Suppose we have an nth degree polynomial, such that
$f(x) = c_n x^n + c_{n-1}x^{n-1} + c_{n-2}x^{n-2} + ... + c_0 = \sum_{i=0}^{n} c_{n-i}x^{n-i}$

By the power rule of differentiation, we can conclude that the first derivative of $f(x)$ is as follows

$$\begin{aligned} f'(x) &= nc_n x^{n-1} + (n-1)c_{n-1}x^{n-2} + (n-2)c_{n-2}x^{n-3} + ... + c_1 \\ &= \sum_{i=0}^{n-1}(n-i)c_{n-i}x^{n-i-1} \end{aligned}$$

And the second derivative is

$$\begin{aligned} f''(x) &= n(n-1)c_n x^{n-2} + (n-1)(n-2)c_{n-1}x^{n-3} + (n-2)(n-3)c_{n-2}x^{n-4} + ... + c_2 \\ &= \sum_{i=0}^{n-2}(i-n)(i-n+1)a_{n-i}x^{n-i-2} \end{aligned}$$

After starting with an initial guess, the next iteration of Halley's method is given by
$x_k - \frac{2f(x_k)f'(x_k)}{2[f'(x_k)]^2 - f(x_k)f''(x_k)}$.
This means that we must first find $f(x)f'(x)$, $[f'(x)]^2$, and $f(x)f''(x)$. These all consist of the multiplication of two series - there is a nice general form to this problem stated below

$$(\sum_{i=0}^{n} x_i)(\sum_{j=0}^{m} y_j) = \sum_{i=0}^{n}\sum_{j=0}^{m} x_i y_j$$

From this, we can conclude that:

$$
\begin{aligned}
f(x)f'(x) &= (\sum_{i=0}^{n} c_{n-i}x^{n-i})(\sum_{i=0}^{n-1}(n-i)c_{n-i}x^{n-i-1}) \\
&= \sum_{i=0}^{n}\sum_{j=0}^{n-1}(n-j)c_{n-i}c_{n-j}x^{2n-i-j-1}
\end{aligned}
$$

$$
\begin{aligned}
[f'(x)]^2 &= (\sum_{i=0}^{n-1}(n-i)c_{n-i}x^{n-i-1})(\sum_{i=0}^{n-1}(n-i)c_{n-i}x^{n-i-1}) \\
&= \sum_{i=0}^{n-1}\sum_{j=0}^{n-1}(n-i)(n-j)c_{n-i}c_{n-j}x^{2(n-1)-i-j}
\end{aligned}
$$

$$
\begin{aligned}
f(x)f''(x) &= (\sum_{i=0}^{n} c_{n-i}x^{n-i})(\sum_{i=0}^{n-2}(i-n)(i-n+1)a_{n-i}x^{n-i-2}) \\
&= \sum_{i=0}^{n}\sum_{j=0}^{n-2}(j-n)(j-n+1)c_{n-i}c_{n-j}x^{2(n-1)-i-j}
\end{aligned}
$$

And all that is left to do is to plug it into the formula for Halley's method, leaving us with the following:

$$
x_{k+1} = x_k - \frac{2\sum_{i=0}^{n}\sum_{j=0}^{n-1}(n-j)c_{n-i}c_{n-j}x^{2n-i-j-1}}{2\sum_{i=0}^{n-1}\sum_{j=0}^{n-1}(n-i)(n-j)c_{n-i}c_{n-j}x^{2(n-1)-i-j} - \sum_{i=0}^{n}\sum_{j=0}^{n-2}(j-n)(j-n+1)c_{n-i}c_{n-j}x^{2(n-1)-i-j}}
$$