Models of the signal response from SiPMs and PMTs

Oleg Kalekin
DPG18, Würzburg
19.03.2018

ERLANGEN CENTRE FOR ASTROPARTICLE PHYSICS
FRIEDRICH-ALEXANDER UNIVERSITÄT ERLANGEN-NÜRNBERG

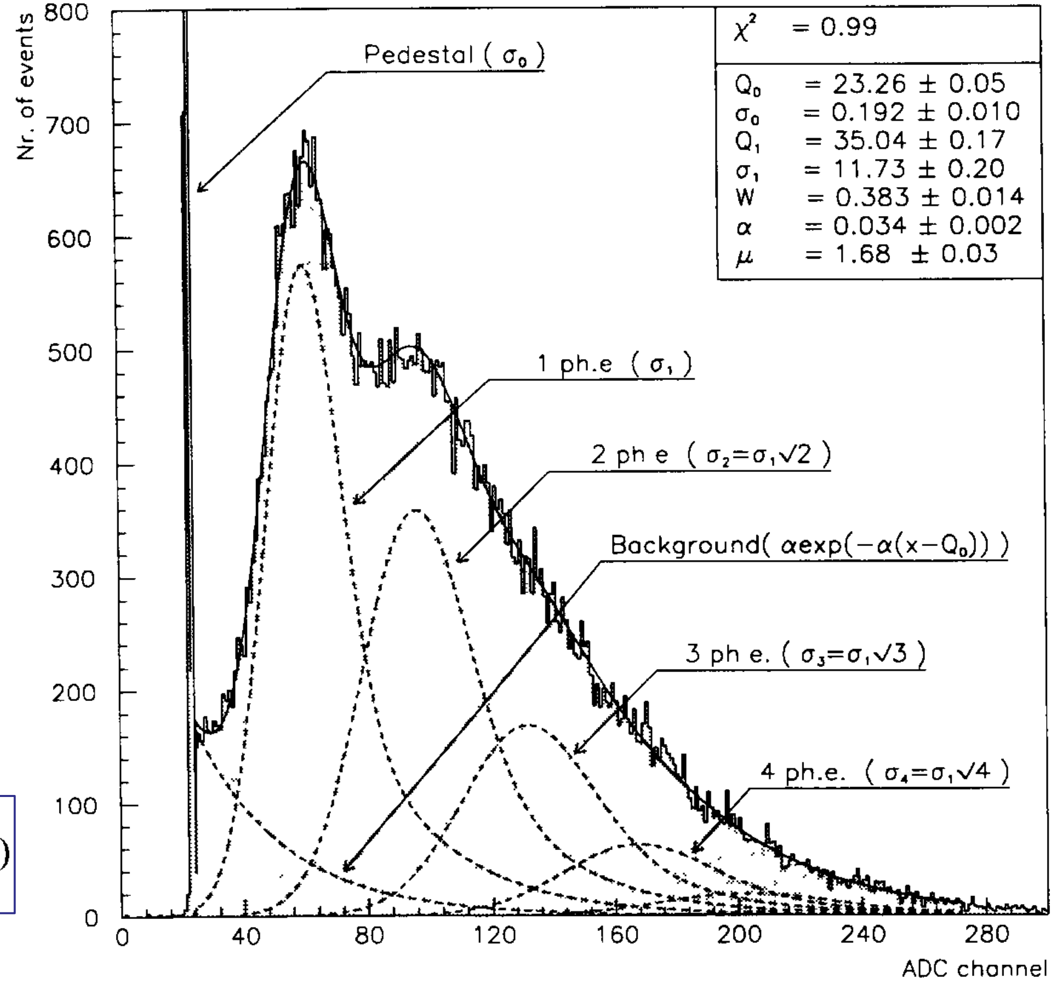
PMT model

Widely used model

- Poisson distribution for number of photoelectrons (p.e.)
- Gauss distribution of single p.e. charge
- Pedestal: Gauss + exp noise

$$S_{\text{ideal}}(x) = P(n;\mu) \otimes G_n(x) = \sum_{n=0}^{\infty} \frac{\mu^n e^{-\mu}}{n!} \frac{1}{\sigma_1\sqrt{2\pi n}} \exp\left(-\frac{(x-nQ_1)^2}{2n\sigma_1^2}\right)$$

$$B(x) = \frac{(1-w)}{\sigma_0\sqrt{2\pi}} \exp\left(-\frac{x^2}{2\sigma_0^2}\right) + w\theta(x)\alpha \exp(-\alpha x)$$

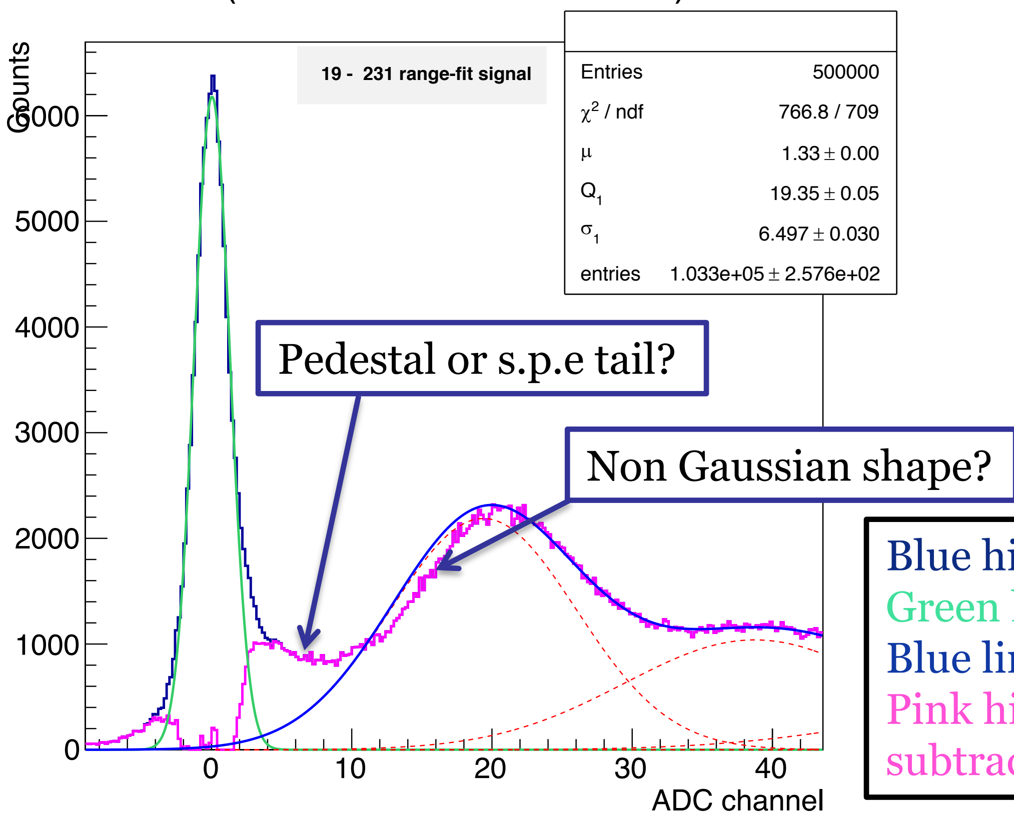


Bellamy et. al. (1994) Absolute calibration and monitoring of a spectrometric channel using a photomultiplier

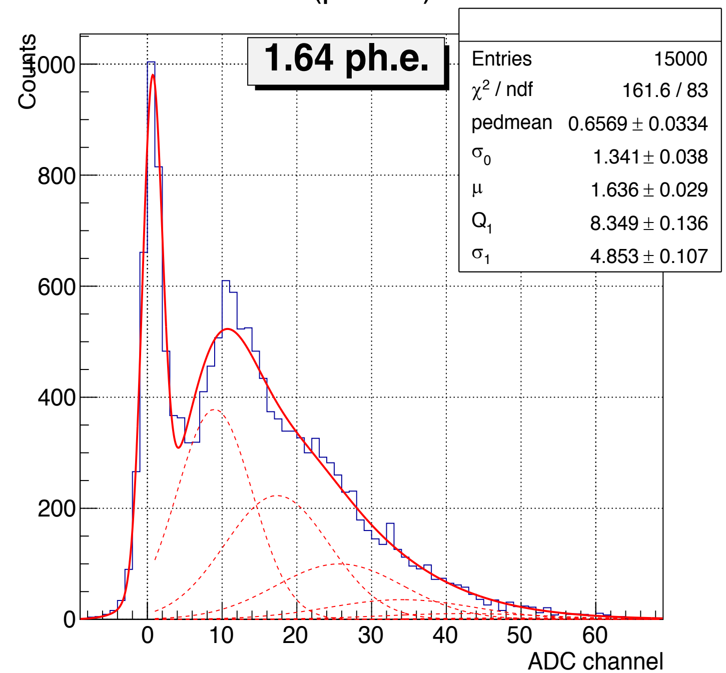
PMT model

The model does not work well in some cases, especially if s.p.e. peak and pedestal are well separated

(ch29 hv+150V fw=3000)



(pzmax)

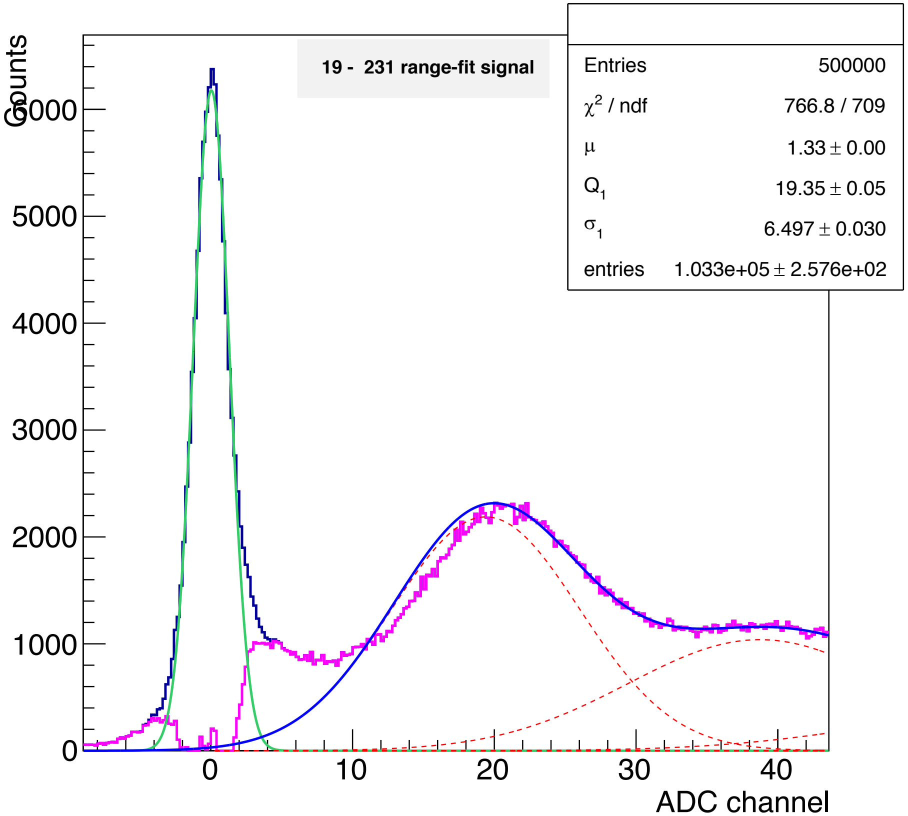


Blue histogram – data
Green line – pedestal Gauss fit
Blue line – PMT model fit (without pedestal)
Pink histogram – data with pedestal Gauss subtracted

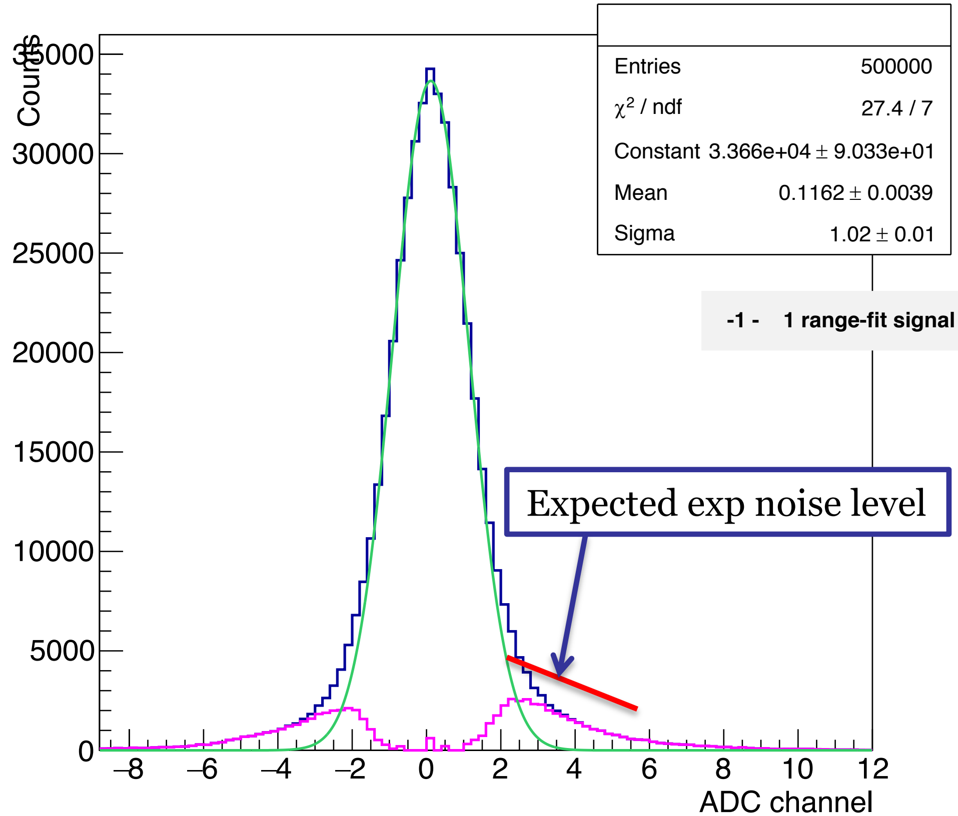
PMT model

- Pedestal distribution is pretty symmetric and can be fit by a sum of two Gaussian functions.
- No evidence for exponential noise tail from pedestal

(ch29 hv+150V fw=3000)



(ch29 hv+0V fw= Laser Off)

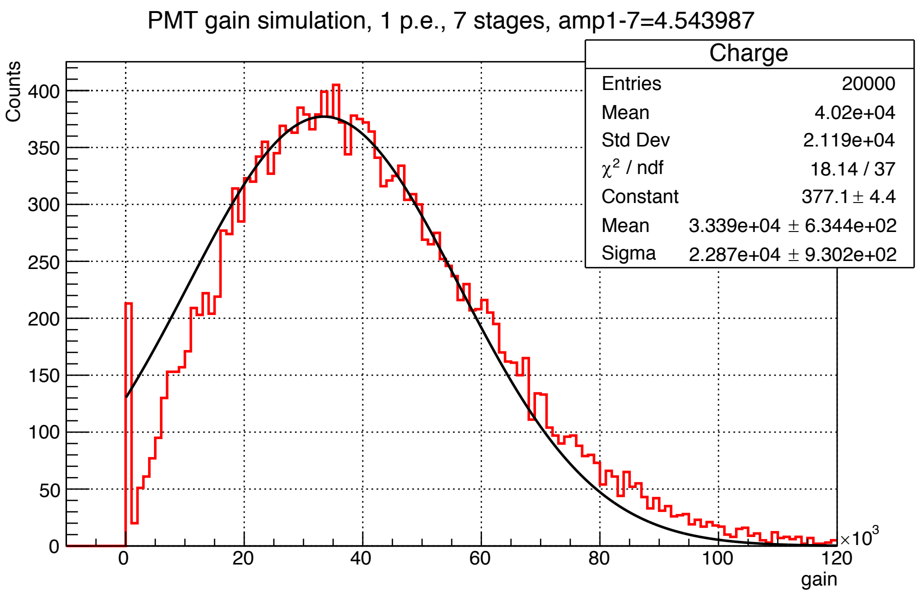


Simulations of s.p.e shape

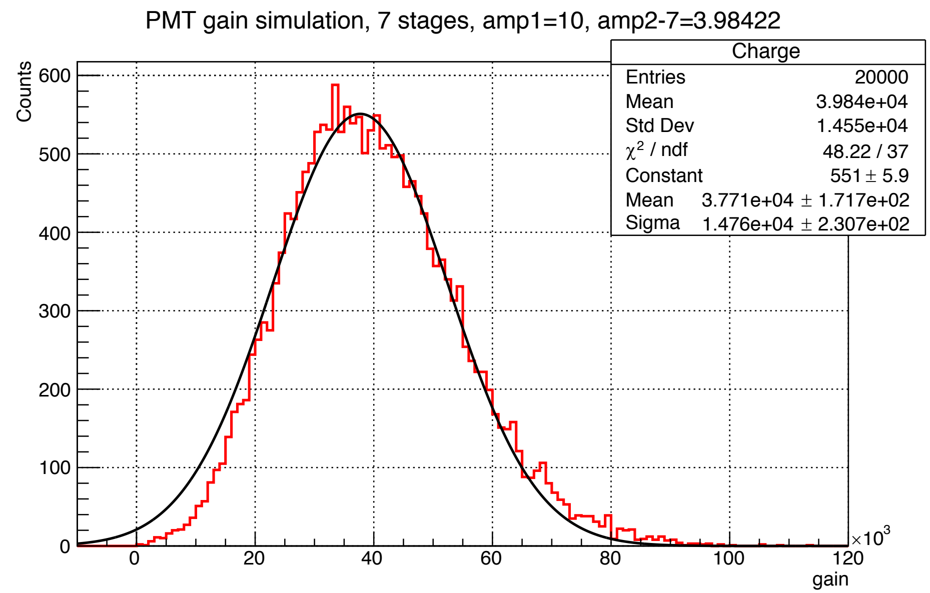
- 7-dynode PMT simulation, gain=4×10^4
- Poisson distribution of amplification factor at each stage

Red histograms – simulated s.p.e. charge distributions
Black lines – Gauss fits in range 20-60x10^3

Equal amplifications at all stages ~4.54



Amplification at 1st stage = 10
All others ~3.98

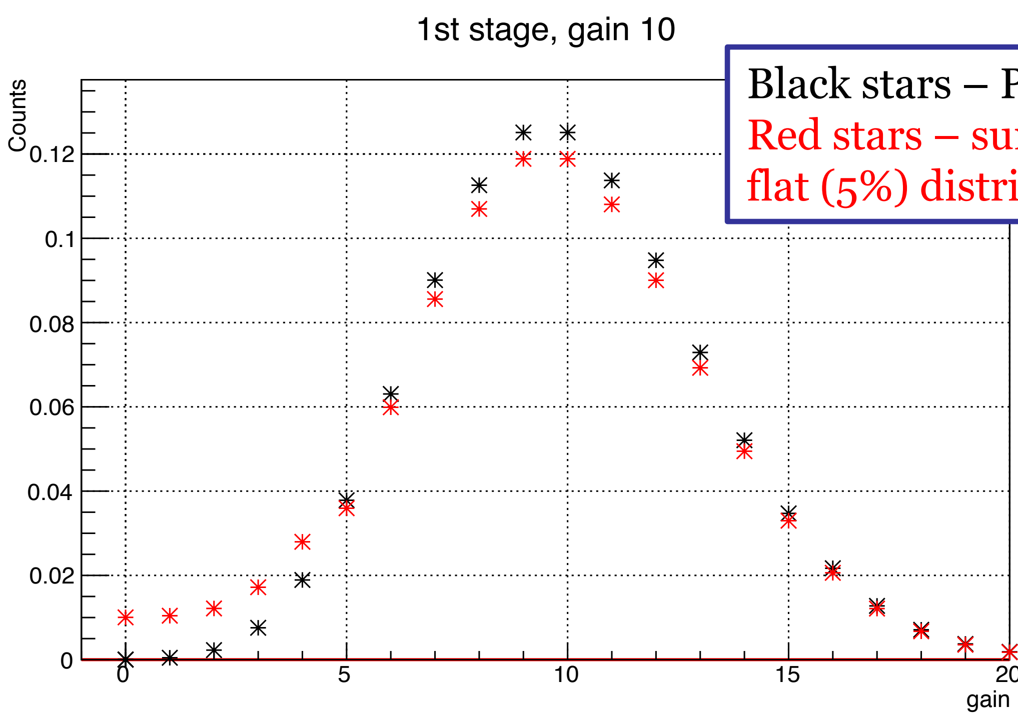


S.p.e shape is not ideal Gaussian even for amplification 10 at the first dynode

Simulations of s.p.e shape

Under-amplification due to partly elastic scattering at first dynode

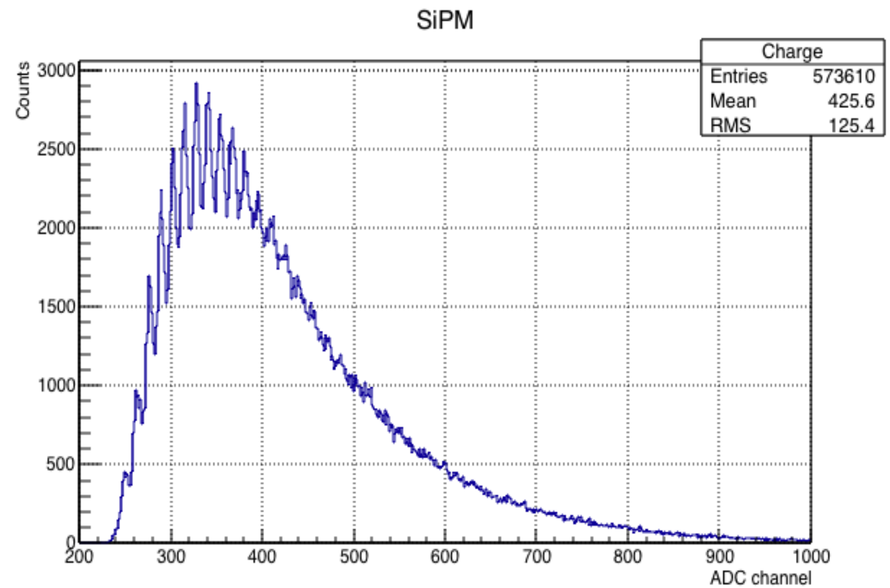
- Signal with under-amplification simulated as a sum of Poisson and flat distributions
  - Amplifications of 0, 1, 2, 3, 4 are equal probable in this flat distribution
- Comparison of simulated signal and PMT response
  - Parameters of new model (1st dynode gain, under amplification probability)
  - PMT response as a sum of basic functions



Black stars – Poisson with mean = 10
Red stars – sum of Poisson (95%) and flat (5%) distributions

SiPM model

- The same basic model: Poisson+Gauss
- Cross-talk as probabilities *P0, P1, …, Pm* that 1 p.e. produces *0, 1, …, m-1* additional p.e.
- Number of p.e. from Poisson changed with cross-talk probabilities using multinomial distribution

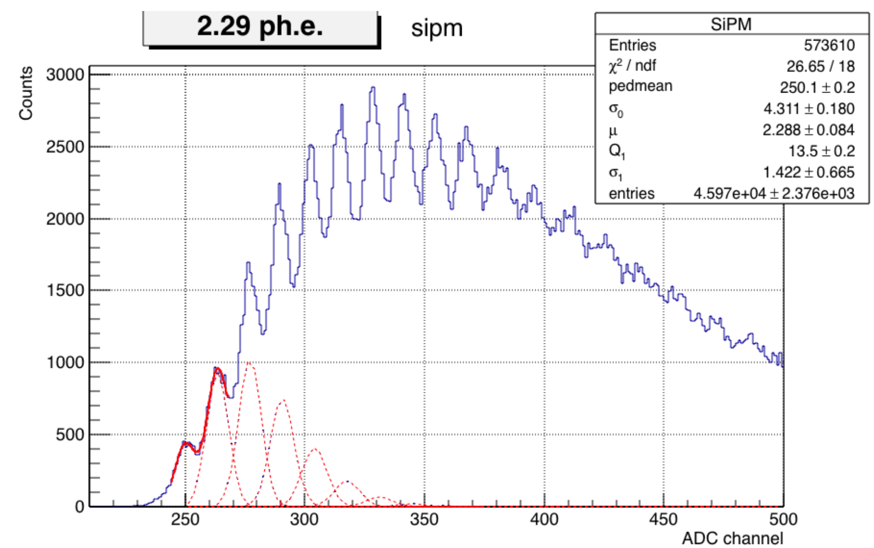


Influence of cross talk:

Expected mean # p.e. easily estimated from $k = 0$ of the Poisson distribution $N_k = \frac{N\mu^k exp^{-\mu}}{k!}$

Mean $\mu = -log\,{}^{N_0}/_{N} = 4.77\ p.e.$
Where N_0 - number of pedestal events and N – total number of events in distribution

If mean is derived from the ration between 0 and 1 p.e. event, it is $\mu = 2.29\ p.e.$ only



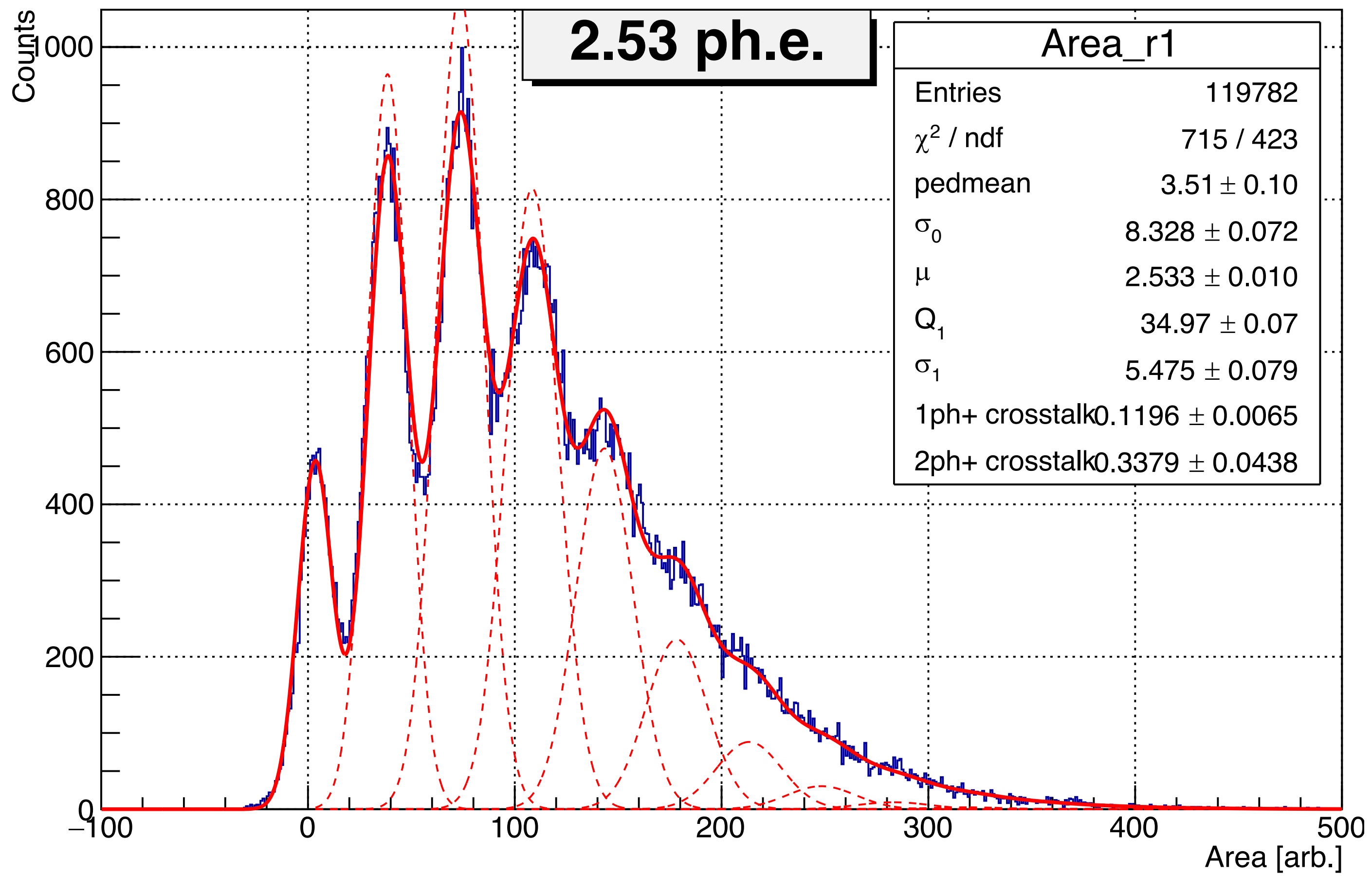
SiPM model

Dashed lines – original Poisson distribution
Solid red line – SiPM model function

Cross talk probabilities:

1ph – 11.6%

2ph – 0.4%



SiPM model (last slide)

Fit procedure is very slow for large # p.e.

Speed up method:

- Distribution is fitted with a sum of free amplitude gauss functions
- Cross talk probabilities are calculated analytically from derived n_0, n_1, … numbers and expected N_0, N_1, … numbers from Poisson distribution
  - $\mu = -\log(N_0/N)$
  - $N_1 = N_1(\mu)$, $n_1 = N_1 - P_{1+}N_1$ => $P_{1+} = (N_1 - n_1)/ N_1$
  - $n_2 = N_2 - P_{1+}N_2 + P_1N_1$ => $P_1 = \ldots$
  - $n_3 = \ldots$
- If higher orders of cross talk are needed, fit with fixed smaller ones

Back up slides

Multinomial distribution

$$f(x_1,\ldots,x_k;n,p_1,\ldots,p_k) = \Pr(X_1 = x_1 \text{ and } \ldots \text{ and } X_k = x_k)$$
$$= \begin{cases} \dfrac{n!}{x_1!\cdots x_k!} p_1^{x_1} \times \cdots \times p_k^{x_k}, & \text{when } \sum_{i=1}^k x_i = n \\ \\ 0 & \text{otherwise,} \end{cases}$$

n – number of p.e. in Poisson

$x_1, \ldots, x_k \Rightarrow 0, \ldots, k\text{-}1$ – number of cross-talk electrons

$p_1, \ldots, p_k$ $(\Sigma p_i=1)$ – probabilities of cross-talks

- **Example:**

Poisson probabilities P_0, P_1, P_2

probabilities of cross-talks $p_0=0.5$, $p_1=0.2$, $p_2=0.2$, $p_3=0.1$

#p.e.	0	1	2	3	4	5	6	7	8
P_0	P_0								
$P_1 \times$		0.5	0.2	0.2	0.1				
$P_2 \times$			0.25	0.2	0.24	0.18	0.08	0.04	0.01

SiPM model

Satisfying of the condition $\Sigma p_i=1$

Redefinition of probabilities of cross-talk in fit as following (each in range [0-1]):

- $P(\geq 1\ p.e.)=Pr(1+) \quad => \quad p_0=1-Pr(1+)$
- $P(\geq 2\ p.e.)=Pr(2+)\times Pr(1+) => p_1=Pr(1+)\times(1-Pr(2+))$
- $P(\geq 3\ p.e.)=Pr(3+)\times Pr(2+)\times Pr(1+) => p_2=Pr(1+)\times Pr(2+)\times(1-Pr(3+))$
- …

Calculated back cross talk probabilities:

1ph – 28.4%

2ph – 5.4%

3ph – 6.2%

4ph – 5.1%

5ph – 0%

6ph – 0%

7ph – 9.2%