

# Lecture 1: Boosting, Feature Importance estimation

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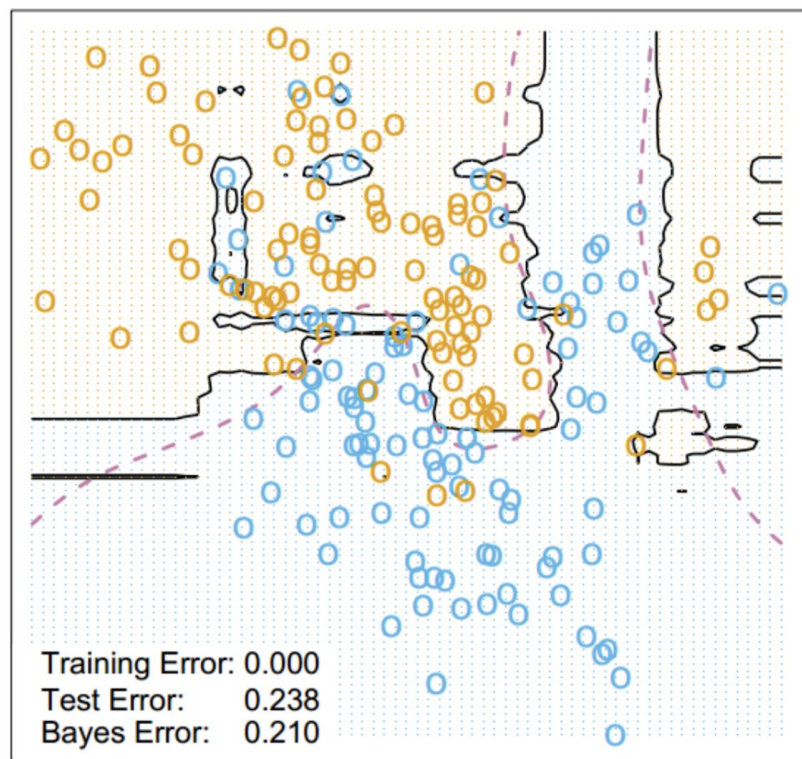
# Outline

1. Ensembling methods recap
2. Boosting intuition
3. Gradient boosting
4. Feature importance estimation
5. Shap values

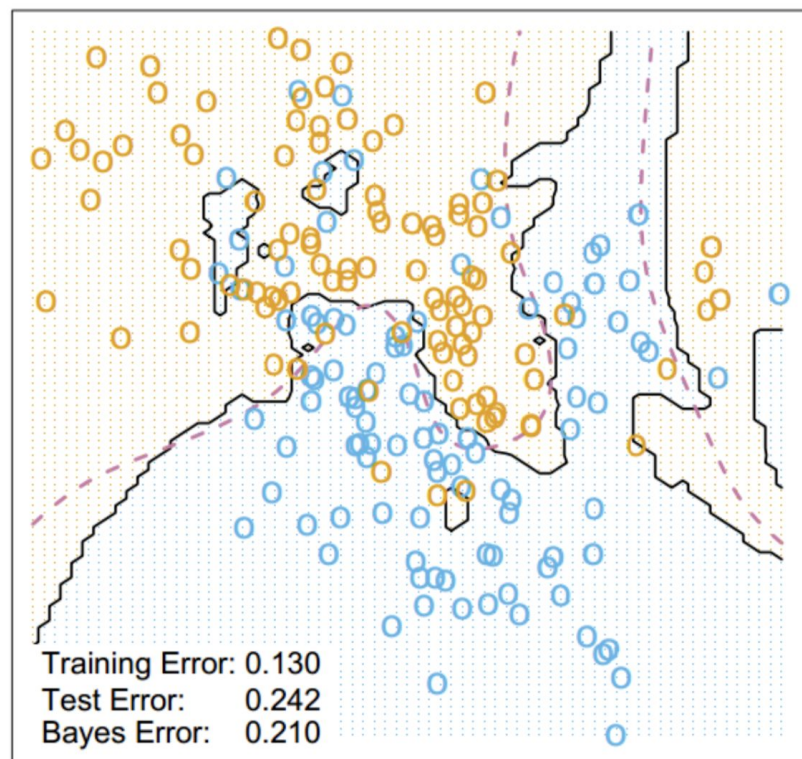
- One of the greatest “universal” models.
- There are some modifications: Extremely Randomized Trees, Isolation Forest, etc.
- Allows to use train data for validation: OOB

$$\text{OOB} = \sum_{i=1}^{\ell} L \left( y_i, \frac{1}{\sum_{n=1}^N [x_i \notin X_n]} \sum_{n=1}^N [x_i \notin X_n] b_n(x_i) \right)$$

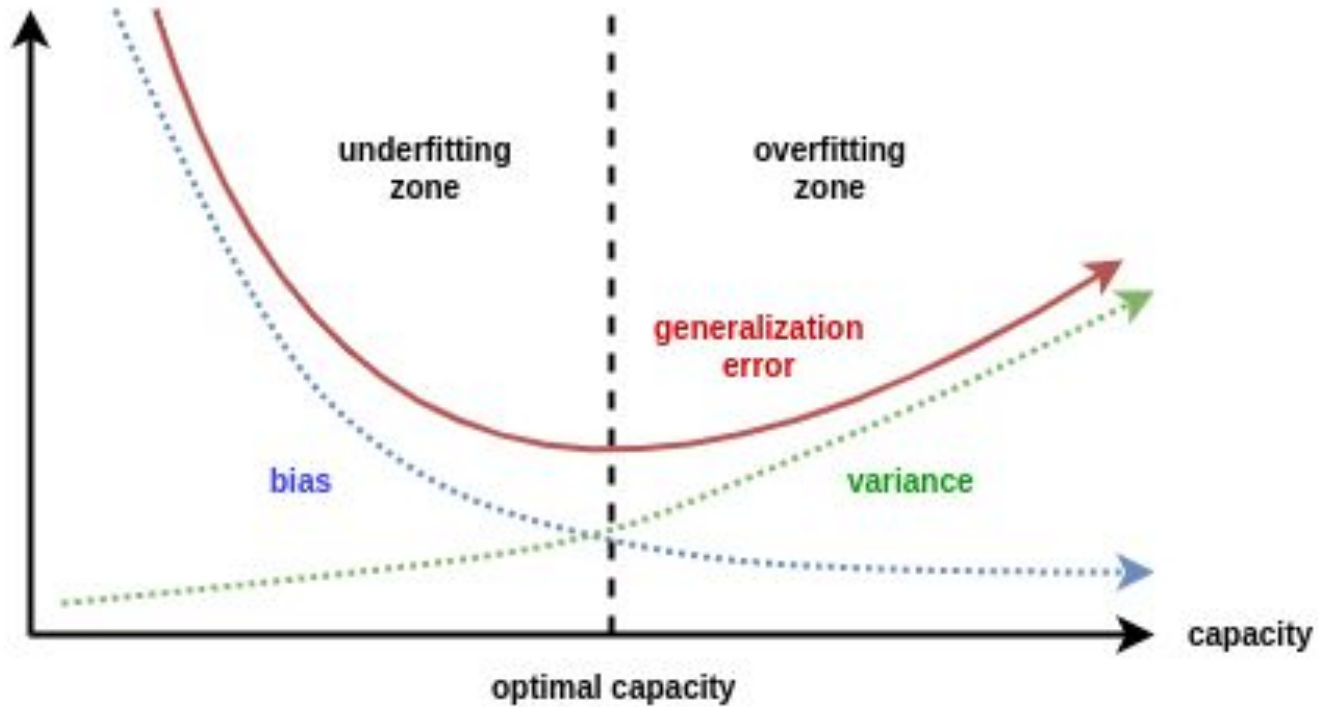
Random Forest Classifier



3-Nearest Neighbors

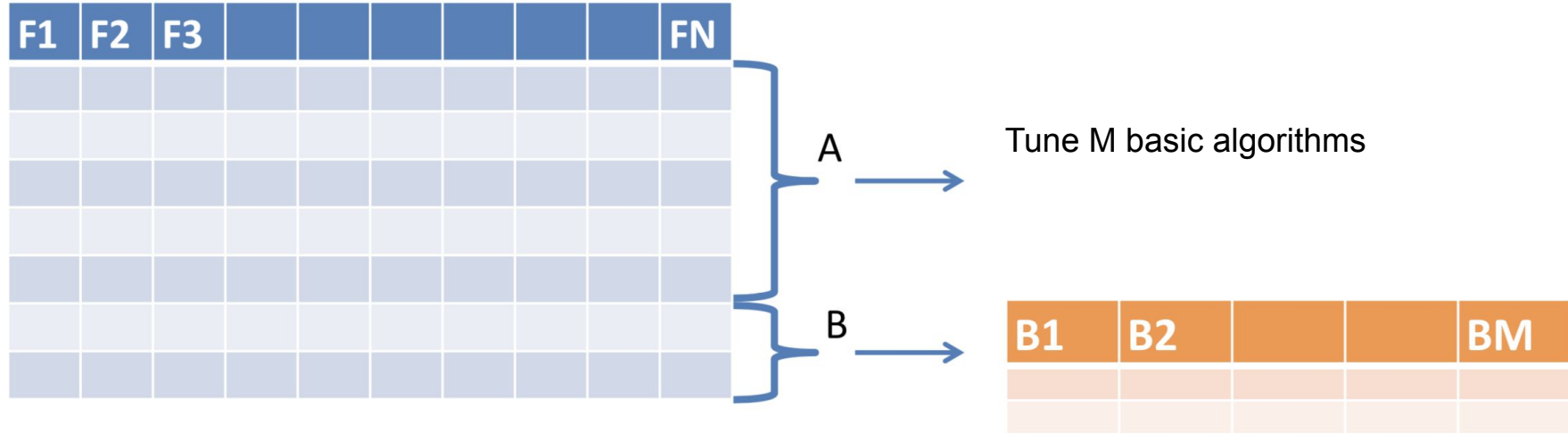


# Bias-variance tradeoff



# Stacking

How to build an ensemble from *different* models?



$$a(x) = \sum_{t=1}^T \alpha_t b_t(x)$$

e.g.

How to build an ensemble from *different* models?

- Use different datasets (or datasets parts) for different level models.
- Experiment with different models (linear, trees ensembles, simple networks, etc.)
- Or just different GBT ensembles (hola, kaggle :)

Just combine several *strong/complex* models.

Weights should sum up to 1  
and come from [0; 1]

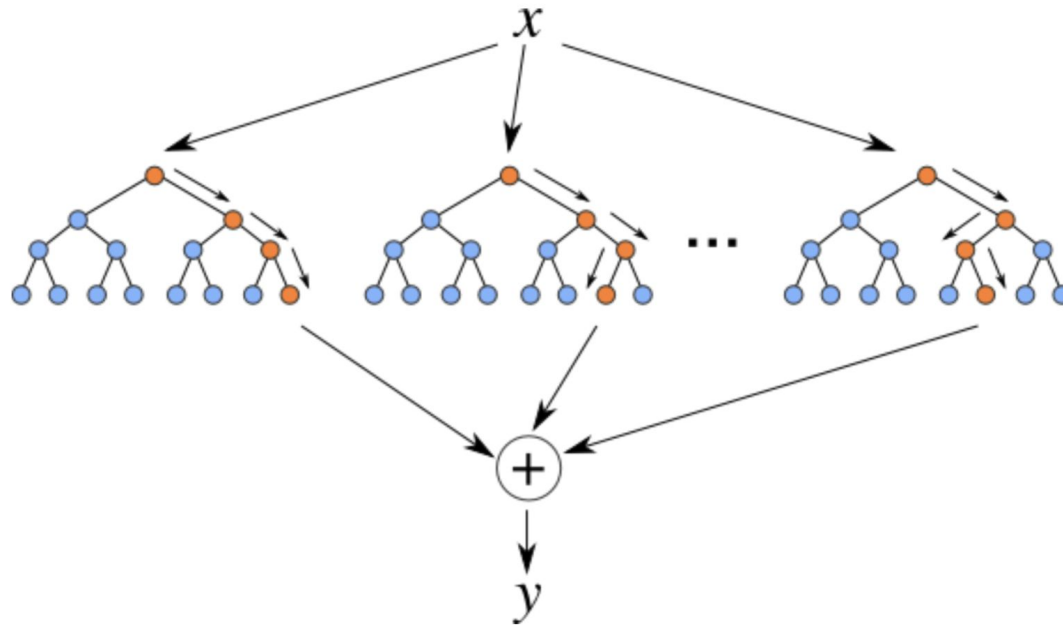
$$a(x) = \sum_{t=1}^T \alpha_t b_t(x)$$

- Simple and intuitive ensembling method.
- Finding optimal weights could be tricky.
- Linear composition is not always enough.



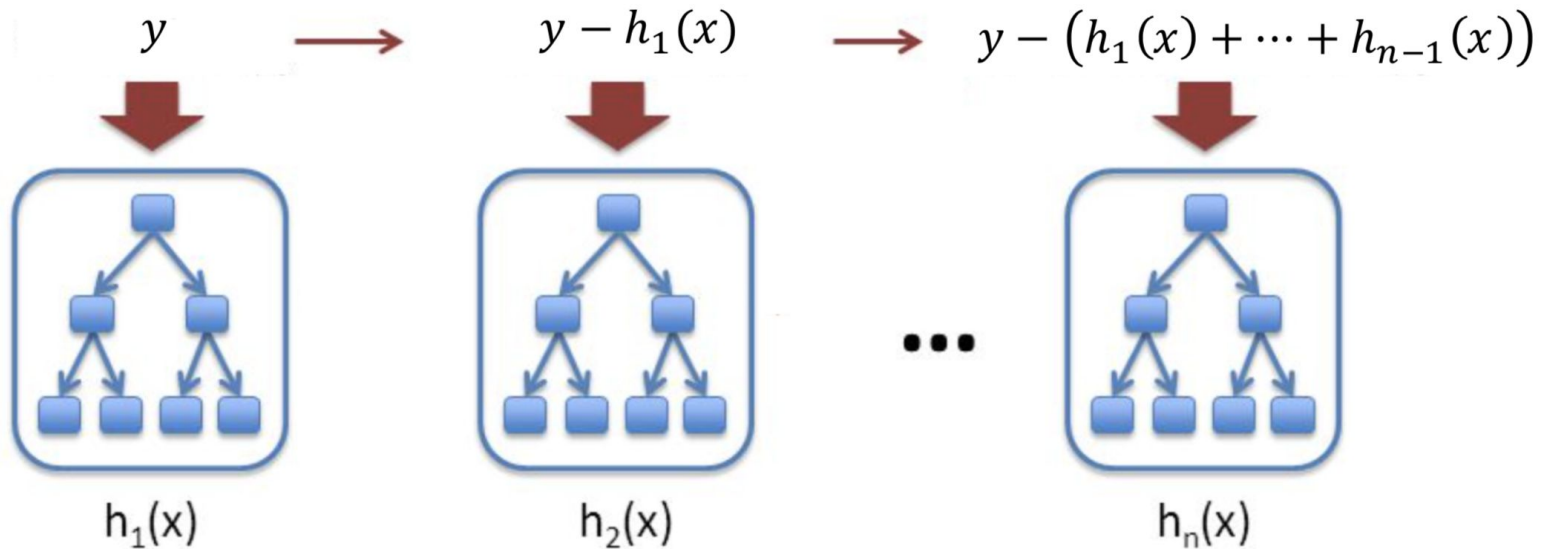
# Random Forest

Bagging + RSM = Random Forest



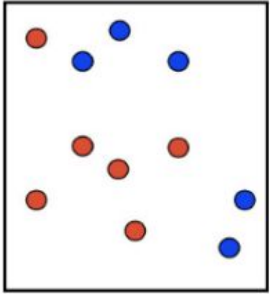
# Gradient boosting

$$a_n(x) = h_1(x) + \dots + h_n(x)$$

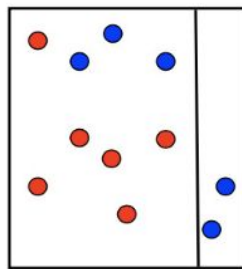
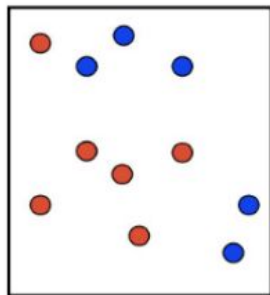


# Boosting: intuition

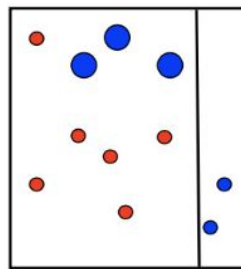
Binary classification problem.  
Models - decision stumps.



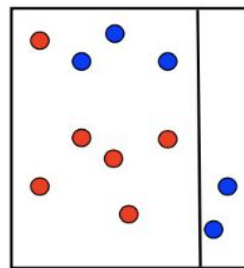
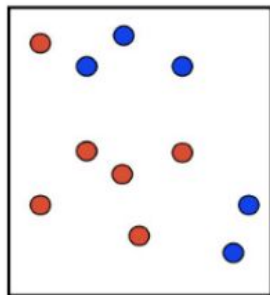
# Boosting: intuition



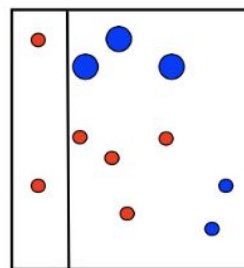
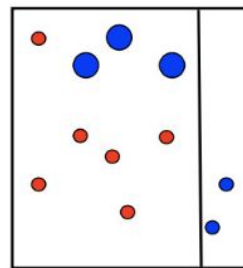
$t = 1$



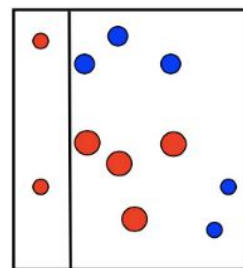
# Boosting: intuition



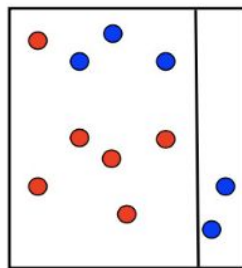
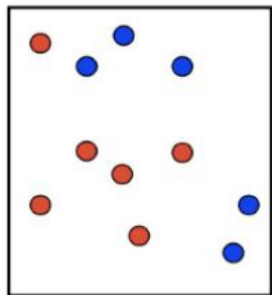
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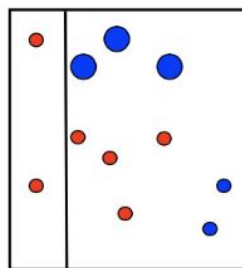
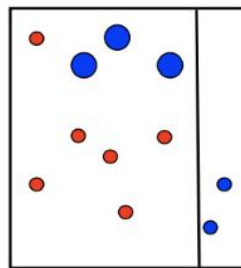
$t = 2$



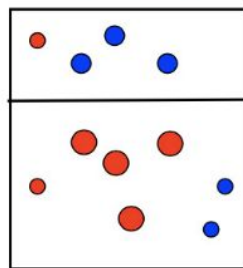
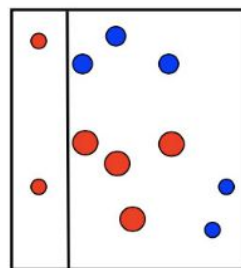
# Boosting: intuition



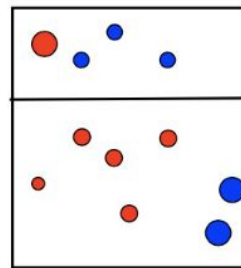
$t = 1$



$t = 2$

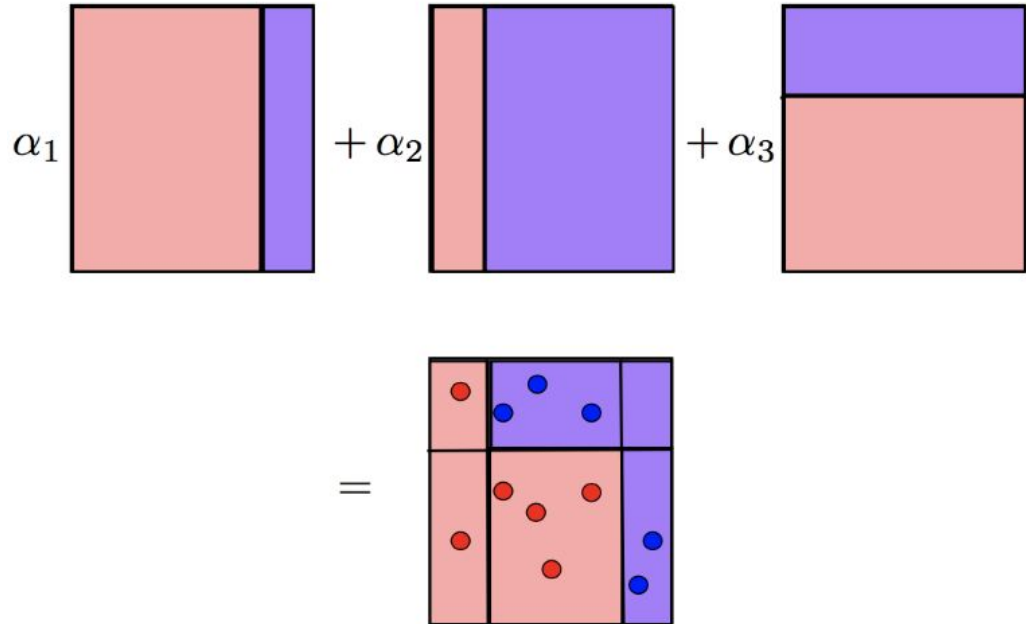
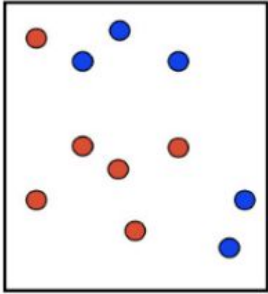


$t = 3$



# Boosting: intuition

Binary classification problem.  
Models - decision stumps.



# Gradient boosting: theory

Denote dataset  $\{(x_i, y_i)\}_{i=1, \dots, n}$ , loss function  $L(y, f)$ .



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Optimal model:

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Let it be from parametric family:  $\hat{f}(x) = f(x, \hat{\theta})$ ,

$$\hat{\theta} = \arg \min_{\theta} \mathbb{E}_{x,y}[L(y, f(x, \theta))]$$

# Gradient boosting: theory

$$\hat{f}(x) = \sum_{i=0}^{t-1} \hat{f}_i(x),$$

$$(\rho_t, \theta_t) = \arg \min_{\rho, \theta} \mathbb{E}_{x,y} [L(y, \hat{f}(x) + \rho \cdot h(x, \theta))],$$

$$\hat{f}_t(x) = \rho_t \cdot h(x, \theta_t)$$

# Gradient boosting: theory

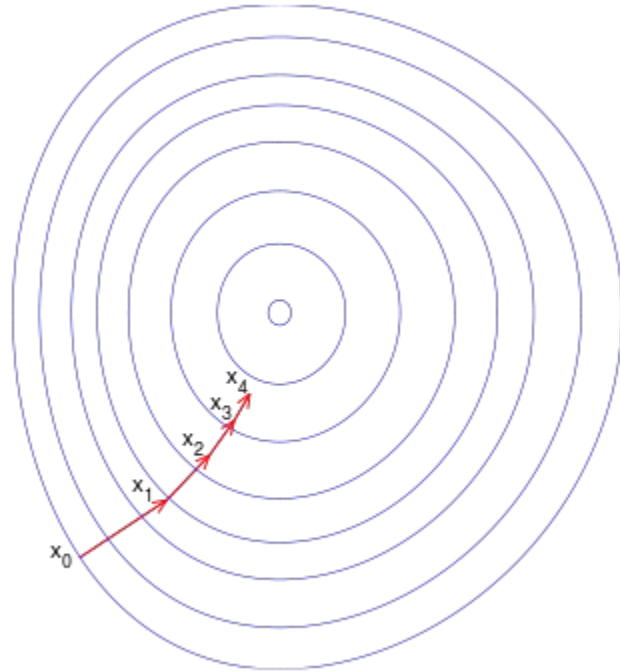
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# Gradient boosting: theory

$$\hat{f}(x) = \sum_{i=0}^{t-1} \hat{f}_i(x),$$

$$r_{it} = - \left[ \frac{\partial L(y_i, f(x_i))}{\partial f(x_i)} \right]_{f(x)=\hat{f}(x)}, \quad \text{for } i = 1, \dots, n,$$

$$\theta_t = \arg \min_{\theta} \sum_{i=1}^n (r_{it} - h(x_i, \theta))^2,$$

$$\rho_t = \arg \min_{\rho} \sum_{i=1}^n L(y_i, \hat{f}(x_i) + \rho \cdot h(x_i, \theta_t))$$

# Gradient boosting: theory

In linear regression case with MSE loss:

$$r_{it} = - \left[ \frac{\partial L(y_i, f(x_i))}{\partial f(x_i)} \right]_{f(x)=\hat{f}(x)} = -2(\hat{y}_i - y_i) \propto \hat{y}_i - y_i$$

# Gradient boosting: theory

What we need:

- Data.
- Loss function and its gradient.
- Family of algorithms (with constraints on hyperparameters if necessary).
- Number of iterations  $M$ .
- Initial value (GBM by Friedman): constant.



# Gradient boosting: example

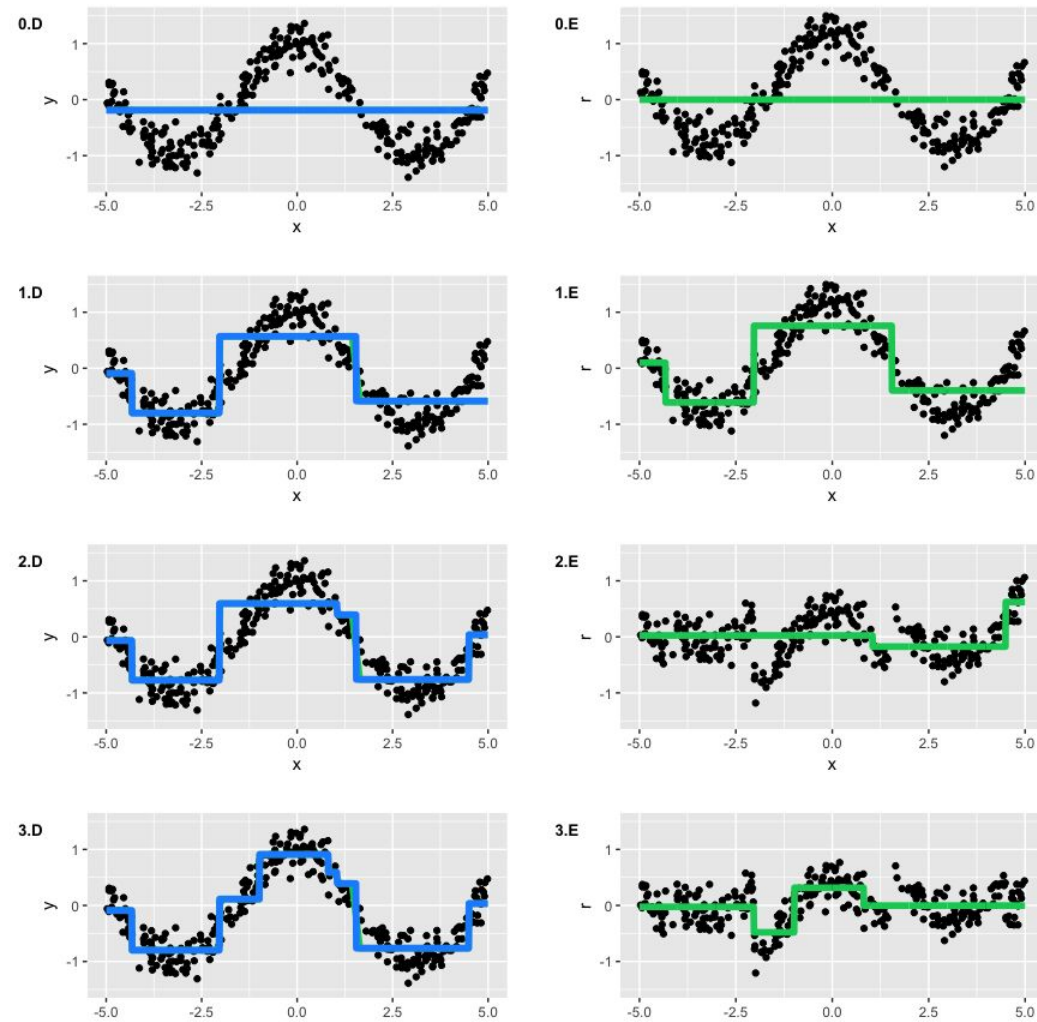
What we need:

- Data: toy dataset  $y = \cos(x) + \epsilon, \epsilon \sim \mathcal{N}(0, \frac{1}{5}), x \in [-5, 5]$
- Loss function: MSE
- Family of algorithms: decision trees with depth 2
- Number of iterations  $M = 3$
- Initial value: just mean value

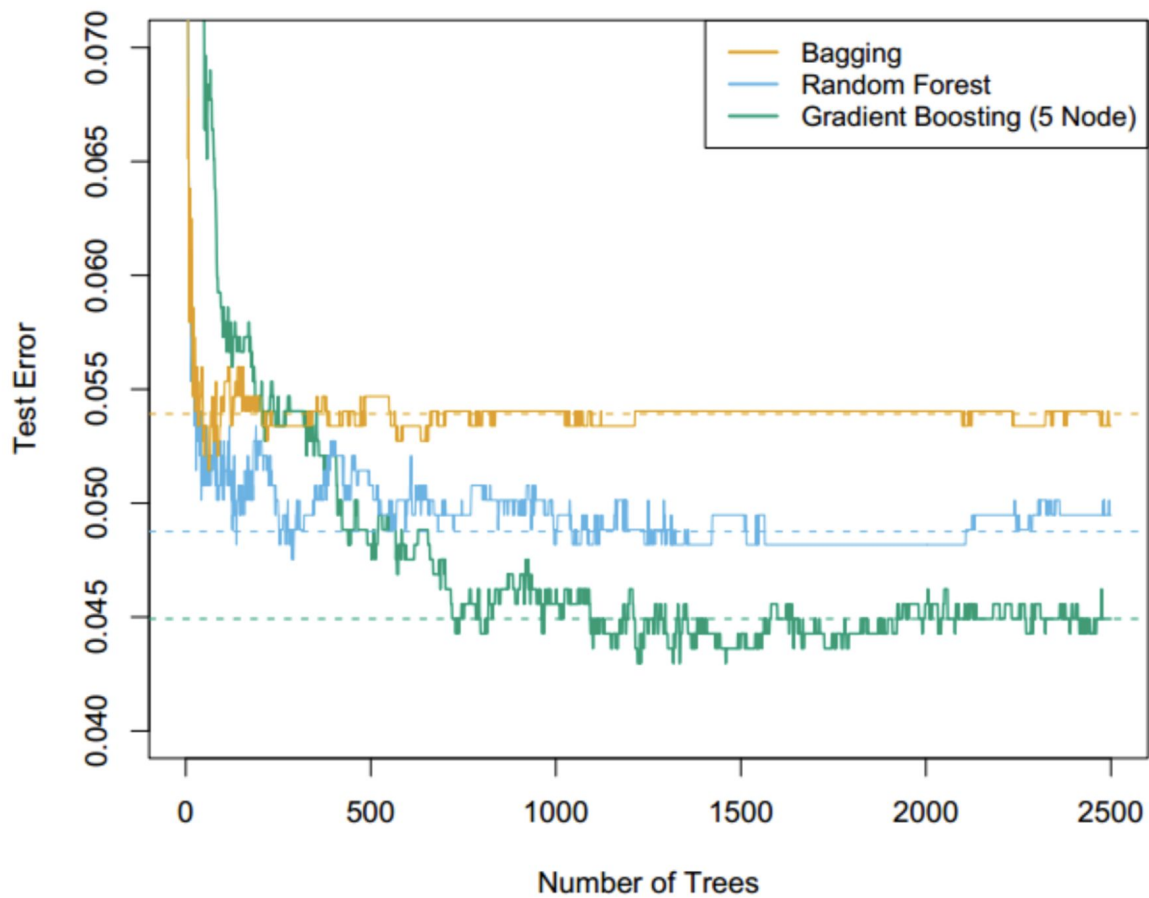
# Gradient boosting: example

Left: full ensemble on each step.

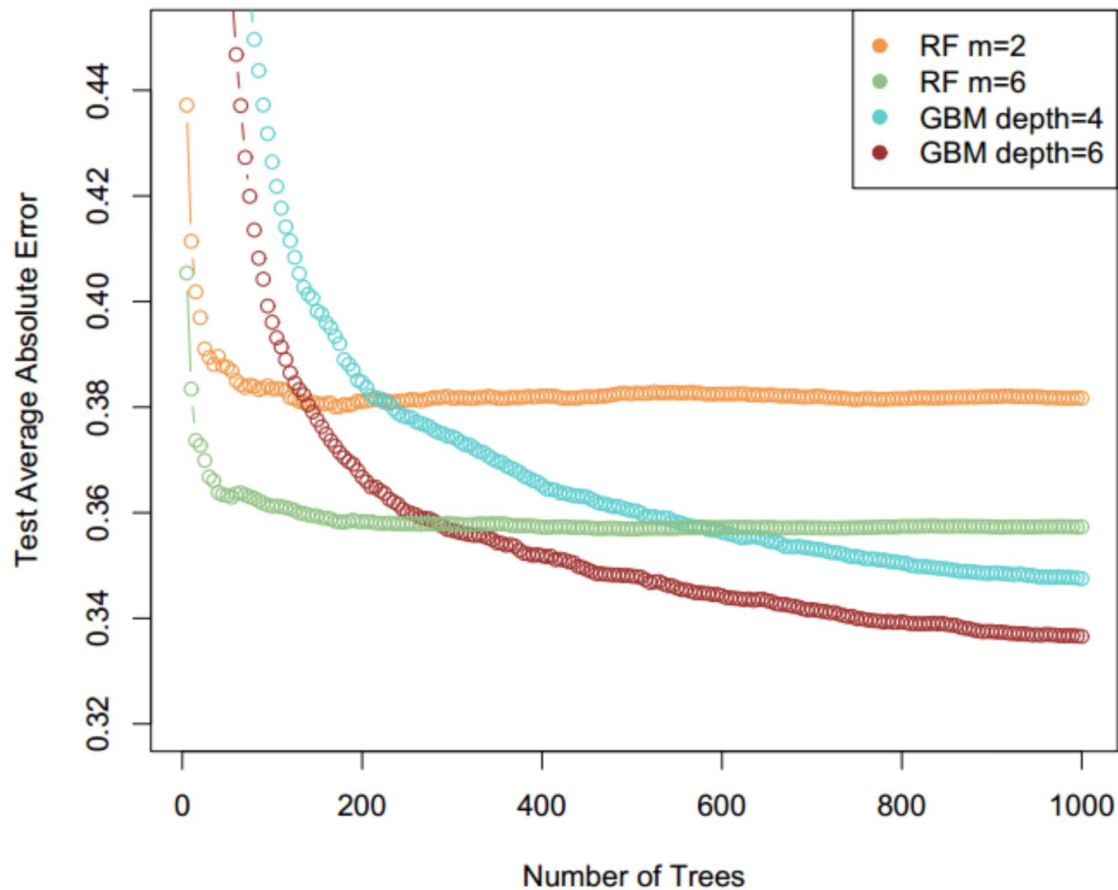
Right: additional tree decisions.



## Spam Data

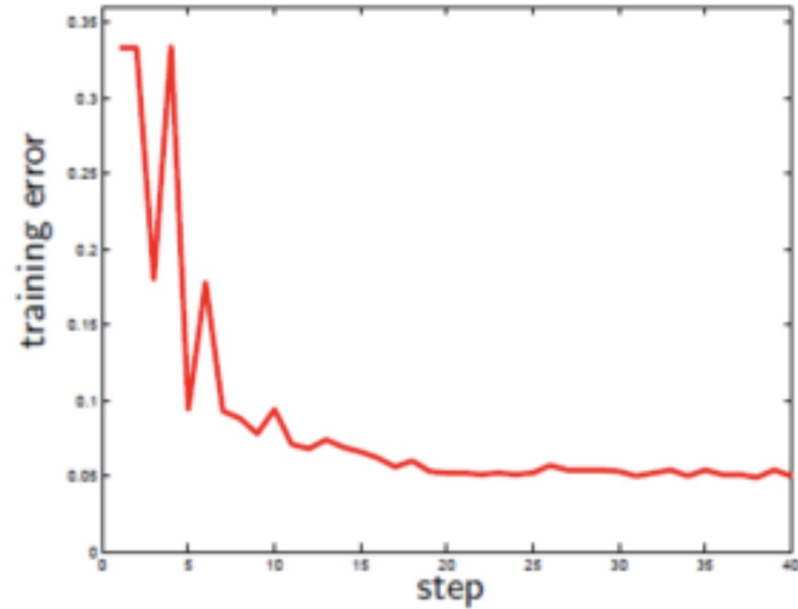
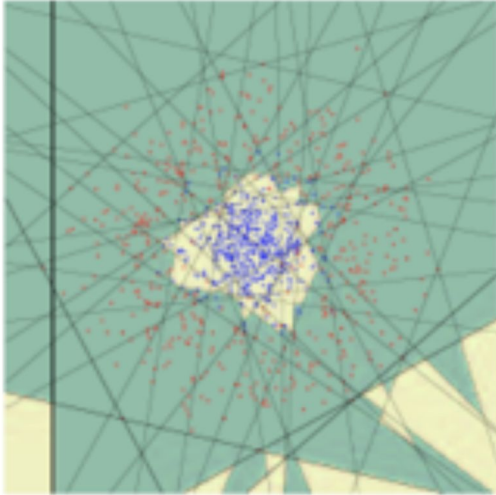


## California Housing Data



# Boosting with linear classification methods

$t = 40$



# Technical side: training in parallel

Which of the ensembling methods could be parallelized?

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- Random Forest: parallel on the forest level (all trees are independent)

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Which of the ensembling methods could be parallelized?

- Random Forest: parallel on the forest level (all trees are independent)
- Gradient boosting: parallel on one tree level

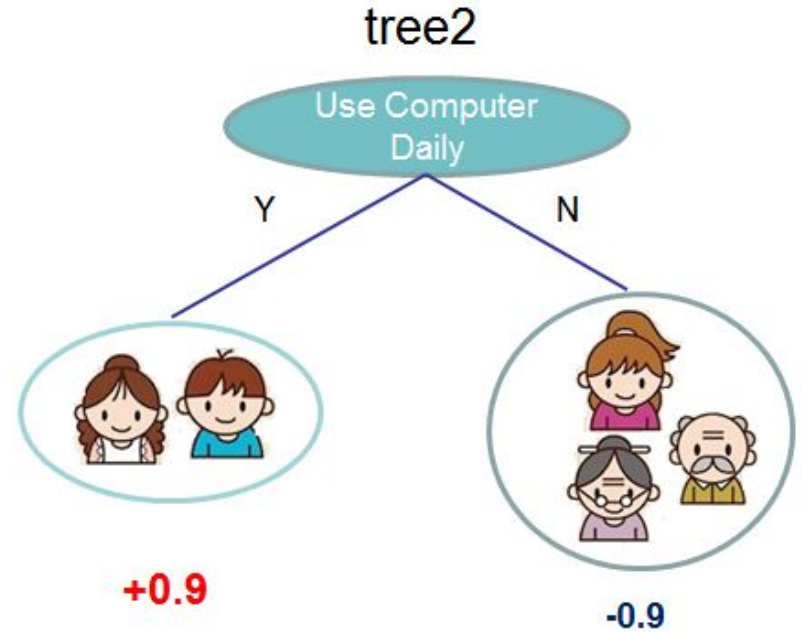
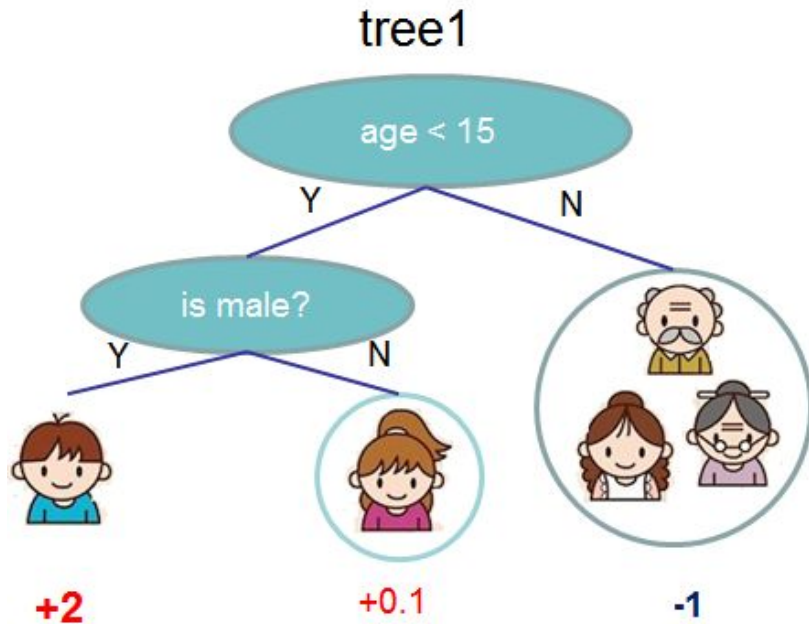


# Recap: ensembling methods

1. Bagging.
2. Random subspace method (RSM).
3. Bagging + RSM + Decision trees = Random Forest.
4. Gradient boosting.
5. Stacking.
6. Blending.

Great demo: [http://arogozhnikov.github.io/2016/06/24/gradient\\_boosting\\_explained.html](http://arogozhnikov.github.io/2016/06/24/gradient_boosting_explained.html)

# Feature importance estimation



$$f(\text{boy icon}) = 2 + 0.9 = 2.9$$

$$f(\text{old man icon}) = -1 - 0.9 = -1.9$$

# Feature importance estimation

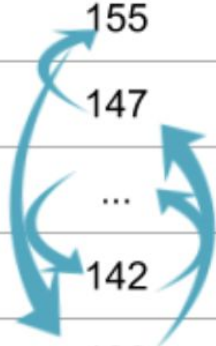
1. Permutation importance
2. Partial Dependence Plots (PDP)
3. Tree specific:
  - a. Gain
  - b. Frequency (Split Count)
  - c. Cover (weighted Split Count)
4. Shap

# Permutation importance

Height at age 20 (cm)	Height at age 10 (cm)	...	Socks owned at age 10
182	155	...	20
175	147	...	10
...	...	...	...
156	142	...	8
153	130	...	24

# Permutation importance

Height at age 20 (cm)	Height at age 10 (cm)	...	Socks owned at age 10
182	155	...	20
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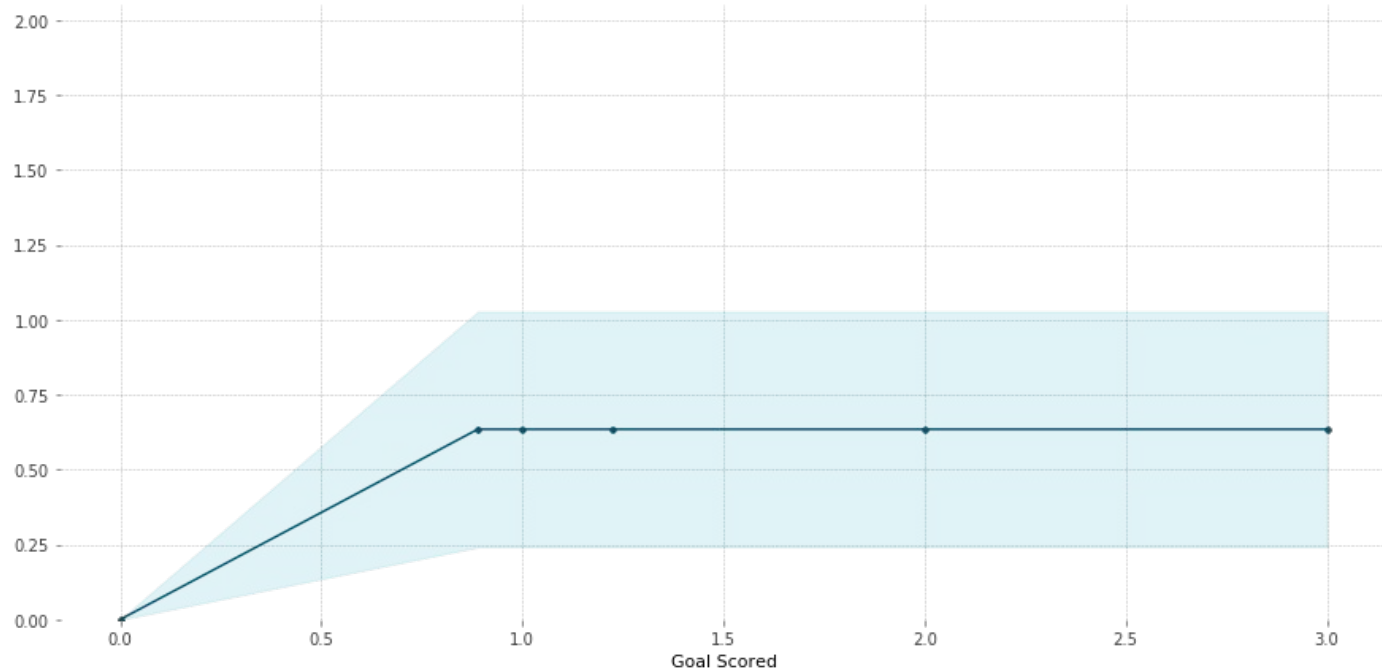
Train model

Observe changes caused by feature random permutations

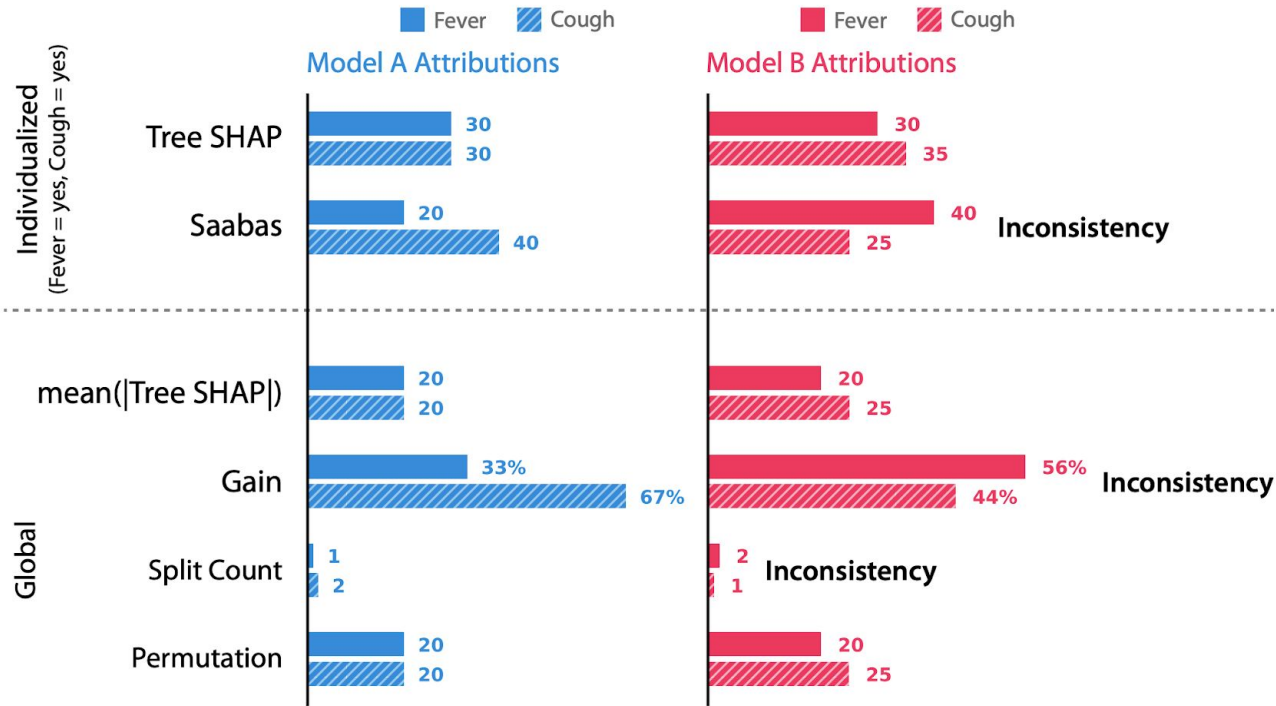
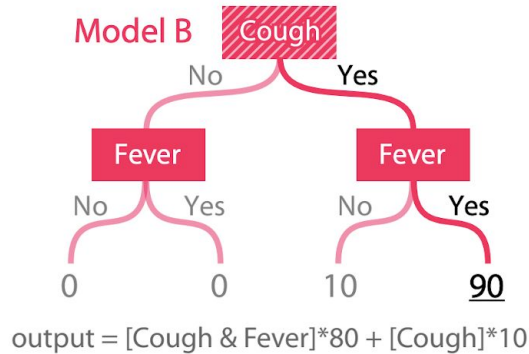
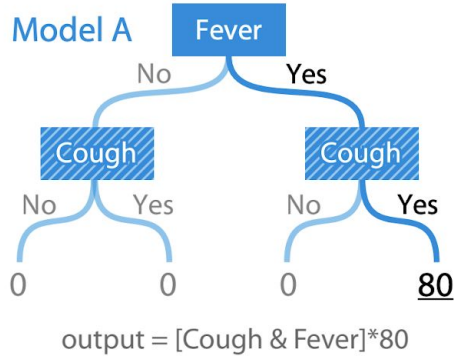
# Partial Dependence Plots

PDP for feature "Goal Scored"

Number of unique grid points: 6



# Importance estimation problems



Consider  $i$ -th feature. Shap value will be

$$\phi_i(p) = \sum_{S \subseteq N/\{i\}} \frac{|S|!(n - |S| - 1)!}{n!} (p(S \cup \{i\}) - p(S))$$

where  $p(S \cup \{i\})$  is model prediction on feature subset  $S$  with  $i$ -th feature added.



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where  $p(S \cup \{i\})$  is model prediction on feature subset  $S$  with  $i$ -th feature added.

*SHAP values are the only consistent and locally accurate individualized feature attributions*

1. Bagging + RSM + Decision trees = Random Forest.
2. Gradient boosting is powerful but prone to overfitting
3. Stacking & Blending are great techniques
4. Consider using SHAP values to estimate feature importances.

Great demo: [http://arogozhnikov.github.io/2016/06/24/gradient\\_boosting\\_explained.html](http://arogozhnikov.github.io/2016/06/24/gradient_boosting_explained.html)