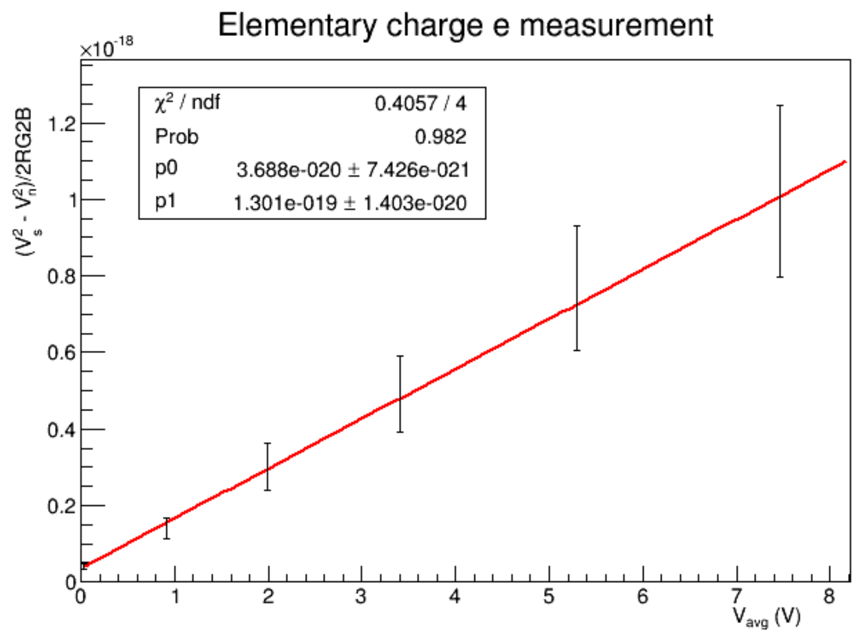



Figure ??: Line fitting results of elementary charge e measurement

From the slope of the plot, we determine the value of $e$ as the following:

$$e = 1.301 \times 10^{-19} \pm 1.403 \times 10^{-20}\mathrm{C}$$

# 4 Summary

| $k$(J/k) | $T_0 \circ C$ | $e(C)$ |
|---|---|---|
| $k$=$1.776\times10^{-23}\pm1.204\times10^{-24}\,\frac{J}{K}$ | $T_0$=$-215.79\pm21.51\circ$C | $e$=$1.301\times10^{-19}\pm1.403\times10^{-20}$C |

Table ??: Results

A summary of our results shows x, y, z.

# 5 Acknowledgements

We want to thank Professor Oh and Dr. Bomze for their help in our experiment. I'd like to thank my lab partner William Willis for the many hours we've spent together working on this experiment.