By Euler's formula:

$$e^{i\theta} = cos(\theta) + i\sin(\theta)$$

$$e^{-i\theta} = cos(\theta) - i\sin(\theta)$$

$$\therefore \sin(\theta) = \frac{e^{i\theta} - e^{-i\theta}}{2i}$$

$$\csc(\theta) = \frac{1}{\sin(\theta)} = \frac{2i}{e^{i\theta} - e^{-i\theta}}$$

$$\text{let} \csc(\theta) = x$$

$$\frac{2i}{x} = e^{i\theta} - e^{-i\theta}$$

$$\frac{2i}{x} e^{i\theta} = (e^{i\theta})^2 - 1$$

$$(e^{i\theta})^2 + (-\frac{2i}{x}) e^{i\theta} - 1 = 0$$

$$e^{i\theta} = \frac{-(\frac{2i}{x}) \pm \sqrt{(\frac{2i}{x})^2 - 4(-1)}}{2} = x^{-1}i \pm \sqrt{1 - x^2}$$

$$i\theta = \ln(x^{-1}i \pm \sqrt{1 - x^2})$$

$$\theta = -i\ln(x^{-1}i \pm \sqrt{1 - x^2})$$

$$\therefore \operatorname{arccsc}(\theta) = -i\ln(i\theta^{-1} \pm \sqrt{1 - \theta^2})$$