Linear Systems TTK4115

Helicopter Lab Report

Bernt Johan Damslora (nr. 759477)
Didrik Rokhaug (nr. 759528)

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NTNU – Trondheim
Norwegian University of
Science and Technology

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1 Part 1

To find a model of the system we started with Newton's 2nd law for rotation, which states that

$$\sum \tau = J\alpha \tag{1}$$

where τ is the external torque, J is the moment of inertia, and α is the angular acceleration. Using this for each of the three axis gives

$$J_p \ddot{p} = L_1 V_d \tag{2a}$$
$$J_e \ddot{e} = L_2 \cos(e) + L_3 V_s \cos(p) \tag{2b}$$
$$J_\lambda \ddot{\lambda} = L_4 V_s \cos(e) \sin(p) \tag{2c}$$

where

$$L_1 = K_f l_p$$
$$L_2 = (m_c l_c - 2m_p l_h)g$$
$$L_3 = K_f l_h$$

As the model in eq. (2) is non-linear we need to linearize the model to be able to design a linear controller. To do this we need a point to linearize around. For this we use $(p^*, e^*, \lambda^*)^T = (0, 0, 0)^T$. We also need to find the voltages V_s^* and V_d^* that makes $(p^*, e^*, \lambda^*)^T$ an equilibrium. Setting eq. (2a) and eq. (2b) to zero gives

$$V_d^* = 0$$
$$V_s^* = -\frac{L_2}{L_3}$$

The next thing we did was a coordinate transform, to simplify the model of the system. The new states are $(\tilde{p}, \tilde{e}, \tilde{\lambda})^T = (p, e, \lambda)^T - (p^*, e^*, \lambda^*)^T$ and the new inputs are $(\tilde{V}_s, \tilde{V}_d)^T = (V_s, V_d)^T - (V_s^*, V_d^*)^T$. This gives the system

$$\ddot{\tilde{p}} = \frac{L_1}{J_p} \tilde{V}_d \tag{5a}$$
$$\ddot{\tilde{e}} = \frac{L_2}{J_e} \cos\tilde{e} + (L_3 \tilde{V}_s - L_2) \cos\tilde{p} \tag{5b}$$
$$\ddot{\tilde{\lambda}} = -\frac{L_2}{J_\lambda} \tilde{p} \tag{5c}$$

Now that the system is on a nice form, we can linearize it. This gives us a system on the form $\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}$, where $\dot{\mathbf{x}} = (\ddot{\tilde{p}}, \ddot{\tilde{e}}, \ddot{\tilde{\lambda}})^T$, and the matrices $\mathbf{A}$ and $\mathbf{B}$ is given by

$$\mathbf{A} = \begin{bmatrix} \frac{\partial \ddot{\tilde{p}}}{\partial \tilde{p}} & \frac{\partial \ddot{\tilde{p}}}{\partial \tilde{e}} & \frac{\partial \ddot{\tilde{p}}}{\partial \tilde{\lambda}} \\ \frac{\partial \ddot{\tilde{e}}}{\partial \tilde{p}} & \frac{\partial \ddot{\tilde{e}}}{\partial \tilde{e}} & \frac{\partial \ddot{\tilde{e}}}{\partial \tilde{\lambda}} \\ \frac{\partial \ddot{\tilde{\lambda}}}{\partial \tilde{p}} & \frac{\partial \ddot{\tilde{\lambda}}}{\partial \tilde{e}} & \frac{\partial \ddot{\tilde{\lambda}}}{\partial \tilde{\lambda}} \end{bmatrix} \Bigg|_{\ddot{\tilde{p}}=0,\, \ddot{\tilde{e}}=0,\, \ddot{\tilde{\lambda}}=0} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -\frac{L_2}{J_\lambda} & 0 & 0 \end{bmatrix} \tag{6a}$$

$$\mathbf{B} = \begin{bmatrix} \frac{\partial \ddot{\tilde{p}}}{\partial \tilde{V}_s} & \frac{\partial \ddot{\tilde{p}}}{\partial \tilde{V}_d} \\ \frac{\partial \ddot{\tilde{e}}}{\partial \tilde{V}_s} & \frac{\partial \ddot{\tilde{e}}}{\partial \tilde{V}_d} \\ \frac{\partial \ddot{\tilde{\lambda}}}{\partial \tilde{V}_s} & \frac{\partial \ddot{\tilde{\lambda}}}{\partial \tilde{V}_d} \end{bmatrix} \Bigg|_{\tilde{V}_s=0,\, \tilde{V}_d=0} = \begin{bmatrix} 0 & \frac{L_1}{J_p} \\ \frac{L_3}{J_e} & 0 \\ 0 & 0 \end{bmatrix} \tag{6b}$$

Expanding eq. (6) we get

$$\ddot{\tilde{p}} = \frac{L_1}{J_p}\tilde{V}_d = K_1\tilde{V}_d \tag{7a}$$

$$\ddot{\tilde{e}} = \frac{L_3}{J_e}\tilde{V}_s = K_2\tilde{V}_s \tag{7b}$$

$$\ddot{\tilde{\lambda}} = -\frac{L_2}{J_\lambda}\tilde{p} = K_3\tilde{p} \tag{7c}$$

Using this model we tried to fly the helicopter using only feed-forward, without any regulator. As expected it was very hard to keep the helicopter stable for more than a few seconds. The helicopter was behaving especially bad with a large elevation, something which according to the linearized model should not be the case. The reason for this is that at a high elevation the force generated by the propellers aren't vertical, and as such the motors need to provide a bigger force to keep the helicopter elevated. This is done by increasing the voltage given to the motors. However the motors can only take so much voltage, and at a high elevation, the motors are close to their's saturation limit. This makes it hard to control the pitch (and travel) without sacrificing elevation.

After this we did some more preparations to be able to control the helicopter. First we added constant values the the output of the encoder so that we would be able to start the helicopter form the table, rather than from all joints at zero. Next we found the motor force constant K_f by finding the voltage that would keep the helicopter horizontal. K_f was found to be 0.14689

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2 Part 2: Monovariable Control

2.1 Problem 1

$$
\begin{aligned}
\tilde{V}_d &= K_{pp}(\tilde{p}_c - \tilde{p}) - K_{pd}\dot{\tilde{p}} && \text{(8a)}\\
\ddot{\tilde{p}} &= K_1(K_{pp}(\tilde{p}_c - \tilde{p}) - K_{pd}\dot{\tilde{p}}) && \text{(8b)}\\
\ddot{\tilde{p}} &= K_1 K_{pp}\tilde{p}_c - K_1 K_{pp}\tilde{p} - K_1 K_{pd}\dot{\tilde{p}} && \text{(8c)}
\end{aligned}
$$

We take the laplace transform:

$$
\begin{aligned}
s^2\hat{\tilde{p}} &= K_1 K_{pp}\hat{\tilde{p}}_c - K_1 K_{pp}\hat{\tilde{p}} - K_1 K_{pd} s\hat{\tilde{p}} && \text{(9a)}\\
(s^2 + sK_1 K_{pd} + K_1 K_{pp})\tilde{p} &= K_1 K_{pp}\tilde{p}_c && \text{(9b)}\\
\frac{\tilde{p}}{\tilde{p}_c} &= \frac{K_1 K_{pp}}{s^2 + sK_1 K_{pd} + K_1 K_{pp}} && \text{(9c)}
\end{aligned}
$$

We write the transfer function as a damped oscillator

$$
\frac{K_1 K_{pp}}{s^2 + K_1 K_{pd} + K_1 K_{pp} s} = \frac{\omega_0^2}{s^2 + 2\zeta\omega_0 s + \omega_0^2} \tag{10}
$$

Giving us the set of equations

$$
\begin{aligned}
K_1 K_{pd} &= 2\zeta\omega_0 && \text{(11a)}\\
K_1 K_{pp} &= \omega_0^2 && \text{(11b)}
\end{aligned}
$$

Solving for K_{pp} and K_{pd} yields

$$
\begin{aligned}
K_{pd} &= \frac{2\zeta\omega_0}{K_1} && \text{(12a)}\\
K_{pp} &= \frac{\omega_0^2}{K_1} && \text{(12b)}
\end{aligned}
$$

We implemented eq. (12a) and eq. (12b) in MATLAB code and started tuning ω_0 and ζ. We let $\zeta = 1$, as we imagined a critically damped system would suit a helicopter. We then increased ω_0 until problems occured. We then tuned both parameters, ending up with $\omega_0 = 4$ and $\zeta = 1.5$. We speculate that the reason for the increased damping factor is the time delay.

A larger K_{pp} should lead to larger absolute value of the eigenvalues and faster response, while a larger K_{pd} should lead to a tighter spacing of the eigenvalues, slower response and less overshoot.

The closed loop PD pitch control was found to be easier than feed-forward control.

2.2 Problem 2

The P-controller is defined as

$$\tilde{p}_c = K_{rp}(\dot{\tilde{\lambda}}_c - \dot{\tilde{\lambda}}) \tag{13}$$

From eq. (7c) we have

$$\ddot{\tilde{\lambda}} = K_3 \tilde{p} \tag{14a}$$

$$\frac{\ddot{\tilde{\lambda}}}{K_3} = \tilde{p} \tag{14b}$$

For this problem we assume $\tilde{p} = \tilde{p}_c$, so by eq. (13) and eq. (14) we have

$$\frac{\ddot{\tilde{\lambda}}}{K_3} = K_{rp}(\dot{\tilde{\lambda}}_c - \dot{\tilde{\lambda}}) \tag{15}$$

Laplace transform of eq. (15) with respect to travel rate yields

$$\frac{s\dot{\tilde{\lambda}}}{K_3} = K_{rp}(\dot{\tilde{\lambda}}_c - \dot{\tilde{\lambda}}) \tag{16a}$$

$$\dot{\tilde{\lambda}}(\frac{s}{K_3} + K_{rp}) = K_{rp}\dot{\tilde{\lambda}}_c \tag{16b}$$

$$\frac{\dot{\tilde{\lambda}}}{\dot{\tilde{\lambda}}_c} = \frac{K_{rp}}{\frac{s}{K_3} + K_{rp}} \tag{16c}$$

$$\frac{\dot{\tilde{\lambda}}}{\dot{\tilde{\lambda}}_c} = \frac{K_{rp}K_3}{s + K_{rp}K_3} \tag{16d}$$

In this case, a K_{rp} value of 1 provided a good result.

References