

3/28/17 ①

E	E	M	P
W	O	U	E
C	Q	T	R
A	M	Y	X

prefix "Com"
used

curX [0]

curY [2]

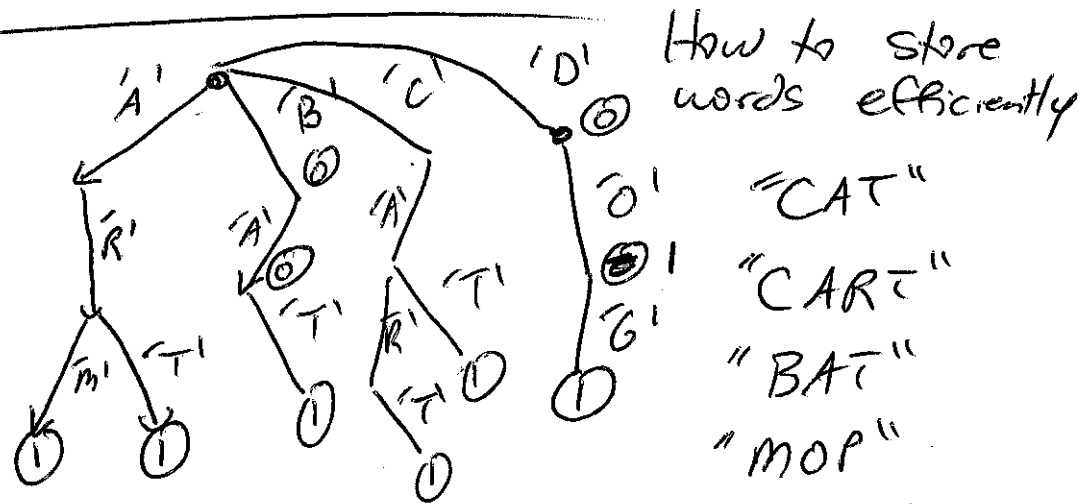
from here go in all 8 directions
skip (a) out of bounds

(b) used tiles

from E → M → P
↓ ↘
U E

if no word starts with "COMU", then
DON'T make the recursive call OR
cut off that branch in the beginning
of the recursive function

Tris



```

struct tri {
    int isWord;
    struct tri* next[26];
};
  
```

How TO INSERT

"DOB" → go down specified path.
 "DO" if no "next" node exists, form it. When we get to the end of the word, put a 1 in its spot.

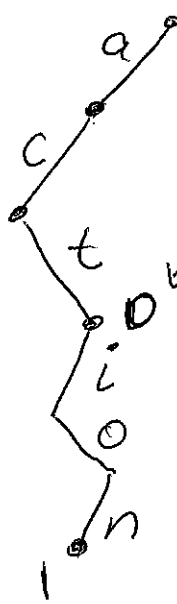
How TO SEARCH

Go down tree. If you get stuck, return 0.
 If you get to the right node return its isWord component.

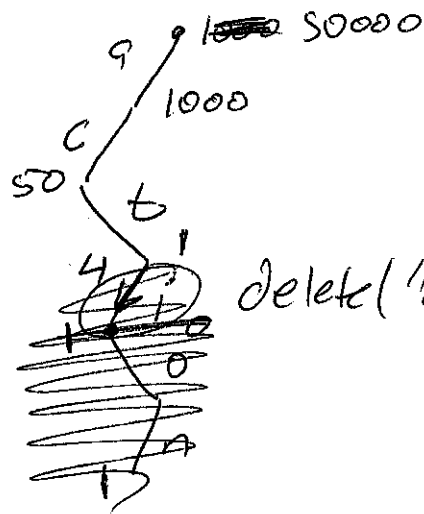
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```
int search(struct tri* myTrie, char word[], int k){  
    if (myTrie == NULL) return 0;  
    if (k == strlen(word))  
        return myTrie->isWord;  
    return search(myTrie->next[word[k]-'a'],  
                  word, k+1);  
}
```

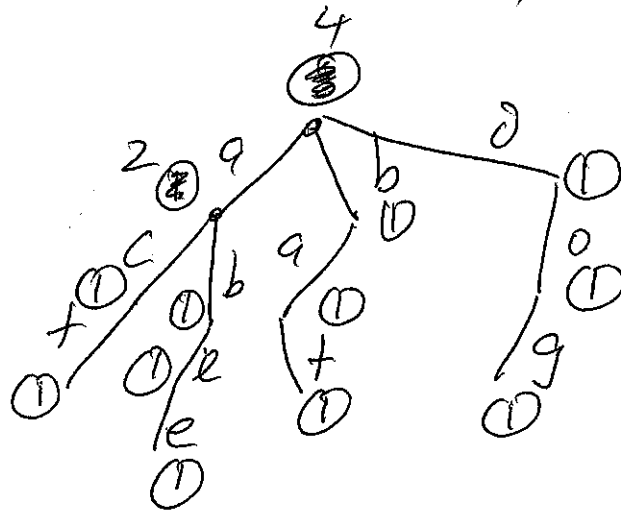
Delete of a word



delete("act")



delete("action")



prefix("zz") → 0

prefix("adj") → 1

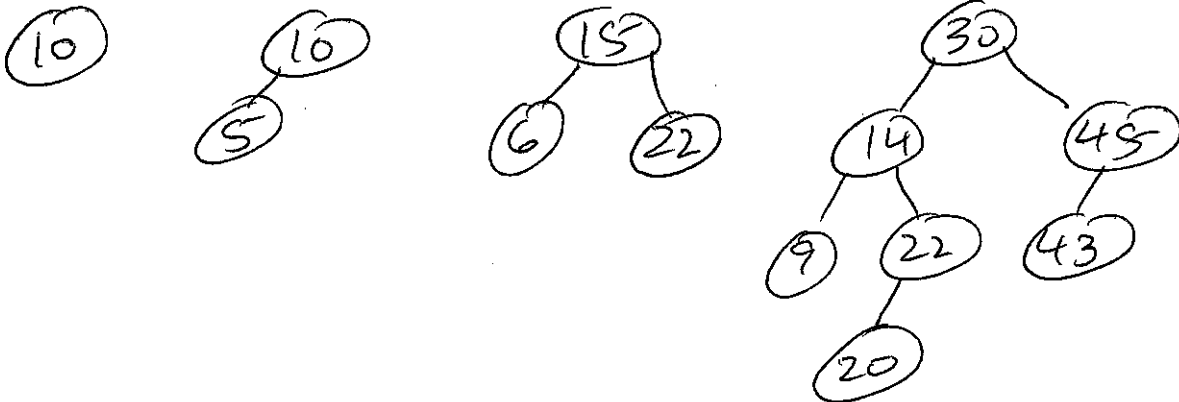
AVL Trees

1st balanced binary tree
 meaning that height = $O(\lg n)$ where
 $n = \# \text{ nodes}$ so insert, delete ~~take~~ take
 $O(\lg n)$ time.

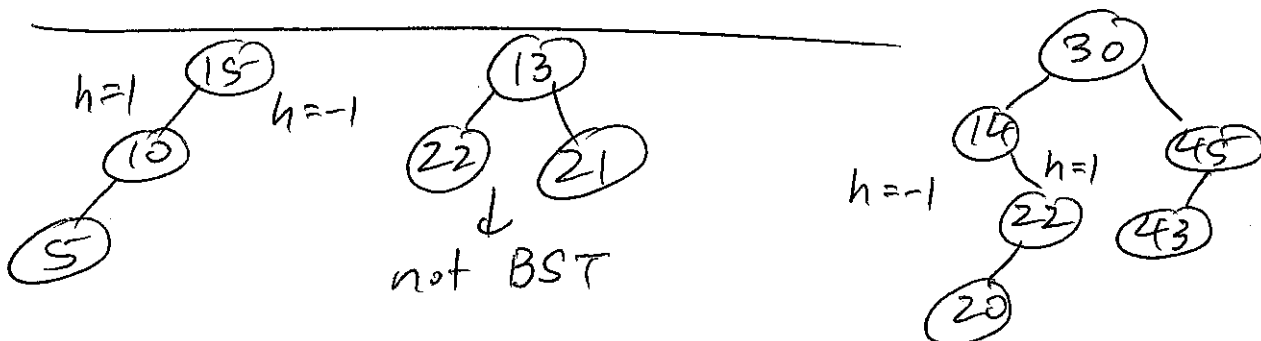
① Binary Search Tree

② for any node the height of its
 left and right subtrees can not
 differ by more than 1.

Valid AVL Trees



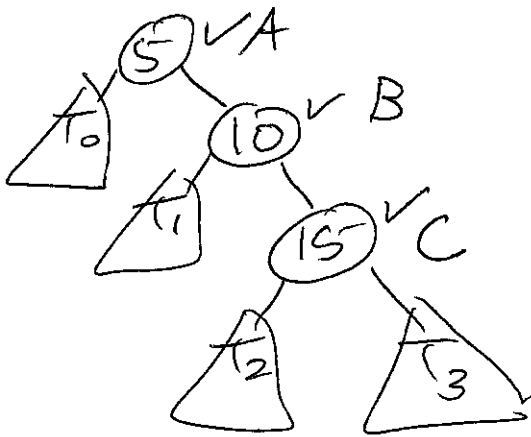
Invalid (NOT AVL)



AVL Tree Insert

3/30/2017 ③

Insert
15

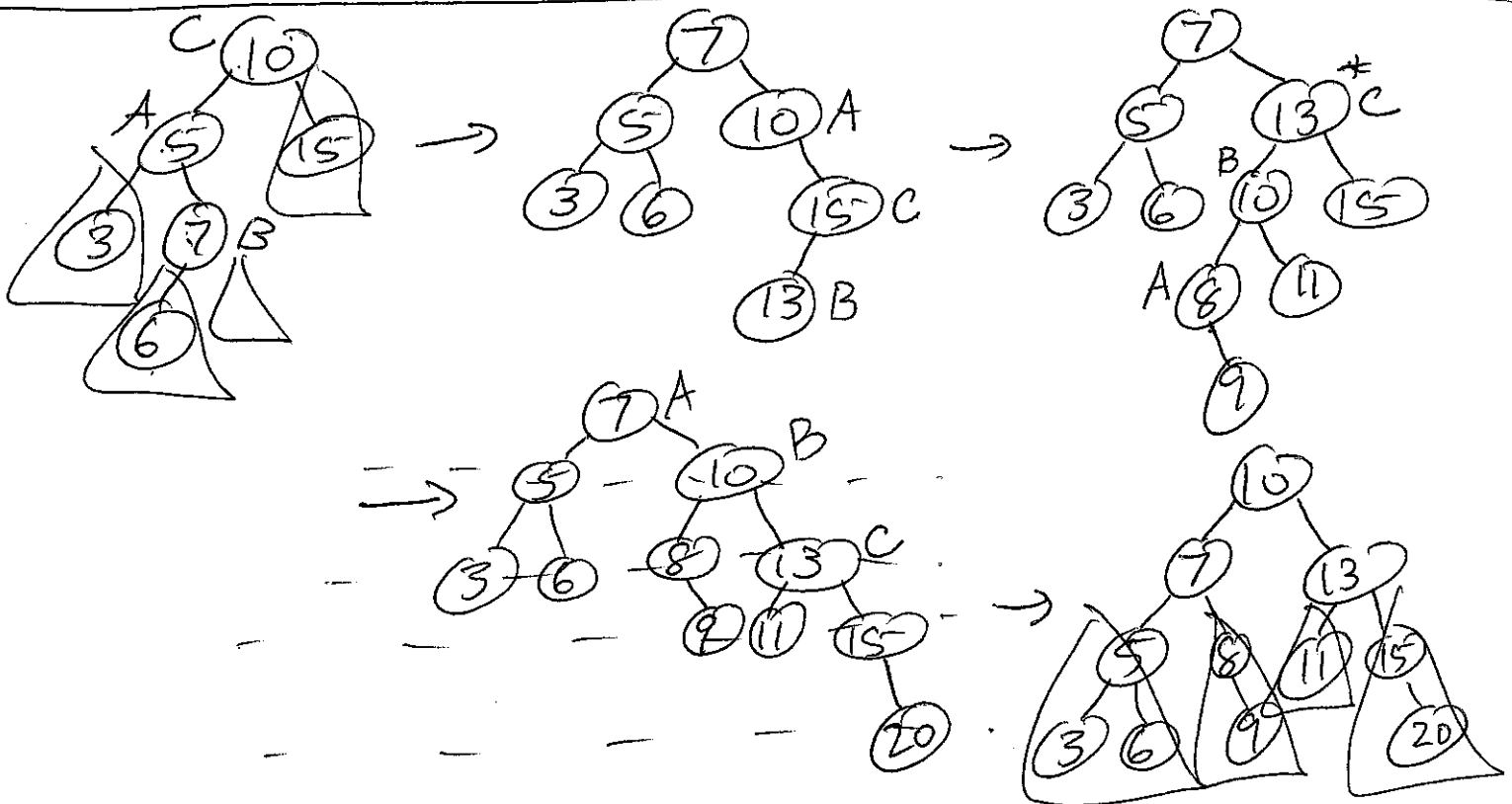
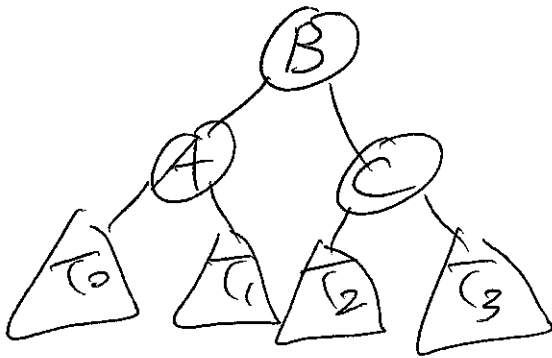


① Regular Insert

② Start marching up the tree. At each node check for imbalance

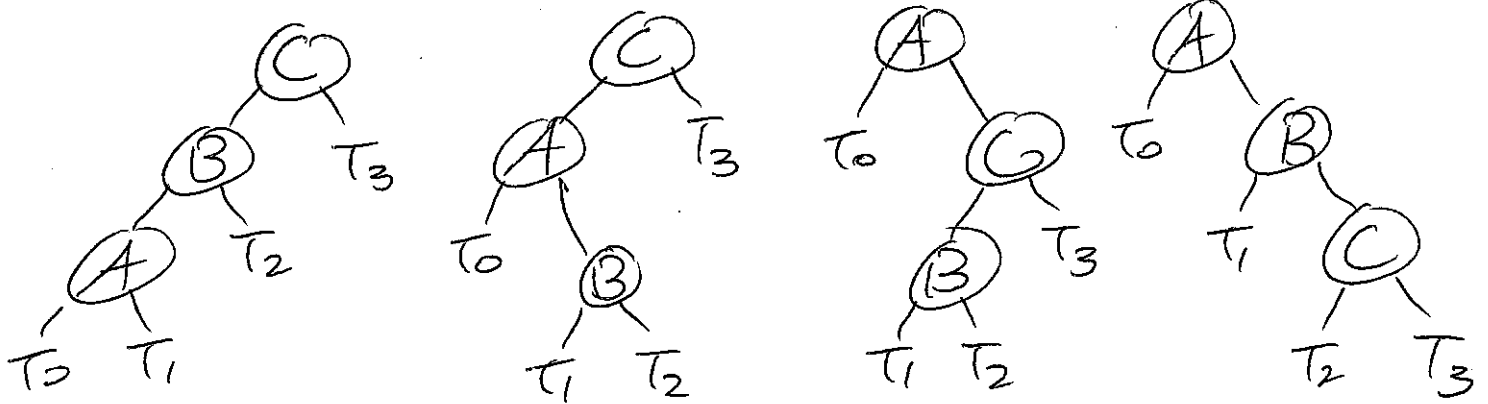
③ If you find one, "rebalance".

$$T_0 < A < T_1 < B < T_2 < C < T_3$$



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When you start tracing up, you may find one of 4 problems:

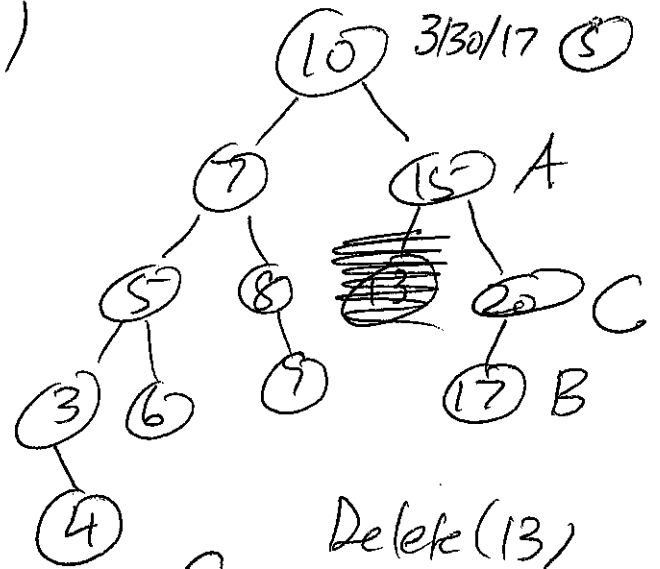
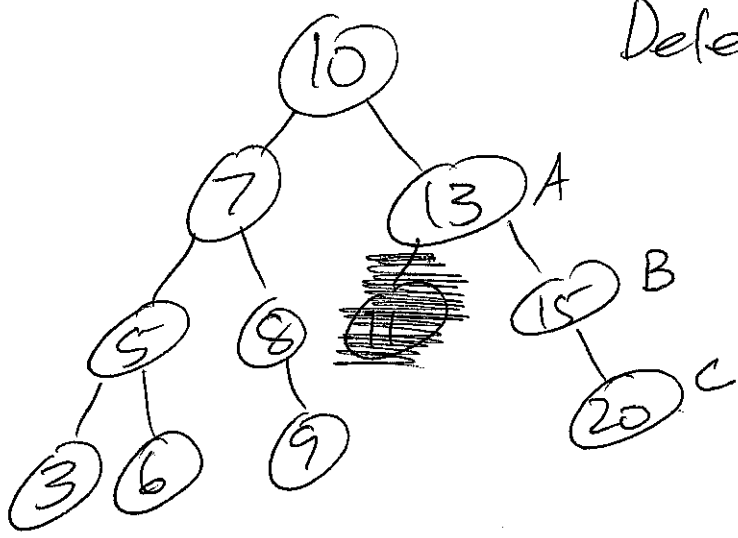


MOST COMMON ERROR

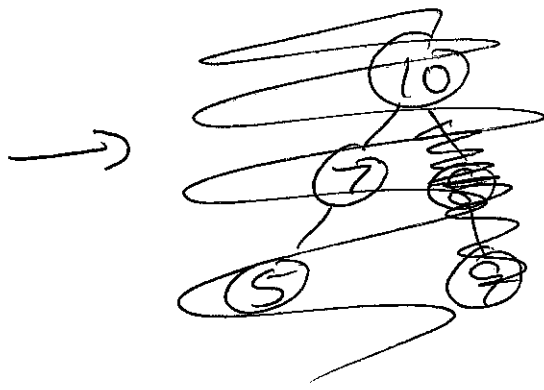
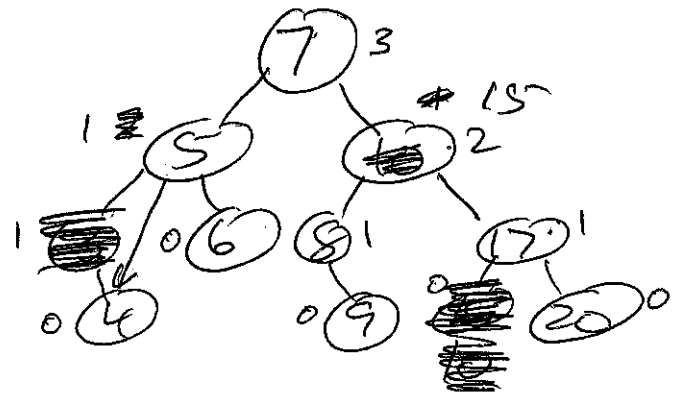
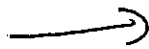
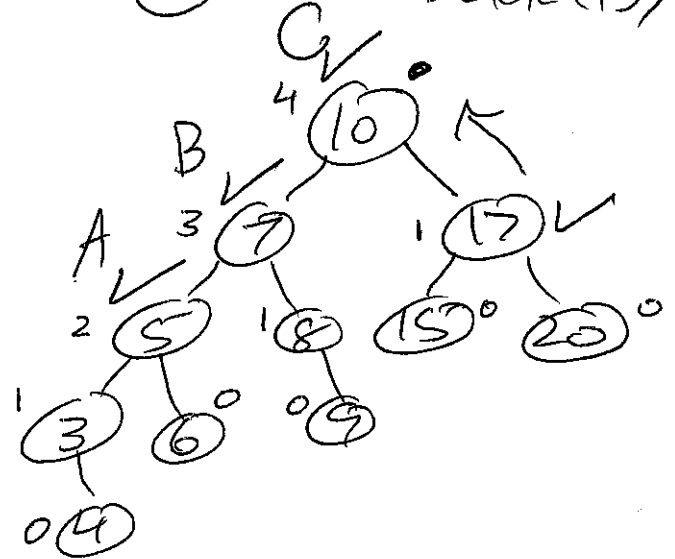
① Not rebalancing at "lowest" node (or 1st node) with imbalance.

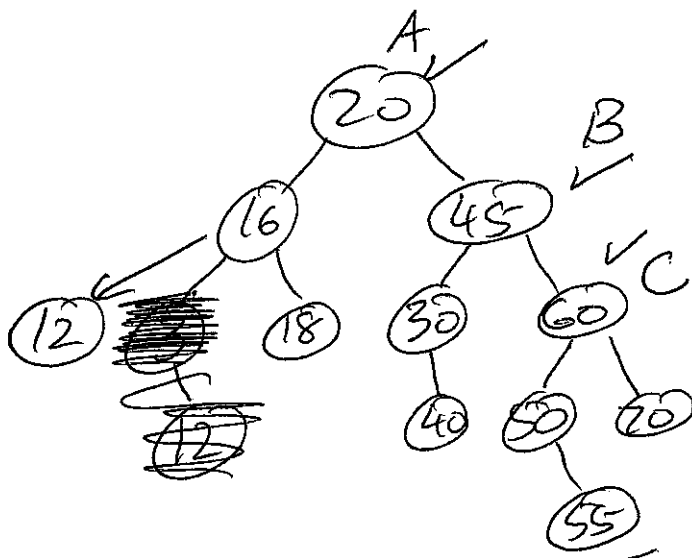
② Answer that isn't a valid binary search tree!

Delete (11)

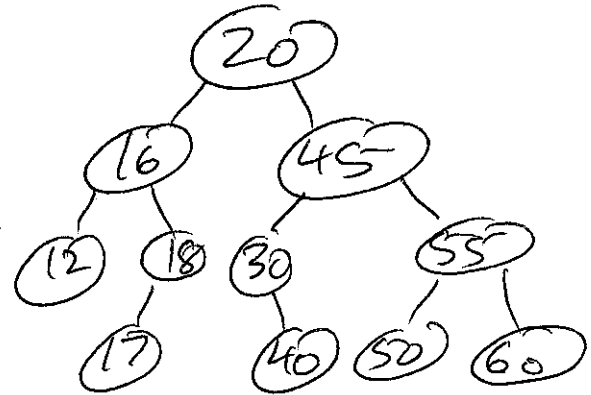
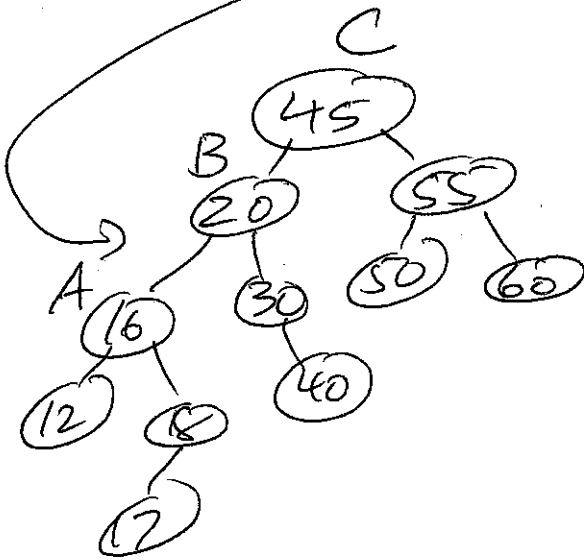
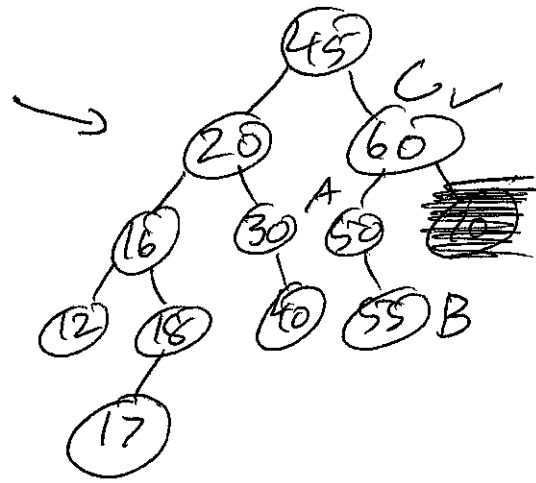


Delete (13)





Delete (3)



3/30/17 ⑦

For an AVL tree of height

h , it contains at least $F_{h+3} - 1$

nodes where $F_n = n^{\text{th}}$ Fibonacci number.

$h=0$ 1 node $F_{0+3} - 1 = F_3 - 1 = 2 - 1 = 1 \checkmark$

$h=1$ 2 node $F_{1+3} - 1 = F_4 - 1 = 3 - 1 = 2 \checkmark$

Assume for all non-neg ints $h \leq k$, where k is an arbitrarily chosen positive integer, that AVL trees of height h have at least $F_{h+3} - 1$ nodes.

Prove for $h = k+1$ that AVL tree of height $k+1$ has at least $F_{k+4} - 1$ nodes.



$$\# \text{ nodes} \geq (\# \text{ nodes tree height } k-1) + (\# \text{ nodes tree height } k) + 1$$

$$\geq (F_{k+2} - 1) + (F_{k+3} - 1) + 1$$

$$= F_{k+2} + F_{k+3} - 1$$

$$= F_{k+4} - 1 \checkmark$$

An AVL tree of height h has
at least $F_{h+3} - 1$ nodes

$$F_n = \frac{1}{\sqrt{5}} \left(\left(\frac{1+\sqrt{5}}{2} \right)^n - \left(\frac{1-\sqrt{5}}{2} \right)^n \right)$$

ϕ

Doesn't affect
the answer much

$$F_n = \frac{1}{\sqrt{5}} \phi^n$$

$n = \#$ of nodes then

$$n \geq \frac{1}{\sqrt{5}} \phi^{h+3} - 1$$

$$n+1 \geq \frac{1}{\sqrt{5}} \phi^{h+3}$$

$$\lg_{\phi} \sqrt{5}(n+1) \geq \lg_{\phi} \phi^{h+3}$$

$$h+3 \leq \lg_{\phi} (\sqrt{5}(n+1))$$

$$h \leq \lg_{\phi} (\sqrt{5}(n+1)) - 3$$

$$h = O(\lg n)$$

3/30/17 (9)

Red - Black Trees

B - Trees

Splay Tree