By Euler's formula:

$$e^{i\theta} = cos(\theta) + i\sin(\theta)$$

$$e^{-i\theta} = cos(\theta) - i\sin(\theta)$$

$$\therefore \sin(\theta) = \frac{e^{i\theta} - e^{-i\theta}}{2i}$$

$$\cos(\theta) = \frac{e^{i\theta} + e^{-i\theta}}{2}$$

$$\tan(\theta) = \frac{\sin(\theta)}{\cos\theta} = -\frac{i(-1+e^{2i\theta})}{1+e^{2i\theta}}$$

$$\text{let} \tan(\theta) = x$$

$$x(1+e^{2i\theta}) = -i(-1+e^{2i\theta})$$

$$x + xe^{2i\theta} = i - ie^{2i\theta}$$

$$xe^{2i\theta} + ie^{2i\theta} = i - x$$

$$e^{2i\theta}(i+x) = i - x$$

$$e^{2i\theta} = \frac{i-x}{i+x}$$

$$2i\theta = \ln(\frac{i-x}{i+x})$$

$$i\theta = \frac{1}{2}\ln(\frac{i-x}{i+x})$$

$$\theta = -\frac{i}{2}\ln(\frac{i-x}{i+x})$$

$$\therefore \arctan(\theta) = -\frac{i}{2}\ln(\frac{i-\theta}{i+\theta})$$