

Question 1

Without using your calculator, write down the sign of the following trigonometric ratios.

$$\begin{aligned}\sec(300^\circ) &= \frac{1}{\cos(300^\circ)} \\ &= \frac{1}{\cos(60^\circ)} \\ \cos(\theta) &> 0 \text{ in the first quadrant } \therefore \sec(300^\circ) > 0\end{aligned}\tag{1a}$$

$$\begin{aligned}\csc(190^\circ) &= \frac{1}{\sin(190^\circ)} \\ &= \frac{1}{\sin(-10^\circ)} \\ &= -\frac{1}{\sin(10^\circ)} \\ \sin(\theta) &> 0 \text{ in the first quadrant } \therefore \csc(190^\circ) < 0\end{aligned}\tag{1b}$$

$$\begin{aligned}\cot(110^\circ) &= \frac{1}{\tan(110^\circ)} \\ &= \frac{1}{-\tan(70^\circ)} \\ &= -\frac{1}{\tan(70^\circ)} \\ \tan(\theta) &> 0 \text{ in the first quadrant } \therefore \cot(110^\circ) < 0\end{aligned}\tag{1c}$$

$$\begin{aligned}\cot(200^\circ) &= \frac{1}{\tan(200^\circ)} \\ &= \frac{1}{\tan(20^\circ)} \\ \tan(\theta) &> 0 \text{ in the first quadrant } \therefore \cot(200^\circ) > 0\end{aligned}\tag{1d}$$

$$\begin{aligned}\sec(95^\circ) &= \frac{1}{\cos(95^\circ)} \\ \cos(\theta) &< 0 \text{ in the second quadrant } \therefore \sec(95^\circ) < 0\end{aligned}\tag{1e}$$

Question 2

Use your calculator to find, to 3 significant figures, the values of:

$$\begin{aligned}\sec(100^\circ) &= \frac{1}{\cos(100^\circ)} \\ &= -5.76 \text{ (3 s.f.)}\end{aligned}\tag{2a}$$

$$\begin{aligned}\csc(260^\circ) &= \frac{1}{\sin(260^\circ)} \\ &= -1.02 \text{ (3 s.f.)}\end{aligned}\tag{2b}$$

$$\begin{aligned}\csc(280^\circ) &= \frac{1}{\sin(280^\circ)} \\ &= -1.02 \text{ (3 s.f.)}\end{aligned}\tag{2c}$$

$$\begin{aligned}\cot(550^\circ) &= \frac{1}{\tan 550^\circ} \\ &= 5.67 \text{ (3 s.f.)}\end{aligned}\tag{2d}$$

$$\begin{aligned}\cot \frac{4}{3}\pi &= \frac{1}{\tan \frac{4}{3}\pi} \\ &= 0.577 \text{ (3 s.f.)}\end{aligned}\tag{2e}$$

$$\begin{aligned}\sec(2.4 \text{ rad}) &= \frac{1}{\cos(2.4 \text{ rad})} \\ &= -1.36 \text{ (3 s.f.)}\end{aligned}\tag{2f}$$

$$\begin{aligned}\csc \frac{11}{10}\pi &= \frac{1}{\sin \frac{11}{10}\pi} \\ &= -3.24 \text{ (3 s.f.)}\end{aligned}\tag{2g}$$

$$\begin{aligned}\sec(6 \text{ rad}) &= \frac{1}{\cos(6 \text{ rad})} \\ &= 1.04 \text{ (3 s.f.)}\end{aligned}\tag{2h}$$

Question 3

Find the exact values (in surd form where appropriate) of the following:

$$\begin{aligned}\csc(90^\circ) &= \frac{1}{\sin(90^\circ)} \\ &= 1\end{aligned}\tag{3a}$$

$$\begin{aligned}\cot(135^\circ) &= \frac{1}{\tan(135^\circ)} \\ &= \frac{1}{-\tan(45^\circ)} \\ &= -1\end{aligned}\tag{3b}$$

$$\begin{aligned}\sec(180^\circ) &= \frac{1}{\cos 180^\circ} \\ &= -1\end{aligned}\tag{3c}$$

$$\begin{aligned}\sec(240^\circ) &= \frac{1}{\cos(240^\circ)} \\ &= \frac{1}{-\cos(60^\circ)} \\ &= \frac{1}{-\frac{1}{2}} \\ &= -2\end{aligned}\tag{3d}$$

$$\begin{aligned}\csc(300^\circ) &= \frac{1}{\sin(300^\circ)} \\ &= \frac{1}{-\sin(60^\circ)} \\ &= \frac{1}{-\frac{\sqrt{3}}{2}} \\ &= -\frac{2}{\sqrt{3}} \\ &= -\frac{2\sqrt{3}}{3}\end{aligned}\tag{3e}$$

$$\begin{aligned}\cot(-45^\circ) &= \frac{1}{\tan(-45^\circ)} \\ &= \frac{1}{-\tan(45^\circ)} \\ &= -1\end{aligned}\tag{3f}$$

$$\begin{aligned}
 \sec(60^\circ) &= \frac{1}{\cos(60^\circ)} \\
 &= \frac{1}{\frac{1}{2}} \\
 &= 2
 \end{aligned}
 \tag{3g}$$

$$\begin{aligned}
 \csc(-210^\circ) &= \frac{1}{\sin(-210^\circ)} \\
 &= \frac{1}{\sin(30^\circ)} \\
 &= \frac{1}{\frac{1}{2}} \\
 &= 2
 \end{aligned}
 \tag{3h}$$

$$\begin{aligned}
 \sec(255^\circ) &= \frac{1}{\cos(255^\circ)} \\
 &= \frac{1}{-\cos(45^\circ)} \\
 &= \frac{1}{-\frac{1}{\sqrt{2}}} \\
 &= -\sqrt{2}
 \end{aligned}
 \tag{3i}$$

$$\begin{aligned}
 \cot \frac{4}{3}\pi &= \frac{1}{\tan \frac{4}{3}\pi} \\
 &= \frac{1}{\tan \frac{\pi}{3}} \\
 &= \frac{1}{\sqrt{3}} \\
 &= \frac{\sqrt{3}}{3}
 \end{aligned}
 \tag{3j}$$

$$\begin{aligned}
 \sec \frac{11}{6}\pi &= \frac{1}{\cos \frac{11}{6}\pi} \\
 &= \frac{1}{\cos \frac{\pi}{6}} \\
 &= \frac{1}{\frac{\sqrt{3}}{2}} \\
 &= \frac{2}{\sqrt{3}} \\
 &= \frac{2\sqrt{3}}{3}
 \end{aligned}
 \tag{3k}$$

$$\begin{aligned}
\csc\left(-\frac{3}{4}\pi\right) &= \frac{1}{\sin\left(-\frac{3}{4}\pi\right)} \\
&= \frac{1}{-\sin\frac{\pi}{4}} \\
&= \frac{1}{-\frac{1}{\sqrt{2}}} \\
&= -\sqrt{2}
\end{aligned} \tag{31}$$

Question 4

Prove that $\csc(\pi - x) \equiv \csc x$.

$$\begin{aligned}
\csc(\pi - x) &= \frac{1}{\sin(\pi - x)} \\
&= \frac{1}{\sin(\pi) \cos(x) - \cos(\pi) \sin(x)} \\
&= \frac{1}{-\sin x} \\
&= \csc x
\end{aligned} \tag{4}$$

Question 5

Show that $\cot(30^\circ) \sec(30^\circ) = 2$.

$$\begin{aligned}
&\frac{\cos(30^\circ)}{\sin(30^\circ)} \times \frac{1}{\cos(30^\circ)} \\
&= \frac{1}{\sin(30^\circ)} \\
&= \frac{1}{\frac{1}{2}} \\
&= 2
\end{aligned} \tag{5}$$

Question 6

Show that $\csc \frac{2}{3}\pi + \sec \frac{2}{3}\pi = a + b\sqrt{3}$ where a and b are real numbers to be found.

$$\begin{aligned}
\csc \frac{2}{3}\pi + \sec \frac{2}{3}\pi &= \frac{1}{\sin \frac{2}{3}\pi} + \frac{1}{\cos \frac{2}{3}\pi} \\
&= \frac{1}{\sin \frac{\pi}{3}} + \frac{1}{-\cos \frac{\pi}{3}} \\
&= \frac{1}{\frac{\sqrt{3}}{2}} + \frac{1}{-\frac{1}{2}} \\
&= -2 + \frac{2}{3}\sqrt{3}
\end{aligned} \tag{6}$$