Pure Book 2, Exercise 6A

Question 1 Without using your calculator, write down the sign of the following trigometric ratios.

$$\sec(300^\circ) = \frac{1}{\cos(300^\circ)} = \frac{1}{\cos(60^\circ)}$$
$$\cos(\theta) > 0 \text{ in the first quadrant } \therefore \sec(300^\circ) > 0 \tag{a}$$

$$\csc(190^\circ) = \frac{1}{\sin(190^\circ)} = \frac{1}{\sin(-10^\circ)} = -\frac{1}{\sin(10^\circ)}$$
$$\sin(\theta) > 0 \text{ in the first quadrant } \therefore \csc(190^\circ) < 0 \tag{b}$$

$$\cot(110^\circ) = \frac{1}{\tan(110^\circ)} = \frac{1}{-\tan(70^\circ)} = -\frac{1}{\tan(70^\circ)}$$
$$\tan(\theta) > 0 \text{ in the first quadrant } \therefore \cot(110^\circ) < 0 \tag{c}$$

$$\cot(200^\circ) = \frac{1}{\tan(200^\circ)} = \frac{1}{\tan(20^\circ)}$$
$$\tan(\theta) > 0 \text{ in the first quadrant } \therefore \cot(200^\circ) > 0 \tag{d}$$

$$\sec(95^\circ) = \frac{1}{\cos(95^\circ)}$$
$$\cos(\theta) < 0 \text{ in the second quadrant } \therefore \sec(95^\circ) < 0 \tag{e}$$

Question 2 Use your calculator to find, to 3 significant figures, the values of:

$$\sec(100^\circ) = \frac{1}{\cos(100^\circ)} = -5.76 \text{ (3 s.f.)} \tag{a}$$

$$\csc(260^\circ) = \frac{1}{\sin(260^\circ)} = -1.02 \text{ (3 s.f.)} \tag{b}$$

$$\csc(280^\circ) = \frac{1}{\sin(280^\circ)} = -1.02 \text{ (3 s.f.)} \tag{c}$$

$$\cot(550^\circ) = \frac{1}{\tan 550^\circ} = 5.67 \text{ (3 s.f.)} \tag{d}$$

$$\cot \tfrac{4}{3}\pi = \tfrac{1}{\tan \frac{4}{3}\pi} = 0.577 \text{ (3 s.f.)} \tag{e}$$

$$\sec(2.4 \text{ rad}) = \tfrac{1}{\cos(2.4 \text{ rad})} = -1.36 \text{ (3 s.f.)} \tag{f}$$

$$\csc \tfrac{11}{10}\pi = \tfrac{1}{\sin \frac{11}{10}\pi} = -3.24 \text{ (3 s.f.)} \tag{g}$$

$$\sec(6 \text{ rad}) = \tfrac{1}{\cos(6 \text{ rad})} = 1.04 \text{ (3 s.f.)} \tag{h}$$

Question 3 Find the exact values (in surd form where appropriate) of the following:

$$\csc(90^\circ) = \tfrac{1}{\sin(90^\circ)} = 1 \tag{a}$$

$$\cot(135^\circ) = \tfrac{1}{\tan(135^\circ)} = \tfrac{1}{-\tan(45^\circ)} = -1 \tag{b}$$

$$\sec(180^\circ) = \tfrac{1}{\cos 180^\circ} = -1 \tag{c}$$

$$\sec(240^\circ) = \tfrac{1}{\cos(240^\circ)} = \tfrac{1}{-\cos(60^\circ)} = \tfrac{1}{-\frac{1}{2}} = -2 \tag{d}$$

$$\csc(300^\circ) = \tfrac{1}{\sin(300^\circ)} = \tfrac{1}{-\sin(60^\circ)} = \tfrac{1}{-\frac{\sqrt{3}}{2}} = -\tfrac{2}{\sqrt{3}} = -\tfrac{2\sqrt{3}}{3} \tag{e}$$

$$\cot(-45^\circ) = \tfrac{1}{\tan(-45^\circ)} = \tfrac{1}{-\tan(45^\circ)} = -1 \tag{f}$$

$$\sec(60^\circ) = \tfrac{1}{\cos(60^\circ)} = \tfrac{1}{\frac{1}{2}} = 2 \tag{g}$$

$$\csc(-210^\circ) = \frac{1}{\sin(-210^\circ)} = \frac{1}{\sin(30^\circ)} = \frac{1}{\frac{1}{2}} = 2 \tag{h}$$

$$\sec(255^\circ) = \frac{1}{\cos(255^\circ)} = \frac{1}{-\cos(45^\circ)} = \frac{1}{-\frac{1}{\sqrt{2}}} = -\sqrt{2} \tag{i}$$

$$\cot\frac{4}{3}\pi = \frac{1}{\tan\frac{4}{3}\pi} = \frac{1}{\tan\frac{\pi}{3}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3} \tag{j}$$

$$\sec\frac{11}{6}\pi = \frac{1}{\cos\frac{11}{6}\pi} = \frac{1}{\cos\frac{\pi}{6}} = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3} \tag{k}$$

$$\csc(-\frac{3}{4}\pi) = \frac{1}{\sin(-\frac{3}{4}\pi)} = \frac{1}{-\sin\frac{\pi}{4}} = \frac{1}{-\frac{1}{\sqrt{2}}} = -\sqrt{2} \tag{l}$$

Question 4 Prove that $\csc(\pi - x) \equiv \csc x$.

$$\csc(\pi - x) = \frac{1}{\sin(\pi - x)} = \frac{1}{\sin(\pi)\cos(x) - \cos(\pi)\sin(x)} = \frac{1}{\sin x} = \csc x$$

Question 5 Show that $\cot(30^\circ)\sec(30^\circ) = 2$.

$$\frac{\cos(30^\circ)}{\sin(30^\circ)} \times \frac{1}{\cos(30^\circ)} = \frac{1}{\sin(30^\circ)} = \frac{1}{\frac{1}{2}} = 2$$

Question 6 Show that $\csc\frac{2}{3}\pi + \sec\frac{2}{3}\pi = a + b\sqrt{3}$ where a and b are real numbers to be found.

$$\csc\frac{2}{3}\pi + \sec\frac{2}{3}\pi = \frac{1}{\sin\frac{2}{3}\pi} + \frac{1}{\cos\frac{2}{3}\pi} = \frac{1}{\sin\frac{\pi}{3}} + \frac{1}{-\cos\frac{\pi}{3}} = \frac{1}{\frac{\sqrt{3}}{2}} + \frac{1}{-\frac{1}{2}} = -2 + \frac{2}{3}\sqrt{3}$$