Linear Systems TTK4115

Helicopter Lab Report

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1 Part 1

To find a model of the system we started with Newton's 2nd law for rotation, which states that

$$\sum \tau = J\alpha \tag{1}$$

where τ is the external torque, J is the moment of inertia, and α is the angular acceleration. Using this for each of the three axis gives

$$J_p\ddot{p} = L_1 V_d \tag{2a}$$
$$J_e\ddot{e} = L_2\cos(e) + L_3 V_s \cos(p) \tag{2b}$$
$$J_\lambda\ddot{\lambda} = L_4 V_s \cos(e)\sin(p) \tag{2c}$$

where

$$\begin{aligned} L_1 &= K_f l_p \\ L_2 &= (m_c l_c - 2m_p l_h)g \\ L_3 &= K_f l_h \end{aligned}$$

As the model in eq. (2) is non-linear we need to linearize the model to be able to design a linear controller. To do this we need a point to linearize around. For this we use $(p^*, e^*, \lambda^*)^T = (0,0,0)^T$. We also need to find the voltages V_s^* and V_d^* that makes $(p^*, e^*, \lambda^*)^T$ an equilibrium. Setting eq. (2a) and eq. (2b) to zero gives

$$\begin{aligned} V_d^* &= 0 \\ V_s^* &= -\frac{L_2}{L_3} \end{aligned}$$

The next thing we did was a coordinate transform, to simplify the model of the system. The new states are $(\tilde{p}, \tilde{e}, \tilde{\lambda})^T = (p, e, \lambda)^T - (p^*, e^*, \lambda^*)^T$ and the new inputs are $(\tilde{V}_s, \tilde{V}_d)^T = (V_s, V_d)^T - (V_s^*, V_d^*)^T$. This gives the system

$$\ddot{\tilde{p}} = \frac{L_1}{J_p}\tilde{V}_d \tag{5a}$$
$$\ddot{\tilde{e}} = \frac{L_2}{J_e}\cos\tilde{e} + (L_3\tilde{V}_s - L_2)\cos\tilde{p} \tag{5b}$$
$$\ddot{\tilde{\lambda}} = \tag{5c}$$

Now that the system is on a nice form, we can linearize it. This gives us a system on the form $\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}$, where the matrices is given by

$$\mathbf{A} = \begin{bmatrix} \frac{\partial \ddot{\tilde{p}}}{\partial \tilde{p}} = 0 & \frac{\partial \ddot{\tilde{p}}}{\partial \tilde{e}} = 0 & \frac{\partial \ddot{\tilde{p}}}{\partial \tilde{\lambda}} = 0 \\ \frac{\partial \ddot{\tilde{e}}}{\partial \tilde{p}} = 0 & \frac{\partial \ddot{\tilde{e}}}{\partial \tilde{e}} = 0 & \frac{\partial \ddot{\tilde{e}}}{\partial \tilde{\lambda}} = 0 \\ \frac{\partial \ddot{\tilde{\lambda}}}{\partial \tilde{p}} = -\frac{L_2}{J_\lambda} & \frac{\partial \ddot{\tilde{\lambda}}}{\partial \tilde{e}} = 0 & \frac{\partial \ddot{\tilde{\lambda}}}{\partial \tilde{\lambda}} = 0 \end{bmatrix} \tag{6a}$$

$$\mathbf{B} = \begin{bmatrix} \frac{\partial \ddot{\tilde{p}}}{\partial \tilde{V}_s} = 0 & \frac{\partial \ddot{\tilde{p}}}{\partial \tilde{V}_d} = \frac{L_1}{J_p} \\ \frac{\partial \ddot{\tilde{e}}}{\partial \tilde{V}_s} = \frac{L_3}{J_e} & \frac{\partial \ddot{\tilde{e}}}{\partial \tilde{V}_d} = 0 \\ \frac{\partial \ddot{\tilde{\lambda}}}{\partial \tilde{V}_s} = 0 & \frac{\partial \ddot{\tilde{\lambda}}}{\partial \tilde{V}_d} = 0 \end{bmatrix} \tag{6b}$$

Writing out eq. (6) we get

$$\ddot{\tilde{p}} = \frac{L_1}{J_p}\tilde{V}_d = K_1\tilde{V}_d \tag{7a}$$

$$\ddot{\tilde{e}} = \frac{L_3}{J_e}\tilde{V}_s = K_2\tilde{V}_s \tag{7b}$$

$$\ddot{\tilde{\lambda}} = -\frac{L_2}{J_\lambda}\tilde{p} = K_3\tilde{p} \tag{7c}$$

2 Part 2: Monovariable Control

2.1 Problem 1

2.1.1 Controllability

We look at the controllability matrix:

References