By Euler's formula:

$$e^{i\theta} = cos(\theta) + i\sin(\theta)$$
$$e^{-i\theta} = cos(\theta) - i\sin(\theta)$$

$$\therefore \cos(\theta) = \frac{e^{i\theta} + e^{-i\theta}}{2}$$
$$\sec(\theta) = \frac{1}{\cos(\theta)} = \frac{2}{e^{i\theta} + e^{-i\theta}}$$

$$\text{let} \quad \sec(\theta) = x$$
$$x(e^{i\theta} + e^{-i\theta}) = 2$$
$$e^{i\theta} + e^{-i\theta} = \frac{2}{x}$$
$$(e^{i\theta})^2 + 1 = \frac{2}{x}e^{i\theta}$$
$$(e^{i\theta})^2 + (-\frac{2}{x})e^{i\theta} + 1 = 0$$

$$e^{i\theta} = \frac{-(-\frac{2}{x}) \pm \sqrt{(-\frac{2}{x})^2 - 4}}{2} = x^{-1} \pm \sqrt{x^{-2} - 1}$$
$$i\theta = \ln(x^{-1} \pm \sqrt{x^{-2} - 1})$$
$$\theta = -i\ln(x^{-1} \pm \sqrt{x^{-2} - 1})$$

$$\therefore \operatorname{arcsec}(\theta) = -i\ln(\theta^{-1} \pm \sqrt{\theta^{-2} - 1})$$