Question 1

Without using your calculator, write down the sign of the following trigometric ratios.

$$
\begin{aligned}
\sec(300°) &= \frac{1}{\cos(300°)} \\
&= \frac{1}{\cos(60°)} \\
\cos(\theta) &> 0 \text{ in the first quadrant} \therefore \sec(300°) > 0
\end{aligned}
\tag{1a}
$$

$$
\begin{aligned}
\csc(190°) &= \frac{1}{\sin(190°)} \\
&= \frac{1}{\sin(-10°)} \\
&= -\frac{1}{\sin(10°)} \\
\sin(\theta) &> 0 \text{ in the first quadrant} \therefore \csc(190°) < 0
\end{aligned}
\tag{1b}
$$

$$
\begin{aligned}
\cot(110°) &= \frac{1}{\tan(110°)} \\
&= \frac{1}{-\tan(70°)} \\
&= -\frac{1}{\tan(70°)} \\
\tan(\theta) &> 0 \text{ in the first quadrant} \therefore \cot(110°) < 0
\end{aligned}
\tag{1c}
$$

$$
\begin{aligned}
\cot(200°) &= \frac{1}{\tan(200°)} \\
&= \frac{1}{\tan(20°)} \\
\tan(\theta) &> 0 \text{ in the first quadrant} \therefore \cot(200°) > 0
\end{aligned}
\tag{1d}
$$

$$
\begin{aligned}
\sec(95°) &= \frac{1}{\cos(95°)} \\
\cos(\theta) &< 0 \text{ in the second quadrant} \therefore \sec(95°) < 0
\end{aligned}
\tag{1e}
$$

Question 2

Use your calculator to find, to 3 significant figures, the values of:

$$\sec(100°) = \frac{1}{\cos(100°)}$$
$$= -5.76 \text{ (3 s.f.)}$$

(2a)

$$\csc(260°) = \frac{1}{\sin(260°)}$$
$$= -1.02 \text{ (3 s.f.)}$$

(2b)

$$\csc(280°) = \frac{1}{\sin(280°)}$$
$$= -1.02 \text{ (3 s.f.)}$$

(2c)

$$\cot(550°) = \frac{1}{\tan 550°}$$
$$= 5.67 \text{ (3 s.f.)}$$

(2d)

$$\cot \frac{4}{3}\pi = \frac{1}{\tan \frac{4}{3}\pi}$$
$$= 0.577 \text{ (3 s.f.)}$$

(2e)

$$\sec(2.4 \text{ rad}) = \frac{1}{\cos(2.4 \text{ rad})}$$
$$= -1.36 \text{ (3 s.f.)}$$

(2f)

$$\csc \frac{11}{10}\pi = \frac{1}{\sin \frac{11}{10}\pi}$$
$$= -3.24 \text{ (3 s.f.)}$$

(2g)

$$\sec(6 \text{ rad}) = \frac{1}{\cos(6 \text{ rad})}$$
$$= 1.04 \text{ (3 s.f.)}$$

(2h)

Question 3

Find the exact values (in surd form where appropriate) of the following:

$$\csc(90°) = \frac{1}{\sin(90°)} \tag{3a}$$
$$= 1$$

$$\cot(135°) = \frac{1}{\tan(135°)}$$
$$= \frac{1}{-\tan(45°)} \tag{3b}$$
$$= -1$$

$$\sec(180°) = \frac{1}{\cos 180°} \tag{3c}$$
$$= -1$$

$$\sec(240°) = \frac{1}{\cos(240°)}$$
$$= \frac{1}{-\cos(60°)} \tag{3d}$$
$$= \frac{1}{-\frac{1}{2}}$$
$$= -2$$

$$\csc(300°) = \frac{1}{\sin(300°)}$$
$$= \frac{1}{-\sin(60°)}$$
$$= \frac{1}{-\frac{\sqrt{3}}{2}} \tag{3e}$$
$$= -\frac{2}{\sqrt{3}}$$
$$= -\frac{2\sqrt{3}}{3}$$

$$\cot(-45°) = \frac{1}{\tan(-45°)}$$
$$= \frac{1}{-\tan(45°)} \tag{3f}$$
$$= -1$$

$$\sec(60°) = \frac{1}{\cos(60°)}$$
$$= \frac{1}{\frac{1}{2}}$$
$$= 2 \tag{3g}$$

$$\csc(-210°)$$
$$= \frac{1}{\sin(-210°)}$$
$$= \frac{1}{\sin(30°)} \tag{3h}$$
$$= \frac{1}{\frac{1}{2}}$$
$$= 2$$

$$\sec(255°) = \frac{1}{\cos(255°)}$$
$$= \frac{1}{-\cos(45°)} \tag{3i}$$
$$= \frac{1}{-\frac{1}{\sqrt{2}}}$$
$$= -\sqrt{2}$$

$$\cot\frac{4}{3}\pi = \frac{1}{\tan\frac{4}{3}\pi}$$
$$= \frac{1}{\tan\frac{\pi}{3}}$$
$$= \frac{1}{\sqrt{3}} \tag{3j}$$
$$= \frac{\sqrt{3}}{3}$$

$$\sec\frac{11}{6}\pi = \frac{1}{\cos\frac{11}{6}\pi}$$
$$= \frac{1}{\cos\frac{\pi}{6}}$$
$$= \frac{1}{\frac{\sqrt{3}}{2}} \tag{3k}$$
$$= \frac{2}{\sqrt{3}}$$
$$= \frac{2\sqrt{3}}{3}$$

$$\csc(-\frac{3}{4}\pi) = \frac{1}{\sin(-\frac{3}{4}\pi)}$$

$$= \frac{1}{-\sin\frac{\pi}{4}} \tag{3l}$$

$$= \frac{1}{-\frac{1}{\sqrt{2}}}$$

$$= -\sqrt{2}$$

Question 4

Prove that $\csc(\pi - x) \equiv \csc x$.

$$\csc(\pi - x) = \frac{1}{\sin(\pi - x)}$$

$$= \frac{1}{\sin(\pi)\cos(x) - \cos(\pi)\sin(x)} \tag{4}$$

$$= \frac{1}{\sin x}$$

$$= \csc x$$

Question 5

Show that $\cot(30°)\sec(30°) = 2$.

$$\frac{\cos(30°)}{\sin(30°)} \times \frac{1}{\cos(30°)}$$

$$= \frac{1}{\sin(30°)} \tag{5}$$

$$= \frac{1}{\frac{1}{2}}$$

$$= 2$$

Question 6

Show that $\csc\frac{2}{3}\pi + \sec\frac{2}{3}\pi = a + b\sqrt{3}$ where a and b are real numbers to be found.

$$\csc\frac{2}{3}\pi + \sec\frac{2}{3}\pi = \frac{1}{\sin\frac{2}{3}\pi} + \frac{1}{\cos\frac{2}{3}\pi}$$

$$= \frac{1}{\sin\frac{\pi}{3}} + \frac{1}{-\cos\frac{\pi}{3}} \tag{6}$$

$$= \frac{1}{\frac{\sqrt{3}}{2}} + \frac{1}{-\frac{1}{2}}$$

$$= -2 + \frac{2}{3}\sqrt{3}$$