Question 1

$$\begin{aligned}\frac{4x^4+5x^2-7x}{x} &= \frac{x(4x^3+5x-7)}{x} \\ &= 4x^3+5x-7\end{aligned} \tag{1a}$$

$$\begin{aligned}\frac{7x^5-5x^5+9x^3+x^2}{x} &= \frac{x(2x^4+9x^2+x)}{x} \\ &= 2x^4+9x^2+x\end{aligned} \tag{1b}$$

$$\begin{aligned}\frac{-x^4+4x^2+6}{x} &= \frac{x(4x-x^3)+6}{x} \\ &= \frac{x(4x-x^3)}{x}+\frac{6}{x} \\ &= 4x-x^3+\frac{6}{x}\end{aligned} \tag{1c}$$

$$\begin{aligned}\frac{7x^5-x^3-4}{x} &= \frac{x(7x^4-x^2)-4}{x} \\ &= \frac{x(7x^4-x^2)}{x}-\frac{4}{x} \\ &= 7x^4-x^2-\frac{4}{x}\end{aligned} \tag{1d}$$

$$\begin{aligned}\frac{8x^4-4x^3+6x}{2x} &= \frac{2x(4x^3-2x^2+3)}{2x} \\ &= 4x^3-2x^2+3\end{aligned} \tag{1e}$$

$$\begin{aligned}\frac{9x^2-12x^3-3x}{3x} &= \frac{3x(3x-4x^2-1)}{3x} \\ &= -4x^2+3x-1\end{aligned} \tag{1f}$$

$$\begin{aligned}\frac{7x^3-x^4-2}{5x} &= \frac{x(7x^2-x^3)}{x(5)}-\frac{2}{5x} \\ &= -\frac{1}{5}x^3+\frac{7}{5}x^2-\frac{2}{5x}\end{aligned} \tag{1g}$$

$$\begin{aligned}\frac{-4x^2+6x^4-2x}{-2x} &= \frac{-2x(2x-3x^3+1)}{-2x} \\ &= -3x^3+2x+1\end{aligned} \tag{1h}$$

$$\frac{-x^8 + 9x^4 - 4x^3 + 6}{-2x} = \frac{-x(x^7 - 9x^3 + 4x^2)}{-x(2)} + \frac{-2(-3)}{-2x} \\ = \frac{1}{2}x^7 - \frac{9}{2}x^3 + 2x^2 - \frac{3}{x} \tag{1i}$$

$$\frac{-9x^9 - 6x^6 + 4x^4 - 2}{-3x} = \frac{-x(9x^8 + 6x^5 - 4x^3)}{-x(3)} - \frac{2}{-3x} \\ = 3x^8 + 2x^5 - \frac{4}{3}x^3 + \frac{2}{3x} \tag{1j}$$

Question 2

$$\frac{(x+3)(x-2)}{(x-2)} = x + 3 \tag{2a}$$

$$\frac{(x+4)(3x-1)}{(3x-1)} = x + 4 \tag{2b}$$

$$\frac{(x+3)^2}{(x+3)} = x + 3 \tag{2c}$$

$$\frac{x^2 + 10x + 21}{(x+3)} = \frac{(x+3)(x+7)}{(x+3)} \\ = x + 7 \tag{2d}$$

$$\frac{x^2 + 9x + 20}{(x+4)} = \frac{(x+4)(x+5)}{(x+4)} \\ = x + 5 \tag{2e}$$

$$\frac{x^2 + x - 12}{(x-3)} = \frac{(x-3)(x+4)}{(x-3)} \\ = x + 4 \tag{2f}$$

$$\frac{x^2 + x - 20}{x^2 + 2x - 15} = \frac{(x-4)(x+5)}{(x-3)(x+5)} \\ = \frac{x-4}{x-3} \tag{2g}$$

$$\frac{x^2 + 3x + 2}{x^2 + 5x + 4} = \frac{(x+1)(x+2)}{(x+1)(x+4)} \\ = \frac{x+2}{x+4} \tag{2h}$$

$$\begin{aligned}\frac{x^2+x-12}{x^2-9x+18} &= \frac{(x-3)(x+4)}{(x-3)(x-6)} \\ &= \frac{x+4}{x-6}\end{aligned} \tag{2i}$$

$$\begin{aligned}\frac{2x^2+7x+6}{(x-5)(x+2)} &= \frac{(2x+3)(x+2)}{(x-5)(x+2)} \\ &= \frac{2x+3}{x-5}\end{aligned} \tag{2j}$$

$$\begin{aligned}\frac{2x^2+9x-18}{(x+6)(x+1)} &= \frac{(2x-3)(x+6)}{(x+6)(x+1)} \\ &= \frac{2x-3}{x+1}\end{aligned} \tag{2k}$$

$$\begin{aligned}\frac{3x^2-7x+2}{(3x-1)(x+2)} &= \frac{(3x-1)(x-2)}{(3x-1)(x+2)} \\ &= \frac{x-2}{x+2}\end{aligned} \tag{2l}$$

$$\begin{aligned}\frac{2x^2+3x+1}{x^2-x-2} &= \frac{(2x+1)(x+1)}{(x-2)(x+1)} \\ &= \frac{2x+1}{x-2}\end{aligned} \tag{2m}$$

$$\begin{aligned}\frac{x^2+6x+8}{3x^2+7x+2} &= \frac{(x+4)(x+2)}{(3x+1)(x+2)} \\ &= \frac{x+4}{3x+1}\end{aligned} \tag{2n}$$

$$\begin{aligned}\frac{2x^2-5x-3}{2x^2-9x+9} &= \frac{(2x+1)(x-3)}{(2x-3)(x-3)} \\ &= \frac{2x+1}{2x-3}\end{aligned} \tag{2o}$$

Question 3

Find a, b, and c:

$$\frac{6x^3+3x^2-84x}{6x^2-33x+42} = \frac{ax(x+b)}{x+c} \tag{3}$$

$$\begin{aligned}\frac{6x^3+3x^2-84x}{6x^2-33x+42} &= \frac{3x(2x^2+x-28)}{3(2x^2-11x+14)} \\ &= \frac{x(2x-7)(x+4)}{(2x-7)(x-2)} \\ &= \frac{x(x+4)}{x-2}\end{aligned}$$

$$\therefore a=1, b=4, c=-2$$