Plotting of Curves and Surfaces (Week 2)

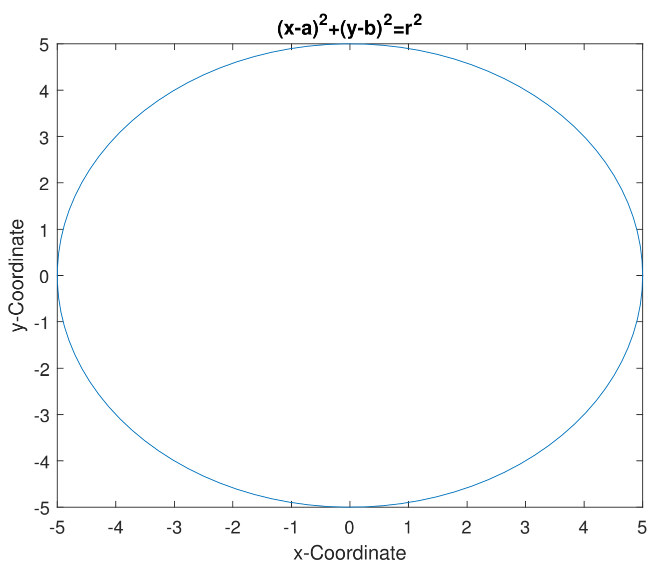
(1) **To Plot the Circle**

Matlab Code

```
clc
clear all
syms r a b
r= input('Enter the radius of the circle')
a= input('Enter the x coordinate of the center')
b= input('Enter the y coordinates of the center')
t = linspace(0, 2*pi, 100);
x = a+r*cos(t);
y = b+r*sin(t);
axis equal
plot(x, y)
xlabel('x-Coordinate')
ylabel('x-Coordinate')
```

title('$(x-a)^2+(y-b)^2=r^2$')

Output



(2) Multiple plots using Hold on Matlab Code

clc

clear all

x = linspace(0, 1, 100);

plot(x, $x.^2$,'r', 'LineWidth',2.0)

hold on

plot(x, cos(x), 'g', 'LineWidth',2.0)

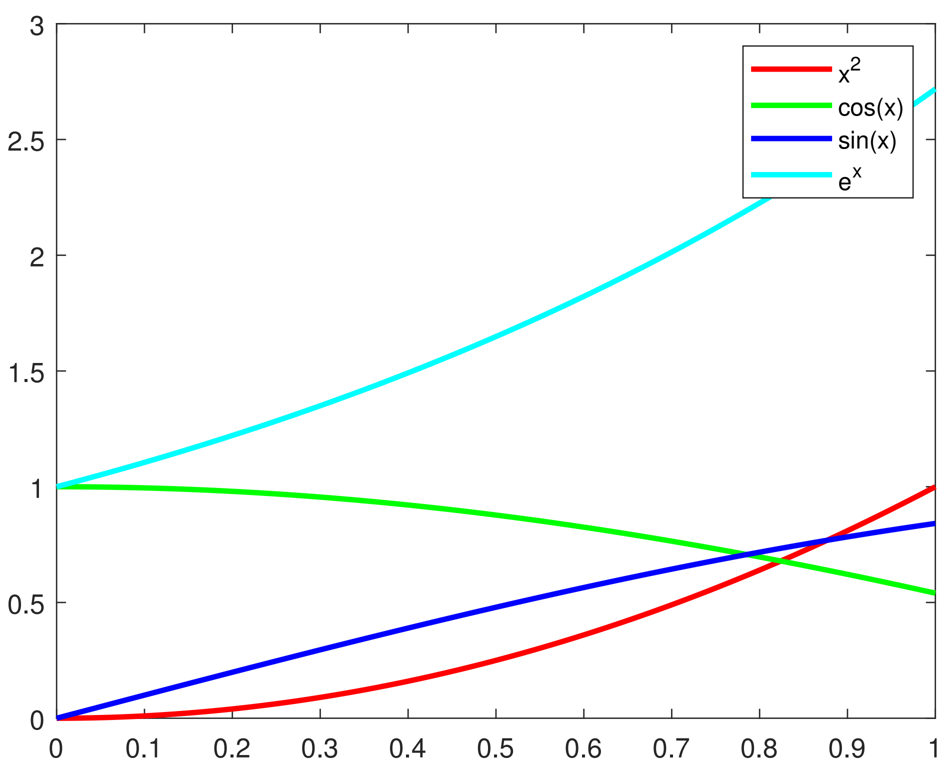
hold on

plot(x, sin(x),'b', 'LineWidth',2.0)

hold on

plot(x, $exp(x)$,'c', 'LineWidth',2.0)

legend('x^2', 'cos(x)' , 'sin(x)' , 'e^x')



(3) **Multiple plots without command "hold on" Matlab Code**

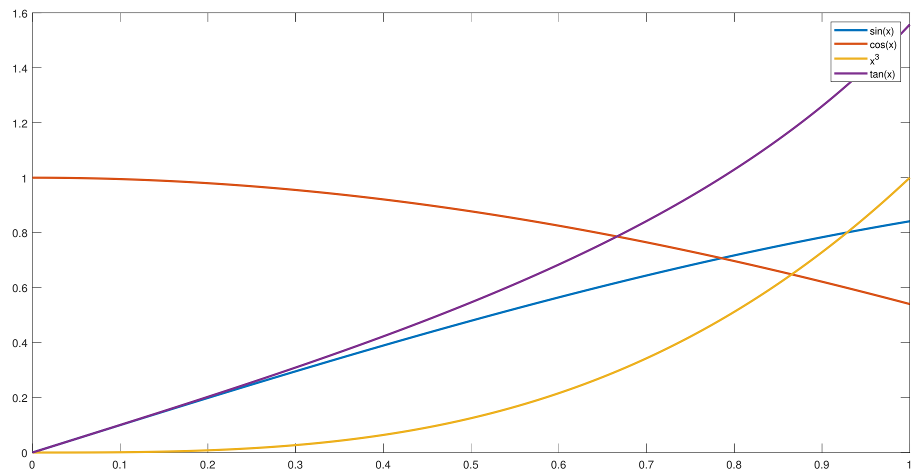
clc

clear all

x = linspace(0, 1, 200);

plot(x, sin(x), x, cos(x), x, x.^{3}, x, tan(x), 'LineWidth',2.0)

legend('sin(x)','cos(x)','x^3','tan(x)') **Output**



(4) **Multiple plots using “subplot ”**

Matlab Code

clc

clear all

x=0:0.1:2*pi;

subplot(2,2,1)

plot(x,sin(x));

title(‘sin(x)’) subplot(2,2,2)

plot(x,cos(x),’r-*’);

title(’cos(x)’) subplot(2,2,3)

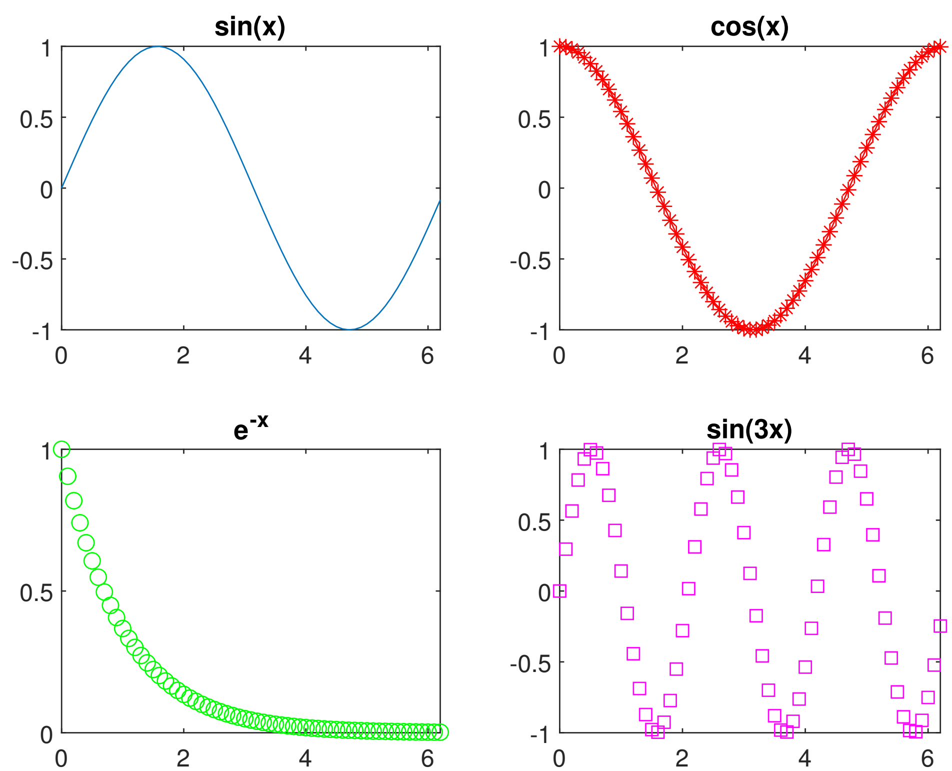
plot(x,$exp(-x)$,’go’)

title(’e^{-x}’) subplot(2,2,4);

plot(x,$sin(3*x)$,’ms’)

title(‘sin(3x)’)

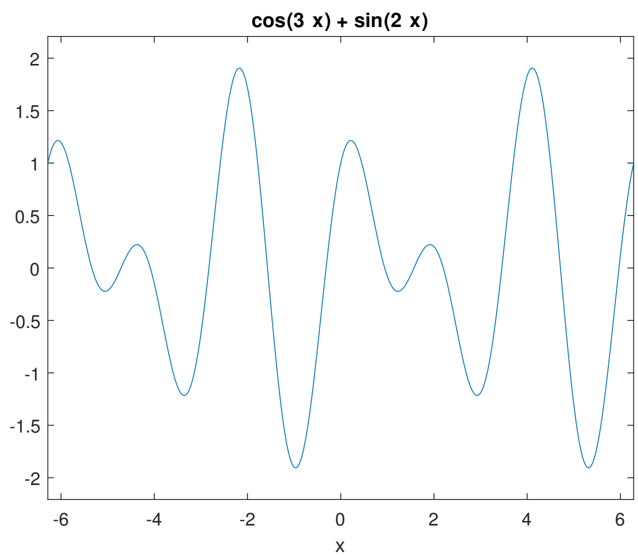
Output

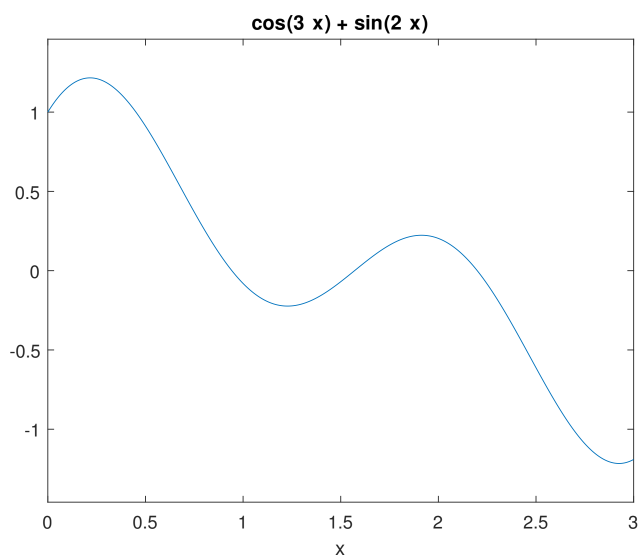


(5) **Graph of the curve using "ezplot "**

Matlab Code

```
clc
clear all
syms x
f=sin(2*x)+cos(3*x)
figure(1)
ezplot(f)
figure(2)
ezplot(f,[0,3])
```



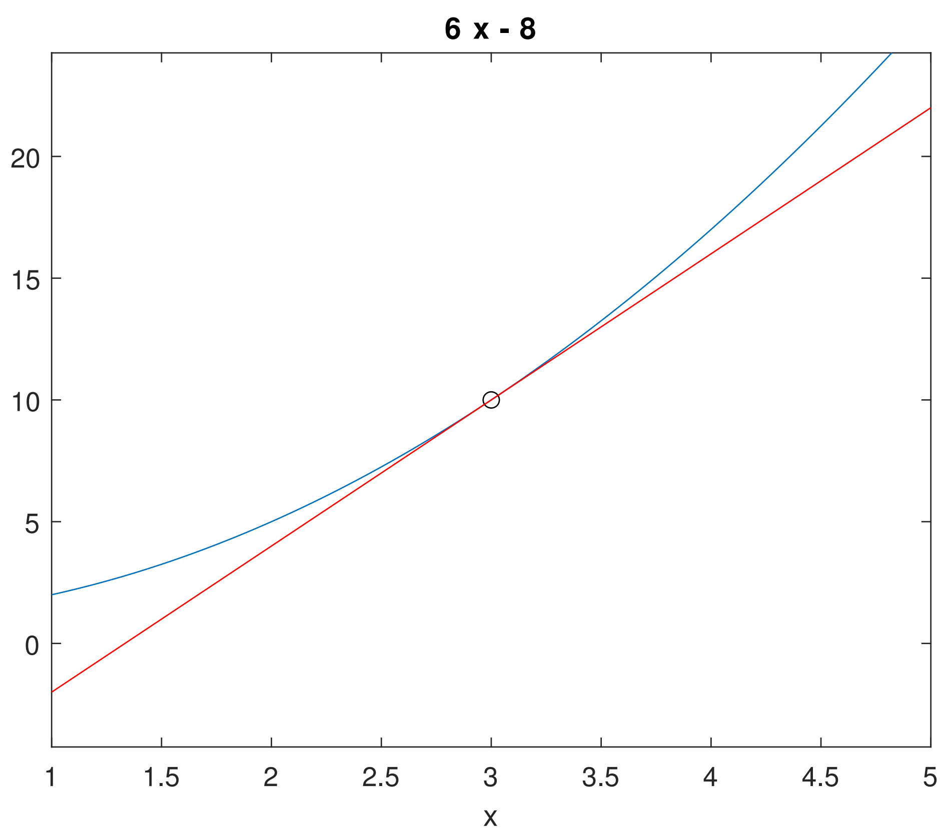


(6) Graph of a curve and its tangent line in the neighbourhood D of a point.

Matlab Code

```
clc
clear all
syms x
y=input('enter the function f in terms of x:')
x1 = input('Enter x value at which tangent : ');
D=[x1-2 x1+2]
ezplot(y,D)
hold on
yd = diff(y,x);
slope = subs(yd,x,x1);
y1 = subs(y,x,x1);
plot(x1,y1,'ko')
Tgtline = slope*(x-x1)+y1
```

Expected Output Based on inputs



Practice Problems

(1) Draw the Ellipse and Hyperbola
(2) Draw any 6 and 8 plots using subplots
(3) Draw multiple graphs using Hold on and Hold off.
(4) Make a list of all the new MATLAB - commands you have learned in this class.

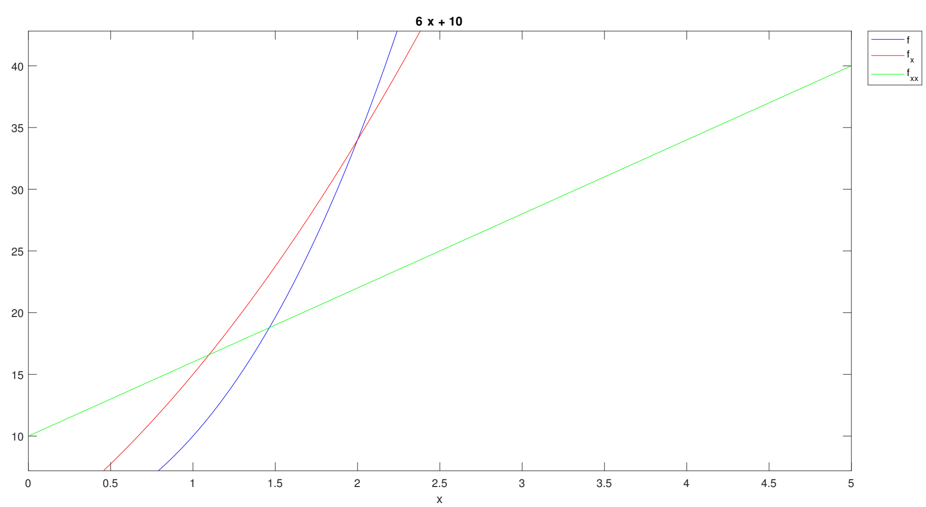
Week 3

(1) **To Plot the function and its derivatives**

Matlab Code

```
clc
clear all
syms x real
f= input('Enter the function f(x):');
fx= diff(f,x)
fxx= diff(fx,x)
D = [0, 5];
l=ezplot(f,D)
set(l,'color','b');
hold on
h=ezplot(fx,D);
set(h,'color','r');
e=ezplot(fxx,D);
set(e,'color','g');
legend('f','f_x','f_xx')
legend('Location','northeastoutside')
```

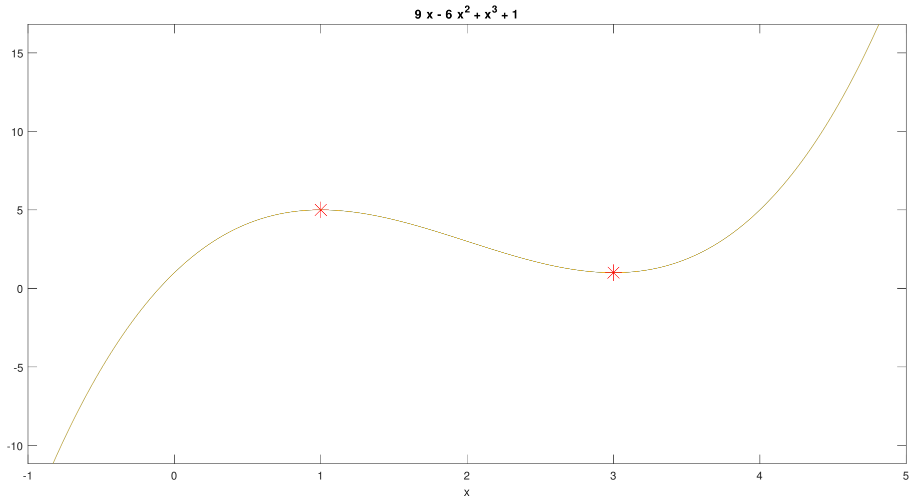
Sample Output



(2) **To find the maxima and minima of the single variable function and visualize it.**

Matlab Code

```
clc
clear all
syms x real
f= input('Enter the function f(x):');
fx= diff(f,x);
fxx= diff(fx,x);
c = solve(fx)
c=double(c);
for i = 1:length(c)
T1 = subs(fxx, x ,c(i) );
T1=double(T1);
T3= subs(f, x, c(i));
T3=double(T3);
if (T1==0)
sprintf('The inflection point is x = %d',c(i))
else
if (T1 < 0)
sprintf('The maximum point x is %d', c(i))
sprintf('The maximum value of the function is %d', T3)
else
sprintf('The minimum point x is %d', c(i))
sprintf('The minimum value of the function is %d', T3)
end
end
cmin = min(c);
cmax = max(c);
D = [cmin-2, cmax+2];
ezplot(f,D)
hold on
plot(c(i), T3, 'g*', 'markersize', 15);
end
```



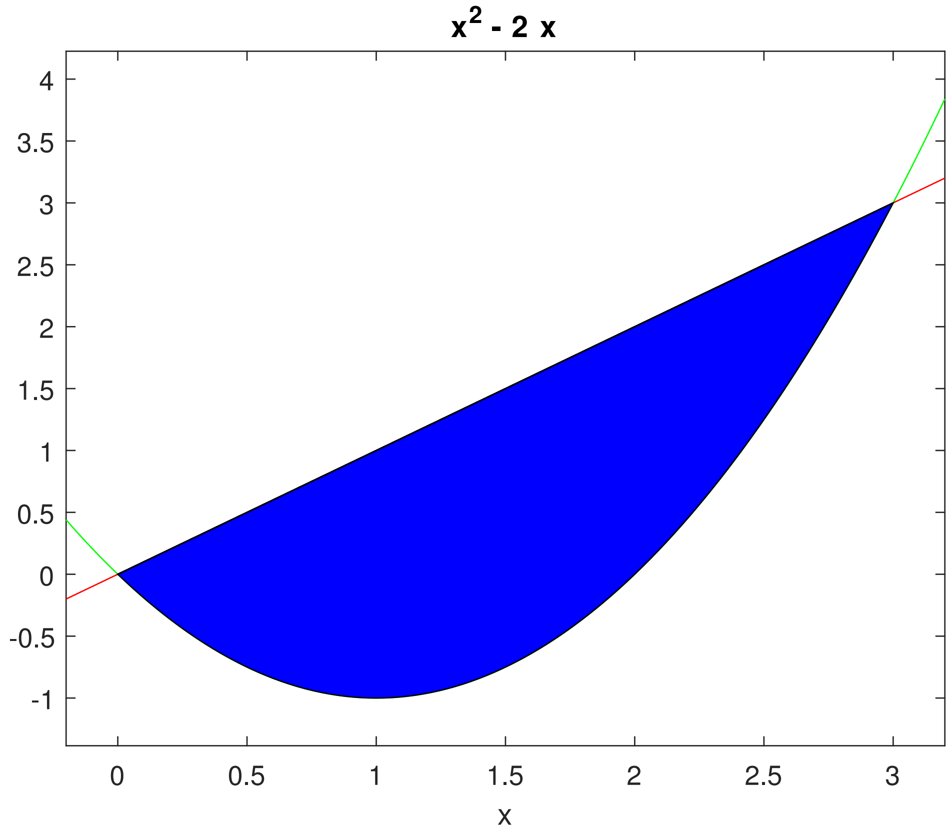
9 x - 6 x^2 + x^3 + 1
15
10
5
0
-5
-10
-1
0
1
2
3
4
5
x

Definite Integrals and its applications

(3) **To find the area of the regions enclosed by curves and visualize it.**

Matlab Code

```
clc
clear
syms x
y1=input('ENTER the upper curve as a function of x : ');
y2=input('ENTER the lower curve as a function of x : ');
t=solve(y1-y2);
t=double(t);
A=int(y1-y2,t(1),t(2))
D=[t(1)-0.2 t(2)+0.2];
ez1=ezplot(y1,D);
set(ez1,'color','r')
hold on
ez2=ezplot(y2,D);
set(ez2,'color','g')
xv = linspace(t(1),t(2));
y1v =subs(y1,x,xv);
y2v = subs(y2,x,xv);
x = [xv,xv];
y = [y1v,y2v];
fill(x,y,'b')
```


$x^2 - 2x$
4
3.5
3
2.5
2
1.5
1
0.5
0
-0.5
-1
0
0.5
1
1.5
2
2.5
3
x

Practice Problems

(a) Find the maximum and minimum of $f(x) = x^3 - 6x^2 + 9x + 1$ on the interval [0, 5].

(b) Find the maximum and minimum of $f(x) = x^3 + 3x^2 + 4x + 5$.

(c) Find the area of the regions enclosed by the curves $y = -x^2 + 4x$, $y = x^2$.

(d) Find the area of the regions enclosed by curves $y = 7 - 2x^2$ and $y = x^2 + 4$.

Volume of the Solid of Revolution (Week 4)

First we defined the symbolic variables

```
clc
clear all
syms x
```

We will take input the function f, limits on which the function is defined, line $y = c$ axis of rotation and limits of integration.

```
f = input('Enter the function: ');
fL = input('Enter the interval on which the function is defined: ');
yr = input('Enter the axis of rotation y = c (enter only c value): ');
iL = input('Enter the integration limits: ');
```

We next find the volume of the 3D surface generated by rotating the function f(x) around the line $y = c$

```
Volume = pi*int((f - yr)^2,iL(1),iL(2));
disp(['Volume is: ', num2str(double(Volume))])
```

We now plot the function $f(x)$ and the line $y = c$, defining the axis of rotation and the area bounded by these curves. In another figure, we plot the rotated region of $f(x)$ in the given integration limits. For this first we convert the given symbolic function f into a MATLAB function by using the inline command of MATLAB. Next we define the x and y ranges on which we have to plot the regions. For plotting the regions using the fill command, we have to flip the range of values of x. For this we use the in built MATLAB command fliplr.

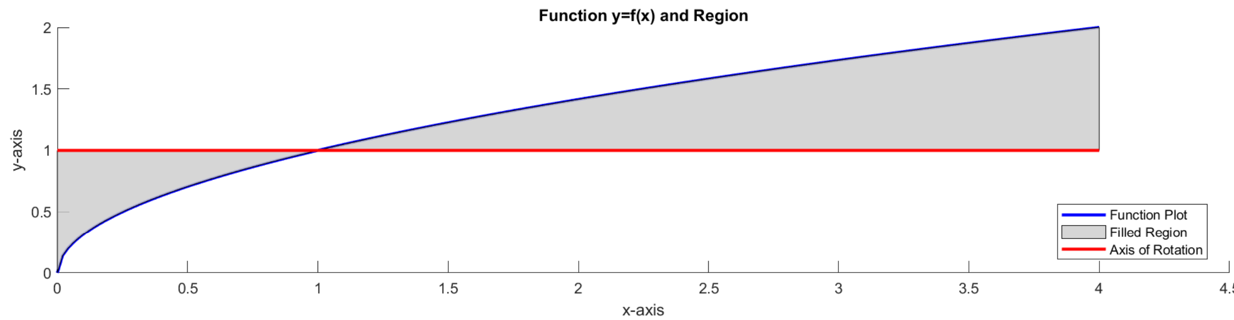
```
fx = inline(vectorize(f));
xvals = linspace(fL(1),fL(2),201);
xvalsr = fliplr(xvals);
xivals = linspace(iL(1),iL(2),201);
xivalsr = fliplr(xivals);
xlim = [fL(1) fL(2)+0.5];
ylim = fx(xlim);
```

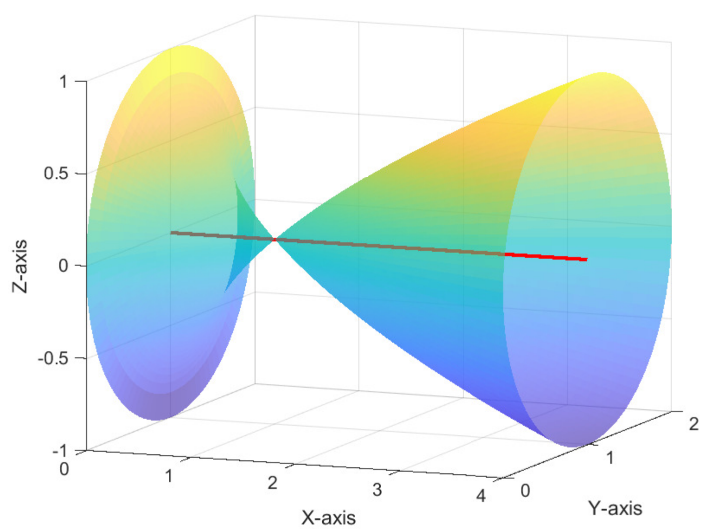
After defining the necessary variables, we will plot the function f(x) in the given function limits.

```
figure('Position',[100 200 560 420])
subplot(2,1,1)
hold on;
plot(xvals,fx(xvals),'-b','LineWidth',2);
```

Finally, we generate the solid of revolution using the MATLAB command cylinder and visualize it using the surf command. We also plot the line, representing the axis of rotation.

```
[X,Y,Z] = cylinder(fx(xivals)-yr,100);
figure('Position',[700 200 560 420])
Z = iL(1) + Z.*(iL(2)-iL(1));
surf(Z,Y+yr,X,'EdgeColor','none','FaceColor','flat','FaceAlpha',0.6);
hold on;
plot([iL(1) iL(2)],[yr yr],'-r','LineWidth',2);
xlabel('X-axis');
ylabel('Y-axis');
zlabel('Z-axis');
view(22,11);
```


Function y=f(x) and Region
y-axis
x-axis
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
Function Plot
Filled Region
Axis of Rotation


Z-axis
X-axis
Y-axis

Practice Problems

(1) Visualize and find the volume of the region in the first quadrant bounded above by the line $y = \sqrt{2}$ below by the curve $y = \sec(x)\tan(x)$, and on the left by the y axis, about the line $y = \sqrt{2}$.

(2) Visualize and find the volume of the solid generated by revolving the region bounded by curve $y = \sin(x)$, $0 \leq x \leq \pi$ about the line $y = 0.5$.

Department of Mathematics
School of Advanced Sciences
BMAT101P – Calculus - Laboratory (MATLAB)
Experiment 2–B

Evaluating maxima and minima of functions of two variables

Aim: To find Maximum and Minimum values (Extreme values) of a function $f(x,y)$ using MATLAB.

Mathematical form:

Let $z = f(x,y)$ be the given function. Critical points are points in the xy – plane where the tangent plane is horizontal. The tangent plane is horizontal, if its normal vector points in the z – direction. Hence, critical points are solutions of the equations: $f_x(x,y) = 0$ and $f_y(a,b) = 0$.

<u>Procedure for finding the maximum or minimum values of $f(x,y)$:</u>

(1) For the given function $f(x,y)$ find $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}$ and equate it to zero and solve them to find the roots (x_1, y_1), (x_2, y_2), ... These points may be maximum or minimum points.

(2) Find the values $r = \frac{\partial^2 f}{\partial x^2}$, $s = \frac{\partial^2 f}{\partial x \partial y}$, $t = \frac{\partial^2 f}{\partial y^2}$ at these points.

(3) (a) If $rt - s^2 > 0$ and $r < 0$ at a certain point, then the function is maximum at that point.

(b)If $rt - s^2 > 0$ and $r > 0$ at a certain point , then the function is minimum at that point.

(c) If $rt - s^2 < 0$ for a certain point, then the function is neither maximum nor minimum at that point. This point is known as saddle point.

(d) If $rt - s^2 = 0$ at a certain point, then nothing can be said whether the function is maximum or minimum at that point. In this case further investigation are required.

MATLAB Syntax used:

`diff`	`diff(expr)` differentiates a symbolic expression expr with respect to its free variable as determined by symvar.
`solve`	Symbolic solution of algebraic equations, The input to solve can be either symbolic expressions or strings
`size`	Dimensions of data and model objects and to access a specific size output.
`figure`	Create figure graphics object, Figure objects are the individual windows on the screen in which the MATLAB software displays graphical output
`double`	Convert to double precision, `double(x)` returns the double-precision value for `X`. If `X` is already a double-precision array, double has no effect.
`sprintf`	Format data into string. It applies the *format* to all elements of array *A* and any additional array arguments in column order, and returns the results to string *str*. `sprintf('%f',var)` is used to format the floating-point number `var` into string.
`solve`	Symbolic solution of algebraic equations, The input to solve can be either symbolic expressions or strings.
`fsurf`	`fsurf(f)` creates a surface plot of the function z = f(x,y) over the default interval `[-5, 5]` for x and y.
`plot3`	The `plot3` function displays a three-dimensional plot of a set of data points.

MATLAB code:

```
clc
clear
syms x y
f(x,y)=input('Enter the function f(x,y):');
p=diff(f,x); q=diff(f,y);
[ax,ay]=solve(p,q);
ax=double(ax);ay=double(ay);
r=diff(p,x);  s=diff(p,y); t=diff(q,y);D=r*t-s^2;
figure
fsurf(f);
legstr={'Function Plot'};% for Legend
for i=1:size(ax)
T1=D(ax(i),ay(i));
T2=r(ax(i),ay(i));
T3=f(ax(i),ay(i));
if(double(T1)==0)
sprintf('At (%f,%f) further investigation is required',ax(i),ay(i))
legstr=[legstr,{'Case of Further investigation'}];
mkr='ko';
elseif (double(T1)<0)
sprintf('The point (%f,%f) is a saddle point', ax(i),ay(i))
legstr=[legstr,{'Saddle Point'}]; % updating Legend
mkr='bv'; % marker
else
if (double(T2) < 0)
sprintf('The maximum value of the function is f(%f,%f)=%f', ax(i),ay(i), T3)
legstr=[legstr,{'Maximum value of the function'}];% updating Legend
mkr='g+';% marker
else
sprintf('The minimum value of the function is f(%f,%f)=%f', ax(i),ay(i), T3)
legstr=[legstr,{'Minimum value of the function'}];% updating Legend
mkr='r*'; % marker
end
end
hold on
plot3(ax(i),ay(i),T3,mkr,'Linewidth',4);
end
legend(legstr,'Location','Best');
```

Example 1. Obtain the maximum and minimum values of $f(x,y)=2(x^2-y^2)-x^4+y^4$

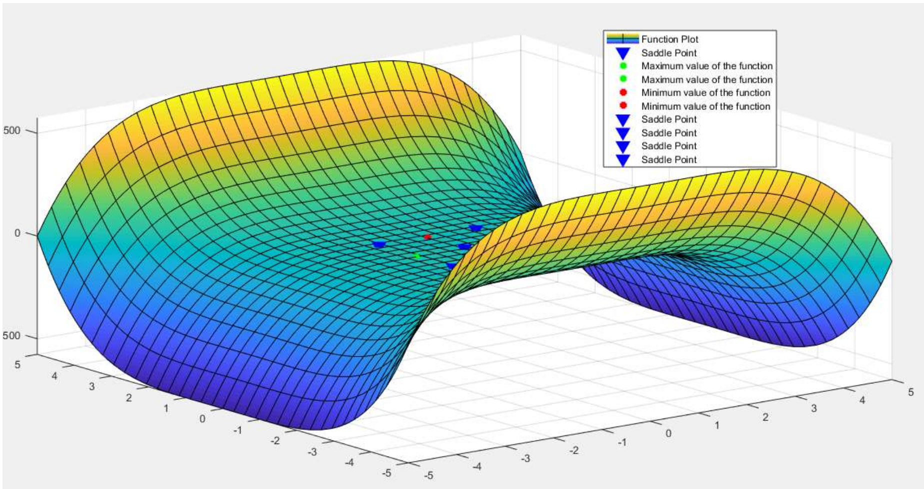
Solution:

S.No.	Critical Points	r	$D=rt-s^2$	Remarks
1	(0,0)	4	$-16<0$	Saddle Point
2	(0,1)	4	32	Minimum
3	(0, –1)	4	32	Minimum
4	(1,0)	-8	32	Maximum
5	(1,1)	-8	$-64<0$	Saddle Point
6	(1, –1)	-8	$-64<0$	Saddle Point
7	(–1,0)	-8	32	Maximum
8	(–1,1)	-8	$-64<0$	Saddle Point
9	(–1, –1)	-8	$-64<0$	Saddle Point

The Minimum value of $f(x,y)$ is -1 at $(0,1)$ & $(0,-1)$ and the Maximum value for $f(x,y)$ is $+1$ at $(1,0)$ & $(-1,0)$

Input:
```
Enter the function f(x,y):2*(x^2-y^2)-x^4+y^4
```

Out Put:
```
 ans =
'The point (0.000000,0.000000) is a saddle point'
ans =
'The maximum value of the function is f(-1.000000,0.000000)=1.000000'
ans =
 'The maximum value of the function is f(1.000000,0.000000)=1.000000'
ans =
'The minimum value of the function is f(0.000000,-1.000000)=-1.000000'
ans =
 'The minimum value of the function is f(0.000000,1.000000)=-1.000000'
ans =
    'The point (-1.000000,-1.000000) is a saddle point'
ans =
    'The point (1.000000,-1.000000) is a saddle point'
ans =
    'The point (-1.000000,1.000000) is a saddle point'
ans =
    'The point (1.000000,1.000000) is a saddle point'
```



Example. 2 Four small towns in a rural area wish to pool their resources to build a television station. If the towns are located at the points $(-5,0)$, $(1,7)$, $(9,0)$ and $(0,-8)$ on a rectangular map grid, where units are in miles, at what point $S(x,\ y)$ should the station be located to minimize the sum of the distances from the towns?

Solution: Let $S(x,y)$ be the location where the television station is to be set up.

The location of the towns are $A(-5,0)$, $B(1,7)$, $C(9,0)$ and $D(0,-8)$.

The point $S(x,y)$ where the sum of the distances from the above points is to be minimized is the same point that minimizes the sum of the squares of the distances; namely,

$$S(x,y)=\left[(x+5)^2+y^2\right]+\left[(x-1)^2+(y-7)^2\right]+\left[(x-9)^2+y^2\right]+\left[x^2+(y+8)^2\right]$$

$$S_x=2(x+5)+2(x-1)+2(x-9)+2x.$$

$$S_y=2y+2(y-7)+2y+2(y+8).$$

Then $S_x = 0 \Rightarrow x = 5/4$ and $S_y = 0 \Rightarrow y = -1/4$.

$r = S_{xx} = 8 > 0$, $s = S_{xy} = 0$ and $t = S_{yy} = 8 > 0$.

Hence $rt - s^2 > 0$, $r > 0$ at $(5/4, -1/4)$.

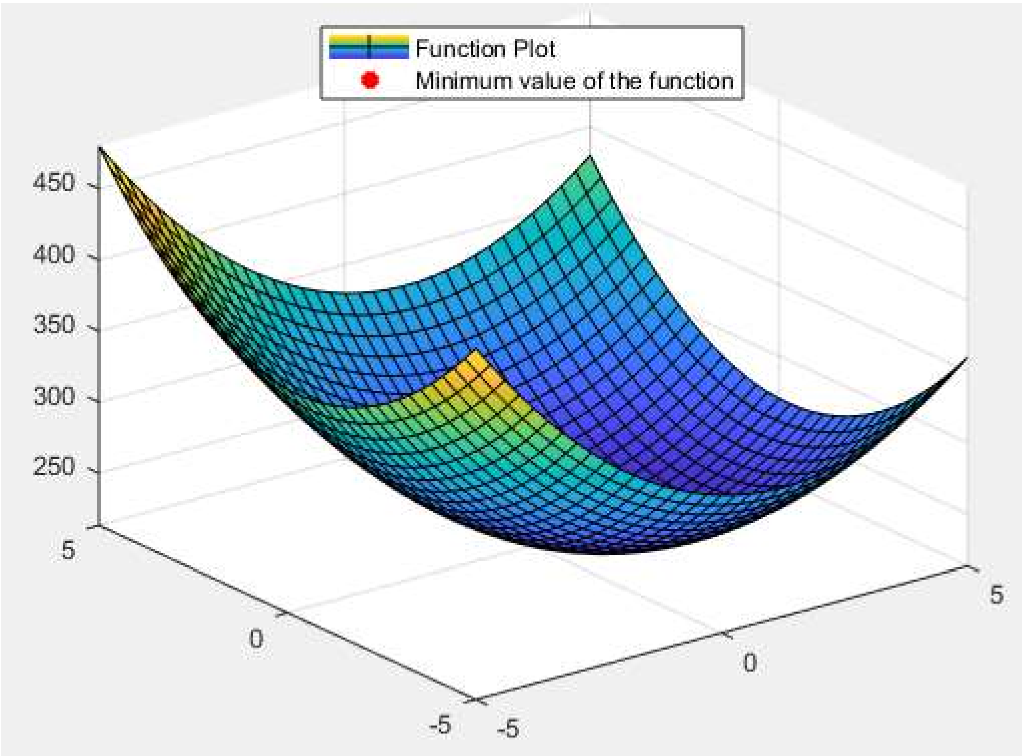
Therefore, the television station can be set up at the location $S(5/4, -1/4)$ on the rectangular map grid such that the distance from S to each of the towns is a minimum.

Input

```
Enter the function f(x,y):
(x+5)^2+y^2+(x-1)^2+(y-7)^2+(x-9)^2+y^2+x^2+(y+8)^2
```

Output

```
ans =
    'The minimum value of the function is f(1.250000,-0.250000)=
                                                          213.500000'
```



Exercise

Find the maxima and minima for the following functions

1. $f(x, y) = x^4 + y^4 - x^2 - y^2 + 1$.
2. $f(x, y) = x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$.

Evaluating Volume under Surfaces

Aim: To evaluate the volume under surface using double integral and to visualize the same using MatLab.

Statement of the problem: Evaluate and visualize the volume represented by the double integral

$$\int_{x_1}^{x_2}\int_{y_1(x)}^{y_2(x)} f(x,y)dydx. \qquad (1)$$

Above integral represents volume of the region below the surface $z = f(x,y)$ and above the plane $z = 0$. This integral can also be setup in the following way (by changing the order of integration of x and y):

$$\int_{y_1}^{y_2}\int_{x_1(y)}^{x_2(y)} f(x,y)dxdy. \qquad (2)$$

Solution Approach: We evaluate the double integrals by repeated application of the symbolic toolbox command for integration that, on applying twice, will read as**:**

$$\text{volume} = \text{int}\big(\text{int}\big(\text{f}(\text{x},\text{y}), y, y_1(x), y_2(x)\big), x, x_1, x_2\big).$$

$$\text{volume} = \text{int}\big(\text{int}\big(\text{f}(\text{x},\text{y}), x, x_1(y), x_2(y)\big), y, y_1, y_2\big)$$

Further, to visualize the volume in MatLab we make use of two additional MatLab functions (provided by MathWorks) viz. “viewSolid” and “viewSolidone”. These supporting function files can be downloaded from the link **ftp://10.30.2.53/MATLAB/**.

The first function “viewSolid” is used to visualize the integrals in which the order of integration is as given in (1) and “viewSolidone” is for the integrals of the form (2).

What follows is the syntax for using “viewSolid” and “viewSolidone” commands:

```
viewSolid(z,0,f(x,y),y,y1(x),y2(x),x,x1,x2)

viewSolidone(z,0,f(x,y),x,x1(y),x2(y),y,y1,y2)
```

It should be observed that the "viewSolid" command is used when y1 and y2 are functions of x whereas x1 and x2 are constants. The "viewSolidone" command is used in the reverse case. Now we consider few examples for illustration of the approach mentioned above.

Example 1:

Set up a double integral to find the volume of a sphere of unit radius.

Solution:

Let the sphere be $x^2 + y^2 + z^2 = 1$. We know that due to the symmetry the volume of the sphere is 8 times its volume in the first octant. Thus we setup a double integral to find the volume below the surface of the sphere in the first octant only and write the total volume as:

$$V = 8\int_0^1 \int_0^{\sqrt{1-x^2}} \sqrt{1-x^2-y^2}\, dy\, dx\, .$$

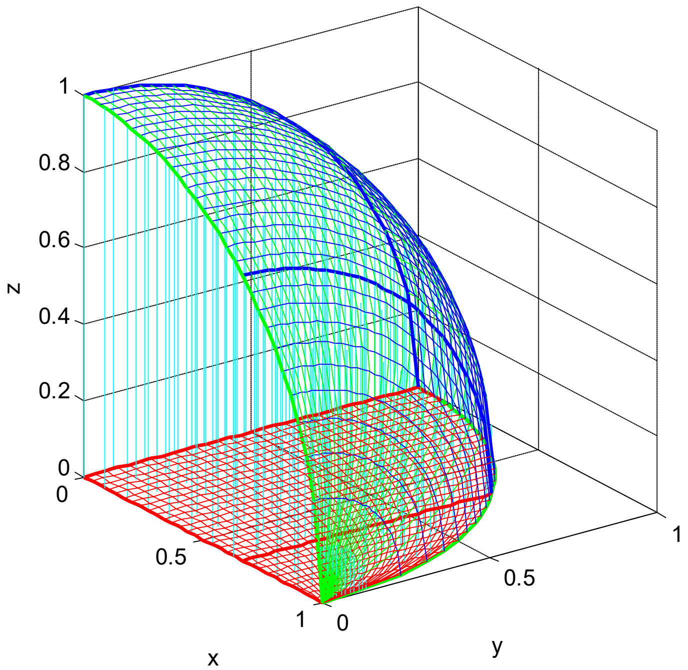
MATLAB Code:

```
clc
clear all
syms x y z
vol=8*int(int(sqrt(1-x^2-y^2),y,0,sqrt(1-x^2)),x,0,1)
viewSolid(z,0+0*x*y,sqrt(1-x^2-y^2),y,0+0*x,sqrt(1-x^2),x,0,1);
axis equal; grid on;
```

```
clc
clear all
syms x y z
vol=8*int(int(sqrt(1-x^2-y^2),y,0,sqrt(1-x^2)),x,0,1)
viewSolid(z,0+0*x*y,sqrt(1-x^2-y^2),y,0+0*x,sqrt(1-x^2),x,0,1); axis equal; grid on;
```

Output:
vol =

(4*pi)/3



Example 2:

Find the volume of the solid that lies under the paraboloid $z = x^2 + y^2$ and above the region D in the xy-plane bounded by the lines $y = 2x$ and the parabola $y = x^2$

Solution:

The double integral for this problem can be setup as:

$$\int_{0}^{4}\int_{y/2}^{\sqrt{y}} (x^2 + y^2)\,dx\,dy$$

```
clc
clear all
syms x y z
vol = int(int(x^2+y^2, x,y/2,sqrt(y)), y, 0, 4)
```

```
viewSolidone(z,0+0*x*y,x^2+y^2,x,y/2,sqrt(y),y,0,4);

grid on;
```

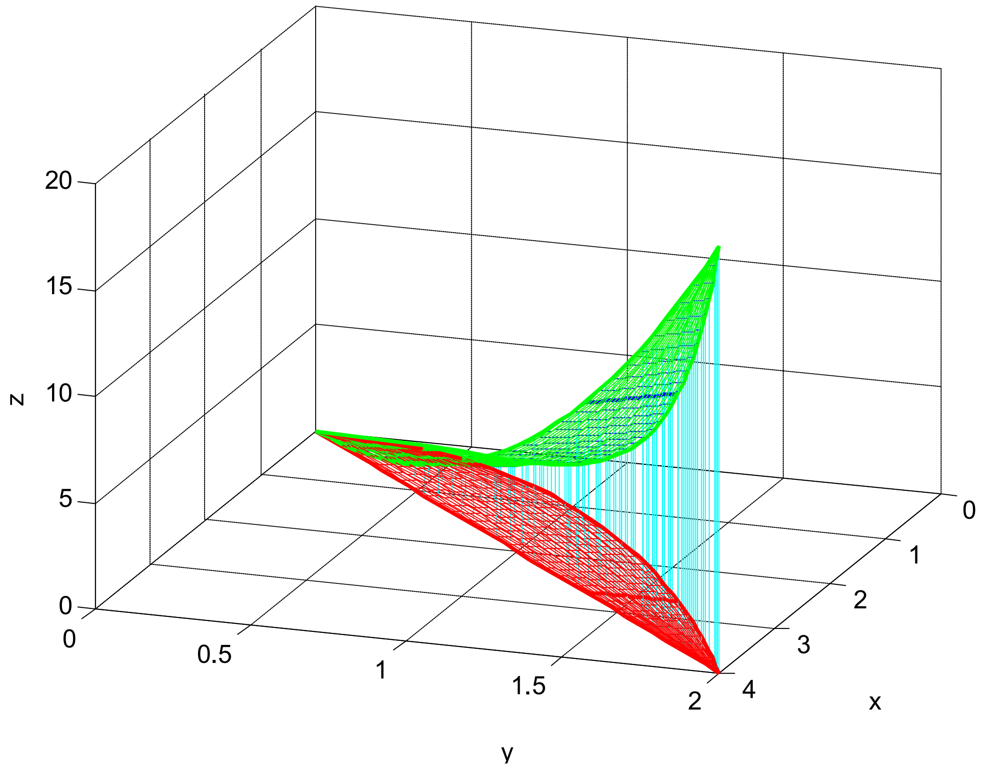
MATLAB Code:

```
Clc
clear all
syms x y z
vol = int(int(x^2+y^2, x,y/2,sqrt(y)), y, 0, 4)
viewSolidone(z,0+0*x*y,x^2+y^2,x,y/2,sqrt(y),y,0,4); grid on
```

Output:

vol =

216/35



Example 3:

Consider the following mathematical problem

$$\text{Evaluate } \iint_R (x - 3y^2)\,dA \text{ where } R = \{(x, y) \mid 0 \le x \le 2,\ 1 \le y \le 2\}$$

Given below is the Matlab code for the above problem.

MATLAB Code:

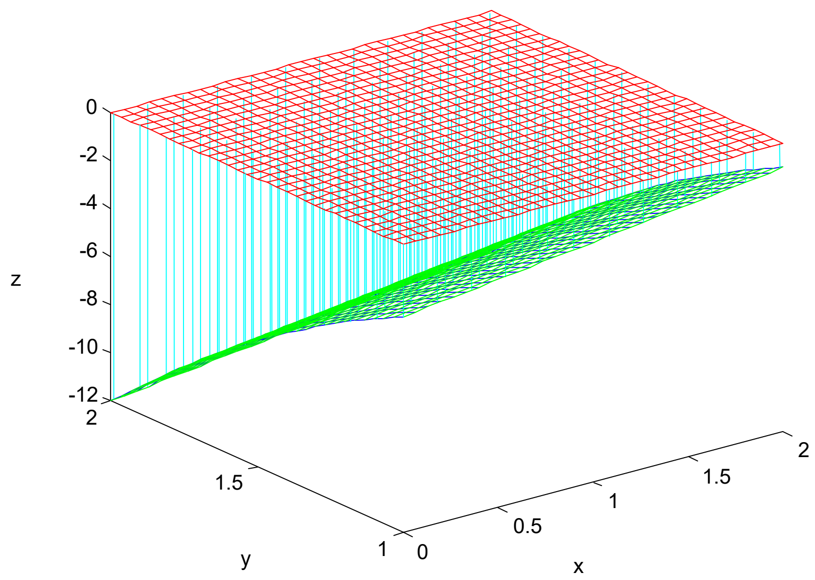
```
clc
clear all
syms x y z
viewSolid(z,0+0*x+0*y,x-3*y^2+0*y,y,1+0*x,2+0*y,x,0,2)
int(int(x-3*y^2+0*y,y,1,2),x,0,2)
```

Output:

In the Command window:

```
>> ans
    -12
```

In the Figure window:



Inference:

In this figure the required volume is below the plane z=o (shown in red) and above the surface z=(x-3y^2)(shown in green) . The reason why the answer is negative is that the surface z=(x-3y^2) is below z=0 for the given domain of integration.

Example 4:

Evaluate $\iint_R y\sin(xy)\ dA$ where $R=[1,2]\times[0,\pi]$

MATLAB Code:

```
clc
clear all
syms x y z
viewSolidone(z,0+0*x+0*y,y*sin(x*y),x,1+0*y,2+0*y,y,0,pi)
int(int(y*sin(x*y),x,1,2),y,0,pi)
```

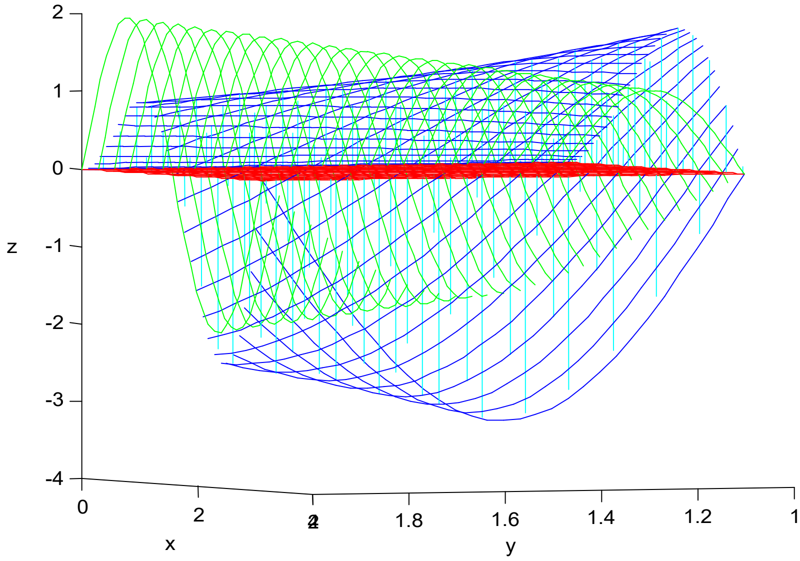
Output:

In the Command window:

>> ans

0

In the Figure window:



Inference:

For a function f(x,y) that takes on both positive and negative values $\iint_R f(x,y)dA$ is a difference of volumes V_1-V_2, V_1 is the volume above R and below the graph of f and V_2 is the volume below R and above the graph. The integral in this example is 0 means V_1=V_2

Converting Cartesian to polar coordinates

Example 5:

Find the volume of the solid bounded by the plane z=0 and the paraboloid $z = 1 - x^2 - y^2$

Sol:

By changing the coordinates from Cartesian to Polar we get

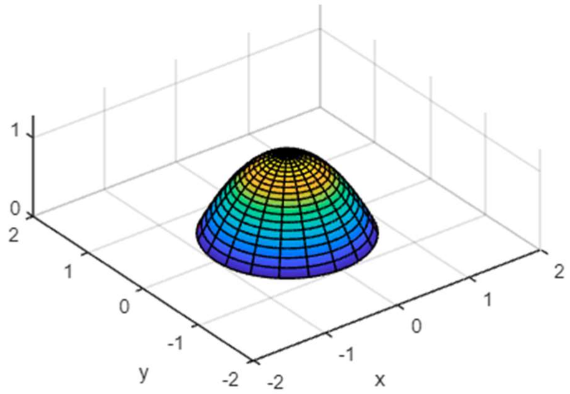
$$V = \iint_D (1 - x^2 - y^2)dA = \int_0^{2\pi} \int_0^{1} (1 - r^2) r dr d\theta$$

```
clc
clear all
syms r theta
V = int(int((1-r^2)*r, r, 0, 1), theta, 0, 2*pi)
fsurf(r*cos(theta),r*sin(theta), 1-r^2, [0 1 0 2*pi], 'MeshDensity', 20)
axis equal; axis([-2 2 -2 2 0 1.3])
xticks(-2:2); yticks(-2:2); zticks(0:1.3)
xlabel('x'); ylabel('y')
```

Output

V = pi/2

Figure window:



Example 6

Find the volume of the solid that lies under the paraboloid $z = x^2 + y^2$ and above the xy- plane, and inside the cylinder $x^2 + y^2 = 2x$

Sol:

By changing the coordinates from Cartesian to Polar we get

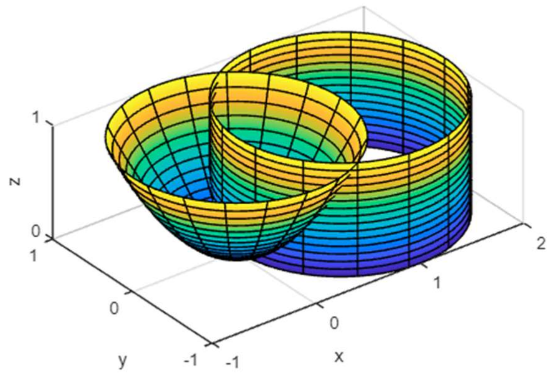
$$V = \iint_D (x^2 + y^2)dA = \int_{-\pi/2}^{\pi/2} \int_0^{2\cos\theta} (r^2) r dr d\theta$$

Matlab code

```
clc
clear all
```

```
syms r theta z r1
V = int(int((r^2)*r, r, 0,2*cos(theta) ), theta, -pi/2, pi/2)
r = 2*cos(theta), x = r*cos(theta), y = r*sin(theta)
fsurf(x,y,z, [0 2*pi 0 1], 'MeshDensity', 16)
axis equal; xlabel('x'); ylabel('y'); zlabel('z')
zticks(0:1.5)
hold on
fsurf(r1*cos(theta),r1*sin(theta),r1^2, [0 1 0 2*pi], 'MeshDensity', 20)
```

Output:
V = (3*pi)/2



Exercise Problems:

1. Set up a double integral to find the volume of the *hoof of Archimedes,* which is the solid region bounded by the planes $z = y$, $z = 0$, and the cylinder $x^2 + y^2 = 1$.
2. Write an iterated integral to view the volume enclosed by the cone $z^2 = x^2 + y^2$ and the plane $z = 0$. Hence find the volume.
3. Find the volume of the solid bounded by the elliptic paraboloid $x^2 + 2y^2 + z = 16$, the planes $x = 2$ and $y = 2$ and the three coordinate planes.
4. Use polar coordinates to find the volume of the solid that lies under the cone $z = \sqrt{x^2 + y^2}$ and above the disk $x^2 + y^2 \leq 4$

Department of Mathematics
School of Advanced Sciences
BMAT 101P – Calculus (MATLAB)
Experiment 4–A
Evaluating Triple Integrals

Triple integrals enable us to solve more general problems such as to calculate the volumes of three – dimensional shapes, the masses and moments of solids of varying density, and the average value of a function over a three –dimensional region.

The triple integral of $f(x, y, z)$ over the region D is given by

$$\iiint_D f(x, y, z)dV$$

where the region D is bounded by the surfaces $x = a$, $x = b$, $y = \psi_1(x)$ to $y = \psi_2(x)$, $z = \phi_1(x, y)$ to $z = \phi_2(x, y)$.

Hence

$$\iiint_D f(x, y, z)dV = \int_a^b \int_{\psi_1(x)}^{\psi_2(x)} \int_{\phi_1(x,y)}^{\phi_2(x,y)} f(x, y, z)dzdydx.$$

Similarly when the region D is bounded by the surfaces $y = c$, $y = d$, $x = \psi_1(y)$ to $x = \psi_2(y)$, $z = \phi_1(x, y)$ to $z = \phi_2(x, y)$.

Hence

$$\iiint_D f(x, y, z)dV = \int_a^b \int_{\psi_1(x)}^{\psi_2(x)} \int_{\phi_1(x,y)}^{\phi_2(x,y)} f(x, y, z)dzdydx.$$

Similarly when the region D is bounded by the surfaces $y = c$, $y = d$, $x = \psi_1(y)$ to $x = \psi_2(y)$, $z = \phi_1(x, y)$ to $z = \phi_2(x, y)$.

Hence

$$\iiint_D f(x, y, z)dV = \int_a^b \int_{\psi_1(y)}^{\psi_2(y)} \int_{\phi_1(x,y)}^{\phi_2(x,y)} f(x, y, z)dzdxdy.$$

Volume using Triple Integral

The volume of a closed, bounded region D in space is given by

$$V = \iiint_D dV$$

Syntax for evaluation of triple integral:

```
int(int(int(f,z,za,zb),y,ya,yb),x,xa,xb)
```

or

```
I=int(int(int(f,z,za,zb),x,xa,xb),y,ya,yb)
```

Syntax for visualization of region bounded by the limits of integration:

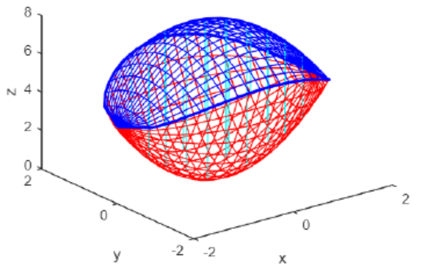
```
viewSolid(z,za,zb,y,ya,yb,x,xa,xb)

viewSolidone(z,za,zb,xa,xb,y,ya,yb)
```

Example 1. Find the volume of the region D enclosed by the surfaces $z = x^2 + 3y^2$ and $z = 8 - x^2 - y^2$.

```
clear
clc
syms x y z
xa=-2;
xb=2;
ya=-sqrt(2-x^2/2);
yb=sqrt(2-x^2/2);
za=x^2+3*y^2;
zb=8-x^2-y^2;
I=int(int(int(1+0*z,z,za,zb),y,ya,yb),x,xa,xb)
viewSolid(z,za,zb,y,ya,yb,x,xa,xb)
```

Output

```
I =
8*pi*2^(1/2)
```



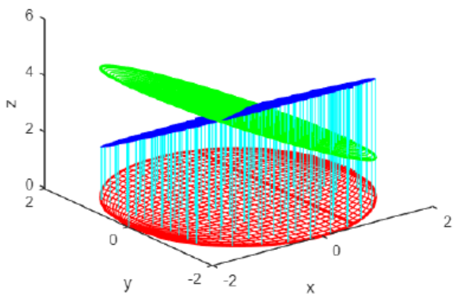
Example 2. Find the volume of the region cut from the cylinder $x^2 + y^2 = 4$ by the plane $z = 0$ and the plane $x + z = 3$.

The limits of integration are $z = 0$ to $3 - x$, $x = -\sqrt{4-y}$ to $\sqrt{4-y}$, $y = -2$ to 2.

```
clear
clc
syms x y z
ya=-2;
yb=2;
xa=-sqrt(4-y^2);
xb=sqrt(4-y^2);
za=0+0*x+0*y;
zb=3-x-0*y;
I=int(int(int(1+0*z,z,za,zb),x,xa,xb),y,ya,yb)
viewSolidone(z,za,zb,x,xa,xb,y,ya,yb)
```

Output

```
I =
12*pi
```



Example 3. Find the volume of the region in the first octant bounded by the coordinate

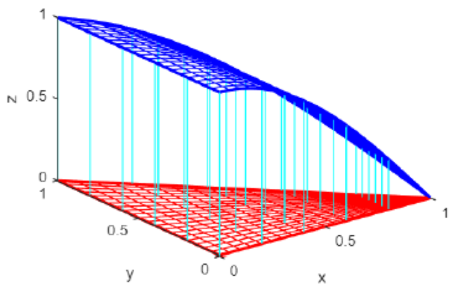
planes, the plane $y = 1 - x$, and the surface $z = \cos(\pi x / 2)$, $0 \le x \le 1$.

The limits of integration are $z = 0$ to $\cos(\pi x / 2)$, $y = 0$ to $1 - x$, $x = 0$ to 1.

```
clear
clc
syms x y z real
xa=0;
xb=1;
ya=0+0*x;
yb=1-x;
za=0*x+0*y;
zb=cos(pi*x/2)+0*y;
I=int(int(int(1+0*z,z,za,zb),y,ya,yb),x,xa,xb)
viewSolid(z,za,zb,y,ya,yb,x,xa,xb)
```

Output.

```
I =

4/pi^2
```



Exercise.

1. Find the volume of the region bounded between the planes $x + y + 2z = 2$ and $2x + 2y + z = 4$ in the first octant.
2. Find the volume of the region cut from the solid elliptical cylinder $x^2 + 4y^2 \le 4$ by the $xy-$ plane and the plane $z = x + 2$.
3. The finite region bounded by the planes $z = x$, $x + z = 8$, $z = y$, $y = 8$ and $z = 0$.

9. GRADIENT, DIVERGENCE AND CURL

Aim:

- To write Matlab codes to visualize the vector field of 2-Dimension as well as 3-Dimension.
- To find the gradient vector and visualize it with contour curves.
- To find divergence, curl and scalar potential

Mathematical form:

Draw the two dimensional vector field for the vector $\vec{F}=\vec{f_1}(x, y) + \vec{f_2}(x,y)$

- Draw the three dimensional vector field for the vector $\vec{F}=\vec{f_1}(x,y,z)+\vec{f_2}(x,y,z)+\vec{f_3}(x,y,z)$
- Find the gradient vector for the following function F(x,y) at the point (x_1,y_1) .
  let the given function be f(x,y). grad (f)=$(\partial f/\partial x)\vec{i} + (\partial f/\partial y)\vec{j}$. Then [grad (f)] at (a,b) is $(\partial f/\partial x)_{(a,b)}\ \vec{i} + (\partial f/\partial y)_{(a,b)}\vec{j}$.
- Find the directional derivative of the function F(x,y,z) in the direction of the vector $\vec{V} = V_1\vec{i} + V_2 j + V_3\vec{k}$ the point (x_1,y_1,z_1).
  Let the given function be F(x,y,z). Find[grad f] at (x_1,y_1,z_1). Find the unit tangent normal by $\overrightarrow{[V/|V|]}$ at(x_1,y_1,z_1). Then directional derivative is given by (gradf).($\overrightarrow{[V/|V|]}$].

MATLAB Syntax used:

inline(expr)	Constructs an inline function object from the MATLAB expression contained in the string expr.
vectorize(fun)	Inserts a . before any ^, * or / in s. The result is a character string
quiver(x,y,u,v)	Displays velocity vectors as arrows with components (u,v) at the points (x,y)
quiver3(x,y,z,u,v,w)	Plots vectors with components (u,v,w) at the points (x,y,z))
vectarrow(p0,p1)	Plots a line vector with arrow pointing from point p0 to point p1. The function can plot both 2D and 3D vector with arrow depending on the dimension of the input
g = gradient(f)	finds the gradient vector of the scalar function f with respect to a vector constructed from all symbolic scalar variables found in f. The order of variables in this vector is defined by

	symvar.

Example 1:

Draw the two dimensional vector field for the vector $\boldsymbol{x\vec{i}} + \boldsymbol{y\vec{j}}$

MATLAB Code:

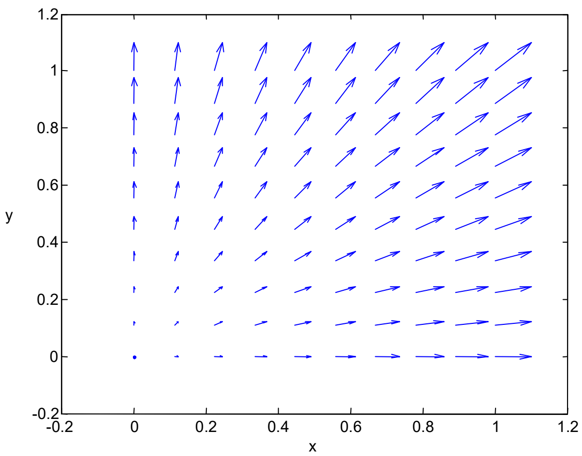
```
clc
clear all
syms x  y
F=input( 'enter the vector as i, and j order in vector form:');
P = inline(vectorize(F(1)), 'x', 'y');
Q = inline(vectorize(F(2)), 'x', 'y');
x = linspace(-1, 1, 10);
y = x;
[X,Y] = meshgrid(x,y);
U = P(X,Y);
V = Q(X,Y);
quiver(X,Y,U,V,1)
axis on
xlabel('x')
ylabel('y')
```

Output:

In the Command window:

Enter the vector as i,j and k order in vector form:[x y]

In the Figure window:



Example 2:

Draw the three dimensional vector field for the vector $\boldsymbol{x\vec{i}}$ $-\boldsymbol{y\vec{j}}$ $+\boldsymbol{z\vec{k}}$

MATLAB Code:

```
syms x  y  z
F=input( 'enter the vector as i,j and korder in vector form:')
P = inline(vectorize(F(1)), 'x', 'y','z');
Q = inline(vectorize(F(2)), 'x', 'y','z');
R = inline(vectorize(F(3)), 'x', 'y','z');
x = linspace(-1, 1, 5); y = x;
z=x;
[X,Y,Z] = meshgrid(x,y,z);
U = P(X,Y,Z);
V = Q(X,Y,Z);
W = R(X,Y,Z);
quiver3(X,Y,Z,U,V,W,1.5)
axis on
xlabel('x')
ylabel('y')
zlabel('z')
```

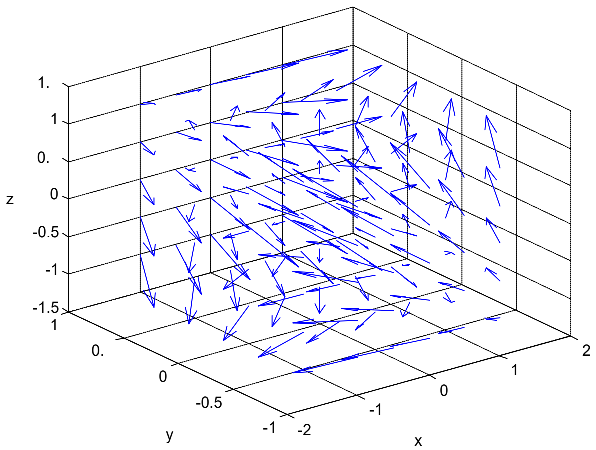
Output:

In the Command Window:

enter the vector as i, j and k order in vector form:[x -y   z]

F =[x, -y, z]

In the figure window:



Example 3:

Find the gradient vector field of $(x, y) = x^2y - y^3$. Plot the gradient vector field together with a contour map of f. How are they related?

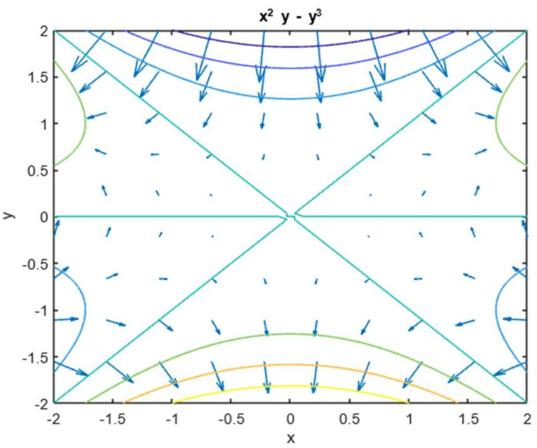
MATLAB Code:

```
clc
clear all
syms x y
f=input( 'enter the function f(x,y):');
F=gradient(f)
P = inline(vectorize(F(1)), 'x', 'y');
Q = inline(vectorize(F(2)), 'x','y');
x = linspace(-2, 2, 10);
y = x;
[X,Y] = meshgrid(x,y);
U = P(X,Y);
V = Q(X,Y);
quiver(X,Y,U,V,1)
axis on
xlabel('x')
ylabel('y')
hold on
ezcontour(f,[-2 2])
```

Output:

Command Window:

```
enter the function f(x,y):
x^2*y-y^3
```



Inference:
The gradient vectors are orthogonal to the contours.

Example 4

Find (a) the curl and (b) the divergence of the vector field.

$$\mathbf{F}(x, y, z) = x^2yz\,\mathbf{i} + xy^2z\,\mathbf{j} + xyz^2\,\mathbf{k}$$

Matlab code

```
clc
clear all
syms x y z real
F=input( 'enter the vector as i, j and k order in vector form:')
curl_F = curl(F, [x y z])
div_F = divergence(F, [x y z])
```

Output:

```
enter the vector as i,j and k order in vector form:
[x^2*y*z x*y^2*z x*y*z^2]
F =
[x^2*y*z, x*y^2*z, x*y*z^2]
curl_F =

  x*z^2 - x*y^2
  y*x^2 - y*z^2
- z*x^2 + z*y^2

div_F =

6*x*y*z
```

Example 5

Determine whether or not the vector field $\mathbf{F}(x, y, z) = y^2z^3\,\mathbf{i} + 2xyz^3\,\mathbf{j} + 3xy^2z^2\,\mathbf{k}$ is conservative. If it is conservative, find a function f such that $\mathbf{F} = \nabla f$.

Matlab code

```
clc
clear all
syms x y z real
F=input( 'enter the vector as i,j and k order in vector form:')
curl_F = curl(F, [x y z])
if (curl_F ==[0 0 0])
```

```
 f = potential(F, [x y z])
else
 sprintf('curl_F is not equal to zero')
end
```

Output:

```
curl_F =
0
0
0
f =
x*y^2*z^3
```

Exercise

1. Plot the gradient vector field of f together with a contour map of f. Explain how they are related to each other
   (a) $f = sin(x) + sin(y)$ (b) $f = sin(x + y)$ (c) $f = x^2 + y^2$

2. Determine whether or not the vector field $\bar{F} = (2xy + z^3)\bar{i} + x^2\,\bar{j} + 3xz^2\,\bar{k}$ is conservative. If it is conservative, find a function f such that $\mathbf{F} = \nabla f$.

<u>**LINE INTEGRAL AND** GREEN'S THEOREM</u>

Definition: let $\vec{F}$ be a continuous vector filed defined on a smooth curve C given by a vector function $\vec{r}(t), a \le t \le b$. Then the line integral of $\vec{F}$ along C is integral $\int_C \vec{F} \cdot d\vec{r} = \int_a^b \vec{F}(\vec{r}(t)) \cdot \vec{r}\,'(t)dt$

inline(expr)	Constructs an inline function object from the MATLAB expression contained in the string expr.
vectorize(fun)	Inserts a . before any ^, * or / in s. The result is a character string
quiver(x,y,u,v)	Displays velocity vectors as arrows with components (u,v) at the points (x,y)
quiver3(x,y,z,u,v,w)	Plots vectors with components (u,v,w) at the points (x,y,z))
vectarrow(p0,p1)	Plots a line vector with arrow pointing from point p0 to point p1. The function can plot both 2D and 3D vector with arrow depending on the dimension of the input

Matlab code

```
clc
clear
syms t x y
f=input('enter the  f vector as i and j order in vector form:');
rbar = input('enter the r vector as i and j order in vector
form:');
lim=input('enter the limit of integration:');
vecfi=input('enter the vector field range'); % knowledge of the
curve is essential
drbar=diff(rbar,t);
sub = subs(f,[x,y],rbar);
f1=dot(sub,drbar)
int(f1,t,lim(1),lim(2))
P = inline(vectorize(f(1)), 'x', 'y');
Q = inline(vectorize(f(2)), 'x', 'y')
x = linspace(vecfi(1),vecfi(2), 10); y = x;
```

```
[X,Y] = meshgrid(x,y);
U = P(X,Y);
V = Q(X,Y);
quiver(X,Y,U,V)
hold on
fplot(rbar(1),rbar(2),[lim(1),lim(2)])
axis on
xlabel('x')
ylabel('y')
```

Example 1

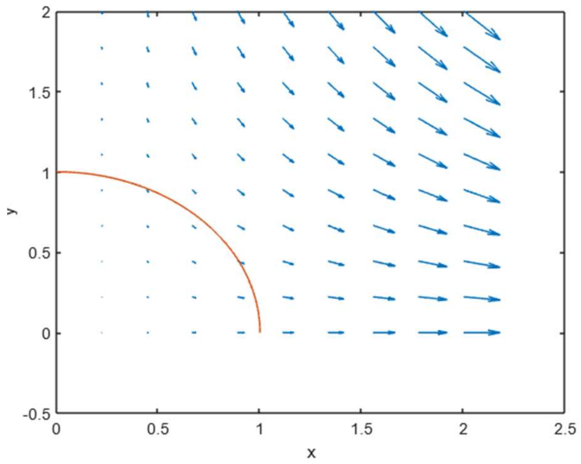
Find the work done by the force field $\vec{F}(x,y) = x^2\vec{\imath} - xy\vec{\jmath}$ in moving a particle along the quarter – circle $\vec{r}(t) = \cos(t)\,\vec{\imath} + \sin(t)\vec{\jmath}$, $0 \le t \le \frac{\pi}{2}$

Sol:

$$\int_C \vec{F}\cdot d\vec{r} = \int_0^{\frac{\pi}{2}} \vec{F}(\vec{r}(t))\cdot \vec{r}\,'(t)dt = \int_0^{\frac{\pi}{2}}(-2cos^2 t\ \sin t)\,dt$$

In the command window

enter the f vector as i and j order in vector form:
[x^2 -x*y]
enter the r vector as i and j order in vector form:
[cos(t) sin(t)]
enter the limit of integration:
[0 pi/2]
enter the vector field range
[0 2]
ans = -2/3
In the figure window



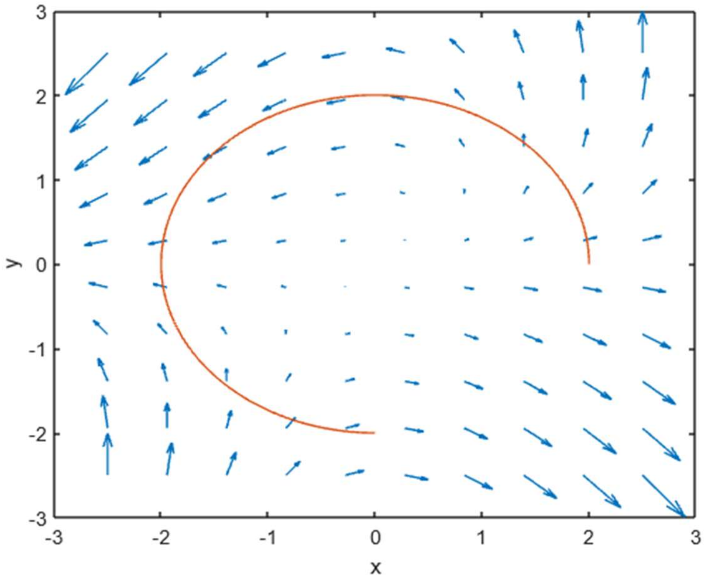
The above figure shows the force field and the curve. The work done is negative because the field impedes movement along the curve.

Example 2

Evaluate the line integral $\int_C \vec{F} \cdot d\vec{r}$, $\vec{F}(x, y) = (x - y)\vec{\imath} + xy\vec{\jmath}$, C is the arc of the circle $x^2 + y^2 = 4$ traversed counter-clockwise from (2, 0) to(0, -2)

In the command window

enter the f vector as i and j order in vector form:
[x-y x*y]
enter the r vector as i and j order in vector form:
[2*cos(t) 2*sin(t)]
enter the limit of integration:
[0, 3*pi/2]
enter the vector field range
[-2.5 2.5]
ans = 3*pi + 2/3



The above figure shows the force field and the curve. The work done is positive.

Exercise

1. Find the work done by the force field $\vec{F}(x, y) = (y + z,\ x + z, x + y)$ on a particle that moves along the line segment (1, 0, 0) to (3, 4, 2)

2. Find the work done by the force field $\vec{F}(x, y) = x\ sin(y)\vec{\imath} + y\vec{\jmath}$ on a particle that moves along the curve $y = x^2$ from (-1, 1) to (2, 4)

3.Evaluate $\int_C \vec{F} \cdot d\vec{r}$, $\vec{F}(x, y, z) = y\ sin(z)\vec{\imath} + z\ sin(x)\vec{\jmath} + x\ sin(y)\overrightarrow{k}$ and $\vec{r}(t) = \cos(t)\,\vec{\imath} + \sin(t)\,\vec{\jmath} + \sin\,(5t)\,\overrightarrow{k}$, $0 \leq t \leq \pi$.

GREEN'S THEOREM

Let C be a positively oriented, piecewise-smooth, simple closed curve in the plane and let D be the region bounded by C. If P and Q have continuous partial derivatives on an open region that contains D, then

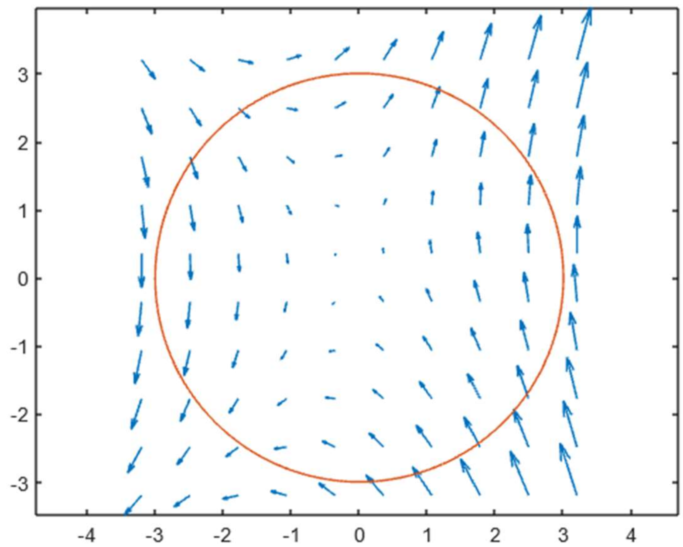
$$\int_C \; P dx + Q\, dy = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA$$

Example 1

Evaluate $\oint_C \left(3y - e^{\sin(x)}\right)dx + \left(7x + \sqrt{y^4 + 1}\right)dy$, where C is the circle $x^2 + y^2 = 9$.

```
clc
clear all
syms x y r t
F=input('enter the  F vector as i and j order in vector form:');
integrand=diff(F(2),x)-diff(F(1),y);
polarint=r*subs(integrand,[x,y],[r*cos(t),r*sin(t)]);
sol=int(int(polarint,r,0,3),t,0,2*pi);
P = inline(vectorize(F(1)), 'x', 'y');
Q = inline(vectorize(F(2)), 'x', 'y')
x = linspace(-3.2,3.2, 10); y = x;
[X,Y] = meshgrid(x,y);
U = P(X,Y);
V = Q(X,Y);
quiver(X,Y,U,V)
hold on
fplot(3*cos(t),3*sin(t),[0,2*pi])
axis equal
```

```
In the command window
enter the  F vector as i and j order in vector form:
[3*y-exp(sin(x))  7*x+ sqrt(y^4+1)]
sol = 36*pi
In the figure window
```



Example 2

Evaluate $\oint_C (y^2)dx + (3xy)dy$, where C is the boundary of the semiannular region D in the upper –plane between the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$.

Matlab code

```
clc
clear all
syms x y r t
F=input('enter the  F vector as i and j order in vector form:');
integrand=diff(F(2),x)-diff(F(1),y);
polarint=r*subs(integrand,[x,y],[r*cos(t),r*sin(t)]);
sol=int(int(polarint,r,1,2),t,0,pi);
P = inline(vectorize(F(1)), 'x', 'y');
Q = inline(vectorize(F(2)), 'x', 'y')
x = linspace(-3.2,3.2,10); y = x;
[X,Y] = meshgrid(x,y);
U = P(X,Y);
V = Q(X,Y);
quiver(X,Y,U,V)
hold on
fplot(1*cos(t),1*sin(t),[0,pi])
fplot(2*cos(t),2*sin(t),[0,pi])
axis equal
```

```
enter the  F vector as i and j order in vector form:
[y^2  3*x*y]

sol = 14/3
```