A new equation for the accurate calculation of sound speed in all oceans

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A new equation is proposed for the calculation of sound speed in seawater as a function of temperature, salinity, depth, and latitude in all oceans and open seas, including the Baltic and the Black Sea. The proposed equation agrees to better than ± 0.2 m/s with two reference complex equations, each fitting the best available data corresponding to existing waters of different salinities. The only exceptions are isolated hot brine spots that may be found at the bottom of some seas. The equation is of polynomial form, with 14 terms and coefficients of between one and three significant figures. This is a substantial reduction in complexity compared to the more complex equations using pressure that need to be calculated according to depth and location. The equation uses the 1990 universal temperature scale (an elementary transformation is given for data based on the 1968 temperature scale). It is hoped that the equation will be useful to those who need to calculate sound speed in applications of marine acoustics.
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I. INTRODUCTION

The accurate calculation of the speed of sound in seawater is needed in various applications of underwater acoustics. Quite a number of equations for this purpose have been developed in the second half of the past century. Comprehensive reviews of the available data and equations have been published by Dushaw *et al.*[1] and Leroy.[2] The equations for sound speed may be clearly divided into two categories. The first are based on precise laboratory measurements where seawater samples of various salinities have been placed under a number of temperature and pressure combinations. These equations are complex polynomials best fits to the whole set of corresponding measured values. They use pressure (the measured parameter) as a variable, while many applications require the use of depth in the sea instead. In that case, a preliminary conversion of depth into pressure is required, with the conversion depending on location on the earth. The exact procedure, developed by oceanographers who needed extreme accuracy, is somewhat tedious and hard to apply. For this reason, a number of simpler equations were developed, most of the time by fitting the first ones over a limited domain of the variables. These simpler equations used depth Z (and eventually latitude Φ) instead of pressure P. The equations of this second category use a reduced number of terms and simpler coefficients, mainly because personal computers were rare or even not existing at the time they were developed.

This set of available equations is, however, not satisfactory to many users. The simple equations do not cover all existing waters and are not accurate enough for many applications. There are also problems when using the two main equations derived from fitting accurate measured data, namely, the "Del Grosso algorithm" or NRL II equation[3] and the UNESCO equation.[4] The former is based on absolute measurements with seawater limited to the salinity range 30%–41%. The latter, though determined over a broader range of salinities, is based on measurements by comparison with pure water but uses an incorrect equation for calculating the speed of sound in pure water under pressure. In all, the attempts to provide a universal accurate equation suffered from the lack of accurate measurements under pressure in both pure water and low-saline seawater. Recently however, new light was cast on the subject by an equation for sound speed in pure water under pressure based on new measurements up to 60 MPa, published by Belogol'skii *et al.*[5] These days, those wishing to calculate sound speed may attempt to implement the suitable equations in their own software, and there have also been attempts to provide online calculators.[6] However, the situation outlined above means that such attempts have to consider the two "complex" equations mentioned above plus an algorithm for calculating pressure from depth,[7,8] in addition to the two best available simple equations, those of Coppens[9] and Mackenzie.[10] Clearly, it would be more satisfactory to devise a single (and ideally simple) equation that gives essentially the same results as the best values obtained from data and equations in their relevant combinations of temperature T, salinity S, and pressure P calculated from depth (plus latitude Φ if necessary). At the same time, considering the fact that a new equation for pure water under pressure had been published, it is clearly of interest to see if it could not be used to replace the incorrect part of the UNESCO equation. It was then a pleasant surprise to discover that such a substitution could make the UNESCO equation be in much better agreement with Del Grosso's data for pressures up to some 30 MPa. Therefore, the use of this merged equation could provide reliable values for the miss-

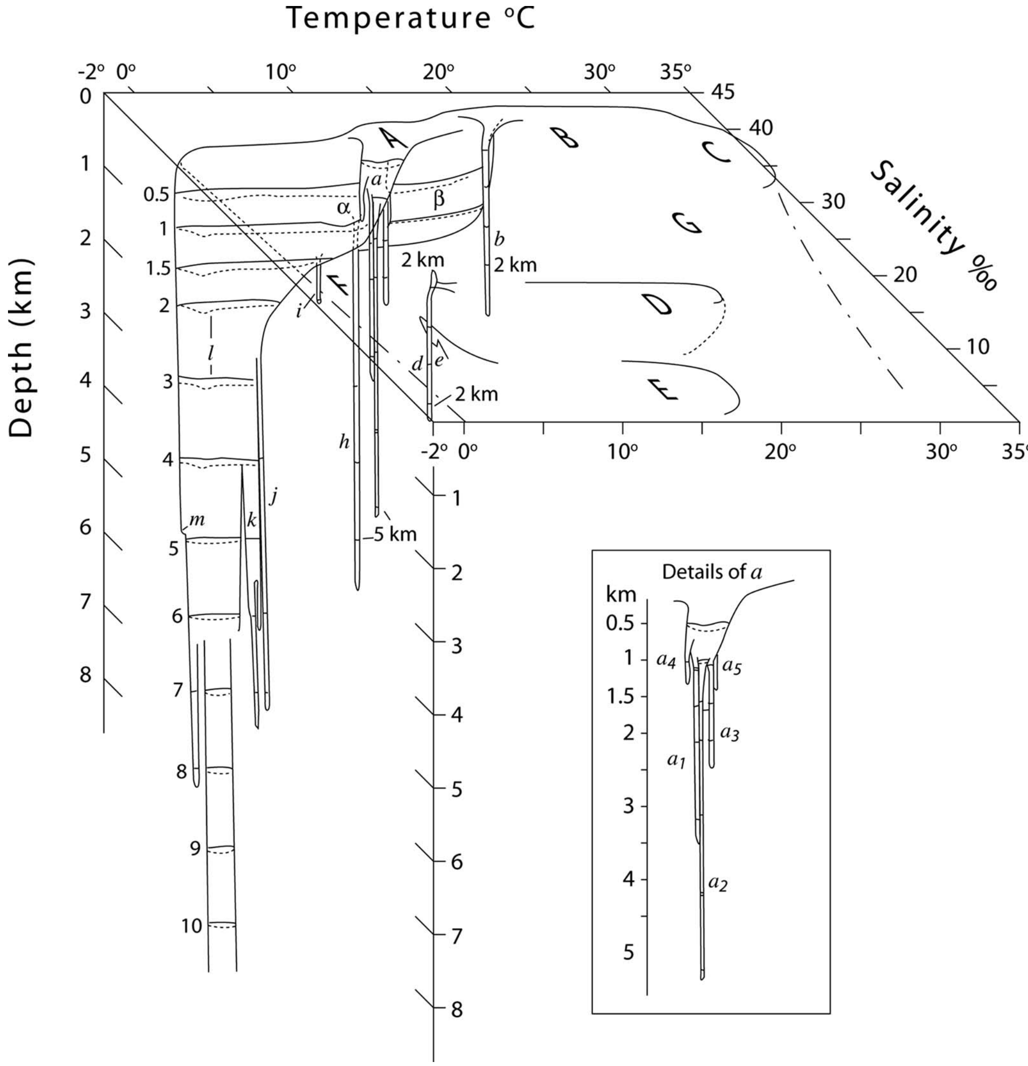

FIG. 1. Three dimensional representation of the temperature-salinity-depth volume of subspace enclosed by real Neptunian water.[11] The key is given Table I.

ing low-saline water under pressure found in the Baltic and the Black Sea, hence providing the possibility of developing a new equation that would be valid in all existing seawater conditions, from nearly pure water to salinities up to 42%. In view of the wide use of depth as a parameter in echo sounding, telemetry, acoustic tomography, ray tracing, and sonar applications, it was decided to use depth and latitude instead of pressure. An equation has thus been developed, which provides an accuracy always better than ± 0.2 m/s with respect to the most appropriate complex equation derived from fitting measured data (either NRL II or the merged equation, according to the required salinity range). The purpose of this paper is to present this equation and compare its results with those from the two reference equations, which are then discussed with respect to the accurate data. A comparison is also made with the equations of Coppens and Mackenzie. To make clear the motivation for the new equation, some discussion is first presented regarding the existing equations.

TABLE I. The key for Figure 1 showing the seas and oceans covered.

KEY	Surface water	Deep water	Water flowing into adjacent oceans
Mediterranean	A	*a*	α
Red Sea	B	*b*	β
Persian Gulf	C		
Black Sea	D	*d*	
Arctic coastal waters	F		
Tropical coast waters	G		
Sulu Sea		*h*	
Halmahera Basin		*i*	
“American Mediterranean”		*j*	
East Indian Basins		*k*	
Sea of Japan		*l*	
Arctic Ocean		*m*	

II. THE BASIC DATA AND THE EXISTING EQUATIONS

A. General

A valuable equation for sound speed in seawater should primarily fit the best available data for combinations of temperature, salinity, and pressure that exist in the oceans and seas. The subject of these existing *TSP* combinations in the oceans has already been examined by Leroy[11] who proposed the terminology "Neptunian" waters to eliminate inland seas of extremely high salinities. These waters must, however, also eliminate the "hot spots" that exist in a few places at the bottom of various seas (Mediterranean, Red Sea, and Gulf of Mexico), where abnormal temperatures and salinities exist ($>200\%$). In any case, the speed of sound has never been measured under such conditions, and if its knowledge was required, a separate formula should be developed from the results of specific measurements under these conditions. The reduced *TSP* domain of seawater of interest should be born in mind and is presented in Fig. 1 (see Table I), reproduced from Ref. 11.

In practice, the existing systematic, accurate, and reliable measurements of sound speed in seawater reduce to two sets only: those of Del Grosso and Mader (performed with Weissler or Mader),[12,13] leading to the NRL II equation, and those of Millero and Kubinski[14] and Chen and Millero,[15] leading to the UNESCO equation. A third series of earlier systematic measurements, leading to two now abandoned equations, had been performed by Wilson.[16,17] Unfortunately, it was found that some sets of his measurements were made at salinities or temperatures erroneously determined, and that his velocimeter was not reliable enough under high pressures. The remaining two main sets of data are quite different both in measuring methods and in contents and will be now examined, with comments on the resulting equations.

B. Del Grosso and the NRL II equation

All the measurements of Del Grosso were absolute measurements of sound speed performed with utmost care using extremely accurate laboratory sound velocimeters especially designed and built by himself and his team. In particular, the design of his pressure velocimeter allowed for the suppression of all problems due to the unavoidable deformations of the equipment under high pressure. For a greater accuracy, all Del Grosso's measurements at any nominal *TSP* combination were repeated (between four and seven times in almost all cases), temperature inside the acoustic cell being each time measured within ± 0.001 °C. The explored *TSP* combinations almost completely covered the regular Neptunian domain down to depths of 9000 m, but limited to $30 < S < 41$ at $Z < 2000$, and to $33 < S < 38$ and $0 < T < 15$ at $Z > 2000$ (salinity quoted in percent, depth in meters, and temperature in °C). The conditions encountered in the Baltic, the Black Sea, and the Red Sea were therefore excluded. In all, these measurements covered 110 different *TSP* combinations, with 627 individual measurements (data points). Figure 2(a) gives an example of the distribution of these measurements.

Anderson then established the NRL II equation by taking the Del Grosso data points and adding seven additional values calculated at 0, 5, 10, 15, 20, 25, and 30 °C and $S = 0\%$ from the "148 point" equation developed by Del Grosso and Mader for pure water at atmospheric pressure,[18] each with a weight of 5 (i.e., the equivalent of 35 data points). The 1968 international temperature scale was used, and the $\mathrm{kg/cm^2}$ was the unit for pressure. This equation was subsequently readjusted by Wong and Zhu[19] for use with *T* values using the 1990 universal temperature scale and *P* values given in bars.

C. Chen and Millero and the UNESCO equation

In contrast to Del Grosso's measurements, those of Chen and Millero[15] (preceded by some from Millero and Kubinsky[14] at atmospheric pressure) were differential measurements performed with a commercial velocimeter and with hardly any repeated measurements. To calibrate the velocimeter, pure water was used at a range of pressures and temperatures, the latter being assumed to be the setting of the automatic controller (no independent measurements were made). Comparative measurements of velocity in seawater of various salinities were then made at similar temperature settings and pressures. The only repeated measurements were duplications and were only performed in four cases. In all, 340 different *TSP* combinations were investigated with a total of 375 data points. The *TSP* domain covered by these data is a "cubic" one, with temperatures between 0 to 40 °C, salinities between 5% and 40%, and pressures regularly spaced in 10 MPa intervals from 0 to 100 MPa, about half of these data corresponding to seawater conditions which do not exist on earth. Figure 2(b) gives an example of the distribution of these measurements for values around the same pressure as for Del Grosso and Mader, illustrating the great difference between the two approaches.

The set of data thus obtained was fitted by a polynomial equation giving the sound speed difference, ΔC, between saline and pure water as a function of *T*, *S*, and *P* as follows: $\Delta C = \{C(S,T,P) - C(0,T,P)\}$. To obtain from this an equation for sound speed in seawater, an equation giving the absolute values for pure water was required. There existed at the time only two such equations, one from Barlow and Yazgan[20] and one from Wilson.[21] (Del Grosso had not been given the time to make measurements in pure water under pressure.[22]) Barlow and Yazgan's equation was rejected because most temperatures considered were out of the domain of interest (a total of 8 values between 16 and 94 °C), and that of Wilson could not be used because evidence had proved that his data were unreliable. Chen and Millero chose to correct the data from Wilson, assuming that their departures from a smooth variation were due to errors in the temperature measurements or sound speed shifts specific to each temperature, but disregarding possible errors in pressure measurements or due to pressure effects.[23] This re-evaluation of Wilson's data led Chen and Millero to a new equation for sound speed in pure water and hence to an equation for seawater, later referred to as the UNESCO equation. This equation used the 1968 temperature scale and the bar as the unit for pressure. It was later recalculated by Wong and Zhu[19] for use with the 1990 temperature scale.

D. The discrepancies between the NRL II and the UNESCO equations

It is easy to observe that the NRL II and the UNESCO equations give different results for seawater under pressure. As the latter was intended to have a larger coverage includ-

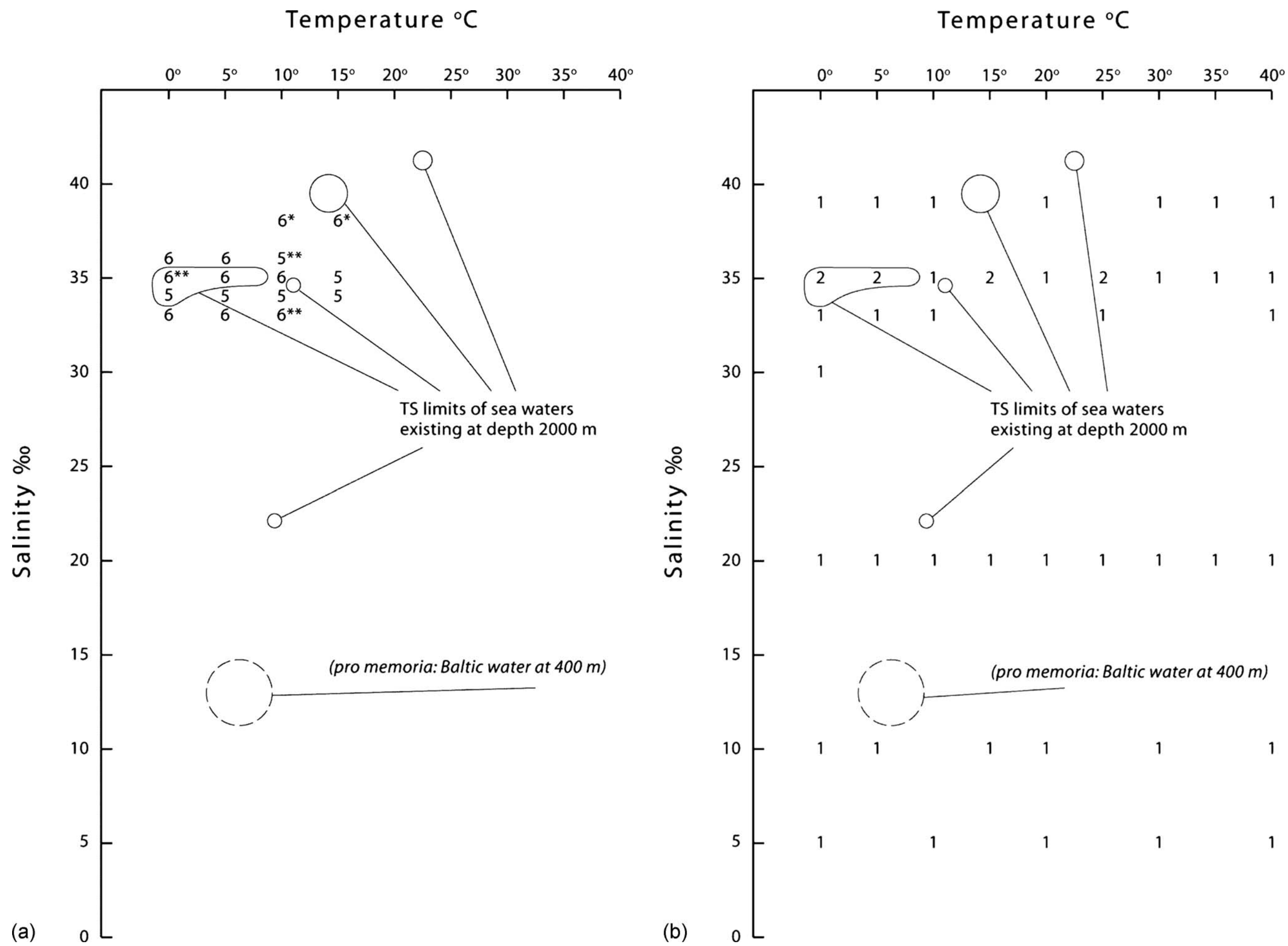

FIG. 2. (a) Distribution of the data points of Del Grosso and Mader as a function of T and S for comparison with existing TSP combinations. The values shown indicate the number of data points at $P=3000$ psi (gauge), while those labeled with * are for $P=2000$ psi (gauge) and those labeled with ** are for $P=4000$ psi (gauge). (b) Distribution of the data points of Chen and Millero at around 20 MPa as a function of T and S for comparison with existing TSP combinations. The values shown indicate the data points at $P=199.67$ bars.

ing low-saline seawater under pressure, it could be preferred by some users, at least in the corresponding seas. The fact that the UNESCO equation gave incorrect values under high pressure was proved by a number of acoustic experiments involving sound paths reaching great depths, and it was found that better agreements between observations and calculations were obtained using the NRL II equation.[24,25] This could obviously be due to the fact that the equation for pure water obtained from the re-evaluated data of Wilson gave incorrect values. This was first stressed by Dushaw *et al.*[1] and later agreed upon by Millero himself in a paper with Li.[22] They pointed out that making corrections independent of pressure was responsible for the errors and proposed a correction to apply to their equation in order to approach the values obtained with NRL II. This proposal, however, has not been adopted for correcting the UNESCO equation. In all, reliable data of sound speed in pure water under pressure were first needed before new comparisons with Del Grosso and with sea measurements involving great depths. The equation for pure water under pressure of Belogol'skii *et al.*[5] casts new light on this aspect of the problem.

E. The equation and data of Belogol'skii *et al.*

The equation of Belogol'skii *et al.* is based on differential measurements (with respect to atmospheric pressure conditions) performed with distilled water in the temperature range 0.4 to 40 °C and at pressures from 0 to 60 MPa by a quite different method than previous approaches. An acoustically transparent vessel filled with pure water was placed in a tank between a transmitter and a receiver and was pressurized at various values. Travel time differences with respect to atmospheric pressure conditions were measured and used to obtain differential values of sound speed, knowing the dimensions of the vessel, and its deformation with pressure calculated from the elastic properties of its material. Details regarding the overall accuracy of the method are, however, limited. A total of 231 data points are cited, resulting from measurements at 11 different temperatures and 21 pressures including some duplications. This set was used to obtain a matrix form polynomial development giving the differential values of sound ΔC as a function of temperature and pressure, to which the universally adopted equation of Del Grosso for sound speed in pure water at atmospheric pressure[18] was added to obtain the final equation. The equa-

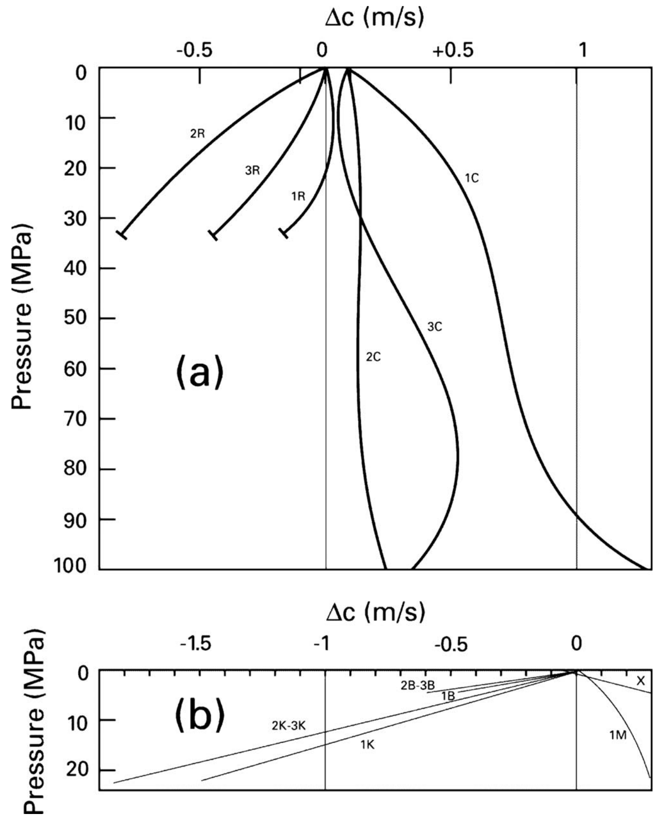

FIG. 3. Differences between sound speed values given by various equations with respect to the NRL II equation which is taken as the reference. The curves are numbered as follows: (1) UNESCO equation, (2) UNESCO equation as corrected by Millero and Li, and (3) UNESCO equation when replacing the pure water part of the equation by that of Belogol'skii *et al.* ("merged equation"). For the upper plot, (a) curves 1C, 2C, and 3C are the common oceans at depth ($S=35\%$, $T=2$ °C); curves 1R, 2R, and 3R are for the Red Sea ($S=40.5\%$, $T=21.5$ °C). The lower plot, (b), shows results for lower saline water, curves 1K, 2K, and 3K are for the Black Sea ($S=22\%$, $T=8.8$ °C); curves 1B, 2B, and 3B are for the Baltic ($S=13\%$, $T=5$ °C). In part (b) of the figure, two curves show the differences when taking the merged equation as the reference: curve 1M, UNESCO equation; curve X, NRLII.

tion for the differences ΔC is claimed to fit the original data with a standard deviation of 0.042 m/s and an arithmetic deviation smaller than 1 cm/s in absolute value.

F. A merged equation and its interest for low-saline water under pressure

Considering the accuracy claimed by the equation of Belogol'skii *et al.*, it is perfectly justifiable to try using it to replace the controversial part for pure water of the UNESCO equation, keeping the polynomial to be added for seawater unchanged. This gives rise to a merged equation for seawater, limited to pressures not exceeding 60 MPa. Using NRL II as the reference, Fig. 3 gives examples of the differences between the sound speeds obtained using three other equations: (i) the UNESCO equation, (ii) the UNESCO equation corrected according to Millero and Li, and (iii) the merged equation just described. One can observe that, in common cases, the departure of the UNESCO equation from NRL II grows markedly with increasing pressure, in particular, in the first 30 MPa. Such a behavior is not found with the merged equation which agrees rather well with NRL II until about 30 MPa. Overall, the results obtained with this merged equation are not entirely convincing, but provide an approximation to NRL II more closely than does the UNESCO equation. It can also be observed that the correction term of Millero and Li gives good results in common oceans but completely fails at high salinities under pressure (as in the case of the Red Sea). Therefore, for the work reported here, the results obtained with NRL II were chosen as the reference for *TSP* combinations encountered in the open oceans and seas with salinities higher than 30%.

The situation is very different with low-saline seawater under pressure. Del Grosso made no measurements at the so-called intermediate salinities, those between 0% and 30%. At atmospheric pressure, however, he had made measurements with distilled water leading to an equation that is universally accepted. As these values were included in the set of data fitted by NRL II, this equation gives good results at intermediate salinities at atmospheric pressure and is in good agreement with the UNESCO equation. However, as soon as these low-saline waters are placed under pressure, NRL II gives erroneous results because the equation is then employed in what corresponds to an extrapolation far outside the domain of the measurements. Figure 3(b) shows the departures between the equations at the low salinity *TSP* combinations found in the Black Sea and the Baltic. As pressure never exceeds 23 MPa in such waters, the use of the merged equation as the reference is then plainly justifiable. On Fig. 3(b), the curves marked "X" and "1M" are drawn using this reference instead of NRL II. The UNESCO equation is in good agreement, while Del Grosso's algorithm gives unacceptable results (outlined on the figure).

III. DEVELOPMENT OF THE NEW EQUATION

Following the observations presented above, it was decided to develop an equation that would give results in accordance with the Del Grosso algorithm in all oceans and seas, with the exception of the Black Sea and the Baltic where it should agree with the merged Belogol'skii, *et al.*-UNESCO equation. It was also decided to use the 1990 temperature scale, so that the equation would be consistent with Wong and Zhu's revisions. As many oceanographic data may employ the 1968 temperature scale, the user of the equation should check the scale and apply an elementary transformation if necessary. In the temperature range of existing seawater, this transformation is simply given by

$$T_{90} = 0.99975 \cdot T_{68}, \tag{1}$$

as it can be found from Fig. 1 and Table 6 of Ref. 26.

The use of results from the two reference equations instead of the original data for establishing the new equation could be considered an unusual approach, but in the end it is plainly justifiable.

This important point will be examined in Sec. IV.

The equation was developed with the aim of fitting all sound speed values that could be found in the oceans and seas below a salinity of 42%. For example, at salinities between 34% and 35% and at 5 °C, sound speed values must be accurate at depths from 0 to 2000 m only, but at all latitudes lower than 60° (open oceans). Instead, in the same salinity range and at $T=-1$ °C, the accuracy must be ensured up to a depth $Z=4800$ m, at latitudes higher than 60° (Arctic Ocean), while at 10 °C it must be ensured up 5600 m of depth, but at 8° of latitude only (Sulu basin). The development of the equation followed the above approach, there being no need for it to give precise values of sound speed at TSZ and Φ combinations where seawater does not exist. It was also necessary that the maximum departures between the new equation and the reference ones should not exceed a given value, and that the agreement with the original data—when they were inside the existing seawater domain—should be satisfactory. The target for the maximum adopted acceptable departure from reference equations was then set at ± 0.3 m/s, with the hope of reaching a better value. It was also decided to use coefficients with reduced values of significant figures whenever possible. This point will be commented below.

The development of the equation was made using computer software through successive approaches, exploring each time the effect of modifying the coefficients in order to adjust them and simplify them if possible. At each given TSZ and Φ combination, the program calculated the pressure in MPa according to Leroy–Parthiot algorithm,[7,8] converted it to the required unit (kg/cm^2 or bars), and calculated the speed of sound given by the reference equation (NRL II or merged equation). The speed of sound given by the new equation was then calculated and compared with the reference, and plots of the differences were thus obtained and examined. The study was carried out systematically. For example, in the common oceans, the exploration was made by steps of 1000 m at each given $TS\Phi$ combination, steps of 0.5 or 1 °C at each selected $SZ\Phi$ combination, etc. In other cases (very deep trenches or closed basins), the specific values of T,S,Z and Φ were used and the proper correction term of the Leroy–Parthiot algorithm employed for calculating pressure.

It has been said that "the age of simple equations is over," a statement based on the fact that any personal computer can handle complex equations.[2] However, the analysis of the first results of the research, starting with common cases plus low-saline water, showed that coefficients with reduced numbers of significant figures could indeed be used with acceptable accuracy, and a decision was taken to proceed on that basis. This was even reinforced when it was found that an accuracy better than ± 0.2 m/s could be achieved. Some help in the development of the equation was provided by the examination of NRL II and Mackenzie's equation.[10] For example, there is no T^2P term in NRL II but a T^3P term is present. Similarly, there is no TZ^2 term in Mackenzie's equation but a TZ^3 term. However, the introduction of latitude is necessary, though it can be done without the use of trigonometric functions.

The new proposed equation is, with temperature T in °C (1990 universal temperature scale), salinity S in%, depth Z in meters, and latitude Φ in degrees, shown in

$$\begin{aligned} c = {} & 1402.5 + 5T - 5.44 \times 10^{-2}T^2 + 2.1 \times 10^{-4}T^3 \\ & + 1.33S - 1.23 \times 10^{-2}ST + 8.7 \times 10^{-5}ST^2 \\ & + 1.56 \times 10^{-2}Z + 2.55 \times 10^{-7}Z^2 - 7.3 \times 10^{-12}Z^3 \\ & + 1.2 \times 10^{-6}Z(\Phi - 45) - 9.5 \times 10^{-13}TZ^3 \\ & + 3 \times 10^{-7}T^2Z + 1.43 \times 10^{-5}SZ. \end{aligned} \tag{2}$$

IV. DISCUSSION

A. The use of reference equations for the adjustment

The use of the values given by reference equations instead of the original measured data points was found preferable for various reasons. First, each data point corresponds to a unique measured pressure, and that pressure can be found at various depths according to location. For example, a pressure of 6000 psi (gauge) can be found at 4053 m (Arctic Ocean) and at 4078 m (equatorial water), a difference of 23 m leading to a speed variation of about 0.35 m/s at similar temperature and salinity values. For this reason, classical computer calculations of coefficients—for instance, by a minimization of the standard deviation of the model-data differences—are not well adapted, since the equation needs to use two variables, Z and Φ, while the data have only one parameter P. Another point in favor of reference equations for the adjustment of the new one is the fact that at high pressure, data points are slightly outside the range of existing temperatures. At depths greater than 5000 m, seawater only exists between 1.5 and 2.5 °C, with only two exceptions (American Mediterranean and Sulu Sea). At such pressures, the measurements were only made at two nominal temperatures, viz., 0 and 5 °C. There was no real need for the new equation to accurately fit the data at these temperatures while the aim was its accuracy between 1.5 and 2.5 °C. In fact, the NRL II equation can give good values in that range because it "inherits" the nonlinear variation of speed with temperature from data at lower pressures.

The above arguments are only valid provided the reference equations be in almost perfect agreement with the data they are based on. This is the case with NRL II which is claimed to give a standard deviation of 0.05 m/s with all Del Grosso's data points corresponding to "realistic combinations of the parameters" TSP for seawater and to temperature values from 0 to 30 °C in pure water at atmospheric pressure. This remarkably good fitting is, however, merely mentioned in the original paper.[3] It only appears in the abstract, and no more comment is given thereafter, nor are values of the mean and maximum deviations with data provided either. In the development of the new equation reported here, calculations have been made of these values for a total of 523 seawater data points of Del Grosso. These corresponded to all his measurements at atmospheric pressure and those under pressure at salinities of 34% and 35%. At the other salinities, some measurements were not considered if the combination of temperature and pressure was not realistic. The mean deviation of NRL II was then found to be +0.001 m/s only, with maximum departures of +0.16 and −0.2 m/s, hence the justification to use it at salinities higher than 33%.

TABLE II. Comparison between Eq. (2) (C_1) and NRL II as recalculated by Wong and Zhu (C_0) for common oceans as a function of depth at three latitudes.

Z (m)	P (MPa)	C_1 (m/s)	C_0 (m/s)	C_1-C_0 (m/s)
Common oceans at $\Phi=0°$, $S=35\%$, $T=2$ °C				
0	0.00	1457.985	1458.010	−0.025
1000	10.06	1474.279	1474.300	−0.021
2000	20.19	1491.027	1491.044	−0.017
3000	30.35	1508.175	1508.181	−0.006
4000	40.57	1525.667	1525.661	0.006
5000	50.83	1543.449	1543.434	0.015
6000	61.14	1561.464	1561.447	0.017
7000	71.49	1579.659	1579.646	0.013
8000	81.89	1597.976	1597.976	0.000
9000	92.33	1616.363	1616.381	−0.018
10000	102.81	1634.762	1634.802	−0.040
11000	113.34	1653.120	1653.181	−0.061
12000	123.90	1671.380	1671.456	−0.076
Common oceans at $\Phi=30°$, $S=35\%$, $T=2$ °C				
0	0.00	1457.985	1458.010	−0.025
1000	10.08	1474.315	1474.322	−0.007
2000	20.21	1491.099	1491.089	0.010
3000	30.40	1508.283	1508.250	0.033
4000	40.63	1525.811	1525.755	0.056
5000	50.90	1543.629	1543.553	0.076
6000	61.22	1561.680	1561.591	0.089
7000	71.59	1579.911	1579.815	0.096
8000	82.00	1598.265	1598.171	0.094
9000	92.45	1616.687	1616.601	0.086
10000	102.95	1635.122	1635.046	0.076
11000	113.49	1653.516	1653.448	0.068
Common oceans at $\Phi=60°$, $S=35\%$, $T=2$ °C				
0	0.00	1457.985	1458.010	−0.025
1000	10.10	1474.351	1474.365	−0.014
2000	20.27	1491.171	1491.179	−0.008
3000	30.48	1508.391	1508.387	0.004
4000	40.73	1525.955	1525.942	0.013
5000	51.04	1543.809	1543.790	0.019
6000	61.39	1561.896	1561.879	0.017
7000	71.78	1580.162	1580.154	0.008
8000	82.22	1598.553	1598.560	−0.007

TABLE III. Comparison between Eq. (2) (C_1) and NRL II as recalculated by Wong and Zhu (C_0) for common oceans at a latitude of 30° as a function of temperature and salinity for two depths.

T (°C)	C_1 (m/s)	C_0 (m/s)	C_1-C_0 (m/s)
Common oceans, $\Phi=30°$, $P=80$ MPa ($Z=7808.13$ m), $S=34.7\%$			
1.0	1590.335	1590.272	0.063
1.5	1592.335	1592.281	0.054
2.0	1594.311	1594.272	0.039
2.5	1596.262	1596.246	0.016
3.0	1598.190	1598.202	−0.012
Common oceans, $\Phi=30°$, $P=80$ MPa ($Z=7808.13$ m), $T=2$ °C			
S (‰)	C_1 (m/s)	C_0 (m/s)	C_1-C_0 (m/s)
33.5	1592.610	1592.80	−0.193
34.0	1593.318	1593.41	−0.095
34.5	1594.027	1594.03	0.001
35.0	1594.736	1594.64	0.093
35.5	1595.445	1595.26	0.182
Common oceans, $\Phi=30°$, $P=5$ MPa ($Z=497.12$ m), $S=35\%$			
T (°C)	C_1 (m/s)	C_0 (m/s)	C_1-C_0 (m/s)
−2	1447.762	1447.727	0.035
0	1457.107	1457.097	0.010
2	1466.043	1466.055	−0.012
4	1474.579	1474.608	−0.029
6	1482.725	1482.767	−0.042
8	1490.492	1490.542	−0.049
10	1497.890	1497.941	−0.051
12	1504.929	1504.975	−0.047
14	1511.618	1511.654	−0.035
16	1517.969	1517.986	−0.017
18	1523.990	1523.981	0.009
20	1529.692	1529.649	0.043
Common oceans, $\Phi=30°$, $P=5$ MPa ($Z=497.12$ m), $T=8$ °C			
S (‰)	C_1 (m/s)	C_0 (m/s)	C_1-C_0 (m/s)
33	1488.004	1488.066	−0.062
34	1489.248	1489.302	−0.053
35	1490.492	1490.542	−0.049
36	1491.737	1491.786	−0.049
37	1492.981	1493.034	−0.053

As for the merged equation, there are no "direct" data points for this since the measurements with seawater were differential ones with respect to pure water, and those in pure water under pressure were not communicated. However, considering the accuracy claimed for the equation of Belogol'skii *et al.*, one can attempt to "reconstitute" data points from the Chen–Millero and Millero–Kubinski published differential values, adding to them the speed of sound calculated from the equation at each temperature and pressure. This work has been done for each realistic combination of temperature, salinity, and pressure. Unfortunately, due to the high number of combinations outside the existing domain and to the fact that only a single value was given for each combination, this corresponds to 18 data points only. Using these data, the merged equation gives a standard error of 0.15 m/s, a mean deviation of +0.023 m/s, and maximum departures of +0.22 and −0.32 m/s.

B. Comparison of the equation with the reference ones and with others

The comparison of the equation with the reference ones has been done systematically in order to cover all existing seawater conditions, with the only exception being the anomalous sea bottom hot spots with salinities far exceeding 42%. As said before, the Leroy–Parthiot algorithm has been used for transforming depth and latitude into pressure, each time with the appropriate term accounting for the gravitational anomaly. Tables II–IV give examples of such calculations for deep oceans and seas, and Table V for calculations with Baltic and Black Sea waters. The maximum departure with the reference equations was verified to stay within ±0.2 m/s. This accuracy of the equation is fully illustrated in Fig. 4.

With respect to the existing equations that use depth instead of pressure, the authors believe that the new equation is of much broader use and is more accurate. Simple equations such as those of Medwin[27] or Coppens[9] are either not

TABLE IV. Comparison between Eq. (2) (C_1) and NRL II as recalculated by Wong and Zhu (C_0) as a function of depth for the Eastern Mediterranean, Sulu Sea, Red Sea, and Arctic basin.

Z (m)	P (MPa)	C_1 (m/s)	C_0 (m/s)	C_1-C_0 (m/s)
Eastern Mediterranean at $\Phi=35°$, $S=38.7\%$, $T=13.6$ °C				
0	0.00	1506.586	1506.546	0.040
500	5.05	1514.746	1514.844	−0.098
1000	10.10	1523.018	1523.189	−0.171
1500	15.17	1531.387	1531.578	−0.191
2000	20.25	1539.838	1540.010	−0.172
2500	25.35	1548.356	1548.483	−0.127
3000	30.45	1556.926	1556.996	−0.070
3500	35.56	1565.532	1565.548	−0.016
4000	40.69	1574.160	1574.137	0.023
4500	45.82	1582.794	1582.761	0.033
5000	50.97	1591.418	1591.420	−0.002
5500	56.13	1600.019	1600.110	−0.091
Sulu Sea at $\Phi=9°$, $S=34.5\%$, $T=10.3$ °C				
0	0.00	1490.291	1490.256	0.035
500	5.02	1498.393	1498.441	−0.048
1000	10.05	1506.611	1506.702	−0.091
1500	15.10	1514.930	1515.024	−0.094
2000	20.16	1523.338	1523.405	−0.067
2500	25.24	1531.823	1531.841	−0.018
3000	30.32	1540.370	1540.330	0.040
3500	35.42	1548.969	1548.869	0.100
4000	40.52	1557.605	1557.455	0.150
4500	45.64	1566.266	1566.086	0.180
5000	50.77	1574.940	1574.757	0.183
5500	55.90	1583.613	1583.466	0.147
6000	61.05	1592.272	1592.211	0.061
Red Sea at $\Phi=20°$, $S=40.5\%$, $T=21.5$ °C				
0	0.00	1531.724	1531.641	0.083
500	5.04	1539.928	1540.011	−0.083
1000	10.09	1548.239	1548.402	−0.163
1500	15.15	1556.636	1556.817	−0.181
2000	20.22	1565.098	1565.257	−0.159
2500	25.31	1573.604	1573.723	−0.119
3000	30.40	1582.134	1582.219	−0.085
3500	35.51	1590.667	1590.745	−0.078
Arctic basin at $\Phi=80°$, $S=35\%$, $T=-1$ °C				
0	0.00	1444.429	1444.466	−0.037
1000	10.13	1460.820	1460.797	0.023
2000	20.31	1477.684	1477.663	0.021
3000	30.54	1494.981	1495.007	−0.026
4000	40.82	1512.674	1512.768	−0.094
5000	51.14	1530.724	1530.885	−0.161
5500	56.32	1539.872	1540.057	−0.185

accurate enough or of limited use (depths not exceeding 4000 m for the latter). Instead, a detailed comparison with the equation of Mackenzie[10] has been made, as this equation is more accurate and recognized as the best of those using depth. Compared with NRL II, the equation of Mackenzie is found to give values within ±0.3 m/s in the common oceans at depths up to 8000 m, but it departs thereafter, reaching as much as +1.4 m/s at 10000 m and +2.3 m/s at 11000 m. In the Artic ocean, the departure reaches −0.74 m/s at its maximum depth of 4800 m, and in the Red Sea sound speed differences are between −0.52 and +0.33 m/s. As for the low-saline seas that were not taken into account, the departures of Mackenzie's equation from the merged reference equation are as follows: (i) in Baltic water from zero to −0.61 m/s (ii) in the Black Sea from −0.35 to +0.185 m/s. All in all, the new equation proposed here stays within ±0.2 m/s of the reference equations, providing enhanced accuracy.

TABLE V. Comparison between Eq. (2) (C_1) and the merged equation (C_0) as a function of depth for the Black Sea and Baltic Sea (three temperatures).

Z (m)	P (MPa)	C_1 (m/s)	C_0 (m/s)	C_1-C_0 (m/s)
Black Sea at $\Phi=43°$, $S=22\%$, $T=8.8$ °C				
0	0.00	1469.457	1469.523	−0.066
500	4.99	1477.487	1477.570	−0.083
1000	9.99	1485.632	1485.709	−0.077
1500	15.01	1493.881	1493.936	−0.055
2000	20.03	1502.223	1502.247	−0.024
2500	25.07	1510.645	1510.639	0.006
Baltic Sea at $\Phi=58°$, $S=12\%$, $T=4$ °C				
0	0.00	1437.029	1437.144	−0.115
250	2.48	1440.993	1441.017	−0.024
500	4.96	1444.988	1444.930	0.085
Baltic Sea at $\Phi=58°$, $S=13\%$, $T=5$ °C				
0	0.00	1442.685	1442.808	−0.123
250	2.48	1446.653	1446.704	−0.051
500	4.96	1450.652	1450.636	0.016
Baltic Sea at $\Phi=58°$, $S=14\%$, $T=6$ °C				
0	0.00	1448.218	1448.349	−0.131
250	2.48	1452.190	1452.265	−0.075
500	4.96	1456.193	1456.214	−0.021

V. CONCLUSION

A simple equation [Eq. (2)] is proposed for the calculation of sound speed in seawater as a function of temperature, salinity, depth, and latitude. The equation uses the 1990 universal temperature scale. An elementary transformation [Eq. (1)] may be used if input data are based on the 1968 temperature scale. The accuracy of the equation with respect to the Del Grosso algorithm at salinities greater than 30% and to a merged equation based on the UNESCO and the equations of Belogol'skii *et al.* at lower salinities is better than ±0.2 m/s anywhere in the oceans and seas, and whatever the depth down to the greatest abysses. This is illustrated in Fig. 4 and in Tables II–V. The only exceptions to the validity of the equation in existing seawater are abnormal waters of salinity much higher than 42% (some inland "seas" and hot brine spots at the bottom of some seas).

The equation is of pure polynomial form, with 14 terms and coefficients of only one to three significant figures. Compared with a complete calculation using the Leroy–Parthiot algorithm to transform depth into pressure (9 terms, coefficients of 1–6 significant figures, 11 different corrective terms) and either the NRL II equation (19 terms, all coefficients with 12 significant figures) or the merged equation (41 terms with coefficients of 3–10 significant figures), this is a

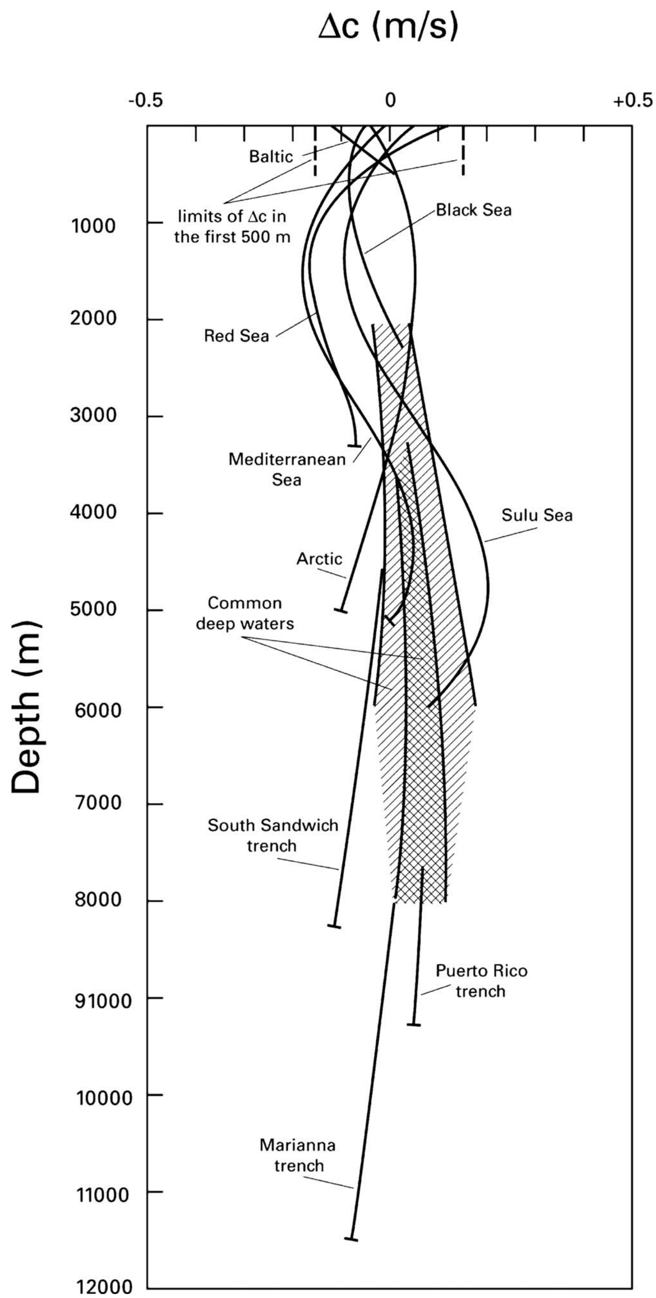

FIG. 4. Agreement of Eq. (2) with complex equations. Differences between the sound speed values (extreme cases) given by Eq. (2) and those obtained by transforming depth into pressure with the Leroy–Parthiot algorithm, and then using the NRLII equation (recalculated by Wong and Zhu). For the Black Sea and Baltic water (and those only), the reference was the merged equation.

substantial simplification. It is hoped that the equation will be welcomed by many of those who need to calculate sound speed in underwater acoustics applications.

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