By Euler's formula:

$$e^{i\theta} = cos(\theta) + i\sin(\theta)$$
$$e^{-i\theta} = cos(\theta) - i\sin(\theta)$$

$$\therefore \sin(\theta) = \frac{e^{i\theta} - e^{-i\theta}}{2i}$$
$$\cos(\theta) = \frac{e^{i\theta} + e^{-i\theta}}{2}$$
$$\tan(\theta) = \frac{\sin(\theta)}{\cos\theta} = -\frac{i(-1 + e^{2i\theta})}{1 + e^{2i\theta}}$$

$$\text{let} \quad \tan(\theta) = x$$
$$x(1 + e^{2i\theta}) = -i(-1 + e^{2i\theta})$$
$$x + xe^{2i\theta} = i - ie^{2i\theta}$$
$$xe^{2i\theta} + ie^{2i\theta} = i - x$$
$$e^{2i\theta}(i + x) = i - x$$
$$e^{2i\theta} = \frac{i - x}{i + x}$$
$$2i\theta = \ln(\frac{i - x}{i + x})$$
$$i\theta = \frac{1}{2}\ln(\frac{i - x}{i + x})$$
$$\theta = -\frac{i}{2}\ln(\frac{i - x}{i + x})$$

$$\therefore \arctan(\theta) = -\frac{i}{2}\ln(\frac{i - \theta}{i + \theta})$$