By Euler's formula:

$$e^{i\theta} = cos(\theta) + i\sin(\theta)$$

$$e^{-i\theta} = cos(\theta) - i\sin(\theta)$$

$$\therefore \sin(\theta) = \frac{e^{i\theta} - e^{-i\theta}}{2i}$$

$$\cos(\theta) = \frac{e^{i\theta} + e^{-i\theta}}{2}$$

$$\tan(\theta) = \frac{\sin(\theta)}{\cos\theta} = -\frac{i(-1 + e^{2i\theta})}{1 + e^{2i\theta}}$$

$$\cot(\theta) = \frac{1}{\tan(\theta)} = -\frac{1 + e^{2i\theta}}{i(-1 + e^{2i\theta})}$$

$$\text{let} \quad \cot(\theta) = x$$

$$x(i + ie^{2i\theta}) = -1(1 + e^{2i\theta})$$

$$ix + ixe^{2i\theta} = -e^{2i\theta} - 1$$

$$ixe^{2i\theta} + e^{2i\theta} = -1 - ix$$

$$(ix + 1)e^{2i\theta} = -1 - ix$$

$$e^{2i\theta} = -\frac{1 - ix}{1 + ix}$$

$$2i\theta = \ln(\frac{x + i}{x - i})$$

$$i\theta = \frac{1}{2}\ln(\frac{x + i}{x - i})$$

$$\theta = -\frac{i}{2}\ln(\frac{x + i}{x - i})$$

$$\therefore \operatorname{arccot}(\theta) = -\frac{i}{2}\ln(\frac{\theta + i}{\theta - i})$$